Lattice QCD and Hadron Spectroscopy

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exp: Paolo Gandini (previous talk)

Outline:

conventional and exotic hadrons

their masses, decay widths and other properties

from ab-initio lattice QCD

Hadrons

G=gluon, *q*=quark=*u*,*d*,*s*,*c*,*b*

Today we know (from exp and theory) that hadrons with the following minimal quark and gluon contents.

There may be more categories, but these are not reliably confirmed yet.

minimal quark (q) and gluon (G) contents

conventional hadrons

$\bar{q}\ q$ meson

$q\ q\ q$ baryon

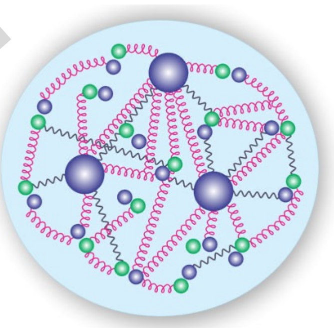


image:
https://physicstoday.scitation.org/do/10.1063/PT.5.7167/full/

exotic hadrons

experimentaly discoveded example

$q\ q\ \bar{q}\ \bar{q}$ tetraquark — $cc\bar{u}\bar{d} = T_{cc}$ [LHCb 2021]

$q\ q\ q\ \bar{q}\ q$ pentaquark — $uud\bar{c}c = P_c$ [LHCb 2015]

$\bar{q}\ G\ q$ hybrid — $\bar{u}Gd = \pi_1$ [several exp.]

$J^{PC} = 1^{-+}$

QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$ $\qquad g_s \not\ll 1$ at hadronic energy scale

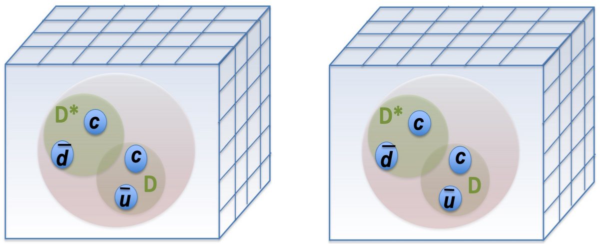
Lattice QCD: nonperturbative approach to QCD

Main quantity extracted: eigen-energy E_n

$\sum_n |n\rangle\langle n|$ $\qquad e^{-iE_n t_M}$ → Eucledian time

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\, e^{-E_n t} \langle n | \mathcal{O}_j^+ | 0 \rangle$$

t, J^P, I $\qquad$ t=0, J^P, I

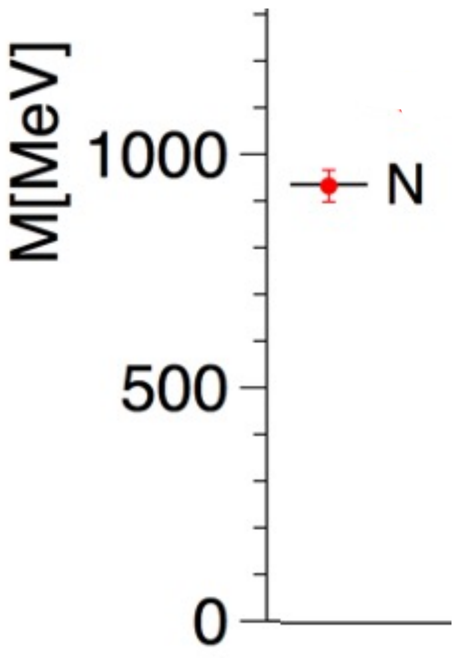


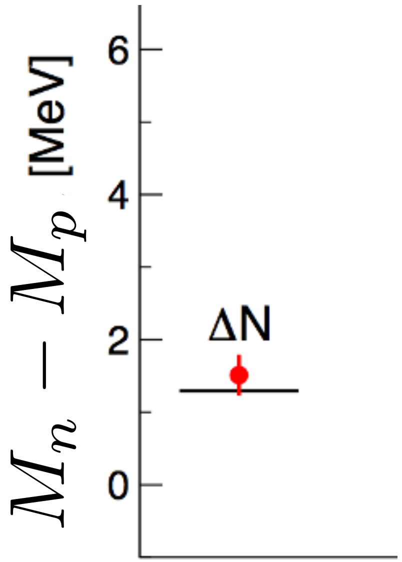
$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S^E_{QCD}/\hbar} \qquad S^E_{QCD} = \int d^4 x_E\, \mathcal{L}^E_{QCD}(m_q, g_s)$$

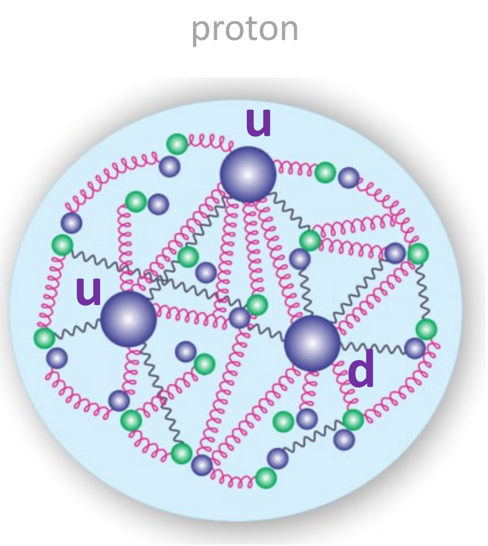
often "non-precision" studies: single a, $\quad m_{u/d} > m_{u/d}^{phy}\,,\ m_\pi > 140$ MeV

strongly stable hadron well below threshold: straightforward $\qquad m = E\ (\vec{p}=0)$

Proton and neutron mass constitute more than 99% of the bright universe mass







exp

lattice QCD m_u=m_d

BMW collaboration
Science 322, 2008

exp

lattice QCD+QED $m_u \neq m_d$

BMW collaboration
Science 347,2015

Contribution of the Higgs mechanism

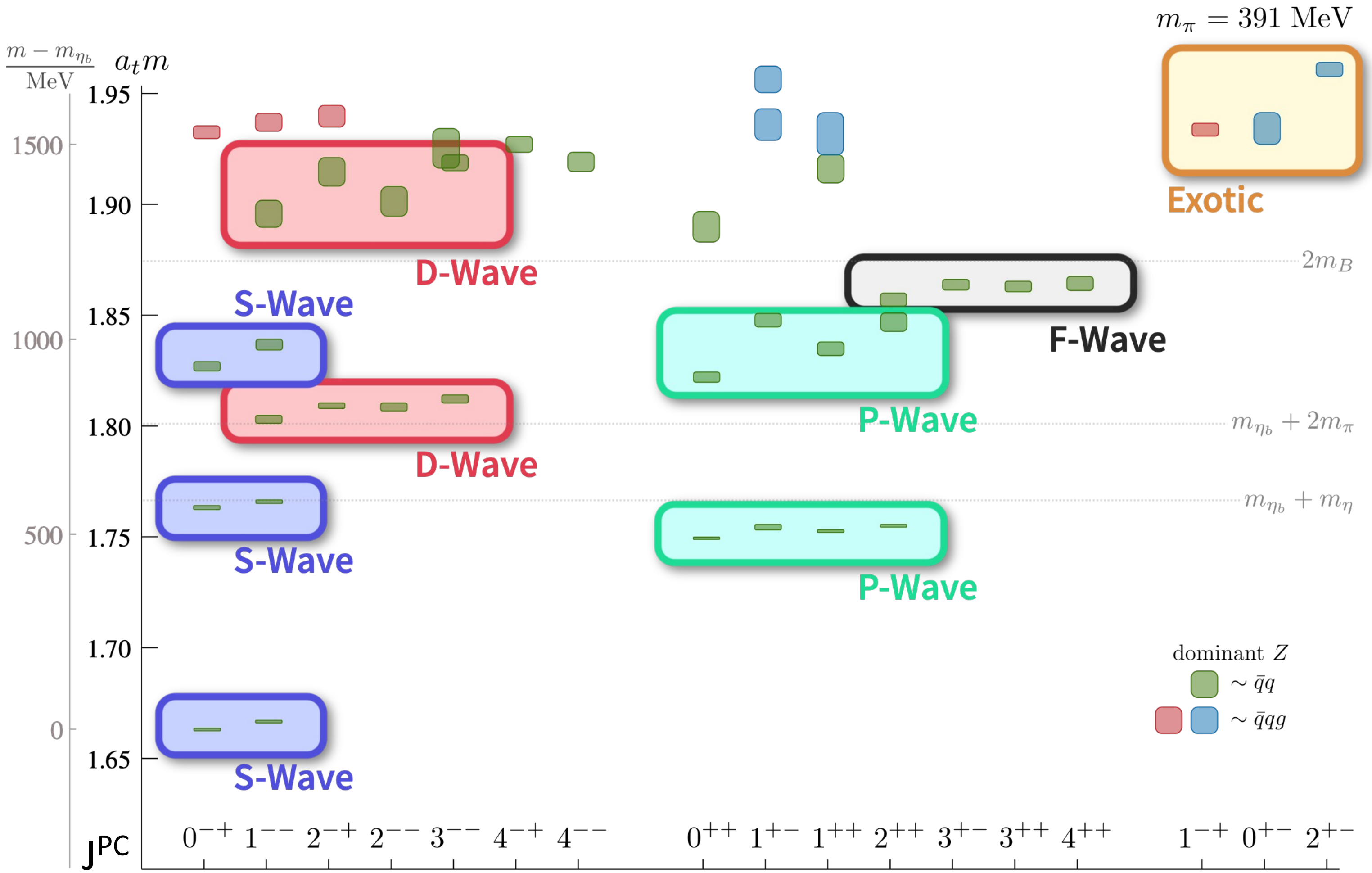
to the valence quark masses

$2m_u+m_d \cong 10$ MeV [PDG]

image:
https://physicstoday.scitation.org/do/10.1063/PT.5.7167/full/

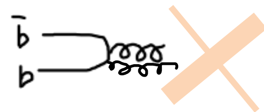
Bottomonia and bottomonium hybrids

$\bar{b}\, b$ $\bar{b}Gb$



Lattice QCD
relativistic b quarks
Ryan & Wilson (HadSpec) 2008.02656, JHEP

strongly stable below BB if bb annihilation is omitted



$m_{hybrid} \geq 10.9$ GeV

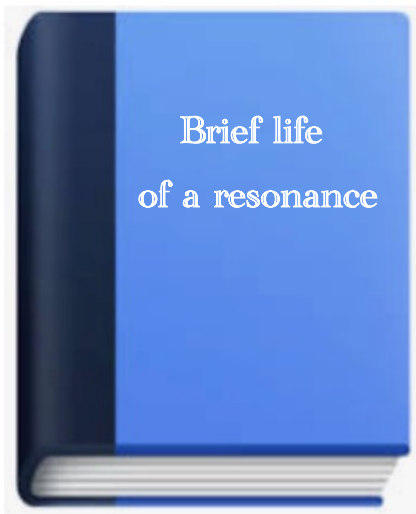
other predictions of hybrids:

from quenched lattice static potentials / EFT

Brambilla et al, 1805.07713, 1908.11699, PRD

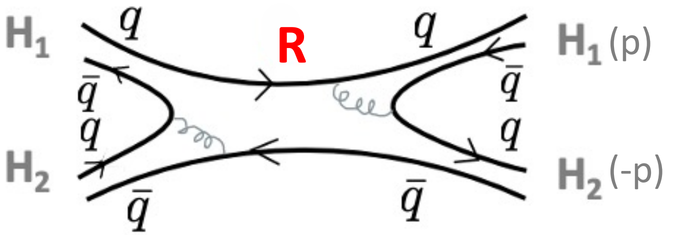
Schlosser & Wagner, 2111.00741

Strongly decaying hadronic resonances

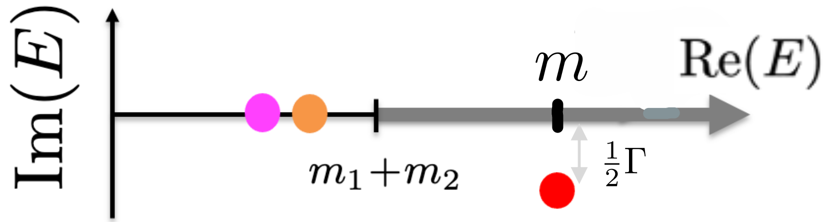


hotemoji.com

Resonances R→ H_1 H_2 , bound states near threshold



scattering matrix T(E)

