Form factors for $b \to s\ell\ell$ and $b \to c\ell\bar{\nu}$ transitions from lattice QCD

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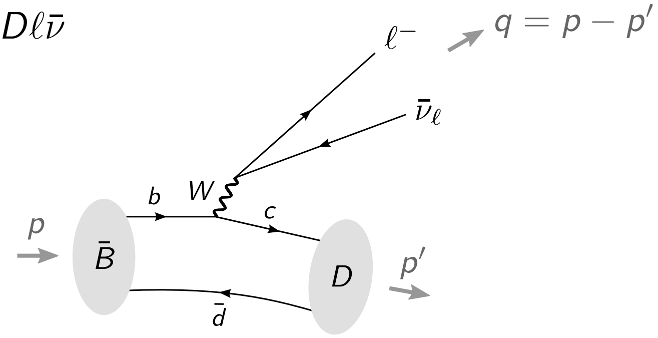
Outline

1. $b \to c\ell\bar{\nu}$
2. $b \to s\ell\ell$

Example: $\bar{B} \to D\ell\bar{\nu}$



Differential decay rate in the Standard Model:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2|V_{cb}|^2}{24\pi^3 m_B^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |\mathbf{p}_D| \left[\left(1+\frac{m_\ell^2}{2q^2}\right) m_B^2 \mathbf{p}_D^2 |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_B^2-m_D^2)^2 |f_0(q^2)|^2\right]$$

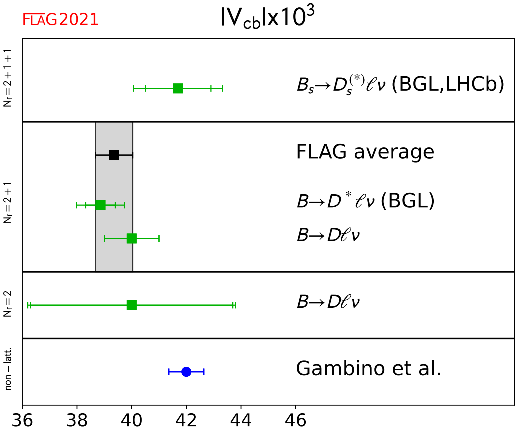
where the form factors $f_+(q^2)$ and $f_0(q^2)$ come from the QCD matrix element

$$\langle D|\bar{c}\gamma^\mu b|\bar{B}\rangle = \left[(p+p')^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right] f_+(q^2) + \frac{m_B^2 - m_D^2}{q^2} q^\mu f_0(q^2)$$

$|V_{cb}|$

Exclusive

Inclusive

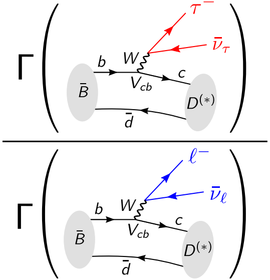


Note that, using CKM unitarity, e.g.

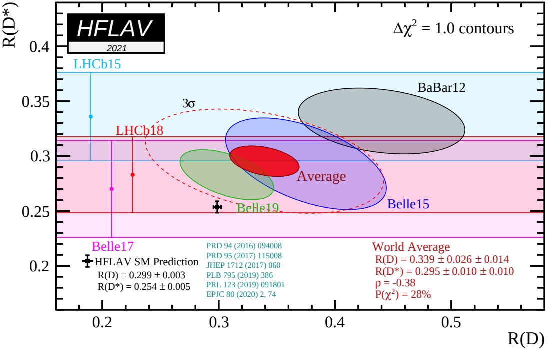
$$\mathcal{B}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} \propto V_{cb}^2, \quad \mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} \propto V_{cb}^4, \quad (\epsilon_K)_{\mathrm{SM}} \propto V_{cb}^{3.4}.$$

