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SCHWARZSCHILD RADIUS FOR ELECTRIC CHARGE

INTRODUCTION

Modern science has a large number of theories about black holes, their origin and development. The study of black holes is of interest from the view point of the Universe origin understanding. For the first time about very massive bodies, in which the force of gravity is so great that even light cannot escape from them, the English astronomer John Mitchell wrote. Thanks to Michel, this hypothesis appeared in 1784 [8]. For stationary solutions of Einstein's equations, only three variable characteristics are used that correspond to one or another type of black hole: mass *M*, momentum *L*, and electric charge *e*. At first Schwarzschild had decided of the Einstein's equations for a static spherically symmetric mass and he had obtained the solution in the form [5]

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi),$$

where $r_g = \frac{2GM}{c^2};$ r_g = gravitational radius; s = interval;
M = mass; G = gravitational constant; c = velocity of light in vacuum; r, Θ, φ = spherical coordinates.

The Reissner-Nordström solution was next the Einstein's equations olution. This is a static solution of the spherically symmetric black hole which has mass and electric charge, but it hasn't the rotation. We can write the metrics of the Reissner-Nordström black hole in the form [4]

$$ds^2 = Fc^2dT^2 - F^{-1}dR^2 - R^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

where

$$F = 1 - \frac{2GM}{c^2R} + \frac{GQ^2}{c^4R^2}.$$

The Kerr solution describes the rotating black hole, which has mass *M* and momentum *L*, but which hasn't the electric charge [10]

$$ds^2 = \left(1 - \frac{r_g r}{r^2 + a^2\cos^2\theta}\right)c^2dt^2 - \frac{r^2 + a^2\cos^2\theta}{r^2 - r_g r + a^2}dr^2 - (r^2 + a^2\cos^2\theta)d\theta^2 +$$

$$+ \frac{2r_g ra\sin^2\theta}{r^2 + a^2\cos^2\theta}cdtd\varphi - (r^2 + a^2 + \frac{r_g ra^2\sin^2\theta}{r^2 + a^2\cos^2\theta})\sin^2\theta d\varphi^2.$$

The Kerr-Newman solution was obtained subsequently. The Kerr-Newman solution describes the black hole, which has mass *M*, momentum *L* and electric charge *e*.

Modern science is actively looking for the black holes. If we put the probe inside a black hole, we will get any information from it. That is, everything inside of the black hole cannot escape from it. For a static spherically symmetric mass it is possible to construct a Schwarzschild radius [6], which determines the black hole formation possibility from a specific mass

$$r_g = \frac{2GM}{c^2}.$$

A similar radius exists for a static spherically symmetric electric charge. And, as follows from the content of this work, this radius for the electric charge can determine the information non-return scope from the volume of a particular electric charge. In other words, just as in gravity we cannot obtain information from a black hole, so in electricity we cannot obtain information from the electric charge sphere which is limited by the Schwarzschild radius analog.

Looking at the previous formula, we can assume that the formula for the Schwarzschild radius analog will consist of a combination of two constants and an electric charge.

FORMULATION OF THE PROBLEM

This work aims to prove the formula for the Schwarzschild radius analog in case of a static spherically symmetric electric charge.

RESULTS

The formula for the Schwarzschild radius analog is obtained

$$r_\lambda = \frac{e}{4\pi\varepsilon_0\lambda},$$

where e = electric charge; ε_0 = electric constant;

λ = some constant ($\lambda \approx 10^6$ V).

The formula for the Schwarzschild radius is written more precise

$$r_g = \frac{GM}{c^2}.$$

The Coulomb's law application limit is explained.

The physical quantities converting method is described, which allows finding unknown dependencies in physics. This work is another step towards the standardization of physics and its formulas from completely different sections of physics.

The radiation temperature expression is proposed for a spherically symmetric black hole with a static electric charge, but without rotation

$$\theta = \frac{\hbar c \vec{E}}{2\pi\lambda k},$$

where $\hbar$ = Planck's constant; $\vec{E}$ = electric field strength;

k = Boltzmann's constant.

An expression is obtained for the particles production probability w_e estimating in a static spherically symmetric electric field of a black hole

$$w_e = A \exp \frac{-\beta E}{2\pi k \theta},$$

where $E = \lambda e$; β = dimensionless constant of the order of unity; A = pre-exponential factor depends on (as well as β) the more detailed characteristics of the field.

