



THE GRAVITATIONAL WAVE MEMORY EFFECT

A New Era Of Astronomy Begins

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General Theory of Relativity

General Theory of Relativity

A small overview of General Theory of Relativity



The curvature of spacetime is directly related to the energy and momentum of whatever matter and radiation are present. The relation is specified by the Einstein field equations, a system of second order partial differential equations.

In 1916 Einstein published a relativistic theory of gravitation in Annalen where he derived a field equation for gravity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu}$$

General relativity is conceptually different from Newton's theory as it introduces the notion of spacetime and its geometry.

- ▶ One of the basic differences of the two theories concerns the speed of propagation of any change in a gravitational field
- ▶ the information of the differing gravitational field propagates with finite speed, the speed of light, as a ripple in the fabric of spacetime. These are the "*gravitational waves*".

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GRAVITATIONAL WAVES

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The gravitational waves were detected on September 14, 2015 at 5:51 a.m. Eastern Daylight Time (09:51 UTC) by both of the twin Laser Interferometer Gravitational-wave Observatory (LIGO) detectors, located in Livingston, Louisiana, and Hanford, Washington, USA.

Gravitational waves (GWs) are “ripples” in space-time which are produced by changes associated with massive gravitating objects.

- ▶ Gravitational waves are created when there is a non-spherical acceleration of mass energy distributions
- ▶ We know Electromagnetic radiation is dipole radiation while gravitational waves are Quadrupole radiation.

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WEAK FIELD LIMIT:

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People typically use the following approximation for calculating stuff about gravitational waves

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

here $h_{\mu\nu}$ tensor describes the variations induced in the spacetime metric and $\eta_{\mu\nu}$ is the space-time metric. In the Transverse-Traceless gauge $h_{\mu\nu}^{TT}$ the constraints of a plain monochromatic wave are : •

$$h_{xx}^{TT} = -h_{yy}^{TT} = \mathbf{R}(A_+ e^{-i\omega(t-z)})$$

$$h_{xy}^{TT} = h_{yx}^{TT} = \mathbf{R}(A_x e^{-i\omega(t-z)})$$

where A_+ and A_x represents plus and cross polarizations

Polarization of GWs are 45 degrees to each other.

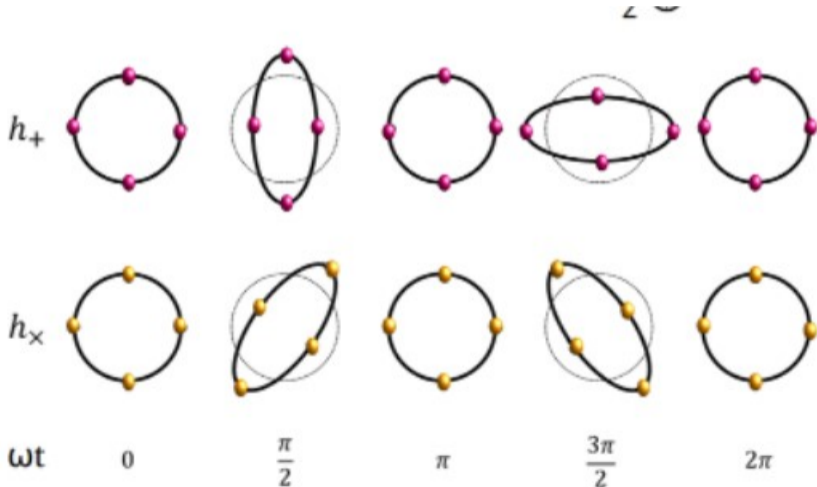
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PLUS AND CROSS POLARIZATION

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There are two types of states of linear polarisation of GW they are CROSS(x) and PLUS(+)
polarisation.



