QUANTUM SIMULATION OF LATTICE GAUGE THEORIES – REQUIREMENTS, CHALLENGES AND METHODS

EREZ ZOHAR

Racah Institute of Physics, The Hebrew University of Jerusalem
and the
Hebrew University Quantum Center

HUQ
HUJI QUANTUM CENTER

מכון רקח
The Racah Institute
לפיסיקה
of Physics

האוניברסיטה העברית בירושלים
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MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

Quantum technologies in particle physics

Theme issue compiled and edited by Steven D. Bass and Erez Zohar

Quantum simulation of lattice gauge theories in more than one space dimension—requirements, challenges and methods

Erez Zohar

Racah Institute of Physics, The Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel

EZ, 0000-0001-6993-6569

Over recent years, the relatively young field of quantum simulation of lattice gauge theories, aiming at implementing simulators of gauge theories with quantum platforms, has gone through a rapid development process. Nowadays, it is not only of interest to the quantum information and technology communities. It is also seen as a valid tool for tackling hard, non-perturbative gauge theory problems by particle and nuclear physicists. Along the theoretical progress, nowadays more and more experiments implementing such simulators are being reported, manifesting beautiful results, but mostly on 1+1 dimensional physics. In this article, we review the essential ingredients and requirements of lattice gauge theories in more dimensions and discuss their meanings, the challenges they pose and how they could be dealt with, potentially aiming at the next steps of this field towards simulating challenging physical problems in analogue, or analogue-digital ways.

This article is part of the theme issue 'Quantum technologies in particle physics'.

Lattice Gauge Theories

- Discretization of both space and time
- Monte Carlo computations on a Wick-rotated, Euclidean lattice

$$\left\langle \hat{A}\left(\hat{\Phi}\right)\right\rangle = \frac{\int \mathcal{D}\phi A(\phi) e^{iS_M}}{\int \mathcal{D}\phi e^{iS_M}}$$

$$\xrightarrow[t\to -i\tau]{} \frac{\int \mathcal{D}\phi A(\phi) e^{-S_E}}{\int \mathcal{D}\phi e^{-S_E}} \equiv \int \mathcal{D}\phi A\left(\phi\right) p\left(\phi\right)$$

- **Very (very) successful for many applications, e.g. the hadronic spectrum**
- **Problems:**
  - **Real-Time evolution:**
    - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - **Sign problem:**
    - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime

$\longrightarrow$ **New approaches: quantum simulation and computation, tensor networks (among others).**

Quantum Simulation

- Take a model, which is either
  - Theoretically unsolvable
  - Numerically problematic
  - Experimentally inaccessible

- Map it to a fully controllable quantum system – quantum simulator

- Study the simulator experimentally

Quantum Simulation of LGTs

- **Real-Time evolution:**
  - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - Exists by default in a real experiment done in a **quantum simulator**: prepare some initial state and the appropriate Hamiltonian (in terms of the simulator degrees of freedom), and let it evolve
- **Sign problem:**
  - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime
  - In real experiments, as those carried out by a **quantum simulator**, fermions are simply fermions, and no path integral is calculated: **nature does not calculate determinants**.

Quantum Simulation of LGTs

- "Step 1" review papers:
  - Wiese, Annalen der Physik 525, 777 (2013)
  - Zohar, Cirac, Reznik, Rep. Prog. Phys. 79, 014401 (2013)
  - Dalmonte, Montangero, Cont. Phys. 57, 388 (2016)

Summarizing the first theoretical proposals for simulating Kogut-Susskind Hamiltonian, using ultracold atoms in optical lattices.

- Experiments:
  - Martinez, Muschik et al, Nature 534, 516 (2016)
  - Kokail et al, Nature 569, 365 (2019)
  - Schweizer et al, Nature Physics 15, 1168 (2019)
  - Mil et al, Science 367, 648, 1168 (2020)
  - Yang at al, Nature 587, 392 (2020)
  - Semeghini et al, Science 374, 1242 (2021)
  - Zhou et al, arXiv:2107.13563 (2021)
  - Riechert at al, Phys. Rev. B 105, 205141 (2022)

Implementations of one dimensional / small size Abelian theories, getting more and more scalable

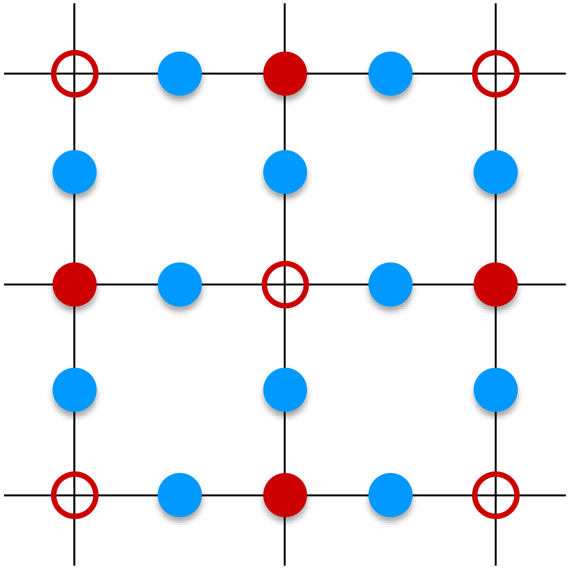
- Contemporary review / "roadmap" papers:
  - Zohar, Nature 534, 7608 (2016)
  - Bañuls et al, Eur. Phys. J. D 74, 165 (2020)
  - Aidelsburger et al, Phil. Trans. R-Soc. A 380, 20210064 (2022)
  - Zohar, Phil. Trans. R-Soc. A 380, 20210069 (2022)
  - Klco, Rogger, Savage, Rep. Prog. Phys. 85, 064301 (2022)

Higher dimensions and plaquette interactions, dealing with the fermionic matter, dual formulations, quantum computing algorithms, …

Hamiltonian LGTs

- **The lattice is spatial**: time is a continuous, real coordinate.
- **Matter particles** (mostly fermions) – on the **vertices**.
- **Gauge fields** – on the lattice's **links**

Challenge 1:
Our simulated platform needs to describe both fermionic and non-fermionic physics.



Kogut and Susskind, Phys. Rev. D 11, 395 (1975)

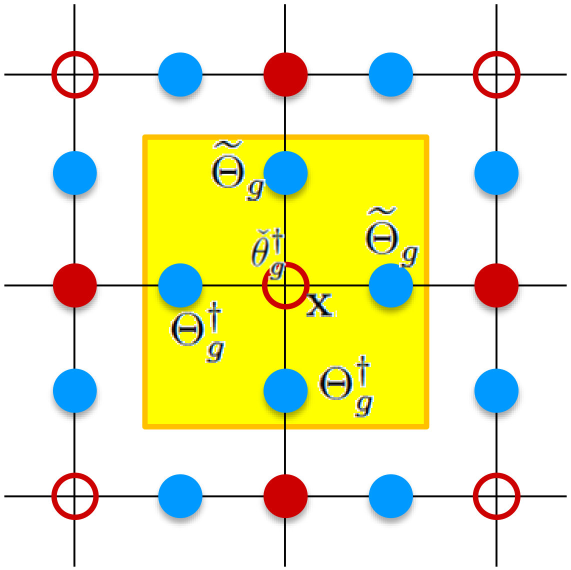
Gauge Transformations

- Act on both the matter and gauge degrees of freedom.

- **Local** : a unique transformation (depending on a unique element of the **gauge group**) may be chosen for each site

- The states are **invariant under each local transformation separately**.

