

# Bayesian inference in heavy ion collisions



Jean-François Paquet (Duke University)

May 17, 2022

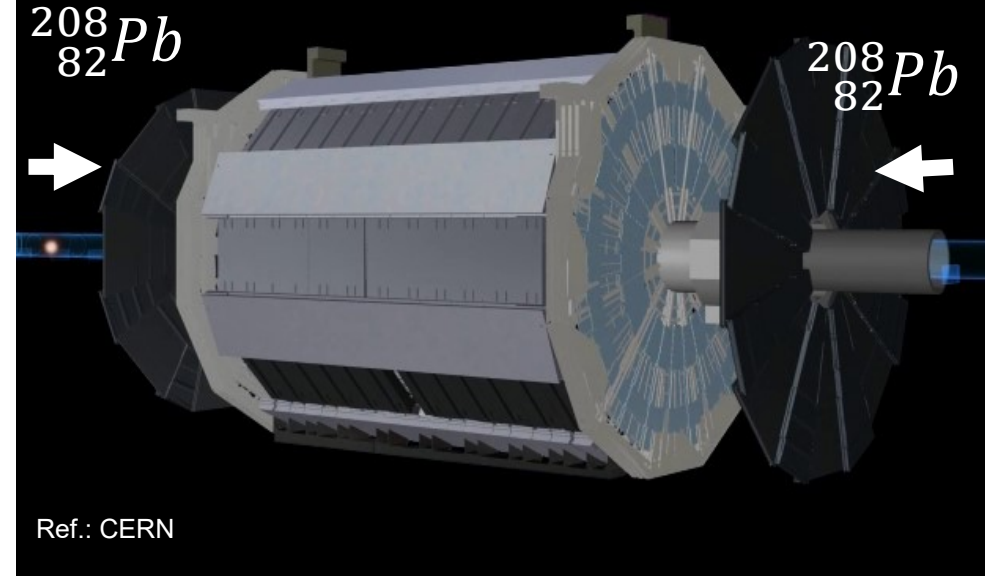


Live from Durham,  
North Carolina

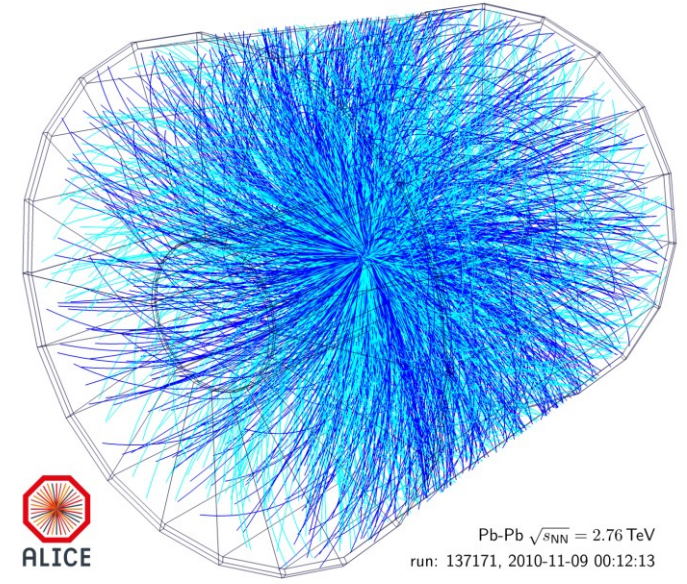
The 10th Annual  
Large Hadron Collider Physics Conference  
May 16-21, 2022



