Shedding light on exclusive B decays

M. Beneke (TU München)

Humboldt Kolleg "Clues to a mysterious Universe - exploring the interface of particle, gravity and quantum physics"

Kitzbühel, June 28 - July 2, 2022

Systematic treatment of hadron structure-dependent QED effects on heavy meson decays

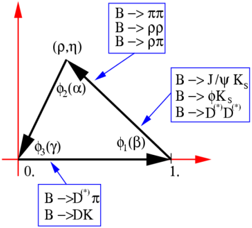
MB, Bobeth, Szafron, 1708.09152, 1908.07011, MB, Böer, Toelstede, Vos, 2008.10615, MB, Böer, Finauri, Vos, 2107.03819, MB, Böer, Toelstede, Vos, 2108.05589, 2204.09091

TECHNISCHE UNIVERSITÄT MÜNCHEN

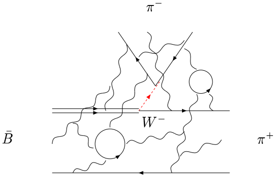
Sino-German CRC110 "Emergence of Structure in QCD"

Motivation: exclusive B decays

Three interests in exclusive (hadronic, semi-hadronic) *B* decays



CKM angles
Detailed investigations of various flavour-violating transitions

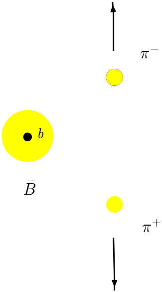


Challenging strong interaction problem

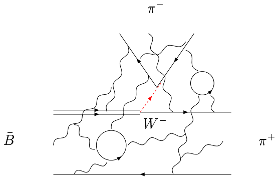
New Physics

Review of factorization

Exclusive B decays: Simple kinematics, complicated dynamics



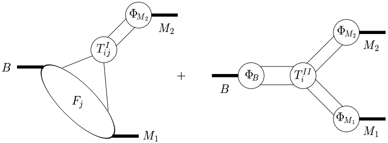
$$\langle \pi^- \pi^+ | \bar{B} \rangle_{\mathcal{L}_{\rm SM}} \quad = $$



Problem with multiple scales: $M_W, m_b, \sqrt{m_b \Lambda_{\rm QCD}}, \Lambda_{\rm QCD}$

Factorization = theory of calculating effects from short-distance scales and parameterising long-distance effects in "universal" quantities.

QCD factorization formula [MB, Buchalla, Neubert, Sachrajda, 1999-2001]



Form factor term +
Spectator scattering

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} = \sum_{\text{terms}} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\text{I}}(\mu_h, \mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$

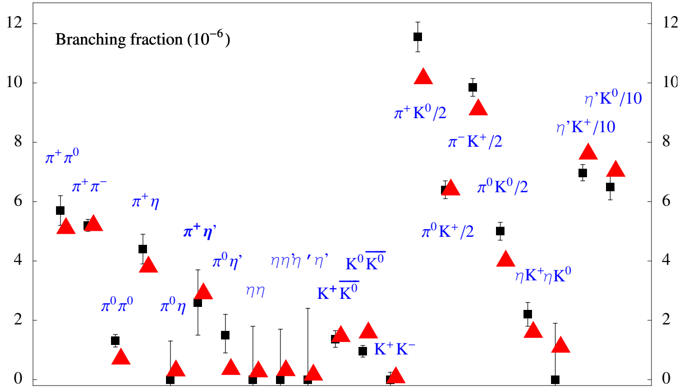
$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\text{II}}(\mu_h, \mu_I)}_{1+\ldots} \star \underbrace{J^{\text{II}}(\mu_I, \mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\} + \mathcal{O}(1/m_b)$$

- Rigorous at leading power in Λ_{QCD}/m_b
- Strong rescattering phases are $\delta \sim \mathcal{O}(\alpha_s(m_b), \Lambda/m_b)$. SCET_{I} matching coefficients only. Direct CP asymmetry is calculable at LP

$$A_{\text{CP}}(M_1 M_2) = \underbrace{a_1 \alpha_s}_{1999} + \underbrace{a_2 \alpha_s^2}_{2020} + \ldots + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$$