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$

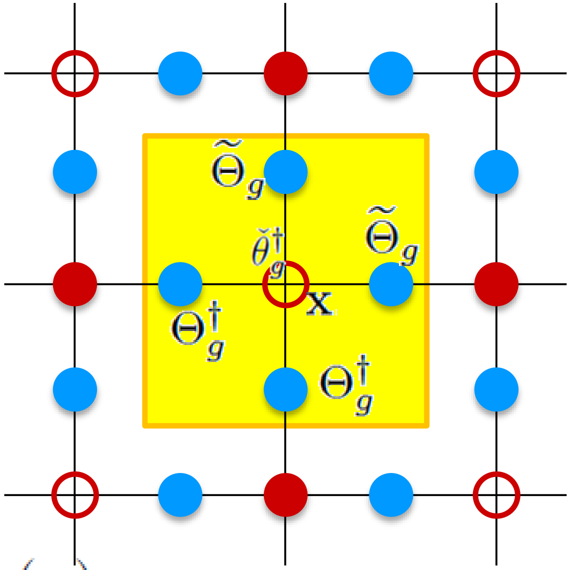


- **Transformation rules on the links**

$$\{|g\rangle\}_{g\in G}$$

$$\Theta_g |h\rangle = |hg^{-1}\rangle \quad \Theta_g = e^{i\phi_a(g)R_a}$$

$$\widetilde{\Theta}_g |h\rangle = |g^{-1}h\rangle \quad \widetilde{\Theta}_g = e^{i\phi_a(g)L_a}$$



- **Gauge Transformations:**

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$

$$\hat{\Theta}_g(\mathbf{x})|\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

- **Compact Lie Group → Generators → Gauss law , left and right E fields:**

$$G_a(\mathbf{x}) = \sum_{k=1\ldots d}\left(L_a(\mathbf{x},k) - R_a\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) - Q_a(\mathbf{x})$$

$$G_a(\mathbf{x})|\Psi\rangle = 0 \quad [G_a(\mathbf{x}),H]=0 \quad \forall \mathbf{x}, a$$

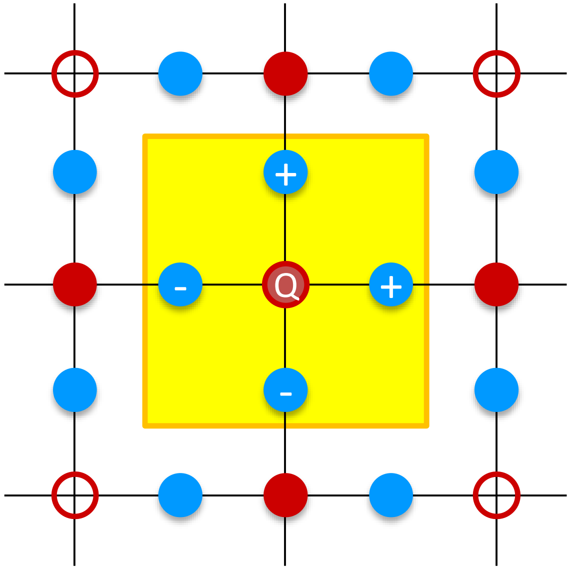
Challenge 2:
Impose / maintain / surpass gauge invariance

Structure of the Hilbert Space

- Generators of gauge transformations (cQED):

$$G(\mathbf{x}) = \mathrm{div} L(\mathbf{x}) - Q(\mathbf{x})$$

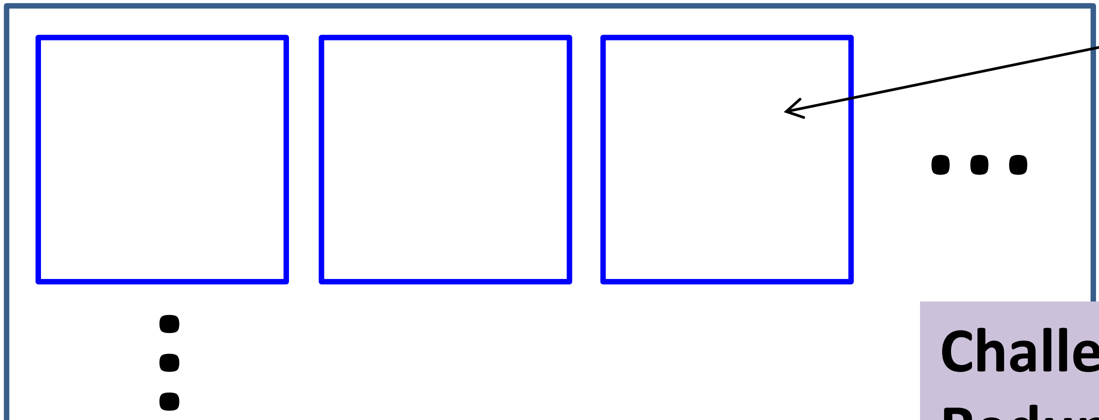
$$\equiv \sum_k \left(L_k(\mathbf{x}) - L_k(\mathbf{x} - \hat{\mathbf{e}}_k) \right) - Q(\mathbf{x})$$



Gauss' Law $G(\mathbf{x})|\psi\rangle = q(\mathbf{x})|\psi\rangle$

$$[G(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}$$

Sectors with fixed Static charge configurations

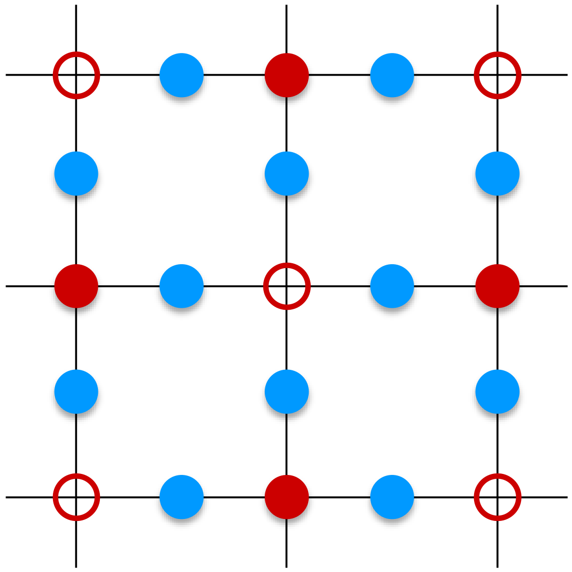


$$\mathcal{H} = \bigoplus \mathcal{H}(\{q(\mathbf{x})\})$$

Challenge 2.1:
Redundant Hilbert Space – Waste of computational resources.

Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)

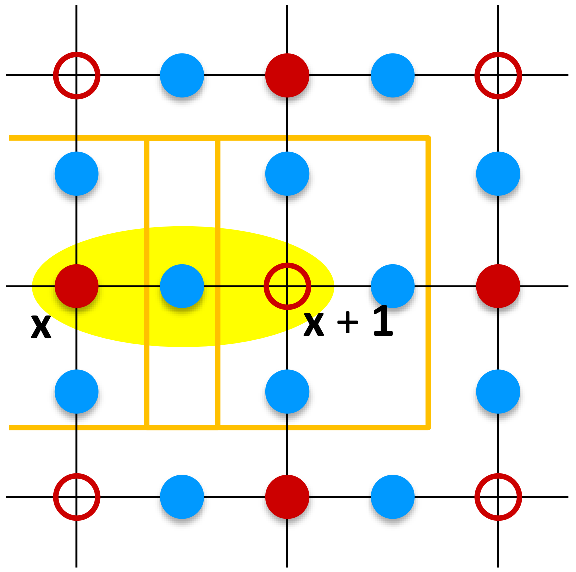


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- <u>First option</u>: Link (**matter**-**gauge**) interaction:

$$\psi_m^\dagger\left(\mathrm{x}\right)U_{mn}\left(\mathrm{x},k\right)\psi_n\left(\mathrm{x}+\hat{\mathrm{k}}\right)$$

  – A **fermion** hops to a **neighboring site**, and the **flux on the link in the middle changes** to preserve **Gauss laws on the two relevant sites**

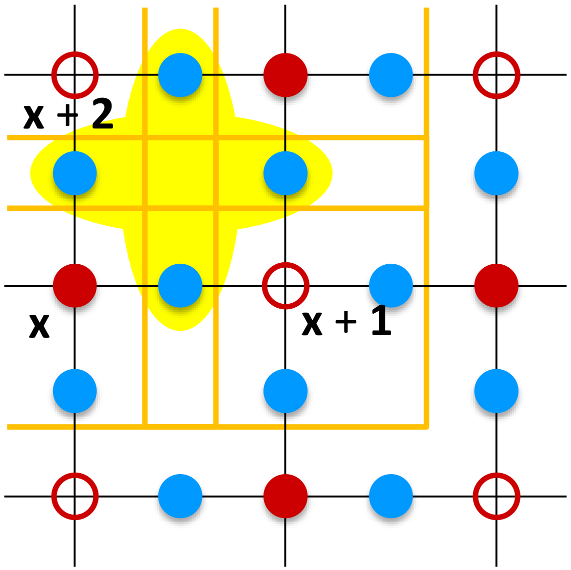


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- <u>Second option</u>: plaquette interaction:

$$\mathrm{Tr}\left(U(\mathrm{x},1)U(\mathrm{x}+\hat{1},2)U^\dagger(\mathrm{x}+\hat{2},1)U^\dagger(\mathrm{x},2)\right)$$