Tests of lepton-flavor universality

$R(D^{(*)})_{\text{SM}} =$



where $\ell = \mu$ or $\ell = e$



$b \to c\ell\bar{\nu}$ form factors from lattice QCD: main references

Transition	References
$B \to D$	FNAL/MILC `1503.07237`, HPQCD `1505.03925`
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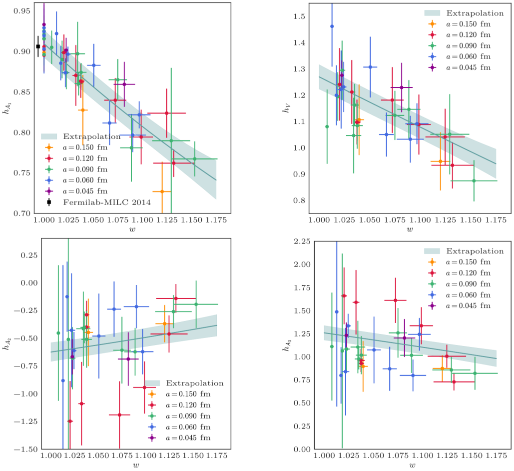
*at zero recoil only

In addition,

- Preliminary results for $B \to D^*$ at nonzero recoil: JLQCD [link], HPQCD [link]
- Fits to lattice+exp. results for $B_{(s)} \to D_{(s)}^{(*)}\ell\nu$ including dispersive bounds: Martinelli, Naviglio, Simula, Vittorio, `2105.07851`, `2105.08674`, `2204.05925`

$B \to D^*$ form factors at nonzero recoil [A. Bazavov *et al.* (FNAL/MILC), `2105.14019`]

First calculation at nonzero recoil, “first-generation” lattice methods – asqtad u, d, s and Fermilab-clover c, b



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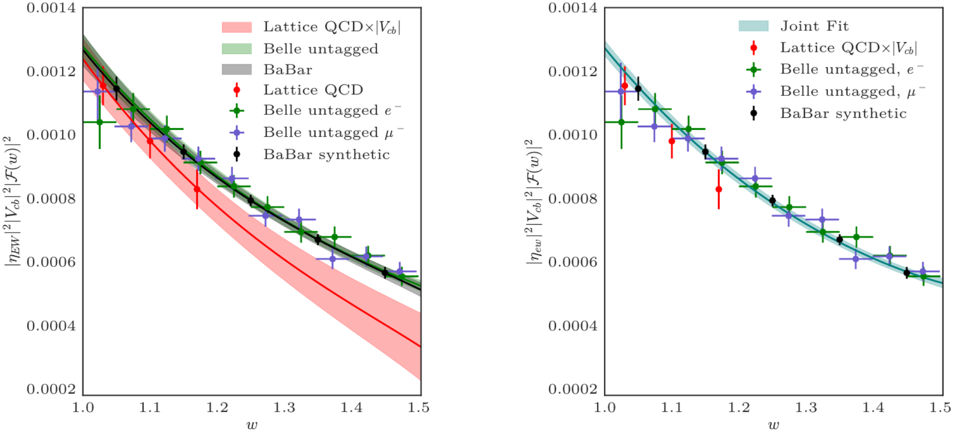
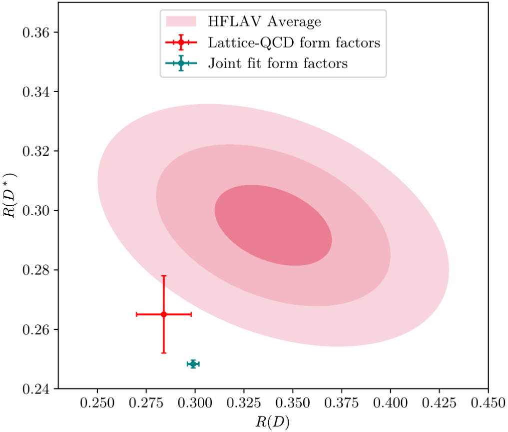


$$|V_{cb}| = (38.40 \pm 0.74) \times 10^{-3}$$

$R(D^{(*)})$



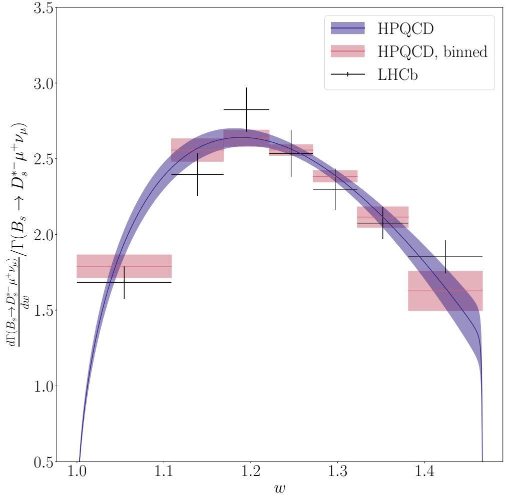
[FNAL/MILC, 2105.14019; HFLAV, 1909.12524/EPJC 2021]