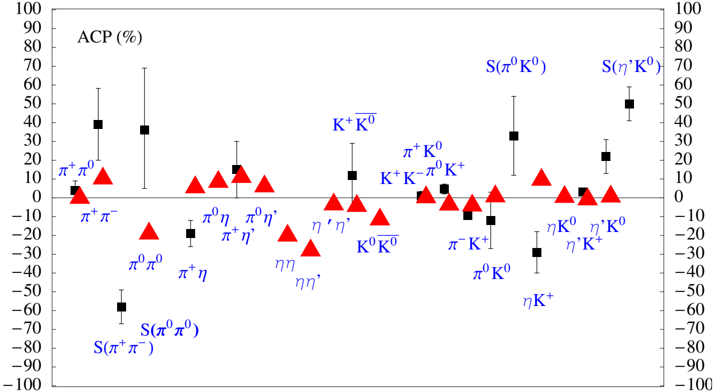
[Bell, MB, Huber, Li]

CP-averaged $B \to PP$ branching fractions



Red triangles: Theory (S4) from MB, M. Neubert, Nucl. Phys. B675 (2003) 333

CP-averaged $B \to PP$ direct CP asymmetries



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Including QED: Motivation

- Precision: Traditionally focus on hadronic uncertainties. Time to look at QED.
- QED effects violate isospin symmetry and can cause large "lepton-flavour violating" logarithms, $\log m_\ell$.
- Photons couple weakly to strongly interacting quarks $\rightarrow$ probe of hadronic physics, requires factorization theorems, which mostly don't exist yet.
- Photons have long-range interactions with the charged particles in the initial/final state $\rightarrow$ QED factorization is more complicated than QCD factorization.

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$\hookrightarrow$ *Systematic treatment of electromagnetic corrections with the tools of factorization and effective theory*

MB, Bobeth, Szafron, 1708.09152, 1908.07011 [$B_s \to \mu^+\mu^-$]

MB, Böer, Toelstede, Vos, 2008.10615 [$B \to \pi K$, charmless]

MB, Böer, Finauri, Vos, 2107.03819 [$B \to D_{(s)}^{(*)+} L^-$, colour-allowed + semi-leptonic]

MB, Böer, Toelstede, Vos, 2108.05589 + 2204.09091 [LCDAs of light and heavy mesons]

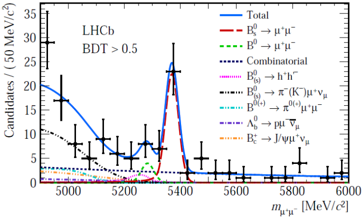
Observables

- IR finite observable is

$$\Gamma_{\rm phys} = \sum_{n=0}^{\infty} \Gamma(B \to f + n\gamma, \sum_n E_{\gamma,n} < \Delta E)$$
$$\equiv \omega(\Delta E) \times \Gamma_{\rm non-rad.}(B \to f)$$

- Assume $\Delta \ll \Lambda_{\rm QCD} \sim$ size of hadrons. $\rightarrow$ Large log ΔE.
- Collinear logs are $\log m_f^2$ (hadron masses)

Signal window $|m_B - m_f| < \Delta$
$\Longrightarrow \quad \Delta E = \Delta$

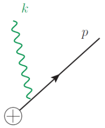


[LHCb, $B_s \to \mu^+\mu^-$, 1703.05748]

(Ultra-) Soft photons and the point-like approximation

Universal soft radiative amplitude

$$A^{i\to f+\gamma}(p_j,k) = A^{i\to f}(p_j) \times \sum_{j=\text{legs}} \frac{-eQ_j p_j^\mu}{\eta_j p_j\cdot k + i\epsilon}$$



The amplitude implies that the charged particles (B-meson, pion, lepton, ...) are treated as point-like. Exponentiates for the decay rate, but the virtual correction is UV divergent in the soft limit. Cut-off Λ.

$$\Gamma = \Gamma^{i\to f}_{\text{tree}} \times \left(\frac{2\Delta E}{\Lambda}\right)^{-\frac{\alpha}{\pi}\sum_{i,j} Q_i Q_j f(\beta_{ij})}$$

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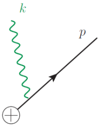

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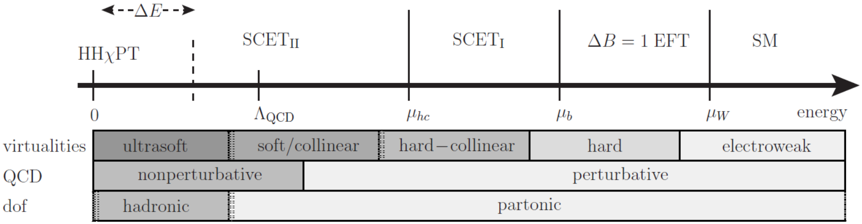
What is Λ? — Derivation implies that $\Lambda \ll \Lambda_{\text{QCD}} \sim$ size of the hadron (B-meson). Otherwise virtual corrections resolve the structure of the hadron and higher-multipole couplings are unsuppressed.