# Before the collision

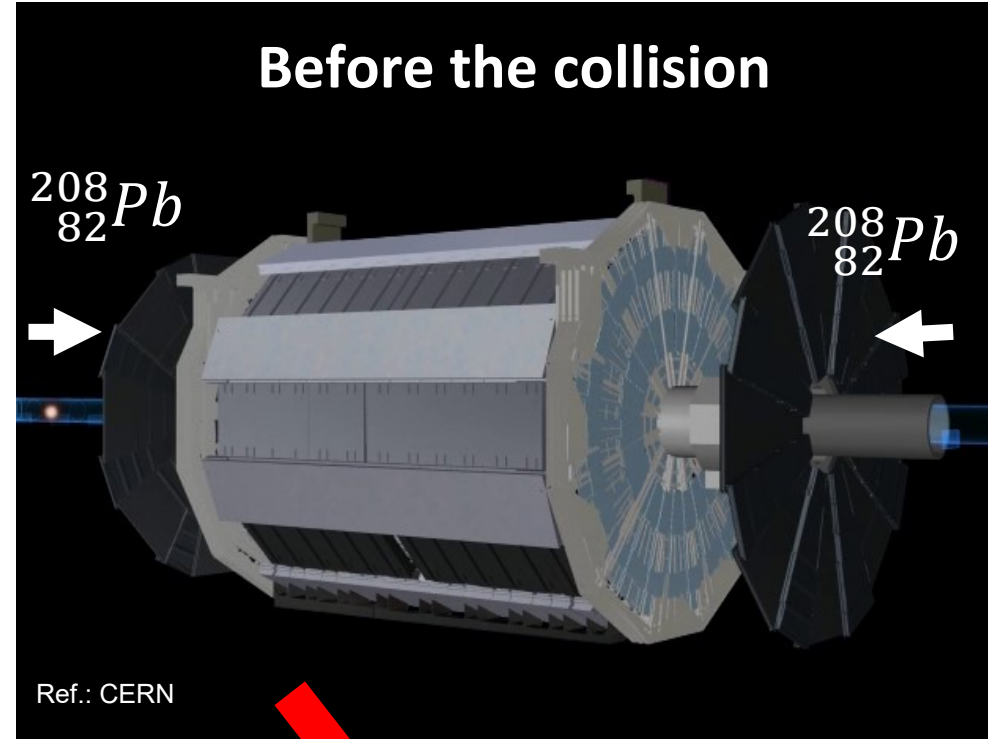


# After the collisions

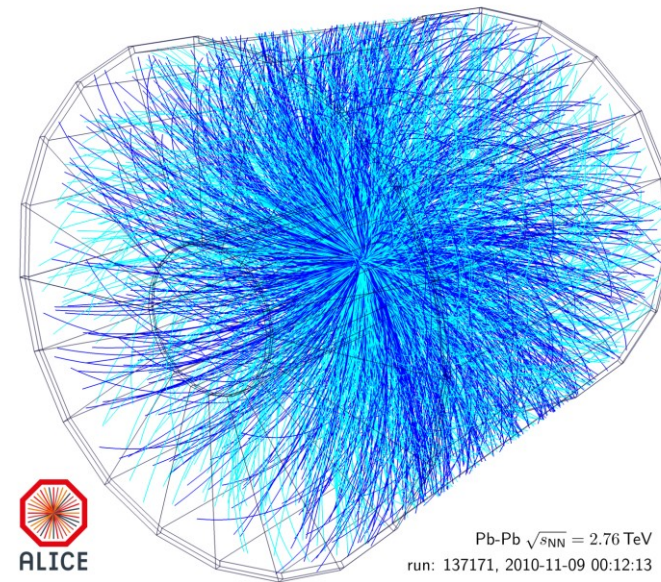




# Before the collision

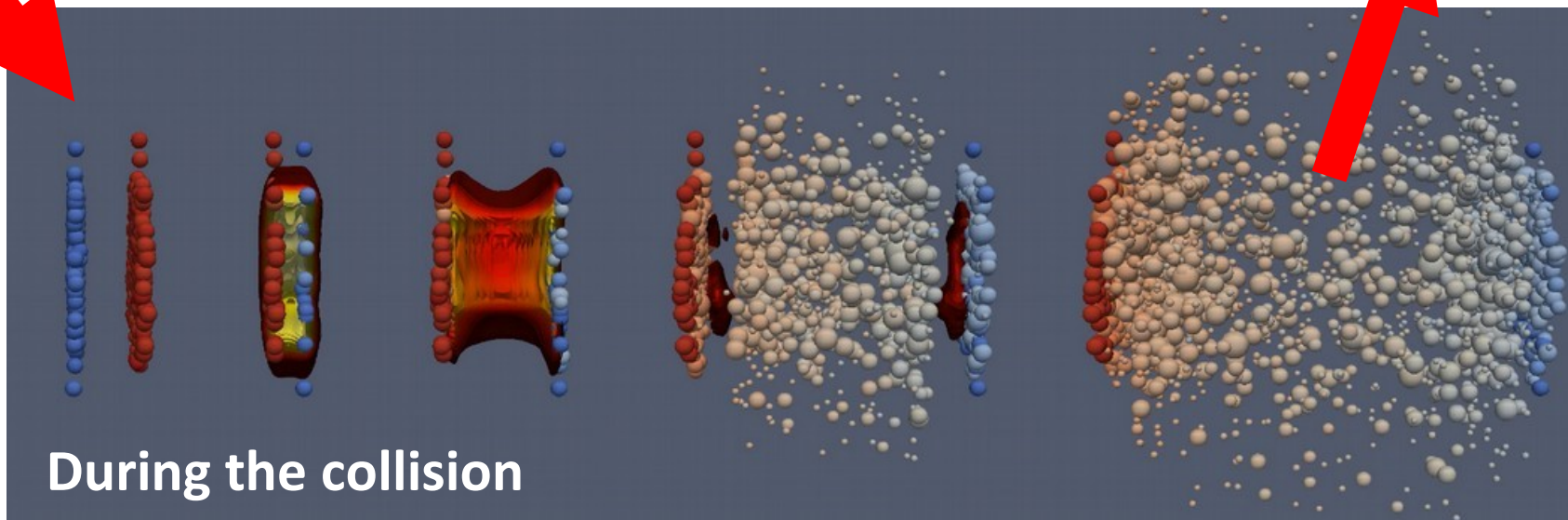


# After the collisions

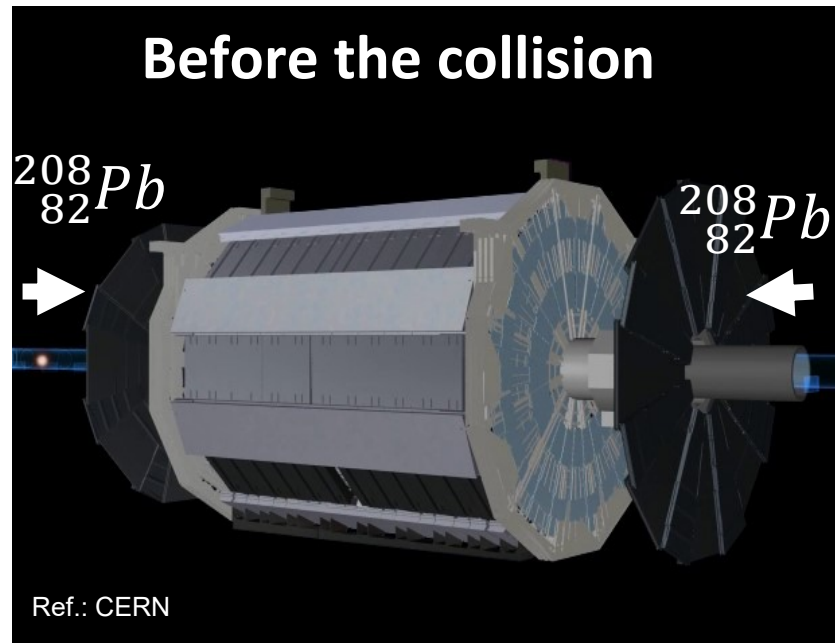


Ref: MADAI  
collaboration,  
Hannah Elfner and  
Jonah Bernhard

# During the collision

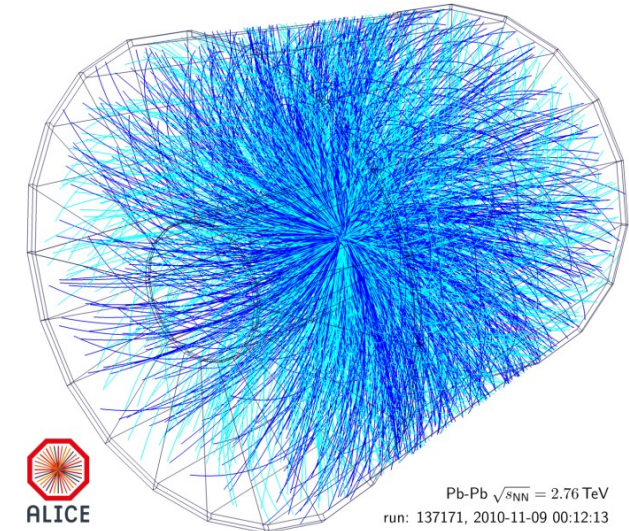


# Many-body properties of nuclear matter



Simulation of  
the collisions  
(Sensitive to  
many-body  
properties of  
nuclear matter)

**After the collisions**



What properties of hot nuclear matter are consistent with the measurements?

- Properties of the quark-gluon plasma (equation of state, viscosity, ...)
- Properties of energetic partons interacting with the quark-gluon plasma (“ $\hat{q}$ ”, ...)

# Near-equilibrium properties of the quark-gluon plasma

- Equilibrium properties of fluid:  
thermodynamics, equation of state
- Near-equilibrium properties?



Ref: Wikipedia

Wave propagating at the  
speed of sound  $c_s$

$$\text{Perturbation} \propto \exp \left[ i c_s k t - i \vec{k} \cdot \vec{x} - \underbrace{\left( \frac{4}{3} \frac{\eta}{s} + \frac{\zeta}{s} \right) \frac{k}{2T}}_{\text{Decay rate of perturbation}} k t \right]$$

