

The 10th Annual  
Large Hadron Collider Physics Conference  
May 16-21, 2022

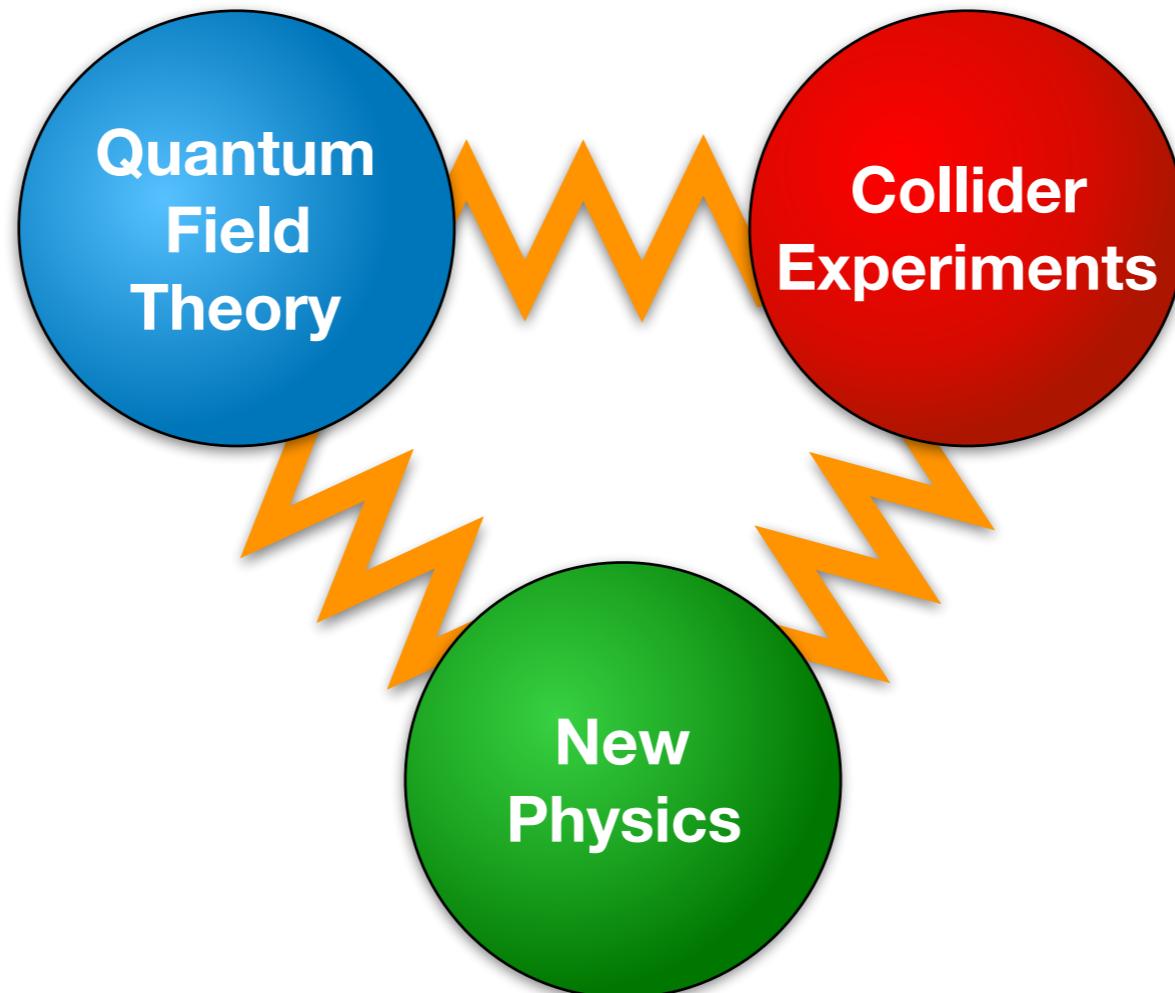


# High pT QCD Theory

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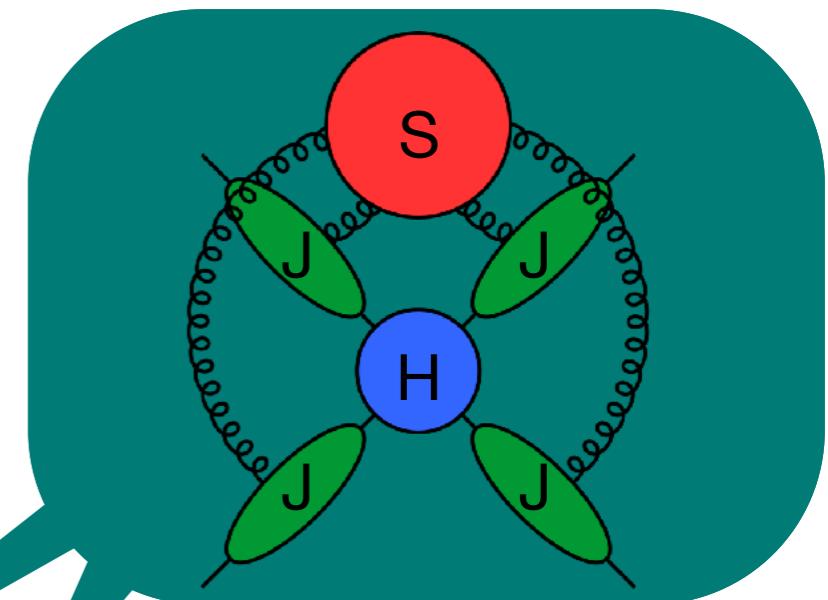
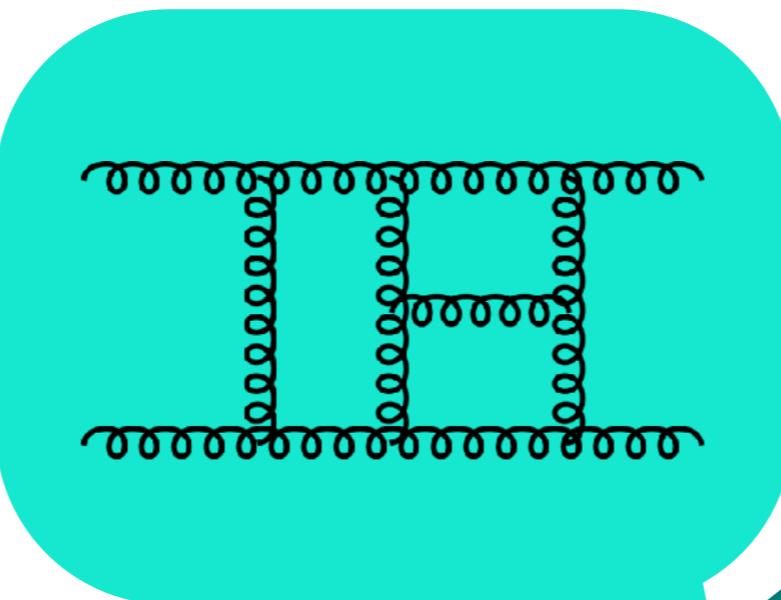
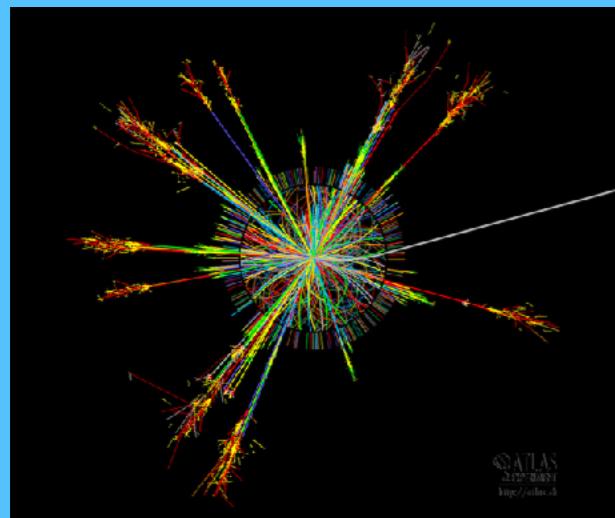
LHCP 2022, May 20

# Why precision QCD



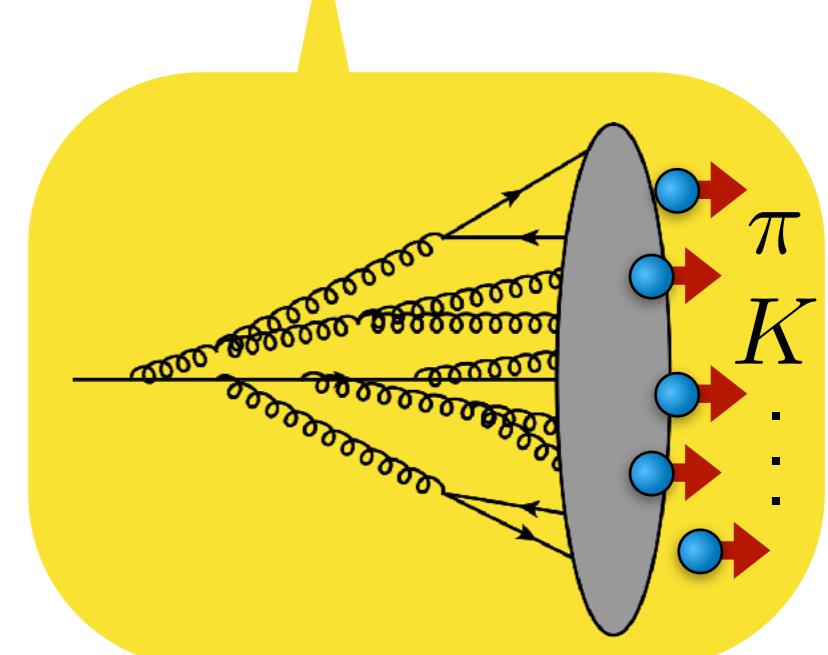
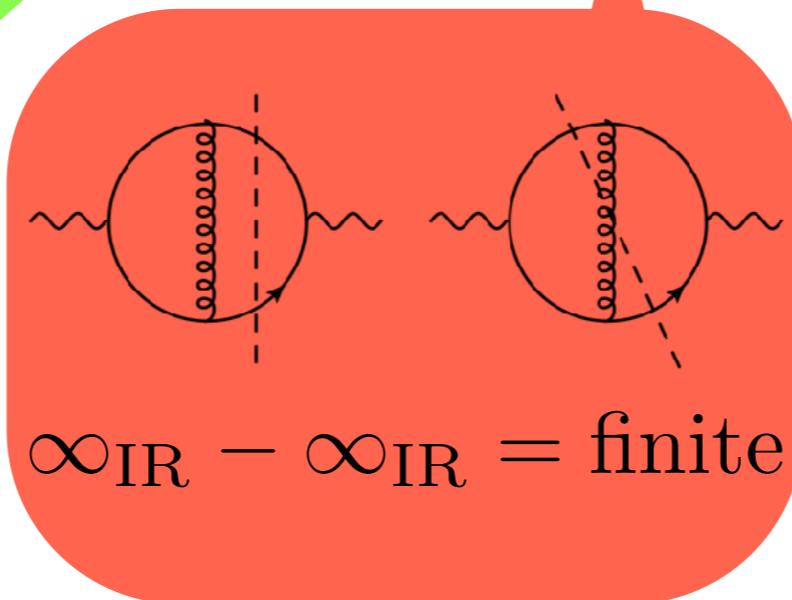
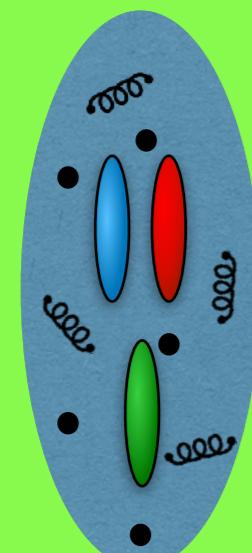
- Quantum field theory is our language of nature, of which **precision** is a hallmark. We are fortunate to be living in a time when precision can be constantly improved, sometimes revolutionized thanks to the ingenuities and hard works of physicists.
- Collider experiments, such as the LHC, are the **roots** of our field. They constantly challenging our theory capability, pushing theorists to achieve things not even imaginable.
- Through this process, we hope to **discover** new physics phenomena and new physics principle, and we are constantly awarded by nature, sometimes big, sometimes small.