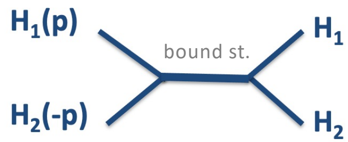


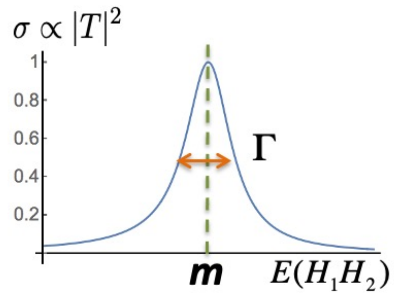
Virtual bound st. | Bound st. | Resonance

p = - i |p| | p = i |p|

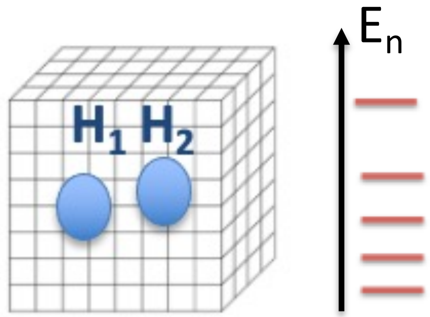
$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$





Scattering matrix T(E) from lattice QCD



many resonances and bound states extracted in this way by now
(appologies for not covering all)

real E | for real E | for complex E

$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

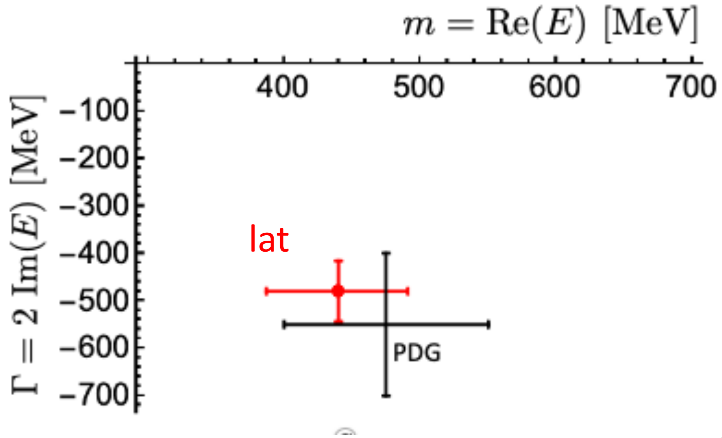
analytic contin.
to complex E

Resonances R→ H_1 H_2

$$\Gamma = g^2 \frac{p^{2l+1}}{E^2}$$

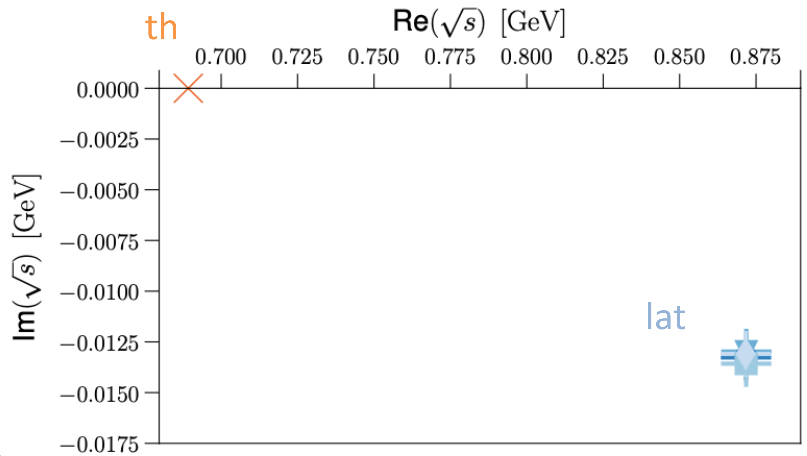
GWU, 1803.02897 $m_\pi = 227, 315$ MeV $\to m_\pi^{phy}$ UChPT

$\sigma \to \pi\pi$



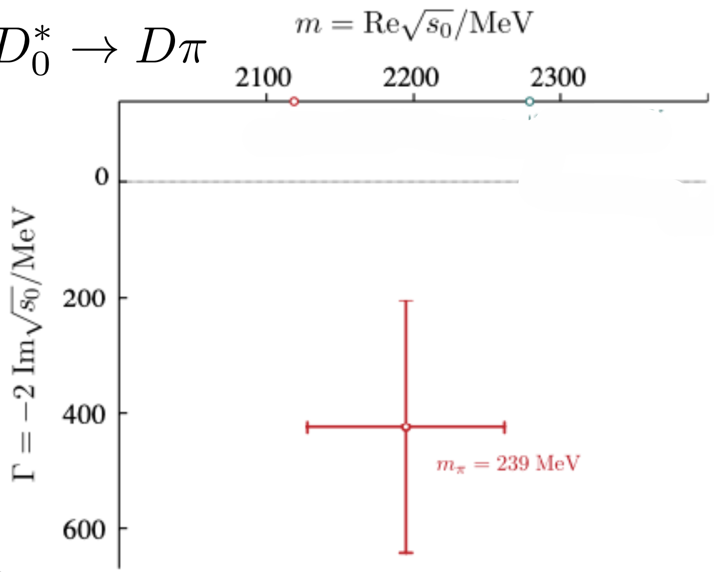
Rendon et al: 2006.14035 $m_\pi \simeq 176$ MeV

$K^* \to K\pi$ coupling g that parametrizes the width agrees with exp.



HadSpec, 2102.04973 JHEP $m_\pi \simeq 239$ MeV

$D_0^* \to D\pi$

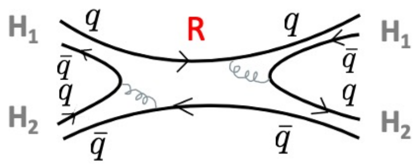


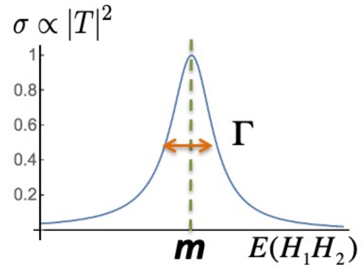
PDG: puzzle

$m[D_0^*] \simeq m[D_{s0}^*(2317)]$

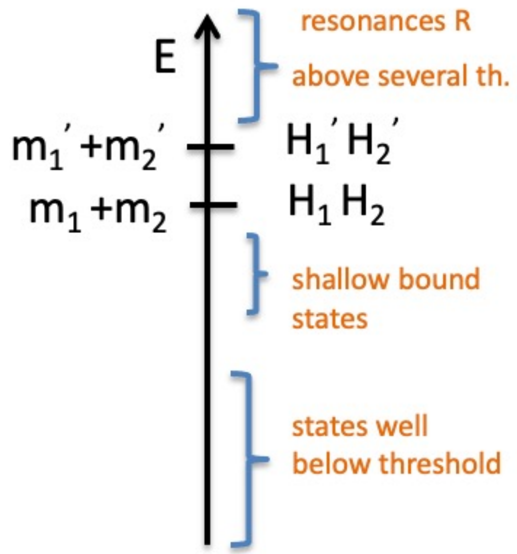
lat: OK

$m[D_0^*] < m[D_{s0}^*(2317)]$





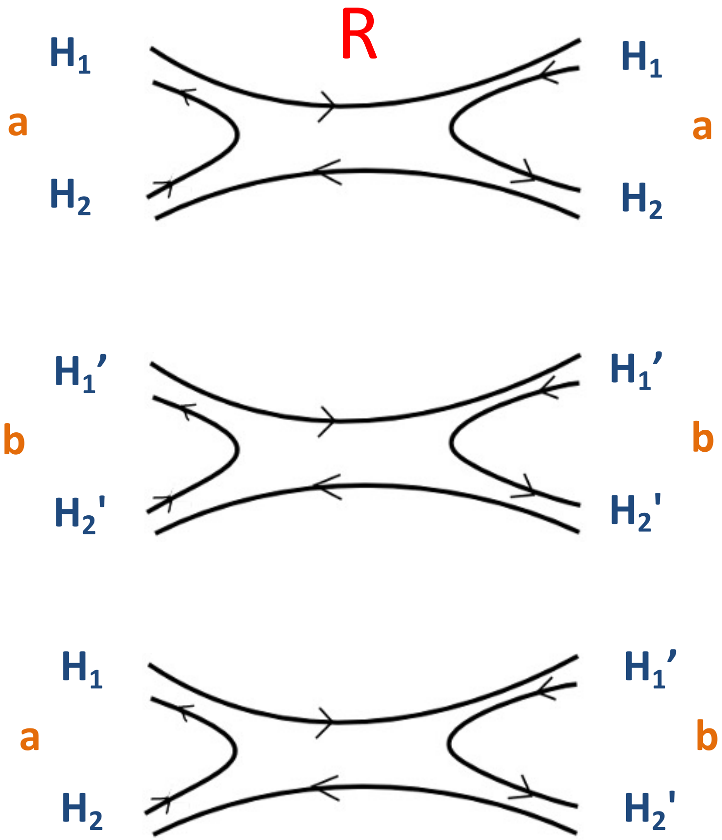
Resonances from coupled-channel scattering



$$R \to H_1 H_2,\ \ H_1' H_2', \ldots$$

channel a: $H_1 H_2$

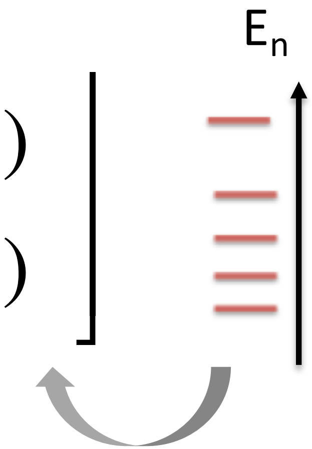
channel b: $H_1' H_2'$



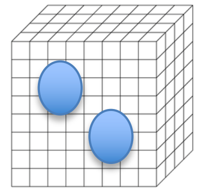
a -> a   a -> b

$$T(E) = \begin{bmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{bmatrix}$$

b->a   b -> b



generalized.
Luscher's rel.

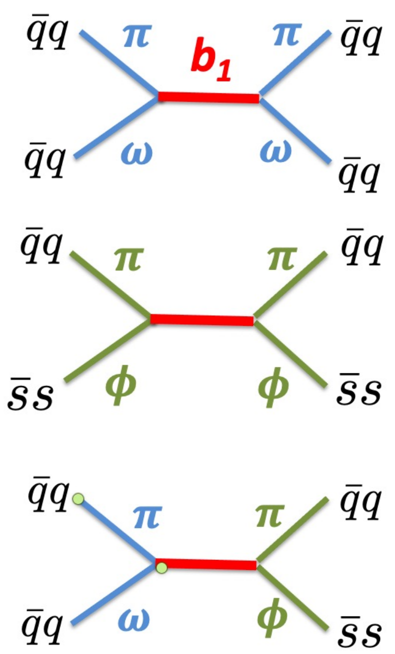


- lattice QCD studies extracted T(E) for several resonances
- most results by HadSpec. coll.

