TOWARDS AN ISING UNIVERSE

Fotis Koutroulis
University of Warsaw
Institute of Theoretical Physics

Kitzbühel Humboldt Kolleg, 30 June 2022

In collaboration with Nikos Irges, Antonis Kalogirou and Alex Kehagias
NTU of Athens

CONTENTS

INTRO

- It is generally believed that the Universe in its early stage underwent a period of rapid expansion

The expansion can be modelled by de Sitter (dS) space, if reasonable assumptions are made about the isometries of space-time

- The dS epoch was followed by an almost flat, Minkowski epoch, to a good approximation. Both of these were phases of finite temperature, beyond little doubt

N. D. Birrell and P. C. W. Davies, '82
V. F. Mukhanov and S. Winitzki Cambridge University Press (2013)
E. T. Akhmedov, Int. J. Mod. Phys. D23 (2014) 1430001

- A simple model that could explain some of the observed features of the CMB is a real scalar field ϕ in fixed dS background

Considering QFT in a fixed de Sitter background includes a possible inadequacy reflected by the uncontrollable infrared (IR) divergences that appear in observables

- Our motivation comes from a model with similar perturbative feature: pure Yang-Mills theory in dimensions $d \geq 5$

Perturbatively non-renormalizable system → *Observables or RG flows at a short interval inside its phase diagram make no sense*

↓

Appearance of uncontrollable ultraviolet (UV) divergences in Feynman diagrams

INTRO

- No sensible continuum description in the interior of the phase diagram. Things are different near at least two points:

  1) In the Coulomb phase, the Gaussian point. Still problematic however

  2) The point of a first order phase transition somewhere in the interior, beyond which a Confined phase appears.

- 2) is a consistent picture since there the system seems to start behaving regularly, as if it was renormalizable due to the finite UV cut-off that the first order transition imposes on the effective theory

  N. Irges and F.K., Eur. Phys. J. C 81 (2021) 2

- An analogy can be now drawn: Consider the de Sitter and Minkowski epochs as two phases of a single cosmological phase diagram with the transition from the one phase into the other proceeding via a first order phase transition

  1. *dS background is a good approximation near the beginning and near the end of the expanding phase*
  2. *Certain properties of a UV character in the YM theory are to be replaced by IR properties in the cosmological model*

- The aim is to construct propagators of the field ϕ at finite temperature and analyze their effects on cosmological observables

- QFT in dS space:

  Conformally flat metric with a time-like coordinate $\tau \in (-\infty, 0]$

  An observer with coordinate $\tau = -\infty$ *is called an "in" observer with* $|in\rangle$ *the associated vacuum state*

  An observer with coordinate $\tau = 0$ *is called an "out" observer with* $|out\rangle$ *the associated vacuum state*

- The $\tau = 0$ surface is also called the Horizon of the expanding Poincare patch of dS space

- According to our picture the Horizon is identified with a first order phase transition

  It is not a sharp point but rather an interval around the Horizon, $\tau \in (0 - \delta, 0 + \delta)$ *with* $0 < \delta << 1$

- Physics for $\tau < -\delta$ is described by thermal QFT in a dS background and for $\tau > \delta$ by thermal QFT in a Minkowski background

  By this regularization the out observer is of course located at $\tau = -\delta$ *rather than at* $\tau = 0$

THE THERMAL PROPAGATORS

- The action which we will quantize taking into account finite temperature effects is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (m^2 + \xi\mathcal{R})\phi^2\right]$$

- Consider a $d+1$ dimensional FRW spacetime with metric and (de Sitter) scale factor

$$ds^2 = a^2\left(d\tau^2 - d\mathbf{x}^2\right) \quad \text{and} \quad \alpha(\tau) = -\frac{1}{H\tau}$$

- The classical e.o.m. for the scalar field mode in d-dimensional momentum space, $\phi_{|\mathbf{k}|} = \frac{\chi_{|\mathbf{k}|}}{\alpha}$, in this background is

$$\ddot{\chi}_{\mathbf{k}} + \omega^2_{|\mathbf{k}|}\chi_{\mathbf{k}} = 0 \qquad \textit{with} \;\; \omega^2_{|\mathbf{k}|} = |\mathbf{k}|^2 + m^2_{\mathrm{dS}} \qquad \textit{the time-dependent mass} \; m^2_{\mathrm{dS}} = \frac{1}{\tau}\left(M^2 - \frac{d^2-1}{4}\right),\; M^2 = \frac{m^2}{H^2} + 12\xi$$

the inverse curvature parameter of dS space, H, satisfying $\mathcal{R} = 12H^2$

- Linear combinations of the Hankel function $H_{\nu_{cl}}(\tau, |\mathbf{k}|)$ and its complex conjugate, solve the e.o.m. with weight

$$\nu_{\mathrm{cl}} = \frac{d}{2}\sqrt{1 - \frac{4M^2}{d^2}}$$

- Quantization of this system results in the notion of a time-dependent vacuum state and a doubled Hilbert space

