# Overview of mixed QCD-EW corrections for Drell-Yan

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# Outline

Organisational structure: higher-order corrections + phase-space region

Mixed QCD-EW corrections to the Drell-Yan processes

- Resonant region: Z + W
- Off-shell region: NCDY
  - Extra: development on higher order calculations (amplitudes)

Outlook and Conclusions

Disclaimer:
in this talk I will strictly cover only fixed-order results

# Motivation (personal selection)

## Theory uncertainty

QCD: estimated via scale variation

EW: renormalized on shell. No scale dependence. Input scheme change

$$d\sigma = d\sigma_0 \left( 1 + \overbrace{\frac{\alpha_s}{2\pi}\delta^{(1,0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \delta^{(2,0)}}^{\text{"LO" from EW side}} + \frac{\alpha}{2\pi}\delta^{(0,1)} + \frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi}\delta^{(1,1)} + \ldots \right)$$

residual EW input scheme dependence

Reduction of EW scheme dependence

Extensive study for on-shell Z production [Bonciani, FB, Rana, Vicini 2111.12694]

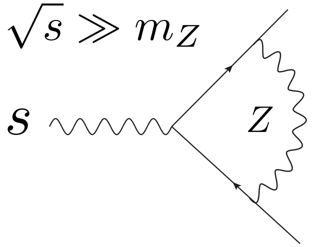
## Interplay of effects

on-shell: IS hard QCD radiation + QED FS radiation off the leptons: normalization + shape effects

off-shell: Naive power counting: $\alpha_s \sim 0.1$ and $\alpha \sim 0.01$, thus expecting QCDxEW at permille level

Really: QCD corrections +20% and EW power counting spoiled by large Sudakov logs [Kuhn, Penin, Smirnov '00][Ciafaloni, Ciafaloni, Comelli '01][Denner, Pozzorini '01]

$\sqrt{s} \gg m_Z$

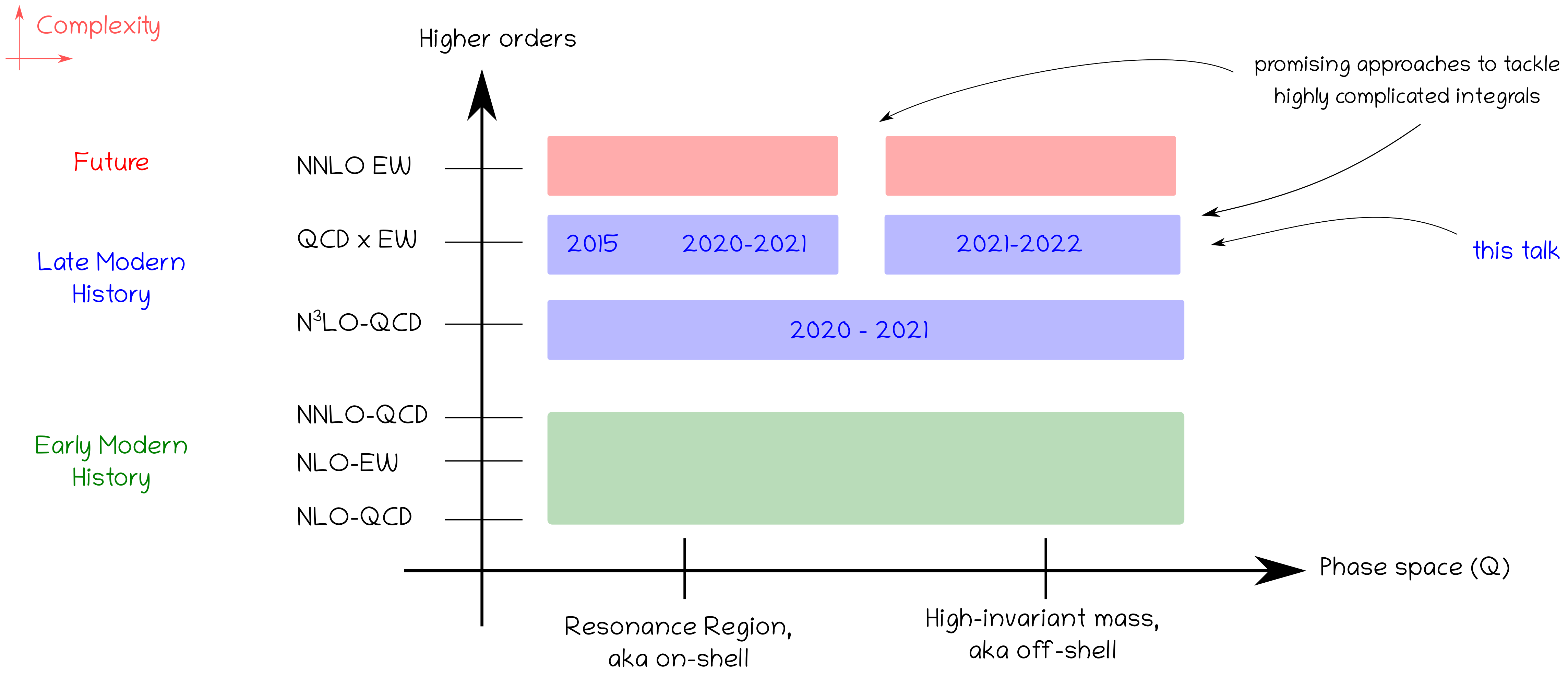



$$\frac{\alpha}{4\pi \sin^2\theta_w} \log^2\left(\frac{s}{m_Z^2}\right) \sim 10\% \qquad \frac{\alpha}{4\pi \sin^2\theta_w} \log\left(\frac{s}{m_Z^2}\right) \sim 1.6\%$$

at $s \sim (2\ \text{TeV})^2$

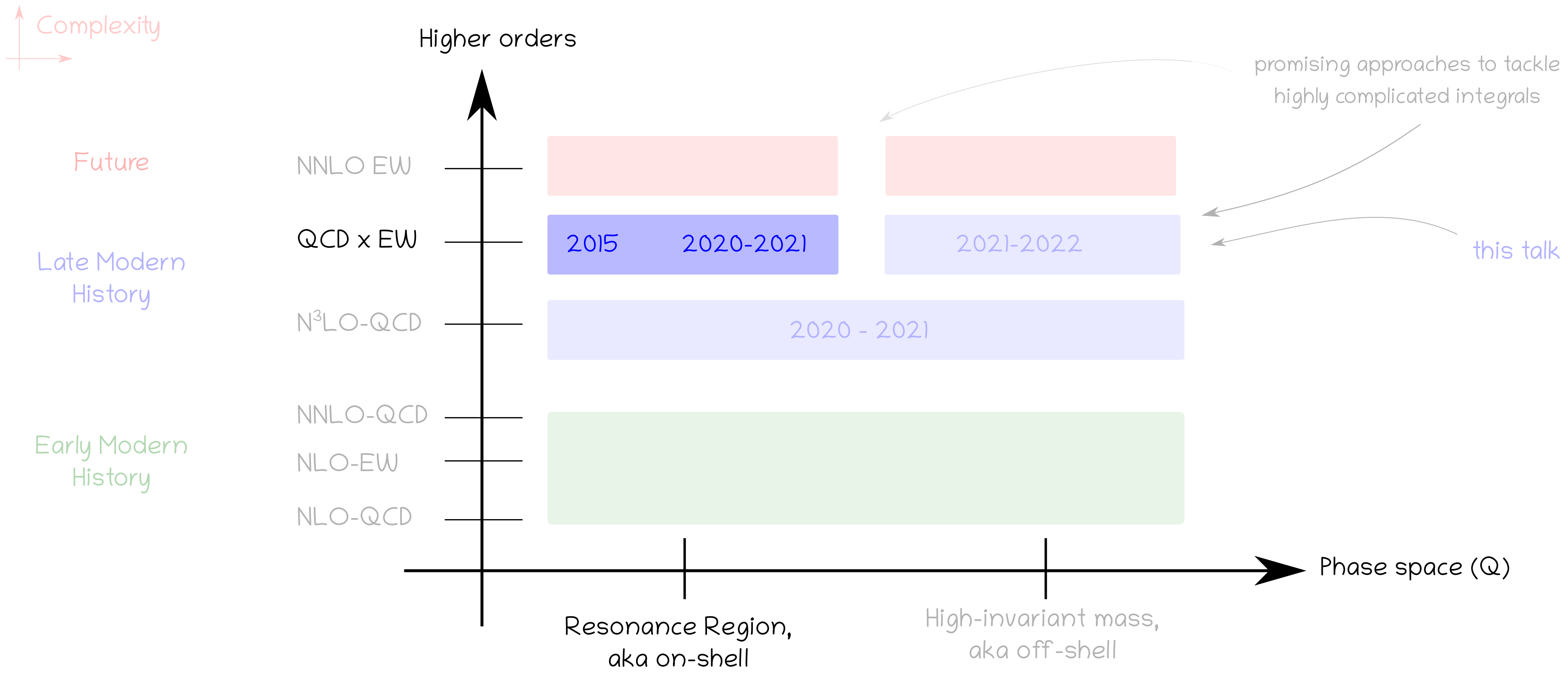
QCDxEW potentially ~ 2% in the high energy region

# Higher-order corrections and phase space




Focus on QCDxEW corrections
(vast literature and constantly richer pheno)

# Higher-order corrections and phase space




Focus on QCDxEW corrections
(vast literature and constantly richer pheno)

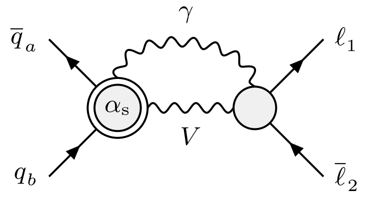
# Drell-Yan in the resonance region

Remarkable technical simplification wrt fully off-shell case. One can work in the narrow-width approximation [Fadin,Khoze,Martin hep-ph/9309234]

$$\sigma = \text{prod} \times \text{dec} + \mathcal{O}\left(\frac{\Gamma}{M}\right)$$

