Exact Top-Quark Mass Dependence in Gluon Fusion at NNLO

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TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

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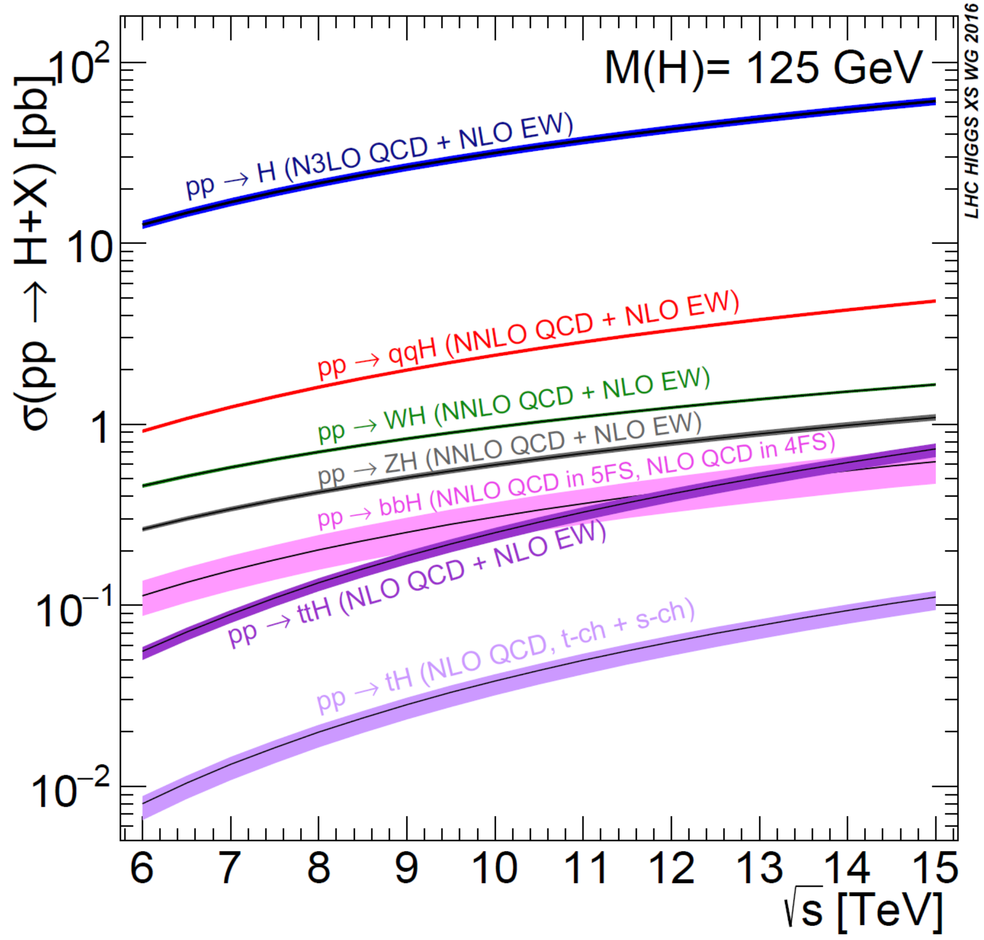
Institute for Theoretical Particle Physics and Cosmology

RWTH AACHEN UNIVERSITY

Motivation

- Gluon fusion is the predominant Higgs-boson production mode at the LHC
- Higgs-boson plays unique role in the SM:
  - Only scalar particle
  - Only particle with Yukawa interactions to fermions



Handbook of LHC Higgs cross sections:
4. Deciphering the nature of the Higgs sector
Report of the LHC Higgs Cross Section Working Group `16