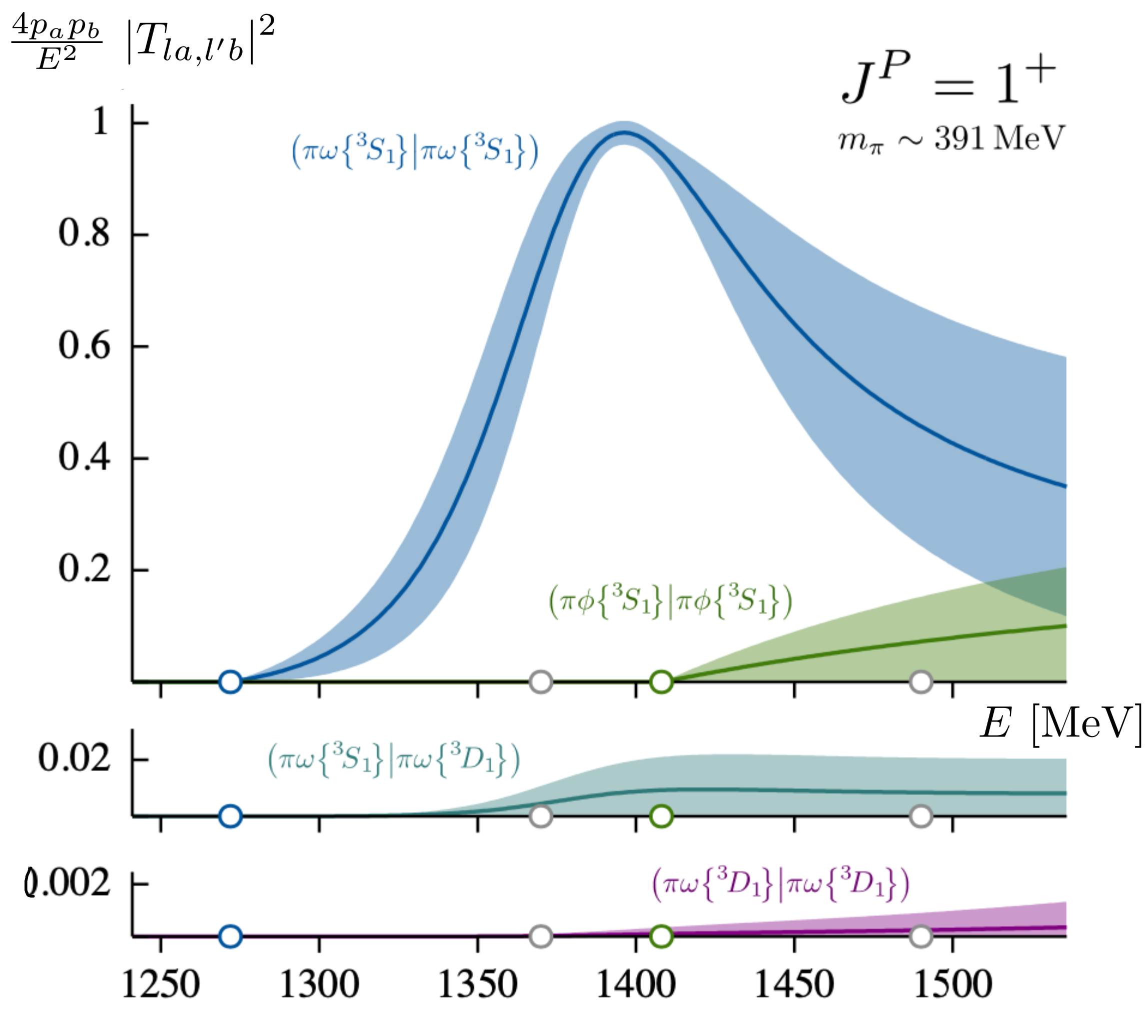
b_1 resonance from lattice

conventional

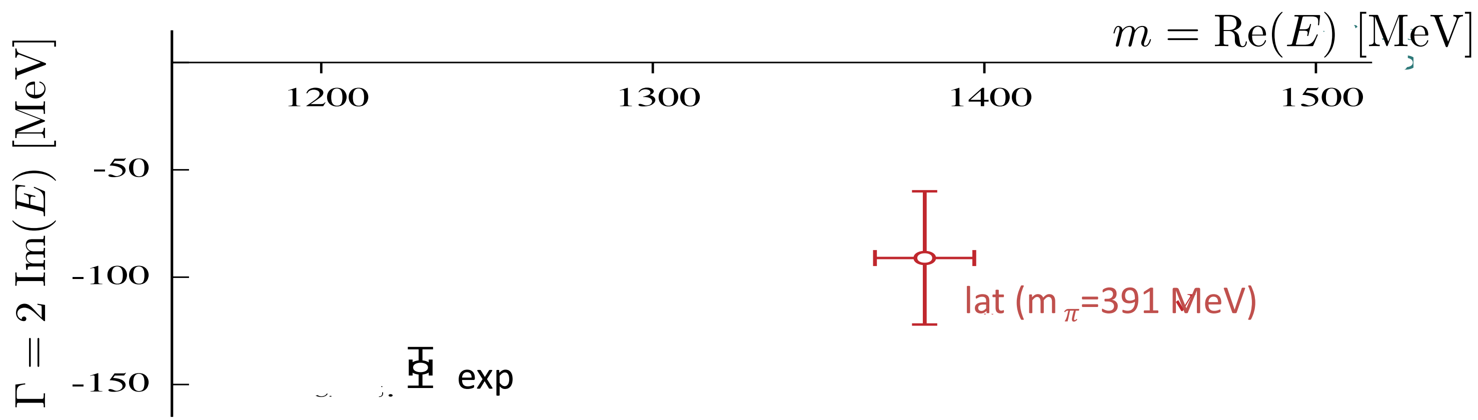
$\bar{d}u$ $J^{PC} = 1^{+-}$



q=u,d

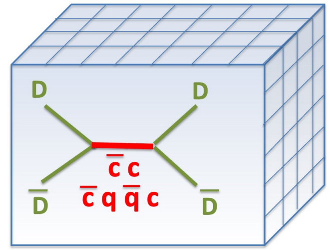


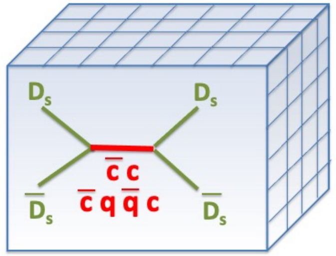
Woss et al, HadSpec, 1904.04136, PRD

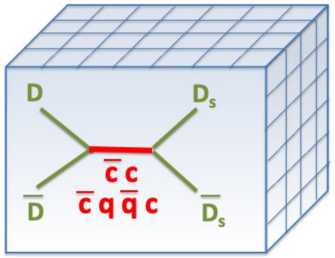


Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$







$\bar{D}_s D_s$ $J^P=0^+$ state

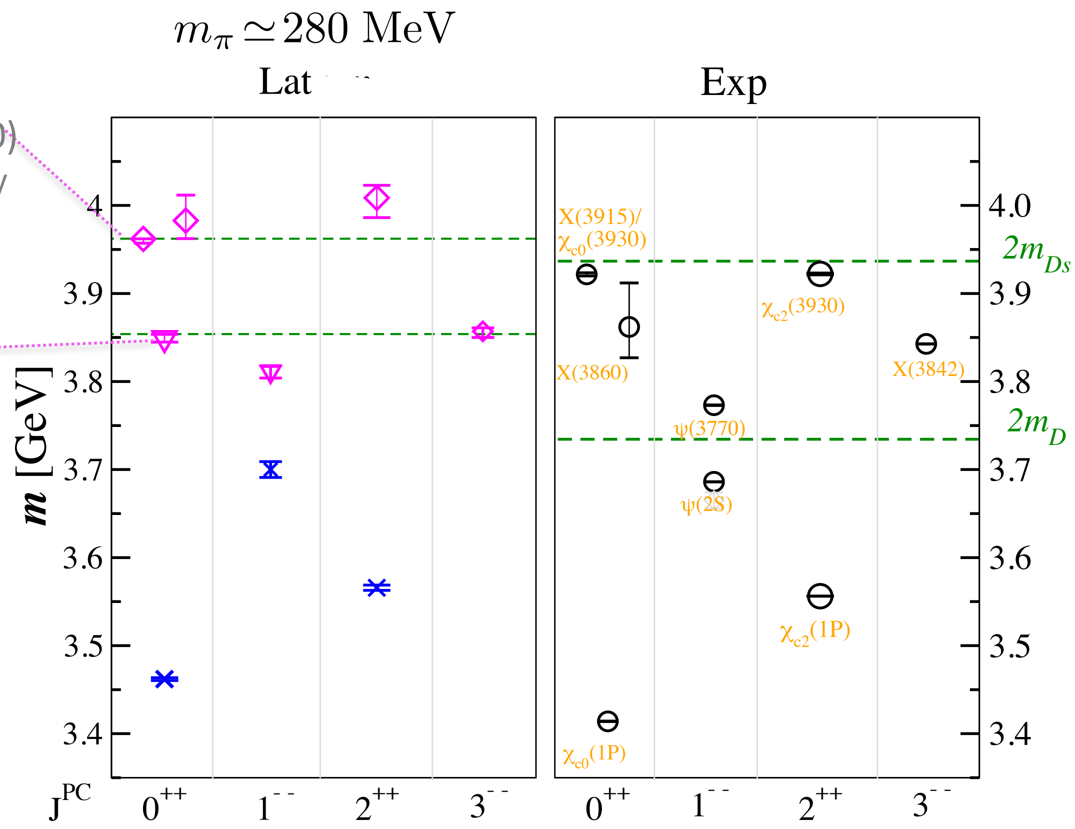
likely related to X(3915) / χ_{c0}(3930) [BaBar, LHCb 2009.00026]; explaining why it has narrow width to DD. Predicted by Lebed, Polosa 1602.08421

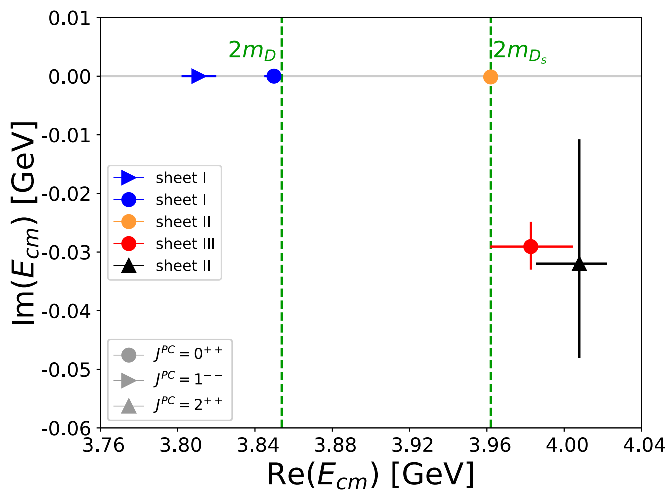
$\bar{D}D$ $J^P=0^+$ state

predicted in models [Oset et al, 0612179 PRD, Hildago Duque et al 1305.4487, Baru et al 1605.09649 PLB]

seen in dispersive analysis of exp. data [Deineka, Danilkin et al 2111.15033]

+ expected conventional charmonia





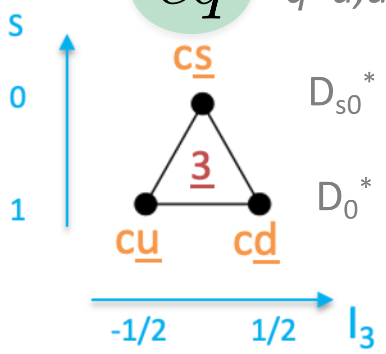
S.P. , Collins, Padmanath, Mohler, Piemonte

2011.02542 JHEP, 1905.03506 PRD, 2111.02934

Scalar heavy-light mesons $J^P = 0^+$

Conventional quark model

$c\bar{q}$ q=u,d,s



new paradigm supported by:

- lattice
- effective models ChPT+HQET
- reanalysis of exp data
- states circled by blue seem to feature in the spectrum

Scattering on the lattice

S=1 Mohler et al, 1308.3175, PRL
Lang et al, 1403.8103, PRD
RQCD, 1706.01247, PRD
HadSpec 2008.06432, JHEP

S=0 Mohler et al. 1208.4059, PRD
(see backup)
HadSpec, 1607.07093, JHEP
HadSpec 2102.04973, JHEP

S=-1 HadSpec, 2008.06432, JHEP

New paradigm

Du et al, 1712.07957, PRD
Albaladejo et al, 1610.06727, PLB
Lutz et al (2003), 0307133, PLB

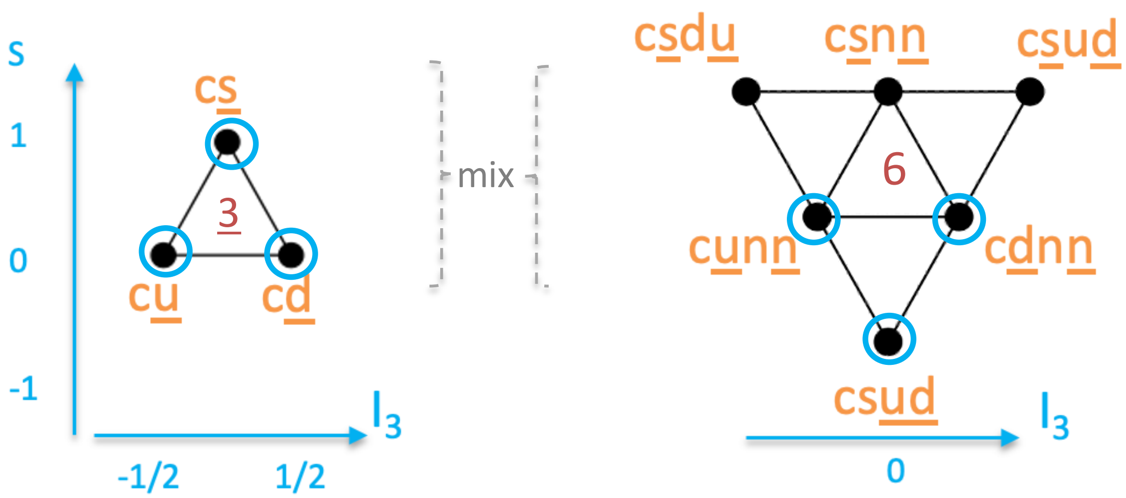
$c\bar{q}$ + $c\bar{q}\ q\bar{q}$ q=u,d,s n=u,d

$\underline{3} \otimes 8 = \underline{3} \oplus 6 \oplus 15$ $SU(3)_F$

Beveren, Rupp; Dmitrasinovic

2.3 GeV

lat: 2.1-2.2 GeV (pole)
PDG: 2.3 GeV (BW)
(see backup)



no state (mix with repulsive 15)

2.4-2.5 GeV
reanalysis of lat 1607.07093 by Albaladejo 1610.06727

virtual bound state
HadSpec 2008.06432
partner of X(2900)
[LHCb 2009.00025] ?