  - The **flux on the links of a single plaquette changes** such that the **Gauss laws on the four relevant sites** is preserved.
  - **Magnetic interaction.**

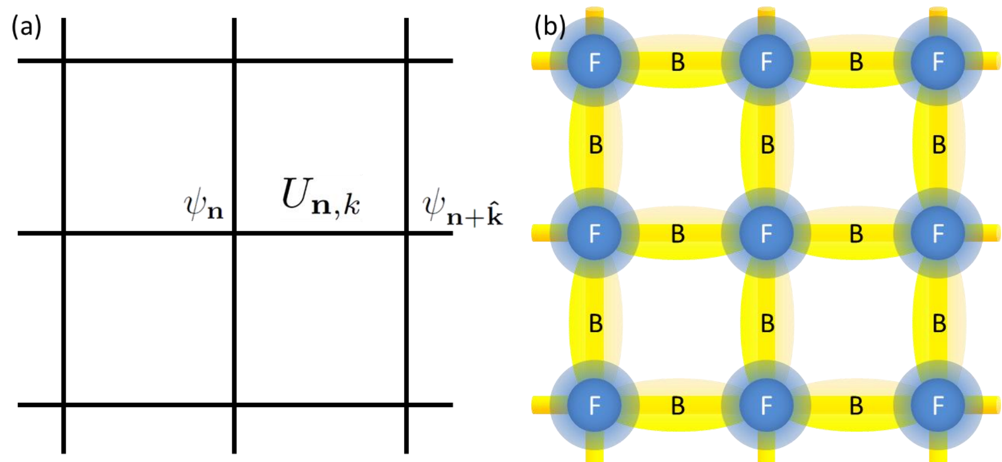


Challenge 3:
Complicated four-body interactions

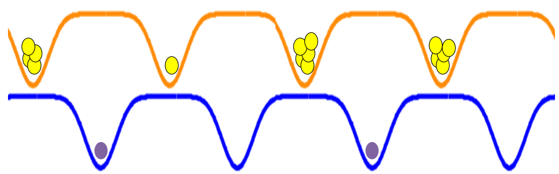
Dealing with the challenges “Naively”

Fermions and Bosons → Cold Atoms in Optical Lattices

- **Fermionic** matter fields
- (Bosonic) gauge fields



Super-lattice:



Atomic internal (**hyperfine**) levels

0 (ultracold)

$$\mathbf{F} = \mathbf{I} + \mathbf{L} + \mathbf{S} \qquad \mathbf{F}^2|F, m_{\mathrm{F}}\rangle = F(F+1)|F, m_{\mathrm{F}}\rangle \qquad F_z|F, m_{\mathrm{F}}\rangle = m_{\mathrm{F}}|F, m_{\mathrm{F}}\rangle$$

$$\mathcal{H} = \sum_{\alpha,\beta} \Phi_\alpha^\dagger(\mathbf{x}) \left(\delta^{\alpha\beta} \left(-\frac{\nabla^2}{2m} + V_{\mathrm{op}}^\alpha(\mathbf{x}) + V_{\mathrm{T}}(\mathbf{x}) \right) + \Omega^{\alpha\beta}(\mathbf{x}) \right) \Phi_\beta(\mathbf{x})$$
$$+ \sum_{\alpha,\beta,\gamma,\delta} \int \mathrm{d}^3x' \Phi_\alpha^\dagger(\mathbf{x}') \Phi_\beta^\dagger(\mathbf{x}) V_{\alpha\beta\gamma\delta}(\mathbf{x} - \mathbf{x}') \Phi_\gamma(\mathbf{x}) \Phi_\delta(\mathbf{x}')$$

Analog Approach I: Effective Local Gauge Invariance

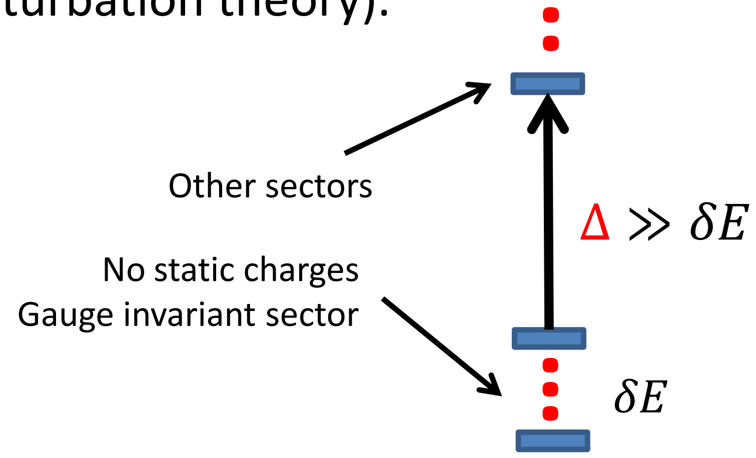
<u>Gauss law</u> is added to the Hamiltonian as a constraint (penalty term, proportional to the square of the symmetry generator).

Leaving a gauge invariant sector of Hilbert space costs too much Energy.

<u>Low energy sector with an effective gauge invariant Hamiltonian</u>.

Emerging plaquette interactions (second order perturbation theory).

- **E. Zohar**, B. Reznik, Phys. Rev. Lett. 107, 275301 (2011)
- **E. Zohar**, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 109, 125302 (2012)
- D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, P. Zoller, Phys. Rev. Lett. 109, 175302 (2012)
- **E. Zohar**, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)
- D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese, P. Zoller, Phys. Rev. Lett. 110, 125303 (2013)



Revisited and simplified later:

With dissipation –

K. Stannigel, P. Hauke, D. Marcos, M. Hafezi, M. Dalmonte and P. Zoller, Phys. Rev. Lett. 112, 120406 (2014)

With linear constraints –

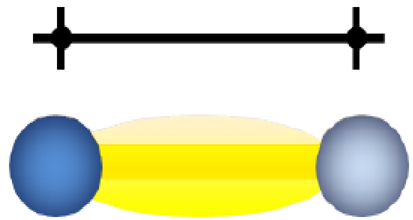
J.C. Halimeh, H. Lang, J. Mildenberger, Z. Jiang, P. Hauke, PRX Quantum 2, 040311 (2021)

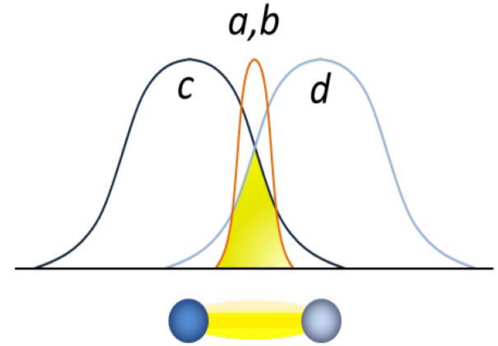
With dynamical decoupling –

V. Kasper, T. V. Zache, F. Jendrzejewski, M. Lewenstein, E. Zohar, arXiv:2012.08620 (2020)

Analog Approach II:
Atomic Symmetries → Local Gauge Invariance

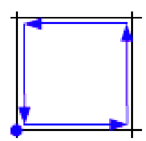
$$\psi_L^\dagger U \psi_R$$

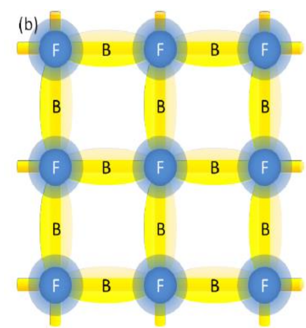




- Links $\leftrightarrow$ atomic scattering : gauge invariance is a <u>fundamental</u> symmetry

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$





- Plaquettes $\leftrightarrow$ gauge invariant links $\leftrightarrow$ virtual loops of ancillary fermions.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