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Generation of gravitational waves:

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Here we know using the compact-source approximation

$$\bar{h}^{00} = -\frac{4GM}{c^2 r}$$

The remaining (spatial) components of the gravitational field are the integrated stress within the source, which may be written in terms of the quadrupole formula

$$\bar{h}^{ij} = -\frac{2GM}{c^6 r} \left[\frac{d^2 I^{ij}(ct')}{dt'^2} \right]_r$$

The quadrupole moment tensor of the source is $I^{ij}(ct) = c^2 \int T^{00}(ct, \vec{y}) y^i y^j d^3 y$ For slowly moving source particles we have $T_{00} = \rho c^2$

So, the gravitational wave produced by an isolated non-relativistic source is proportional to the second derivative of the quadrupole moment of the matter density distribution.

$$I^{ij}(ct) = c^2 \int \rho(ct, \vec{x}) x^i x^j d^3 x$$

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Post Newtonian Wave Generation Formalism:

Post Newtonian Wave Generation Formalism:



The asymptotic waveform h_{ij}^{TT} can be decomposed into two sets of symmetric trace-free (STF) radiative multipole moments as It is then straightforward to show that:

Now

$$h_{ij}^{TT} = \frac{4G}{c^2 R} \Pi_{ijmn} \sum_{\ell=2}^{\infty} \left\{ \frac{1}{c^{\ell} \ell!} \mathcal{U}_{mnL-2}(T_R) N_{L-2} \right. \\ \left. + \frac{2\ell}{c^{\ell+1} (\ell+1)!} \epsilon_{pq(m} \mathcal{V}_{n)pL-2}(T_R) N_{qL-2} \right\} \quad (1)$$

$$h_{+} - ih_{\times} = m_m^{*} m_n^{*} h_{mn}^{TT}$$

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Post Newtonian Wave Generation Formalism:

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where $\bar{}$ denotes complex conjugation. It will now be shown how

$$h_+ - ih_\times$$

can be decomposed into modes using spin-weighted spherical harmonics of weight -2

$$h_+ - ih_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h^{\ell m}_{-2} Y^{\ell m}(\Theta, \Phi) \quad (2)$$

This equation (2) has been used to study the waveform from a hyperbolic binary as an example of linear memory. where $_{-2}Y^{\ell m}$ is spin-weighted spherical harmonic and $h^{\ell m}$ is the spin weighted spherical harmonic component which is given by

$$h^{\ell m} = \frac{G}{\sqrt{2}Rc^{\ell+2}} \left(U^{\ell m}(T_R) - \frac{i}{c} V^{\ell m}(T_R) \right)$$



Spherical Harmonics:

Spherical Harmonics:



We know there are three kinds OF Spherical Harmonics: Scalar Spherical Harmonics, Vector Spherical Harmonics, Tensor Spherical Harmonics. Here we have used Scalar Spherical Harmonic for further calculations.

For the calculation of memory effect we require two kinds of Spherical Harmonics those are: Scalar Spherical Harmonics, Spin-Weighted spherical harmonic.