DISCUSSION

PART 1

A

Let's write the formula for the Schwarzschild radius

$$r_g = \frac{2GM}{c^2}. \qquad (1)$$

Let's try to prove the same formula. The gravitational energy E of resting mass m located at a distance r_g from the centre of mass M is equal to

$$E = \frac{GMm}{r_g}.$$

On the other hand, the same energy can be written in this form $E = mc^2$. Let's equate them.

$$mc^2 = \frac{GMm}{r_g}.$$

Whence we obtain the formula (2)

$$r_g = \frac{GM}{c^2}. \qquad (2)$$

The equation (3) is used in the proof of the formula for the Schwarzschild radius [2]

$$\frac{mv^2}{2} = \frac{GMm}{r_g}. \qquad (3)$$

Kinetic energy is implied on the left-hand side of this equation. But the formula for kinetic energy in classical physics differs from the formula for relativistic kinetic energy. And if we assume that the velocity of the mass m equals to c, then we must also use the formula for the relativistic kinetic energy E_k

$$E_k = E - E_0\,,$$

where E = total energy; E_0 = rest energy.

That is, a simple substitution of $mv^2/2$ for $mc^2/2$ is impossible. Therefore, the formula (3) is incorrect in relativistic physics. Therefore, expression (1) is also incorrect. And the expression (2) validity should be recognized.

B

Continuing the electric charge non-invariance theme, proposed by the author in 2006, the author offers a proof of the formula for the Schwarzschild radius analog (but not for mass, but for electric charge).

Let's carry out similar reasoning for an electric charge.
On the one hand, the energy E of the resting electric charge q, located at a distance r from another electric charge e, is calculated by the formula

$$E = \frac{qe}{4\pi\varepsilon_0 r}. \tag{4}$$

On the other hand, according to [7] one can be written so

$$E = \lambda q, \tag{5}$$

where E = energy of electric charge q; λ = constant.
Let’s equate expressions (4) and (5).

$$\lambda q = \frac{qe}{4\pi\varepsilon_0 r}\,.$$

Then we get r

$$r = \frac{e}{4\pi\varepsilon_0\lambda}.$$

If we take r on the Schwarzschild sphere, then $r = r_\lambda$. And now we can write the final form of the formula for the Schwarzschild radius analog in case of a static spherically symmetric electric charge

$$r_\lambda = \frac{e}{4\pi\varepsilon_0\lambda}. \qquad (6)$$

C

Knowing the positron and electron annihilation energy, in the first approximation, we can calculate the value of λ. And knowing λ, you can calculate the Schwarzschild radius analog for electric charge.

For example, let's calculate the Schwarzschild radius analog for an electron having an almost elementary electric charge. Let's mark this radius $r_{\lambda e}$. Firstly calculate λ. The energy ($E \approx 1$ MeV) is released during the positron and electron annihilation. This energy can be expressed in joules. $E \approx 1.6\times10^{-13}$ J. Then we find λ from this formula
$E = \lambda e$.

$$\lambda = E/e.$$

$$\lambda = \frac{1.6\times10^{-13}\,J}{1.6\times10^{-19}\,C}.$$

$$\lambda = 10^6\ \mathrm{V}.$$

We got λ. Now we can calculate the Schwarzschild radius analog for the electron from (6)

$$r_{\lambda e} = \frac{1.6\times10^{-19}}{4\pi\times8.85\times10^{-12}\times10^6}\,\mathrm{m}.$$

$$r_{\lambda e} \approx 1.44\times10^{-15}\ \mathrm{m} \approx 10^{-15}\ \mathrm{m}.$$

We got the Schwarzschild radius $r_{\lambda e}$ analog for the electron. And the atomic nuclei dimensions are just of such order (about 10^{-15} m [11]). At this distance, the Coulomb's repulsion of protons in the nucleus no longer prevents them from approaching, and a strong interaction begins to appear. It turns out that it is precisely at distances of this order the Coulomb's force becomes vanishingly small, either by itself or in comparison with the forces of another nature manifested at such distances. It is well known that the deviations from the Coulomb's law occur at distances of this order. In other words, calculating the Schwarzschild radius analog for an electron, we see that this radius coincides with the Coulomb's law application limit.