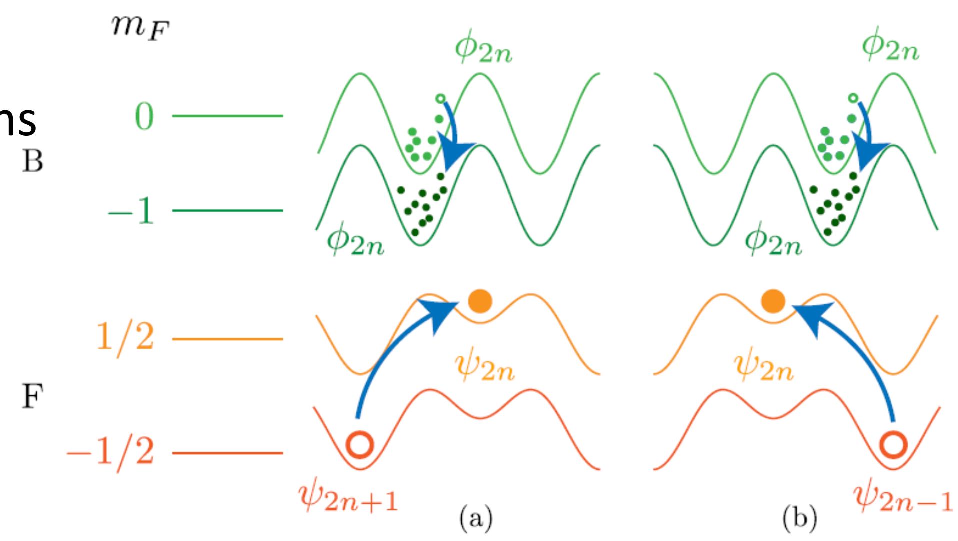
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

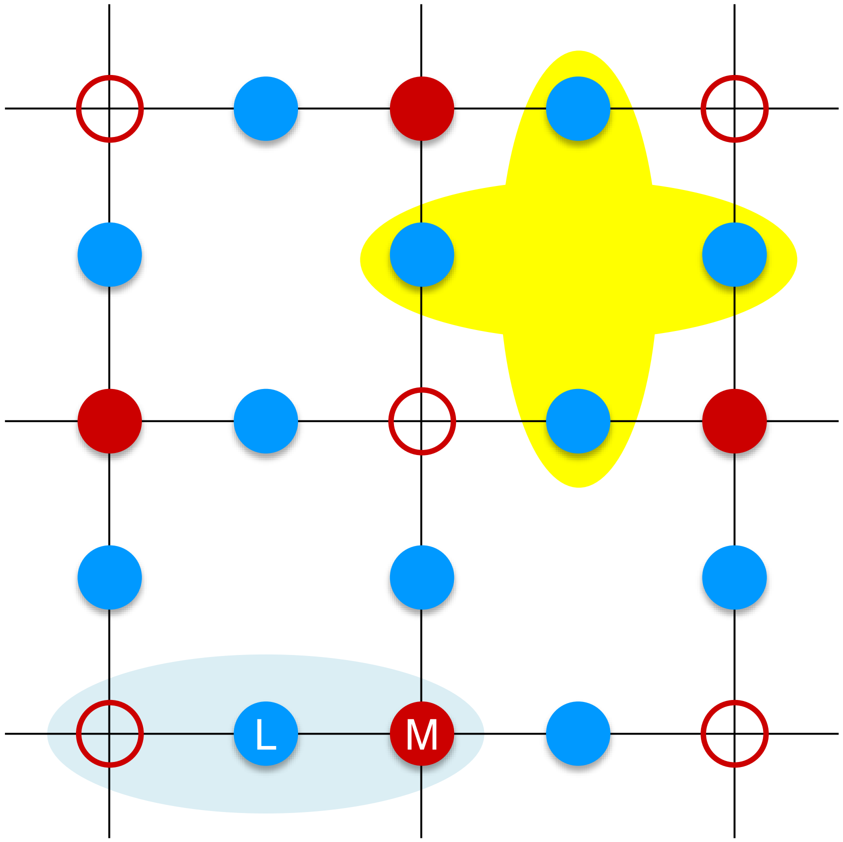
Heidelberg Implementation

- **Kasper, Hebenstreit, Jendrzejewski, Oberthaler, Berges** NJP 19 023030 (2017) – very exciting results:
  - Matter: $F = ½$ ^{6}Li atoms
  - Gauge field: $F = 1$ ^{23}Na atoms
  - No Feshbach resonance!
  - On the links, around 100 atomic bosons – very high electric field truncation (±50)



- **Experimental realization** of a single building block in a similar way: **Mil, Zache, Hegde, Xia, Bhatt, Oberthaler, Hauke, Berges and Jendrzejewski**, Science 367, 6482 (2020)

Digital Plaquette Generation



The Z_2 example:

- Plaquette interactions

$$\sigma_x(\mathbf{x},1)\,\sigma_x(\mathbf{x}+\hat{\mathbf{1}},2)\,\sigma_x(\mathbf{x}+\hat{\mathbf{2}},1)\,\sigma_x(\mathbf{x},2)$$

- Link interactions

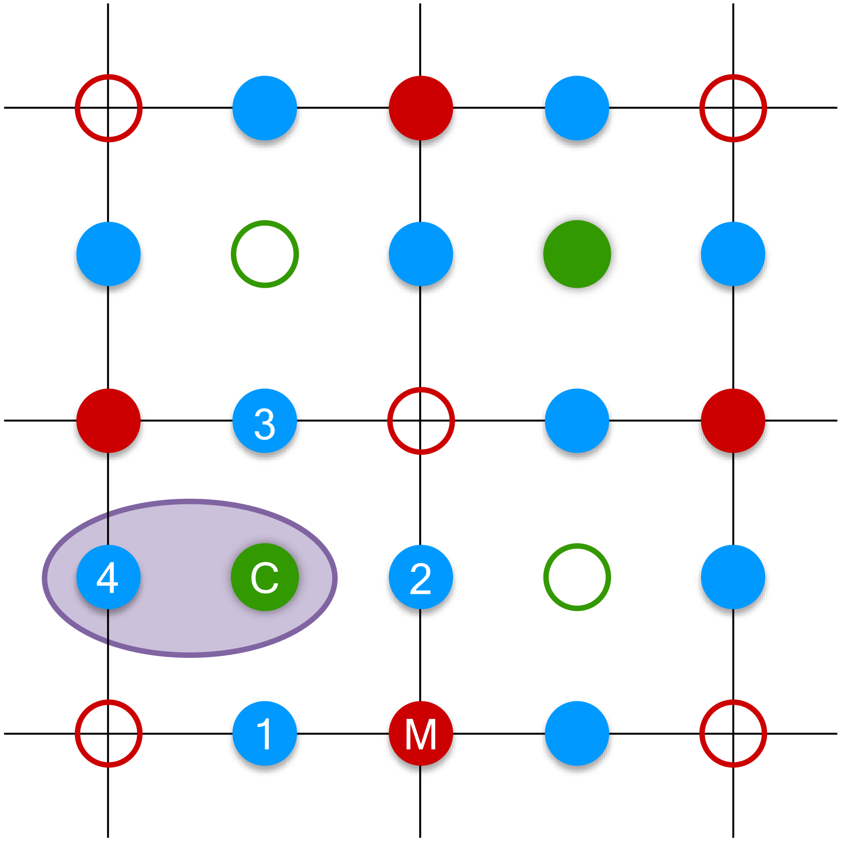
$$\psi^\dagger(\mathbf{x})\sigma_x(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Digital Plaquette Generation