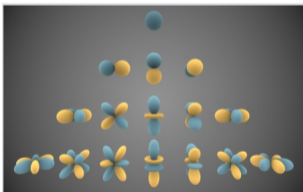
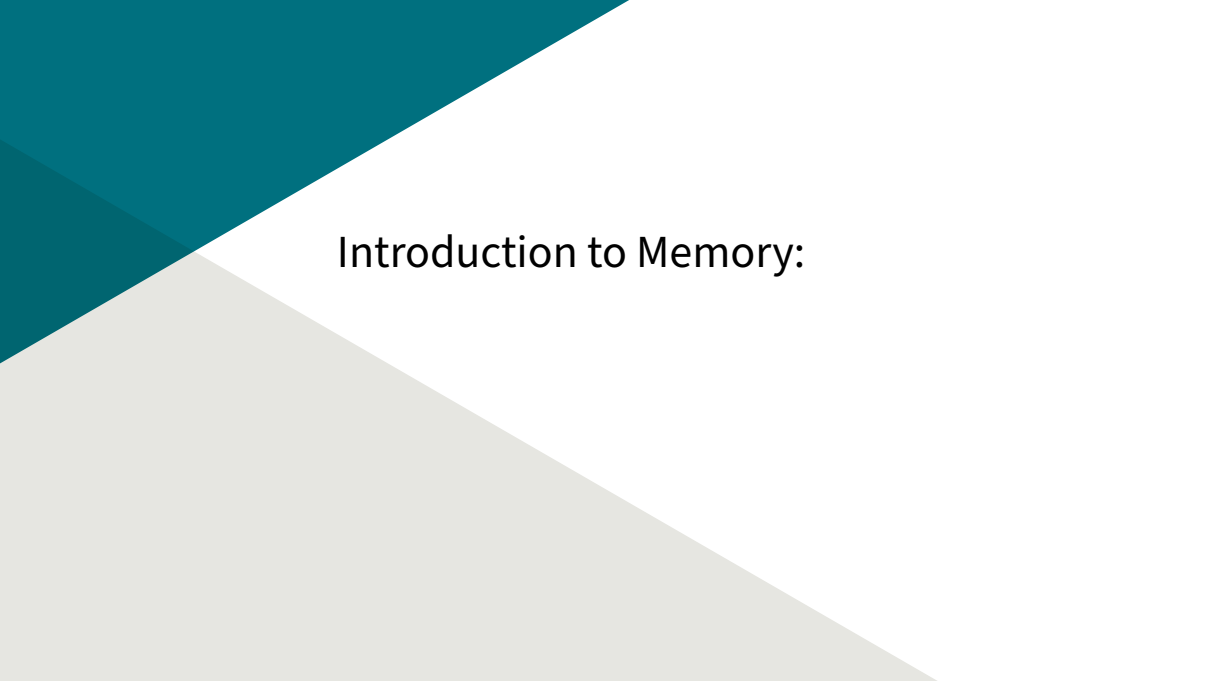


Figure 1: Visual representations of the first few real spherical harmonics

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Introduction to Memory:

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General Relativity predicts that gravitational waves will have an oscillatory component as well as a memory component. The memory and oscillatory components are polarised in the plus and cross direction.

- ▶ These polarizations are related by a 45 degree rotation comparison to the 90 degree rotation for electromagnetic radiation.
- ▶ The polarizations have an oscillatory and non-oscillatory component.
- ▶ For example, in the case of GWS produced during inspiral of binary stars, there is a non-oscillatory component to the "+" polarization which makes the amplitude of the gravitational wave end with a non-zero value. The "non-zero" amplitude represents the "gravitational wave memory", a weak stretching that permanently alters the spacetime fabric

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Gravitational Wave Memory Effect:

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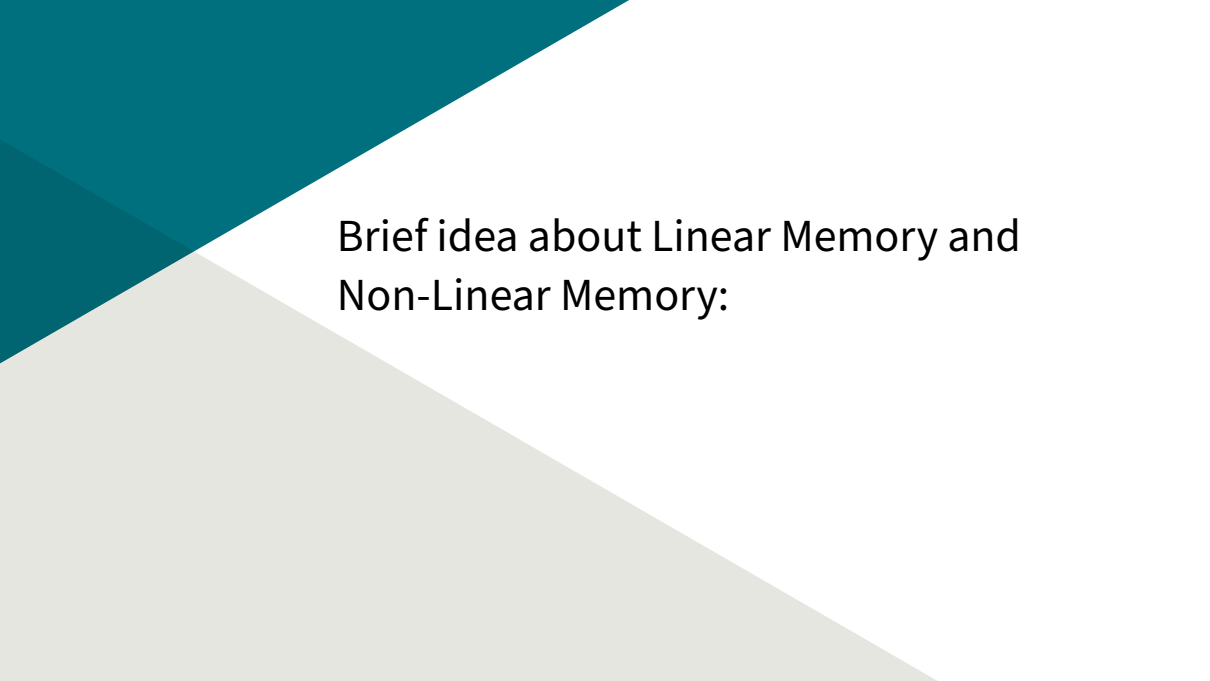


GW memory is a result of a non-oscillatory component to one of the polarisation states "+" and "x" of a GW.

- ▶ GW signal displays a “memory” that causes the signal amplitude to decay to a nonzero value
- ▶ The GW signal from a ‘source with memory’ has the property that the late-time and early-time values of at least one of the GW polarizations differ from zero, corresponding to an observed difference in the metric perturbation

$$\Delta h_{+, \times}^{\text{mem}} = \lim_{t \rightarrow +\infty} h_{+, \times}(t) - \lim_{t \rightarrow -\infty} h_{+, \times}(t)$$

here $h_{+, \times}(t)$ are the plus and cross polarization.

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Brief idea about Linear Memory and
Non-Linear Memory:

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In the 1970's Linear Memory was discovered.

- ▶ Arises from the non-oscillatory motion of a source, especially due to unbound masses.
- ▶ It arises from the non-zero frequency changes in the time-derivatives of the source's multipole moment.
- ▶ Ex: hyperbolic orbits, mass/neutrino ejection in supernovas/GRBs .
- ▶ Linear and Non-Linear memory depend on the form of General Theory field equations, a set of ten coupled non-linear differential equations that describe gravity as a result of spacetime being curved by mass and energy

We here tried to formulate the relationship between **multiple moments and metric perturbation** through various examples and as an example of linear memory we shall study the **memory effects on hyperbolic binaries and "N" gravitationally unbounded masses**. Now we shall use this formulae to study further about **memory of neutrino detection** .

Brief idea about Linear Memory and Non-Linear Memory:

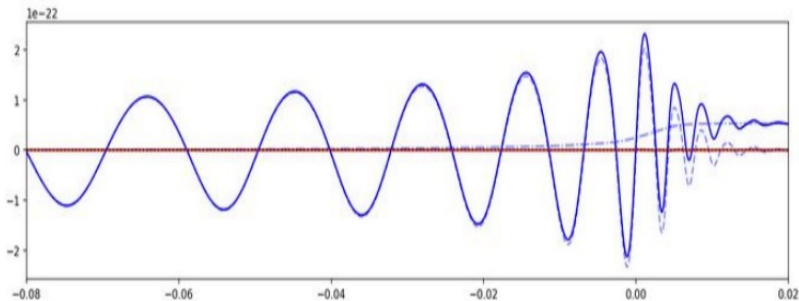
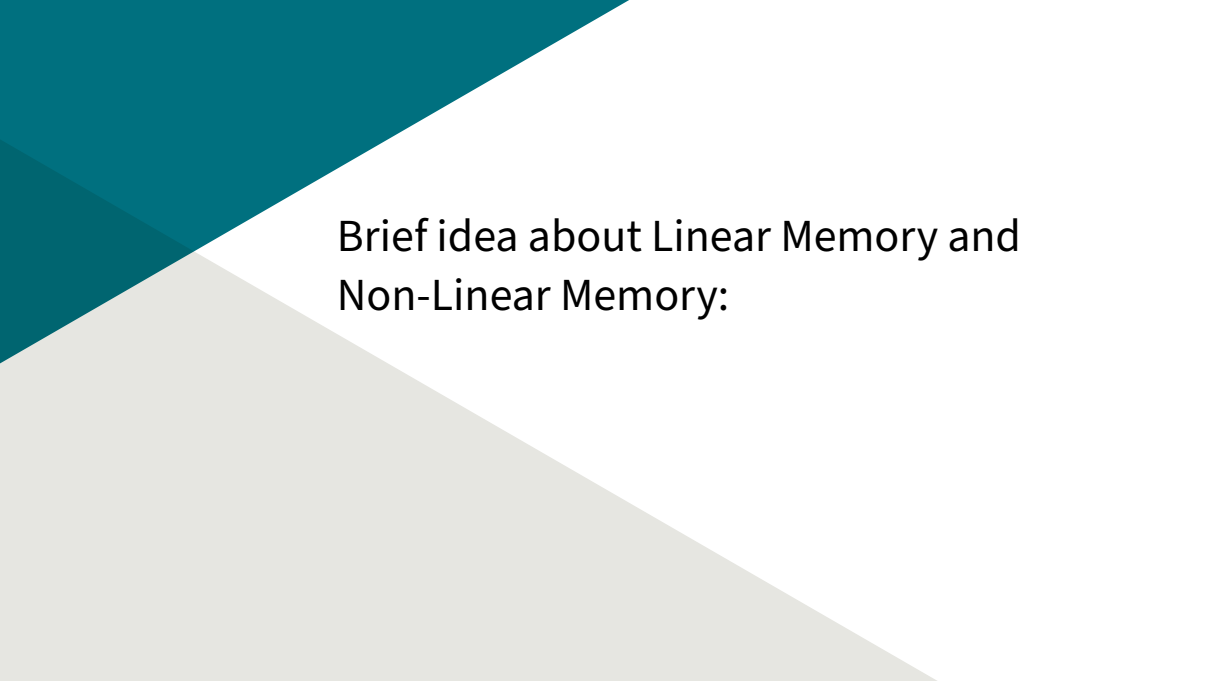


Figure 2: Waveform showing oscillatory binary black hole merger waveform with memory (solid blue line) and without memory (–line).

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Brief idea about Linear Memory and
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Non-Linear Memory: In the 1990's a nonlinear form of memory was discovered independently by Blanchet Damour and Christodoulou:

- ▶ Due to the contribution of the emitted GW's to the changing quadrupole and higher mass moment the non-linear memory arises from it.
- ▶ As discussed in [5] the non linear memory or Christodoulou's effect comes from a linear memory in which the unbound masses are the individual radiated gravitons. This implies that nearly all GW sources are sources with memory
- ▶ All GW's sources are sources with memory
- ▶ We would study The Non-linear memory effect considering "Unbound Particles" as "individual gravitons"[5]

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Example of Linear Memory:

Example of Linear Memory:



As an example of linear memory we need to study the waveform from a hyperbolic binary
The leading order multipolar contribution to the polarization is as shown in [2]:

$$h_+ - ih_x = \sum_{m=-2}^{m=2} \frac{I_{2m}^{(2)}}{R\sqrt{2}} {}_{-2}Y^{\ell m}(\Theta, \Phi)$$

We have obtained the waveform The waveforms are written as;

$$I_{20}^{(2)} = -8\sqrt{\frac{\pi}{15}} \frac{\eta M^2}{p} e_0 (e_0 + \cos \nu) \quad (3)$$

The detailed calculation of $I_{20}^{(2)}$ can be found in appendix I of our dissertation .