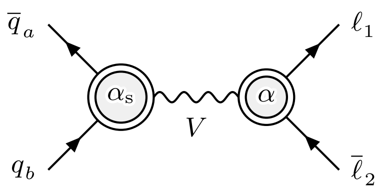
The pole approximation [Stuart '91] is well suited for describing (mixed qcd-) electroweak effects near the resonance region

Reference results for mixed QCDxEW corrections in the resonance region in pole approximation [Dittmaier, Huss, Schwinn '14,'15]



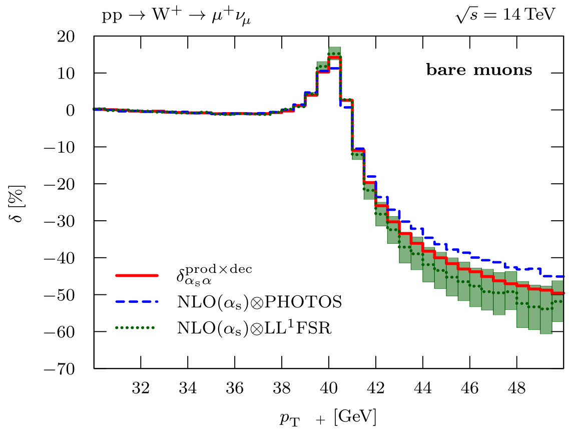

[Dittmaier, Huss, Schwinn, 1403.3216]

IS-FS non-factorisable contributions subdominant in the resonance region. Phenomenologically negligible



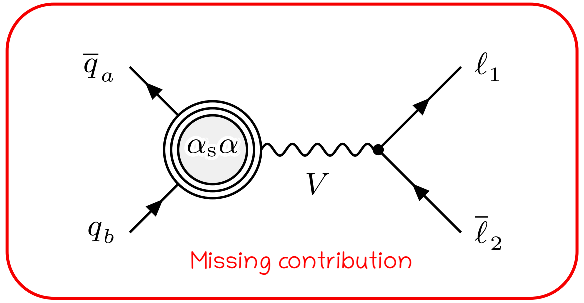

[Dittmaier, Huss, Schwinn, 1511.08016]

Dominant effects from IS-FS factorisable contributions. Authors consider initial-final and final-final (technically an NLO problem)



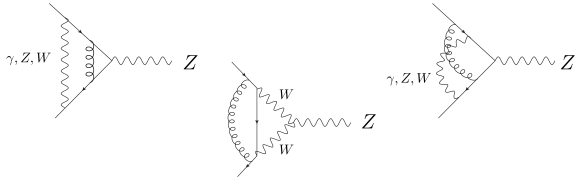

Comparison of IS-FS corrections with NLO QCD (IS) x QED PS (FS). Generally good agreement

(one of the) Outcome of the study: Mixed QCDxEW corrections can have an impact as 14 MeV in the extraction of the W mass



# Mixed QCD-EW to on-shell Z: fully inclusive XS

Analytic calculation of the fully inclusive cross section for the production of an on-shell Z boson [Bonciani, F.B, Rana, Vicini 2007.06518]




NLO-QCD NNLO-QCD

$$A_1 = \sigma^{\mathrm{LO}} + \sigma^{(1,0)} + \sigma^{(2,0)}$$

NLO-EW

$$B_2 = \sigma^{\mathrm{LO}} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)}$$

$$B_3 = \sigma^{\mathrm{LO}} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)} + \sigma^{(1,1)}$$

QCD-EW

$\mu_R = \mu_F = M_Z$ (results expressed in pb)

| order | $G_\mu$ | $\alpha(0)$ | $\delta_{G_\mu - \alpha(0)}$ (%) |
|---|---|---|---|
| $A_1$ | 55787 | 53884 | 3.53 |
| $B_2$ | 55501 | 55015 | 0.88 |
| $B_3$ | 55469 | 55340 | 0.23 |

Reduction of uncertainties due to EW input scheme

Comparing of $G_\mu$ and $\alpha(0)$ schemes is a very conservative choice.
It maximises the spread of the results

Clearly, this has to be complemented by studies of uncertainties from QCD (convential scale variation?), N$^3$LO QCD essential

Detailed study of impact and QCD-EW corrections on inclusive XS and related uncertainties (including PDFs) [Bonciani, F.B., Rana, Vicini 2111.12694]

7-pt scale variations around central scale $\mu = m_Z$

| collider | $\sigma_{QCD}$ | $\delta_{7pts}$ | $\sigma_{QCD-EW}$ | $\delta_{7pts}$ |
|---|---|---|---|---|
| $p\bar{p}$ 1.96 TeV | 7710.0 | $^{+0.48\%}_{-0.66\%}$ | 7649.5 | $^{+0.37\%}_{-0.64\%}$ |
| LHC 7 TeV | 29356.2 | $^{+0.52\%}_{-0.26\%}$ | 29120.4 | $^{+0.49\%}_{-0.31\%}$ |
| LHC 8 TeV | 34116.0 | $^{+0.58\%}_{-0.30\%}$ | 33840.2 | $^{+0.56\%}_{-0.35\%}$ |
| LHC 13 TeV | 57769.1 | $^{+0.78\%}_{-0.45\%}$ | 57287.6 | $^{+0.76\%}_{-0.49\%}$ |
| LHC 14 TeV | 62454.4 | $^{+0.80\%}_{-0.47\%}$ | 61931.2 | $^{+0.79\%}_{-0.50\%}$ |
| LHC 100 TeV | 418617 | $^{+1.26\%}_{-1.16\%}$ | 412815 | $^{+1.26\%}_{-1.18\%}$ |

NNLO-QCD mostly responsabile for SV uncertainties

XS computed using different PDF sets.
"env" ~ envelope of 3 sets in the QCD model only

| collider | PDF set | $\sigma_{QCD}$ | $\sigma_{QCD-EW}$ | $\delta_{QCD-EW}$ | $\Delta_{env}$ | $\delta_{PDF}$ |
|---|---|---|---|---|---|---|
| LHC 13 TeV | NNPDF3.1 | 57769.1 | 57287.6 | -0.8 | 1.1% | $^{+0.8\%}_{-0.8\%}$ |
| | CT18 | 57152.1 | 56898.9 | -0.4 | | $^{+1.9\%}_{-2.5\%}$ |
| | MMHT2015 | 57564.8 | 56899.3 | -1.2 | | $^{+2.1\%}_{-2.1\%}$ |
| LHC 14 TeV | NNPDF3.1 | 62454.4 | 61931.2 | -0.8 | 1.0% | $^{+0.8\%}_{-0.8\%}$ |
| | CT18 | 61840.8 | 61568.1 | -0.4 | | $^{+2.0\%}_{-2.5\%}$ |
| | MMHT2015 | 62278.6 | 61553.7 | -1.2 | | $^{+2.2\%}_{-2.2\%}$ |

δ in units of %

Take home message: availability of N$^3$LOQCD + mixed NNLO QCD-EW bring theoretical uncertainties on cross-sections to the per-mille region!

# QCDxEW corrections to on-shell Z/W production, differential

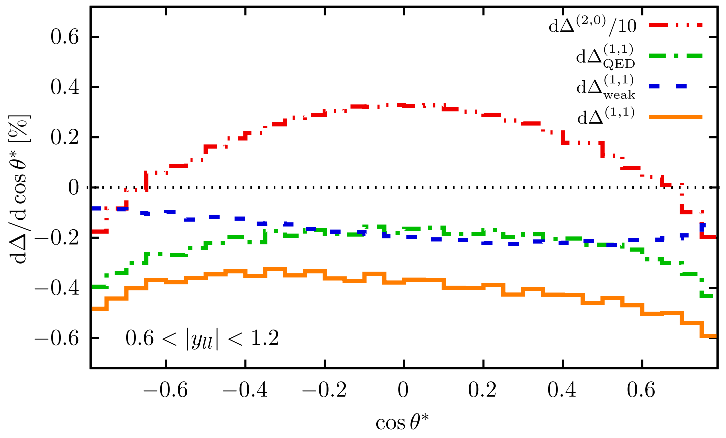
Calculation of mixed QCD-EW to on-shell Z-[F.B, Caola, Delto, Jaquier, Melnikov, Roentsch 2005.10221] and W-bosons production [Behring, F.B, Caola, Delto, Jaquier, Melnikov, Roentsch 2009.10386]

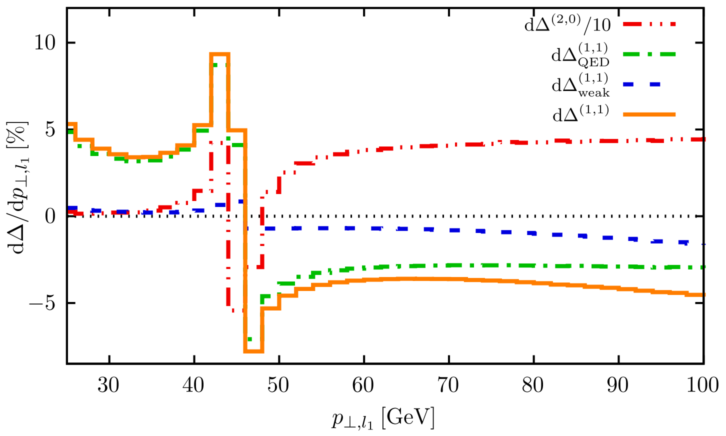
Framework: nested-soft collinear subtraction + analytic two-loop form factors

$$\mathrm{d}\Delta^{(i,j)} = \frac{\mathrm{d}\sigma^{(i,j)}}{\mathrm{d}\sigma^{(0,0)} + \mathrm{d}\sigma^{(1,0)}}$$