Lattice-QCD-only SM predictions:

$$\begin{aligned} R(D^*)_{\text{FNAL/MILC 2021}} &= 0.265 \pm 0.013 \\ R(D)_{\text{FNAL/MILC 2015}} &= 0.284 \pm 0.014 \\ R(D)_{\text{FLAG 2021}} &= 0.2934 \pm 0.0053 \end{aligned}$$

$B_s \to D_s^*$ form factors [J. Harrison and C. Davies (HPQCD), `2105.11433`]

First calculation at nonzero recoil, using second-generation MILC configurations, both b and c quarks treated using the same lattice action (HISQ) as the light quarks, which requires extrapolations in m_b but largely eliminates the renormalization uncertainty.



$|V_{cb}|$ result using LHCb measurements [`2001.03225`/PRD 2020]:

$$|V_{cb}| = 42.2(1.5)_{\mathrm{latt}}(1.7)_{\mathrm{exp}}(0.4)_{\mathrm{EM}} \times 10^{-3}$$

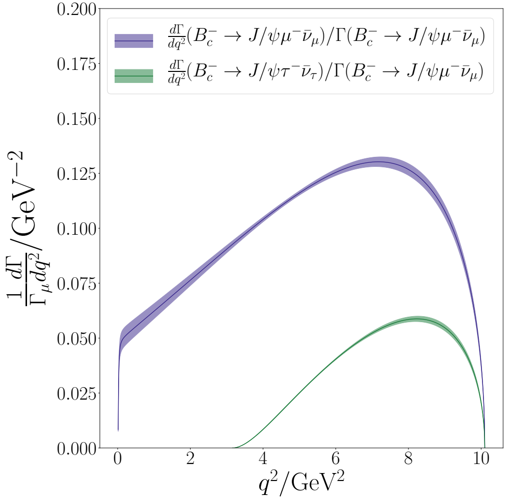
SM prediction for τ-vs.-μ ratio:

$$R(D_s^*)_{\mathrm{SM}} = 0.2490(60)_{\mathrm{latt}}(35)_{\mathrm{EM}}$$

$B_c \to J/\psi$ form factors

[J. Harrison, C. Davies, A. Lytle (HPQCD), `2007.06957`/PRD 2020; `2007.06956`/PRL 2020]

First calculation; same lattice methods as discussed on the previous page.



SM prediction for τ-vs.-μ ratio:

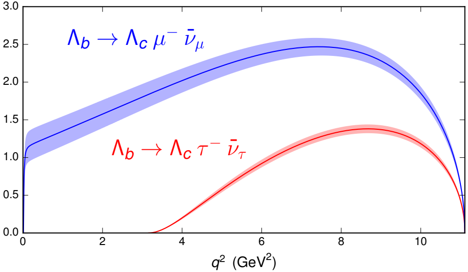
$$R(J/\psi)_{\mathsf{SM}} = 0.2582(38)$$

For comparison, the LHCb result is

$$R(J/\psi)_{\mathsf{expt.}} = 0.71(17)(18)$$

[`1711.05623`/PRL 2018]

$\Lambda_b \to \Lambda_c$ form factors

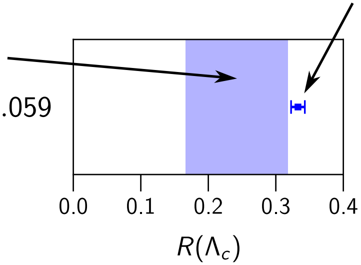


$$R(\Lambda_c)_{\mathrm{SM}} = 0.3328 \pm 0.0074_{\,\mathrm{stat}} \pm 0.0070_{\,\mathrm{syst}}$$