THE THERMAL PROPAGATORS

- For the former $|\text{in}\rangle$ and $|\text{out}\rangle$ are empty vacua from the perspective of corresponding local (in conformal time) observers

  → *Maximally symmetric Bunch-Davies vacuum*

  N. A. Chernikov and E. A. Tagirov, '68
  B. Allen, Phys. Rev. D32 (1985) 3136

- $|\text{in}\rangle$ ↔ $|\text{out}\rangle$ via the Bogolyubov Transformation (BT): $< J|\mathbf{\Phi}^{\mathbf{I}} = < I|\mathbf{\Phi}^{\mathbf{J}}$

  $I, J = \text{in}, \text{out}$ *and* $\mathbf{\Phi}^{\mathbf{I}}$ *is the field operator with eigenvalue* $\chi^{I}_{|\mathbf{k}|}$ ($\chi^{\text{in}}_{|\mathbf{k}|} = u_{|\mathbf{k}|}$, $\chi^{\text{out}}_{|\mathbf{k}|} = v_{|\mathbf{k}|}$)

- The doubled Hilbert space can be understood in the context of the Schwinger-Keldysh path integral as being related to a + (or forward) branch and a − (or backward) branch in conformal time evolution

- The main quantity: The field propagator $\mathcal{D}$ in this basis (2×2 matrix structure)

$$\langle 0|\,\Phi^+(\tau_2)\Phi^-(\tau_1)\,|0\rangle = \mathcal{D}_<(\tau_1;\tau_2) \qquad \langle 0|\,\mathcal{T}^*[\Phi^+(\tau_1)\Phi^+(\tau_2)]\,|0\rangle = \mathcal{D}_{++}(\tau_1;\tau_2)$$

$$\langle 0|\,\Phi^-(\tau_1)\Phi^+(\tau_2)\,|0\rangle = \mathcal{D}_>(\tau_1;\tau_2) \qquad \langle 0|\,\mathcal{T}[\Phi^-(\tau_1)\Phi^-(\tau_2)]\,|0\rangle = \mathcal{D}_{--}(\tau_1;\tau_2)$$

$$\mathcal{D}_>(\tau_1;\tau_2) = \mathcal{D}^*_<(\tau_1;\tau_2),\ \mathcal{D}_{--}(\tau_1;\tau_2) = \mathcal{D}^*_{++}(\tau_1;\tau_2) \quad \rightarrow$$

$$\mathcal{D}_<(\tau_1;\tau_2) = \chi_{|\mathbf{k}|}(\tau_1)\chi^*_{|\mathbf{k}|}(\tau_2)$$

$$\mathcal{D}_{++}(\tau_1;\tau_2) = \theta(\tau_1-\tau_2)\mathcal{D}_<(\tau_1;\tau_2) + \theta(\tau_2-\tau_1)\mathcal{D}_>(\tau_1;\tau_2)$$