Decay rate of perturbation

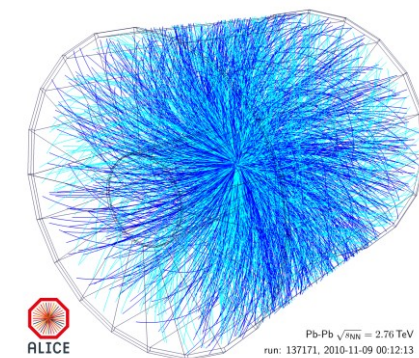
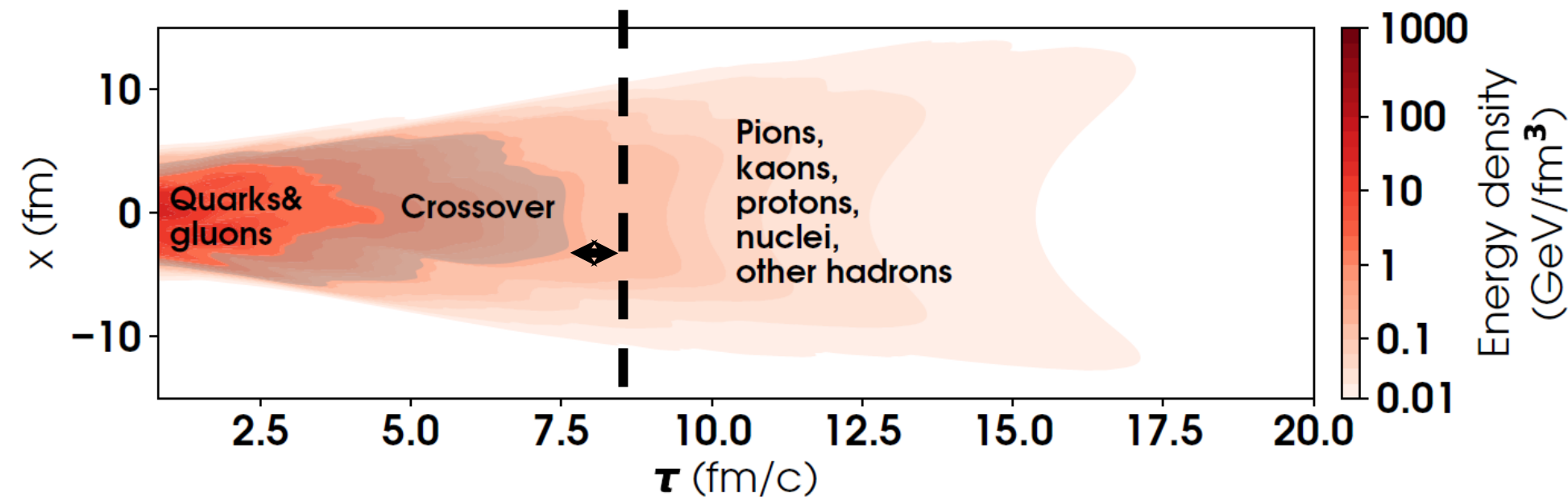
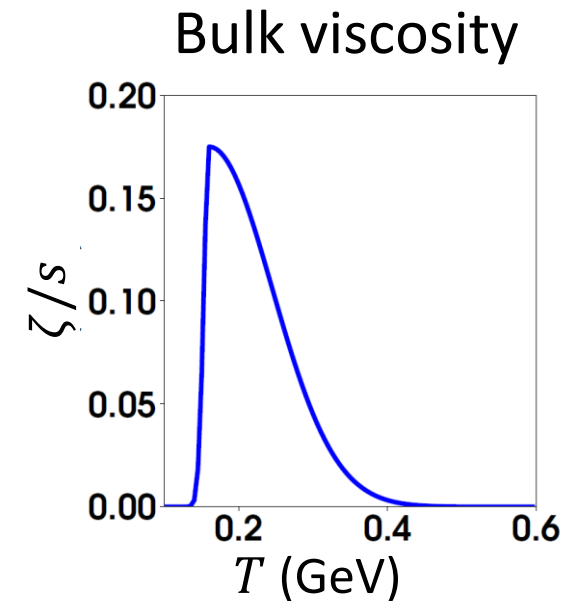
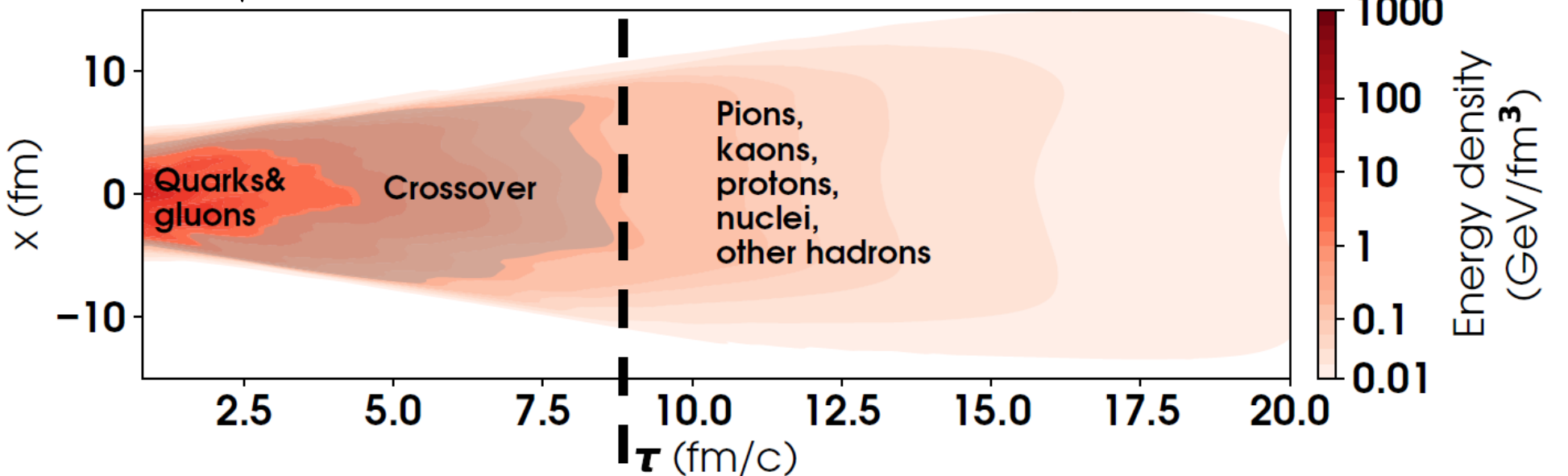
- Specific **shear viscosity**  $\eta/s$  and **bulk viscosity**  $\zeta/s$  are “**transport coefficients**”  
 (“s” is the entropy density)



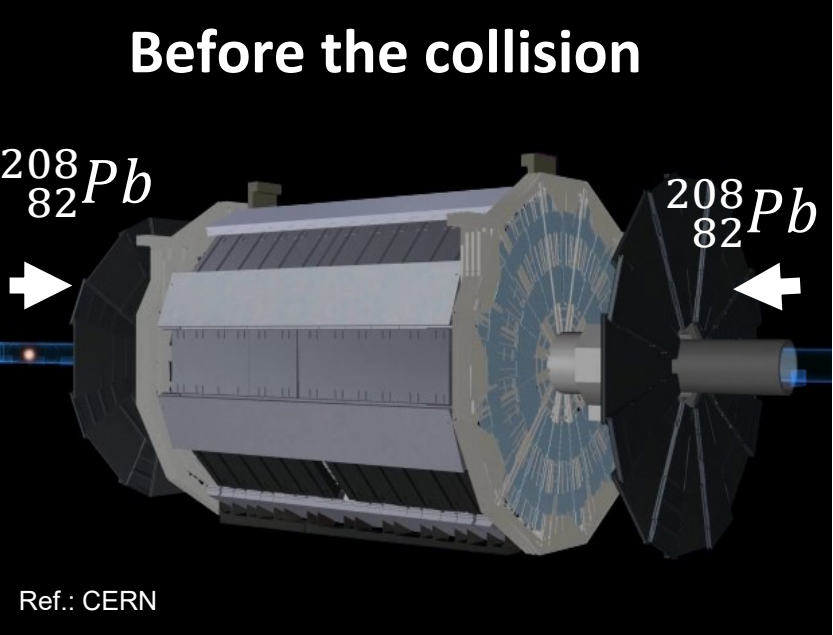
# Viscosities affect the evolution of the plasma