But:

- Present treatment of QED effects in B decays sets $\Lambda = m_B$ (e.g. using a theory of point-like mesons), neglecting structure-dependent effects
- Experimental analyses uses the PHOTOS Monte Carlo [Golonka, Was, 2005]

Scales and Effective Field theories (EFTs)

Multiple scales: m_W, m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, m_μ, ΔE



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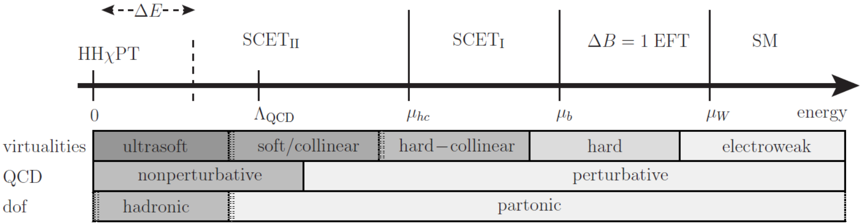
Short-distance QED at $\mu \gtrsim m_b$ can be included in the usual weak effective Lagrangian (extended Fermi theory) + renormalization group.

Far IR (ultrasoft scale) described by theory of point-like hadrons.



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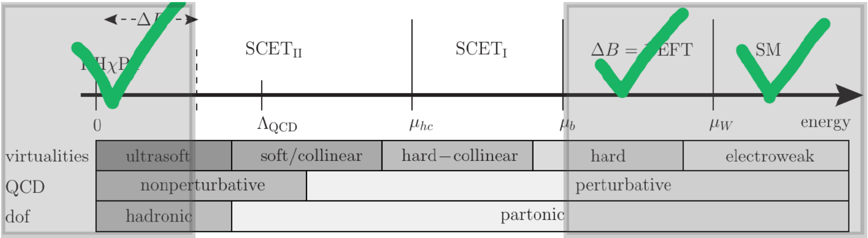
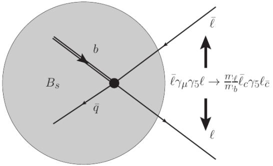
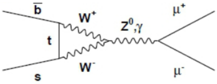


Goal: Theory for QED corrections between the scales m_b and $\Lambda_{\rm QCD}$ (structure-dependent effects).

Example: $B_s \to \mu^+\mu^-$ [MB, Bobeth, Szafron, 2017]





$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$$

Local annihilation and helicity flip.

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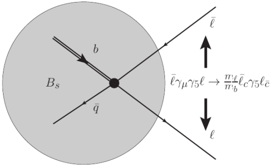

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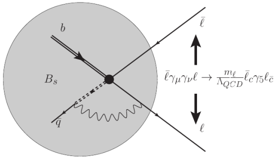

$$\langle 0|\int d^4x\, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

Helicity-flip and annihilation delocalized by a hard-collinear distance

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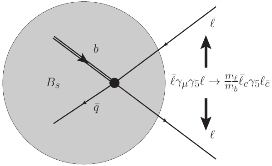

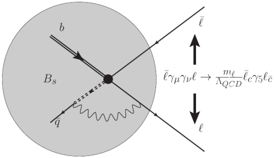

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Local annihilation and helicity flip.

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Helicity-flip and annihilation delocalized
by a hard-collinear distance

- The virtual photon probes the B meson structure. Annihilation/helicity-suppression is "smeared out" over light-like distance $1/\sqrt{m_B\Lambda}$. Still short-distance [→ B-meson LCDA] .
- Structure-dependent effects is a m_B/Λ power-enhanced and (double) logarithmically enhanced, purely virtual correction. Not the standard soft logarithms.