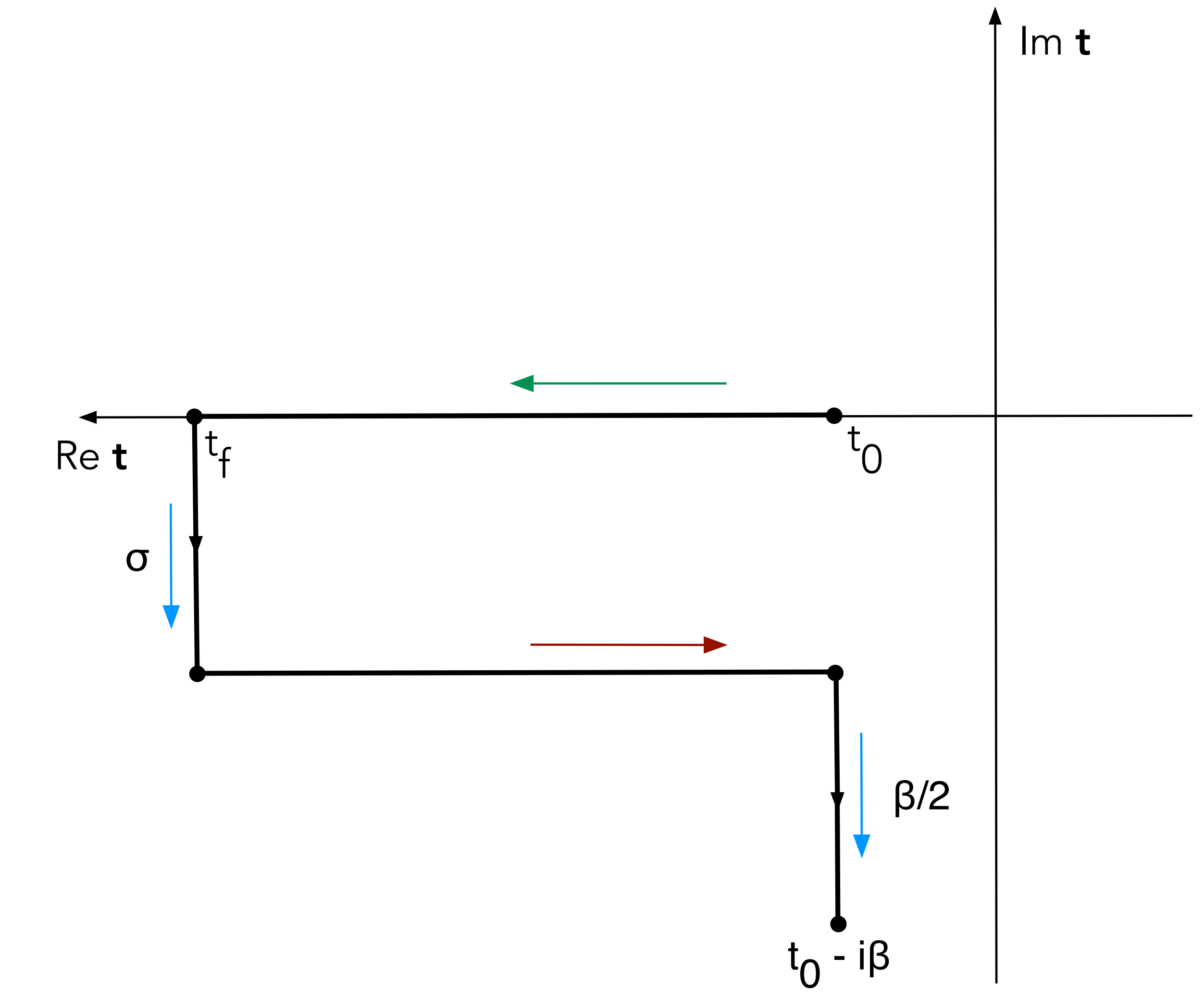
THE THERMAL PROPAGATORS

- The advantage is that Schwinger-Keldysh ≡ Thermofield dynamics (TD)

- The passage to finite temperature is via the transformation $\mathcal{D}_\beta = U_\beta \, \mathcal{D} \, U_\beta^T$

$$\textit{with } \beta = \frac{1}{T} \textit{ and } U_\beta = \begin{pmatrix} \cosh\theta_{|\mathbf{k}|} & \sinh\theta_{|\mathbf{k}|} \\ \sinh\theta_{|\mathbf{k}|} & \cosh\theta_{|\mathbf{k}|} \end{pmatrix}$$

$$\textit{A BT with coefficients } \cosh\theta_{|\mathbf{k}|} = \frac{1}{\sqrt{1-e^{-\beta\omega_{|\mathbf{k}|}}}} \textit{ and } \sinh\theta_{|\mathbf{k}|} = \sqrt{\cosh^2\theta_{|\mathbf{k}|} - 1}$$



The Schwinger-Keldysh contour equivalent of Thermofield dynamics

L. V. Keldysh, JETP 20(4): (1965) 1018–1026.
J. Schwinger, Journal of Mathematical Physics 2(3): (1961) 407–432.
A. Das, Topics in Finite Temperature Field Theory, hep-ph/0004125

- All the allowed thermal transformations of $\mathcal{D}$ are correlators of the form

$$\mathcal{D}^I_{J,\alpha} = \langle J;\alpha| \, \mathcal{T}[\mathbf{\Phi}^I (\mathbf{\Phi}^I)^T] \, |J;\alpha\rangle$$

- The doublet field in TD language is $(\mathbf{\Phi}^I)^T = (\Phi^{+,I}, \Phi^{-,I})$ and α is a thermal index, associated with any combination of thermal transformations of the form of U_β

THE THERMAL PROPAGATORS

- 2 relevant types of thermal transformations

  1. The insertion of an explicit density matrix $\rightarrow |I;\beta> = U_\beta |I>$

  2. The Gibbons-Hawking (GH) effect (for which we will momentarily use the parameter δ to distinguish it from β) $\rightarrow |I> = |J;\delta>, \; I \neq J$

- $D^{\text{in}}_{\text{out},\beta} = D^{\text{in}}_{\text{in},\alpha} \rightarrow$ Near the horizon ($m^2_{\text{dS}} \rightarrow \infty$) the thermal parameters are related

$$\begin{aligned} \beta < \delta: \qquad \alpha &= \beta + \delta e^{-\frac{|\beta-\delta|}{2} m^2_{\text{dS}}} + \cdots \\ \beta > \delta: \qquad \alpha &= \delta + \beta e^{-\frac{|\beta-\delta|}{2} m^2_{\text{dS}}} + \cdots \end{aligned}$$