Comparing a gravitational radius value for electron ($r_{ge} \approx 10^{-57}$m, this is a tabular value) and the Schwarzschild radius analog value for electron ($r_{\lambda e} \approx 10^{-15}$ m), we see that the black hole formation conditions for electron as an electric charge occur at much greater distances than for electron as the mass owner.

Modern science denies the long-lived charged black holes existence possibility, believing that charges of the same sign will quickly scatter in different directions. However, it has to remember that under the black hole conditions, the Coulomb's repulsion forces become vanishingly small (a complete analogy with protons in a nucleus). Therefore, the charge in a black hole can persist for an arbitrarily long time.

Knowing expression (2), we can get expression (6) in another way. We can come to expression (6) by logical considerations. We can assume that mass *M* will naturally transform into an electric charge *e*, gravitational constant *G* will change into electric constant ε_0 with multiplier 4π and for two reasons it will be in the denominator of the sought formula. Firstly the gravitational constant *G* is numerically equals to the force with which two unit masses interact, when they are at a unit distance from each other. Having written down Coulomb's law, we can see that the electric constant ε_0 cannot be imagined as the force with which two unit charges interact at a unit distance from each other. But this fraction $1/(4\pi\varepsilon_0)$ will already equal numerically to the interaction force of two unit charges located at a unit distance from each other. And, secondly, as can be seen visually at once, in the universal gravitation law and in the Coulomb's law these constants are in the different structural positions: *G* is in the numerator and $4\pi\varepsilon_0$ is in the denominator. It remains to understand what can replace c^2. And here we have to remember the constants that assign energy in accordance to each mass and each electric charge. Einstein assigned the energy *E* for each mass *M* through the coefficient of proportionality c^2. In the work [7] for each electric charge *e* the author assigned the energy *E* through the coefficient λ of proportionality. In electricity the constant λ plays the same role as the constant c^2 in gravity.

Then c^2 can be replaced by the constant λ in the Schwarzschild radius formula. Therefore, using expression (2), we can write the formula for the Schwarzschild radius analog in case of a static spherically symmetric electric charge in this form

$$r_\lambda = \frac{e}{4\pi\varepsilon_0\lambda},$$

and this completely coincides with the expression (6). Thus, we arrived at the proven formula by simple logical reasoning.

These simple substitutions ($M – e; c^2 – \lambda; 1/G$ - $4\pi\varepsilon_0$) can become a very effective original method in the unknown dependencies search in physics.

One interesting remark can be made. The specific energy of binding for many nuclei turns out by a constant value, which equals ≈ 8 MeV [11]. You can see that the constant λ differs from this value by about 8 times. It is very likely that in the values specifying case of these two physical quantities, it may turn out that their values coincide. And the specific energy of binding will turn out by the physical meaning of the constant λ. The specific energy Y constancy of binding indicates that the nucleus energy $E_{(Z, A)}$ is proportional to the number A of nucleons [11]

$$E_{(Z, A)} \sim YA.$$

PART 2

There is an expression for the Schwarzschild radius square [3], but for an electric charge

$$r_Q^2 = \frac{e^2 G}{4\pi\varepsilon_0 c^4}. \qquad (7)$$

Let's make some remarks about the expressions (2) and (7).

1. Let's think a little about the coefficient 2 in expression (1). There are three independent paths that lead to the same result. Firstly, there is the expression (2) proof in this work. Secondly, if in the expression (7) we replace e with M, as well as this fraction $1/(4\pi\varepsilon_0)$ with G, re-

spectively, then we get the expression for the Schwarzschild radius in such form

$$r_g = \frac{GM}{c^2}. \tag{8}$$

We see that expressions (8) and (1) are in contradiction with each other. They differ by a multiplier of 2. This means that some result of the Einstein's equations solving is not correct: either the expression (1) or the expressions (7) and (8).