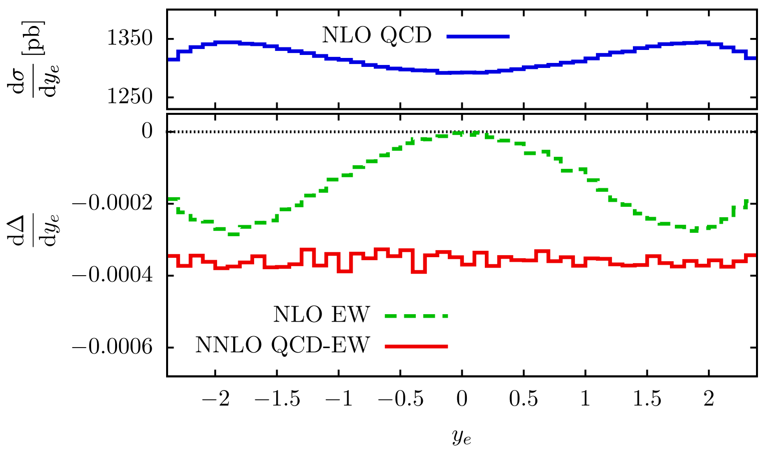
bin-by-bin ratios of NLO-EW, NNLO-QCD and NNLO QCD-EW corrections

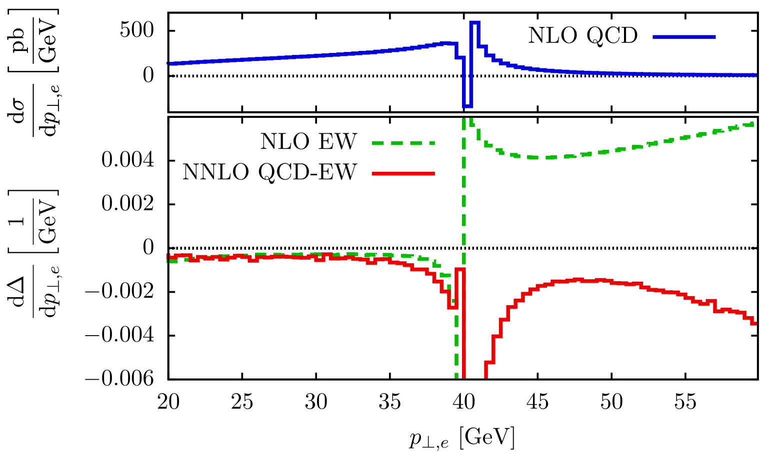
Z-case




W-case




General comments:

- corrections sensitive to selection cuts (asymmetric cuts in Z case)
- corrections are observable dependent
- QCD-EW corrections could be larger than NLO-EW in certain regions

# Estimate of QCDxEW corrections on $m_W$ extraction

Availability of QCD-EW corrections on Z and W productions allows for estimates of the impact of these corrections on the extraction of $m_W$ [Behring et al' 2103.02671]

Stress once more: the results from [Behring et al' 2103.02671] have to be interpreted as an estimate

W mass measurements rely on excellent control of Z production process: to calibrate detectors, tune the generators, etc.

Basic idea: estimate effect of QCD-EW corrections on $m_W$ extraction, due to decorrelations in Z and W production

Simple theoretical model: correlation between average transverse momentum of leptons and mass of bosons

$$\frac{m_W}{m_Z} = \frac{\langle p_{T,l}^W \rangle}{\langle p_{T,l}^Z \rangle} \Rightarrow m_W^{\text{meas.}} = m_Z \frac{\langle p_{T,l}^{W,\text{meas.}} \rangle}{\langle p_{T,l}^{Z,\text{meas.}} \rangle} C_{\text{th.}} \qquad C_{\text{th.}} = \frac{m_W^{\text{in}}}{m_Z^{\text{in}}} \frac{\langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

assume input masses,
evaluate W-mass and compare with input

Estimate impact of decorrelations in W and Z spectra from higher order corrections

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

Very strong sensitivity on selection cuts:

- "Atlas" cuts: $\Delta m_W$ = -17 MeV
- "Tuned" cuts: $\Delta m_W$ = -1 MeV

$p_{T,\ell}^Z > 25\ \text{GeV};\ |\eta_\ell^Z| < 2.4$

"ATLAS" cuts: $\mathrm{p}_{T,\ell}^W > 30\ \text{GeV};\ \mathrm{p}_{\mathrm{T,miss}}^{\mathrm{W}} > 30\ \text{GeV};\ |\eta_\ell^{\mathrm{W}}| < 2.4.$

"Tuned" cuts: $\mathrm{p}_{T,\ell}^W > 25.44\ \text{GeV};\ \mathrm{p}_{\mathrm{T,miss}}^{\mathrm{W}} > 25.44\ \text{GeV};\ |\eta_\ell^{\mathrm{W}}| < 2.4.$

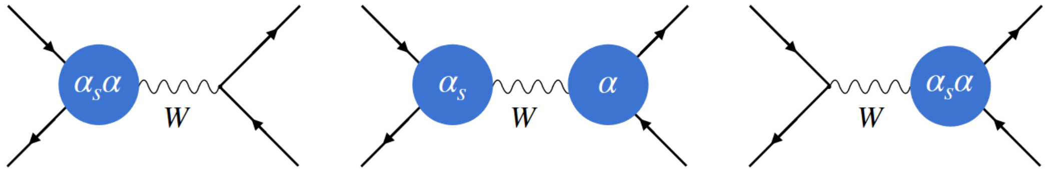
Take home:

- Yet again: these are estimates
- Decorrelation effects can be significant thus relevant for target 0.1 permille precision
- QCD-EW shifts potentially relevant for target precision of 8 MeV

# Mixed QCDxEW corrections to CCDY

Mixed QCDxEW corrections to pp → $\ell\nu_\ell$ production (CCDY) [Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539] (leptons are massive, μ)

Two-loop amplitude expanded in Pole Approximation. Similarly to [Dittmaier, Huss, Schwinn 1511.08016] including IS-IS NNLO corrections




**Initial-Initial**: extracted from mixed QCD-EW form factor for the W boson *[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]*

**Initial-Final**: computed using the one-loop provider RECOLA

**Final-Final**: finite renormalisation constant *[Dittmaier, Huss, and Schwinn (2015)]*

from Luca Buonocore, EPS 2021

Framework: qT slicing [Catani, Grazzini hep-ph/0703012] [Buonocore, Grazzini, Tramontano 1911.10166] + 2-loop amplitudes improved by reweighting techqnique

all other real and real-virtual contributions are included without approximation

Setup: ($\sqrt{s}$ = 14TeV)

- massive muons
- bare muons (no muon-photon recombination)
- NNPDF31_nnlo_as_0118_luxqed
- $p_{T,\mu} > 25\,\mathrm{GeV}$ $\quad p_{T,\nu} > 25\,\mathrm{GeV}$ $\quad |y_\mu| < 2.5$
- fixed scale μ=$m_W$

| $\sigma$ [pb] | $\sigma_{\mathrm{LO}}$ | $\sigma^{(1,0)}$ | $\sigma^{(0,1)}$ | $\sigma^{(2,0)}$ | $\sigma^{(1,1)}$ |
|---|---|---|---|---|---|
| $q\bar{q}$ | 5029.2 | 970.5(3) | −143.61(15) | 251(4) | − 7.0(1.2) |
| $qg$ | — | −1079.86(12) | — | −377(3) | 39.0(4) |
| $q(g)\gamma$ | — | — | 2.823(1) | — | 0.055(5) |
| $q(\bar{q})q'$ | — | — | — | 44.2(7) | 1.2382(3) |
| $gg$ | — | — | — | 100.8(8) | — |
| tot | 5029.2 | −109.4(4) | −140.8(2) | 19(5) | 33.3(1.3) |
| $\sigma^{(m,n)}/\sigma_{\mathrm{LO}}$ | | −2.2 % | −2.8 % | +0.4 % | +0.6 % |

from Luca Buonocore, EPS 2021

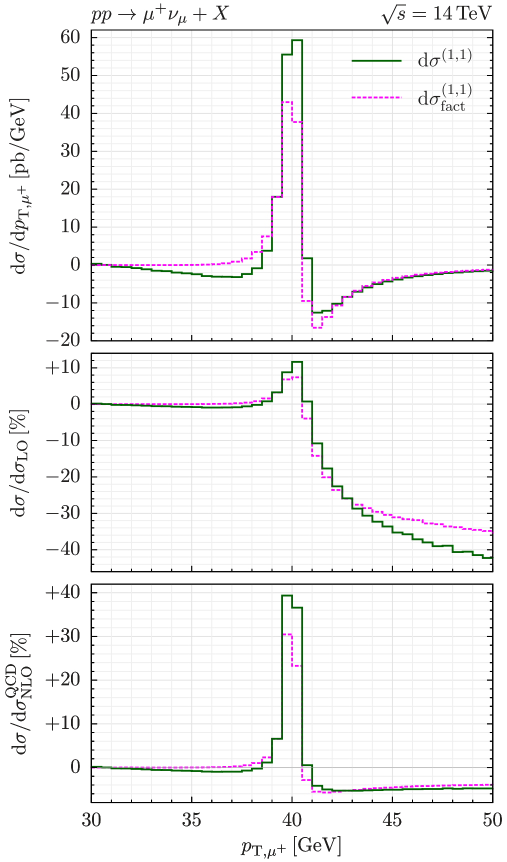
- Large cancellations in NLO- and NNLO-QCD between qq+gg and qg
- For the chosen setup, mixed QCD-EW corrections dominated by qg (no approximation for this channel)