- Through inspection and due to no backreaction of the scalar $\rightarrow$ dS space can only sustain the GH temperature

- In and out observers agree on the observed physical thermal effects only if $\dfrac{1}{\beta} = T = T_{\text{dS}} = \dfrac{H}{2\pi} = \dfrac{1}{\delta}$

- Key feature is the form of the thermal dS-scalar propagator

$$\mathcal{D}^I_{J,\alpha} = \langle J;\alpha|\, \mathcal{T}[\mathbf{\Phi}^I(\mathbf{\Phi}^I)^T]\, |J;\alpha\rangle$$

$$\mathcal{D}_\beta = \mathcal{D} + \left(\mathcal{D}_{++} + \mathcal{D}^*_{++}\right)(s^2 + sc)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

with $s \equiv \sinh\theta_{|\mathbf{k}|}$ *and* $c \equiv \cosh\theta_{|\mathbf{k}|}$ *for some generic* β

THE COSMOLOGICAL OBSERVABLES

- Go to equal space-time points, at the time of horizon exit $|H\tau| = 1$ and for horizon exiting modes $|\mathbf{k}\tau| = 1$

$$\mathcal{D}_\beta = \mathcal{D} + \left(\mathcal{D}_{++} + \mathcal{D}^*_{++}\right)(s^2 + sc)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- The thermal propagator determines several important observables (n_S, n_S', n_S'' and f_{NL} relevant here) through the power spectrum ($\mathbf{1}$ is the 2×2 matrix with unit elements)

$$P_{S,\beta}\mathbf{1} = \mathcal{D}_\beta \mathbf{1}|_{\tau_1=\tau_2}$$

- The picture is that of a two-step BT process $\mathcal{D}^{\text{in}}_{\text{in}} \leftrightarrow \mathcal{D}^{\text{out}}_{\text{out}} \leftrightarrow \mathcal{D}^{\text{out}}_{\text{out},\beta}$ $\quad (D^{\text{out}}_{\text{out},\beta} = D_\beta)$

$\mathcal{D}^{\text{in}}_{\text{in}} \leftrightarrow \mathcal{D}^{\text{out}}_{\text{out}}$ → *Fixes the observables in terms of a single parameter* $\kappa \equiv \omega_{|\mathbf{k}|}|\tau|\Big|_{|\mathbf{k}\tau|=1} = \sqrt{\frac{5-d^2}{4} + M^2}$ → $d = 3, M^2 = 0 \rightarrow \kappa = i$ or $\nu_{\text{cl}} = \frac{3}{2}$

Time-independent BT $\rightarrow P_S(\tau) = \left(\frac{H}{2\pi}\right)^2\left(1 + |\mathbf{k}\tau|\right) \Rightarrow P_S(0) = \left(\frac{H}{2\pi}\right)^2$, *a scale invariant spectrum*

P. R. Anderson, C. Molina-Paris and Emil Mottola, Phys. Rev. D 72 (2005) 043515
P. R. Anderson and E. Mottola, Phys. Rev. D 89 (2014) 104038

THE COSMOLOGICAL OBSERVABLES

- $\mathcal{D}^{\text{out}}_{\text{out}} \leftrightarrow \mathcal{D}^{\text{out}}_{\text{out},\beta}$ → *Time-dependent BT with* $\Omega_{|\mathbf{k}|} = \omega_{|\mathbf{k}|}(|c|^2 + |s|^2)$

B. Garbrecht, T. Prokopec and M. G. Schmidt, Eur. Phys. J. C38 (2004) 135-143

- The single parameter now $\kappa \to \Lambda$ (under defining $x = \dfrac{\pi H}{2\pi T}$, for $x \in [\pi, \infty]$ which eventually will be fixed to $x = \pi$, its natural dS value where $T = T_{\text{dS}}$) is

$$\Lambda = \kappa\left(1 + 2\frac{e^{-2x\kappa}}{1 - e^{-2x\kappa}}\right) = \kappa\coth(x\kappa)$$

- The spectral index of scalar curvature fluctuations, n_S, is shifted due to finite temperature effects → $P_{S,\beta} = P_S[1 + 2(s^2 + sc)]$

$$n_{S,\beta} = 1 + \frac{d\ln\left(|\mathbf{k}|^3 P_{S,\beta}\right)}{d\ln|\mathbf{k}|} \quad \longrightarrow \quad \delta n_S \equiv n_{S,\beta} - 1 = -\frac{2x}{\Lambda}\left[\frac{e^{-x\Lambda}}{1 - e^{-2x\Lambda}}\right]$$