# Golden rule of precision QCD



R. Poncelet's talk

$$d\sigma(p_T, M^2, \dots) = \text{PDFs} \otimes \sum_j |\mathcal{M}_{i \rightarrow j}|^2 \otimes J \otimes \dots$$

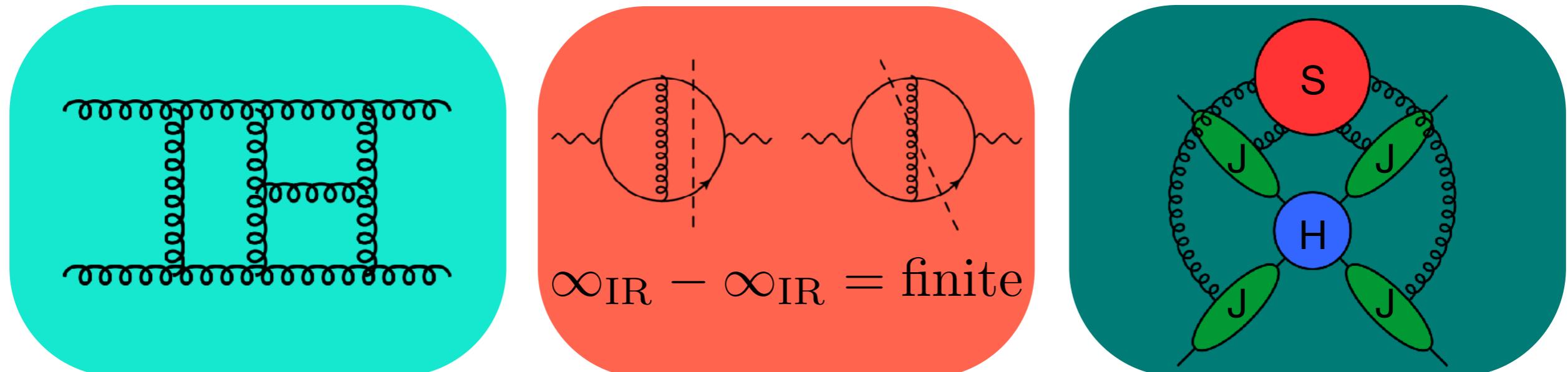


P. Nadolsky's talk

R. Poncelet's talk

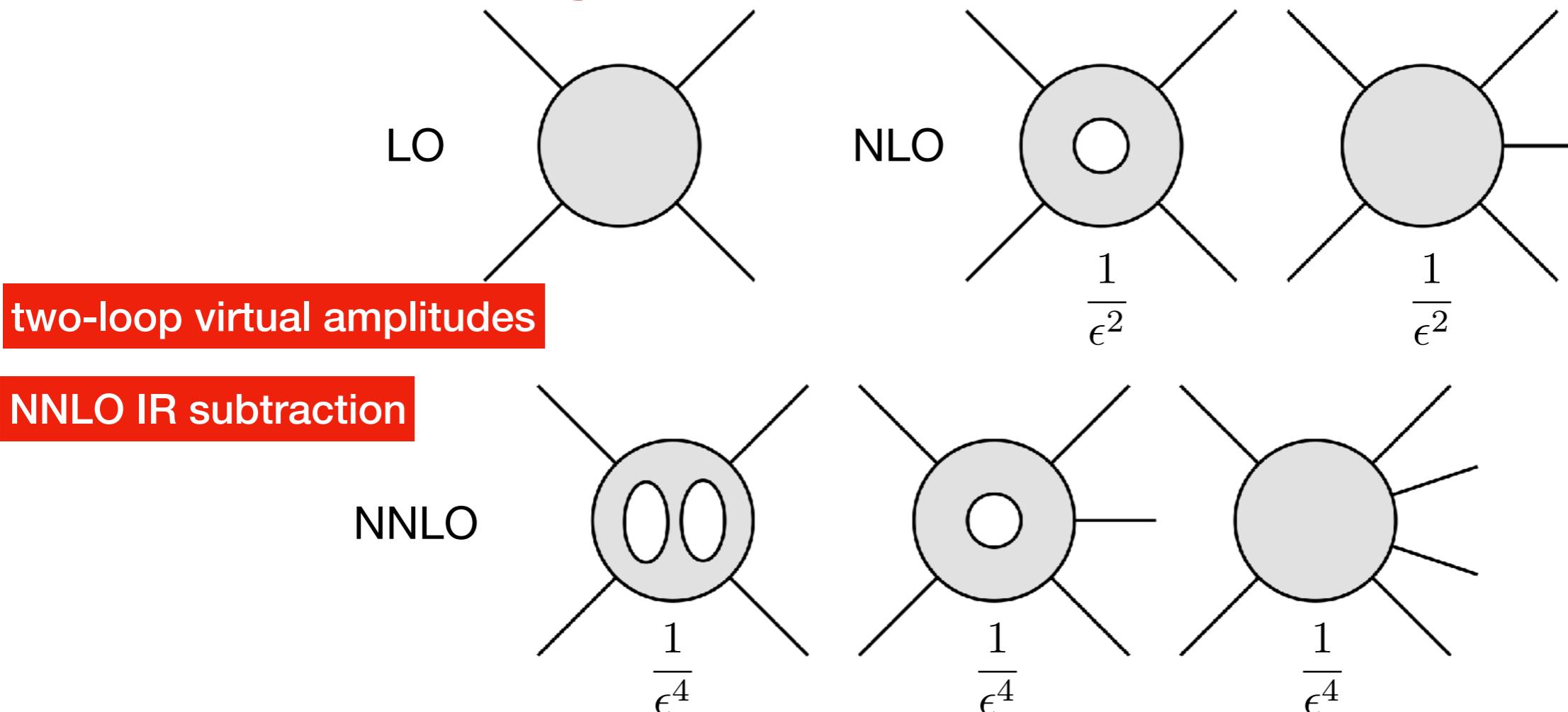
X.H. Liu's talk

# Outline



- Fixed-order perturbation theory
- All-order resummation

# Anatomy of NNLO calculation



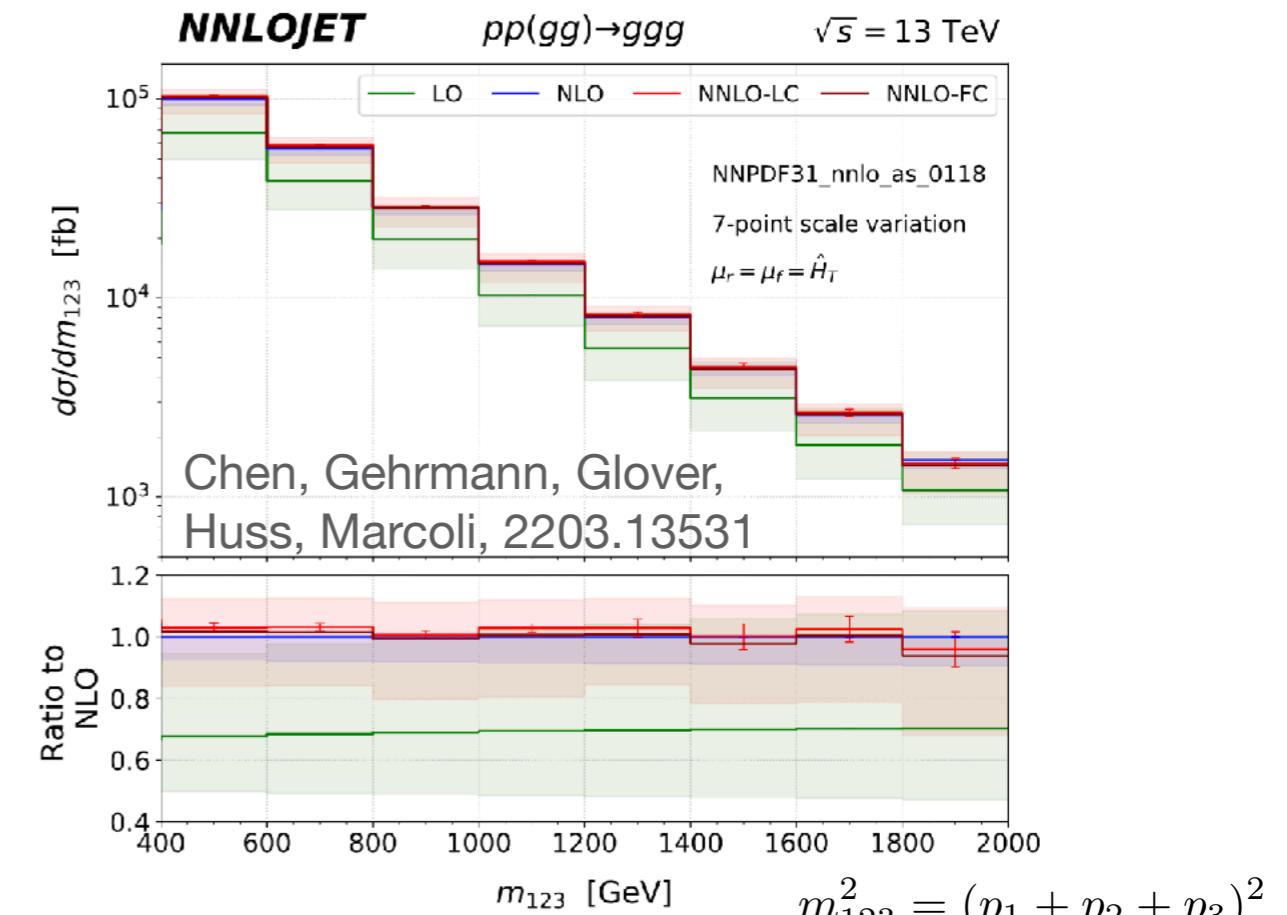
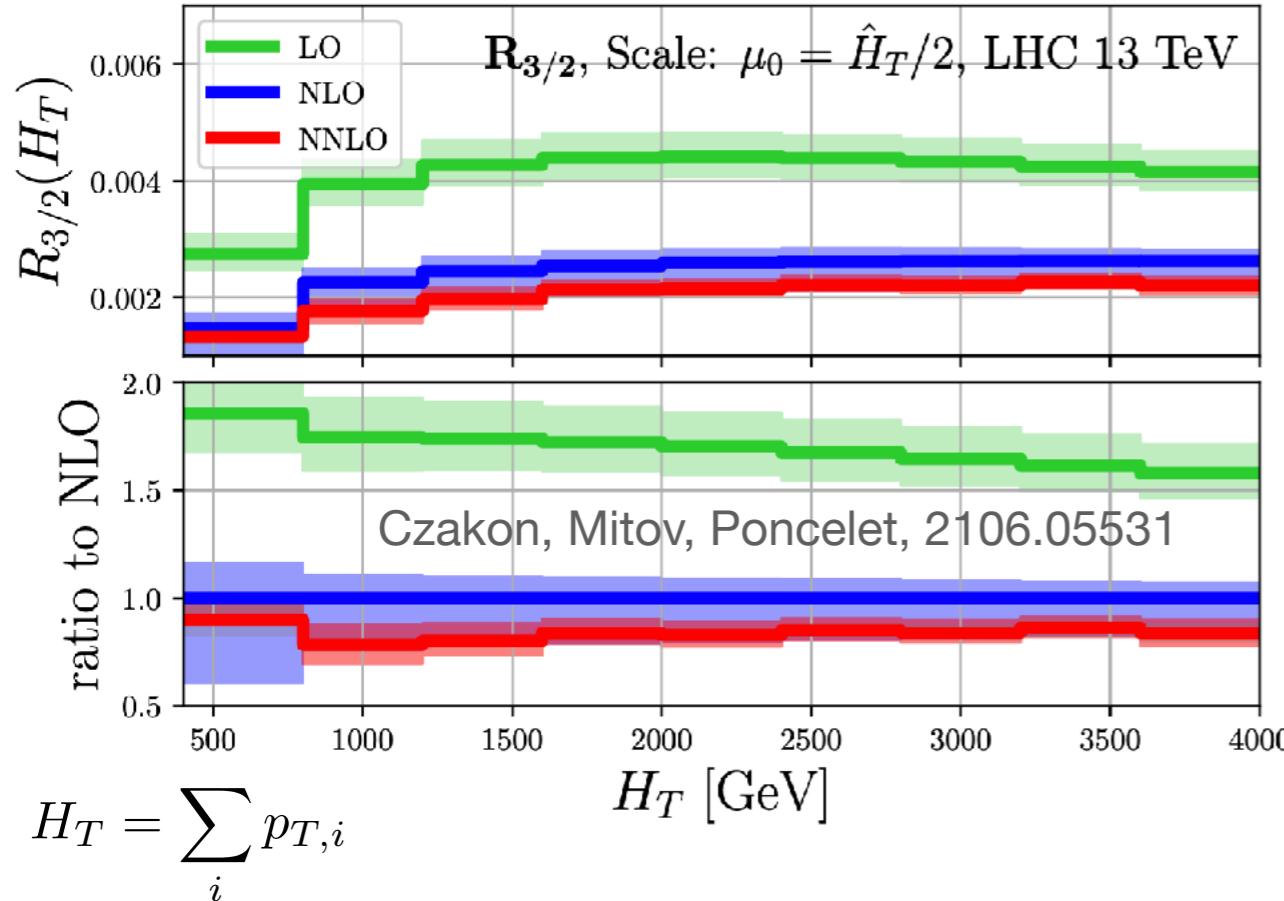
- Many well established general purpose methods to combine virtual and real corrections at NNLO:
  - Antenna subtraction [A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover]; colorful subtraction [G. Somogyi, Z. Trocsanyi, V. Del Duca]; **N-jettiness** [R. Boughezal, C. Focke, X. Liu, F. Petriello; J. Gaunt, M. Stahlhofen, F. J. Tackmann, J. R. Walsh]; **Projection-to-born** [M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam, G. Zanderighi]; **qT subtraction** [S. Catani, M. Grazzini]; **sector subtraction** [C. Anastasiou, K. Melnikov, F. Petriello]; **sector-improved residue subtraction** [M. Czakon]
  - Even for complicated processes such as multi-jet production, antenna subtraction and sector-improved residue subtraction have been shown to work successfully.
  - Challenges: two-loop amplitudes for processes with high multiplicities or/and many mass scale.

# High-multiplicity processes at NNLO

- An impressive number of 2 to 3 processes calculated at NNLO in the past few years.
  - $\text{pp} \rightarrow 3 \text{ jet}$  M. Czakon, A. Mitov, R. Poncelet; X. Chen, T. Gehrmann, N. Glover, A. Huss, M. Marcoli
  - $\text{pp} \rightarrow \gamma\gamma + \text{jet}$  H. Chawdry, M. Czakon, A. Mitov, R. Poncelet; S. Badger, T. Gehrmann, M. Marcoli, R. Moddie
  - $\text{pp} \rightarrow \gamma\gamma\gamma$  H. Chawdry, M. Czakon, A. Mitov, R. Poncelet; S. Kallweit, V. Sotnikov, M. Wiesemann
  - $\text{pp} \rightarrow Wbb$  H. Hartanto, R. Poncelet, A. Popescu, S. Zoia
