Towards a Learning Based Error Mitigation Method for Simulating Quantum Fields

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Motivation

- **Gauge theories are fundamental to our understanding of interactions between the elementary constituents of matter as mediated by gauge bosons.**
- **However, simulating the real-time dynamics of gauge theories is a notorious challenge classically**
- **This has recently stimulated effort, using Feynman's idea of a quantum simulator to devise schemes for simulating such theories on engineered quantum mechanical devices.**
- Practical quantum computing holds clear promise in addressing problems not generally tractable with classical simulation techniques, and some key physically interesting applications are those of real-time dynamics in strongly coupled lattice gauge theories.
- In particular, The QCD conundrum is well known. Due to its running coupling, at high energies, it is weakly coupled (asymptotically free), enabling perturbative treatment. At low-energy, it is a strongly interacting, non-perturbative theory leading to the problem of quark *confinement.*
- *O*ther questions and problems of importance include : properties and dynamics of finite-density systems or the fragmentation of high-energy quarks and gluons into hadrons.
- Quantum computers offer potential solutions in these systems that are inaccessible with conventional computing
- Existing and near-term quantum hardware is imperfect, with a small number of qubits, sparse qubit connectivity, and noisy quantum gates—so called NISQ (noisy intermediate-scale quantum) era devices.
- These technical imperfections constrain the circuit depth and dimensionality of problems that can be solved on available quantum computers.
- Error mitigation strategies are essential to take advantage of capabilities of current and near term devices

Talk Outline

- Motivation: Realizing Feyman's Dream
- The Quantum Advantage
- Challenges to Quantum Simulation of LGTs
- Plaquette Models and Real Time Dynamics
- The Noise Problem on Quantum Hardware
- Why Learning-based Quantum Error Mitigation

Simulating Quantum Gauge Fields: Requirements

- In the Hamiltonian approach to LGTs, we only discretize space, and time is a continuous, real coordinate. The lattice sites, hosting the matter fields (usually fermionic, but can be bosonic too) are labelled by $\mathbf{x} \in \mathrm{Z}_d$ $\{\mathbf{e}_i\}^{d}{}_{i=1}$ are unit vectors in the positive directions.
- The links, which hosts the gauge field degrees of freedom, are denoted by pairs of a starting site and a direction, $(\mathbf{x}, i)$.
- This fulfils the first requirement from quantum simulation of LGTs: must include two different types of degrees of freedom—gauge field and matter, residing on the links and the sites, respectively.
- The second requirement is: quantum simulators of lattice gauge theories should be gauge-invariant—that is, manifest a local symmetry parametrized by the gauge group G. Gauge transformations $\Theta g(\mathbf{x})$ should be well defined on each site $\mathbf{x} \in \mathrm{Z}_d$ and for any $g \in G$;
- The system has to be prepared initially in a gauge-invariant state and the dynamics should include gauge-invariant interactions.
- *In more than* $1 + 1d$*, the gauge field cannot be eliminated, and has to implement the complicated four-body plaquette interactions.*