[W. Detmold, C. Lehner, S. Meinel,
1503.01421/PRD 2015]

$$R(\Lambda_c)_{\mathrm{expt.}} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$

[LHCb Collaboration, 2201.03497/PRL 2022]



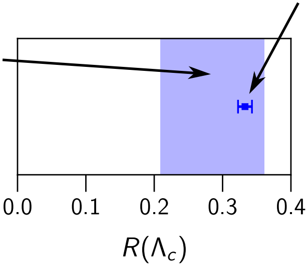
$\Lambda_b \to \Lambda_c$ form factors

The third and dominant uncertainty of the LHCb $R(\Lambda_c)_{\text{expt.}}$ is from the DELPHI result $\mathcal{B}(\Lambda_b \to \Lambda_c \mu \bar{\nu})_{\text{DELPHI}} = 6.2(1.4)\,\%$. When fitting a model that has no new physics in the muon mode, one may replace the DELPHI measurement by the SM prediction using lattice QCD,

$$\mathcal{B}(\Lambda_b \to \Lambda_c \mu \bar{\nu})_{\text{SM}} = 5.26(49)\,\%$$

(using $\tau_{\Lambda_b} = 1.471(9)$ ps and $|V_{cb}| = 40.8(1.4) \cdot 10^{-3}$). Then

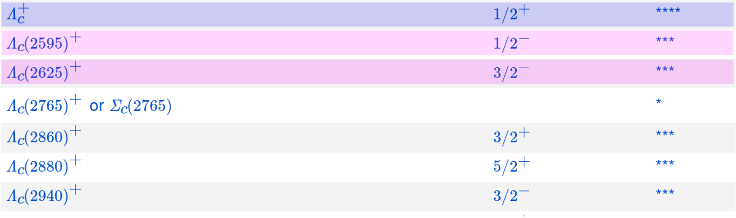
$$R(\Lambda_c)^{\text{new}}_{\text{expt.}} = \ 0.285 \pm 0.030 \pm 0.047 \pm 0.051$$



Standard Model (1σ)

Experiment (1σ)
with $\mathcal{B}(\Lambda_b \to \Lambda_c \mu \bar{\nu})$ from SM

Λ_c^* baryons $(C = +1,\ S = 0,\ I = 0)$



Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$ or $\Sigma_c(2765)$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***

$\Gamma_{\Lambda_c^*(2595)} \approx 2.6\ \text{MeV}$

$\Gamma_{\Lambda_c^*(2625)} < 1.0\ \text{MeV}$

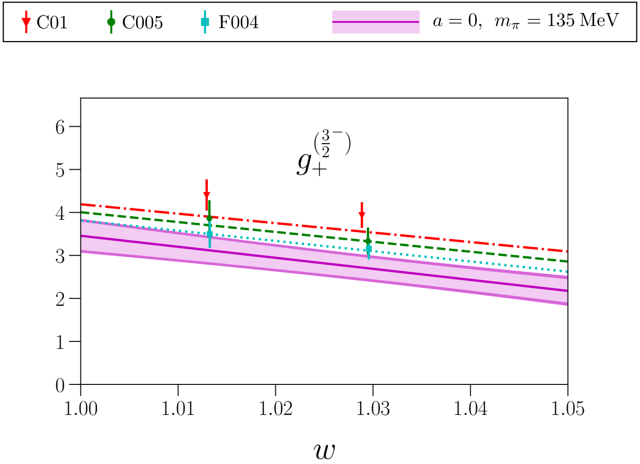
[PDG]

Note: for $m_c \to \infty$ the $\Lambda_c^*(2595)$ and $\Lambda_c^*(2625)$ baryons become mass-degenerate heavy-quark spin-symmetry partners. However, they are NOT related to the Λ_c by heavy-quark spin symmetry (the light degrees of freedom have different angular momentum).