- Remarkable progress in the past few years thanks to break-through calculations for two-loop five point amplitudes: master integral calculation (analytic or numerical) and efficient integral reduction. Very impressive community efforts!

Abreu, Agawal, Badger, Bern, Bohm, Bonetti, Bonnum-Hansen, Buccioni, Canko, Chawdry, Chicherin, Czakon, de Laurentis, Dixon, Dormans, Duhr, Febres Cordero, Gehrmann, Georgoudis, Georgoudis, Gluza, Guan, Hartanto, Heinrich, Heller, Henn, Hermann, Hidding, Ita, Jones, Kadja, Kardos, Klinkert, Kosher, Kraus, Kreer, Krys, Larsen, Liu, Lo Presti, Ma, Maitre, Marcoli, Mitov, Moddie, Moriello, Page, Panzer, Papadopoulos, Pascual, Peraro, Poncelet, Ruf, Schabinger, Schulze, Smirnov, Sotnikov, Syrrakos, Tancredi, Tommasini, Tschernow, von Manteuffel, Wang, Wasser, Weinzierl, Wever, Zeng, Zhang, Zoia, .....

# Three-jet production at leading color



- NNLO for three-jet production has been achieved using different subtraction scheme: sector-improved residue subtraction (left) v.s. antenna (right).
- Main bottleneck to full color is virtual two-loop amplitudes (master integrals known).
- 3/2 jet ratio is important for alphas measurement. Giant NLO K factor. NNLO stabilize the ratio and smaller scale uncertainties. How to better estimate scale uncertainties?
- Small corrections to triple jet invariant mass at NNLO, <10% event at 2 TeV.

# Wbb production at leading color

- First NNLO corrections for a 1-mass 5 particle processes.
- Irreducible background to  $\text{pp} \rightarrow \text{WH}$  and  $\text{pp} \rightarrow \text{bt}$ .
- Test ground for b-quark scheme.
- Substantial contributions from two-loop virtual corrections: 5%(incl.) v.s. 10%(excl.)

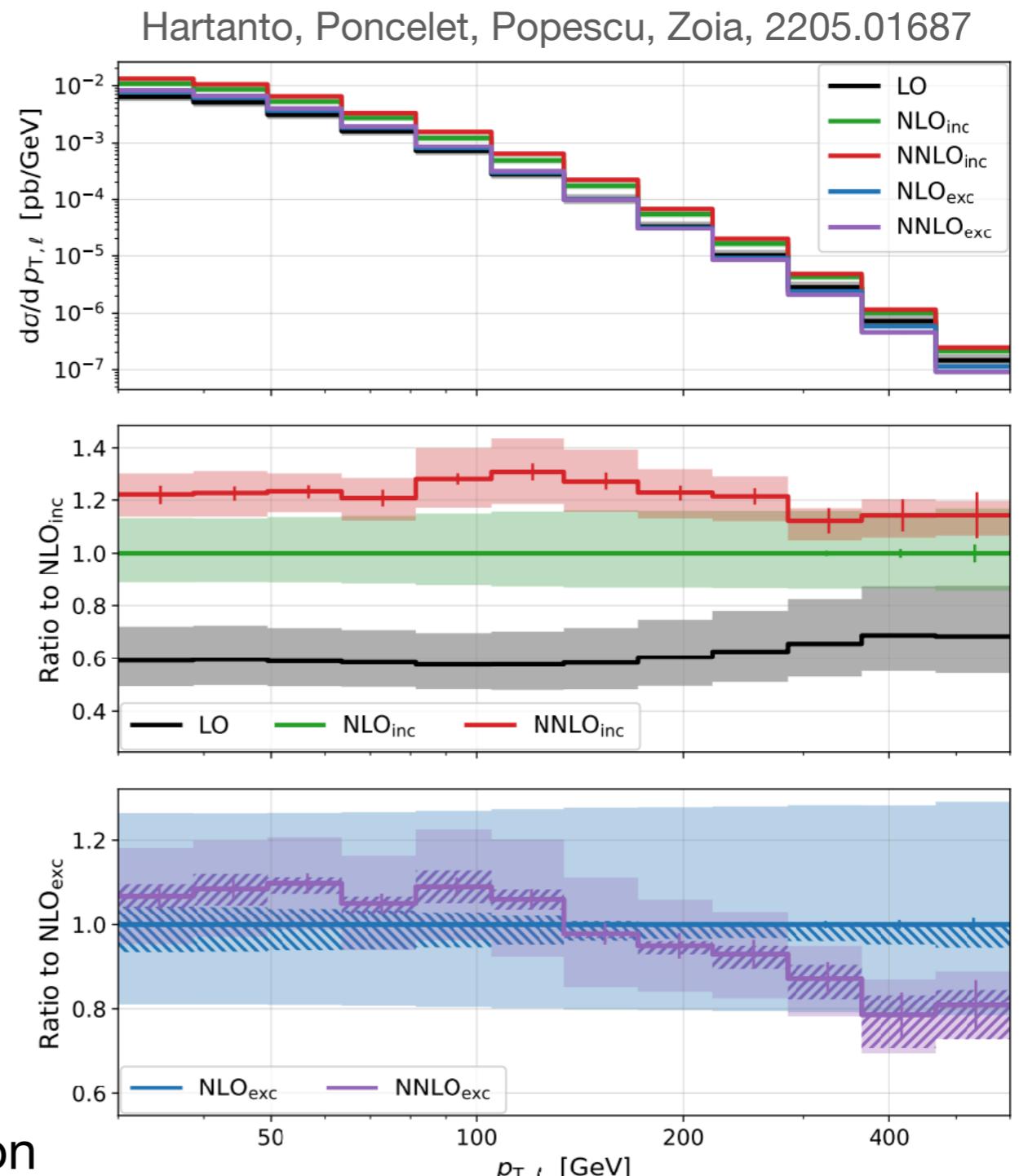
**at least 2 b-jet    exactly 2 b-jet**

	inclusive [fb]	$\mathcal{K}_{\text{inc}}$	exclusive [fb]	$\mathcal{K}_{\text{exc}}$
$\sigma_{\text{LO}}$	$213.2(1)^{+21.4\%}_{-16.1\%}$	-	$213.2(1)^{+21.4\%}_{-16.1\%}$	-
$\sigma_{\text{NLO}}$	$362.0(6)^{+13.7\%}_{-11.4\%}$	1.7	$249.8(4)^{+3.9(+27)\%}_{-6.0(-19)\%}$	1.17
$\sigma_{\text{NNLO}}$	$445(5)^{+6.7\%}_{-7.0\%}$	1.23	$267(3)^{+1.8(+11)\%}_{-2.5(-11)\%}$	1.067