- All the freedom is included in Λ which admits its natural value when $x = \pi$

THE COSMOLOGICAL OBSERVABLES

$$n^{(1)}_{S,\beta} = \frac{dn_{S,\beta}}{d\ln|\mathbf{k}|}\,, \qquad n^{(2)}_{S,\beta} = \frac{dn^{(1)}_{S,\beta}}{d\ln|\mathbf{k}|}$$

$$n^{(1)}_{S,\beta} = \delta n_S\left[2 - \frac{1}{\Lambda^2} - \frac{x}{\Lambda}\left(1 + \frac{2e^{-2x\Lambda}}{1-e^{-2x\Lambda}}\right)\right]$$

$$f_{NL} = -\frac{5\left[x(-1+\Lambda^2)^2\left(1 + x\Lambda\cot\left(\frac{x\Lambda}{2}\right)\right) + 2\Lambda^3\sinh(x\Lambda)\right]}{6\Lambda^2\left[x(-1+\Lambda^2) + \Lambda\sinh(x\Lambda)\right]}$$

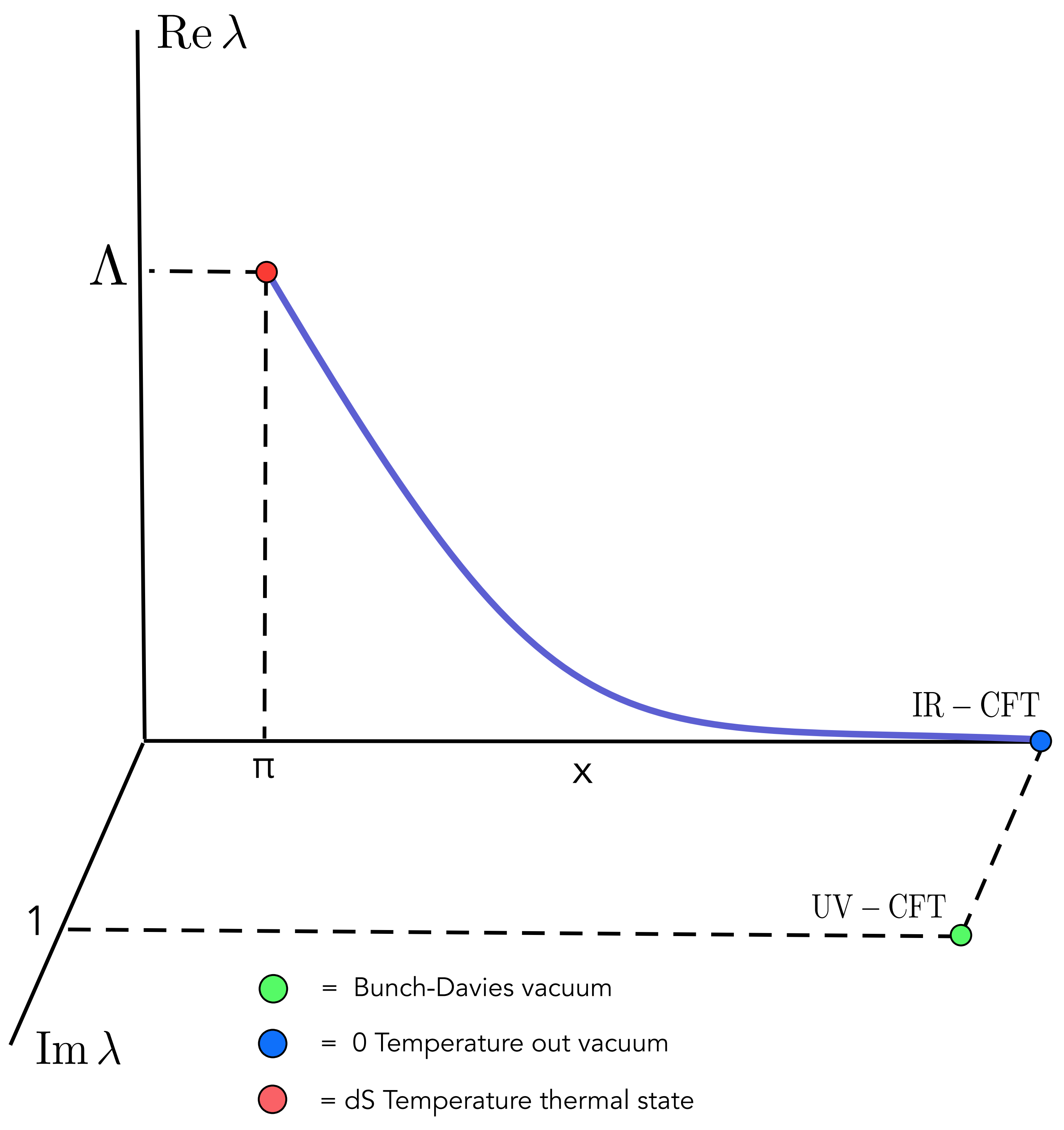
$$n^{(2)}_{S,\beta} = \frac{\left(n^{(1)}_{S,\beta}\right)^2}{\delta n_S} + \delta n_S\left[-\frac{2}{\Lambda^2} + \frac{2}{\Lambda^4} - \frac{x}{\Lambda}\left(2 - \frac{1}{\Lambda^2}\right)\left(1 + \frac{2e^{-2x\Lambda}}{1-e^{-2x\Lambda}}\right) + \frac{4x^2}{\Lambda^2}\frac{e^{-2x\Lambda}}{(1-e^{-2x\Lambda})^2}\right]$$