Doubly heavy tetraquarks

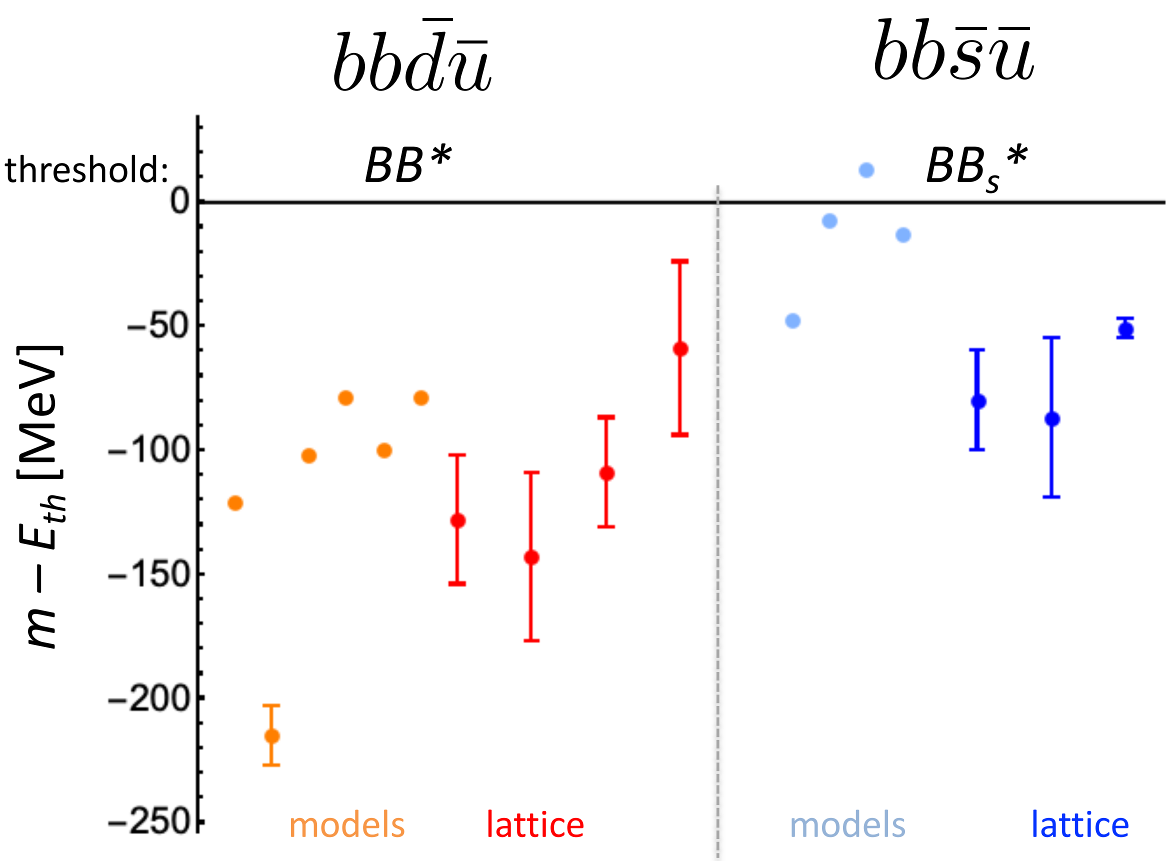
$$QQ'\bar{q}\bar{q}'$$

$Q=c,b \quad q=u,d,s$

Doubly bottom tetraquarks

not found in exp, difficult to find

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$



references from left to right

models (many more references):
Eichten and Quigg (2017) 1707.09575 PRL
Karliner and Rosner (2017) 1707.07666 PRL
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)
Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results
Leskovec, Meinel, Pflaumer, Wagner (2019) 1904.04197
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary
Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)
Eichten and Quigg (2017) 1707.09575 PRL
Parket al. (2018) 1809.05257
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)

lattice: most updated results
Pflaumer, Leskovec,Meinel,Wagner (2021) 2108.10704
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

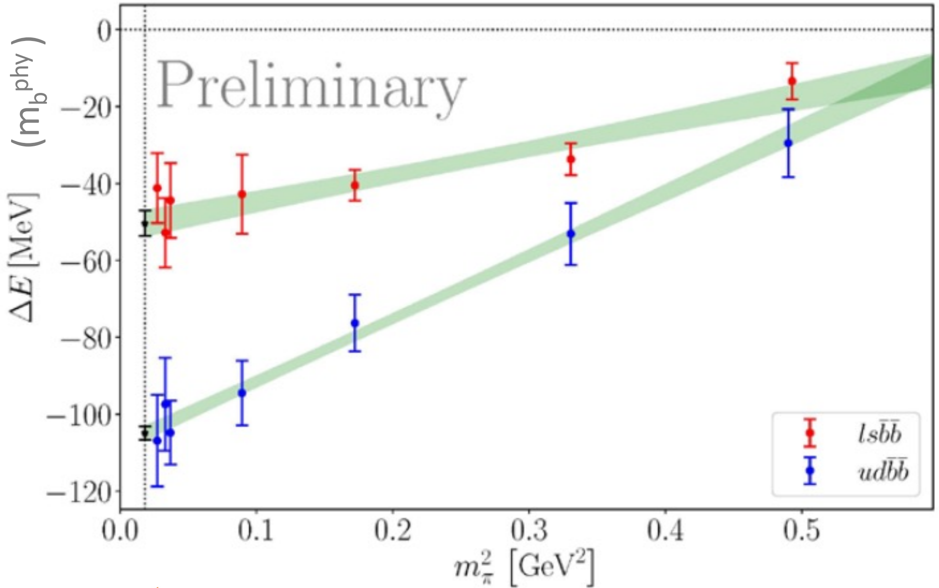
lattice	$m_{u/d}$	a [fm]
Leskovec , Pflaumer et al	$m_{u/d}$ -> $m_{u/d}^{phy}$	0.08-0.11
Junnarkar et al.	$m_{u/d}$ -> $m_{u/d}^{phy}$	a -> 0
Francis et al	$m_{u/d}$ -> $m_{u/d}^{phy}$	0.09
Bicudo et al.	$m_{u/d}$ -> $m_{u/d}^{phy}$	0.08

earlier results of Frances et al in 1810.10550, 1607.05214 PRL

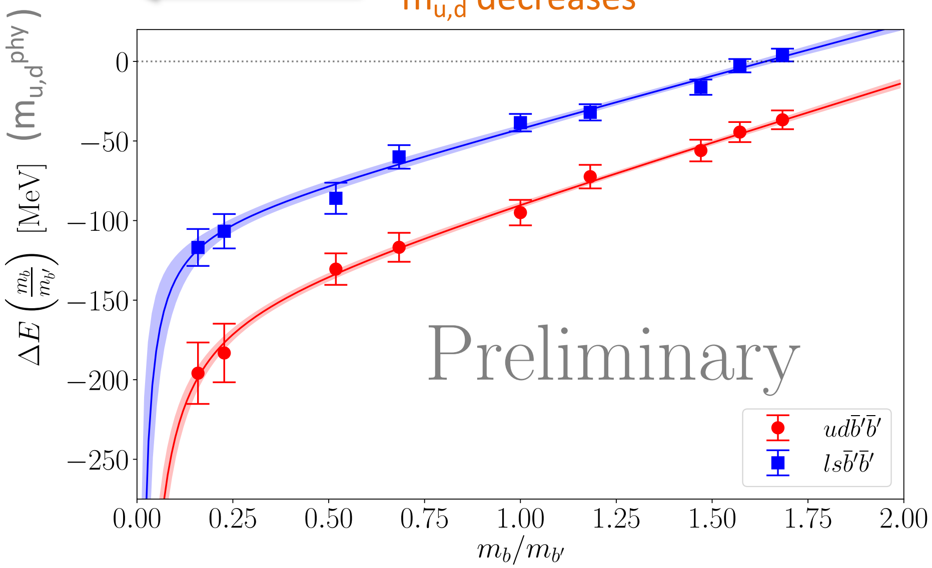
Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



$m_{u,d}$ decreases



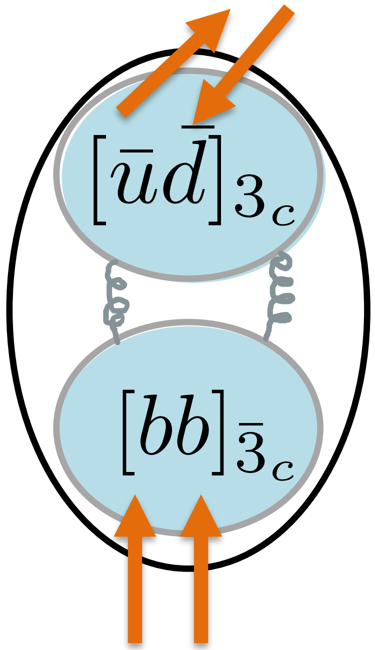
$m_{b'}$ increases

preliminary lattice results of Frances, Colquhoun, Hudspith, Maltman (2021) @Lattice2021, Hadron2021

supports internal structure below

supported also by almost all model studes

Karliner and Rosner (2017), Janc and Rosina (2004), ...



$S^P = 0^+$

good diquark

$S^P = 1^+$

color Coulomb

strongly binds bb

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bc\bar{d}\bar{u}$ $cc\bar{d}\bar{u}$

Theoretically expected near or above threshold

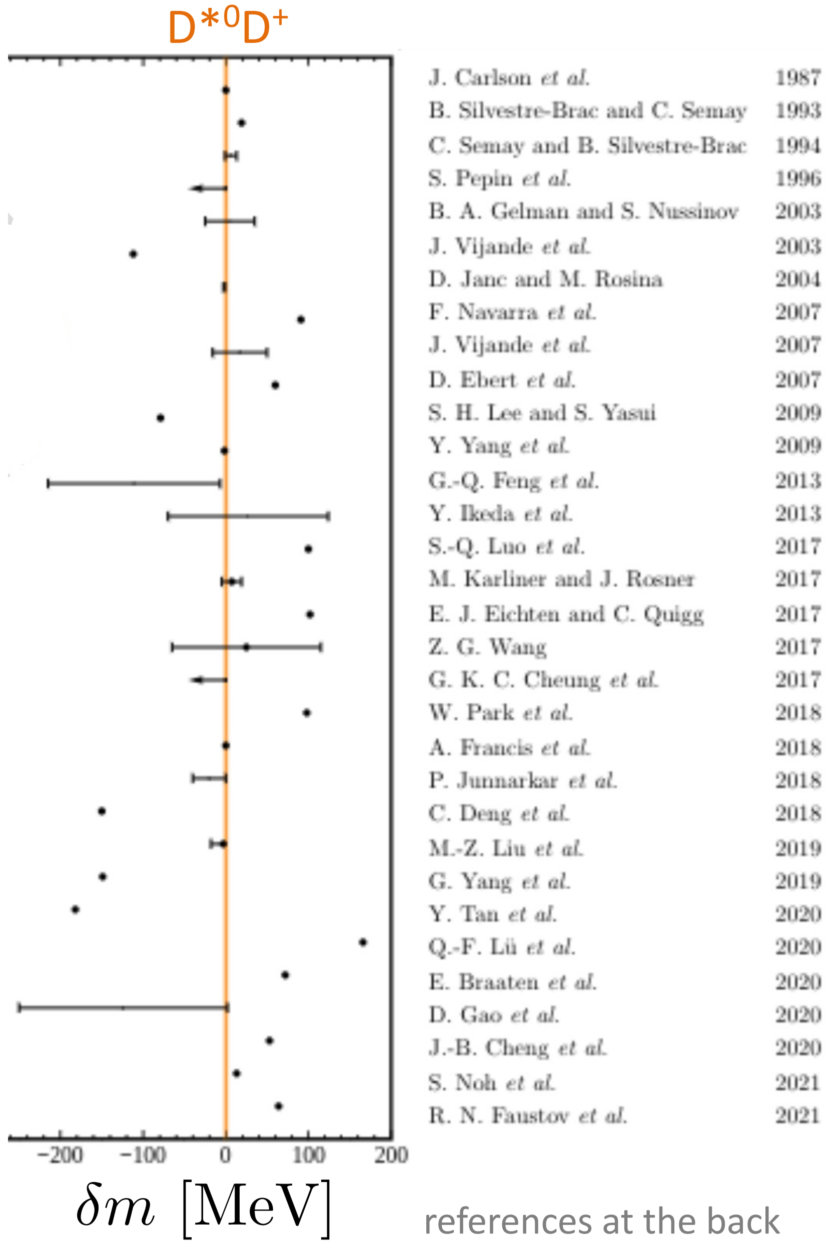
States near or above threshold have to be identified as poles in scattering T(E)