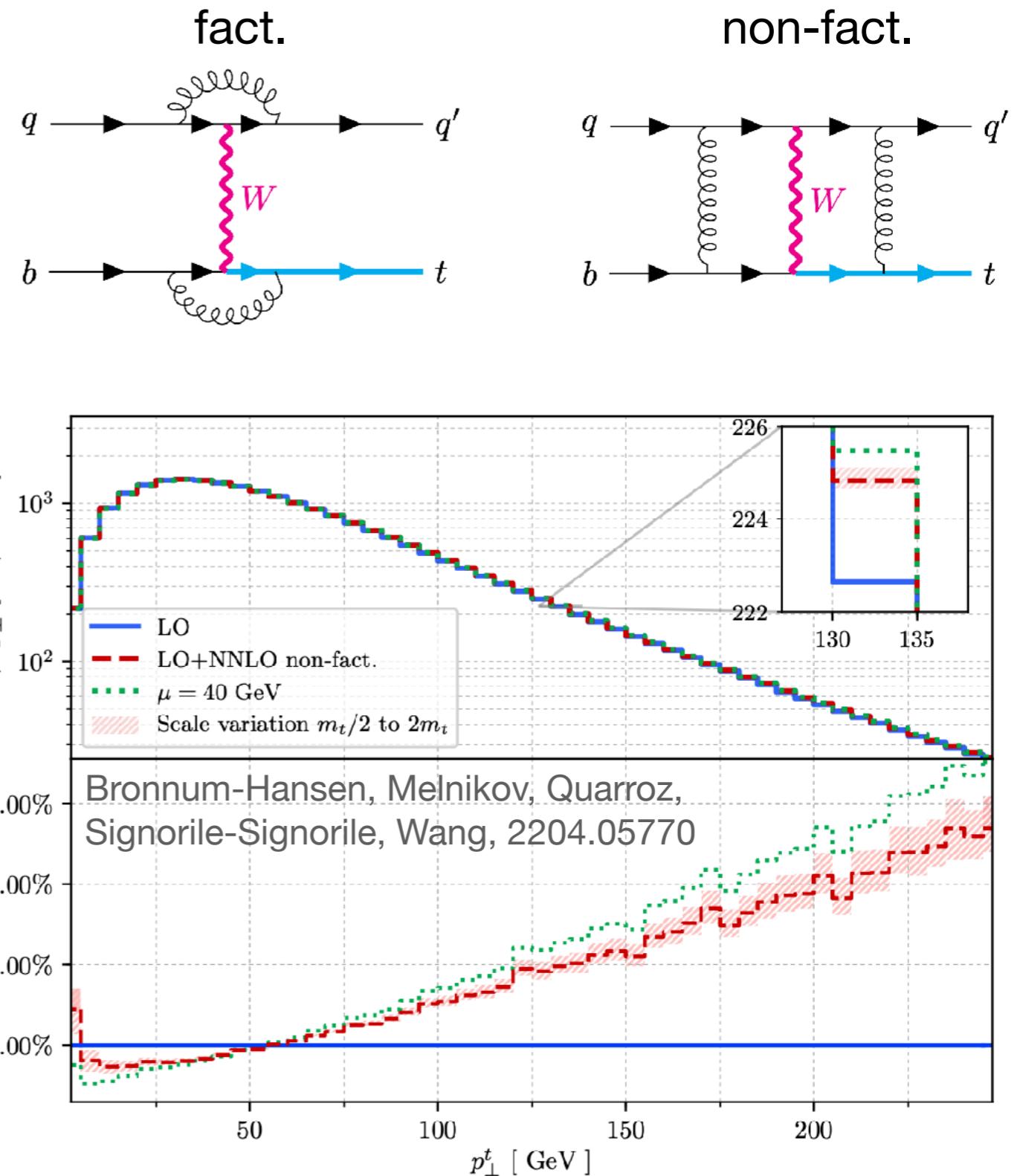
uncorrelated scale variation

Stewart, Tackmann, 1107.2117



# More mass scale: non-fact. single top

- Large rate for single-top at the LHC can be used to measure properties of top quark and CKM.
- Previously NNLO corrections for **factorisable only**: [M. Brucherseifer, F. Caola, K. Melnikov; E. Berger, J. Gao, C.-P. Yuan, H.X. Zhu; J. Campbell, T. Neumann, Z. Sullivan].
- Main bottleneck was two-loop virtual non-factorizable diagrams, recently become available [C. Bronnum-Hansen, K. Melnikov, J. Quarroz, C.Y. Wang] using auxiliary mass flow method [X. Liu, Y.Q. Ma, C.Y. Wang].
- Only slightly smaller than factorisable corrections: 0.4% for inclusive Xsec, O(1-2)% for kinematical corrections. Need to be taken into account in future percent-level measurements.



# N3LO frontier

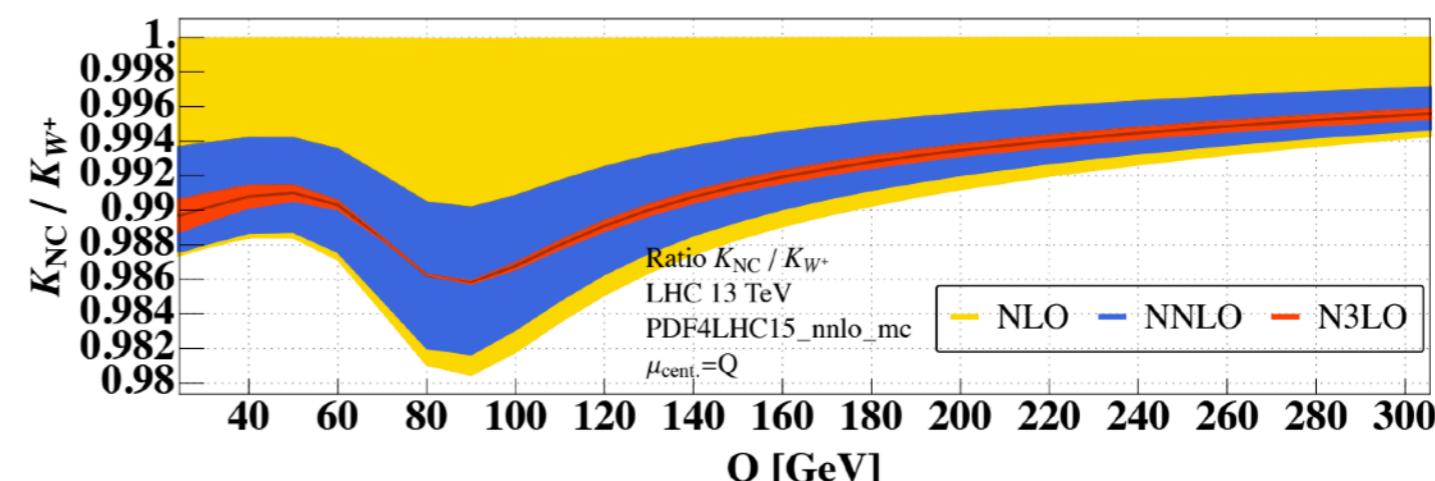
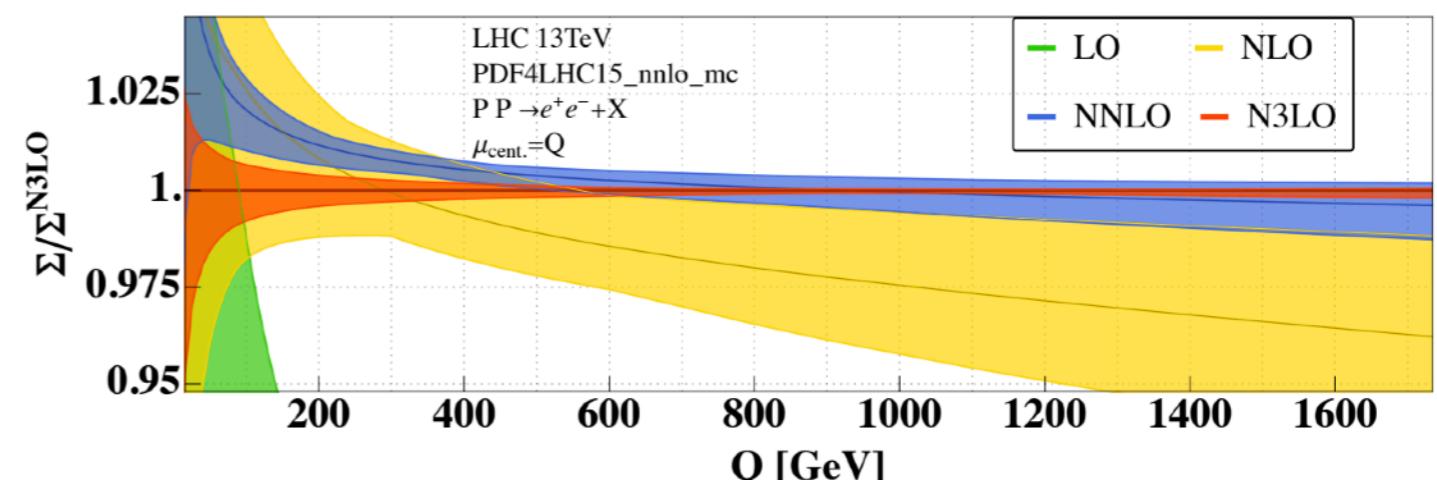
- Rapid developments of N3LO calculations in the past few years.
  - **gg $\rightarrow$ H:** C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger; X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss, B. Mistlberger, A. Pelloni; B. Mistlberger; F. Dulat, B. Mistlberger, A. Pelloni; L. Cieri, X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss; G. Billis, B. Dehnadi, M. A. Ebert, J. K. L. Michel, and F. J. Tackmann
  - **bb $\rightarrow$ H:** C. Duhr, F. Dulat, B. Mistlberger; Duhr, F. Dulat, V. Hirschi, B. Mistlberger
  - **pp $\rightarrow$ H+2j:** F. Dreyer, A. Karlberg
  - **DIS jet production:** J. Currie, T. Gehrmann, E. W. N. Glover, A. Huss, J. Niehues, A. Vogt; T. Gehrmann, A. Huss, J. Niehues, A. Vogt, and D. M. Walker
  - **gg $\rightarrow$ HH:** L.B. Chen, H.T. Li, H.S. Shao, J. Wang
  - **Neutral-current Drell-Yan:** C. Duhr, F. Dulat, B. Mistlberger; X. Chen, T. Gehrmann, E.W.N. Glover, A. Huss, T.Z. Yang, HXZ; C. Duhr, B. Mistlberger; X. Chen, T. Gehrmann, E.W.N. Glover, A. Huss, P. Monni, L. Rottoli, E. Re, P. Torrielli
  - **Charged-current Drell-Yan:** C. Duhr, F. Dulat, B. Mistlberger
- Three working methods for infrared subtraction: analytic, qT subtraction, Project-to-Born
- More differential, color singlet production processes (s-channel or t-channel).