Ref.: Gale, JFP, Schenke and Shen, PRC 2021

Pb-Pb  $\sqrt{s_{NN}}=2.76$  TeV; 20-30% centrality

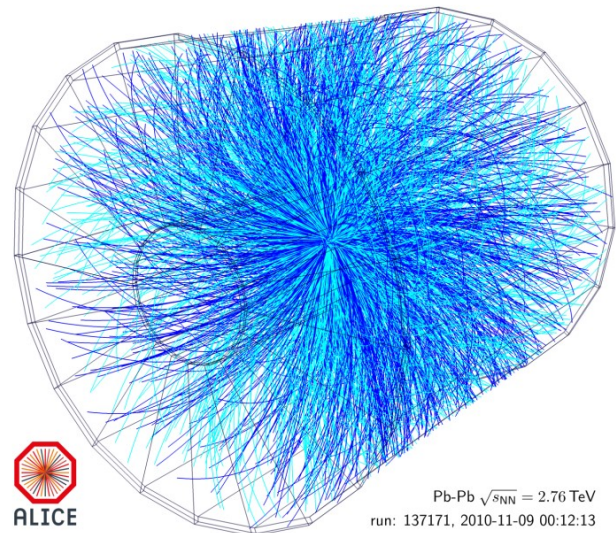


# Bayesian inference & inverse problem in heavy ion collisions

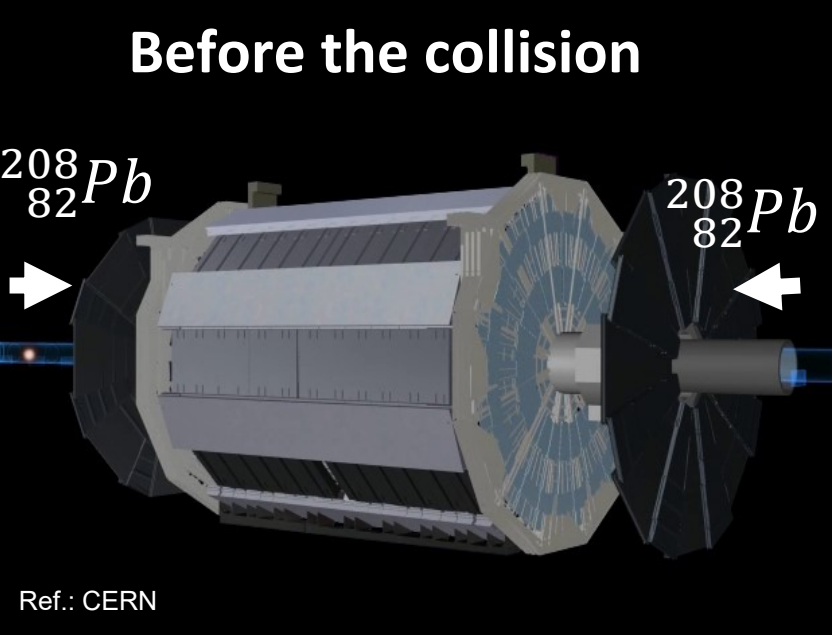


Simulation of the collisions  
(Sensitive to many-body properties of nuclear matter)

**After the collisions**

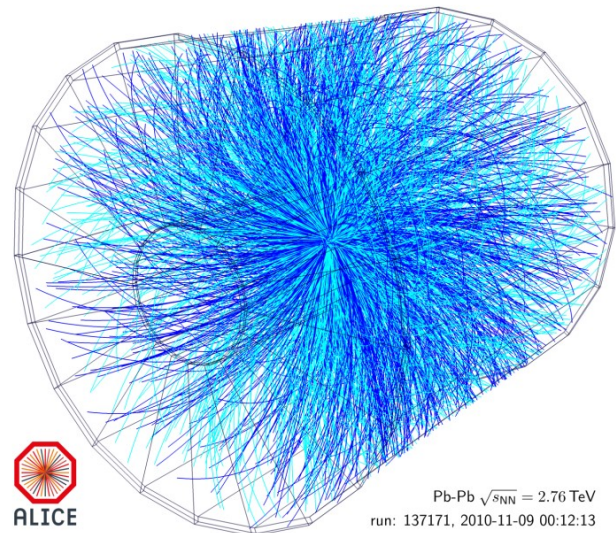


# Bayesian inference & inverse problem in heavy ion collisions



Simulation of the collisions  
(Sensitive to many-body properties of nuclear matter)

After the collisions



Bayes' theorem

$$\text{prob}(\text{obs}) \times \text{prob}(\text{param}|\text{obs}) = \text{prob}(\text{p, obs}) = \text{prob}(\text{param}) \times \text{prob}(\text{obs}|\text{param})$$

Evidence × Posterior

= Joint

= Prior × Likelihood

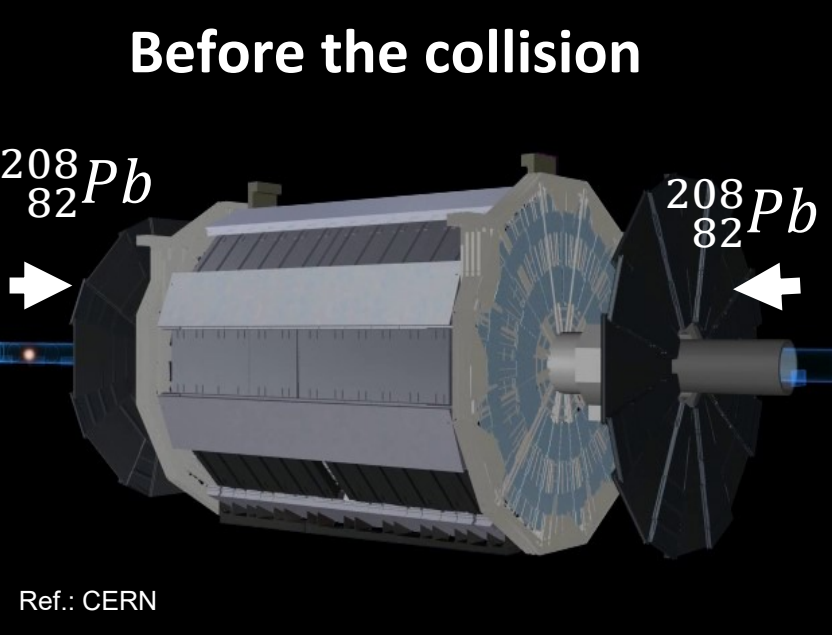
Outputs

Inputs

Adapted from: John Skilling - "Foundations and algorithms" in Bayesian Methods in Cosmology, Cambridge University Press, 2009.

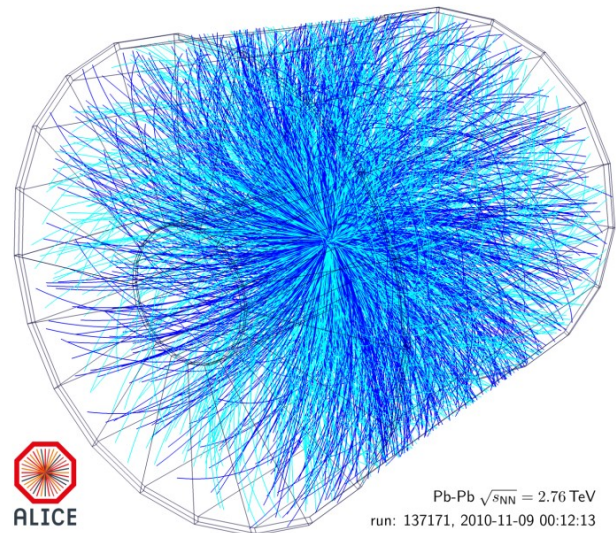


# Bayesian inference & inverse problem in heavy ion collisions



Simulation of the collisions  
(Sensitive to many-body properties of nuclear matter)