Prototype: $\mathbb{Z}$(2) Lattice Gauge Theory

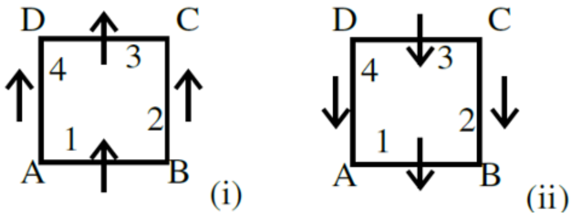
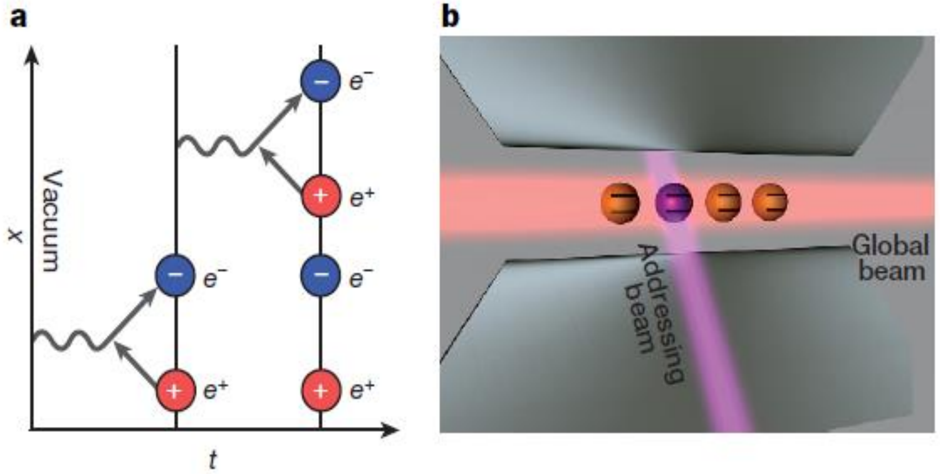


Fig. 1
Four-spin interaction that defines the Hamiltonian



Quantum simulation of coherent real-time dynamics of particle–antiparticle creation by in the Schwinger model (one-dimensional quantum electrodynamics) on a lattice

Z(2) Gauge Theory

On a square lattice, we can define a lattice gauge theory through a four-spin interaction that defines a loop around a unit square.

$$H = -g \sum_{\square} U_{\square} - \Gamma \sum_i S_i^x \,, \tag{1}$$

$$U_{\square} = S^z_{r,\mu} S^z_{r+\mu,\nu} S^z_{r+\nu,\mu} S^z_{r,\nu} \,. \tag{2}$$

Subject to Gauge symmetry

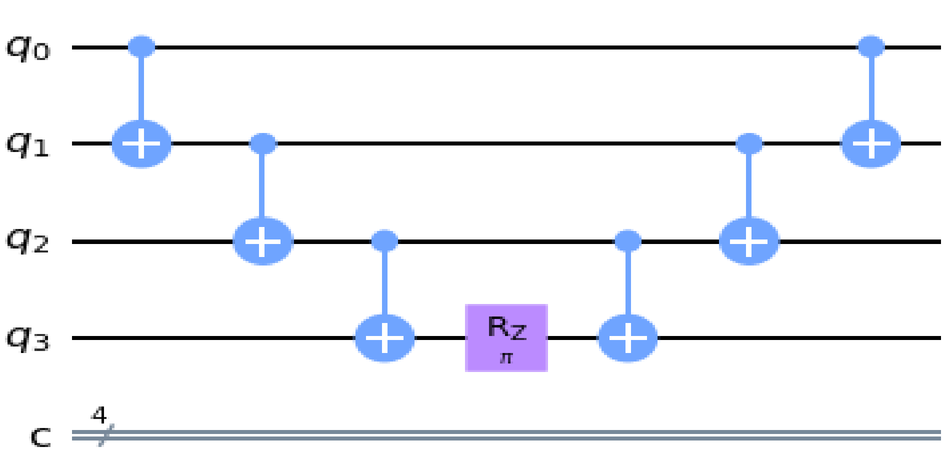
$$\begin{aligned} V_r &= \sigma^x_{r,\mu}\sigma^x_{r,\nu}\sigma^x_{r-\mu,\mu}\sigma^x_{r-\nu,\nu} \\ &= \exp\left[i\pi \sum_{\mu} (S^x_{r,\mu} - S^x_{r-\mu,\mu}) \right] . \end{aligned} \tag{3}$$