# Z/ $\gamma^*$ at N3LO

- The last missing contribution to neutral-current DY become available [C. Duhr, B. Mistlberger].
- The main new ingredient is the singlet axial-current contribution, treated in Larin scheme. Three-loop virtual amplitudes are only known recently [T. Ahmed, L. Chen, M. Czakon; L. Chen, M. Czakon, M. Niggetiedt; T. Gehrmann, A. Primo].
- The size of corrections and scale dependence behavior is similar to previous  $\gamma^*$  or W production.
- In particular, scale variation band does not overlap between NNLO and N3LO.
- The ratio of NC-DY and W production shows a few percent difference in K factor. This demonstrate that care must be taken when making precision statement for W production using knowledge from NC-DY.

Duhr, Mistlberger, 2111.10379

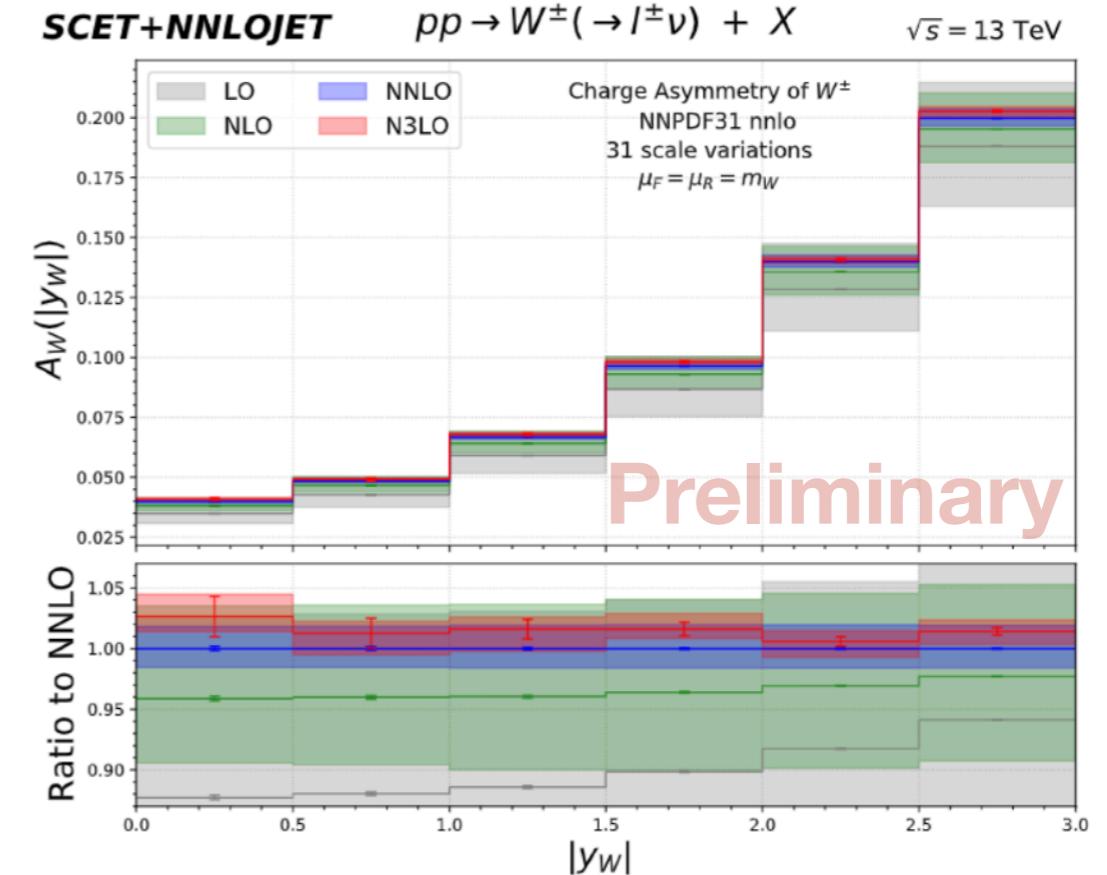
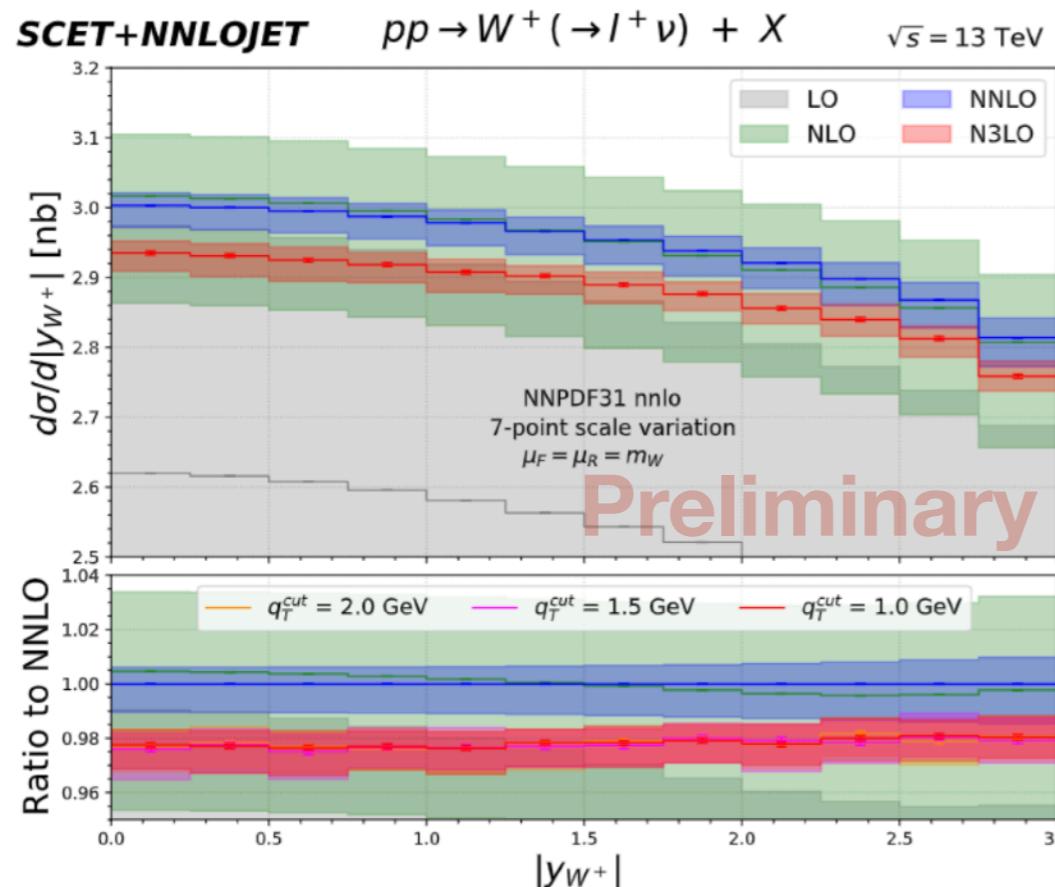
$Q$ [GeV]	$\Sigma^{\text{N}3\text{LO}}$ [pb]	$K^{\text{N}3\text{LO}}$	$\delta(\text{PDF}-\alpha_S)$	$\delta(\text{PDF-TH})$
30	$531.7^{+1.53\%}_{-2.54\%}$	0.952	+3.7%	2.8%
60	$112.636^{+0.97\%}_{-1.29\%}$	0.97	-3.8%	2.5%
91.1876	$21756.4^{+0.7\%}_{-0.86\%}$	0.977	+2.8%	2.5%
100	$458.473^{+0.66\%}_{-0.79\%}$	0.979	-2.5%	2.5%
300	$1.24661^{+0.26\%}_{-0.29\%}$	0.992	+2.2%	1.7%



# W boson rapidity at N3LO



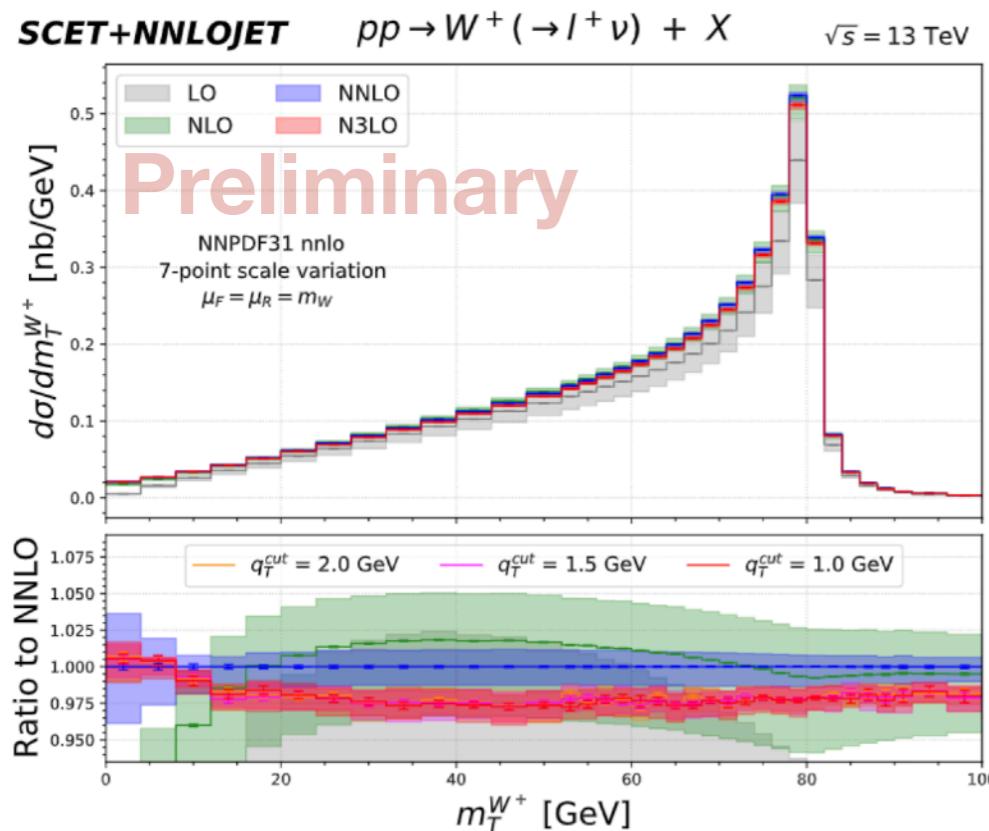
Chen, Gehrmann, Glover, Huss, Yang, HXZ, to appear



- Relatively flat N3LO corrections as a function of rapidity, about -2.5%.
- Similar to inclusive distribution, rapidity distribution shows non-overlap scale variation band from NNLO to N3LO.
- Charge asymmetry sensitive to PDFs.
- Scale uncertainties varying independently in the numerator and denominator. N3LO corrections depends on rapidity. Overlap of scale variation band observed in charge asymmetry.