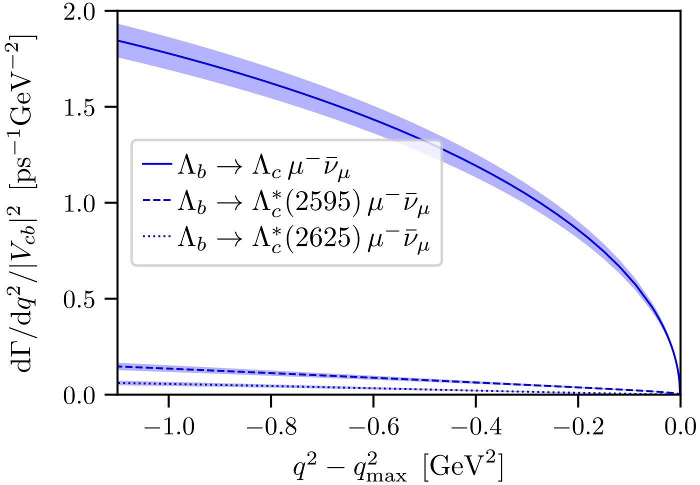
$\Lambda_b \to \Lambda_c^*(2595)$ and $\Lambda_b \to \Lambda_c^*(2625)$ form factors

[S. Meinel and G. Rendon, `2103.08775`/PRD 2021 and `2107.13140`/PRD 2022].

One of the 14 form factors for $\Lambda_b \to \Lambda_c^*(2625)$ is show below:



$\Lambda_b \to \Lambda_c^{(*)} \mu^- \bar{\nu}_\mu$ differential decay rates near $q^2_{\max}$ from lattice QCD



The relative size of $\frac{1}{2}^-$ and $\frac{3}{2}^-$ differential decay rates is opposite to the expectation from LO HQET.

Outline

1. $b \to c\ell\bar{\nu}$
2. $b \to s\ell\ell$

Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= (e\, m_b)/(16\pi^2) \;\; \bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= e^2/(16\pi^2) \;\; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= e^2/(16\pi^2) \;\; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\;\;\ldots
\end{aligned}$$

In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	…
−0.288	1.010	−0.336	4.275	−4.160	…

[Computed using EOS, `https://eos.github.io/`]

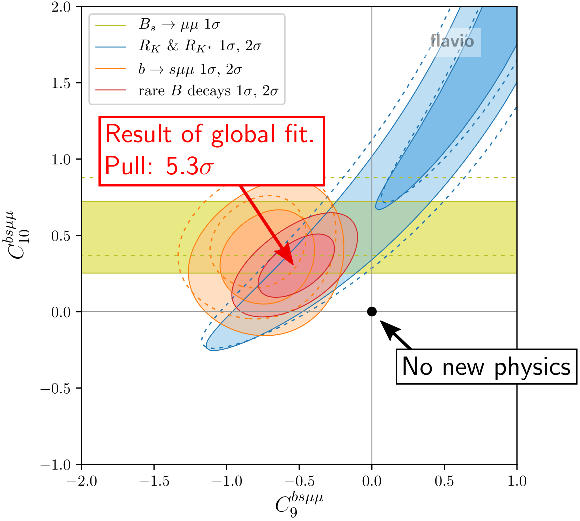
Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

For a generic decay $H_b \to H_s\ell^+\ell^-$:

Contributions from O_7, O_9, O_{10}: $\langle H_s(p')|\, \bar{s}\Gamma b \,|H_b(p)\rangle$. These local matrix elements can be calculated straightforwardly in lattice QCD.

Contributions from $O_{1,\ldots,6}$, O_8: $\int \mathrm{d}^4x \;\; e^{iq\cdot x} \, \langle H_s(p')|\, \mathrm{T}\; O_i(0)\; J^{\mu}_{\mathrm{e.m.}}(x) \,|H_b(p)\rangle$.
Calculating these nonlocal matrix elements directly in lattice QCD is extremely difficult because of the use of imaginary time (see [K. Nakayama, T. Ishikawa, S. Hashimoto, 2001.10911] for first steps). The state of the art is to use a local OPE at high q^2, and QCDF/light-cone OPE at low q^2.