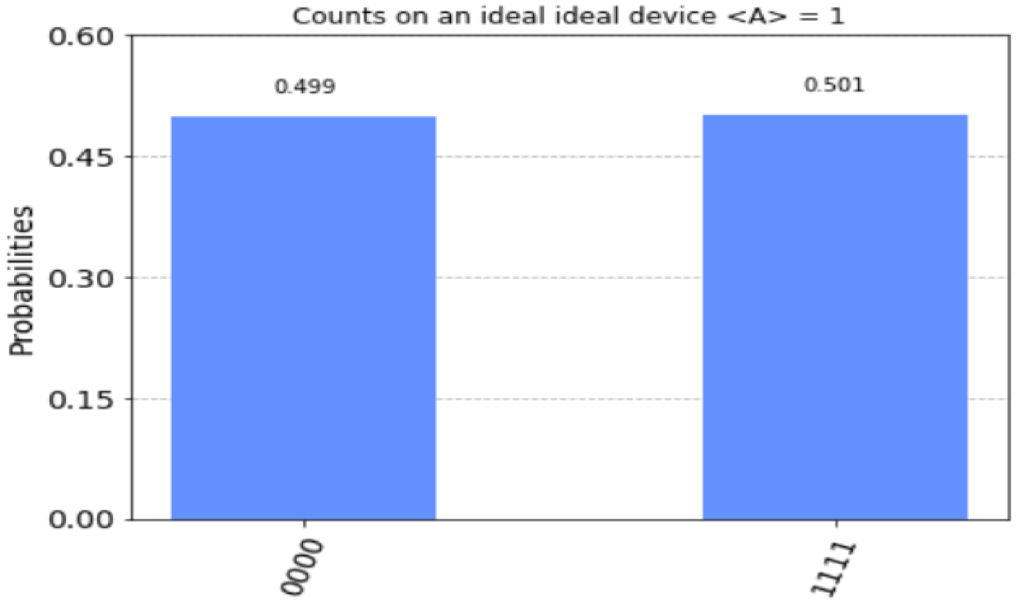
Consider the sector spanned by

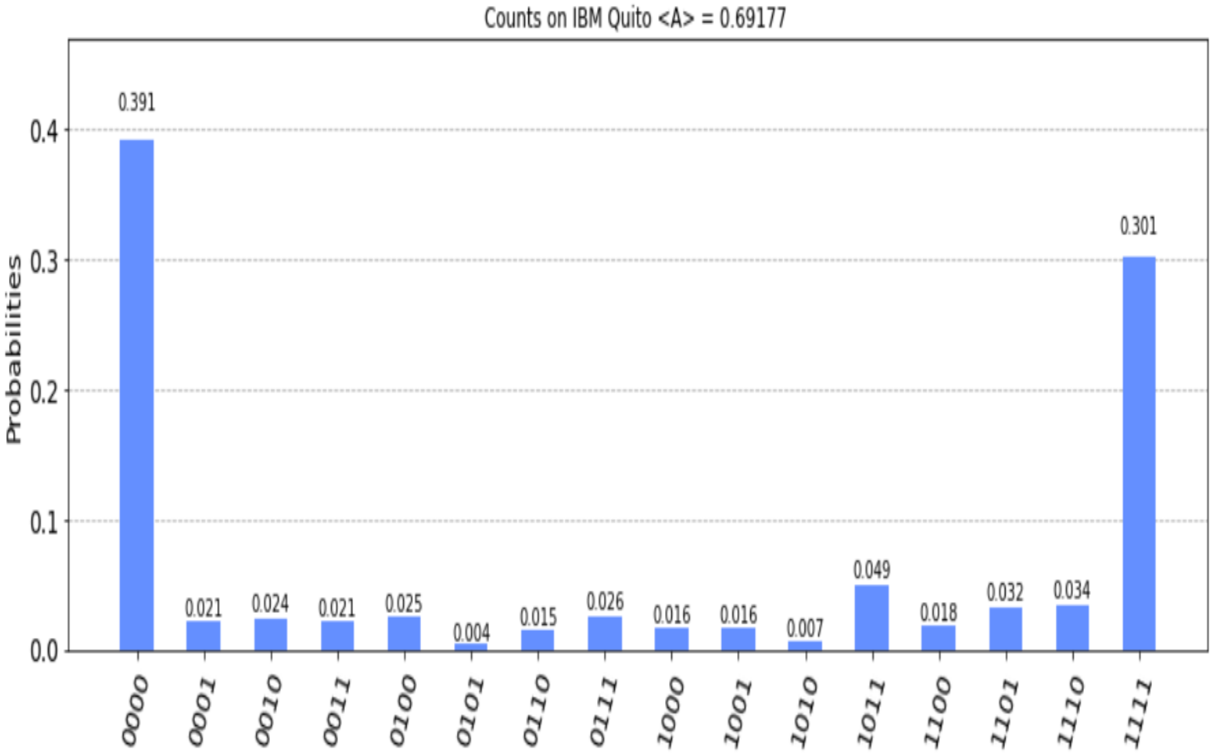
$$\begin{aligned} |\Psi_1\rangle &= (|1111\rangle + |0000\rangle)/\sqrt{2}, \\ |\Psi_2\rangle &= (|1111\rangle - |0000\rangle)/\sqrt{2}. \end{aligned} \tag{4}$$