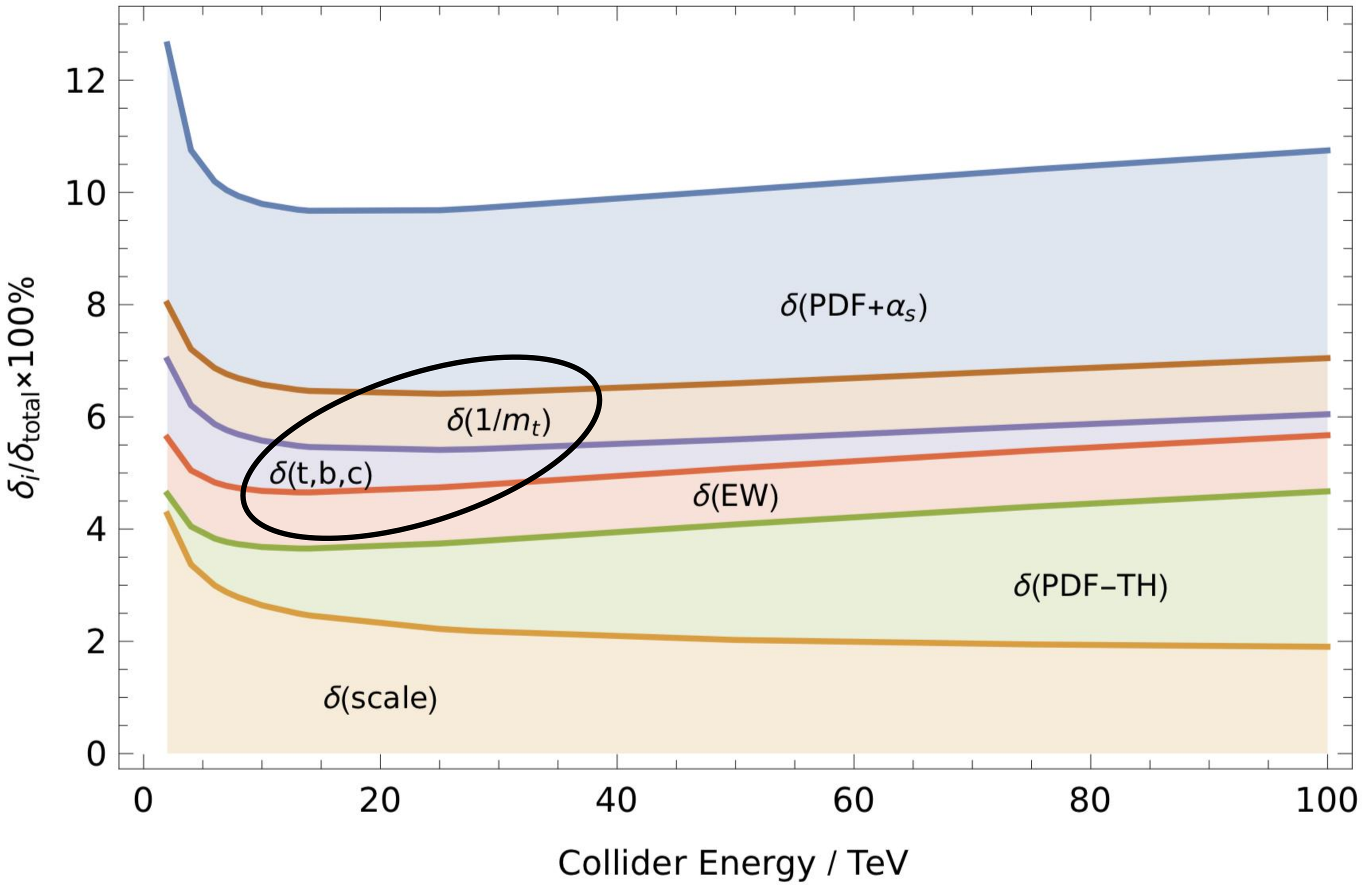
LHC @13 TeV

$$\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\mathrm{pb}\ (3.20\%)\ (\text{PDF}+\alpha_s)$$

Theory uncertainties

- δ(scale) and δ(PDF-TH) due to missing higher-order terms in $\hat{\sigma}$ and PDFs   Anastasiou, et al. `15
- δ(trunc) has been removed   Mistlberger `18
- δ(EW) was addressed recently
  Bonetti, Melnikov, Tancredi `18
  Anastasiou, del Duca, et al. `19
  Becchetti, Bonciani, et al. `21
- δ(t,b,c) and $\delta(1/m_t)$ related to quark mass effects

Higgs Physics at the HL-LHC and HE-LHC
Report from Working Group 2 on the Physics of the HL-LHC, and Perspectives at the HE-LHC `19



δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
+0.10 pb −1.15 pb	±0.18 pb	±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
+0.21% −2.37%	±0.37%	±1.16%	±1%	±0.83%	±1%

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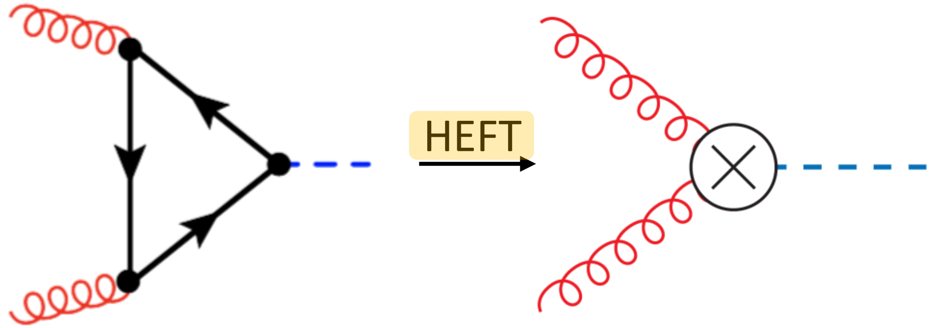
Contributions to σ_{tot}



$$
\begin{array}{lrrl}
48.58\,\text{pb} = & 16.00\,\text{pb} & (+32.9\%) & (\text{LO, rEFT}) \\
 & +\,20.84\,\text{pb} & (+42.9\%) & (\text{NLO, rEFT}) \\
 & -\;\;2.05\,\text{pb} & (-4.2\%) & ((t,b,c),\ \text{exact NLO}) \\
 & +\;\;9.56\,\text{pb} & (+19.7\%) & (\text{NNLO, rEFT}) \\
 & +\;\;0.34\,\text{pb} & (+0.7\%) & (\text{NNLO}, 1/m_t) \\
 & +\;\;2.40\,\text{pb} & (+4.9\%) & (\text{EW, QCD-EW}) \\
 & +\;\;1.49\,\text{pb} & (+3.1\%) & (\text{N}^3\text{LO, rEFT})
\end{array}
$$

"Born-improved" total cross section:

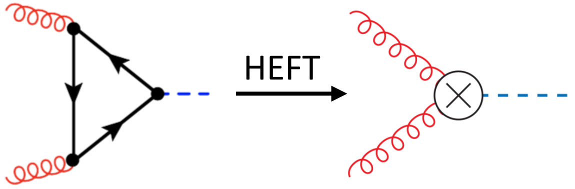
$$
\boxed{\sigma^{\rm HO}_{\rm HEFT} = \left(\frac{\sigma^{\rm HO}}{\sigma^{\rm LO}}\right)_{M_{\rm t}\to\infty} \sigma^{\rm LO}}
$$

- Gluon-fusion is induced by quark loops
  - NLO result available for arbitrary quark masses Graudenz, Spira, Zerwas \`93
  - Radiative corrections beyond NLO restricted to top-loop induced terms
- Dominant effect of top-loop induced terms can be accounted for in HEFT approximation

Anastasiou, Melnikov \`02
Harlander, Kilgore \`02
Ravindran, Smith, van Neerven \`03
Marzani, Ball, Del Duca, et al. \`08
Harlander, Mantler, Marzani, et al. \`09
Pak, Rogal, Steinhauser \`09

HEFT

- Introduce effective Higgs-gluon vertex
  - → reduce number of loops by one
  - → reduce number of scales by one



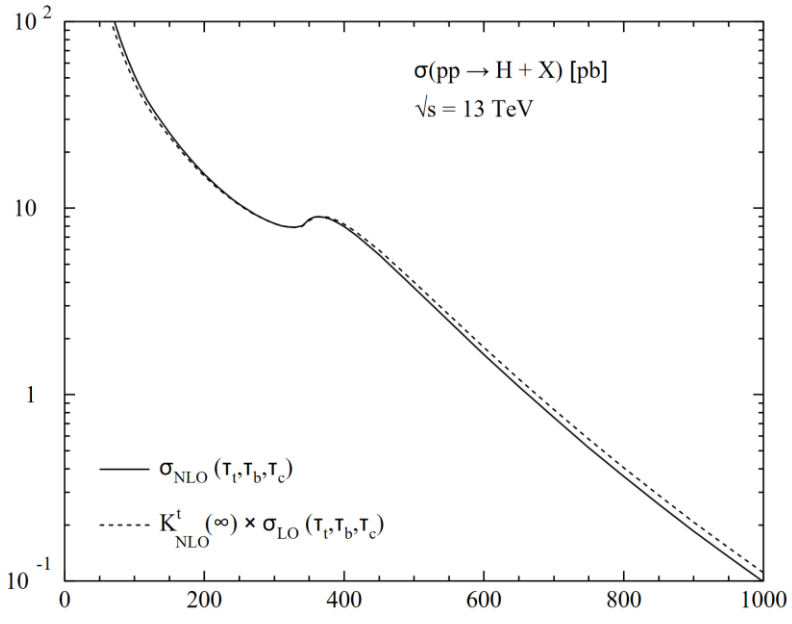
- Very good agreement with exact result at NLO

→ Remarkable, because
  - M_t being the largest scale is invalid over large range of $\sqrt{\hat{s}}$
  - $M_t \to \infty$ is applied to more than 50% of total cross section
  - HEFT fails to capture top-mass effects for partonic quark channels

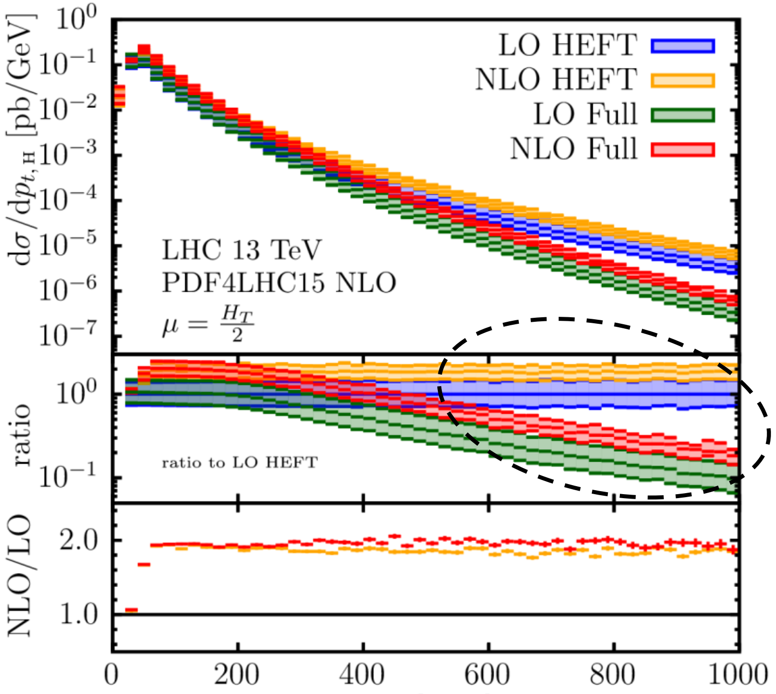
- Qualitative explanation:
  - Suppression of large-$\hat{s}$ region by PDFs
  - Dominance of the soft region

- Only estimate of top-mass effects beyond HEFT at NNLO based on combination of $1/M_t$-expansion with leading terms in large-$\hat{s}$ limit

  → Eliminate this uncertainty with exact calculation of top-quark mass effects

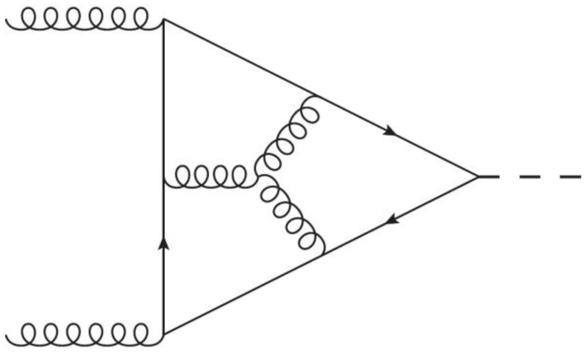
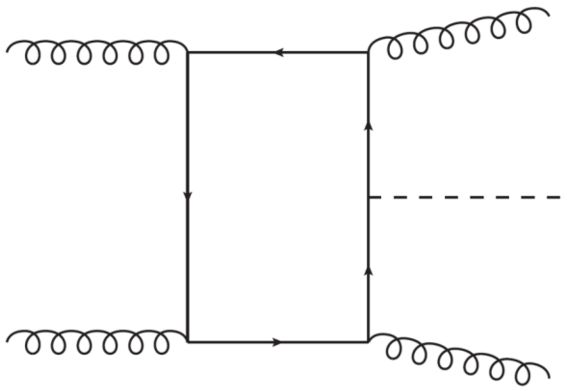


Spira `16



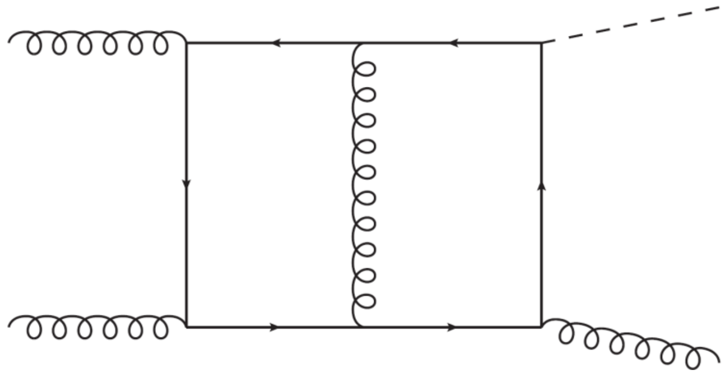
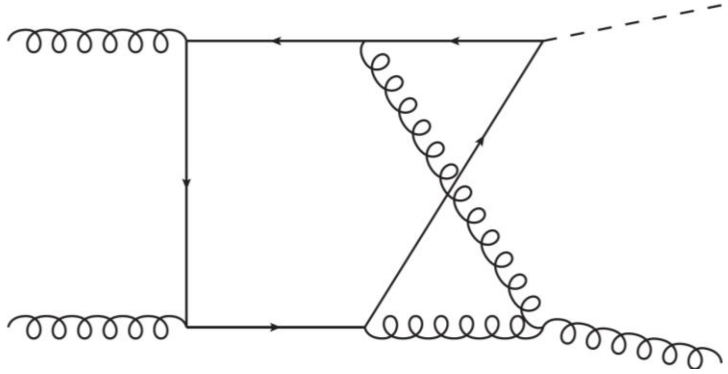
Jones, Kerner, Luisoni `18

Ingredients

- 3-loop virtual corrections (✓)
  → Davies, Gröber, et al. `19
  Czakon, MN `20
- Double-real corrections (✓)
  → Del Duca, Kilgore, et al. `01
- 2-loop real-virtual corrections:
  - 4 scales (m_q^2, s, t, u or m_H^2)
    + dimension $\mathrm{d} = 4 - 2\varepsilon$
  - 282 diagrams for gg → Hg
  - 49 diagrams for $q\bar{q}$ → Hg

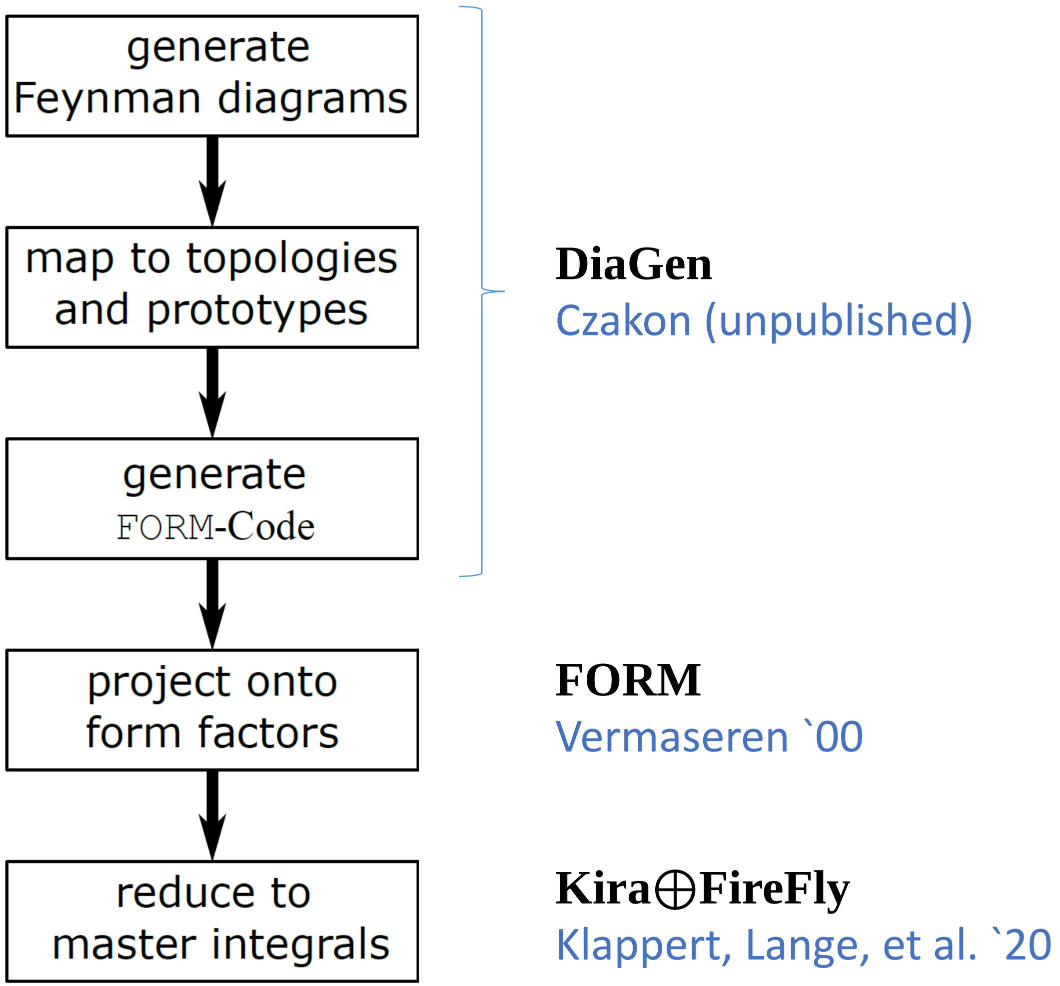
→ Master integrals have been computed
Frellesvig, Hidding, et al. `19

Workflow of the computation

- Get rid of tensor/colour structure to end up with a linear combination of scalar integrals with rational function coefficients in front

- Reduce the scalar integrals to a linearly independent set of master integrals (MI) (447 master integrals for gg→Hg)
- Reduction is highly non-trivial since rational coefficients depend on 5 variables!
  → Use finite fields to reconstruct symbolic coefficients from numerical probes of the system of equations



Computing the MI

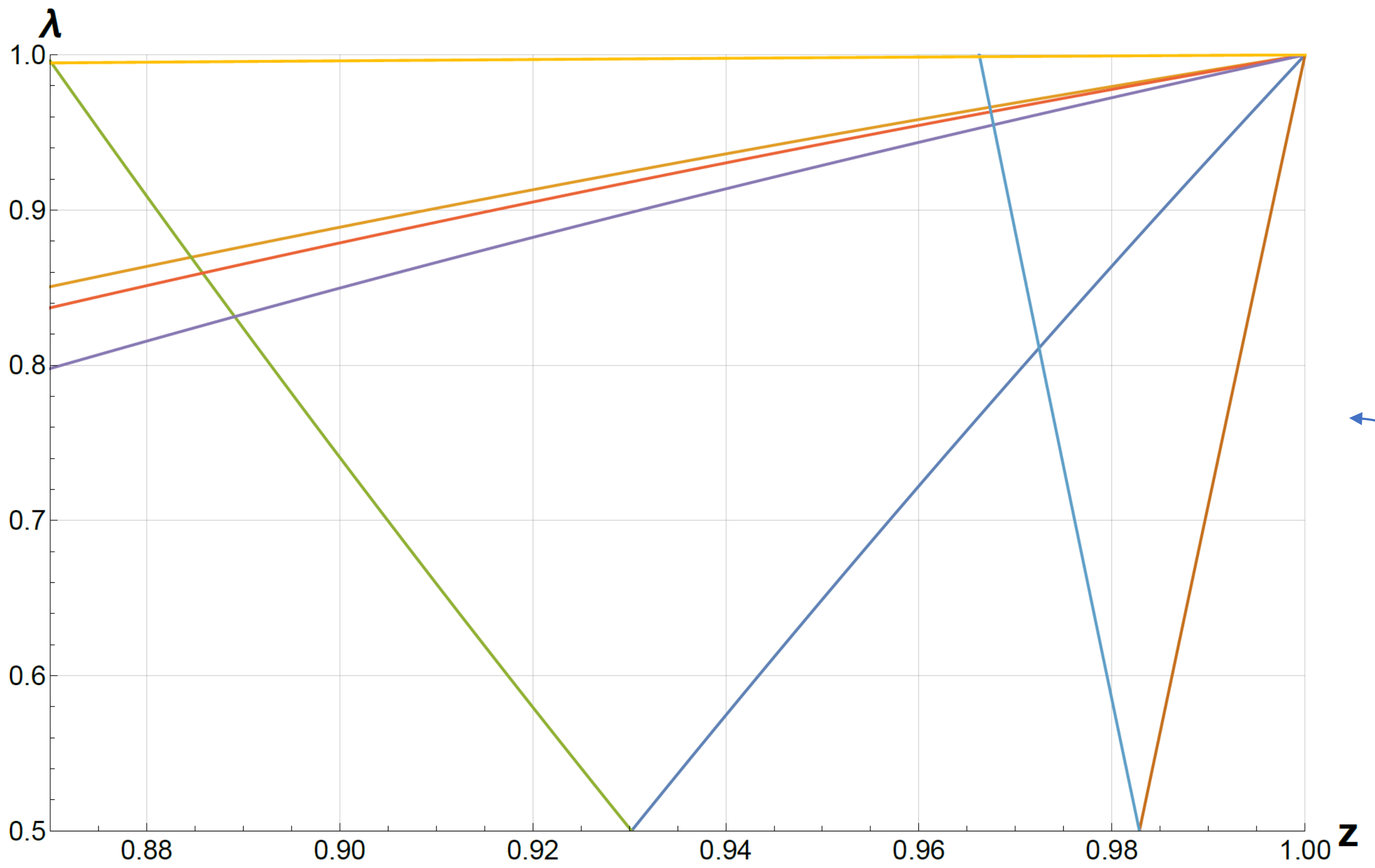
- Refrain from computing the MI analytically
- Two popular methods for calculating MI numerically:
  - Sector decomposition
  - Via differential equations

- Take the derivative of the MI with respect to the kinematic invariants and reduce the result to obtain a system of differential equations

- Provide boundary conditions in the limit $m_q^2 \to \infty$ and let the system evolve

Computation in detail

Above top-quark threshold



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

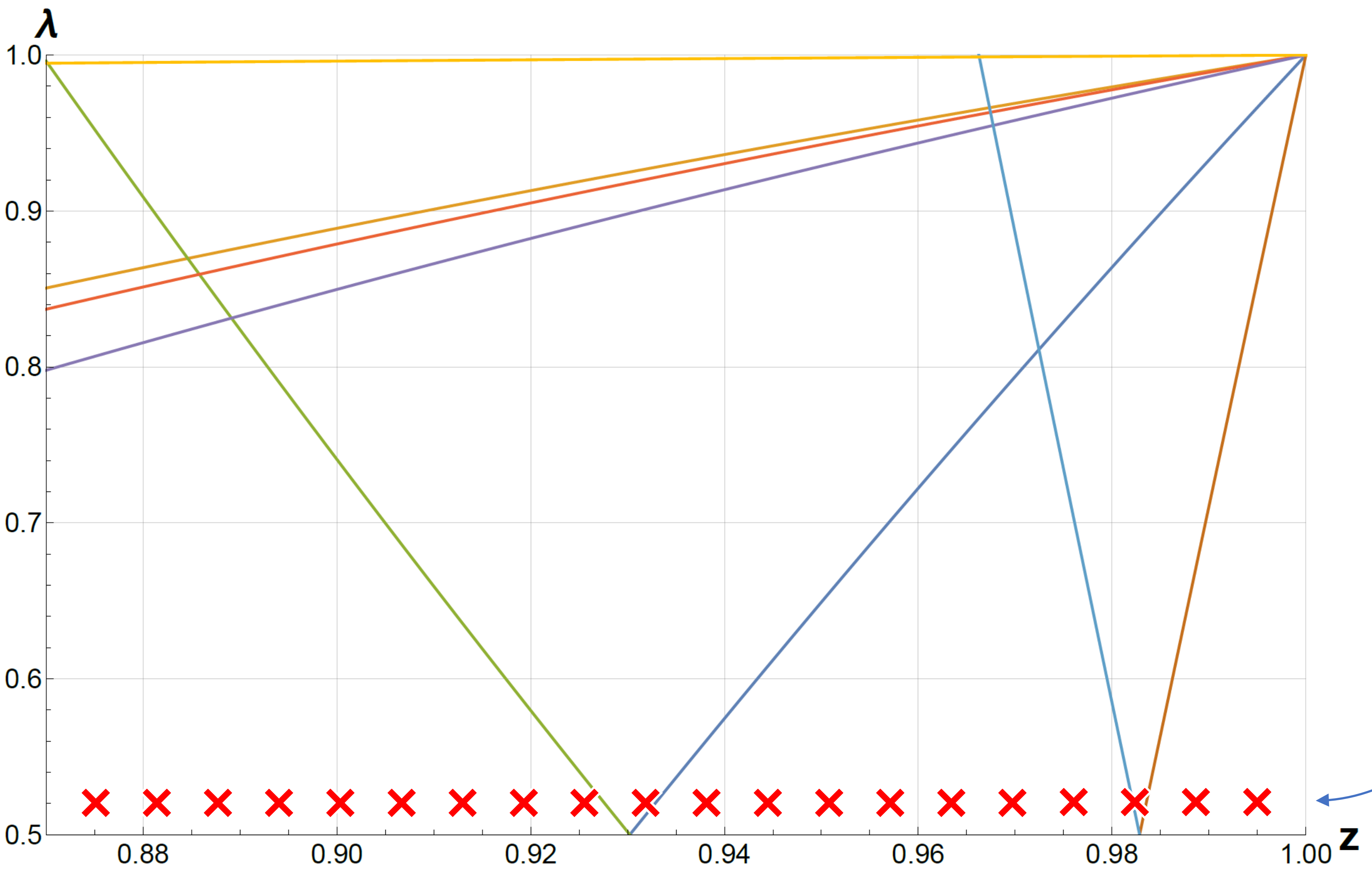
Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Poles of differential equations in λ

Computation in detail

Above top-quark threshold



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

Range of parameters:

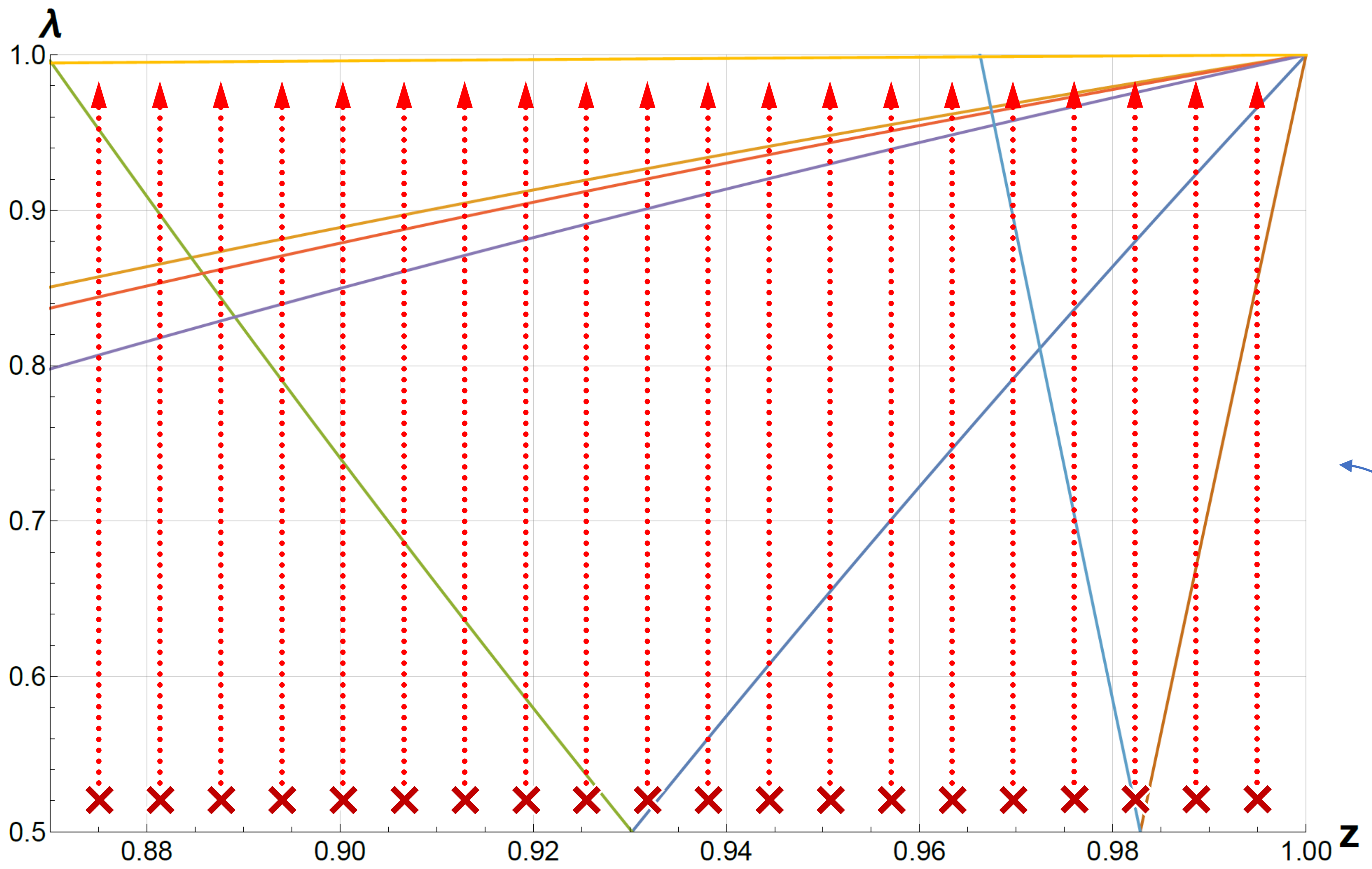
- $\lambda \in (0,1)$
- $z \in (0,1)$

Integrate differential equations in m_q^2 from boundaries at $m_q^2 \to \infty$ to top-quark mass

Boundaries for numerical integration

Computation in detail

Above top-quark threshold



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

Range of parameters:

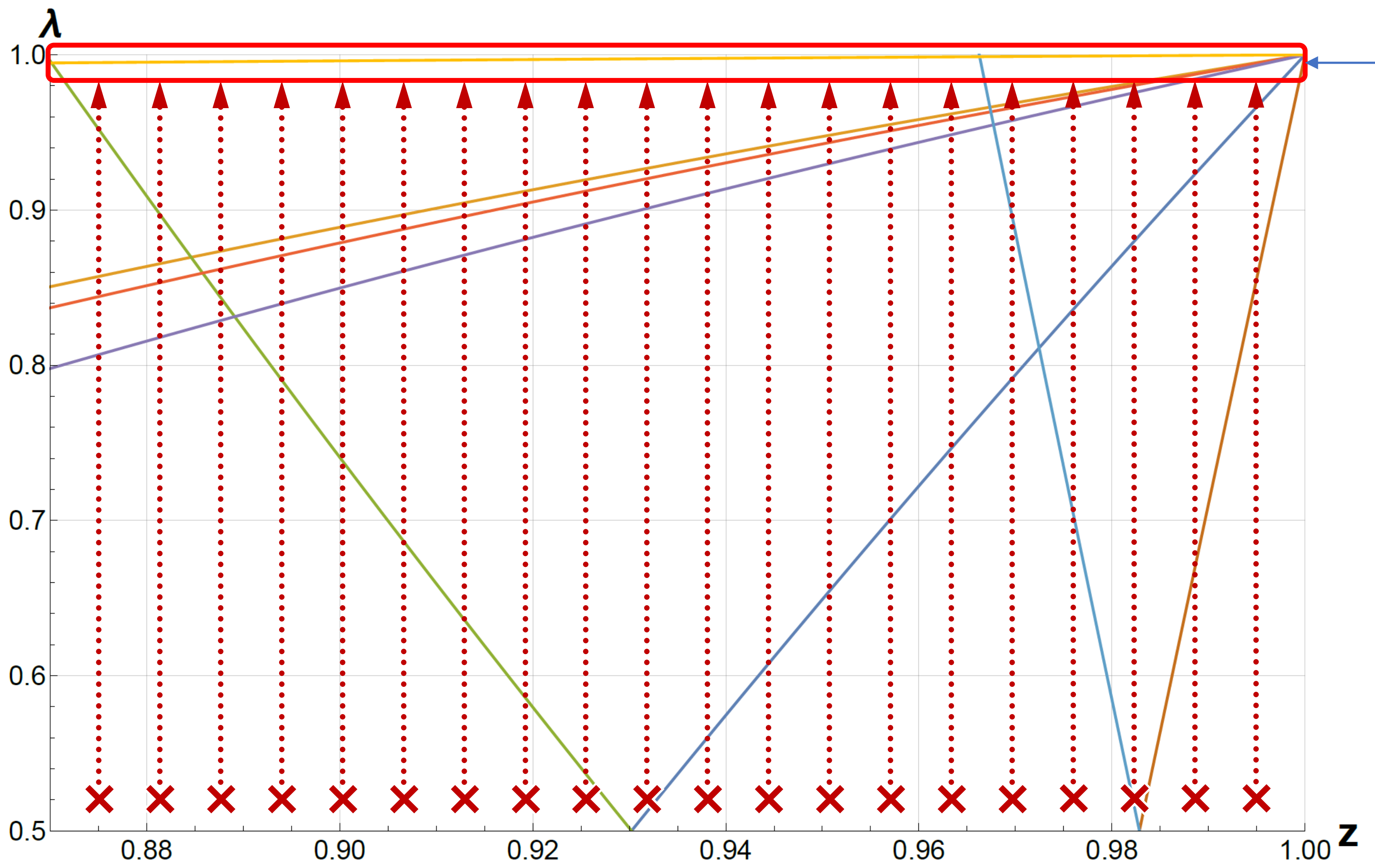
- $\lambda \in (0,1)$
- $z \in (0,1)$

Integrate differential equations in λ to the edge at $\lambda \sim 1$

Collect numerical samples for MI along straight integration contours

Computation in detail

Above top-quark threshold



z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$

m_t^2/m_H^2 = 23/12

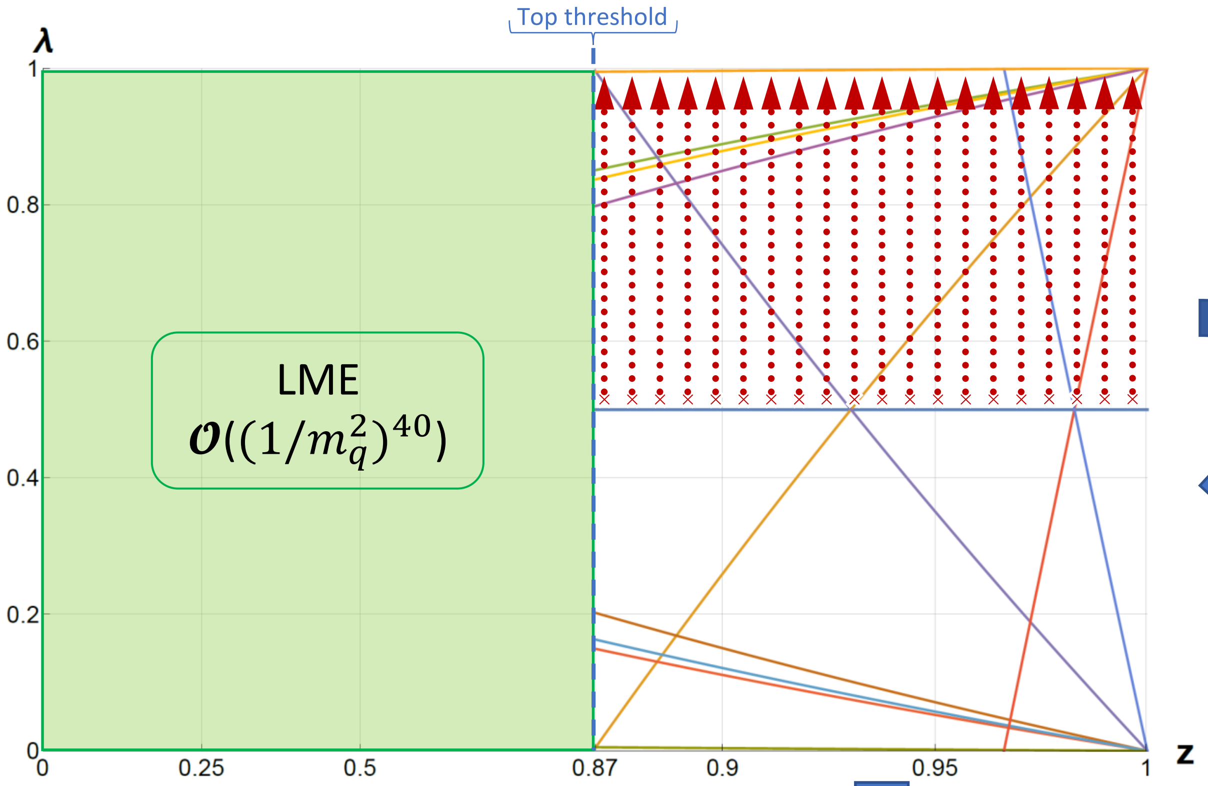
Range of parameters:

- $\lambda \in (0,1)$
- z $\in (0,1)$

Extrapolation to the collinear limit

➢ Ansatz: Power-Log-Series

Computation in detail



Problem is symmetric with respect to λ = ½

- reflect numerical samples to cover entire region above top-quark threshold

- Construct large-mass expansion up to order $(1/m_q^2)^{40}$ with full symbolic dependence
- Method to obtain expansion coefficients inspired by interpolation techniques
- Sufficient to cover region below top-quark threshold

Collect numerical samples

Insert into amplitudes

Subtraction for gg → gH

$z = 1 - m_H^2/\hat{s}$
$\lambda = \hat{t}/(\hat{t}+\hat{u})$
$m_t^2/m_H^2 = 23/12$

- Interested in finite results
  - Soft and collinear divergences
  - Amplitudes have to be regulated

- Directly evaluate difference between HEFT and exact result:

Free of divergences!

NLO: $$\langle M^{(1)}_{\rm exact}|M^{(1)}_{\rm exact}\rangle\big|_{\rm regulated} \equiv \langle M^{(1)}_{\rm exact}|M^{(1)}_{\rm exact}\rangle - \left[\langle M^{(1)}_{\rm HEFT}|M^{(1)}_{\rm HEFT}\rangle\right]$$

NNLO: $$\langle M^{(1)}_{\rm exact}|M^{(2)}_{\rm exact}\rangle\big|_{\rm regulated} \equiv \langle M^{(1)}_{\rm exact}|M^{(2)}_{\rm exact}\rangle - \left[\langle M^{(1)}_{\rm HEFT}|M^{(2)}_{\rm HEFT}\rangle + \frac{8\pi\alpha_s}{\hat{t}}\left\langle P^{(0)}_{gg}\left(\frac{\hat{s}}{\hat{s}+\hat{u}}\right)\right\rangle \langle F^{(1)}|(F^{(2)}_{\rm exact} - F^{(2)}_{\rm HEFT})\rangle\right]$$

Splitting-function regulates residual soft and collinear divergences

- Better numerical stability!
- Appearance of divergences delayed!
  - Simple counterterms!

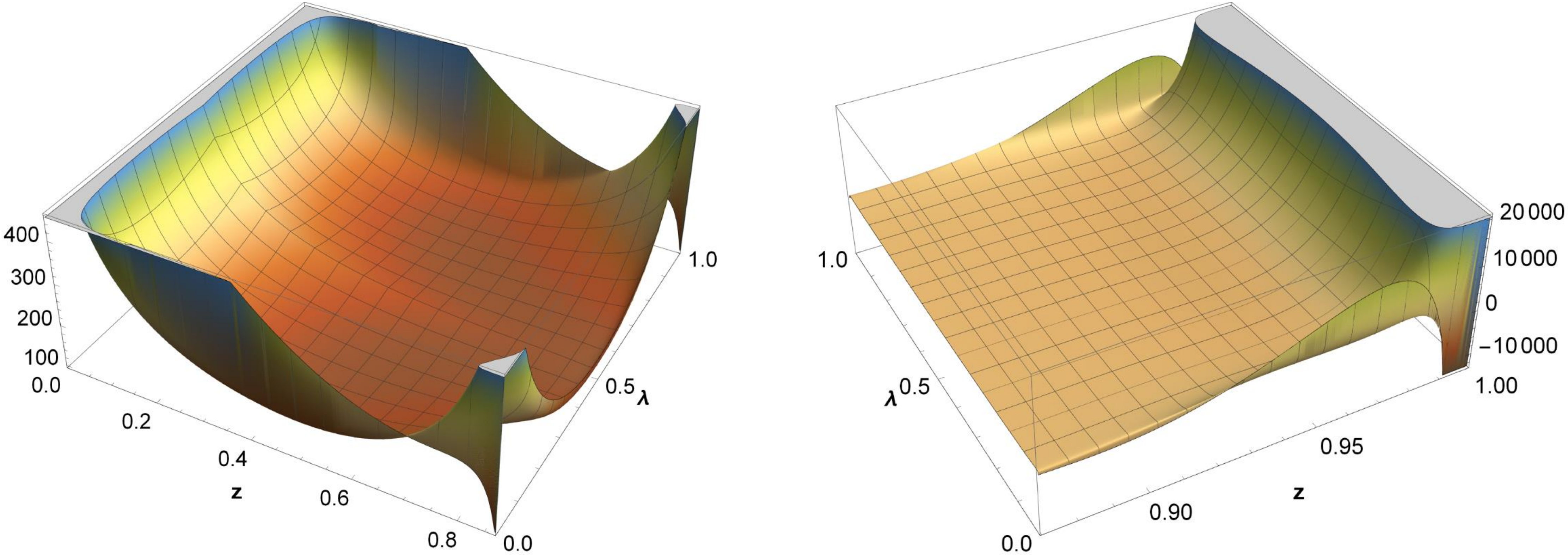
One of the reasons for the smallness of top-quark mass effects beyond HEFT

Subtraction for gg → gH

$z = 1 - m_H^2/\hat{s}$
$\lambda = \hat{t}/(\hat{t}+\hat{u})$
$m_t^2/m_H^2 = 23/12$

$$\langle M^{(1)}_{\text{exact}} | M^{(2)}_{\text{exact}} \rangle\big|_{\text{regulated}} \equiv \langle M^{(1)}_{\text{exact}} | M^{(2)}_{\text{exact}} \rangle - \left[\langle M^{(1)}_{\text{HEFT}} | M^{(2)}_{\text{HEFT}} \rangle + \frac{8\pi\alpha_s}{\hat{t}} \left\langle P^{(0)}_{gg} \left(\frac{\hat{s}}{\hat{s}+\hat{u}} \right) \right\rangle \langle F^{(1)} | \left(F^{(2)}_{\text{exact}} - F^{(2)}_{\text{HEFT}} \right) \rangle \right]$$

- Real part of the regulated quantity at $\mu_R = m_H/2$:



- Integrate in λ and convolute with PDFs to obtain contribution to σ_{tot}
- Subtraction term and other contributions are computed with Monte Carlo methods using **Stripper** Czakon (unpublished)

Results

- Effects of a finite top-quark mass on the total hadronic Higgs-boson production cross section for the LHC
  - PDF set: NNPDF31_nnlo_as_0118
  - $\mu_R = \mu_F = m_H/2$
  - M_H = 125 GeV $\Rightarrow M_t \approx$ 173.055 GeV

channel	$\sigma_{\text{HEFT}}^{\text{NNLO}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma_{\text{exact}}^{\text{NNLO}}-\sigma_{\text{HEFT}}^{\text{NNLO}})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma_{\text{exact}}^{\text{NNLO}}/\sigma_{\text{HEFT}}^{\text{NNLO}}-1)$ [%]
		$\sqrt{s}=8\,\text{TeV}$		
gg	$7.39+8.58+3.88$	$+0.0353$	$+0.0879\pm0.0005$	$+0.62$
qg	$0.55+0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01+0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39+9.15+4.18$	-0.0873	$+0.0667\pm0.0007$	-0.10
		$\sqrt{s}=13\,\text{TeV}$		
gg	$16.30+19.64+8.76$	$+0.0345$	$+0.2431\pm0.0020$	$+0.62$
qg	$1.49+0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02+0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30+21.15+9.79$	-0.3029	$+0.1815\pm0.0023$	-0.26

Results

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channel	$\sigma_{\text{HEFT}}^{\text{NNLO}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma_{\text{exact}}^{\text{NNLO}} - \sigma_{\text{HEFT}}^{\text{NNLO}})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma_{\text{exact}}^{\text{NNLO}}/\sigma_{\text{HEFT}}^{\text{NNLO}} - 1)$ [%]		
$\sqrt{s} = 8\,\text{TeV}$						
gg	$7.39 + 8.58 + 3.88$	$+0.0353$	$+0.0879 \pm 0.0005$	$+0.62$	(= 0.18	+ 0.44)
qg	$0.55 + 0.26$	-0.1397	-0.0021 ± 0.0005	-18	(= -17.5	- 0.5)
qq	$0.01 + 0.04$	$+0.0171$	-0.0191 ± 0.0002	-4	(= +34	- 38)
total	$7.39 + 9.15 + 4.18$	-0.0873	$+0.0667 \pm 0.0007$	-0.10	(= -0.42	+ 0.32)
$\sqrt{s} = 13\,\text{TeV}$						
gg	$16.30 + 19.64 + 8.76$	$+0.0345$	$+0.2431 \pm 0.0020$	$+0.62$	(= 0.08	+ 0.54)
qg	$1.49 + 0.84$	-0.3696	-0.0115 ± 0.0010	-16	(= -15.5	- 0.5)
qq	$0.02 + 0.10$	$+0.0322$	-0.0501 ± 0.0006	-15	(= +27	- 42)
total	$16.30 + 21.15 + 9.79$	-0.3029	$+0.1815 \pm 0.0023$	-0.26	(= -0.64	+ 0.38)

Conclusions

✓ The hadronic Higgs production cross section including the full top-quark mass dependence was computed!

✓ Slight decrease relative to HEFT at NNLO

- -0.26% at 13 TeV
- -0.10% at 8 TeV

✓ The result confirms and eliminates the uncertainty estimate from the lack of knowledge of the exact top-quark mass effects!

➢ Same techniques can be applied to compute bottom-quark mass effects…