$$f_{NL} = \frac{5}{6}\frac{N_{\rho\rho}}{N_\rho^2} \qquad N_\rho = \frac{\partial N}{\partial \rho},\ N_{\rho\rho} = \frac{\partial^2 N}{\partial \rho^2} \text{ and } \rho \equiv P_{S,\beta}$$

$N = \int_{t_i}^{t_f} dt H$ the number of e-folds

P. Creminelli and M. Zaldarriaga, JCAP 10 (2004) 006
A. Kehagias and A. Riotto, Nucl. Phys. B868 (2013) 577-595

Λ	x
$\to 0$	$\to \infty$
10^{-6}	$3.5 \cdot 10^7$
0.01	1600
0.5	14.8
1.5117	π



- The physical case $x = \pi, \Lambda = 1.5117$

$n_{S,\beta} \equiv n_S \approx 1 - 0.036 = 0.964$ ✓
(0.9625 ± 0.0048)

$n^{(1)}_{S,\beta} \approx 0.0186$ ✓
(0.013 ± 0.012)

$f_{NL} \approx -1.7138$ ✓ ?

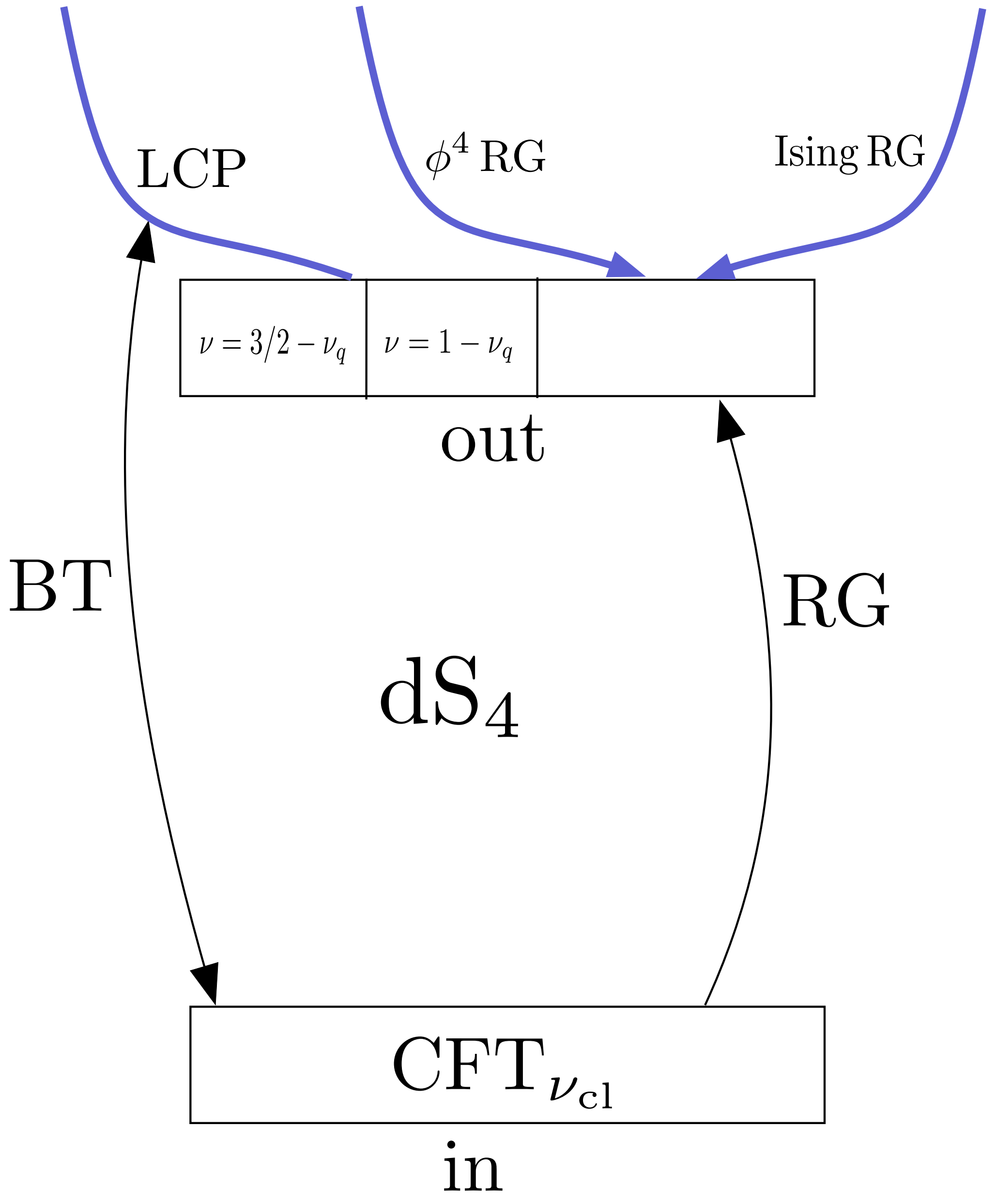
$n^{(2)}_{S,\beta} \approx 0.1250$ ★
(0.022 ± 0.012)