Time Evolution and The Noise Problems

The unitary evolution e^{-iHt} is implemented by the equivalent to the circuit







Learning Based Quantum Error Mitigation Method & Advantages

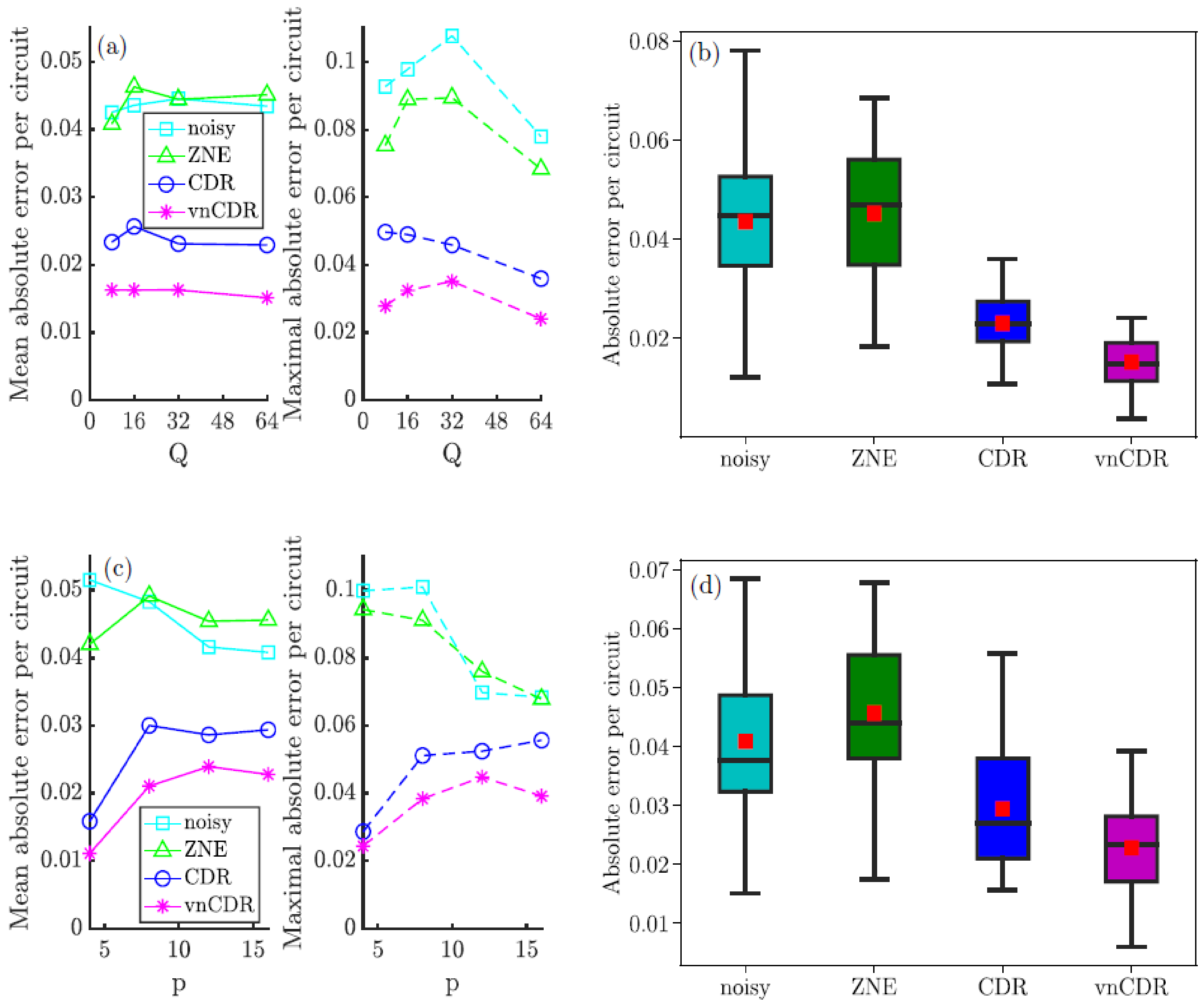
The Goal: Learn a function which takes noisy expectation values to their unmitigated values