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^{\dagger} = |\tilde{\uparrow}\rangle\langle\tilde{\uparrow}| + \sigma^{x} \otimes |\tilde{\downarrow}\rangle\langle\tilde{\downarrow}|$



$$|\tilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + |\tilde{\downarrow}\rangle\right)$$

$$\mathcal{U}_4^{\dagger}|\tilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_4^{x} \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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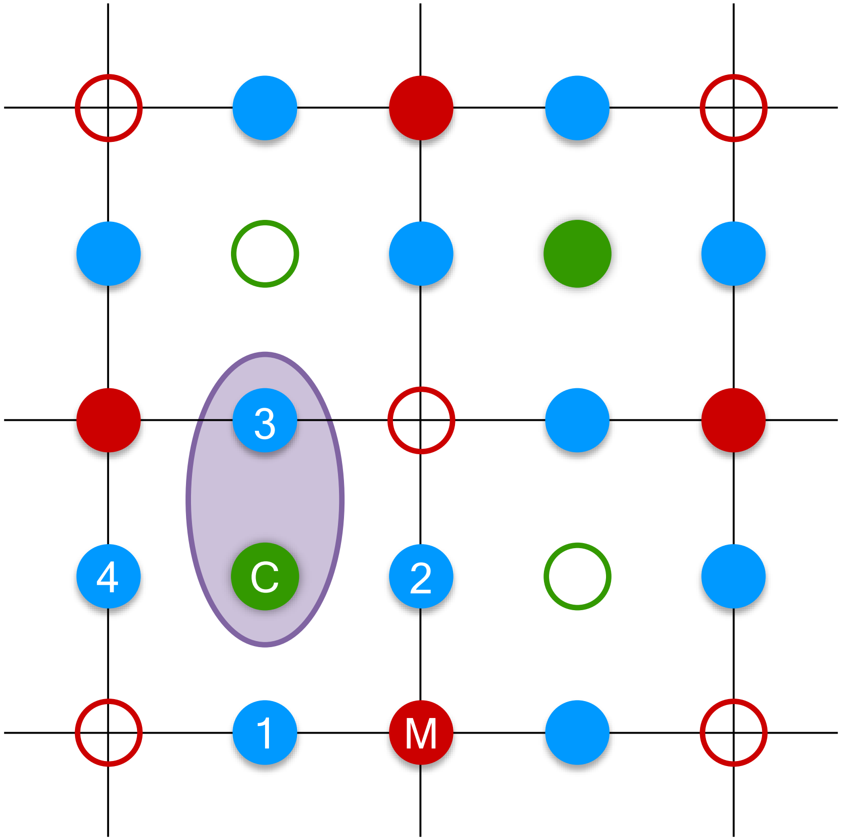

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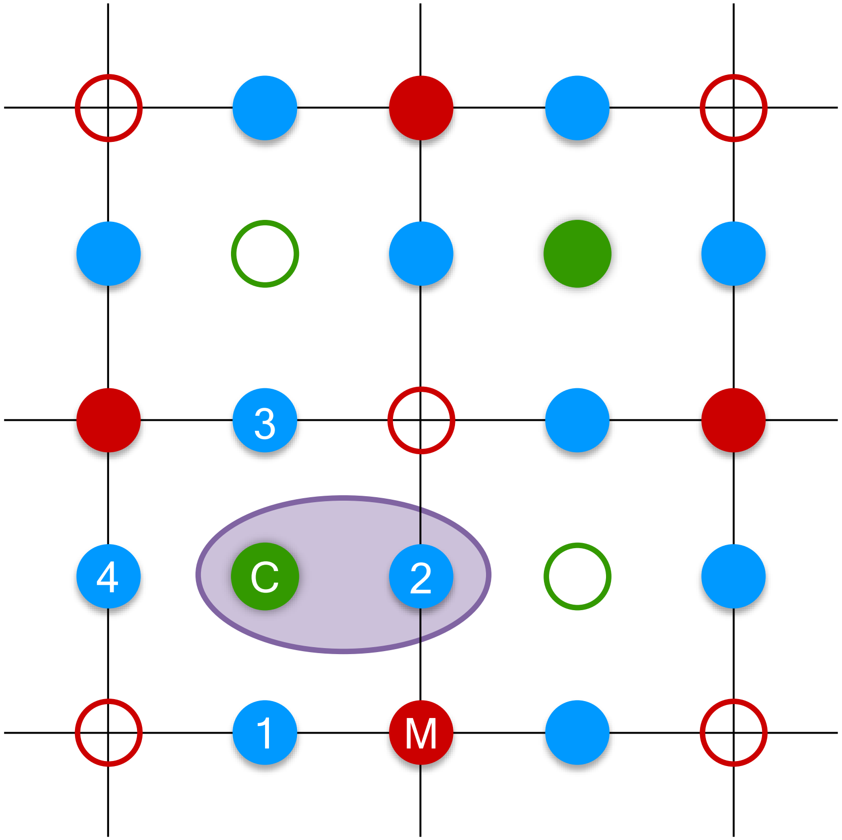

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$$\mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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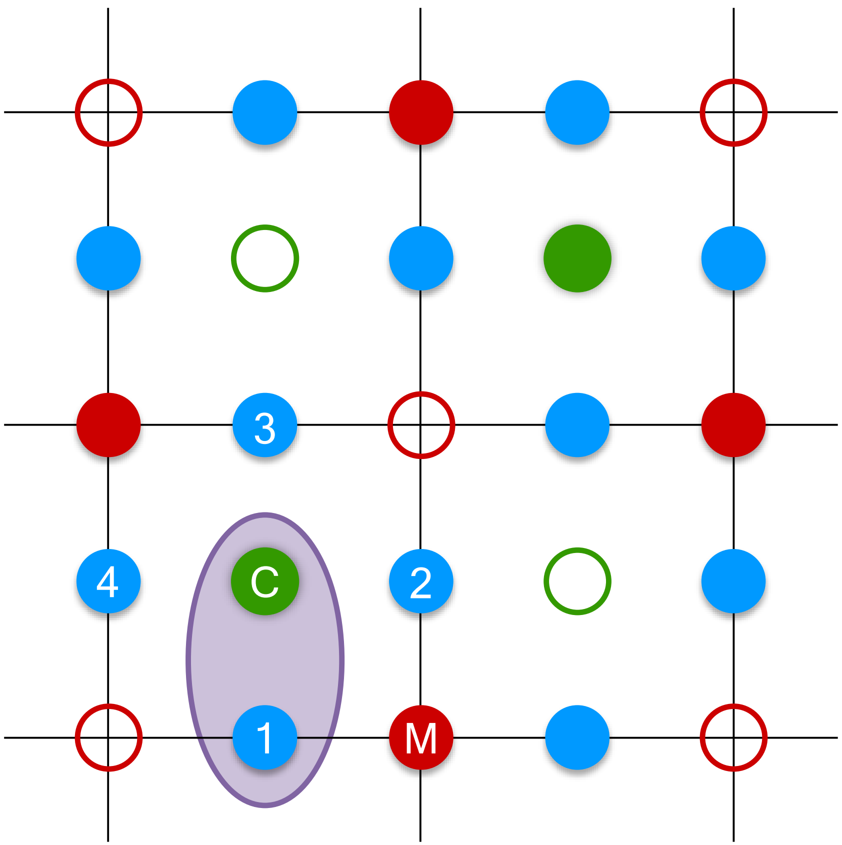

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$$\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^{\dagger}\mathcal{U}_4^{\dagger}\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_1^{x}\sigma_2^{x}\sigma_3^{x}\sigma_4^{x}\otimes\left|\tilde{\downarrow}\right\rangle\right)$$

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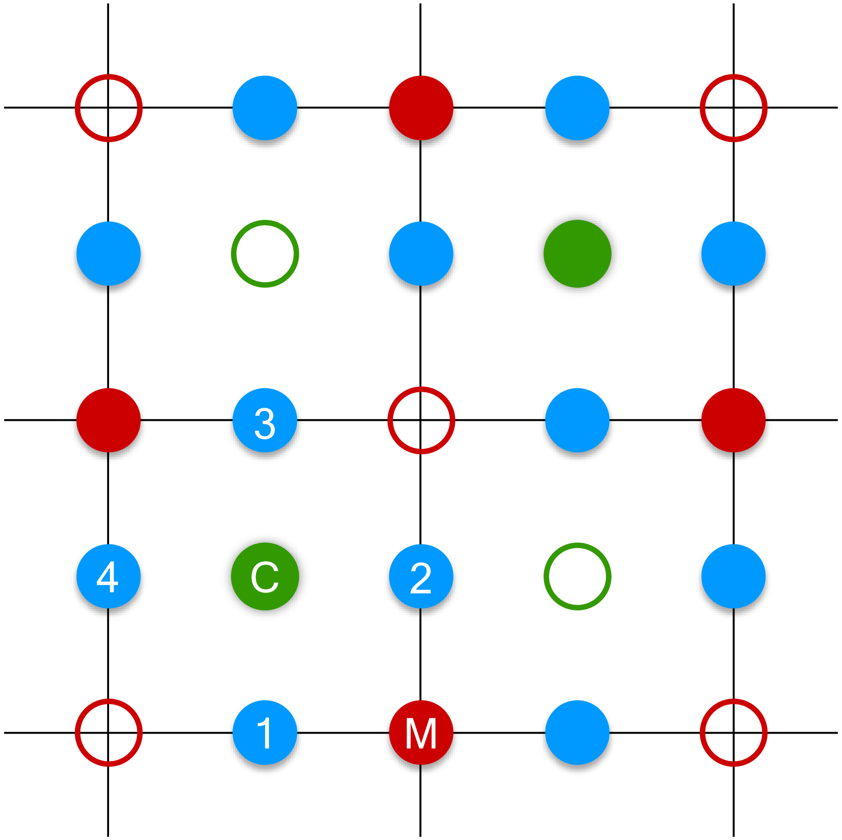