Thirdly, if in the formula (6) for the Schwarzschild radius analog in case of a static spherically symmetric electric charge we do the analogous substitutions, namely e with M, this fraction $1/(4\pi\varepsilon_0)$ with G as well as λ with c^2, respectively, then we get the expression for the Schwarzschild radius in this form

$$r_g = \frac{GM}{c^2}.$$

Thus we see that the proven expression (2), the expression (7) and the formula (6) for the Schwarzschild radius analog in case of a static spherically symmetric electric charge lead to the same expression for the Schwarzschild radius. That is, three different paths lead to the same result. From this we can conclude that the coefficient 2 should not be in the formula for the gravitational radius. Therefore the gravitational radius r_g formula will be more correct in such form

$$r_g = \frac{GM}{c^2}.$$

2. Let's try to compare expressions (6) and (7). To do this, we divide the square of expression (6) by square of expression (7)

$$\frac{r_\lambda^2}{r_Q^2} = \frac{e^2}{16\pi^2\varepsilon_0^2\lambda^2} \times \frac{4\pi\varepsilon_0 c^4}{e^2 G} .$$

$$r_\lambda^2/r_Q^2 \approx 10^{42}.$$

We get r_λ $\quad r_\lambda \approx 10^{21} r_Q.$

That is the Schwarzschild radius analog is 10^{21} times greater than the one given by the expression (7). Let's write it down for an electron

$$r_{\lambda e} \approx 10^{21} r_{Qe}.$$

With this in mind $\quad r_{\lambda e} \approx 10^{-15}$ m,

we get $\quad r_{Qe} \approx 10^{-36}$ m.

The difference between these two values is huge. Comparing the values of $r_{\lambda e}$ and r_{Qe}, on the author's opinion, one should opt for the value of $r_{\lambda e}$. It should be done for two reasons. Firstly, the value of $r_{\lambda e}$ is confirmed by the facts that are presented in this work. Secondly, the value of $r_{\lambda e}$ explains the Coulomb's law application limit existence. What explains the value of r_{Qe} - it is unknown.

3. Let's try to divide the expression (7) into two parts: electrical and gravitational. Let's transform expression (7) to a form in which only electrical characteristics will participate, without the participation of G and c. We replace G with fraction $1/(4\pi\varepsilon_0)$ and replace c^4 with λ^2. After these replacements, it can be assumed that the left-hand side of expression (7) will already be r_λ instead of r_Q. Then

$$r_\lambda^2 = \frac{e^2}{16\pi^2\varepsilon_0^2\lambda^2} .$$

And we get r_λ $$r_\lambda = \frac{e}{4\pi\varepsilon_0\lambda}. \qquad (9)$$

From this it is clear that after such transformation, expression (9) completely coincides with expression (6) - the formula for the Schwarzschild radius analog in case of a static spherically symmetric electric charge. That is, the formula for the Schwarzschild radius analog in case of a static spherically symmetric electric charge is in excellent agreement with the Reissner-Nordström solution result (from [3]).

And the gravitational part of the expression (7) will represent the Schwarzschild radius. Let's show it. After the substitution in this expression of e with M, as well as of the fraction $1/(4\pi\varepsilon_0)$ with G, respectively, we can write

$$r_g = \frac{GM}{c^2},$$

what coincides with the proven expression (2) for the gravitational radius.

Thus we see that expression (7) is not a final product, it is a semi-finished product (semi-manufacture). With a certain refinement, expression (7) is transformed either into the Schwarzschild radius or into the Schwarzschild radius analog. The expression (7) use in practical calculations can lead to erroneous results.

Such transformations of the Reissner-Nordström solution results in the Schwarzschild radius, as well as the transformations of the Schwarzschild radius in the Schwarzschild radius analog and vice versa, are a good confirmation of the formula truth for the Schwarzschild radius analog, which is in full compliance with the existing solutions of the Einstein's equations. Consequently, it is possible to assert about validity of the formula for connection between an electric charge and its energy $E = \lambda e$, proposed by the author in 2006 and on the basis of which the formula for the Schwarzschild radius analog was obtained.

PART 3

In 1974-1975, Hawking obtained the radiation temperature expression for a spherically symmetric black hole without an electric charge and rotation. A black hole creates and emits particles in the same way as a black body heated to a temperature Θ [9]

$$\theta = \frac{\hbar \kappa}{2\pi c k}, \qquad (10)$$

where κ = a value that characterizes the gravitational field strength near the black hole surface; k = Boltzmann's constant.

A similar expression of temperature can be obtained for a spherically symmetric black hole with an electric charge, but without rotation (naturally excluding mass). The electric field strength $\vec{E}$ is the same force characteristic of the electric field, as the value of κ is the force characteristic of an gravitational field. Therefore, instead of the value of κ we write the electric field strength $\vec{E}$.

