GRAVITATIONAL ENTROPY, BLACK HOLES AND COSMOLOGY

Gravitational Physics And Cosmology Conference, 2022

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Introduction

- Entropy is the degree of disorder of a system.

- Cosmologically, the early stages of the universe had a lower entropy state than late cosmological times.

- At times $t \approx 0$, the universe was homogeneous and isotropic, i.e. was a perfect FLRW model.

- As time progresses, structure formations increase -- increasing entropy.

- Penrose conjectured that the increasing entropy was linked to the Weyl curvature.
Introduction

• The Weyl tensor measures tidal distortions -- gravitational effects on matter.

• Clumping matter increases anisotropies -- increasing Weyl curvature.

• At the initial singularity, the Weyl curvature is zero -- perfect FLRW universe=conformally flat spacetime.

• Define some sort of gravitational entropy -- must follow some conditions to be consistent with thermodynamic entropy.
Gravitational Entropy

- Gravitational entropy conditions:
  - 1. Must vanish only for conformally flat spacetimes (zero Weyl curvature),
  - 2. Must account for structure formations -- increasing anisotropies,
- FLRW model -- Petrov type O spacetime: zero Weyl curvature.
- Therefore gravitational entropy in FLRW spacetime is zero.
Gravitational Entropy

• For other spacetimes, Penrose's hypothesis can be stated as:

  "For cosmologies with matter, Weyl curvature monotonically increases, and the corresponding gravitational entropy also strictly increases."

• This is simply a form of gravitational analog of the second law of thermodynamics:

  • Thermodynamic second law: entropy must always increase.
  
  • Gravitational second law of thermodynamics: gravitational entropy must always increase -- i.e. Weyl curvature must always increase with time.
Gravitational Entropy

- The initial focus of Penrose's hypothesis was not purely cosmological -- how is gravitational entropy mathematically described at all?

- Clifton, Ellis and Tavakol (2013) showed that the gravitational entropy could be written in terms of gravitational analogs to thermodynamic parameters -- gravitational pressure, temperature, internal energy and entropy.

- Rudjord and Gron (2006) showed that the Weyl invariant could be used to describe the entropy via a surface integral over the event horizon of the black hole.
Gravitational Entropy

- The Weyl invariant based proposal focuses on the equivalence between the Hawking-Bekenstein entropy and the gravitational entropy for black holes.

- The proposal then describes gravitational entropy using a surface integral over the surface of the horizon in terms of a scalar in terms of the Weyl invariant.

- Further extensions involve Ricci tensor based and Riemann tensor based invariants -- extension from Ricci invariant due to isotropic singularities.
Clifton, Ellis And Tavakol Proposal

• Background: Weyl curvature is non-zero -- increases monotonically with time.

• Newmann-Penrose formalism -- form a tetrad, gravitational thermodynamic variables: and finally gravitational entropy.

• Gravitational entropy would be \( S_g = \int_V \frac{\rho_g}{T_g} \, dV \)

• Consider an FLRW cosmology.
Clifton, Ellis And Tavakol Proposal

• But first.... Petrov classification?

• Based on the Weyl scalars, the Petrov classification is:

\[
\begin{align*}
\Psi_0 &= 0 \quad \text{Petrov type I} \\
\Psi_0 &= \Psi_1 = 0 \quad \text{Petrov type II} \\
\Psi_0 &= \Psi_1 = \Psi_3 = \Psi_4 = 0 \quad \text{Petrov type D} \\
\Psi_0 &= \Psi_1 = \Psi_2 = 0 \quad \text{Petrov type III} \\
\Psi_0 &= \Psi_1 = \Psi_2 = \Psi_3 = 0 \quad \text{Petrov type N} \\
\Psi_0 &= \Psi_1 = \Psi_2 = \Psi_3 = \Psi_4 = 0 \quad \text{Petrov type O}
\end{align*}
\]

• Classification and subclasses:
Clifton, Ellis And Tavakol Proposal

- Arrows represent classes:

```
  I  
  ↓  
  II → D
  ↓  
  III → N → O
```

- CET is based on Petrov type D and N -- both classes under type O.

- So.... FLRW cosmologies?
Clifton, Ellis And Tavakol Proposal

- FLRW cosmologies are Petrov type O -- all Weyl scalars are zero.

\[ \rho_g = \frac{\alpha}{4\pi} |\Psi_2| \]

- Therefore, gravitational energy density also vanishes -- gravitational temperature non-zero.

- .... And so gravitational entropy is zero from CET. Not very surprising, since Weyl tensor is zero in FLRW anyway!
Clifton, Ellis And Tavakol Proposal

• Summary of the CET approach for Petrov type D:

• Find a tetrad satisfying \( \rho_g = \frac{\alpha}{4\pi} |\Psi_2| \)

• Next, find gravitational energy density and temperature.

• Gravitational entropy variation can be found out.

• But.... black holes?
Weyl Invariant Based Proposal

• Bekenstein showed that the entropy of black holes must be related to the information encoded on the surface of the event horizon.

• Hawking and Bekenstein showed that the entropy of black holes and the area of the event horizon were related by a factor of 1/4 (in natural units).

• For cosmological horizons, the entropy is of a form similar to the Hawking-Bekenstein relation -- the radius would be of a cosmological horizon rather than an event horizon (Gibbons-Hawking).