The Motivation: Lowe et al in [1] proved that we achieve zero loss on all arbitrary circuits if we obtain zero loss on training data composed of all possible Clifford circuits.

Method: For each training circuit ρ_i^{train} evaluate classically a noiseless expectation value of E, $y_i = Tr(\rho_i E)$ and its noisy expectation values $x_{i,l}$ using a quantum computer with several noise rates

The Gist: Fit the expectation values of the training circuits with a linear ansatz given by $y = f(x_1, x_2, \ldots, x_m)$. Where $f(x_1, x_2, \ldots, x_m) = \sum_{l=1}^{m} x_l a_l$

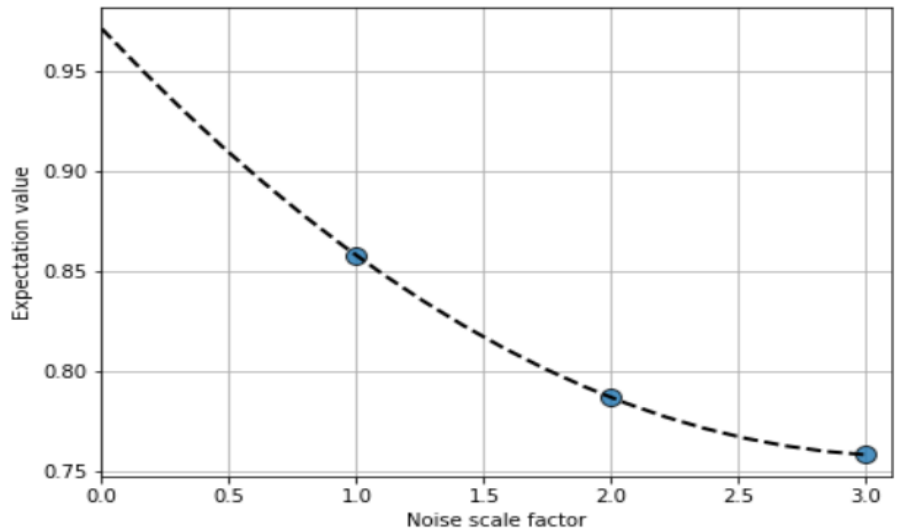
The Magic: Use the fitted ansatz to correct the noisy expectation