After the collisions



Bayes' theorem

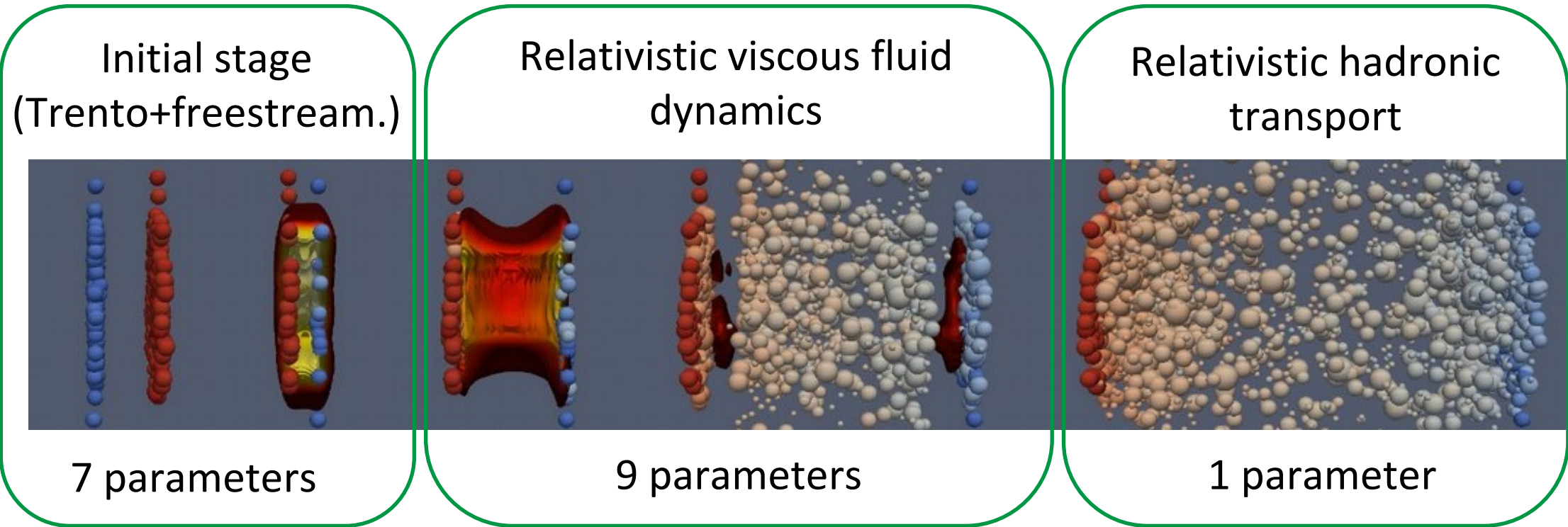
$$\text{prob}(\text{obs}) \times \text{prob}(\text{param}|\text{obs}) = \text{prob}(\text{p, obs}) = \text{prob}(\text{param}) \times \text{prob}(\text{obs}|\text{param})$$

Evidence      ×      Posterior      =      Joint      =      Prior      ×      Likelihood

$$\propto \exp \left[ - \frac{(\text{obs} - \text{model}(\text{param}))^T (\text{covariance matrix})^{-1} (\text{obs} - \text{model}(\text{param}))}{2} \right]$$

# Example: shear and bulk viscosity of the quark-gluon plasma

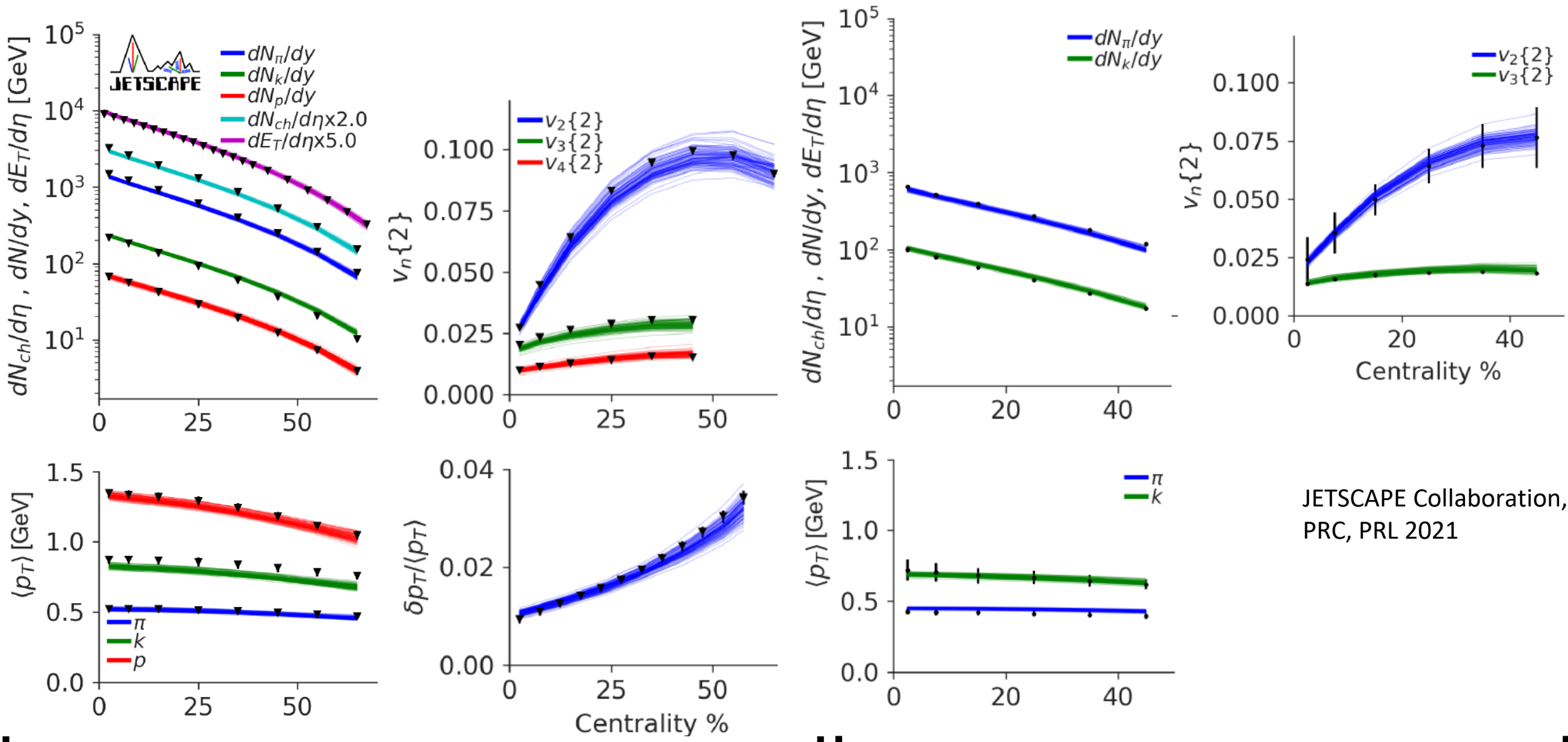
Model:



Data: all at midrapidity, from STAR Au-Au  $\sqrt{s_{NN}}=200$  GeV and ALICE Pb-Pb  $\sqrt{s_{NN}}=2760$  GeV

- Pion, kaon & proton multiplicity at midrapidity
- Pion, kaon & proton mean transverse momentum at midrapidity
- Charged hadron anisotropic flow ( $v_{2/3/4}\{2\}$ )
- Mean transverse momentum fluctuations; transverse energy

# Example: shear and bulk viscosity of the quark-gluon plasma



JETSCAPE Collaboration,  
PRC, PRL 2021

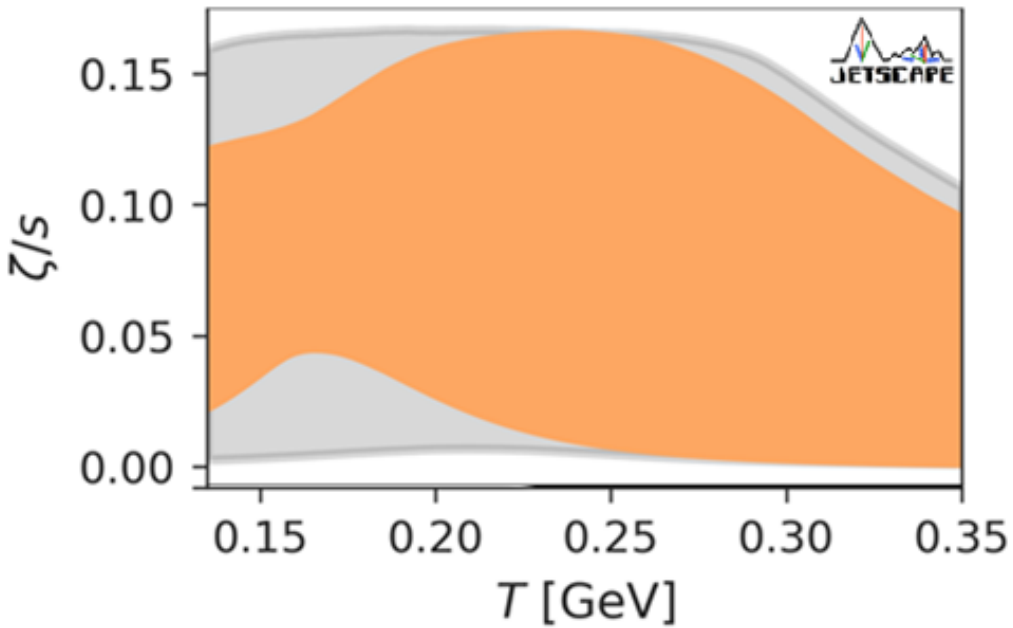
PbPb  $\sqrt{s_{NN}} = 2760$  GeV, ALICE Collaboration (LHC)