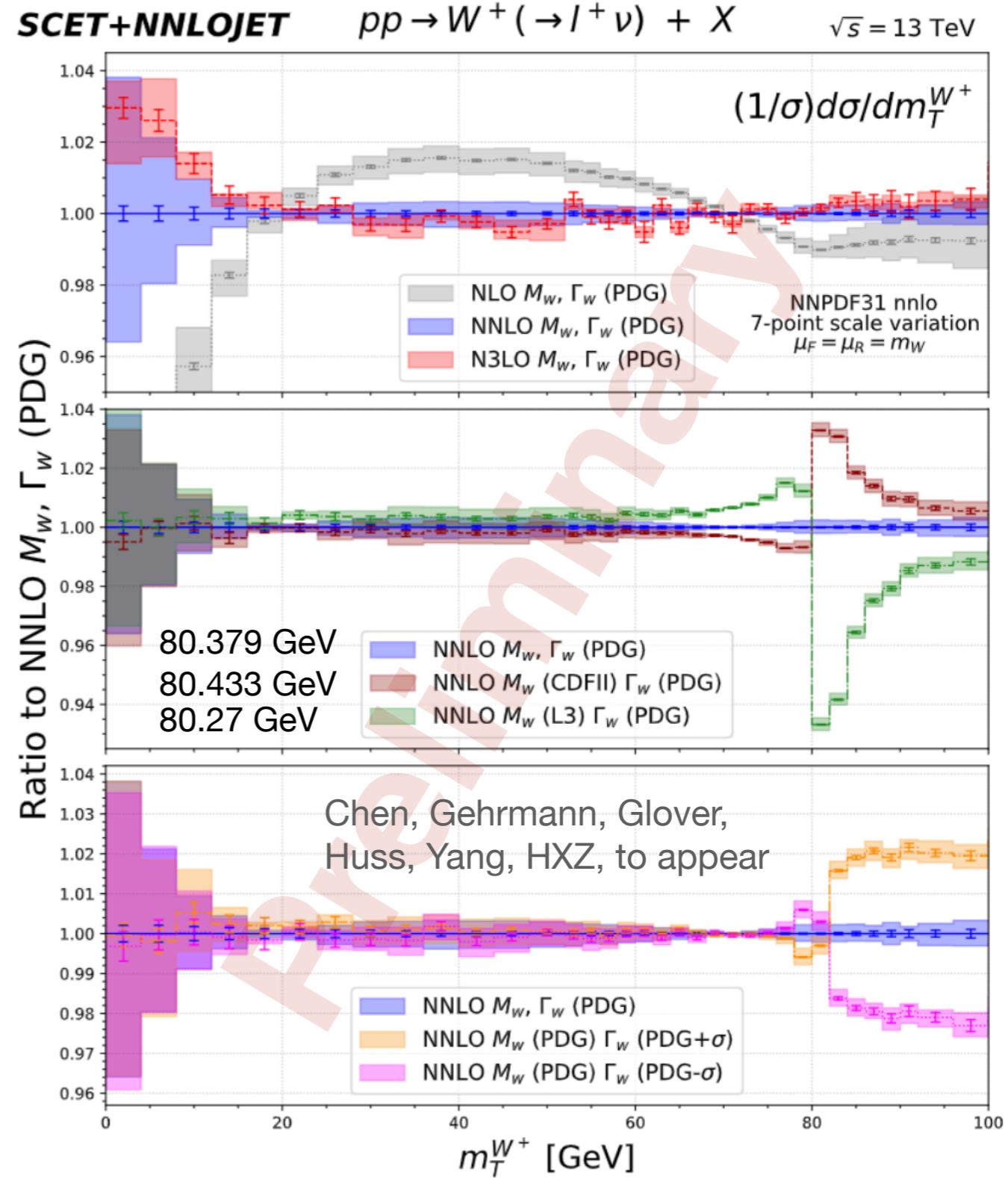
$$A_W(|y_W|) = \frac{d\sigma/d|y_{W^+}| - d\sigma/d|y_{W^-}|}{d\sigma/d|y_{W^+}| + d\sigma/d|y_{W^-}|}$$

# W transverse mass at N3LO

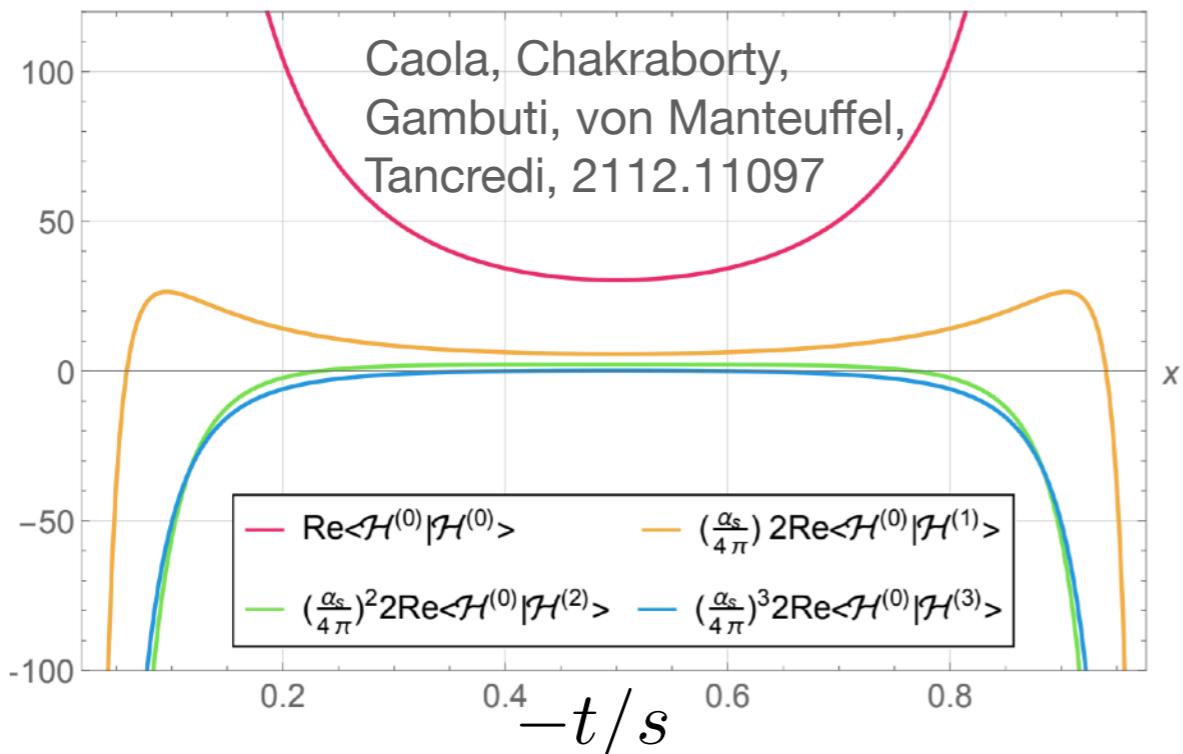


- W boson transverse mass distribution play significant role in recent CDFII W mass determination.
- Normalized distribution show convergence of perturbation series at N3LO (less than 1% in the peak region), in contrast to large corrections from NLO to NNLO.
- It suggests that sensitivity to EW parameter can be reliably estimated using NNLO calculation.
- Variation of EW input parameters  $M_W$  and  $\Gamma_W$  leads to leads to change of normalized shape by 2-6%.

Chen, Gehrmann, Glover, Huss, Yang, HXZ, to appear



# N3LO beyond DY-type processes



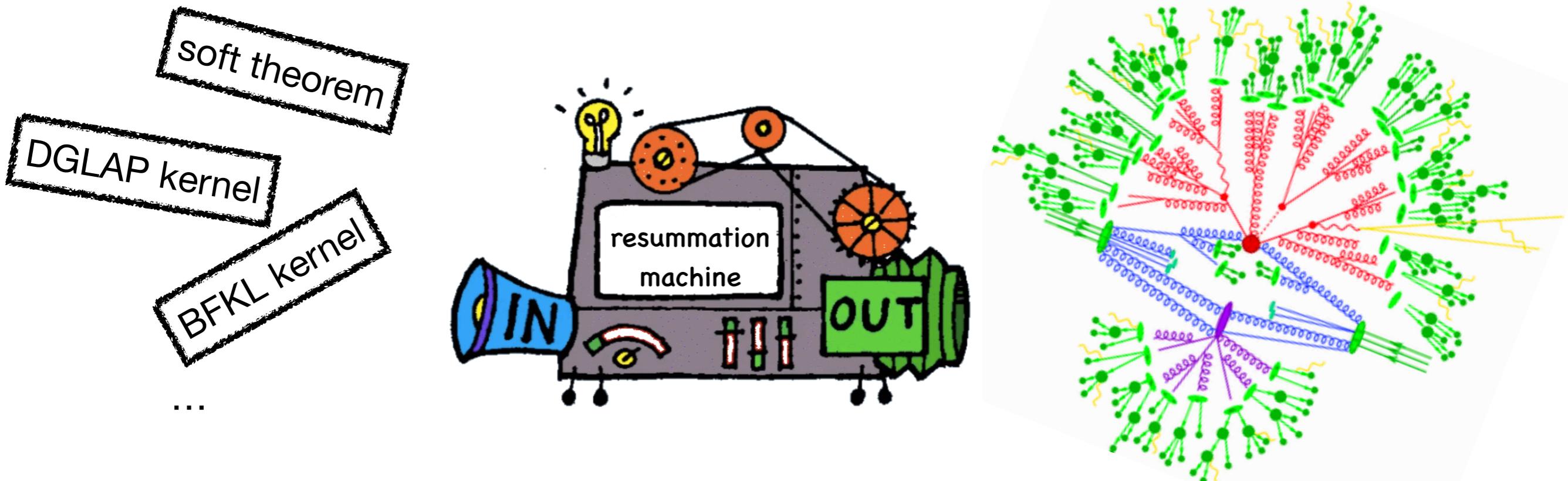
- Three-loop amplitudes for dijet production become available in the last year [F. Caola, A. Chakraborty, G. Gambuti, A. von Manteuffel, L. Tancredi]<sup>2</sup>.
- Virtual amplitudes alone show large corrections, comparable with two-loop amplitudes.
- Regge trajectory extracted to three loops for the first time using the recently defined cut scheme [G. Falcioni, E. Gardi, N. Maher, C. Milloy, L. Vernazza]<sup>2</sup>.