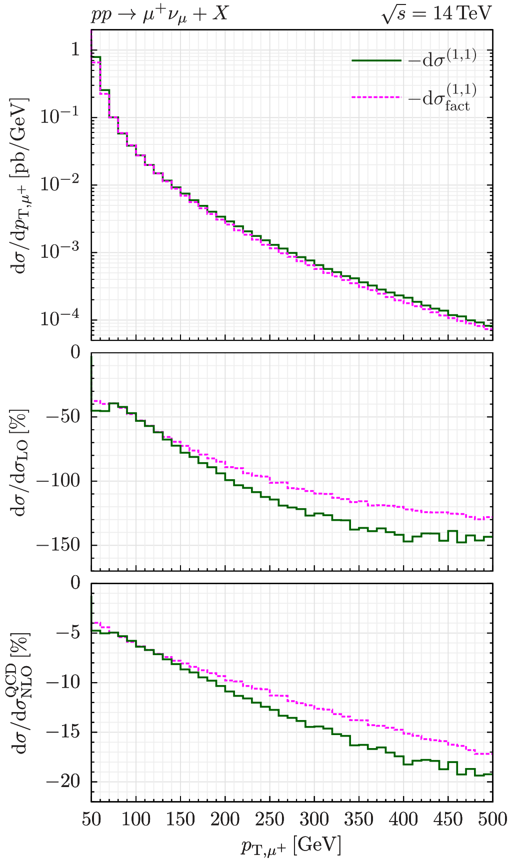
# Mixed QCDxEW corrections to CCDY

Mixed QCDxEW corrections to pp → $\ell\nu_\ell$ production (CCDY) [Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539] (leptons are massive, $\mu$)

Main focus on the observable $p_{T,\mu}$

At the Jacobian peak $p_{T,\mu}$ dominated by resonant W boson



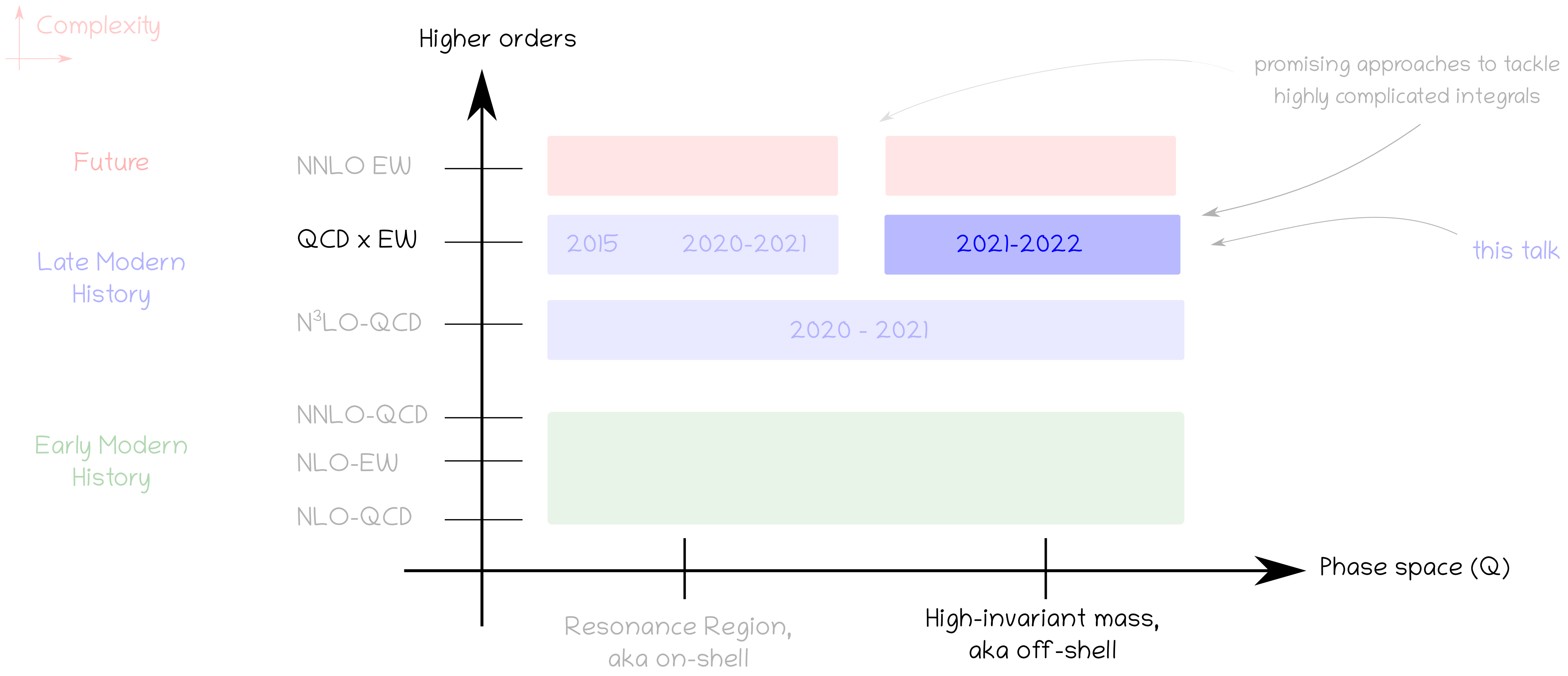

In the high $p_{T,\mu}$ region born-like contributions are suppressed

Dominant mechanism: (resonant) W + hard jet

Thus the high $p_{T,\mu}$ tail, well described by W+jet. The authors find that the dominant contribution comes from the qg channel

In principle possible to investigate also transverse mass distribution (template fit) + impact of measurments of $p_{T,\mu}$ on $m_W$ thus it would be interesting to compare to previous outcome from [Dittmaier, Huss, Schwinn 1511.08016]

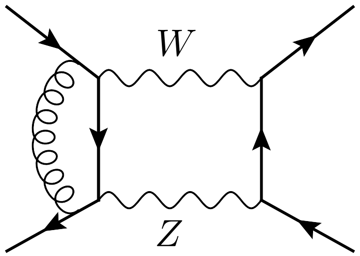
# Higher-order corrections and phase space




Focus on QCDxEW corrections
(vast literature and constantly richer pheno)

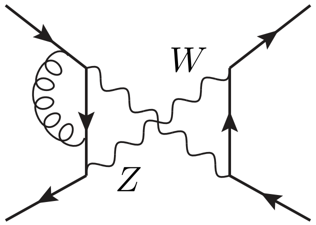
# Fully off-shell Drell-Yan processes

The complete calculation of mixed QCDxEW corrections




Amplitudes/Integrals

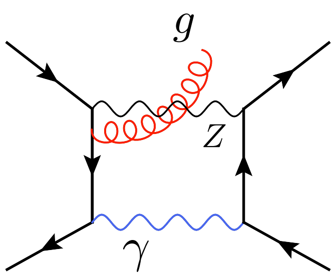
Two-loop integrals involving several energy scales




Two-loop amplitudes for CCDY not available yet

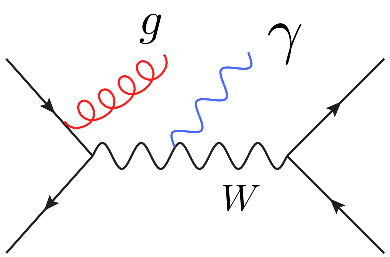
Recent developments:

Complete two-loop amplitudes for NCDY [Heller, Manteuffel, Schabinger, Spiesberger 2012.05918] [Armadillo, Bonciani, Devoto, Rana, Vicini 2201.01754]




Subtraction of IR singularities

Complex cancellation pattern of IR singularities




Complete fully off-shell CCDY not available yet

(although in sight)

Recent developments:

- QCDxEWK corrections to l+l- production (qT slicing) [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini 12106.11953]
- QCDxEWK corrections to l+l- production (nested-soft collinear subtraction) [F.B., Caola, Chawdhry, Devoto, Heller, Manteuffel, Melnikov, Roentsch, Signorile-Signorile, 2203.11237]

# Two-loop amplitudes for mixed QCDxEW corrections

Two main results, two completely independent approaches

1) Fully analytic calculation of two-loop integrals and helicity amplitudes [Heller, von Manteuffel, Schabinger, Spiesberger 2012.05918]

   Analytic results expressed in terms of well known functions: Goncharov Polylogarithms (GPLs). Many programs available for numerical evaluation

   - Fast evaluation: $\sim$ 0.8 s/psp on a standard laptop
   - It can manipulated, hence numerical stability can be easily improved
   - Run-time evaluation for any psp

2) Numerical evaluation of tree-loop interference [Armadillo, Bonciani, Devoto, Rana, Vicini 2201.01754]

   Two-loop integrals evaluated via series expansions. Procedure inspired by DiffExp [Hidding 2006.05510], but (novelty) extended to complex-masses

   Analytic boundary conditions + series expansion: makes it possible to achieve an arbitrary number of digits

   - Public packages: DiffExp [Hidding 2006.05510] and SeaSyde [Armadillo, Bonciani, Devoto, Rana, Vicini 2205.03345]
   - The method is general and applicable to other processes in practice (potential for QCDxEW CCDY and NNLO-EWK)
   - Easier to evaluate integrals in various regions, thresholds not a practical issue

# Mixed QCDxEW corrections, fully off-shell

★ First calculation of complete mixed QCD-EW corrections to the Drell-Yan process [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini 2106.11953]

Framework: semi-analytical amplitudes [Armadillo et al' 201.01754] + qT slicing [Catani, Grazzini hep-ph/0703012] [Buonocore, Grazzini, Tramontano 1911.10166]

Setup: (√s = 14TeV)

- massive leptons

  bare muons (no muon-photon recombination)

- NNPDF31_nnlo_as_0118_luxqed
- symmetric cuts: $M_{\mu^+\mu^-} > 50\,\mathrm{GeV}$ $p_{T,\mu^\pm} > 25\,\mathrm{GeV}$ $|y_{\mu^\pm}| < 2.5$
- fixed scale $\mu = m_Z$

Very complicated calculations.
Having two totally independent approaches is highly desirable!

★ Complete mixed QCD-EW corrections to the Drell-Yan process, high invariant mass [F.B., Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Roentsch, Signorile-Signorile 2203.11237]

Framework: analytical amplitudes [Heller et al' 2012.05918] + nested soft collinear subtraction [Caola, Melnikov, Roentsch 1702.01352]

Setup: (√s = 13.6TeV)

- massless electrons

  dressed leptons: electron-photon recombination (fixed cone)

- NNPDF31_nnlo_as_0118_luxqed
- symmetric + product cuts: $M_{\ell^+\ell^-} > 200\,\mathrm{GeV}$ $p_{T,\ell^\pm} > 30\mathrm{GeV}$ $\sqrt{p_{T,\ell^-} p_{T,\ell^+}} > 35\mathrm{GeV}$ $|y_{\ell^\pm}| < 2.5$
- dynamic scale $\mu = m_{ll}/2$

Inspired from [Salam,Slade 2106.08329]

# Results: cross sections

★ qT slicing & massive muons

$M_{\mu^+\mu^-} > 50\,\mathrm{GeV} \quad p_{T,\mu^\pm} > 25\,\mathrm{GeV} \quad |y_{\mu^\pm}| < 2.5$

| $\sigma$ [pb] | $\sigma_{\mathrm{LO}}$ | $\sigma^{(1,0)}$ | $\sigma^{(0,1)}$ | $\sigma^{(2,0)}$ | $\sigma^{(1,1)}$ |
|---|---|---|---|---|---|
| $q\bar{q}$ | 809.56(1) | 191.85(1) | −33.76(1) | 49.9(7) | −4.8(3) |
| $qg$ | — | −158.08(2) | — | −74.8(5) | 8.6(1) |
| $q(g)\gamma$ | — | — | −0.839(2) | — | 0.084(3) |
| $q(\bar{q})q'$ | — | — | — | 6.3(1) | 0.19(0) |
| $gg$ | — | — | — | 18.1(2) | — |
| $\gamma\gamma$ | 1.42(0) | — | −0.0117(4) | — | — |
| tot | 810.98(1) | 33.77(2) | −34.61(1) | −0.5(9) | 4.0(3) |
| | | +4.2% | -4.3% | -0.6‰ | +0.5% |

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \dots$$

LO NLO-QCD NLO-EWK NNLO-QCD NNLO-QCDxEWK

- accidental cancellation NLO-QCD vs NLO-EW
- tiny (accidental) NNLO-QCD: very large cancellations
- mixed QCDxEW comparable to previous orders (larger than NNLO QCD)

★ local nested-soft/coll subtraction & massless leptons

$M_{\ell^+\ell^-} > 200\,\mathrm{GeV} \quad p_{T,\ell^\pm} > 30\mathrm{GeV} \quad \sqrt{p_{T,\ell^-}p_{T,\ell^+}} > 35\mathrm{GeV} \quad |y_{\ell^\pm}| < 2.5$

| $\sigma$[fb] | $\sigma^{(0,0)}$ | $\delta\sigma^{(1,0)}$ | $\delta\sigma^{(0,1)}$ | $\delta\sigma^{(2,0)}$ | $\delta\sigma^{(1,1)}$ |
|---|---|---|---|---|---|
| $q\bar{q}$ | 1561.42 | 340.31 | −49.907 | 44.60 | −16.80 |
| $\gamma\gamma$ | 59.645 | | 3.166 | | |
| $qg$ | | 0.060 | | −32.66 | 1.03 |
| $q\gamma$ | | | −0.305 | | −0.207 |
| $g\gamma$ | | | | | 0.2668 |
| $gg$ | | | | 1.934 | |
| sum | 1621.06 | 340.37 | −47.046 | 13.88 | −15.71 |
| | | +22% | -2.9% | +0.8% | -0.9% |

- NLO: +20% from QCD, -3% from EW

  NLO EW (and mixed) corrections could reduced by increasing $R_{\gamma l}$: -3% @ R=0.1 -> -1.6% @ R=0.4

  experimentally feasible/meaningful? like R=0.2

- NNLO QCD: < 1%, accidentally small, qq vs qg cancellation
- QCDxEWK: ~ 1% quite large and beyond naive power counting.

  larger than NNLO-QCD. qq channel by far dominant

# Results: invariant mass windows

lIntegrated XS in different invariant mass windows [F.B., Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Roentsch, Signorile-Signorile 2203.11237]

At high invariant mass, mixed effects should be well described by the product of QCD and electroweak corrections

Factorized approximation:

$$\delta\sigma^{(1,1)}_{\text{fact}} = \delta^{(1,0)}_{\text{NLO}}\, \delta^{(0,1)}_{\text{NLO}}\, \sigma^{(0,0)} \qquad \delta^{(1,0)}_{\text{NLO}} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}}, \qquad \delta^{(0,1)}_{\text{NLO}} = \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}}$$

$\Phi^{(1)}$ : 200 GeV $< m_{\ell\ell} <$ 300 GeV  $\Phi^{(2)}$ : 300 GeV $< m_{\ell\ell} <$ 500 GeV

$\Phi^{(3)}$ : 500 GeV $< m_{\ell\ell} <$ 1.5 TeV  $\Phi^{(4)}$ : 1.5 TeV $< m_{\ell\ell} < \infty$

| $\sigma$ [fb] | $\sigma^{(0,0)}$ | $\delta\sigma^{(1,0)}$ | $\delta\sigma^{(0,1)}$ | $\delta\sigma^{(2,0)}$ | $\delta\sigma^{(1,1)}$ | $\delta\sigma^{(1,1)}_{\text{fact.}}$ | $\sigma_{\text{QCD}\times\text{EW}}$ |
|---|---|---|---|---|---|---|---|
| $\Phi^{(1)}$ | 1169.8 | 254.3 | −30.98 | 10.18 | −10.74 | −6.734 | $1392.6^{+0.75\%}_{-0\%}$ |
| $\Phi^{(2)}$ | 368.29 | 71.91 | −11.891 | 2.85 | −4.05 | −2.321 | $427.1^{+0.41\%}_{-0.02\%}$ |
| $\Phi^{(3)}$ | 82.08 | 14.31 | −4.094 | 0.691 | −1.01 | −0.7137 | $91.98^{+0.22\%}_{-0.14\%}$ |
| $\Phi^{(4)} \times 10$ | 9.107 | 1.577 | −1.124 | 0.146 | −0.206 | −0.1946 | $9.500^{+0\%}_{-0.97\%}$ |

Factorized approximation not reliable in the low $10^2$ GeV region

It fails at the level of the theoretical uncertainty

QCD rather flat ~ +18%

EWK increases in size to up ~ -12%

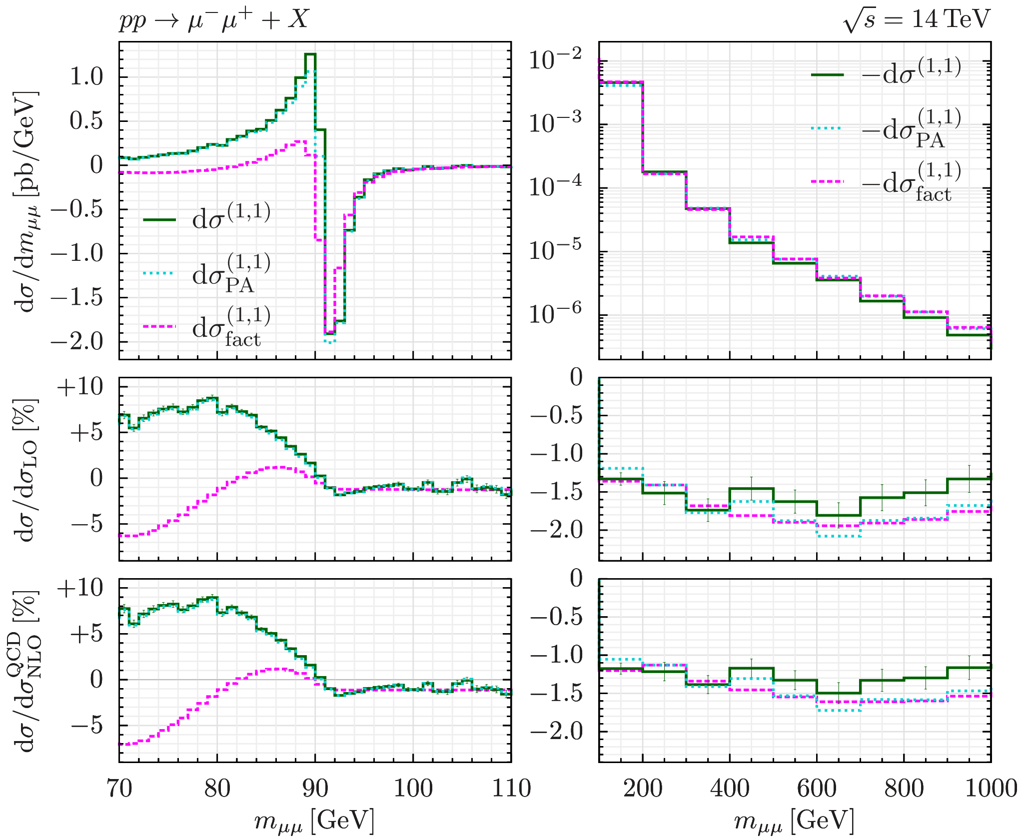
expected from dominant 1-loop Sudakov logs

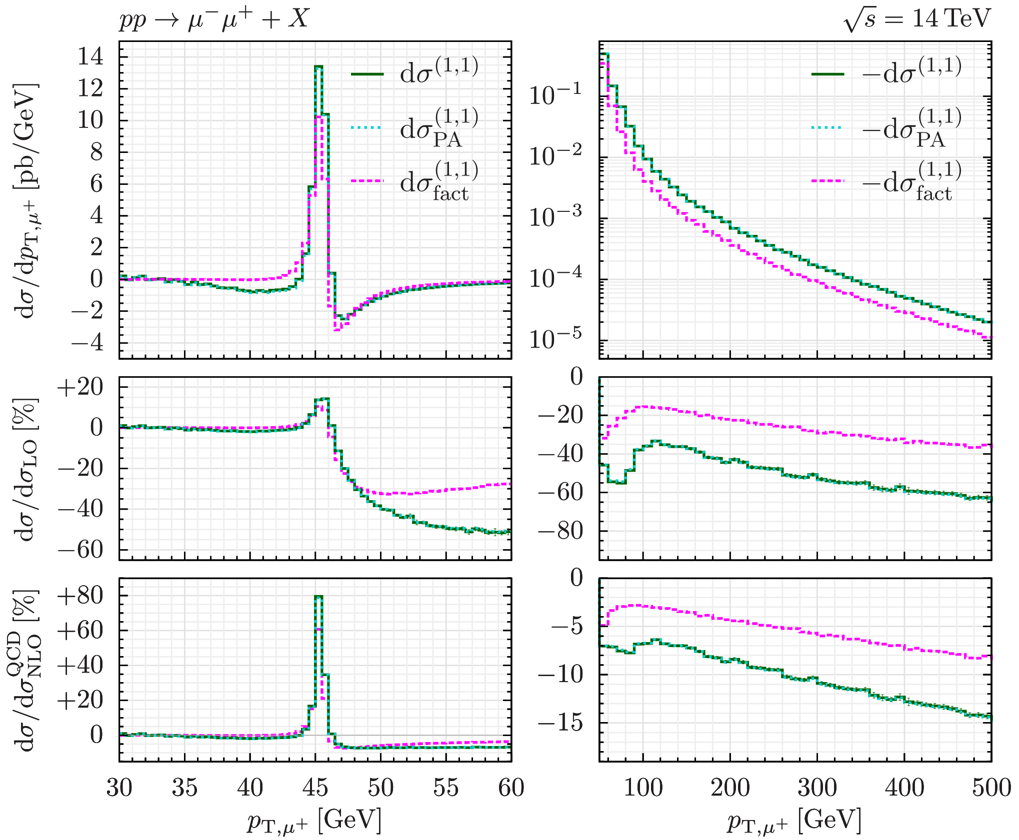
-12% x 18% = -2.2%

Factorized approximation captures 90% of full QCDxEW correction in the TeV region

# Pheno of mixed QCDxEW corrections (qT)

Mixed QCD-EW corrections [Buonocore et al' 2106.11953]: phenomenology both at the Z resonance and in the off-shell region




- QCD and QED effects do not factorise below the resonance
- The pole approximation provides a very good description around the resonance
- O(~1%) corrections in the high-tail of the invariant mass (PA does not capture)

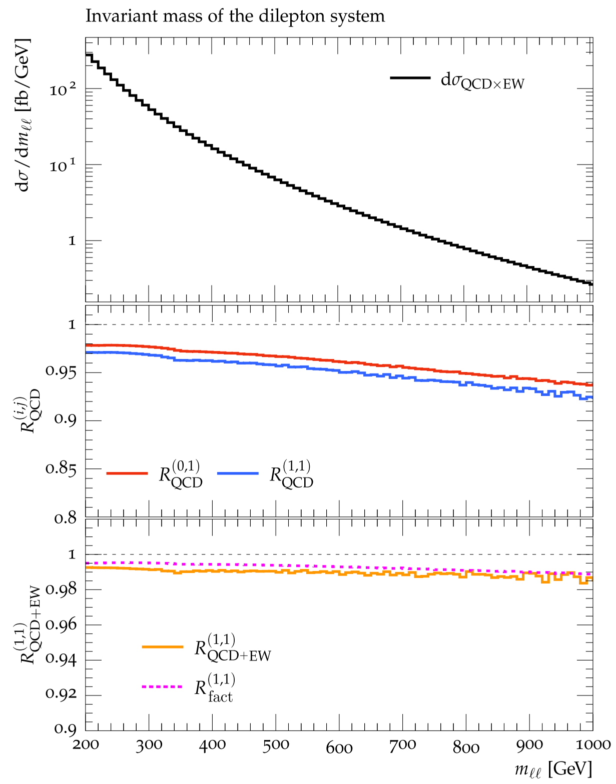
  relevant for NP searches in the high-mass region

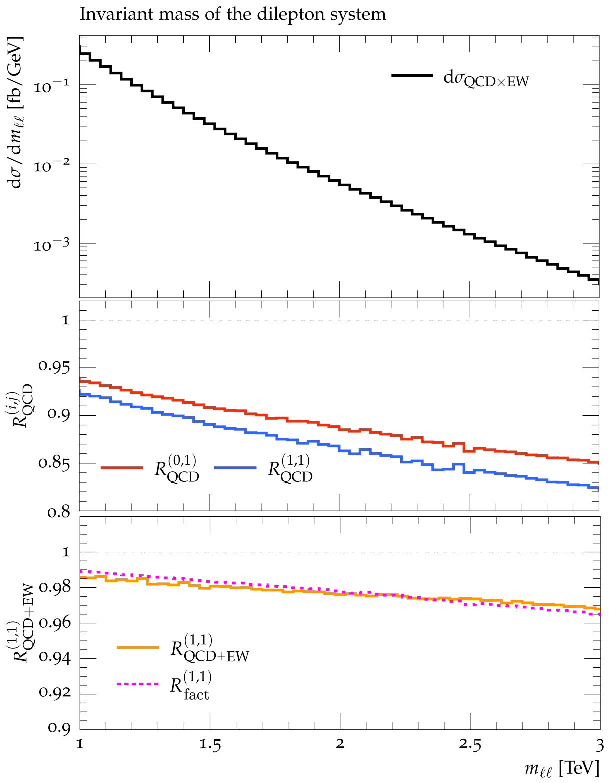
- At the Jacobian peak, dominated by Z resonance
- At high lepton pT, Z+jet is the dominant configuration (Z still near the resonance) ~ Z + jet @ NLO-EW

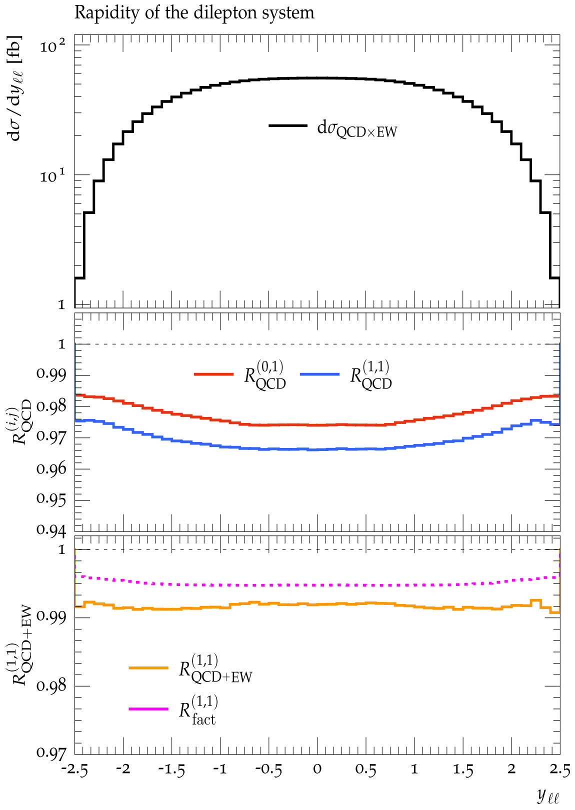
Very strong confirmation of [Dittmaier, Huss, Schwinn 1511.08016], i.e. that Pole Approximation works extremely well at the Z peak

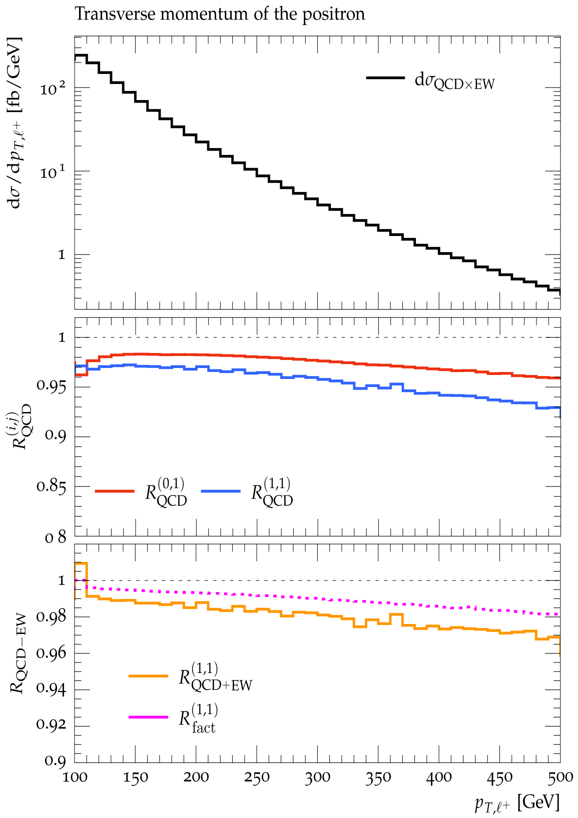
# Pheno of mixed QCDxEW corrections (nested soft-collinear)

Mixed QCD-EW corrections to the Drell-Yan process [F.B., Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Roentsch, Signorile-Signorile 2203.11237]




- QCDxEW shape: not entirely flat: shape driven by Sudakov logs in EW loop amplitudes
- NLO-EW: -15% @ 3 TeV, QCDxEW: -3% @ 3 TeV
- factorized approximation reproduces well the result at higher $m_{ll}$

- In the case of the lepton pT, QCDxEW corrections up to O(-3%) @ pT ~ 500 GeV
- factorized approximation not valid for this type of observables

# Angular variables and $A_{FB}$

$$\cos\theta^* = \frac{p^+_{\ell^-}p^-_{\ell^+} - p^-_{\ell^-}p^+_{\ell^+}}{m_{\ell\ell}\sqrt{m^2_{\ell\ell}+p^2_{\ell\ell,\perp}}} \times \mathrm{sgn}(p_{\ell\ell,z})\,.$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \qquad \sigma_F = \int_0^1 d\cos\theta^* \frac{d\sigma(pp\to\ell^-\ell^+)}{d\cos\theta^*} \qquad \sigma_B = \int_{-1}^0 d\cos\theta^* \frac{d\sigma(pp\to\ell^-\ell^+)}{d\cos\theta^*}$$

Recent measurement of $A_{FB}$ by the CMS collaboration at $\sqrt{s} = 13$ TeV in the high invariant mass region [2202.12327] (including mass windows)

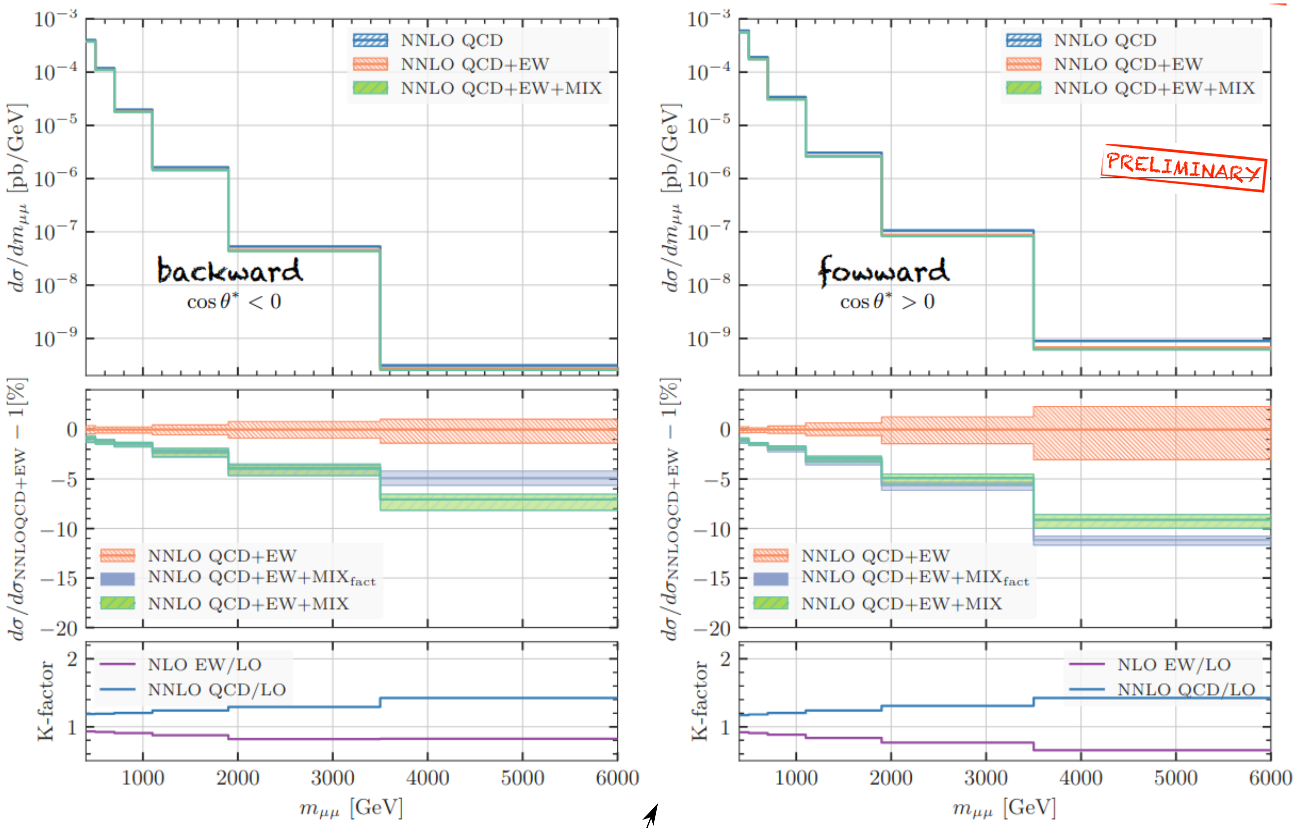
without QCDxEW

including QCDxEW

| | $\tilde{A}_{FB}$ | $A_{FB}$ |
|---|---|---|
| $\Phi^{(1)}$ | $0.1442^{+0.05\%}_{-0.31\%}$ | $0.1440^{+0.11\%}_{-0.09\%}$ |
| $\Phi^{(2)}$ | $0.1852^{+0.08\%}_{-0.40\%}$ | $0.1847^{+0.10\%}_{-0.19\%}$ |
| $\Phi^{(3)}$ | $0.2401^{+0.13\%}_{-0.64\%}$ | $0.2388^{+0.06\%}_{-0.47\%}$ |
| $\Phi^{(4)}$ | $0.3070^{+0.49\%}_{-1.5\%}$ | $0.3031^{+0.19\%}_{-1.2\%}$ |

[F.B., Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Roentsch, Signorile-Signorile 2203.11237]

Plots from Luca Buonocore's talk (Loops&Legs 2022). Preliminary!




- In the TeV region: QCDxEW corrections $\sim 1\%$ to $A_{FB}$: potentially interesting for HL-LHC
- Factorised approximation: justified for for $m_{ll}$ in the TeV region

**SETUP** (LHC @ $\sqrt{s} = 13$ TeV) CMS 2103.02708

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 53$ GeV, $|y_\mu| < 2.4$, $m_{\mu^+\mu^-} > 150$ GeV
- massive muons (no photon lepton recombination)
- $G_\mu$ scheme, complex mass scheme
- dynamic scale $\mu_F = \mu_R = m_{\mu^+\mu^-}$

# Conclusions

$N^3LO$ QCD corrections + mixed QCD-EW → now ranging over the whole phase space

we are witnessing the onset of precision phenomenology: per-mille accuracy in sight     in the resonance region + off-shell region

- EWK precision measurments: resonant region

  Mixed QCD-EW calculations demonstrate that theoretical uncertainty estimate more reliabe

  Impact at the level of the target (sub) per-mille accuracy: these corrections need to be taken into account for a full SM study

  thorough investigation of both Z and W production at the LHC

- New Physics searches: off-shell region

  Foreseen statistical uncertainty at HL-LHC in the O(0.5%) range or below

  Impact of mixed QCD-EW corrections at the same level (or well above in the high-invariant mass region)

  Actually: have proved to be larger than expected: O(-1%) (even away from Sudakov dominated regime)

  at extreme energy, the expected factorization of NLO QCD x universal Sudakov logs is reproduced (mostly/only for cross sections and $m_{ll}$)

- For the near future:

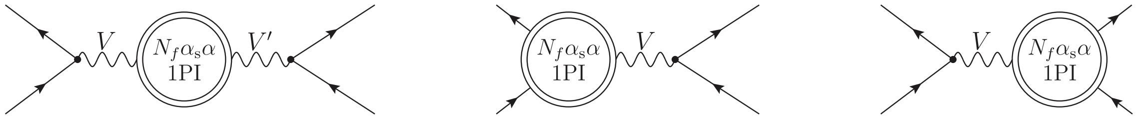
  advisable a synchronised effort from theory + experimental communities to provide best and consistent predictions

  most ingredients of precision physics are now there, let's play with them!

# Backup

# QCDxEW corrections to off-shell DY: $O(n_f \alpha_s \alpha)$ contribution

Mixed QCDxEW corrections to off-shell W and Z production arising from closed fermionic loops [Dittmaier, Schmidt, Schwarz 2009.02229]




Gauge-invariant subset of mixed QCDxEW corrections. It can be investigated on its own.

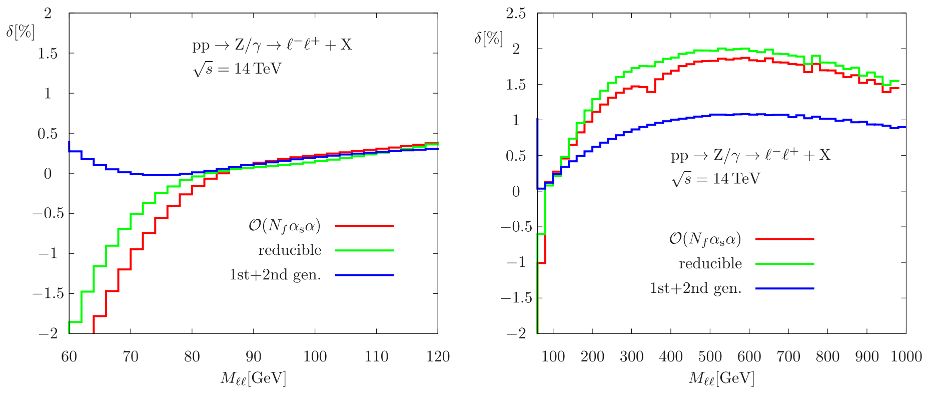
large $\log(m_t^2/s)$ at $s \sim$ TeV

Not yet full-fledged two-loop complexity, but 2 very interesting aspects (at least according to me)

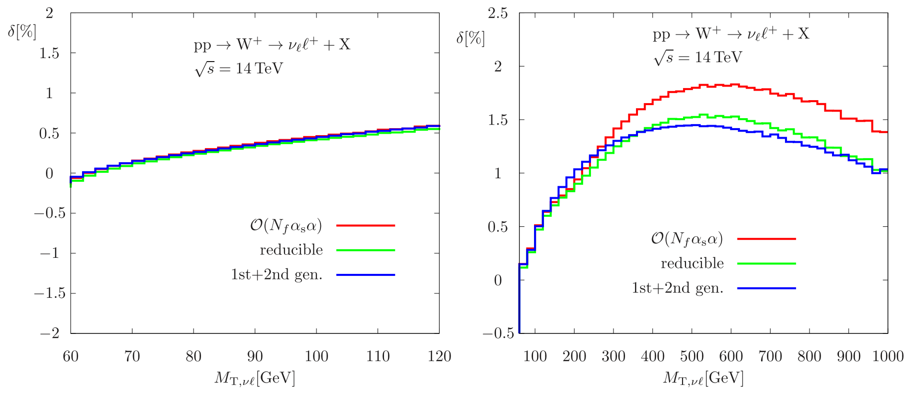
1) $O(n_f \alpha_s \alpha)$ contribution is sufficient for the generalisation of the complex mass scheme for the complete mixed QCDxEW corrections

2) General comment: in regions dominated by the resonance corrections are small O(1 ‰). They increase to O(1-2%) in the off-shell region

NCDY




CCDY




2.1) These contributions are not captured by factorisation, i.e. Tree x QCD x EW-Sudakovs. Thus, they are relevant and need to be included properly!

# Theoretical uncertainty

Mixed QCD-EW corrections to the Drell-Yan process [F.B., Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Roentsch, Signorile-Signorile 2203.11237]

Estimate of theory uncertainty: envelope of QCD + EWK - related uncertainties

$$d\sigma = d\sigma_0 \left(1 + \frac{\alpha_s}{2\pi}\delta^{(1,0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \delta^{(2,0)} + \frac{\alpha}{2\pi}\delta^{(0,1)} + \frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi}\delta^{(1,1)} + \dots\right)$$

Reduction of EW scheme dependence

$p_{T,\ell^\pm} > 30\mathrm{GeV}$ $\quad |y_{\ell^\pm}| < 2.5$

$\sqrt{p_{T,\ell^-} p_{T,\ell^+}} > 35\mathrm{GeV}$

$M_{\ell^-\ell^+} > 200\,\mathrm{GeV}$

Selection cuts

QCD: factor 2 rescaling (up and down) of nominal scale

EWK: variation of input scheme: $G_\mu$-scheme vs $\alpha(M_Z)$-scheme $(\alpha, M_Z, M_W)$

not including QCDxEWK

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%}\ \mathrm{fb}.$$

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%}\ \mathrm{fb}$$

including QCDxEWK

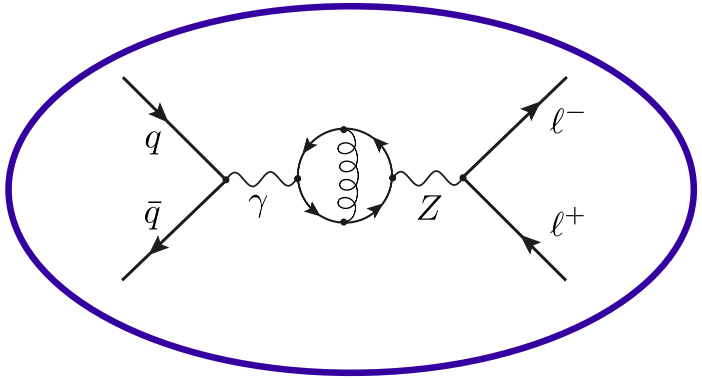
QCDxEW remove large input-scheme dependence from NLO QCD

Theoretical uncertainty below 1% upon including mixed corrections

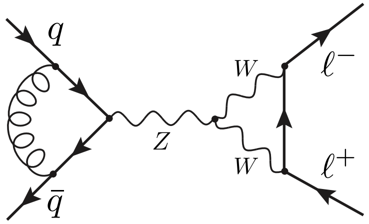
N3LO QCD essential for realistic 1% level accuracy

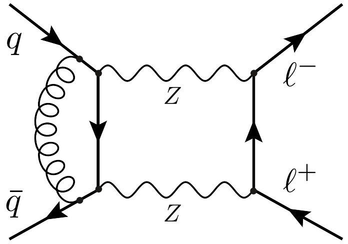
# Two-loop amplitudes: anatomy

Fermionic contributions




Bosonic contributions



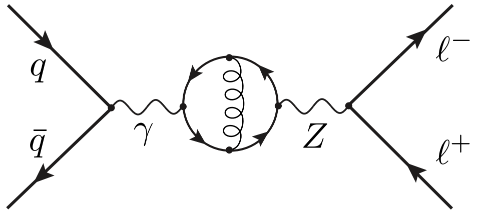

just two-point functions (easy)

Only contribution to $\alpha$ renormalization at $O(\alpha\alpha_s)$

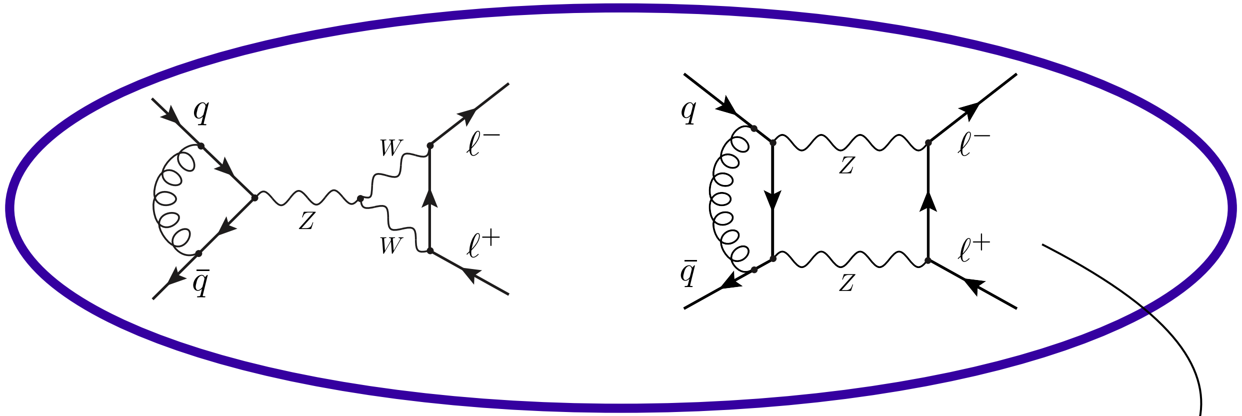
However:
contains $\log^2(m^2_t/s)$.
Grow large @ high invariant mass

# Two-loop amplitudes: anatomy

Fermionic contributions




Bosonic contributions




Much harder: highly complicated two-loop master integrals.
Up to two internal masses (non-trivial resonance structure)

$(s, t, m^2)$

non-rationalizable roots

Fully analytic computation of two-loop helicity amplitudes for dilepton production [Heller, von Manteuffel, Schabinger, Spiesberger 1907.00491,2012.05918]

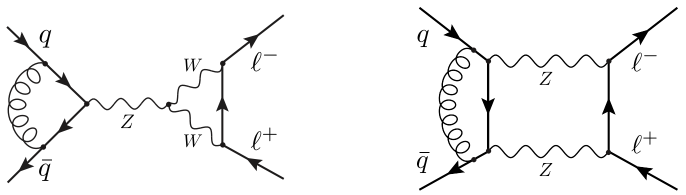
Integration of integrals with non-rationalizable roots in terms of multiple polylogarithms

Result expressed in terms of well known generalised polylogs (GPLs)

Evaluation time: ~ 0.8 s/psp

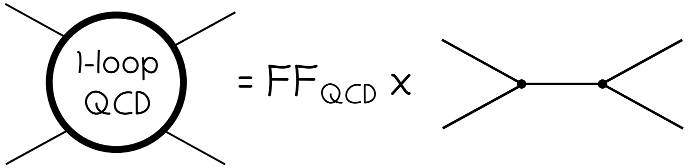
Shall not forget about $\gamma_5$ related issues in dim-reg!

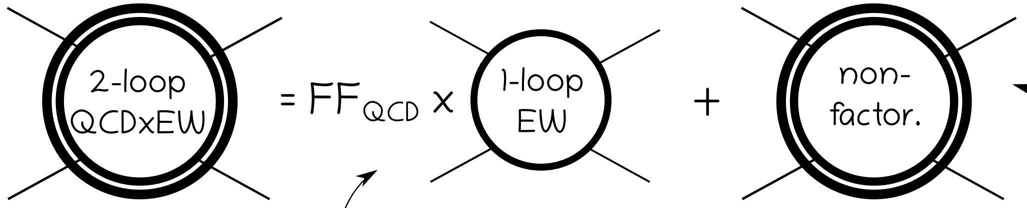
# Two-loop amplitudes: anatomy




Bosonic contributions :
1-loop QCD x 1-loop EWK "factorisable" + a "non-factorisable" bit

Gauge invariant separation




non-factorizable part

factorizable part

one-loop complexity
contains dominant Sudakov logs

genuine two-loop contribution.
Highly complicated (no dominant Sudakov logs)

Factorisable part of the amplitude dominates throughout the phase space we have investigated (8/9 times larger than non-fact)

Pratical advantage:
reduced MC running time for target accuracy

Picture can change close to resonances