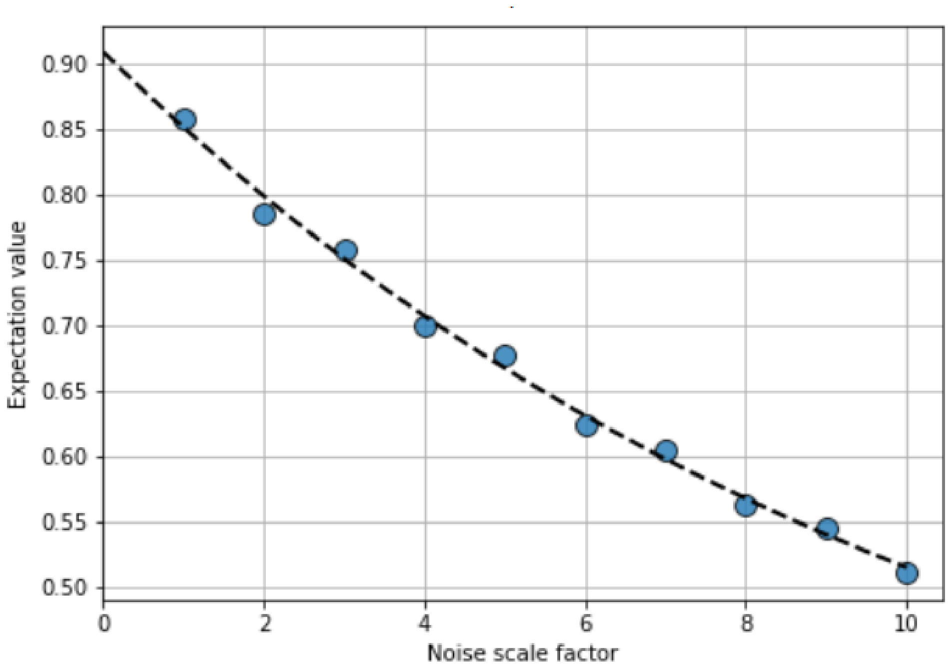


Theorems (Proofs Omitted)

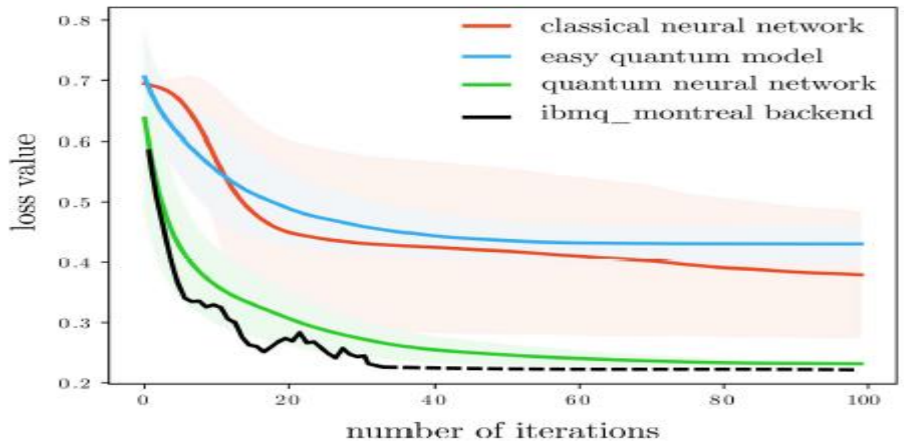
- Theorem 1: Variable Noise Clifford Data Regression (vnCDR) perfectly mitigates global depolarizing noise
- Note that while there are many distinct noise channels, the depolarizing noise model often appropriately describes average noise for large circuits involving many qubits and gates.
- Theorem 2: The Clifford circuit set is sufficient to estimate zero error expectation values of arbitrary circuits.
- In essence, we formulate classical simulation to enble Quantum Simulation!
- Cf: A. Lowe et al. Phys. Rev. Research **3**, 033098