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$$S_\square = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_\square^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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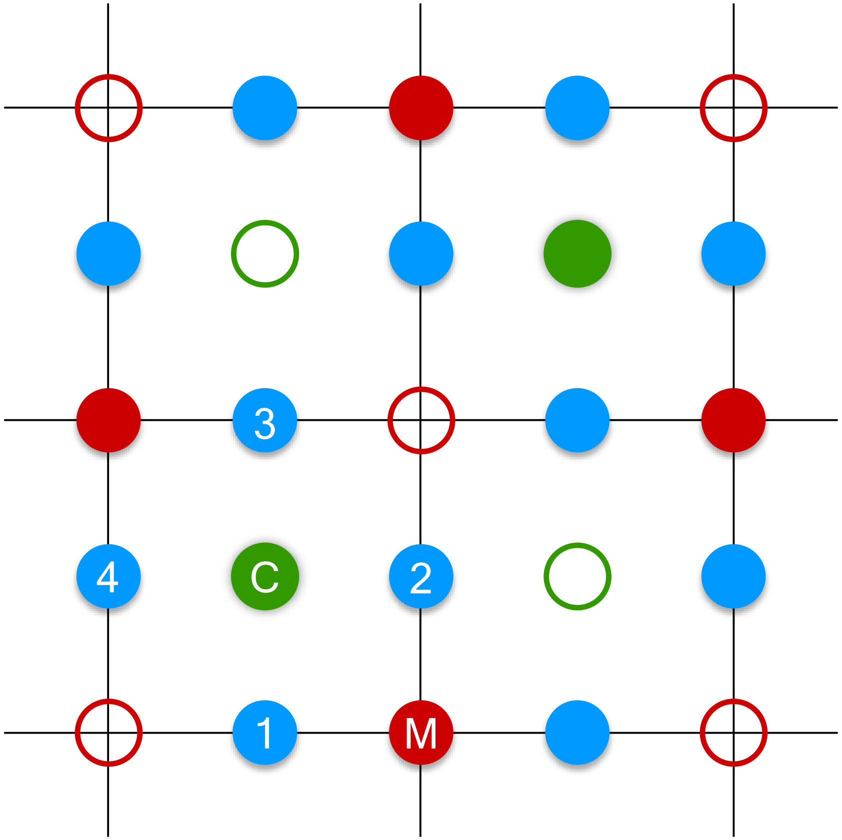
Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^{\dagger} = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^{x} \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$



$$S_{\square} = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma^{x}_{\square} \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\widetilde{\sigma}^{x} S_{\square} = S_{\square}\sigma^{x}_{\square}$$

$$e^{-i\lambda\widetilde{\sigma}^{x}\tau} S_{\square} = S_{\square} e^{-i\lambda\sigma^{x}_{\square}\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^{\dagger}\mathcal{U}_1^{\dagger} e^{-i\lambda\widetilde{\sigma}^{x}\tau} \mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^{\dagger}\mathcal{U}_4^{\dagger} \left|\widetilde{in}\right\rangle = \left|\widetilde{in}\right\rangle e^{-i\lambda\sigma^{x}_{\square}\tau}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Any gauge group – generalization using stators

$$S = \int dg \left|g_A\right\rangle \left\langle g_A\right| \otimes \left|g_B\right\rangle$$

$$\left(U^j_{mn}\right)_B S = S \left(U^j_{mn}\right)_A$$

$$S_\square = \mathcal{U}_\square \left|\widetilde{in}\right\rangle \equiv \mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle \qquad \mathrm{Tr}\left(\widetilde{U^j} + \widetilde{U}^{j\dagger}\right) S_\square = S_\square \mathrm{Tr}\left(U^j_1 U^j_2 U^{j\dagger}_3 U^{j\dagger}_4 + H.c.\right)$$

Feasible for finite or truncated infinite groups

E. Zohar, J. Phys. A. 50 085301 (2017)

Common method for implementing plaquette interactions

For example:

Zohar, Farace, Reznik, Cirac, PRL 2017

Zohar, Farace, Reznik, Cirac, PRA 2017

Zohar, J. Phys. A 2017

Bender, Zohar, Farace, New J. Phys. 2018

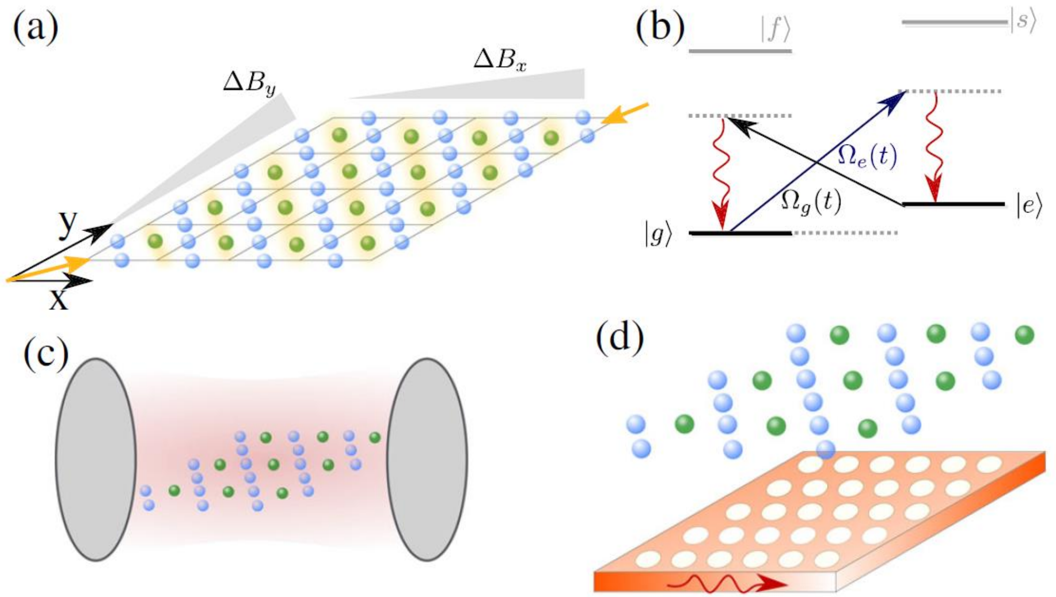
Lamm, Lawrence, Yamauchi, PRD 2019

Zohar, PRD 2020

Gonzalez-Cuadra, Zache, Carrasco, Kraus, Zoller, 2022

Recent Proposal – 2+1d Pure Gauge Z_2

First proposal to simulate a Z2 LGT in nanophotonic or cavity QED setups, naturally allowing for long-range interactions, sparing the need for sequential operation: the control interacts with all four links at once, saving experimental run-time.



Armon, Ashkenazi, Garcia-Moreno,
González-Tudela, **Zohar**,
Physical Review Letters 127 (25), 250501

More Generally: Duality Transformations

- Theoretically, transferring the information to the ancilla and back may be seen as **switching between two dual formulations**.

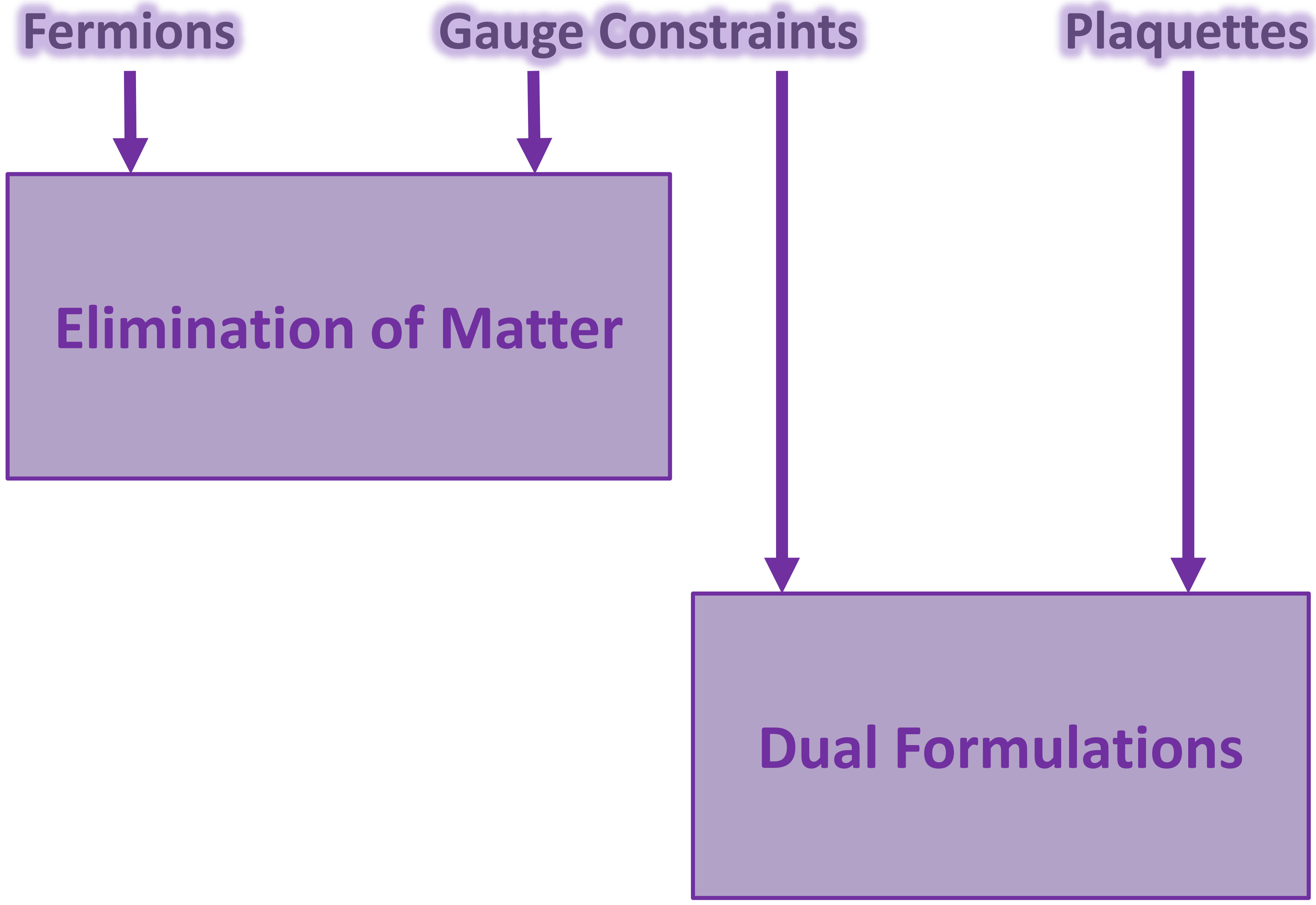
- This may be used as a general tool in the context of quantum simulation: make duality transformations feasible, physical transformations which can be implemented in the lab, using local unitaries and measurements.

- The theory behind it: the original system serves as matter, and the dual one – gauge fields, coupled minimally without dynamics.

- Why duality? Soon.

Shachar Ashkenazi and **Erez Zohar**, Physical Review A 105, 022431 (2022)

Getting rid of challenges



Fermions
Gauge Constraints
Plaquettes
Elimination of Matter
Dual Formulations

Eliminating the fermions

- Fermions are subject to a **global Z_2 symmetry** (parity superselection)
- If this symmetry is **local** (which happens naturally in a lattice gauge theory whose gauge group contains $\mathbf{Z}_2$ as a normal subgroup), it can be used for **locally transferring the statistics information to the gauge field**
- One is left with **hard-core bosonic matter (spins),** with **fermionic statistics taken care of by the gauge field**

$$\psi^{\dagger}\left(\mathbf{x}\right)=c\left(\mathbf{x}\right)\eta^{\dagger}\left(\mathbf{x}\right)$$

Majorana Fermion: Statistics ($c(\mathbf{x})$)

Hardcore Boson: Physics ($\eta^{\dagger}(\mathbf{x})$)

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Eliminating the fermions

- With a **local unitary transformation** which is independent of the space dimension, one can remove the fermions from the Hamiltonian, and stay with **hard-core bosonic matter** and **electric field dependent signs** that preserve the fermionic statistics.

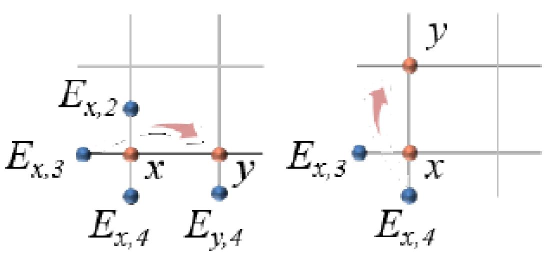
$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\psi^{\dagger}(\mathbf{x}) U(\mathbf{x},i) \psi(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$

$$\psi^{\dagger}(\mathbf{x}) = c(\mathbf{x}) \eta^{\dagger}(\mathbf{x})$$

$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\eta^{\dagger}(\mathbf{x}) c(\mathbf{x}) U(\mathbf{x},i) c(\mathbf{x}+\hat{\mathbf{e}}_i) \eta(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$

Unitary transformation

$$-i\epsilon \sum_{\mathbf{x},i=1,2} \left(\xi_i \sigma_+(\mathbf{x}) U(\mathbf{x},i) \sigma_-(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$



$$\xi_h = e^{i\pi(E_{x,2}+E_{x,3}+E_{x,4}+E_{y,4})}$$

$$\xi_v = e^{i\pi(E_{x,3}+E_{x,4})}$$

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Eliminating the fermions

- This procedure opens the way for **quantum simulation of lattice gauge theories with fermionic matter in 2+1d and more**, even with **simulating systems that do not offer fermionic degrees of freedom**.

- In the U(N) case **the matter can be removed completely**!

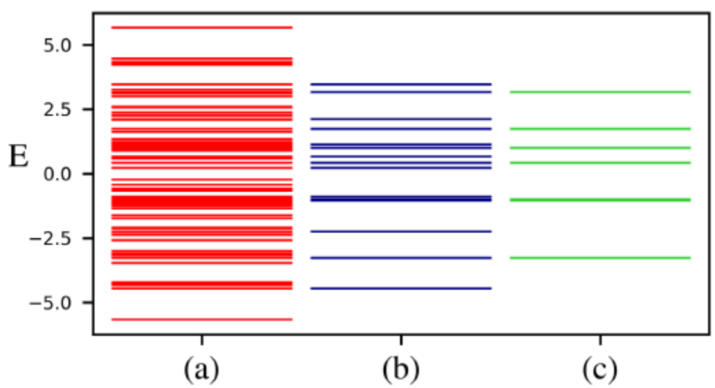
E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)
E. Zohar, J. I. Cirac, Rev. D 99, 114511 (2019)

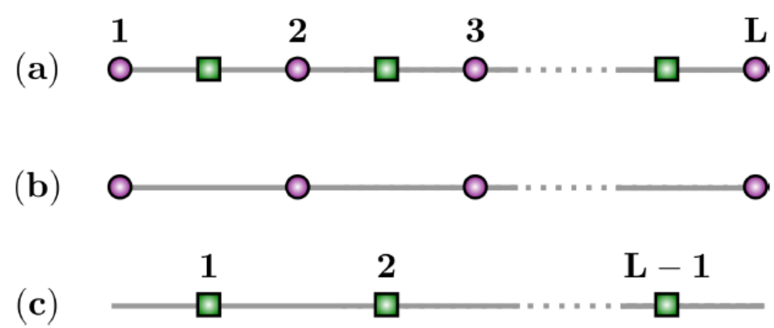
Example: Z_2 quantum simulatoin

- The entire dynamics of a Z_2 LGT with fermionic matter may be rexpressed as a qubit model, only with gauge field qubits, **without explicit fermions** (or any physical degree of freedom representing the matter), **without gauge constraints**, **using simple local single- and two-qubit unitaries**

- This can be done in **arbitrary space dimensions**.