• This is what challenges the Weyl invariant based proposal in the case of de Sitter spacetime!!
Weyl Invariant Based Proposal

• The Rudjord and Gron method uses the equivalence between gravitational entropy and the Hawking-Bekenstein entropy: \( S_g = S_{HB} \)

• Define gravitational entropy via a surface integral -- scalar made of Weyl invariant forms.

\[
S_g = k_s \int_{\sigma} \Psi e_r \, d\sigma
\]

• Simplest case -- the scalar is the square root of the Weyl invariant..

• But.. This form blows up near isotropic singularities (Wainwright and Anderson, 1984) -- introduce factors....
Weyl Invariant Based Proposal

- Alternative form -- introduce a factor via the Ricci invariant.
- Better form -- introduce a factor using the Kretschmann invariant.
- Did Penrose suggest the Weyl tensor is a measure of the gravitational entropy?
Weyl Invariant Based Proposal

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- No!!
Weyl Invariant Based Proposal

- Alternative form -- introduce a factor via the Ricci invariant.
- Better form -- introduce a factor using the Kretschmann invariant.
- Did Penrose suggest the Weyl tensor is a measure of the gravitational entropy?
  - No!!
- We only want the Weyl curvature to be strictly increasing from zero.
Weyl Invariant Based Proposal

• Use the divergence theorem -- transform the surface integral to a volume integral.

• This defines the entropy density (be sure to use absolute brackets).

• The factor in the integral can be found out by requiring $S_g = S_{HB}$

• Example: The Schwarzschild solution
Weyl Invariant Based Proposal

- We are considering the scalar $\Psi = \sqrt[{
\begin{array}{c}
  C_\alpha \beta \gamma \delta \\
  R_\alpha \beta \gamma \delta \\
  C_\alpha \beta \gamma \delta \\
  R_\alpha \beta \gamma \delta 
\end{array}}]{C_\alpha \beta \gamma \delta \ C_\alpha \beta \gamma \delta \ R_\alpha \beta \gamma \delta \ R_\alpha \beta \gamma \delta}$

- In the Schwarzschild solution, both are equal -- the scalar is equal to 1.

- Therefore, we expect the gravitational entropy of Schwarzschild black holes to be maximal.

- Wait.... singularity causes divergence of the integral!

- How do we let the integral be defined if there is a divergence?
**Weyl Invariant Based Proposal**

- Simple: consider a small sphere of some radius around the singularity.

- Next, define the integral for the Schwarzschild radius and the *ad hoc* sphere radius.

- Then, let the "sample" sphere radius tend to zero -- does the gravitational entropy work?

- Using this process, the usual Hawking-Bekenstein entropy is derived -- exactly what we wanted!!
Weyl Invariant Based Proposal

- Does CET obtain this result?
Weyl Invariant Based Proposal

- Does CET obtain this result?

- Yes!!
Weyl Invariant Based Proposal

- Step one: find the tetrad.
- Step two: find the gravitational energy density and the gravitational temperature.
- Step three: gravitational entropy is the integral via the above parameters.
- Step four: find the gravitational entropy density.
- CET gives us the Hawking-Bekenstein entropy too!
CET Vs Weyl Invariant Proposal

• Which is better?

• CET has a limitation -- only works when considering Petrov type D or N.

• Works only in the general relativistic case via the Bel-Robinson tensor.

• But.. The Weyl invariant proposal does not describe the gravitational entropy of de Sitter spacetimes..

• Both need to account for extremal Reissner-Nordstrom black holes.
CET Vs Weyl Invariant Proposal

• Also, modified theories of gravity = modifications to the Hawking-Bekenstein entropy.

• What about cosmologies?

• Again, CET -- Petrov type D or N.

• But many good cosmologies -- LRS Bianchi type I spacetime is an example.

• Astrophysical wormholes -- described by the Weyl invariant proposal well.
CET Vs Weyl Invariant Proposal

- CET also works quite well for higher order black hole solutions -- same formalism, same approach using the Weyl scalar for non-zero forms (Petrov type D).

- Both proposals work quite well for solutions that include charge.

- Rotating charged black holes, accelerating black holes, etc.

- But what can we infer from the entire landscape of gravitational entropy?
Further....

• The gravitational entropy proposal develops the variation of Weyl curvature as some sort of "arrow of time".

• Further aspects of this also includes conformal cyclic cosmologies, also introduced by Penrose.

• But what would we achieve by describing gravitational entropy?
Further....

• The description of gravitational entropy would allow us to describe the evolution of cosmologies under the Weyl curvature hypothesis.

• We would be able to map the link between different parameters and the nature of gravitational entropy of such solutions -- for instance, LRS Bianchi type I spacetimes have two scale factors for the entire spatial metric components.

• In this spacetime, the nature of different scale factors affects the parameters affecting the variation of entropy.
Further....

• Description of wormholes under such proposals would allow us to understand geometric properties of solutions containing exotic matter.

• Also allow us to understand differential geometric aspects of cosmologies that are quasi-regular (O. Stoica, 2012).

• Both proposals need to account for the additional forms of modifications under modified gravity.
Conclusion

- The requirements for gravitational entropy can be summarized as:

- Vanishes only if the Weyl tensor is zero, i.e. iff the spacetime is conformally flat,

- Accounts for structure formations -- anisotropies,

- Reduces to the Hawking-Bekenstein relation for black holes,

- Is strictly increasing in late times of the universe.
Conclusion

- The CET proposal adopts the Newmann-Penrose formalism to describe gravitational entropy.

- The Weyl invariant based proposal adopts the equivalence of the gravitational entropy with the Hawking-Bekenstein relation to describe entropy via Weyl curvature invariant forms with Riemann tensor factors.

- Each of these proposals work consistently in most cases, and further research on understanding models where these proposals work is being done.
Thank you for your attention!