Since we are interested in a black hole with an electric charge, we now need to understand how to enter the constant λ into the formula for temperature. We have repeatedly replaced c^2 with λ and vice versa. Let's make such replacement again: c^2 is replaced by λ. Now we can write the formula for temperature in the case of a spherically symmetric black hole with an electric charge, but without rotation (naturally excluding mass)

$$\theta = \frac{\hbar c \vec{E}}{2\pi\lambda k} . \qquad (11)$$

The particles production probability w in an external static field with intensity Γ is described by an expression of such form [9]

$$w = A \exp \frac{-\beta m^2 c^3}{\hbar g \Gamma} , \qquad (12)$$

where g = the charge of the particles being born;
β = dimensionless constant of the order of unity; A = pre-exponential factor depends on (as well as β) the more detailed characteristics of the field.

For gravitational interaction, the value of its mass acts as the gravitational charge of the system. Therefore, to estimate the particles production probability w in a static gravitational field, we should put on

$g = m$ and $\Gamma = \kappa$, where $\kappa = \dfrac{c^4}{4GM}$ [9].

According to [9], the particles production probability w in the field of a black hole (from (12))

$$w = A \exp \frac{-\beta m c^2 m c}{\hbar g \Gamma} \qquad w = A \exp \frac{-\beta E c}{\hbar \kappa}$$

$$w = A\exp \frac{-\beta E}{2\pi k\Theta}$$

(13)

In this expression we have replaced $\hbar\kappa$ by $2\pi ck\Theta$ (from (10).
If the black hole has an electric charge, then expression (12) will take on a following form. For electrical interaction we should put on $g = e$, $m = e$ and $\Gamma = \vec{E}$. Let's apply the formula again $E = \lambda e$.

After replacing mc^2 with λe we can write for a process that characterizes only electrical interaction.
Let's write formula (12) in such form

$$w = A\exp \frac{-\beta mc^2 mc}{\hbar g\Gamma}$$

Let's make substitutions and obtain

$$w_e = A\exp \frac{-\beta ee\lambda c}{\hbar e\vec{E}} = A\exp \frac{-\beta e\lambda c}{\hbar \vec{E}}.$$

It follows from the formula (11): $\hbar\vec{E} = \dfrac{2\pi\lambda k\theta}{c}$.
We now have a final expression for w_e

$$w_e = A\exp\frac{-\beta e\lambda cc}{2\pi\lambda k\theta} = A\exp\frac{-\beta Ec^2}{2\pi\lambda k\theta} = A\exp\frac{-\beta E}{2\pi k\theta}.$$

As a result, we came to the same expression that was obtained for the Schwarzschild black hole. Only $E = \lambda e$.

That is, if we knew that energy can be calculated by the formula $E = \lambda e$, we could immediately write down the final result for the case of a non-rotating charged black hole. As it was, we had to do some mathematics.

In 1972, J. Bekenstein hypothesized [1] that a black hole has entropy S proportional to its surface area s (for a spherical hole $s = 4\pi r_g^2$)

$$S = \frac{\eta c^3 ks}{G\hbar},$$

where $\eta = 1/4$.

Let's denote N

$$N = \frac{kc^3}{G\hbar}.$$

One can write a similar formula for the entropy S_e of a spherically symmetric black hole with an electric charge, but without rotation. If we make such substitutions (c^2 with λ and G with the familiar fraction $1/(4\pi\varepsilon_0)$, respectively) we can write

$$N_e = \frac{4\pi\varepsilon_0 k\lambda^2}{c\hbar}.$$

And we get S_e

$$S_e = N_e \frac{s}{4} = \frac{\pi\varepsilon_0 k\lambda^2}{c\hbar} s.$$

This is the entropy of a spherically symmetric black hole, due to its electric charge.

Since entropy is an additive quantity, when calculating the total entropy S_{BH} of a charged black hole, you will have to add two of its parts, namely

$$S_{BH} = \frac{kc^3 s}{4G\hbar} + \frac{\pi\varepsilon_0 k\lambda^2 s}{c\hbar}.$$

CONCLUSIONS

The formulas transforming method of electricity and gravity, which is described in the work, allows us to move towards the standardization of physics. A very similar situation is observed in the study of black holes and questions of thermodynamics. There are quite a few considerations indicating the close intertwining of black hole physics and thermodynamics [9].

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