Example of a recent global fit of $b \to s\mu^+\mu^-$ Wilson coefficients



Result of global fit.
Pull: 5.3σ

No new physics

[W. Altmannshofer, P. Stangl, 2103.13370/EPJC 2021]

$b \to s\ell\ell$ form factors from lattice QCD: main references

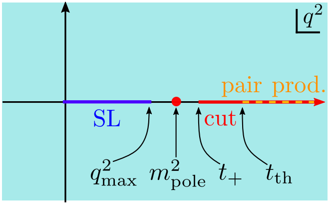
Transition	References
$B \to K$	HPQCD `1306.2384`, FNAL/MILC `1509.06235`
$B \to K^*$	Horgan, Liu, Meinel, Wingate, `1310.3722` (upd. in `1501.00367`)
$B_s \to \phi$	Horgan, Liu, Meinel, Wingate, `1310.3722` (upd. in `1501.00367`)
$B_c \to D_s$	HPQCD, `2108.11242`
$\Lambda_b \to \Lambda$	Detmold and Meinel, `1602.01399`
$\Lambda_b \to \Lambda^*(1520)$	Meinel and Rendon, `2009.09313` (updated in `2107.13140`)

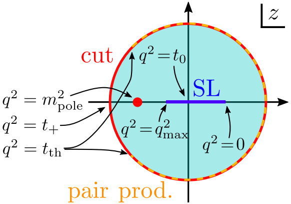
In addition,

- Fits to $\Lambda_b \to \Lambda$ lattice results with dispersive bounds:
  Blake, Meinel, Rahimi, van Dyk, `2205.06041`

Dispersive bounds on $\Lambda_b \to \Lambda$ form factors

[T. Blake, S. Meinel, M. Rahimi, D. van Dyk, 2205.06041]

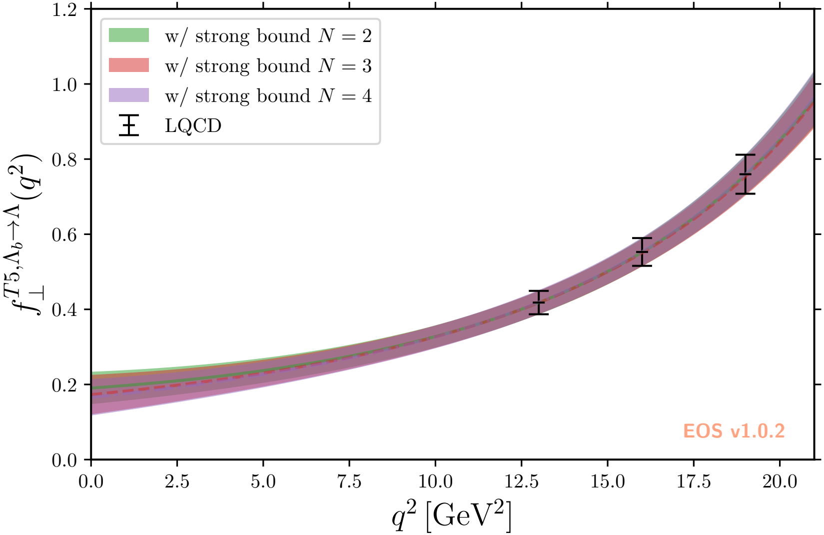




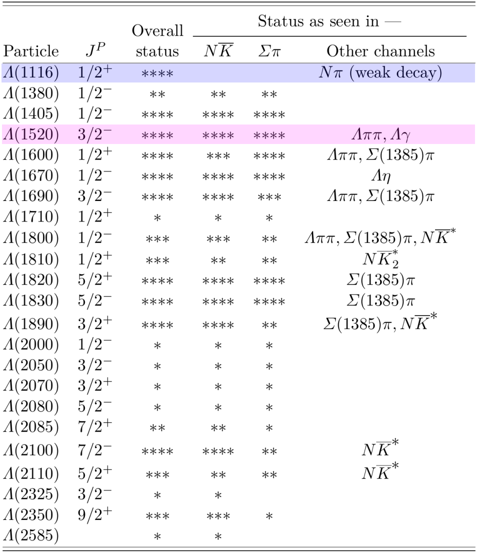
We propose a novel z-parametrization of semileptonic form factors, suitable for cases with $t_+ \equiv t_{\text{cut}} < t_{\text{th}} \equiv (m_{\text{initial}} + m_{\text{final}})^2$, in terms of **orthogonal polynomials on the unit circle**; this diagonalizes the form factor contributions to dispersive bounds.
We tested the method by performing new fits to the $\Lambda_b \to \Lambda$ form factors from lattice QCD. [In that case, $t_+ = (m_B + m_K)^2$, $t_{\text{th}} = (m_{\Lambda_b} + m_\Lambda)^2$.]