$$\begin{aligned} \mathcal{M}_{ij \rightarrow ij}^{(-)} &= \underbrace{\mathcal{M}_{ij \rightarrow ij}^{(-)SR} + \mathcal{M}_{ij \rightarrow ij}^{(-)MR} \Big|_{\text{planar}}}_{\mathcal{M}_{ij \rightarrow ij}^{(-)\text{pole}}} + \mathcal{M}_{ij \rightarrow ij}^{(-)MR} \Big|_{\text{nonplanar}} \\ &= \mathcal{M}_{ij \rightarrow ij}^{(-)\text{pole}} + \mathcal{M}_{ij \rightarrow ij}^{(-)\text{cut}}. \end{aligned} \quad (10)$$

$$\mathcal{M}_{ij \rightarrow ij}^{(-)\text{pole}} = \tilde{C}_i(t) \tilde{C}_j(t) e^{\tilde{\alpha}_g(t) C_A L} \mathcal{M}_{ij \rightarrow ij}^{\text{tree}}$$

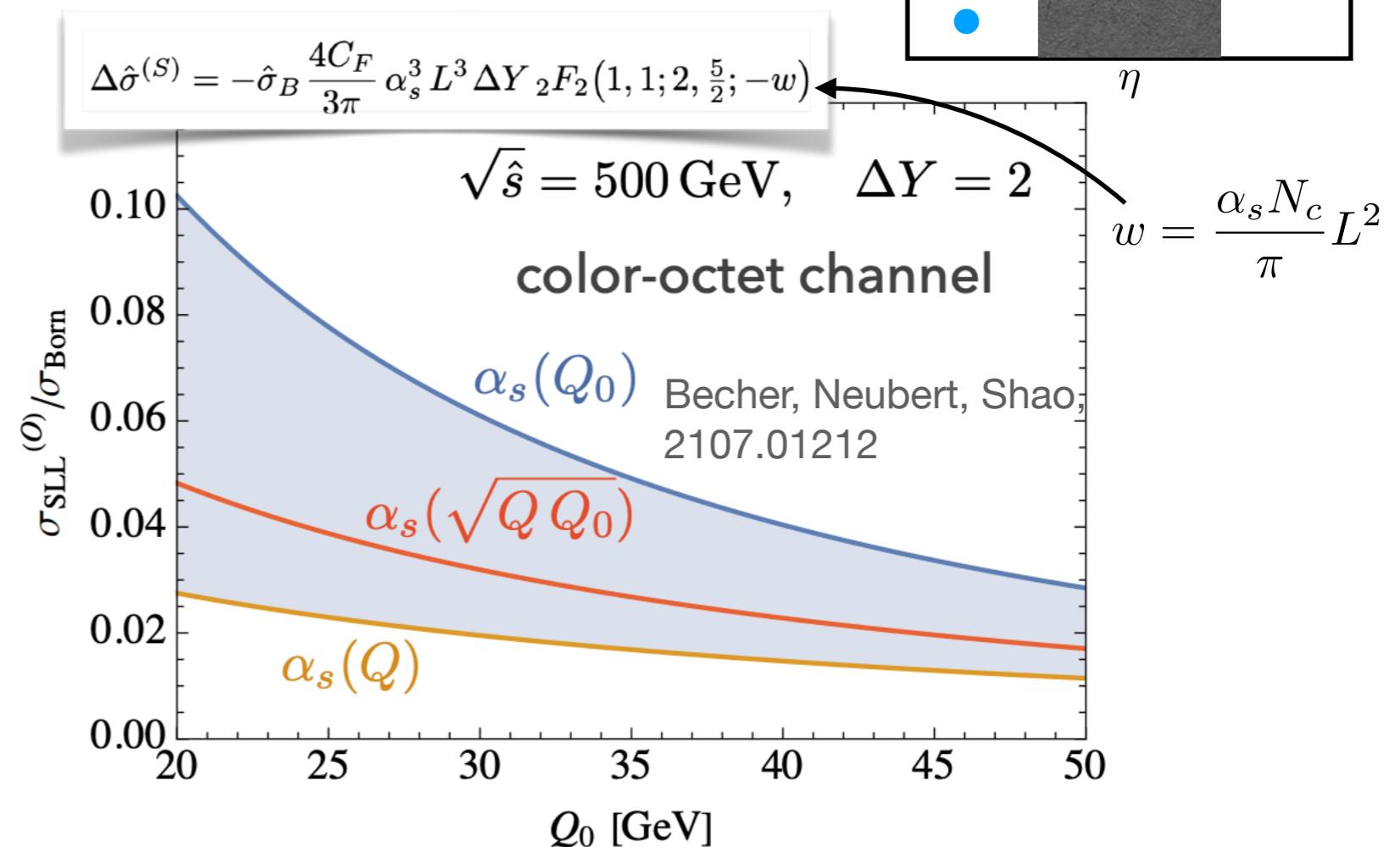
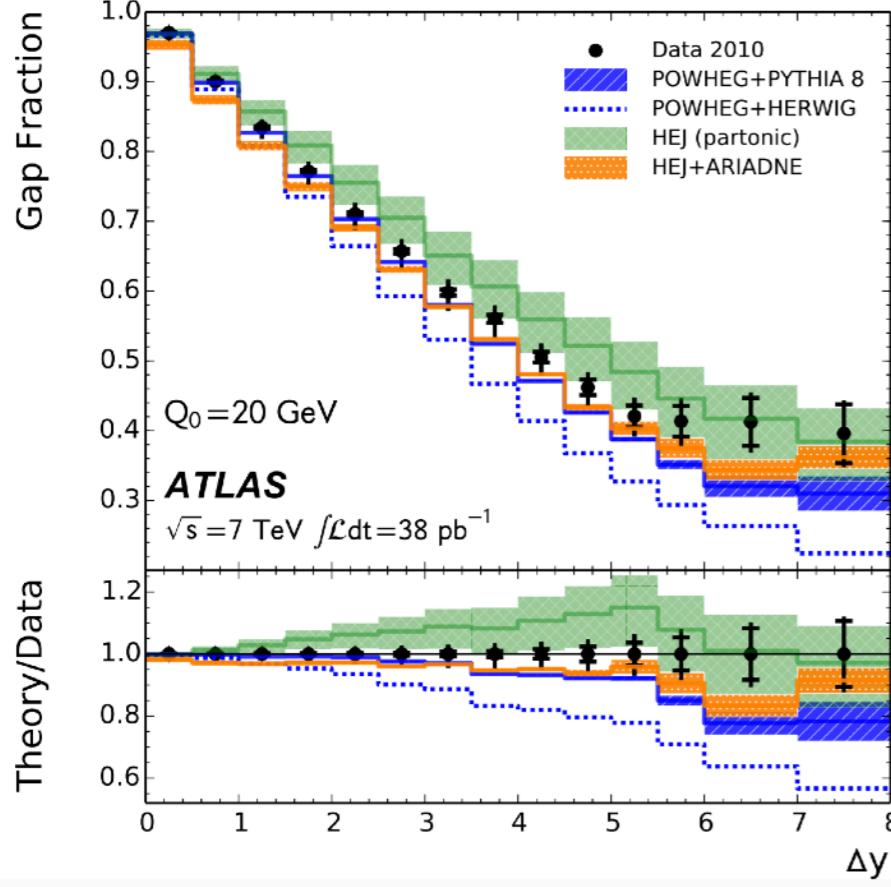
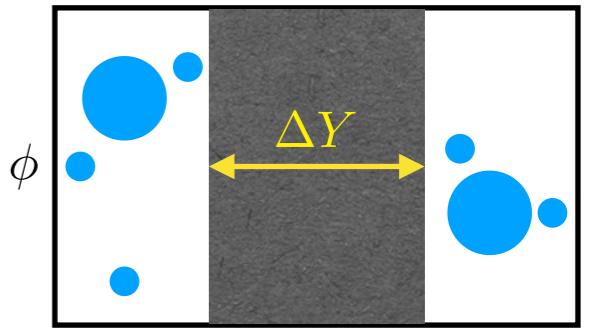
- An important ingredient missing for cross-section level dijet production: N3LO infrared subtraction. Some of the subtraction terms are available, but substantial works remain to construct a full fledged subtraction scheme.
- The best near-term hope to achieve differential N3LO beyond DY-type process is diphoton production. Three-loop virtual amplitudes are known [F. Caola, A. von Manteuffel, L. Tancredi; P. Bargiela, F. Caola, A. von Manteuffel, L. Tancredi], as well as infrared subtraction terms in the qT subtraction scheme [Y. Li, HXZ; M.X. Luo, T.Z. Yang, HXZ, Y.J. Zhu; M. Ebert, B. Mistlberger, G. Vita]. But combining these ingredients requires substantial efforts.

# QCD resummation



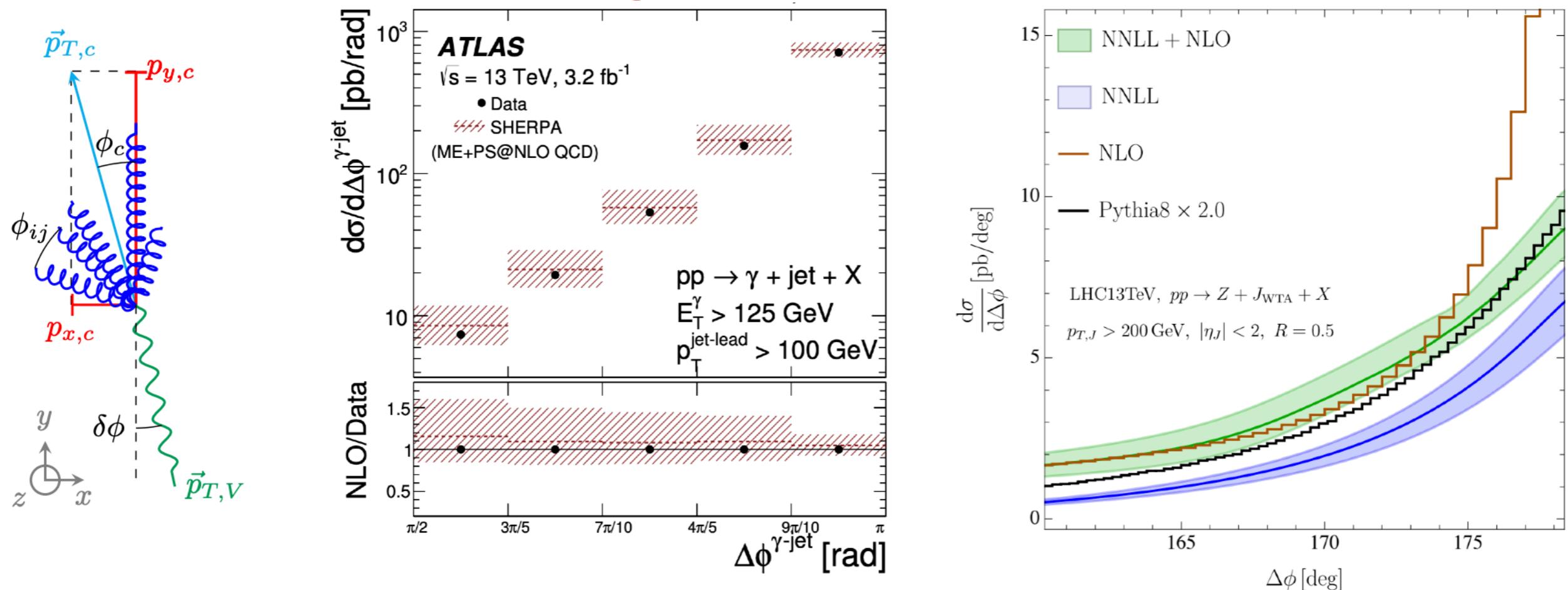
- A characteristic feature of perturbative QCD is multiple soft and collinear radiation at high energy.
- For inclusive cross section, they cancel with virtual terms. But at the boundary of phase space, cancellation is not perfect, and large logarithms are often found for cross section.
- It is necessary to resum all the large logarithms to reach reliable predictions.
- Several different methods available for all-order resummation of large logarithms: branching algorithm, CASER/ARES, SCET, Collins-Soper-Sterman formalism.