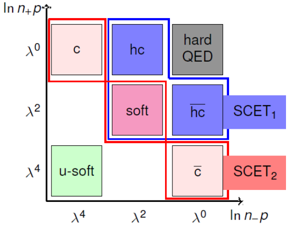
All orders, EFT, summation of logarithms

Back-to-back energetic lepton pair
Collinear (lepton $n_+ p_\ell$ large) and anti-collinear (anti-lepton $n_- p_{\bar{\ell}}$ large) modes

$$n_+^2 = n_-^2 = 0, \quad n_+ \cdot n_- = 2, \qquad p^\mu = n_+ p \,\frac{n_-^\mu}{2} + n_- p\, \frac{n_+^\mu}{2} + p_\perp^\mu$$

$$p = (n_+ p, p_\perp, n_- p), \qquad \lambda \sim \frac{\Lambda_{\rm QCD}}{m_b} \sim \frac{m_\mu}{m_b}$$

- Modes in the EFT classified by virtuality and rapidity
- Matching QCD+QED $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$



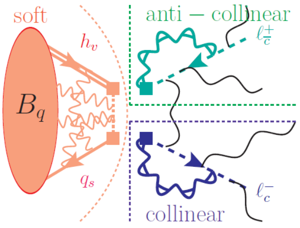
mode	relative scaling	absolute scaling	virtuality k^2
hard	$(1,1,1)$	(m_b, m_b, m_b)	m_b^2
hard-collinear	$(1,\lambda,\lambda^2)$	$(m_b, \sqrt{m_b \Lambda_{\rm QCD}}, \Lambda_{\rm QCD})$	$m_b \Lambda_{\rm QCD}$
anti-hard-collinear	$(\lambda^2,\lambda,1)$	$(\Lambda_{\rm QCD}, \sqrt{m_b \Lambda_{\rm QCD}}, m_b)$	$m_b \Lambda_{\rm QCD}$
collinear	$(1,\lambda^2,\lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
anticollinear	$(\lambda^4,\lambda^2,1)$	$(m_\mu^2/m_b, m_\mu, m_b)$	m_μ^2
soft	$(\lambda^2,\lambda^2,\lambda^2)$	$(\Lambda_{\rm QCD}, \Lambda_{\rm QCD}, \Lambda_{\rm QCD})$	$\Lambda_{\rm QCD}^2$

SCET$_{\text{II}}$ (re-) factorization and soft rearrangement

- Leading long-range soft photon interactions between energetic charged particles can be accounted for by light-like Wilson lines:

$$Y_{\pm}^{(q)} = \exp\left\{-iQ_q e \int_0^\infty ds\, n_\pm A_s(sn_\pm)\right\}$$

- s, c, $\bar{c}$ do not interact in SCET$_{\text{II}}$. Sectors are factorized.
- Anomalous dimensions should be separately well defined, but turn out to be IR divergent.



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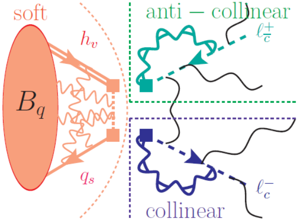


$$\widetilde{\mathcal{J}}^{B1}_{\mathcal{A}\chi}(v,t) = \bar{q}_s(vn_-)Y(vn_-,0)\frac{\not{n}_-}{2}P_L h_v(0)\left[Y_+^\dagger Y_-\right](0)\left[\bar{\ell}_c(0)(2\mathcal{A}_{c\perp}(tn_+)P_R)\ell_{\bar{c}}(0)\right] = \widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}}$$

- **Soft rearrangement** of the Wilson line tadpole: $\langle 0|\left[Y_+^\dagger Y_-\right](0)|0\rangle \equiv R_+ R_-$

$$\widehat{\mathcal{J}}_s \otimes \widehat{\mathcal{J}}_c \otimes \widehat{\mathcal{J}}_{\bar{c}} = \frac{\widehat{\mathcal{J}}_s}{R_+R_-} \otimes R_+\widehat{\mathcal{J}}_c \otimes R_-\widehat{\mathcal{J}}_{\bar{c}}$$

Definition of structure-dependent, non-radiative amplitude and radiation

Amplitude [evolved to $\mu_c \approx$ few $\times\, \Lambda_{\rm QCD}$]

$$\mathcal{A}_{B\to f}(\mu_c) \equiv e^{S_f(\mu_b,\,\mu_c)} \times A_{\text{non-rad}}(\mu_c)$$

– defines the non-radiative amplitude by extracting universal final-state Sudakov exponential
$\mathcal{A}_{B\to f}$ is an exclusive amplitude, therefore IR divergent and scale-dependent

Including ultrasoft photon radiation (+ virtual ultrasoft)

$$\mathcal{A}_{B\to f+X_s}(\Delta E) = \langle f\, X_s | \overline{B}_s \rangle_{E_{X_S}<\Delta E} = \mathcal{A}_{B\to f}(\mu_c)\, \langle X_s | S^\dagger_{v_{f_1}}(0) S_{v_{f_2}}(0) | 0 \rangle (\mu_c)_{E_{X_S}<\Delta E}$$

– $\mathcal{A}_{B\to f}$ should be viewed as a matching coefficient of SCET_{II} onto the theory of point-like hadrons at a scale $\mu_f \lesssim \Lambda_{\rm QCD}$. This matching is non-perturbative.