Dispersive bounds on $\Lambda_b \to \Lambda$ form factors

The strong unitarity bounds allow stable extrapolations to low q^2.



Λ^* baryons ($S = -1$, $I = 0$)



Particle	J^P	Overall status	Status as seen in —		
			$N\overline{K}$	$\Sigma\pi$	Other channels
$\Lambda(1116)$	$1/2^+$	****			$N\pi$ (weak decay)
$\Lambda(1380)$	$1/2^-$	**	**	**	
$\Lambda(1405)$	$1/2^-$	****	****	****	
$\Lambda(1520)$	$3/2^-$	****	****	****	$\Lambda\pi\pi, \Lambda\gamma$
$\Lambda(1600)$	$1/2^+$	****	***	****	$\Lambda\pi\pi, \Sigma(1385)\pi$
$\Lambda(1670)$	$1/2^-$	****	****	****	$\Lambda\eta$
$\Lambda(1690)$	$3/2^-$	****	****	***	$\Lambda\pi\pi, \Sigma(1385)\pi$
$\Lambda(1710)$	$1/2^+$	*	*	*	
$\Lambda(1800)$	$1/2^-$	***	***	**	$\Lambda\pi\pi, \Sigma(1385)\pi, N\overline{K}^*$
$\Lambda(1810)$	$1/2^+$	***	**	**	$N\overline{K}_2^*$
$\Lambda(1820)$	$5/2^+$	****	****	****	$\Sigma(1385)\pi$
$\Lambda(1830)$	$5/2^-$	****	****	****	$\Sigma(1385)\pi$
$\Lambda(1890)$	$3/2^+$	****	****	**	$\Sigma(1385)\pi, N\overline{K}^*$
$\Lambda(2000)$	$1/2^-$	*	*	*	
$\Lambda(2050)$	$3/2^-$	*	*	*	
$\Lambda(2070)$	$3/2^+$	*	*	*	
$\Lambda(2080)$	$5/2^-$	*	*	*	
$\Lambda(2085)$	$7/2^+$	**	**	*	
$\Lambda(2100)$	$7/2^-$	****	****	**	$N\overline{K}^*$
$\Lambda(2110)$	$5/2^+$	***	**	**	$N\overline{K}^*$
$\Lambda(2325)$	$3/2^-$	*	*		
$\Lambda(2350)$	$9/2^+$	***	***	*	
$\Lambda(2585)$		*	*		

$\Gamma_{\Lambda^*(1520)} \approx 16$ MeV

[PDG]

$\Lambda_b \to \Lambda^*(1520)$ form factors

[S. Meinel and G. Rendon, 2009.09313/PRD 2021 and 2107.13140/PRD 2022].

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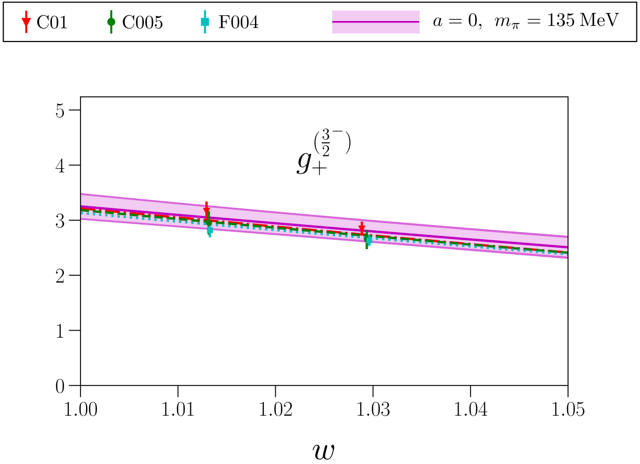
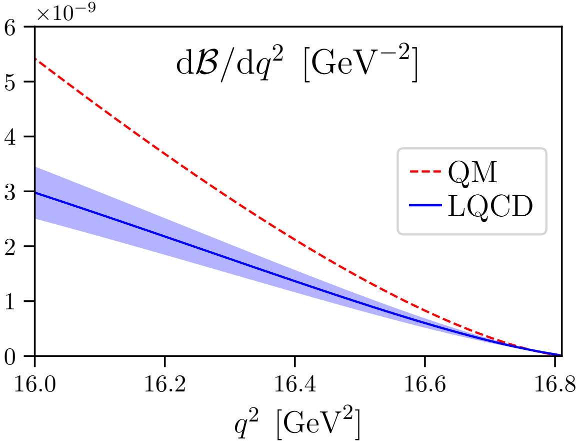