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bc\bar{d}\bar{u}$ $cc\bar{d}\bar{u}$

Theoretically expected near or above threshold

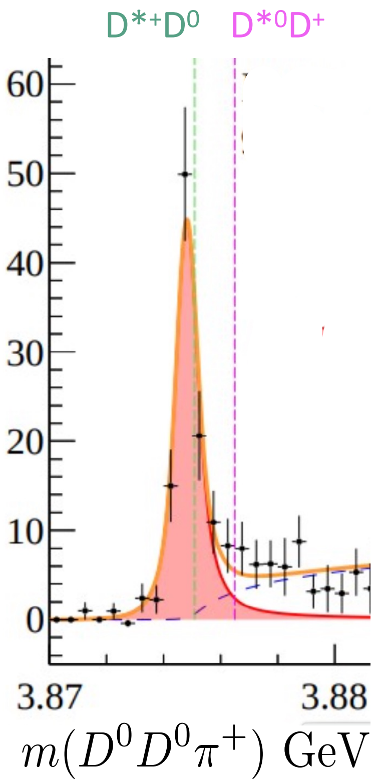
States near or above threshold have to be identified as poles in scattering T(E)



references at the back

Theoretical predictions for T_{cc} mass (I=0, $J^P=1^+$)

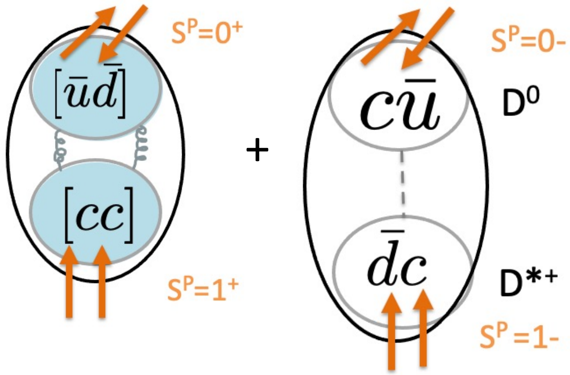
Doubly charm tetraquark T_{cc} $cc\bar{d}\bar{u}$



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics

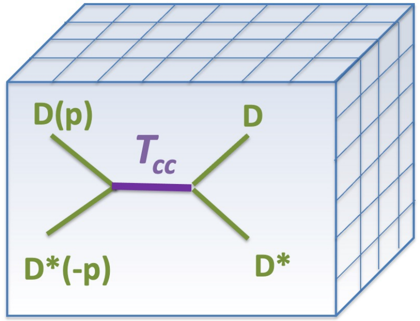


likely dominant

Doubly charm tetraquark T_{cc}
from lattice QCD

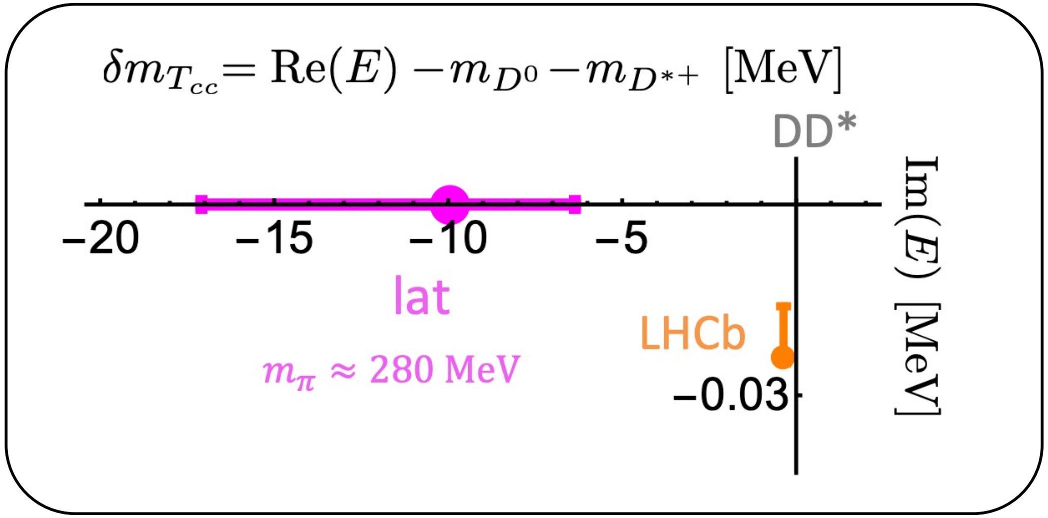
$cc\bar{d}\bar{u}$

$J^P = 1^+, \ I = 0$



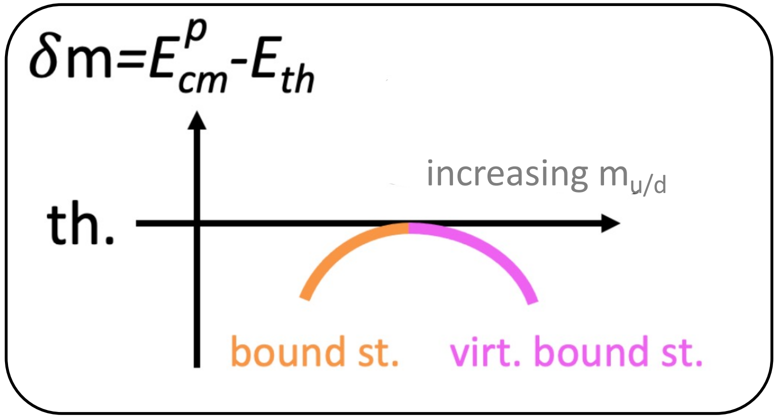
What is virtual bound state? See supplemental of 2202.10110

Pole of T(E) ● virtual bound state pole p = -i|p|



- The only lat. study that extracts T(E): [Padmanath, SP: 2202.10110] $m_\pi \approx 280$ MeV
- Evidence for pole related to Tcc
- For $m_{u,d} > m_{u,d}^{phy}$ one expects decreased attraction
  T_{cc} : bound state becomes virtual bound state
  indeed this is what is found on the lattice

Sketch of expected binding energy



prevoious simulations extracted only eigen-energies: [Junnarkar, Mathur, Padmanath, 1810.12285 PRD ; HadSpec 1709.01417 JHEP]

Bound and virtual bound state: simplest example

scattering in square-well potential in QM

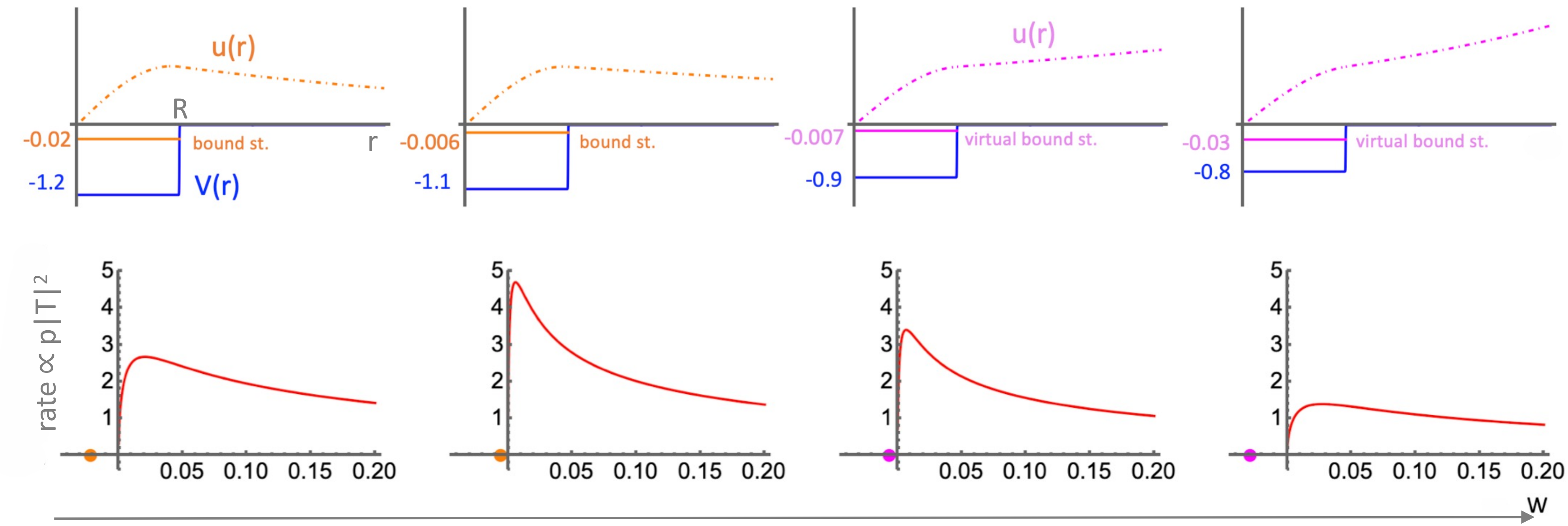
partial wave l=0

$$\delta_l = \arctan[\tan(qR)\tfrac{p}{q}] - pR$$

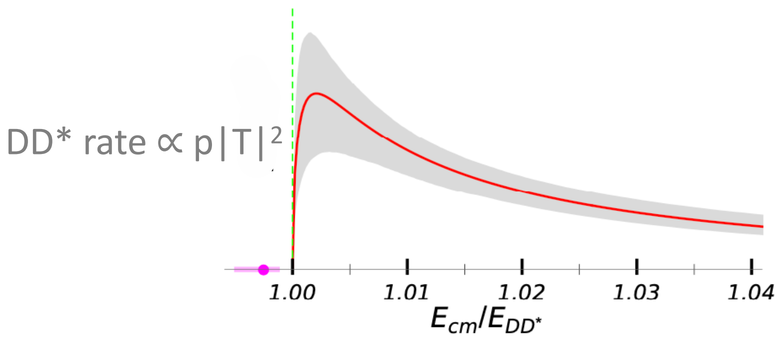
$u(r) = A\sin(qr)$ $u(r) = B\sin(pr+\delta)$

p=i|p| $e^{ipr} = e^{-|p|r}$

p=-i|p| $e^{ipr} = e^{|p|r}$



increasing $m_{u/d}$, decreasing attraction



for T_{cc} extracted on lattice 2202.101101

Exotic hadrons with Q and Q

Q=u,b

Hadrons $\bar{Q}Q\bar{q}q$, $\bar{Q}Qqqq$

Q=c,b q=u,d,s

$Z_b = \bar{b}b\bar{d}u$ [Belle 2011] $P_c = \bar{c}cuud$ [LHCb 2015]

$Z_c = \bar{c}c\bar{d}u$ [BessII, Belle, 2013]

GRRR!