AuAu  $\sqrt{s_{NN}} = 200$  GeV, STAR Collaboration (RHIC)

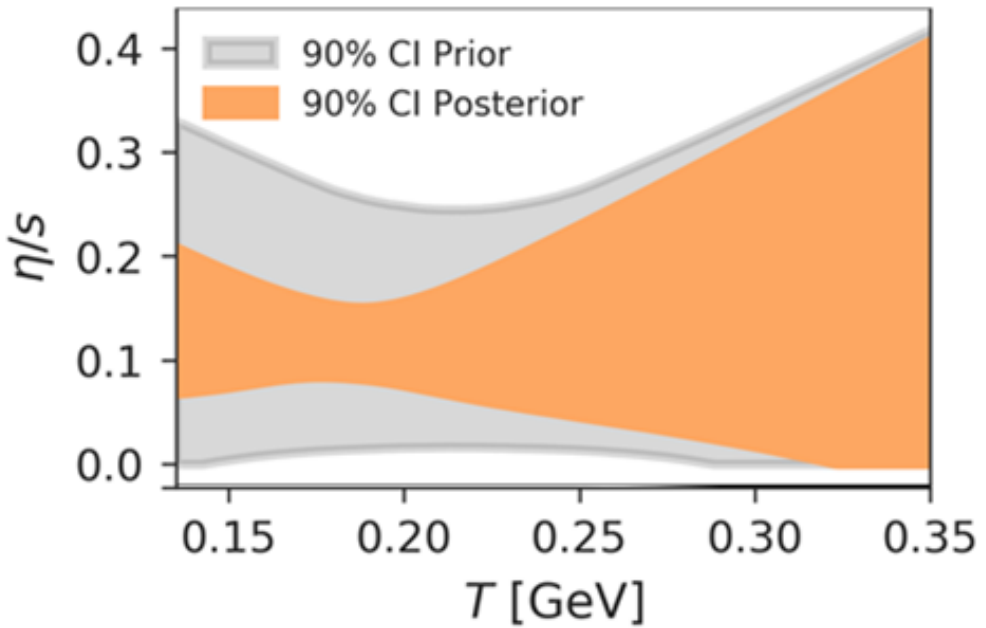


# Example: shear and bulk viscosity of the quark-gluon plasma

Bulk viscosity



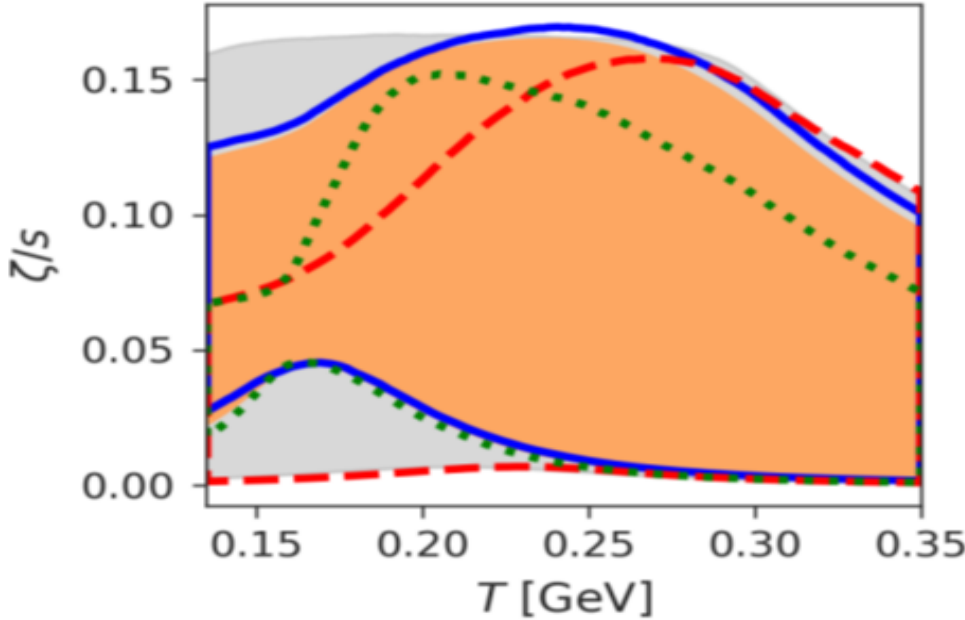
Shear viscosity



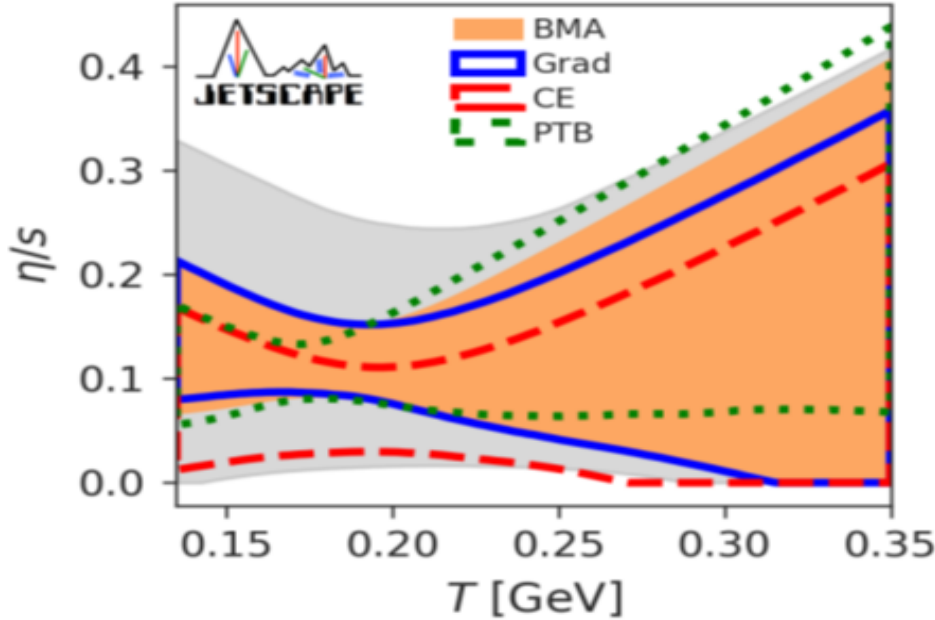
JETSCAPE  
Collaboration,  
PRC,PRL 2021