Decay rate [including ultrasoft photon radiation]

$$\Gamma[B\to f+X_s](\Delta E) = \underbrace{\frac{m_{B_q}}{8\pi}\beta_f \left|A_{\text{non-rad}}\right|^2}_{\text{non-radiative rate}} \times \underbrace{\left|e^{S_\ell(\mu_b,\,\mu_c)}\right|^2 \mathcal{S}(v_\ell, v_{\overline{\ell}}, \Delta E)}_{\text{ultrasoft radiation}} \equiv \Gamma^{(0)}[B\to f] \left(\frac{2\Delta E}{m_{B_q}}\right)^{-\frac{2\alpha}{\pi} f(\ln m^2_{f_i}/m^2_{B_q})}$$

Hadronic B two-body decays
($B \to \pi K$ ..., D^+L^-, ...)

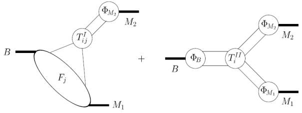
2008.10615 (charmless) and 2107.03819 (heavy-light + semi-leptonic), with P. Böer, G. Finauri, J. Toelstede and K. Vos

Recap: QCD theory

"QCD factorization" [MB, Buchalla, Neubert, Sachrajda, 1999-2001], later understood and formulated as a SCET_{II} problem:

$$\text{QCD} \overset{\text{remove h}}{\longrightarrow} \text{SCET}_{\text{I}} \overset{\text{remove hc}}{\longrightarrow} \text{SCET}_{\text{II}}(c, \bar{c}, s)$$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = \underbrace{F^{BM_1}(0)}_{\text{form factor}} \int_0^1 du\, T_i^I(u) \Phi_{M_2}(u)$$



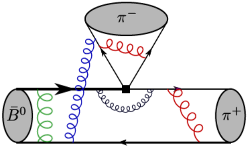
$$+ \int_0^1 dz du\, H_i^{\text{II}}(z,u) \int_0^\infty d\omega \int_0^1 dv\, J(\omega, u, v)\, \underbrace{\Phi_B(\omega) \Phi_{M_1}(v) \Phi_{M_2}(u)}_{\text{LCDAs}}$$

SCET_{I} operators

$$\mathcal{O}^{\text{I}}(t) = [\bar{\chi}_{\bar{C}}(tn_-) \not{n}_- \gamma_5 \chi_{\bar{C}}]\, [\bar{\chi}_C h_v]$$

$$\mathcal{O}^{\text{II}}(t,s) = \underbrace{[\bar{\chi}_{\bar{C}}(tn_-) \not{n}_- \gamma_5 \chi_{\bar{C}}]}_{M_2}\, \underbrace{[\bar{\chi}_C \not{\mathcal{A}}_{C,\perp}(sn_+) h_v]}_{B \to M_1}$$

Including virtual QED effects into the factorization theorem



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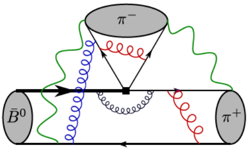


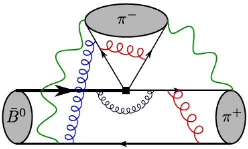
SCET$_{\rm I}$ operators

$$\mathcal{O}^{\rm I}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$\mathcal{O}^{\rm II}(t,s) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\not{\mathcal{A}}_{C,\perp}(sn_+)\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$S_{n_\pm}^{(q)} = \exp\left\{-iQ_q e\int_0^\infty ds\, n_\pm A_s(sn_\pm)\right\}$$

Including virtual QED effects into the factorization theorem



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QCD + QED factorization formula

$$\langle M_1M_2|Q_i|\bar{B}\rangle_{|\mathrm{non-rad.}} = \mathcal{F}_{Q_2}^{BM_1}(0)\int_0^1 du\;\underbrace{T_{i,Q_2}^{\mathrm{I,QCD+QED}}(u)}_{\mathcal{O}(\alpha_\mathrm{em})\text{ corrected SD}}\;f_{M_2}\Phi_{M_2}(u)$$

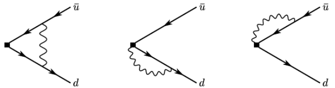
$$+\int_{-\infty}^{\infty}d\omega\int_0^1 dudv\,T_{i,\otimes}^{\mathrm{II,QCD+QED}}(z,u)\,f_B\Phi_{B,\otimes}(\omega)f_{M_1}\Phi_{M_1}(v)f_{M_2}\Phi_{M_2}(u)$$

Formula retains its form, but the hadronic matrix elements are generalized. They become process-dependent through the directions and charges of the *other* particles.