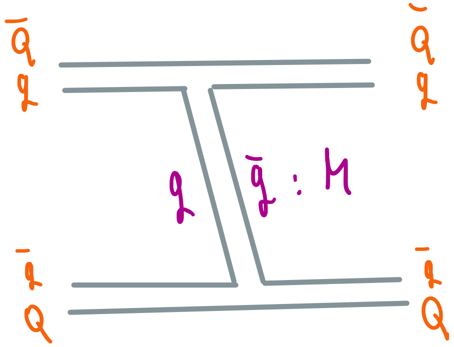
challenging for ab-initio study due to many decay channels:

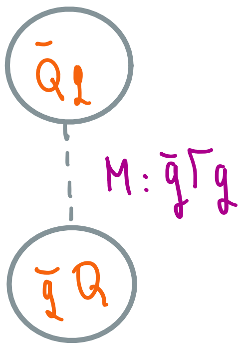
$$\bar{Q}Q\bar{q}q \to \bar{Q}q + \bar{q}Q, \quad \bar{Q}Q + \bar{q}q$$

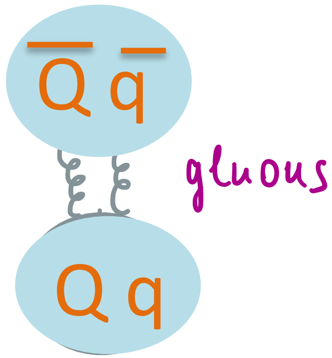
$$\bar{Q}Qqqq \to \bar{Q}q + Qqq, \quad \bar{Q}Q + qqq$$

Only partial conclusions are available from ab-initio approaches [reviewed e.g. in S.P. 2001.01767]
I'll discuss other approaches (also due to lack of time).

hadronic molecule vs. **diquark antidiquark**







quark spins correlated within mesons   diquarks

Examples of conclusons from lattice QCD: still many studies left to do ...

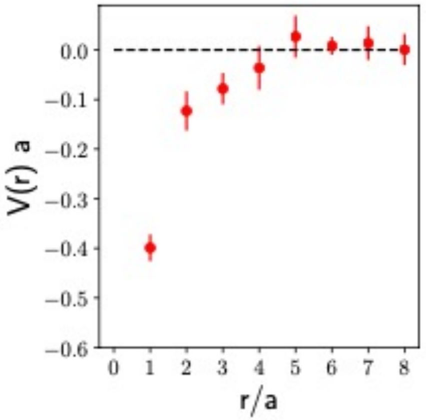
$Z_b = \bar{b}b\bar{d}u$ [Belle 2011]

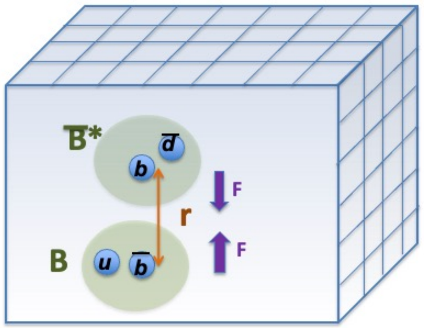
$\bar{b}b\bar{d}u \rightarrow B\bar{B}^*,\ \Upsilon\pi$

static b quarks & Born-Oppenheimer approach

Peter, Wagner, Bicudo

SP, Bahtiyar, Petkovic, Sadl 2019, 2020,





attraction between B and B* likely responsible for Zb

$Z_c = \bar{c}c\bar{d}u$ [BessIII, Belle 2013]

$\bar{c}c\bar{d}u \rightarrow D\bar{D}^*,\ J/\psi\pi$

non-static c quarks

several lattice studies found very small interactions in this system

[Leskovec Mohler Lang SP: 1308.2097,1405.7623
HadSpec 1709.01417
Liuming Liu et al. 1907.03371, 1911.08560]

$\bar{b}\bar{b}bb$

no indication for strongly stable state

lesson: incorporate all (connected) Wick contractions

otherwise false conclusions might be reached

[Hughes, Eichten, Davies, HPQCD, 1710.03236 PRD]

$P_c = \bar{c}cuud$ [LHCb 2015, 2019]

One-channel scattering $J/\psi\ p$ does not lead to observed Pc

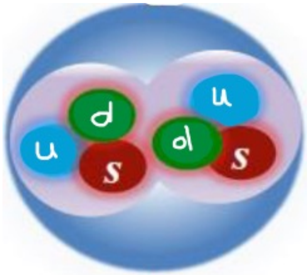
-> the channels $(\bar{c}u)(udc)$ must be crucial

[Skerbis SP PRD 2019]

Di-baryons

binding energy $\Delta E = m - m_{B1} - m_{B2}$

H-dibaryon



$\Lambda \ \Lambda$

many lattice studies:

NPLQCD, CalLAT, Mainz, ...

mostly at mu=md=ms

[Mainz group, 2103.01054 PRL]

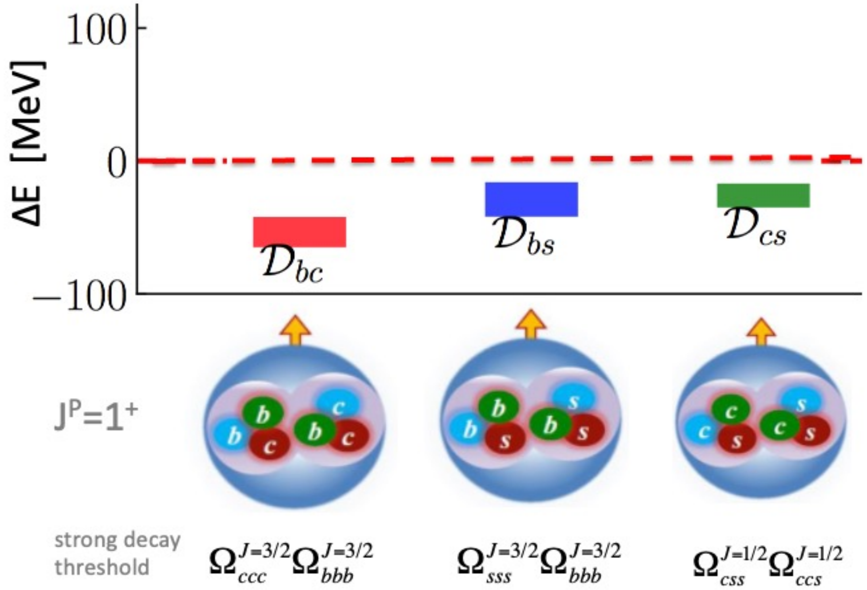
$$m_\pi = m_K \simeq 420 \text{ MeV}$$

$$\Delta E = -\ 4.56 \pm 1.13_{\rm stat} \pm 0.63_{\rm syst} \text{ MeV.}$$

physical m_q : ??
experimentally not confirmed (yet)

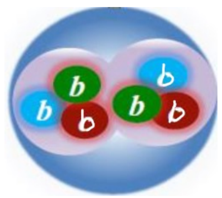
dibaryons with heavy quarks

[Junnarkar Mathur 1906.06054 PRL 2019]



[Mathur, Padmanath, Chakraborty 2205.02862]

$$\Delta E = -89(^{+16}_{-12}) \text{ MeV}$$

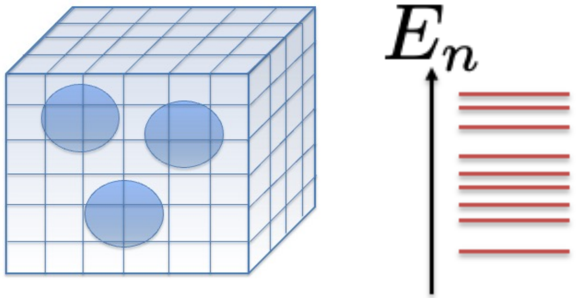


$\Omega_{bbb} \ \Omega_{bbb}$

Scattering of three hadrons $H_1 H_2 H_3$, $R \to H_1 H_2 H_3$

Nature $a_1 \to \pi\pi\pi,\ N(1440) \to N\pi\pi,\ X(3872) \to J/\psi\pi\pi,\ ....$

Formalism



3-body force

$E \to T(E)$

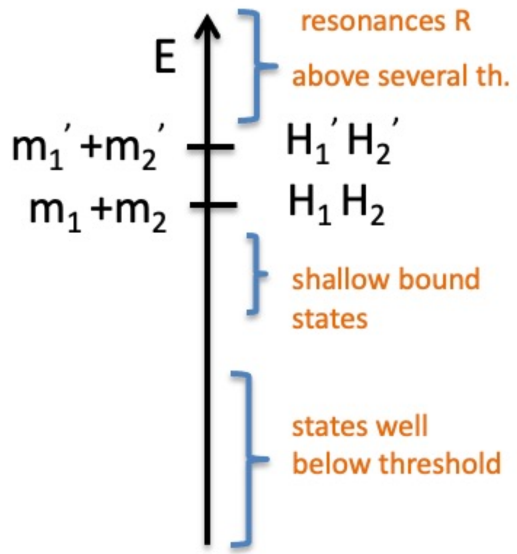
formalism by 3 groups: agreement
Sharpe, Hansen, Briceno, Lopez,...
Doring, Mai,...
Rusetsky, Hammer,...

Status
with actual simulations
impressive progres!!

$\pi^+\pi^+\pi^+,\ K^+K^+K^+,\ \ldots$ [GWU, HadSpec, Hanlon, Horz, NPLQCD, ETMC, .. : 2020 – 2022]

$a_1 \to \pi\pi\pi$ [GWU, 2107.03973, PRL]

Conclusions



Compliments to experiments for GREAT results !!

Status on hadron spectrum from Lattice QCD :

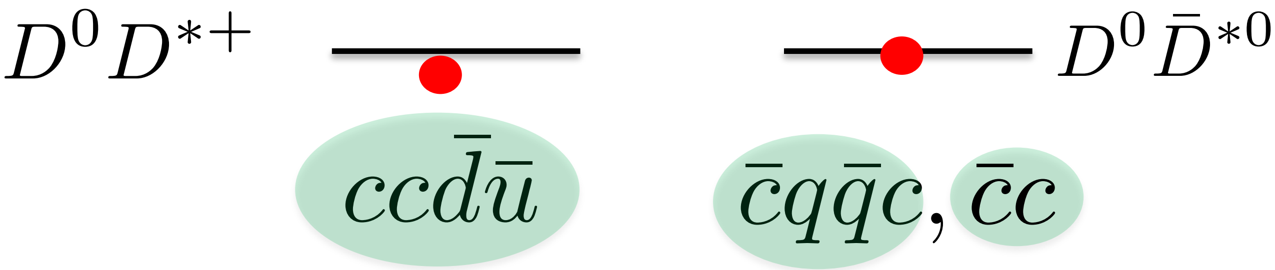
- strongly stable : "straighforward, done"
- strongly decaying to 1 channel "mostly done"
- 2-3 channels "challenging, some of them studied"
- > 3 channels "very challenging, mostly unexplored"
- P_c, Z_c, Z_b, $X(6900)$,... decay via many channels: reason why lattice has only partial conclusions on those
- $R \to H_1 H_2 H_3$ "very difficult"
- why $m_{u,d} > m_{u,d}^{phy}$? Smaller number of decay channels, smaller statistical errors on E and T(E)

Exotic hadrons: one picture can not explain all exotic hadron states; for each exotic hadron there is at least one viable picture

Theory and lattice QCD predict many conventional and exotic hadrons yet to be discovered

backup

A puzzle comparing T_{cc} and *X(3872)*



Why both reside within 1 MeV of threshold in exp ? There are many differences ...