# Example: shear and bulk viscosity of the quark-gluon plasma

Bulk viscosity



Shear viscosity



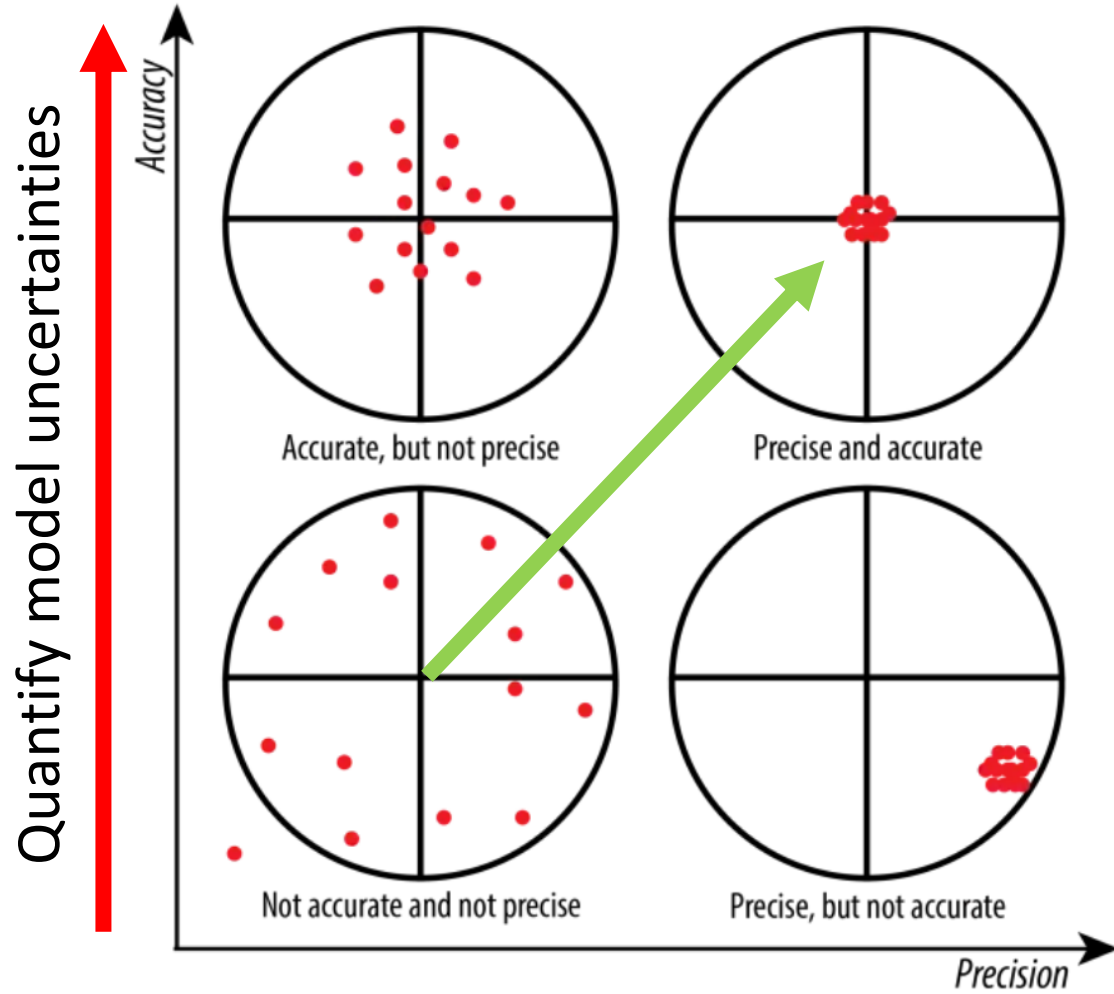
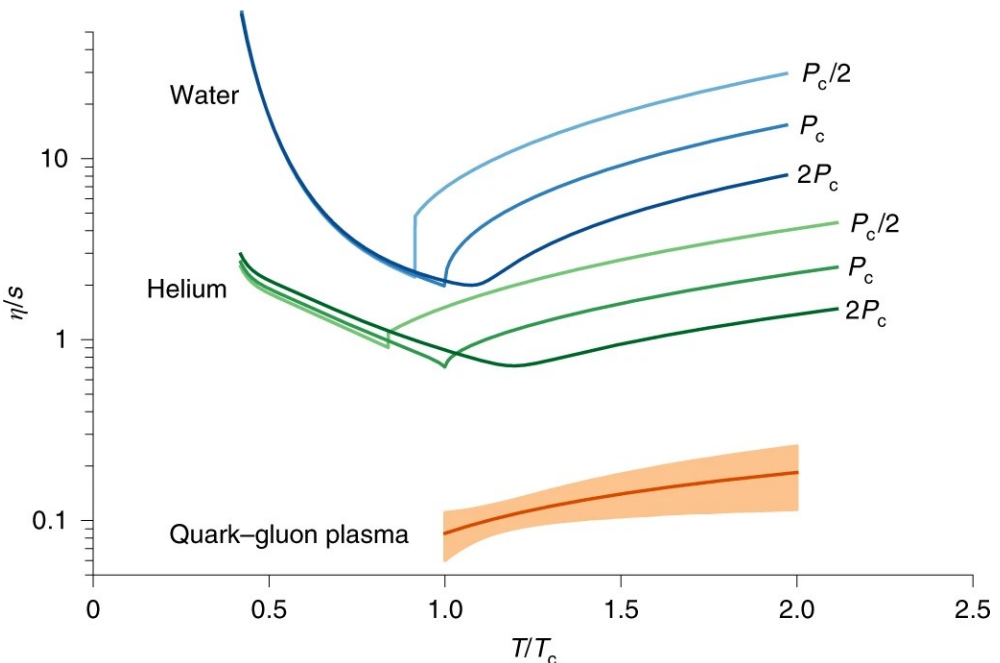
JETSCAPE  
Collaboration,  
PRC,PRL 2021

- Different lines = different transition models from hydrodynamics to hadronic transport
- Shaded average is “Bayesian model average”:  
average weighted by degree of agreement with data (“Bayes factor”/evidence)

# Bayesian inference in heavy ion collisions

## Viscosity of the quark-gluon plasma

- Nijs, van der Schee, Gürsoy & Snellings, PRC, PRL 2021
- Parkkila, Onnerstad, Taghavi, Mordasini & Bilandzic (arXiv:2111.08145)
- Auvinen, Eskola, Huovinen, Niemi, Paatelainen & Petreczky, PRC 2020
- Bernhard, Moreland & Bass, Nature Physics 2019:

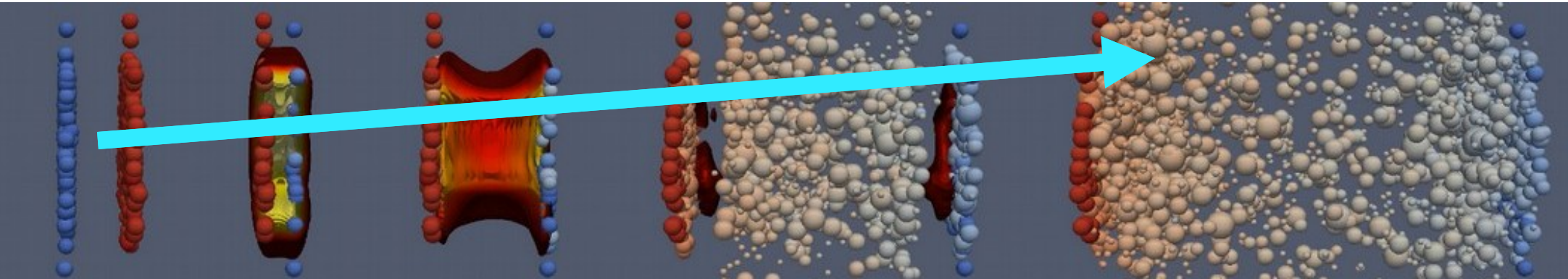


Compare with more data

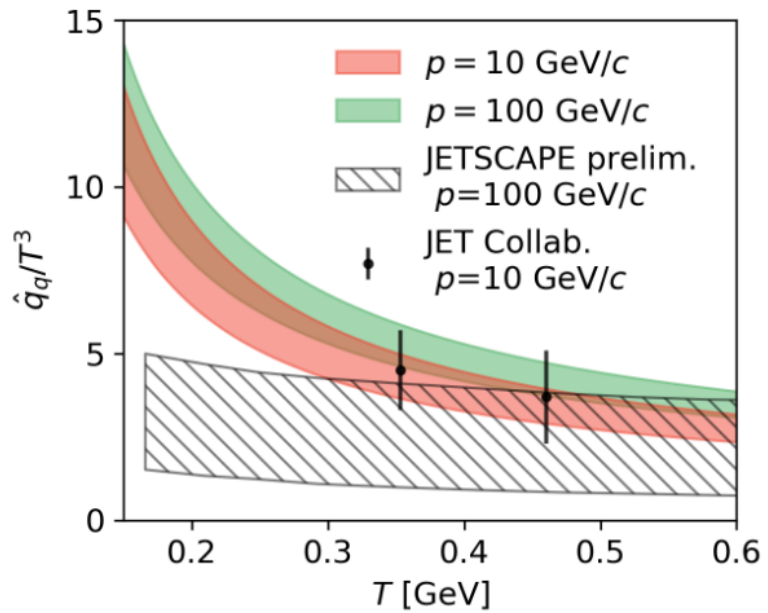
Ref.: <https://wp.stolaf.edu/it/gis-precision-accuracy/>