$\Lambda_b \to \Lambda^*(1520)\mu^+\mu^-$ differential branching fraction near q^2_{max}

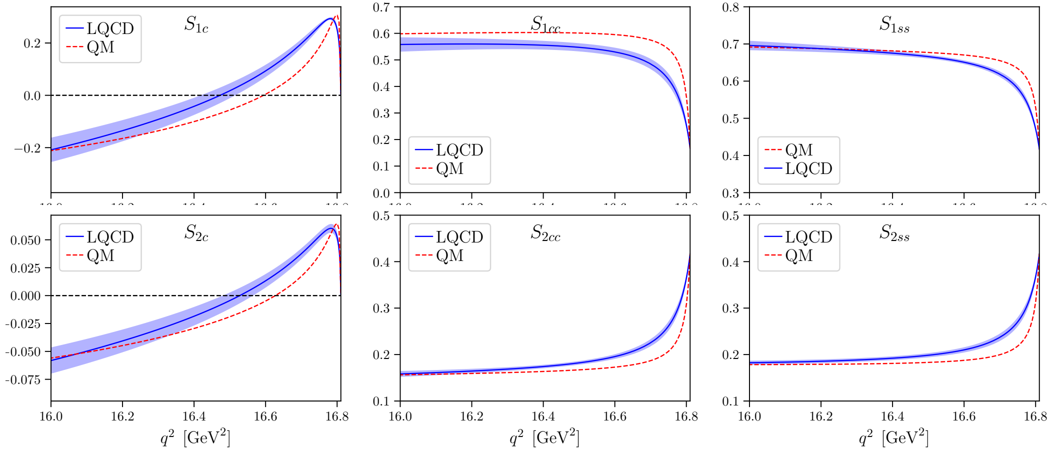
Quark model vs. lattice QCD



QM = using form factors from [L. Mott, W. Roberts, `arXiv:1108.6129`/IJMPA 2012]

$\Lambda_b \to \Lambda^*(1520)(\to pK^-)\mu^+\mu^-$ angular observables near $q^2_{\max}$

Quark model vs. lattice QCD



See [S. Descotes-Genon, M. Novoa-Brunet, 1903.00448/JHEP 2019] for definitions. The lepton mass is neglected here.

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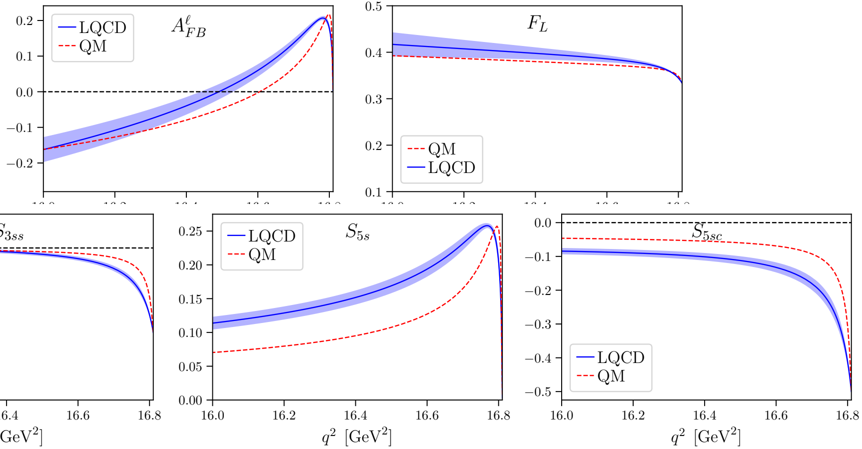
Quark model vs. lattice QCD



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*at zero recoil only

For averages, see `http://flag.unibe.ch`