Similarities:

- $J^P=1^+$, I=0
- in molecular picture: attraction via light vector exchange [e.g. Guo et al. 2101.01021, 2108.02673]

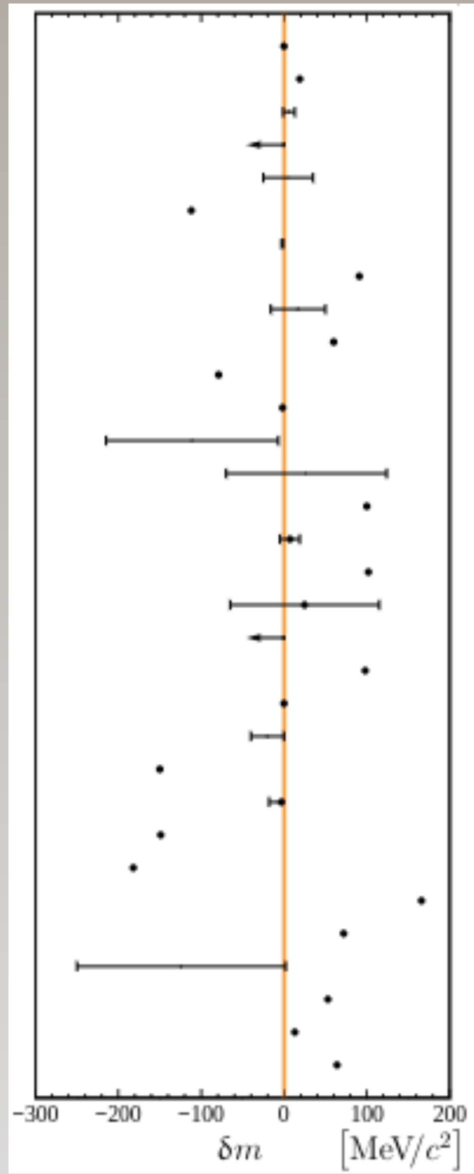
Differences:

- in molecular picture: attraction from one-pion exchange for X(3872) [Tornquist 1994]
  slight attraction from one-pion exchange for Tcc [eg. Du, Guo, Hanhart, 2110.13765]
- presence of Fock component cc for X(3872) [e.g. Padmanath, Lang, SP 1503.03257, PRD]
- presence of Fock component [cc][ud] for Tcc

Tcc theory predictions (taken from Ivan Polyakov, EPS meeting)

Theory predictions

Reference		Year	$\delta' m$ [MeV/c^2]
J. Carlson, L. Heller and J. A. Tjon	[36]	1987	~ 0
B. Silvestre-Brac and C. Semay	[37]	1993	$+19$
C. Semay and B. Silvestre-Brac	[38]	1994	$[-1, +13]$
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	[39]	1996	< 0
B. A. Gelman and S. Nussinov	[40]	2002	$[-25, +35]$
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	[41]	2003	-112
D. Janc and M. Rosina	[42]	2004	$[-3, -1]$
F. Navarra, M. Nielsen and S. H. Lee	[43]	2007	$+91$
J. Vijande, E. Weissman, A. Valcarce	[44]	2007	$[-16, +50]$
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	[45]	2007	$+60$
S. H. Lee and S. Yasui	[46]	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	[47]	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	[48]	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	[49]	2013	$[-70, +124]$
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	[50]	2017	$+100$
M. Karliner and J. Rosner	[51]	2017	$7 \pm 12 \to 1$
E. J. Eichten and C. Quigg	[52]	2017	$+102$
Z. G. Wang	[53]	2017	$+25 \pm 90$
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	[54]	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	[55]	2018	$+98$
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	[56]	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	[57]	2018	$[-40, 0]$
C. Deng, H. Chen and J. Ping	[58]	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	[59]	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	[60]	2019	-149
Y. Tan, W. Lu and J. Ping	[61]	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	[62]	2020	$+166$
E. Braaten, L.-P. He and A. Mohapatra	[63]	2020	$+72$
D. Gao, D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	[64]	2020	$[-250, +2]$
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	[65]	2020	$+53$
S. Noh, W. Park and S. H. Lee	[66]	2021	$+13$
R. N. Faustov, V. O. Galkin and E. M. Savchenko	[67]	2021	$+64$



Tcc theory predictions (taken from Ivan Polyakov, EPS meeting)

Refs. for theory predictions

[36] J. Carlson, L. Heller, and J. A. Tjon, *Stability of dimesons*, Phys. Rev. **D37** (1988) 744. 2

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[41] J. Vijande, F. Fernandez, A. Valcarce, and B. Silvestre-Brac, *Tetraquarks in a chiral constituent quark model*, Eur. Phys. J. **A19** (2004) 383, arXiv:hep-ph/0310007. 2

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[44] J. Vijande, E. Weissman, A. Valcarce, and N. Barnea, *Are there compact heavy four-quark bound states?*, Phys. Rev. **D76** (2007) 094027, arXiv:0710.2516. 2

[45] D. Ebert, R. N. Faustov, V. O. Galkin, and W. Lucha, *Masses of tetraquarks with two heavy quarks in the relativistic quark model*, Phys. Rev. **D76** (2007) 114015, arXiv:0706.3853. 2

[46] S. H. Lee and S. Yasui, *Stable multiquark states with heavy quarks in a diquark model*, Eur. Phys. J. **C64** (2009) 283, arXiv:0901.2977. 2

[47] Y. Yang, C. Deng, J. Ping, and T. Goldman, *ps-wave $QQ\bar{q}\bar{q}$ state in the constituent quark model*, Phys. Rev. **D80** (2009) 114023. 2

[48] G.-Q. Feng, X.-H. Guo, and B.-S. Zou, *$QQ'\bar{u}\bar{d}$ bound state in the Bethe-Salpeter equation approach*, arXiv:1309.7813. 2

[49] Y. Ikeda *et al.*, *Charmed tetraquarks T_{cc} and T_{cs} from dynamical lattice QCD simulations*, Phys. Lett. **B729** (2014) 85, arXiv:1311.6214. 2

[50] S.-Q. Luo *et al.*, *Exotic tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration*, Eur. Phys. J. **C77** (2017) 709, arXiv:1707.01180. 2

[51] M. Karliner and J. L. Rosner, *Discovery of doubly-charmed Ξ_{cc} baryon implies a stable $(bb\bar{u}\bar{d})$ tetraquark*, Phys. Rev. Lett. **119** (2017) 202001, arXiv:1707.07666. 2

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[54] Hadron Spectrum collaboration, G. K. C. Cheung, C. E. Thomas, J. J. Dudek, and R. G. Edwards, *Tetraquark operators in lattice QCD and exotic flavour states in the charm sector*, JHEP **11** (2017) 033, arXiv:1709.01417. 2

[55] W. Park, S. Noh, and S. H. Lee, *Masses of the doubly heavy tetraquarks in a constituent quark model*, Acta Phys. Polon. **B50** (2019) 1151, arXiv:1809.05257. 2

[56] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, *Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD*, Phys. Rev. **D99** (2019) 054505, arXiv:1810.10550. 2

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[62] Q.-F. Lü, D.-Y. Chen, and Y.-B. Dong, *Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model*, Phys. Rev. **D102** (2020) 034012, arXiv:2006.08087. 2

[63] E. Braaten, L.-P. He, and A. Mohapatra, *Masses of doubly heavy tetraquarks with error bars*, Phys. Rev. D **103** (2021) 016001, arXiv:2006.08650. 2

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[65] J.-B. Cheng *et al.*, *Double-heavy tetraquark states with heavy diquark-antiquark symmetry*, arXiv:2008.00737. 2

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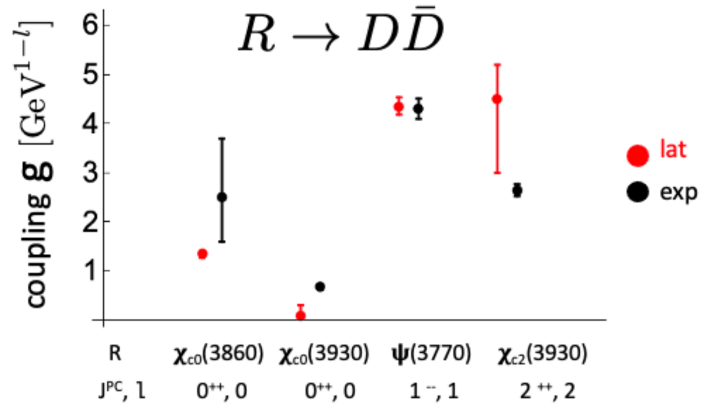
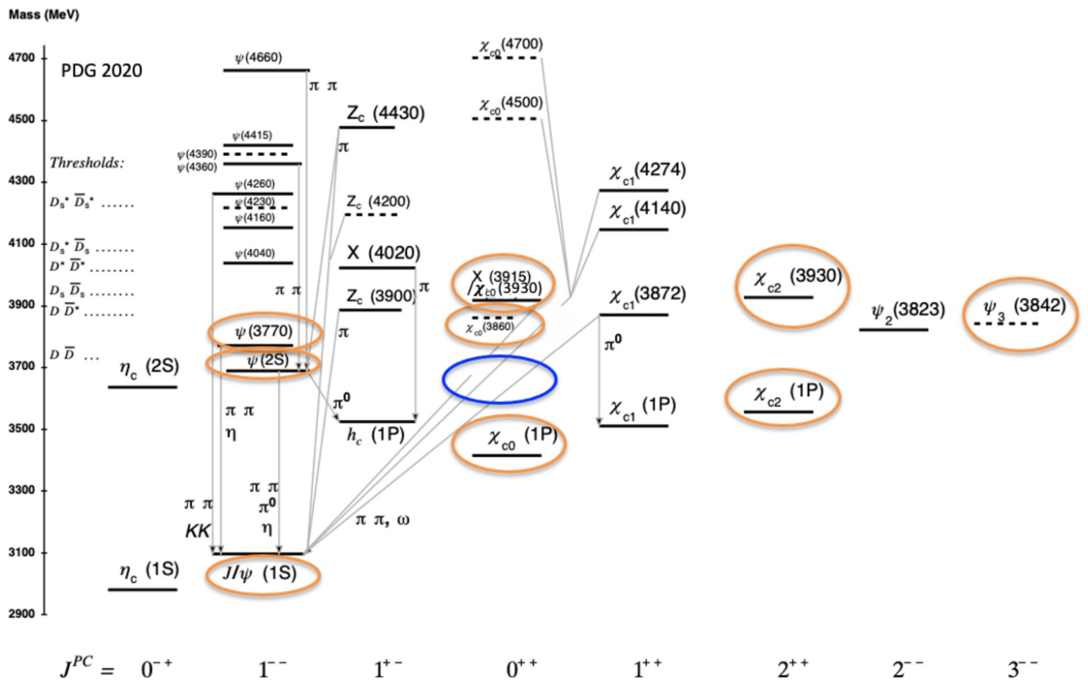
Charmonium resonances from lattice

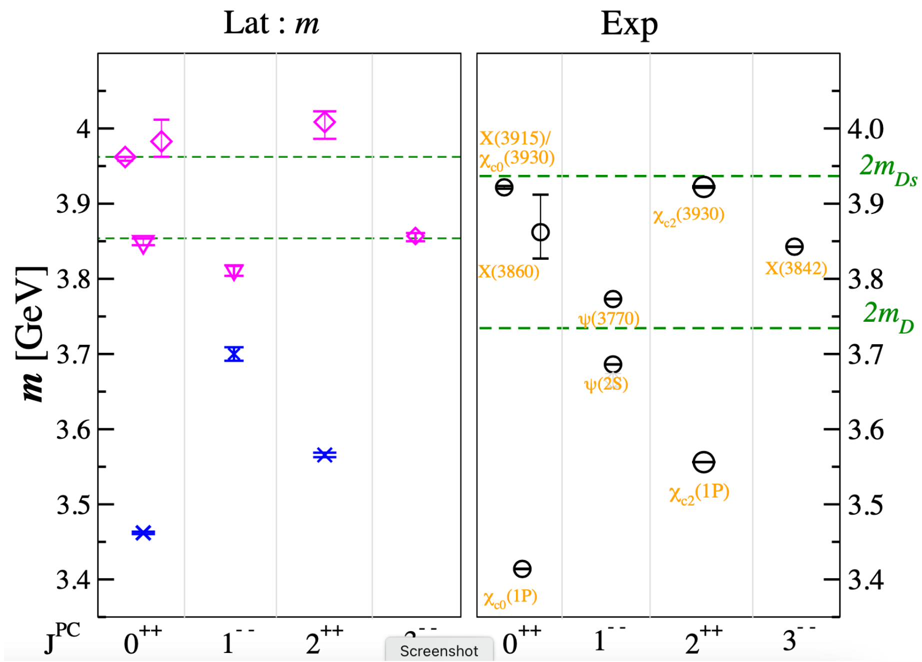
$$\Gamma \equiv g^2 p_D^{2l+1} / m^2$$



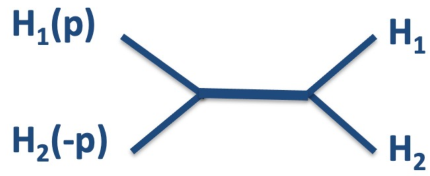
S.P. , Collins, Padmanath, Mohler, Piemonte

2011.02541 JHEP, 1905.03506 PRD, 2111.02934



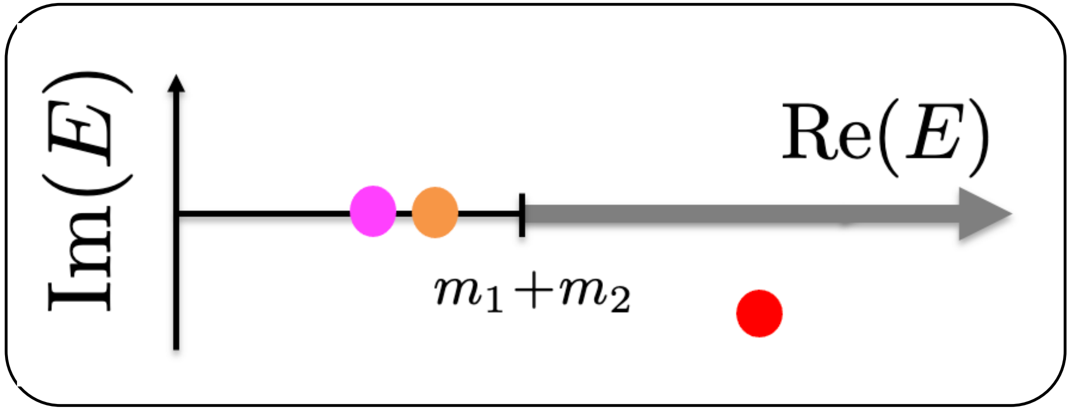


Definitions: bound state, virtual bound state & resonance



$$T(E) \propto \frac{1}{E^2 - m^2} \qquad T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