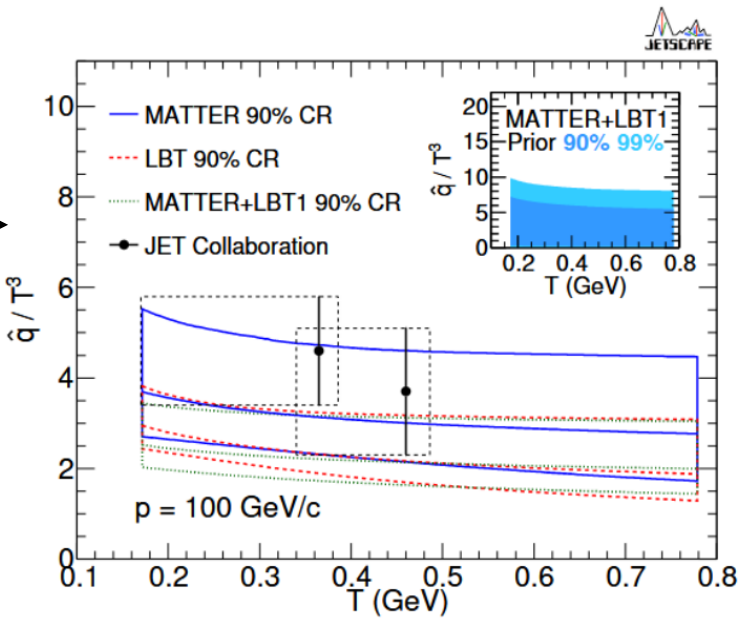
# Bayesian inference in heavy ion collisions: parton energy loss



How do (heavy quarks)/(light quarks)/gluons interact with quark-gluon plasma?



JETSCAPE Collaboration, PRC 2021



Ke, Wang, JHEP 2021

# Summary

Heavy ion collisions well suited for Bayesian inference:

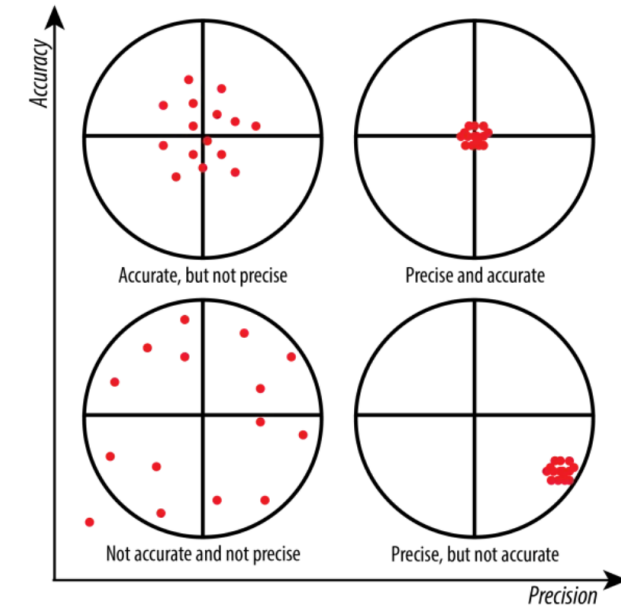
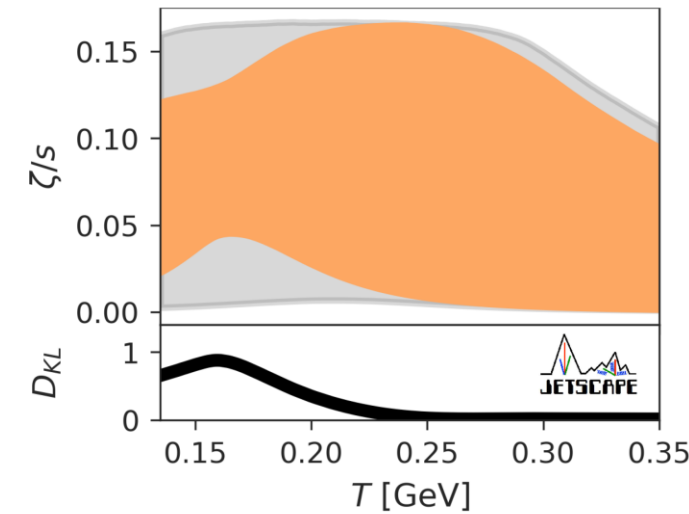
- Large number of measurements with variable uncertainties
- Mature but complex multistage models with multiple unknowns

Contributions from the JETSCAPE analysis:

- Theoretical uncertainty quantification
- Bayes factors & Bayesian model averaging
- Closure tests, priors, sensitivity analysis, ...

Outlook:

- Additional data
- Additional source of theoretical uncertainties must be accounted for
- Experimental uncertainties: correlations between uncertainties [ 2102.11337 ]



# Acknowledgements

Bayesian inference results are from the JETSCAPE Collaboration, with **Derek Everett** (formerly OSU) & **Weiyao Ke** (LANL, formerly UC Berkeley, LBNL & Duke) driving the effort.

## PHYSICAL REVIEW C

Multisystem Bayesian constraints on the transport coefficients of QCD matter

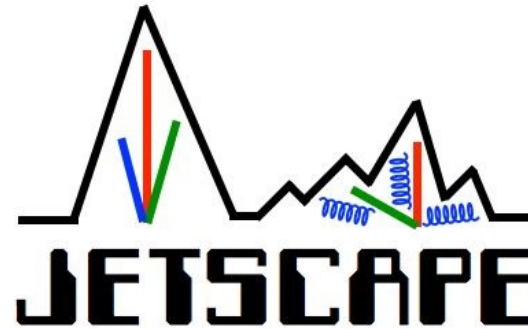
D. Everett *et al.* (JETSCAPE Collaboration)  
Phys. Rev. C **103**, 054904 – Published 14 May 2021

## PHYSICAL REVIEW LETTERS

Editors' Suggestion

Phenomenological Constraints on the Transport Properties of QCD Matter with Data-Driven Model Averaging

D. Everett *et al.* (JETSCAPE Collaboration)  
Phys. Rev. Lett. **126**, 242301 – Published 17 June 2021



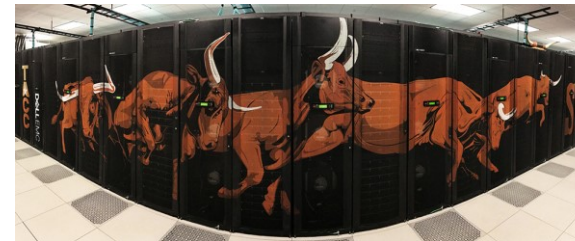
This work was supported by:



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Multi-system Bayesian constraints on the transport coefficients of QCD matter

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Model description

Priors

Closure tests

Bayesian parameter estimation

Observable sensitivity study

Bayesian model selection

Validation against additional data

Validation of hydrodynamics against external solutions

Hadronic transport validation

Questions?