# super leading log



- Jets production with large rapidity gap is important in VBF Higgs production, as well as probing BFKL dynamics.
- It has been known for 15 years the existence of a class of super-leading logarithms in non-global observables [J. Forshaw, A. Kyrieleis, M. Seymour]. Five-loop explicit calculation available, but no resummation formula known. Effects not capture by parton shower.
- Recently an all-order analytic resummation formula become available, allowing for the first time estimating impacts of SLL to gap fraction.

# Boson-jet correlation

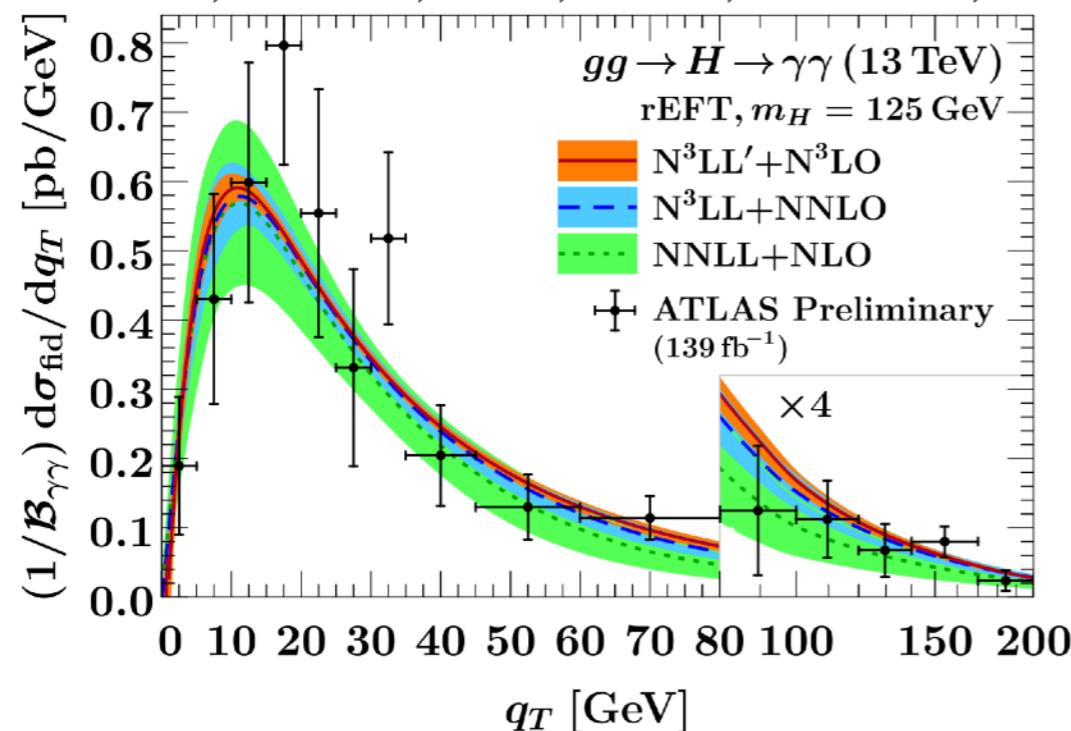


Chien, Rahn, Shao, Waalewijn, Wu, 2205.05104

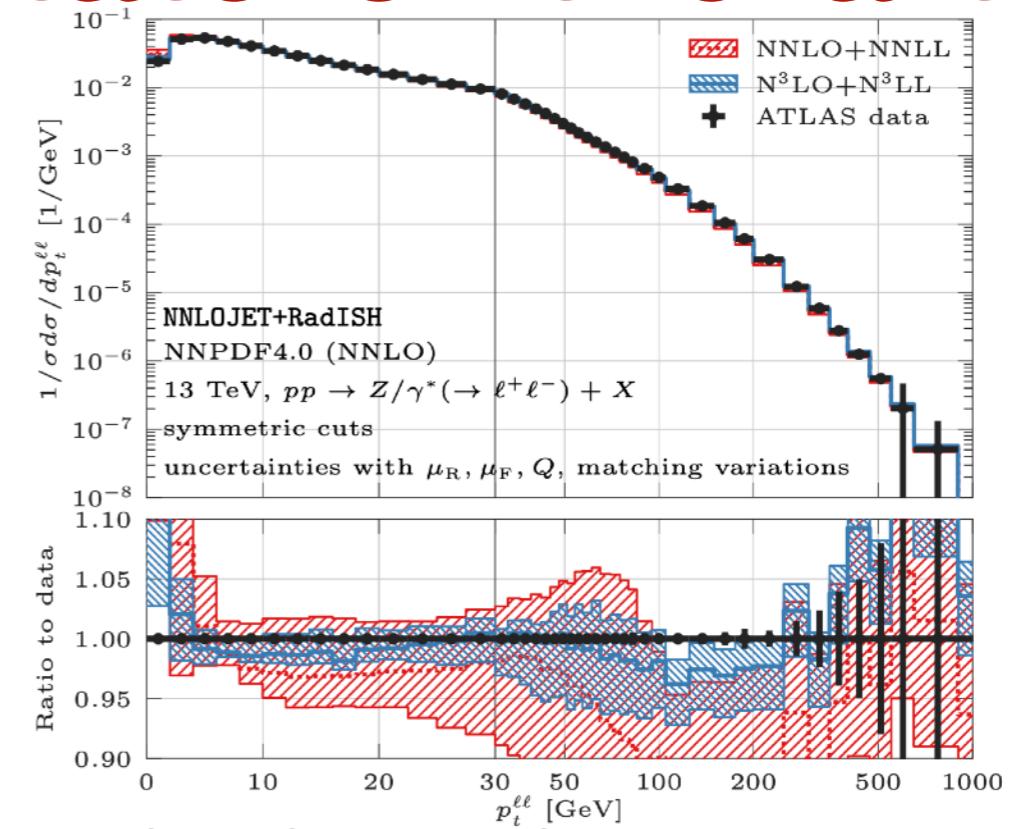
- Jet-boson azimuthal angle correlations are interesting experimentally for studying QCD dynamics, in particular in heavy ion collision [X.H. Liu, F. Ringer, W. Vogelsang, F. Yuan].
- The back-to-back correlation is enhanced by collinear and large angle soft radiations. QCD resummation complicated by the definition of jet.
- Significant simplification using recoil-free jet definition, such as WTA jet.
- First NNLL calculation for jet-boson correlation.

# pT resummation: state-of-the-art

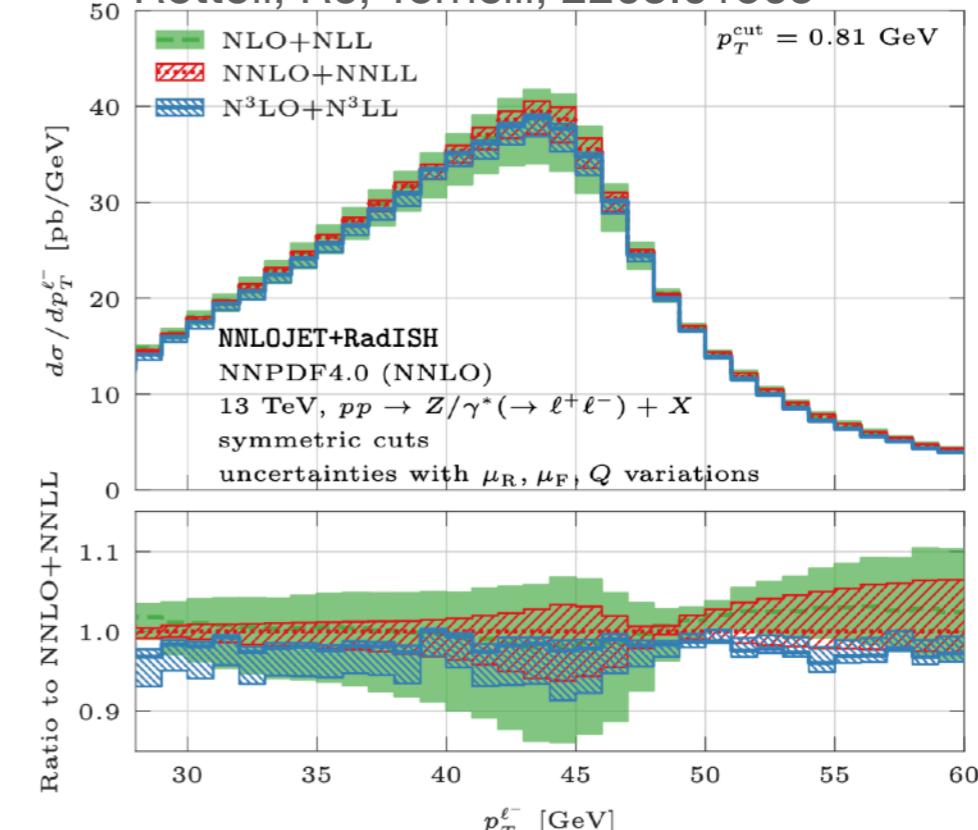
Billis, Dehnadi, Ebert, Michel, Tackmann, 2102.08039



- N3LL'+N3LO become standard for DY and Higgs pT distribution.
- Less than 5% theory uncertainties for DY for pT<50 GeV.
- Excellent agreement with ATLAS for pT spectrum. More efforts required to bring down theory uncertainties to compete with experimental ones.
- Lepton pT important for W mass measurement. Good perturbative convergence with N3LL resummation. Inclusion of EW corrections?



Chen, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli, 2203.01565



# Summary

- Remarkable progress in precision QCD theory. NNLO QCD corrections for high multiplicities processes, N3LO go more differential.
- Many three-loop ingredients for genuine  $2 \rightarrow 2$  at N3LO become available, calls for the development of full fledged N3LO IR subtraction scheme.
- QCD resummation greatly extends its breadth and depth: first LL resummation of super leading logarithms; First NNLL resummation of jet-boson correlation; N3LL'+N3LO becomes standard for pT resummation.
- Looking forward:
  - Conventional scale variation underestimate theory uncertainties for exclusive jet production. Well-known problem, but becomes more striking with N3LO theory data point. Better method of uncertainty estimation? New sources of large logarithms to resum?
  - How would N3LO PDFs change this story?
  - Incorporating EW and mixed QCD-EW corrections?
  - Pushing the limit of logarithmic accuracy for resummation? For pT resummation, several 4-loop anomalous dimension are available. Missing ingredient: 4-loop DGLAP splitting functions.
  - .....