LCDA of a charged pion in QCD×QED [2108.05589, MB, Böer, Toelstede, Vos]

QCD definition

$$\langle \pi^-(p)|\,\bar{d}(tn_+)\frac{\not{n}_+}{2}\gamma_5[tn_+,0]u(0)|0\rangle = -\frac{in_+p}{2}\int_0^1 du\, e^{iu(n_+p)t} f_M\phi_M(u;\mu)$$



ERBL kernel [Efremov, Radyushkin, Brodsky, Lepage, 1979]

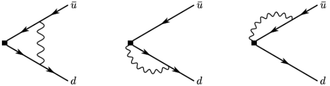
$$\Gamma(u,v;\mu) = -\left(\frac{\alpha_s C_F}{\pi}\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\,\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\,\theta(u-v)\right]_+$$

- LCDA symmetric in $u \leftrightarrow 1-u$
- One-loop kernel diagonalized by Gegenbauer polynomials, asymptotic behaviour $\Phi_\pi(u,\mu) \stackrel{\mu\to\infty}{\to} 6u(1-u)$.

LCDA of a charged pion in QCD×QED [2108.05589, MB, Böer, Toelstede, Vos]

QCD×QED

$$\langle \pi^-(p)| R_c^{(Q_M)} \, (\bar{d} W^{(d)})(tn_+) \frac{\not{n}_+}{2} \gamma_5 [tn_+, 0] (W^{\dagger (u)} u)(0) |0\rangle = -\frac{in_+ p}{2} \int_0^1 du \, e^{iu(n_+p)t} f_M \Phi_M(u;\mu)$$



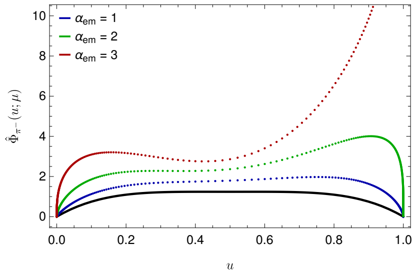
QCD×QED kernel

$$\Gamma(u,v;\mu) = -\frac{\alpha_{\rm em} Q_M}{\pi} \, \delta(u-v) \left(Q_M \left(\ln \frac{\mu}{2E} + \frac{3}{4} \right) - Q_d \ln u + Q_u \ln \bar{u} \right)$$
$$- \left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\rm em}}{\pi} Q_u Q_d \right) \left[\left(1 + \frac{1}{v-u} \right) \frac{u}{v} \, \theta(v-u) + \left(1 + \frac{1}{u-v} \right) \frac{1-u}{1-v} \, \theta(u-v) \right]_+$$

- Cusp anomalous dimension, logarithms $\ln u$, $\ln(1-u)$ and energy dependence are a remnant of the soft physics and breaking of boost invariance.
- Gegenbauer polynomials are no longer eigenfunctions, asymptotic behaviour $\Phi_\pi(u,\mu) \stackrel{\mu\to\infty}{\to} 6u(1-u)$ no longer holds. QED evolution is asymmetric and endpoint behaviour changes from linear.

Endpoint behaviour and numerical QED effect

Large $\alpha_{\rm em}$ for numerical illustration



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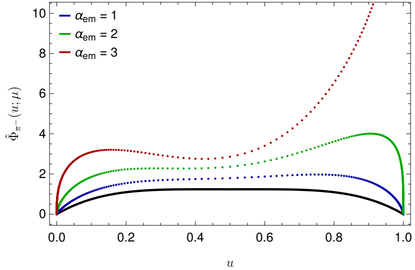
Large $\alpha_{\rm em}$ for numerical illustration



Example for the real world $\alpha_{\rm em}$:

$$\left\langle \bar{u}^{-1} \right\rangle_{M^-}(\mu) = \int_0^1 \frac{du}{1-u}\,\Phi_{M^-}(u;\mu) = 3Z_\ell(\mu)\sum_{n=0}^{\infty} a_n^{M^-}(\mu)$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-}(5.3\,\text{GeV}) = 0.9997\big|^{\text{QED}}_{\text{point charge}}\left(3.285^{+0.05}_{-0.05}\big|_{\text{LL}} - 0.020\big|_{\text{NLL}} + 0.017\big|^{\text{QED}}_{\text{partonic}}\right)$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-}(80.4\,\text{GeV}) = 0.985\big|^{\text{QED}}_{\text{point charge}}\left(3.197^{+0.03}_{-0.03}\big|_{\text{LL}} - 0.022\big|_{\text{NLL}} + 0.042\big|^{\text{QED}}_{\text{partonic}}\right)$$

(Initial value: 3.42 at $\mu = 1$ GeV.