Planck Collaboration, Y. Akrami et al., Astron. Astrophys. 641 (2020)

WHY THIS TITLE?

- The IR CFT can be recognized within perturbation theory as the interacting fixed point of the d = 3 scalar theory with classical Lagrangian $\mathscr{L} = -\frac{1}{2}\sigma\Box\sigma - \lambda\sigma^4$ (σ is the Ising field)

M. Bianchi, D.Z. Freedman and K. Skenderis Nucl. Phys. B 631 (2002) 159
I. Antoniadis, P. O. Mazur and E. Mottola, JCAP 09 (2012) 024



The connection between the in and out vacua can be seen either as a BT or an RG flow from an UV CFT to an IR CFT. The IR limit is a 3d CFT as long as the BT preserves the $SO(4)$ isometry. The claim is that it has to be in the universality class of the interacting 3d Ising model.

WHY THIS TITLE?

- In the dS/CFT correspondence a bulk field ζ with dimension Δ_- is dual to an operator $\mathcal{O}$ of the boundary CFT of dimension Δ_+

$\Delta_+ = \frac{d}{2} + \nu$ $(\Delta_-, \Delta_+)_{\rm cl} = (0, 3)$ $\langle \zeta_{|\mathbf{k}|} \zeta_{-|\mathbf{k}|} \rangle \sim \frac{1}{\langle \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} \rangle}$

J. Maldacena, J. High Energy Phys. 05 (2003) 013
J. M. Maldacena and G. L. Pimentel, JHEP 09 (2011) 045

- Then the spectral index is

$$\begin{aligned} n_S - 1 &= \frac{d}{d \ln |\mathbf{k}|} \left[\ln \left(|\mathbf{k}|^3 P_{S,\beta} \right) \right] = \frac{d}{d \ln |\mathbf{k}|} \ln \left(|\mathbf{k}|^3 \langle \zeta_{|\mathbf{k}|} \zeta_{-|\mathbf{k}|} \rangle \right) \\ &= 3 - \frac{1}{\langle \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} \rangle} \left(\frac{d}{d \ln |\mathbf{k}|} \langle \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} \rangle \right) \end{aligned}$$

- Using the Callan-Symanzik

$$\left(\frac{\partial}{\partial \ln |\mathbf{k}|} - \beta_\lambda \frac{\partial}{\partial \lambda} + (3 - 2\Delta_{\mathcal{O}}) \right) \langle \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} \rangle = 0$$

F. Larsen and R. McNees, JHEP 07 (2003) 051
J. P. van der Schaar, JHEP 01 (2004) 070

$$n_S = 1 - 2\Gamma_{\mathcal{O}} - \beta_\lambda \frac{\partial}{\partial \lambda} \ln \langle \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} \rangle$$

WHY THIS TITLE?

- For us $\mathcal{O} = \Theta \equiv \mathrm{Tr}\, T_{ij}$ with the spectral index

$$n_S = 1 + \frac{\partial}{\partial \ln \mu} \ln\langle\Theta(x_1)\Theta(x_2)\rangle = 1 - \beta_\lambda \frac{\partial}{\partial\lambda} \ln\langle\Theta(x_1)\Theta(x_2)\rangle$$

- For $\Theta = \beta_\lambda \sigma^4$ the holographic interpretation of the running of λ imposes the eigenvalue equation

$$\beta_\lambda \frac{\partial}{\partial\lambda}\langle\Theta\Theta\rangle = \left(\beta_\lambda^2 + 2\frac{\partial\beta_\lambda}{\partial\lambda}\right)\langle\Theta\Theta\rangle$$

F. Larsen and R. McNees, JHEP 07 (2003) 051
J. P. van der Schaar, JHEP 01 (2004) 070

- Very close to the IR Wilson-Fisher fixed point $\beta_\lambda^2 << \frac{\partial\beta_\lambda}{\partial\lambda}$ and $2\frac{\partial\beta_\lambda}{\partial\lambda} \approx 2\gamma_\sigma \equiv \eta$

$$\left(\beta_\lambda \frac{\partial}{\partial\lambda} - \eta\right)\langle\Theta(x_1)\Theta(x_2)\rangle \simeq 0$$