- In 1+1d, unlike the other conventional method where the gauge field is eliminated, **each time step is independent of the system size**.

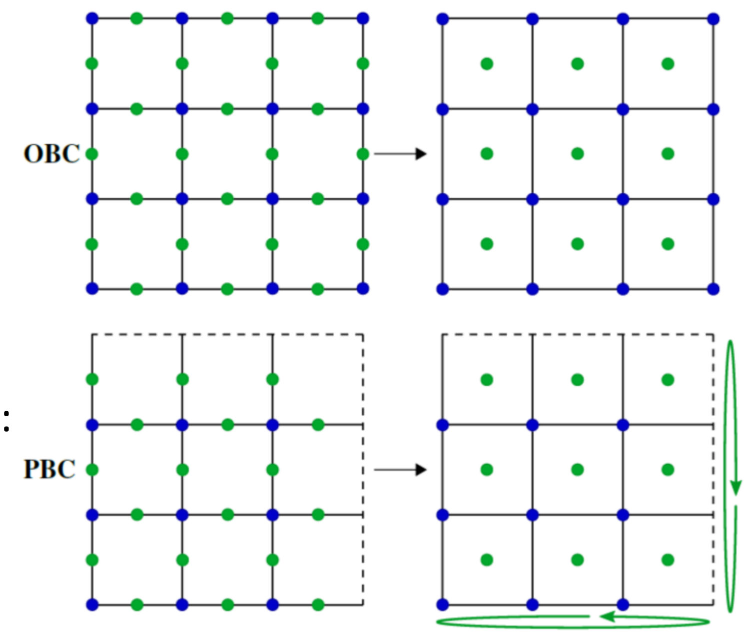




Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, arXiv:2206.00685 (2022)
See also: Reinis Irmejs, Mari Carmen Bañuls , J. Ignacio Cirac, arXiv:2206.08909 (2022)

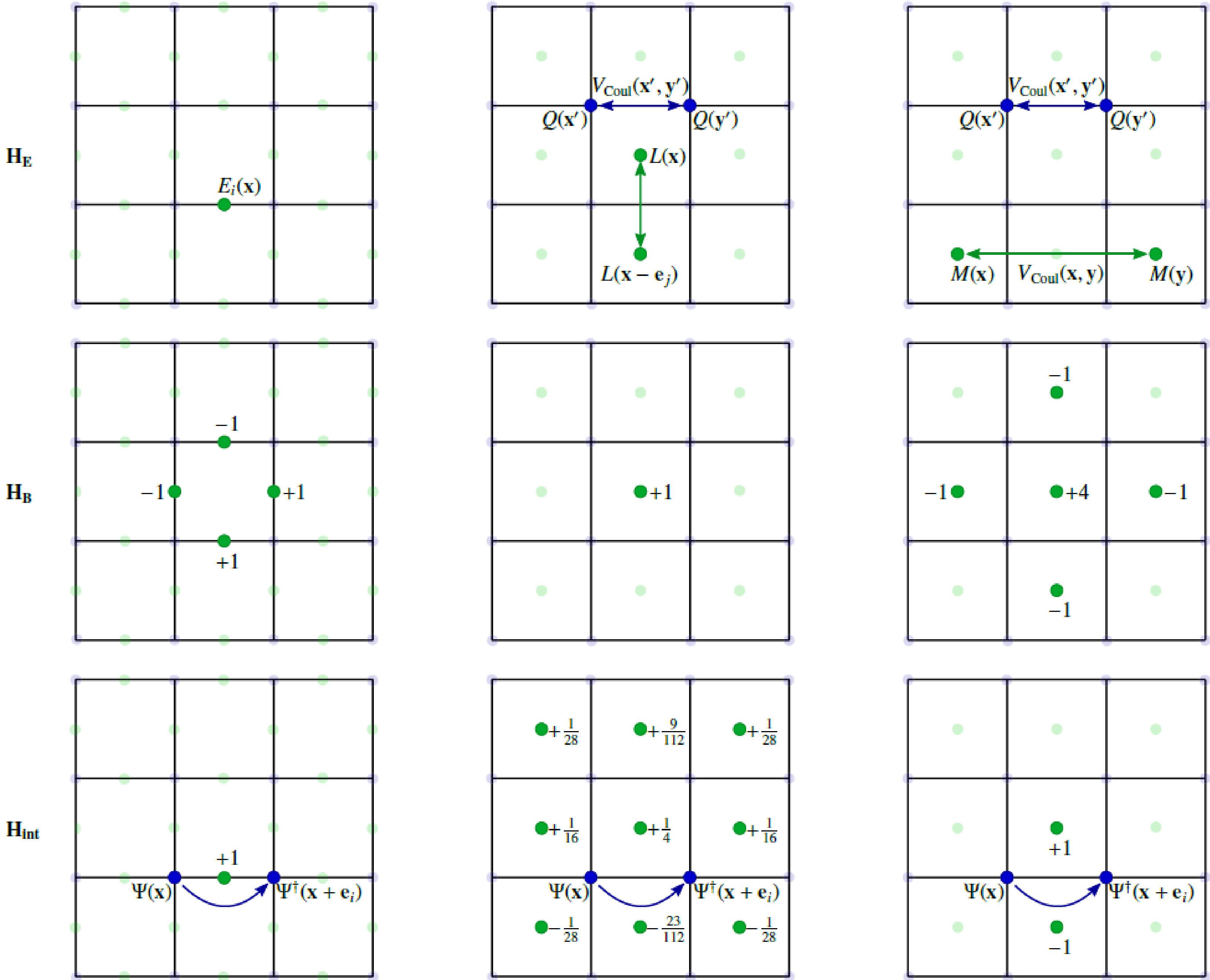
Gauge constraints: resources or redundancies?

- Compact QED in 2+1, dual formalism – no local constraints; In 3+1 – some local constraints are needed in the dual picture too.
- Pure gauge: everything is local:
  - Plaquette, four-body interaction → Non-interacting terms
  - Link terms → Two-body interactions
  - Drell, Quinn, Svetitsky, Weinstein, Phys. Rev. D 19, 619 (1979), Kaplan and Stryker, Phys. Rev. D 102, 094515 (2020)
  - Unmuth-Yockey, Phys. Rev. D 99, 074502 (2019)
  - Bauer & Grabowska, arxiv:2111.08015 (2021)
- With dynamical matter:
  - Haase et al, Quantum 5, 393 (2021), Paulson et al, PRX Quantum 2, 030334 (2021): coupling to matter introduces non-locality in the form of strings (maximal trees)
  - Bender and **Zohar**, Phys Rev. D 102, 114517 (2020): another type of dual formulation, using Green's functions: non-locality does not break spatial symmetries; matter and gauge field Coulomb interactions.



Phys Rev. D 102, 114517 (2020)

Original Formulation | "Regular" dual formulation | Another dual formulation



Julian Bender and **Erez Zohar**, Phys Rev. D 102, 114517 (2020)

Dualities and Quantum Simulation – General Formulation

- Duality maps may be formulated as <u>physical</u> transformations which are feasible on quantum platforms (including NISQ devices).

- This allows one to build a quantum simulator of both sides of a duality map of models admitting one, enjoying the benefits of both in the same simulator.

- Generalizations: Matter, non-Abelian.

Shachar Ashkenazi and **Erez Zohar**, Physical Review A 105, 022431 (2022)

In conclusion,

- Quantum Simulation of (lattice) gauge theories is subject to several challenges:
  - Our simulated platform needs to describe both fermionic and non-fermionic physics.
  - Impose / maintain / surpass gauge invariance
    - Redundant Hilbert Space – Waste of computational resources.
  - Complicated four-body interactions

- These can be addressed in various ways, directly and indirectly, in spite of or thanks to the local constraints.

- Quantum simulation of lattice gauge theories is an exponentially growing field; besides the massive experimental progress, there is still room for exciting theoretical study.

Quantum Information & Many Body Physics Group

Racach Institute of Physics, Hebrew University of Jerusalem, Israel

Quantum Simulation

Shachar Ashkenazi

Guy Pardo

Tomer Greenberg

Erez Zohar

Tensor Networks

Johannes Knaute

Ariel Kelman

Matan Feuerstein