Poles of *T(E)*, *E=E cm*



Virtual bound st. $p = -i\,|p|$

Bound st. $p = i\,|p|$

Resonance

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + (-p)^2} \; < \; m_1 + m_2$$

Bound and virtual bound state: simplest example

scattering in square-well potential in QM

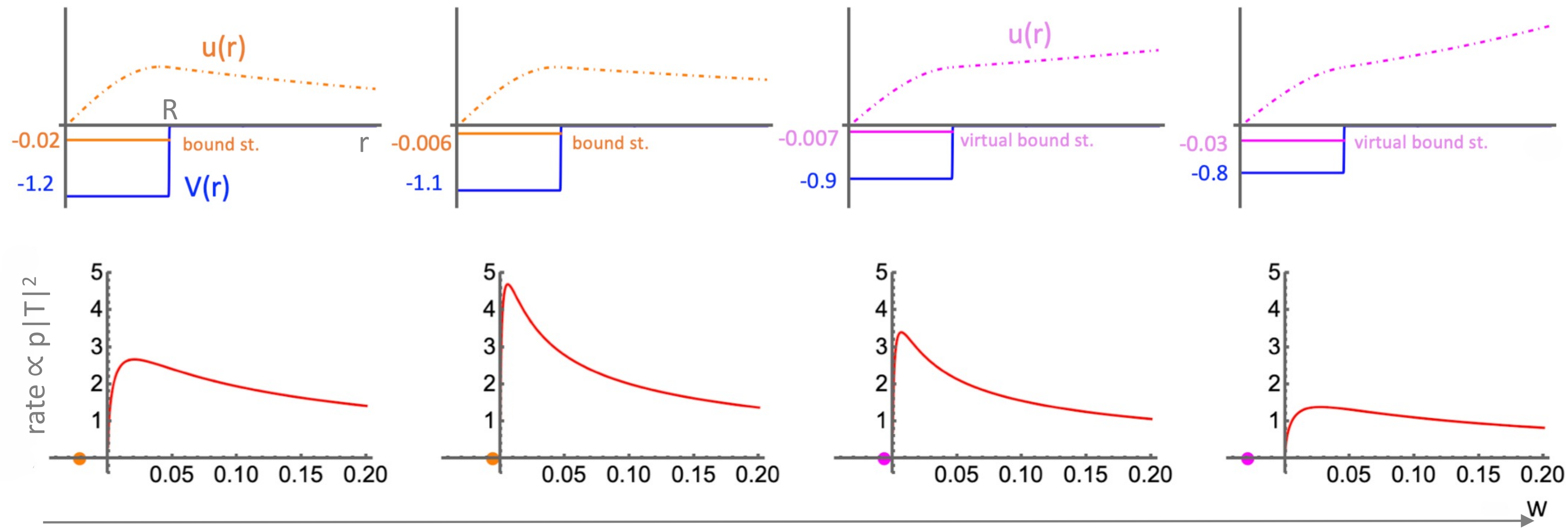
partial wave l=0

$$\delta = \arctan[\tan(qR)\tfrac{p}{q}] - pR$$

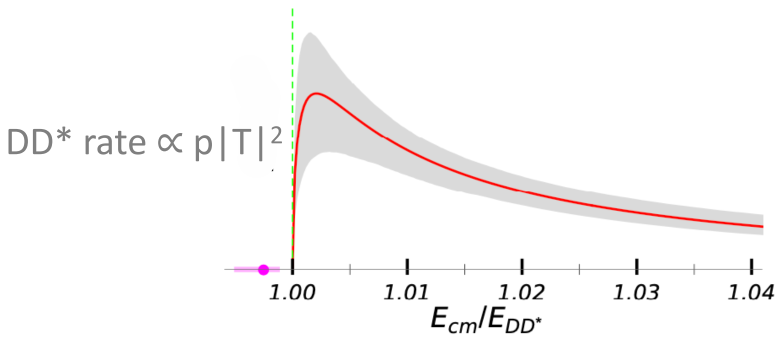
$u(r) = A\sin(qr)$ $u(r) = B\sin(pr+\delta)$

p=i|p| $e^{ipr} = e^{-|p|r}$

p=-i|p| $e^{ipr} = e^{|p|r}$



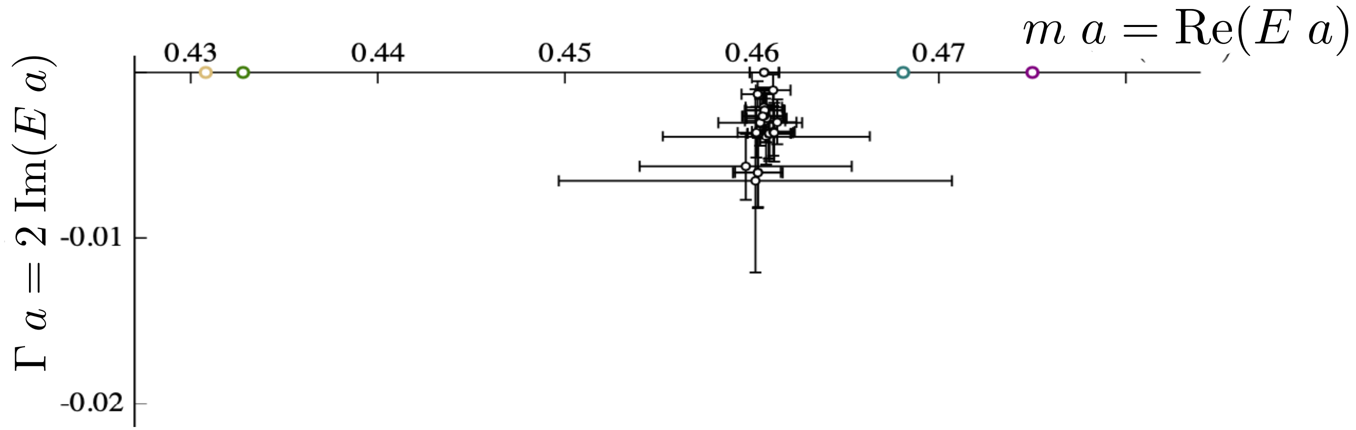
increasing $m_{u/d}$, decreasing attraction

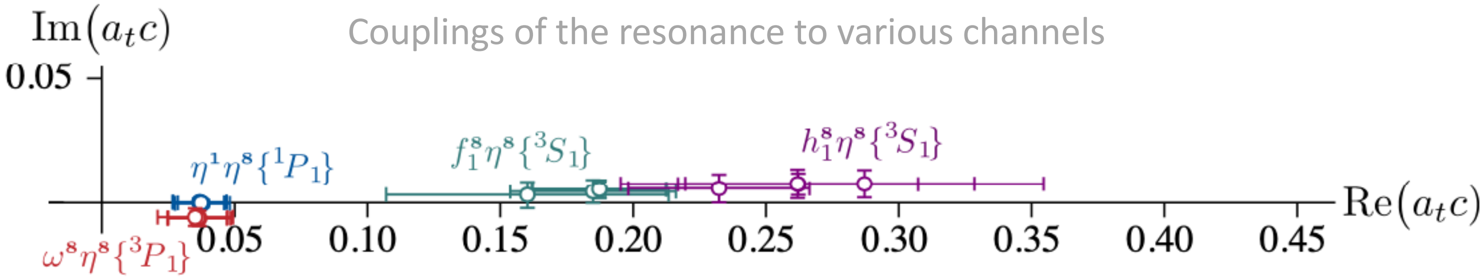


for T_{cc} extracted on lattice

light hybrid meson π_1 from lattice

$\bar{d}Gu$ $J^{PC} = 1^{-+}$





$$T_{ij} \sim \frac{c_i\ c_j}{E_p^2 - E^2}$$

Woss et al. (HadSpec)
2009.10034

$m_u = m_d = m_s$, $m_\pi \approx 700\ MeV$

pheno analysis

physical world

$\rho\ \pi$ $\eta'\ \pi$ $f_1\ \pi$ $b_1\ \pi$

dominant coupling

resemblence to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

P_c pentaquarks

$\bar{c}\,c\,u\,u\,d$

$P_c =$ uud$\bar{c}$c $\rightarrow$ (uud) ($\bar{c}$c): p J/Ψ,...

$\rightarrow$ (udc) ($\bar{c}$u): $\Sigma_c^+ \bar{D}^0$,...

[LHCb 2019, 1904.03947, PRL]

Indications that **$\Sigma_c^+ \bar{D}^{(*)}$ molecular** component is important:

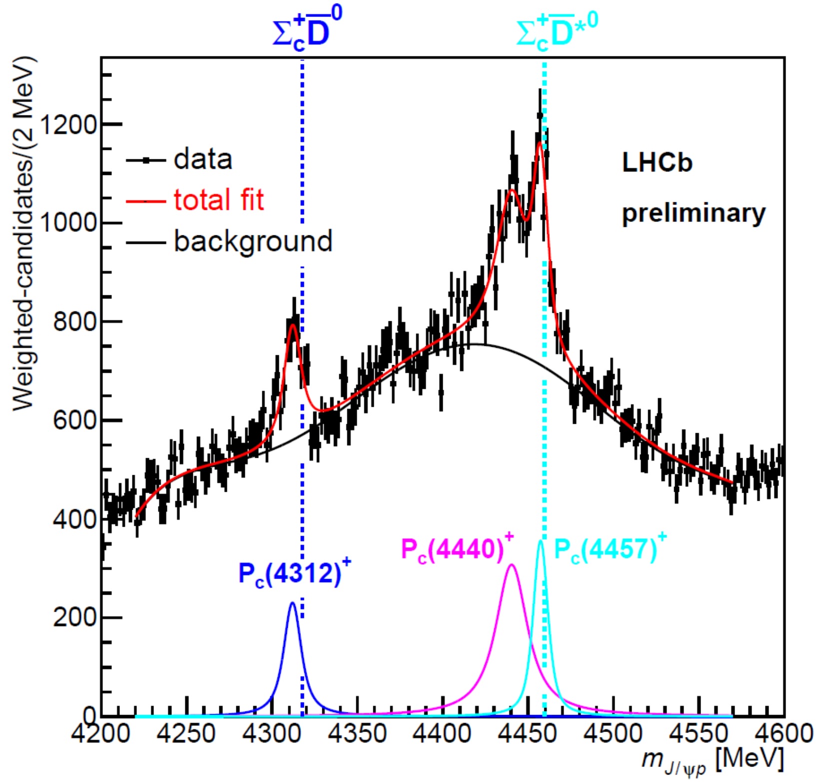
- **experiment** finds them slightly below those thresholds
- supported by **phenomenological models** with ρ/ω exchange

  predicted 2010-2012 [Wu, Molina, Oset, Zou, 1007.0573, PRL; Wu et al., 1202.1036. PRC, Yang et al, 1105.2901, Wang et al, 1101.0453, PRC]

J^P not determined from exp.

Expected J^P for molecule in s-wave:

$\Sigma_c(\tfrac{1}{2}^+)\bar{D}(0^-) \rightarrow J^P = \tfrac{1}{2}^-$ $\quad$ $\Sigma_c(\tfrac{1}{2}^+)\bar{D}^*(1^-) \rightarrow J^P = \tfrac{1}{2}^-, \tfrac{3}{2}^-$



- **Lattice QCD** addressed simplified question:

Do Pc resonances appear in one-channel

$$p \;\; J/\psi \rightarrow P_c \rightarrow p \;\; J/\psi$$

scattering if it is decoupled from other channels ?

Answer: No [Skerbis, S. P., 1811.02285, PRD 2019]

T(E)≈0 within large errors, small interaction, no resonance

This indicates that coupling of p J/ψ channel with other two-hadron channels is likely responsible for Pc in experiment (in line with LHCb result)