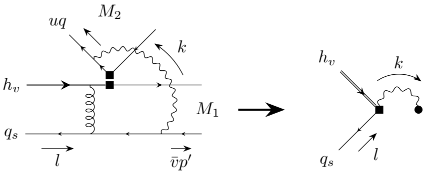
QED effects of similar size as NLL evolution for the inverse moments.

B-LCDA alias soft function for $B \to M_1 M_2$ in QCD×QED

[2204.09091, MB, Böer, Toelstede, Vos]

$$\frac{1}{R_c^{(Q_{M_1})} R_{\bar{c}}^{(Q_{M_2})}} \langle 0 | \bar{q}_s^{(q)}(tn_-)[tn_-, 0]^{(q)} \not{n}_- \gamma_5 h_v(0) S_{n_+}^{\dagger(Q_{M_2})}(0) S_{n_-}^{\dagger(Q_{M_1})}(0) | \bar{B}_v \rangle$$
$$= iF_{\rm stat}(\mu) \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \, \Phi_{B,\otimes}(\omega, \mu)$$

- Four different soft functions depending on final state charges $00, -0, 0-, +-$.



- Not really LCDAs, but soft functions: contain electromagnetic final-state rescattering. Complex-valued.

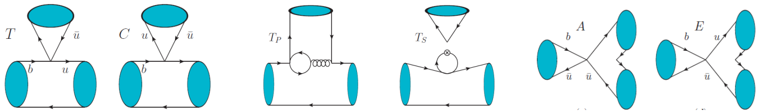
- For $Q_{M_2} \neq 0$, support in $\omega \in]-\infty, \infty[$ (different from QCD where $(\omega > 0)$.

Application to the $B \to \pi K$ final states

- Topological amplitude parameterization

$$T, C, P, P_{\rm EW}, S, E, A, \ldots$$

Can be (mostly) calculated with factorization at leading order in $1/m_b$.



[fig from 1803.04227]

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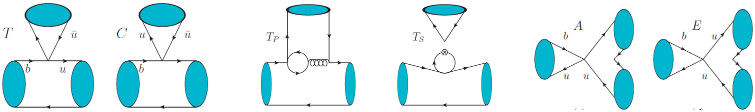

[fig from 1803.04227]

$$\begin{aligned}
\mathcal{A}_{B^-\to\pi^-\bar{K}^0} &= \lambda_c^{(s)}\,[P_c - \frac{1}{3}P_c^{C,EW}] + \lambda_u^{(s)}\,[P_u - \frac{1}{3}P_u^{C,EW}] \\
\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} &= \lambda_c^{(s)}\,[P_c + P_c^{EW} + \frac{2}{3}P_c^{C,EW}] + \lambda_u^{(s)}\,[T + C + P_u + P_u^{EW} + \frac{2}{3}P_u^{C,EW}] \\
\mathcal{A}_{\bar{B}^0\to\pi^+K^-} &= \lambda_c^{(s)}\,[P_c + \frac{2}{3}P_c^{C,EW}] + \lambda_u^{(s)}\,[T + P_u + \frac{2}{3}P_u^{C,EW}] \\
\sqrt{2}\,\mathcal{A}_{\bar{B}^0\to\pi^0\bar{K}^0} &= \lambda_c^{(s)}\,[-P_c + P_c^{EW} + \frac{1}{3}P_c^{C,EW}] + \lambda_u^{(s)}\,[C - P_u + P_u^{EW} + \frac{1}{3}P_u^{C,EW}]
\end{aligned}$$