- η is the critical exponent of the Ising field and non-perturbative admits the numerical value ≈ 0.036 (MC simulation). The spectral index is

$$n_S \simeq 1 - \eta \quad \longrightarrow \quad n_S \simeq 1 - 0.036 = 0.964$$

- So $\Lambda = 1.5117$ is indeed fixed independently (without connection to the inflationary characteristics)

CONCLUSIONS

- We considered a thermal scalar in de Sitter background. Starting from the Bunch-Davies $|\text{in}\rangle$ vacuum, a Bogolyubov Transformation placed us somewhere in the interior of the finite temperature phase diagram.

- Then we took the low temperature limit in such a way that instead of returning to the BD vacuum, we landed on the nearly zero temperature $|\text{out}\rangle$ vacuum, which is connected to an interacting IR CFT, in the universality class of the 3d Ising model.

- This interacting CFT is rather special, in the sense that the boundary operator that couples to the scalar curvature perturbations in the bulk has a classical scaling dimension. The critical exponent η is the order parameter of the breaking of the scale invariant spectrum of curvature fluctuations

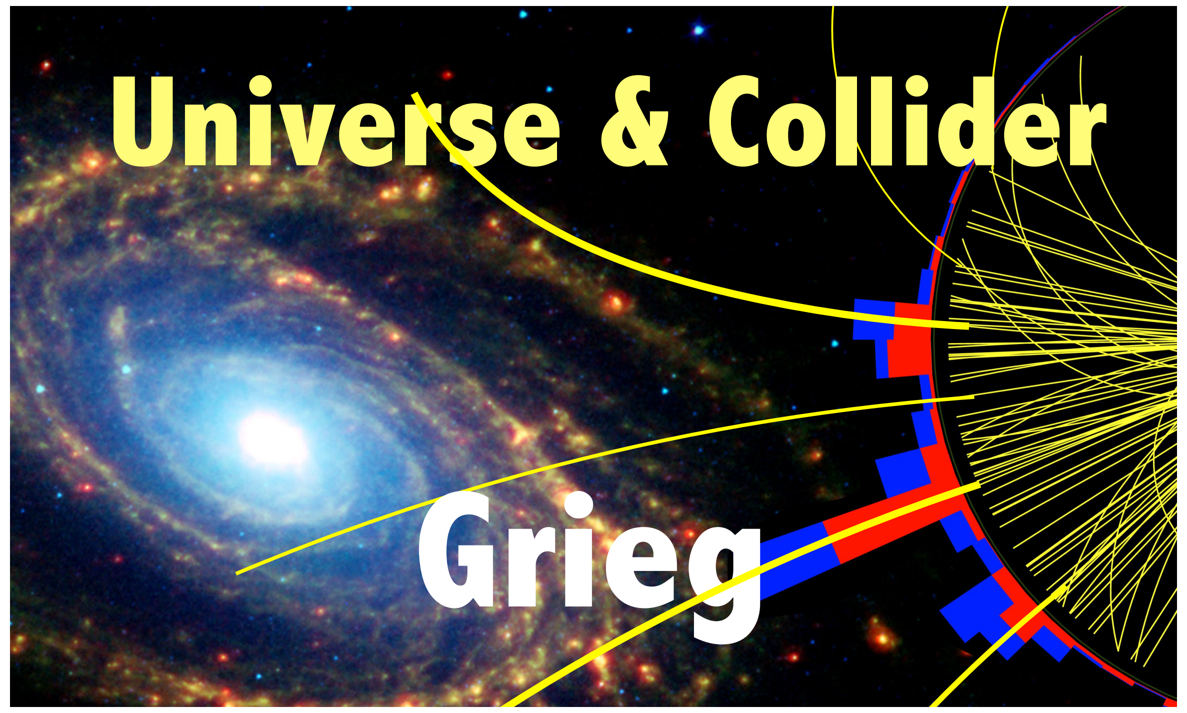
- η fixes the parametric freedom in the dS scalar theory, yielding the prediction $n_{S,\beta} \approx 0.964$, up to errors associated with its lattice Monte Carlo measurements.

- Heating up the system $T = T_{\text{dS}}$ numerically in a controlled way we evaluated additional cosmological observables $n^{(1)}_{S,\beta}$, f_{NL} and $n^{(2)}_{S,\beta}$. We finally note that our predicted values of $n_{S,\beta}$, $n^{(1)}_{S,\beta}$ and f_{NL} are well within current experimental bounds while $n^{(2)}_{S,\beta}$ exceeds them

THANK YOU

The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

Norway grants



Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen