GRAVITATIONAL ENTROPY, BLACK HOLES AND COSMOLOGY

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Introduction

- Entropy is the degree of disorder of a system.
- Cosmologically, the early stages of the universe had a lower entropy state than late cosmological times.
- At times t~0, the universe was homogeneous and isotropic, i.e. was a perfect FLRW model.
- As time progresses, structure formations increase -- increasing entropy.
- Penrose conjectured that the increasing entropy was linked to the Weyl curvature.

Introduction

- The Weyl tensor measures tidal distortions -- gravitational effects on matter.

- Clumping matter increases anisotropies -- increasing Weyl curvature.

- At the initial singularity, the Weyl curvature is zero -- perfect FLRW universe=conformally flat spacetime.

- Define some sort of gravitational entropy -- must follow some conditions to be consistent with thermodynamic entropy.

Gravitational Entropy

- Gravitational entropy conditions:
- 1. Must vanish only for conformally flat spacetimes (zero Weyl curvature),
- 2. Must account for structure formations -- increasing anisotropies,
- 3. Must reduce to Hawking-Bekenstein relation: $S=A/4$.
- FLRW model -- Petrov type O spacetime: zero Weyl curvature.
- Therefore gravitational entropy in FLRW spacetime is zero.

Gravitational Entropy

- For other spacetimes, Penrose's hypothesis can be stated as:
- "*For cosmologies with matter, Weyl curvature monotonically increases, and the corresponding gravitational entropy also strictly increases.*"
- This is simply a form of gravitational analog of the second law of thermodynamics:
- Thermodynamic second law: entropy must always increase.
- Gravitational second law of thermodynamics: gravitational entropy must always increase -- i.e. Weyl curvature must always increase with time.

Gravitational Entropy

- The initial focus of Penrose's hypothesis was not purely cosmological -- how is gravitational entropy mathematically described at all?
- Clifton, Ellis and Tavakol (2013) showed that the gravitational entropy could be written in terms of gravitational analogs to thermodynamic parameters -- gravitational pressure, temperature, internal energy and entropy.
- Rudjord and Gron (2006) showed that the Weyl invariant could be used to describe the entropy via a surface integral over the event horizon of the black hole.

Gravitational Entropy

- The Weyl invariant based proposal focuses on the equivalence between the Hawking-Bekenstein entropy and the gravitational entropy for black holes.

- The proposal then describes gravitational entropy using a surface integral over the surface of the horizon in terms of a scalar in terms of the Weyl invariant.

- Further extensions involve Ricci tensor based and Riemann tensor based invariants -- extension from Ricci invariant due to isotropic singularities.

Clifton, Ellis And Tavakol Proposal

- Background: Weyl curvature is non-zero -- increases monotonically with time.
- Newmann-Penrose formalism -- form a tetrad, gravitational thermodynamic variables: and finally gravitational entropy.
- Gravitational entropy would be $S_g = \int_V \frac{\rho_g}{T_g}\, dV$
- Consider an FLRW cosmology.

Clifton, Ellis And Tavakol Proposal

- But first.... Petrov classification?
- Based on the Weyl scalars, the Petrov classification is:

$\Psi_0 = 0$ – Petrov type I

$\Psi_0 = \Psi_1 = 0$ – Petrov type II

$\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0$ – Petrov type D

$\Psi_0 = \Psi_1 = \Psi_2 = 0$ – Petrov type III

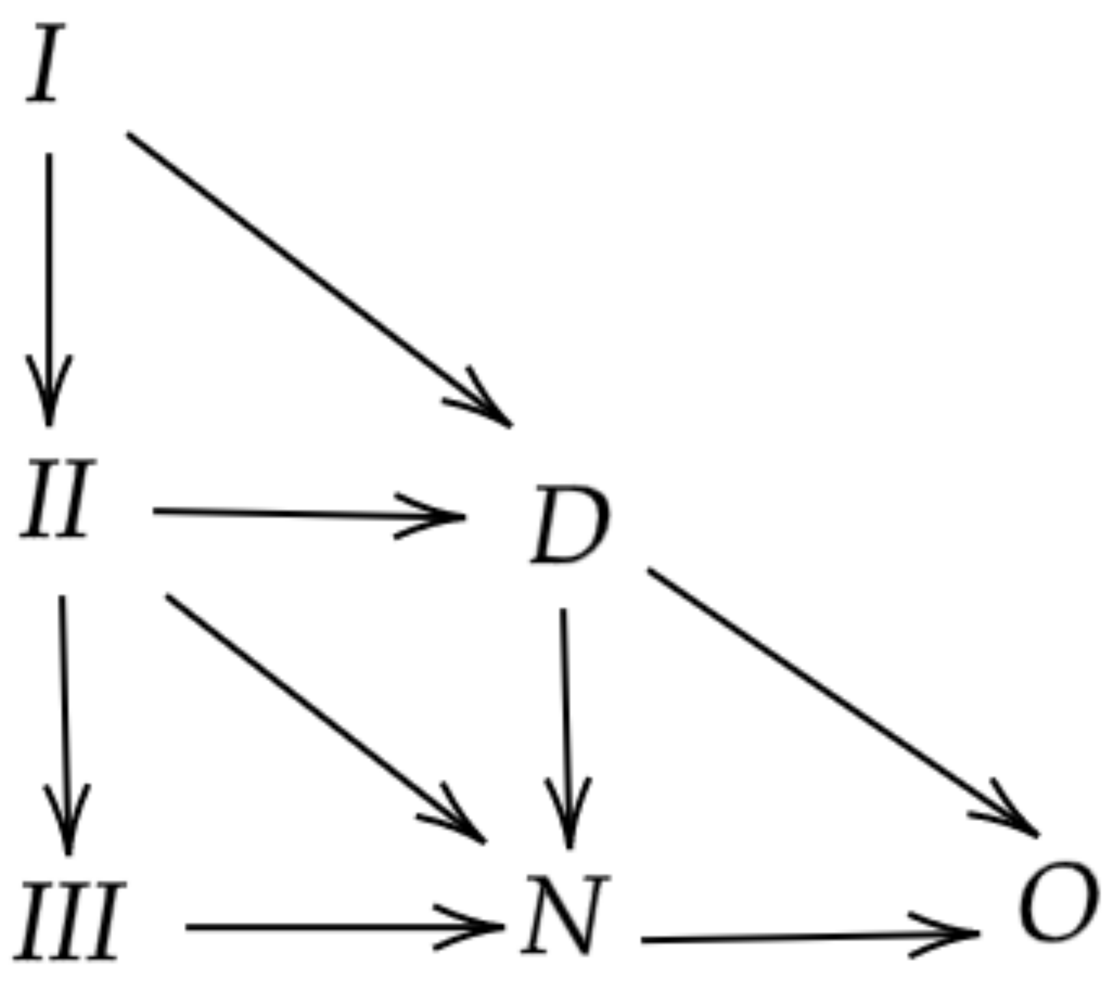
$\Psi_0 = \Psi_1 = \Psi_2 = \Psi_3 = 0$ – Petrov type N

$\Psi_0 = \Psi_1 = \Psi_2 = \Psi_3 = \Psi_4 = 0$ – Petrov type O

- Classification and subclasses:

Clifton, Ellis And Tavakol Proposal

- Arrows represent classes:



- CET is based on Petrov type D and N -- both classes under type O.
- So.... FLRW cosmologies?

Clifton, Ellis And Tavakol Proposal

- FLRW cosmologies are Petrov type O -- all Weyl scalars are zero..

$$\rho_g = \frac{\alpha}{4\pi}|\Psi_2|$$

- Therefore, gravitational energy density also vanishes -- gravitational temperature non-zero.

- .... And so gravitational entropy is zero from CET.. Not very surprising, since Weyl tensor is zero in FLRW anyway!

Clifton, Ellis And Tavakol Proposal

- Summary of the CET approach for Petrov type D:
- Find a tetrad satisfying $\rho_g = \frac{\alpha}{4\pi}|\Psi_2|$
- Next, find gravitational energy density and temperature.
- Gravitational entropy variation can be found out.
- But.... black holes?

Weyl Invariant Based Proposal

- Bekenstein showed that the entropy of black holes must be related to the information encoded on the surface of the event horizon.
- Hawking and Bekenstein showed that the entropy of black holes and the area of the event horizon were related by a factor of 1/4 (in natural units).
- For cosmological horizons, the entropy is of a form similar to the Hawking-Bekenstein relation -- the radius would be of a cosmological horizon rather than an event horizon (Gibbons-Hawking).
- This is what challenges the Weyl invariant based proposal in the case of de Sitter spacetime!!

Weyl Invariant Based Proposal

- The Rudjord and Gron method uses the equivalence between gravitational entropy and the Hawking-Bekenstein entropy: $S_g = S_{HB}$

- Define gravitational entropy via a surface integral -- scalar made of Weyl invariant forms.

$$S_g = k_s \int_\sigma \Psi \mathbf{e}_r \, d\sigma$$

- Simplest case -- the scalar is the square root of the Weyl invariant..

- But.. This form blows up near isotropic singularities (Wainwright and Anderson, 1984) -- introduce factors....

Weyl Invariant Based Proposal

- Alternative form -- introduce a factor via the Ricci invariant.
- Better form -- introduce a factor using the Kretschmann invariant.
- Did Penrose suggest the Weyl tensor is a measure of the gravitational entropy?
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Weyl Invariant Based Proposal

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Weyl Invariant Based Proposal

- Alternative form -- introduce a factor via the Ricci invariant.
- Better form -- introduce a factor using the Kretschmann invariant.
- Did Penrose suggest the Weyl tensor is a measure of the gravitational entropy?
- No!!
- We only want the Weyl curvature to be strictly increasing from zero.

Weyl Invariant Based Proposal

- Use the divergence theorem -- transform the surface integral to a volume integral.
- This defines the entropy density (be sure to use absolute brackets).
- The factor in the integral can be found out by requiring $S_g = S_{HB}$
- Example: The Schwarzschild solution

Weyl Invariant Based Proposal

- We are considering the scalar $\Psi = \sqrt{\dfrac{C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}}{R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}}$
- In the Schwarzschild solution, both are equal -- the scalar is equal to 1.
- Therefore, we expect the gravitational entropy of Schwarzschild black holes to be maximal.
- Wait.... singularity causes divergence of the integral!
- How do we let the integral be defined if there is a divergence?

Weyl Invariant Based Proposal

- Simple: consider a small sphere of some radius around the singularity.
- Next, define the integral for the Schwarzschild radius and the *ad hoc* sphere radius.
- Then, let the "sample" sphere radius tend to zero -- does the gravitational entropy work?
- Using this process, the usual Hawking-Bekenstein entropy is derived -- exactly what we wanted!!

Weyl Invariant Based Proposal

- Does CET obtain this result?
-

Weyl Invariant Based Proposal

- Does CET obtain this result?
- Yes!!

Weyl Invariant Based Proposal

- Step one: find the tetrad.
- Step two: find the gravitational energy density and the gravitational temperature.
- Step three: gravitational entropy is the integral via the above parameters.
- Step four: find the gravitational entropy density.
- CET gives us the Hawking-Bekenstein entropy too!

CET Vs Weyl Invariant Proposal

- Which is better?
- CET has a limitation -- only works when considering Petrov type D or N.
- Works only in the general relativistic case via the Bel-Robinson tensor.
- But.. The Weyl invariant proposal does not describe the gravitational entropy of de Sitter spacetimes..
- Both need to account for extremal Reissner-Nordstrom black holes.

CET Vs Weyl Invariant Proposal

- Also, modified theories of gravity = modifications to the Hawking-Bekenstein entropy.
- What about cosmologies?
- Again, CET -- Petrov type D or N.
- But many good cosmologies -- LRS Bianchi type I spacetime is an example.
- Astrophysical wormholes -- described by the Weyl invariant proposal well.

CET Vs Weyl Invariant Proposal

- CET also works quite well for higher order black hole solutions -- same formalism, same approach using the Weyl scalar for non-zero forms (Petrov type D).
- Both proposals work quite well for solutions that include charge.
- Rotating charged black holes, accelerating black holes, etc.
- But what can we infer from the entire landscape of gravitational entropy?

Further....

- The gravitational entropy proposal develops the variation of Weyl curvature as some sort of "arrow of time".
- Further aspects of this also includes conformal cyclic cosmologies, *also* introduced by Penrose.
- But what would we achieve by describing gravitational entropy?

Further....

- The description of gravitational entropy would allow us to describe the evolution of cosmologies under the Weyl curvature hypothesis.

- We would be able to map the link between different parameters and the nature of gravitational entropy of such solutions -- for instance, LRS Bianchi type I spacetimes have two scale factors for the entire spatial metric components.

- In this spacetime, the nature of different scale factors affects the parameters affecting the variation of entropy.

Further....

- Description of wormholes under such proposals would allow us to understand geometric properties of solutions containing exotic matter.
- Also allow us to understand differential geometric aspects of cosmologies that are quasi-regular (O. Stoica, 2012).
- Both proposals need to account for the additional forms of modifications under modified gravity.

Conclusion

- The requirements for gravitational entropy can be summarized as:
- Vanishes only if the Weyl tensor is zero, i.e. iff the spacetime is conformally flat,
- Accounts for structure formations -- anisotropies,
- Reduces to the Hawking-Bekenstein relation for black holes,
- Is strictly increasing in late times of the universe.

Conclusion

- The CET proposal adopts the Newmann-Penrose formalism to describe gravitational entropy.
- The Weyl invariant based proposal adopts the equivalence of the gravitational entropy with the Hawking-Bekenstein relation to describe entropy via Weyl curvature invariant forms with Riemann tensor factors.
- Each of these proposals work consistently in most cases, and further research on understanding models where these proposals work is being done.

Thank you for your attention!