$B \to \pi K$ ratios and asymmetries

$$R_{00} = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0)}{\Gamma(B^- \to \pi^- \bar{K}^0)} = |1 - r_{\rm EW}|^2 + 2\cos\gamma \,{\rm Re}\, r_C + \ldots$$

$$R_L = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0) + 2\Gamma(B^- \to \pi^0 K^-)}{\Gamma(B^- \to \pi^- \bar{K}^0) + \Gamma(\bar{B}^0 \to \pi^+ K^-)} = 1 + |r_{\rm EW}|^2 - \cos\gamma \,{\rm Re}(r_T\, r_{\rm EW}^*) + \ldots$$

$$\delta A_{\rm CP} = A_{\rm CP}(\pi^0 K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma\Big({\rm Im}(r_C) - {\rm Im}(r_T\, r_{\rm EW})\Big) + \ldots$$

theory: $r_{\rm EW} \approx 0.12 - 0.01i, \qquad r_C \approx 0.03[\times 2?] - 0.02i, \qquad r_T \approx 0.18 - 0.02i$

	theory	data
R_{00}	0.79 ± 0.08	0.85 ± 0.05
R_L	1.01 ± 0.02	1.06 ± 0.04
$\delta A_{\rm CP}$	0.03 ± 0.03	0.111 ± 0.013

$$r_{\rm EW} = \frac{3}{2} R_{\pi K} \frac{\alpha^c_{3,\rm EW}(\pi\bar{K})}{\hat{\alpha}^c_4(\pi\bar{K})}$$

$$r_C = -R_{\pi K} \left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{\alpha_2(\pi\bar{K})}{\hat{\alpha}^c_4(\pi\bar{K})}$$

$$r_T = -\left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \frac{\alpha_1(\pi\bar{K})}{\hat{\alpha}^c_4(\pi\bar{K})}$$

where $R_{\pi K} = (f_\pi/f_K)\cdot(F_0^{B\to K}/F_0^{B\to\pi}) \approx 1.$

Numerical estimate of QED effects for πK final states

Non-radiative amplitude

- Electroweak scale to m_B: QED corrections to Wilson coefficients included
- m_B to μ_c: $\mathcal{O}(\alpha_{\rm em})$ corrections to short-distance kernels included.
  QED effects in form factors and LCDA not included.

Ultrasoft photon radiation

$$U(M_1 M_2) = \left(\frac{2\Delta E}{m_B}\right)^{-\frac{\alpha_{\rm em}}{\pi}\left(Q_B^2 + Q_{M_1}^2\left[1+\ln\frac{m_{M_1}^2}{m_B^2}\right] + Q_{M_2}^2\left[1+\ln\frac{m_{M_2}^2}{m_B^2}\right]\right)} \qquad (M_1, M_2 \text{ light mesons})$$

$$U(\pi^+ K^-) = 0.914$$
$$U(\pi^0 K^-) = U(K^- \pi^0) = 0.976$$
$$U(\pi^- \bar{K}^0) = 0.954 \qquad [\text{for } \Delta E = 60\,\text{MeV}]$$
$$U(\bar{K}^0 \pi^0) = 1$$

Isospin-protected ratios / sum rules for the πK final states

Consider ratios / sums where some QCD uncertainties drop out.

[MB, Neubert, 2003]

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \mathrm{Re}\, \delta_{\mathrm{E}} + \delta_U$$

$$R_L^{\mathrm{QCD}} - 1 \approx (1 \pm 2)\% \qquad \delta_E \approx 0.1\% \qquad \delta_U = 5.8\%$$

QED correction larger than QCD and QCD uncertainty, but short-distance QED negligible.

[Gronau, Rosner, 2006]

$$\Delta(\pi K) \equiv A_{\mathrm{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 K^-) - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\mathrm{QCD}} + \delta\Delta(\pi K)$$

$$\Delta(\pi K)^{\mathrm{QCD}} = (0.5 \pm 1.1)\% \qquad \delta_\Delta(\pi K) \approx -0.4\%$$

QED correction of similar size but small.

Summary

- (I) Theory of structure-dependent QED effects available for the first time.

- (II) QED factorization is more complicated than QCD due to charged external states. SCET applies and we now understand how to systematically include QED effects, but it requires new non-perturbative matrix elements, generalizing the familiar hadronic matrix elements.

- (III) For charmless hadronic decays the QCD $\times$ QED factorization formula takes a similar form as in QCD alone, but the generalized pion (etc.) and B-meson LCDA exhibit novel properties (asymmetric evolution, soft rescattering phases in the B-LCDA)

- (IV) Structure-dependent effects / logarithms turn out to be small ($\lesssim 1\%$)

- (V) Comparison to experiment now requires precise statements how QED effects are treated in the analysis. Ideally compare theoretically well-defined and calculable *radiative* branching fractions and use Monte Carlo generators only to estimate efficiencies.