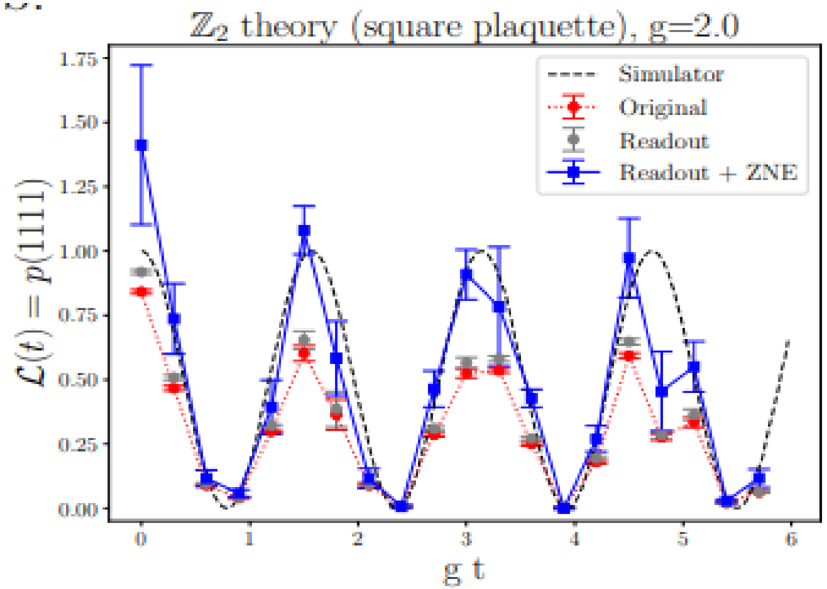
Error Mitigation Results





Noise Scale	1	2	3	4	5	6	7	8	9
Exp. Values	0.7847	0.7578	0.70036	0.67661	0.62346	0.60547	0.56344	0.54577	0.51062





Future work: U(1) Model

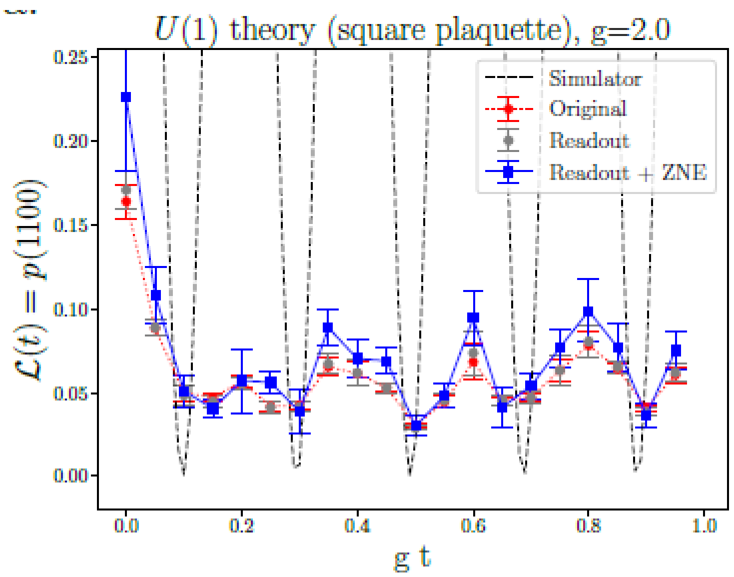
- For single square plaquette we obtain the Hamiltonian:

$$H_{\square} = -\frac{g}{2}\left[\sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x + \sigma_1^y\sigma_2^y\sigma_3^y\sigma_4^y - \sigma_1^x\sigma_2^x\sigma_3^y\sigma_4^y \right.$$
$$-\sigma_1^y\sigma_2^y\sigma_3^x\sigma_4^x + \sigma_1^y\sigma_2^x\sigma_3^y\sigma_4^x + \sigma_1^y\sigma_2^x\sigma_3^x\sigma_4^y$$
$$\left. +\sigma_1^x\sigma_2^y\sigma_3^y\sigma_4^x + \sigma_1^x\sigma_2^y\sigma_3^x\sigma_4^y\right].$$

Gauss law:

$$G_x = \sum_{\mu}\left(E_{x,\mu} - E_{x-\mu,\mu}\right).$$

This operator G_x generates the gauge transformations, which can be expressed as $V = \prod_x \exp\left(-i\alpha_x G_x\right),$
Where α is the (local) parameter associated with the local unitary transformation.



Estimating Quantum Volume:
$V_Q = d*m$
with m = 5 and d = 80, V_Q =400

If a processor can use eight qubits to successfully run a circuit with eight-time steps worth of gates, then we say it has a Quantum Volume of 256 — we raise 2 to the power of the number of qubits (2^n)

Note that *n* from Quantum Volume does not limit you to only *n* qubits with *m* time layers

Discussion