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Now another waveform obtained from ;

$$I_{2\pm 2}^{(2)} = \left(-4\sqrt{\frac{2\pi}{5}}\eta\frac{M^2}{p}e^{\pm 2i\phi(t)}\right)\left(\left((-e_0^2 + 1 + (1 + e_0\cos v)(1 + 2e_0e^{\mp i\nu}))\right)\right)$$

The detailed calculation of $I_{2-2}^{(2)}$ can be found in appendix II of our dissertation.

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LINEAR MEMORY FOR "N"
GRAVITATIONALLY UNBOUND PARTICLES :

LINEAR MEMORY FOR "N" GRAVITATIONALLY UNBOUND PARTICLES :



Green's Function of the Wave equation:

Our starting point is the linearised Einstein equation, re-written using the D'Alembertian operator

$$\square h_{\mu\nu} = -16\pi T_{\mu\nu} \quad (4)$$

which assumes that both the source, in the guise of the energy momentum tensor $T_{\mu\nu}$ and the perturbed metric $h_{\mu\nu}$ are small. This is simply a bunch of decoupled wave equations. The general solution to (4), the linearised field equations in Lorentz gauge, outside of Σ can be given using the (retarded) Green's function; it is

$$\bar{h}_{\mu\nu}(t, \vec{x}) = \int_{\Sigma} \frac{4T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' \quad (5)$$

LINEAR MEMORY FOR "N" GRAVITATIONALLY UNBOUND PARTICLES :



where $|x - x'| = r$ and taking the Taylor expansion about the source this perturbation metric is in terms of $t_{ret} = t - |x - x'|$

$$\bar{h}_{\mu\nu}(t, \vec{x}) = \int_{\Sigma} \frac{4T_{\mu\nu}(t_{rel}, x')}{r} d^3x' \quad (6)$$

We have arrived in this equation:

$$\Delta h_{jk}^{TT} = \Delta \sum_{A=1}^N \frac{4M_A}{r \sqrt{1 - v_A^2}} \left[\frac{v_A^j v_A^k}{1 - v_A \cdot N} \right]^{TT}$$

Here N points from the source to the observer and Δ is the difference between late and early time values [2] Detailed calculation of derivation of the above equation is shown in our dissertation.



Memory detection of supernovae:

Memory detection of supernovae:



Now we have used (4) to calculate the memory of a individual radiated neutrinos. We can solve the wave equation by using the retarded Green's function corresponding to the D'Alembert operator in four-space time dimensions,

$$h_{\mu\nu} = 4G \int \frac{S_{\mu\nu}(\vec{x}', t - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|} d^3\vec{x}' \quad (8)$$

where where the effective stress-energy tensor (in presence of matter) is

$S_{\mu\nu} = T_{\mu\nu} - \frac{\eta_{\mu\nu} T_{\lambda}^{\lambda}}{2}$ We have arrived in this equation :

$$h_{TT}^{xx} = h(r, t) = \frac{2G}{rc^4} \int_{-\infty}^{t-r/c} dt' L_{\nu}(t') \alpha(t') \quad (9)$$

where c is the speed of light, t is the time post bounce and G is the universal gravitational constant. L_{ν} is the all-flavors neutrino luminosity α is the time-varying anisotropy parameter as mentioned in [4]

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Summary:

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- ▶ The post Newtonian approximation is a slow-motion, approximation to general relativity. For quasi-circular compact binaries- the non linear memory has a large contribution to the time-domain waveform amplitude.
- ▶ As an example of linear memory we have studied the waveform from a hyperbolic binary.
- ▶ As an application of linear memory we have studied linear memory for "N" Gravitationally Unbound Particles.

There is also great potential that LISA (to be launched in 2030) will be able to greatly enhance the detectability of gravitational wave memory due to its capability to measure much lower frequencies which will open more pathways for Gravitational Astronomy.

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[5] [2] [4] [3] [1]