Extended Relaxation Time Approximation (ERTA) and Relativistic Dissipative Hydrodynamic

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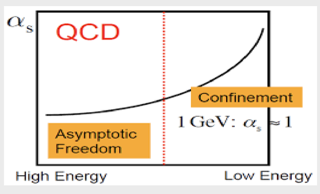
Abstract

Development of a new framework for the derivation of order-by-order hydrodynamics from the Boltzmann equation is necessary as the widely used Anderson-Witting formalism leads to violation of fundamental conservation laws when the relaxation-time depends on particle energy, or in a hydrodynamic frame other than the Landau frame. We generalize an existing framework for the consistent derivation of relativistic dissipative hydrodynamics from the Boltzmann equation with an energy-dependent relaxation-time by extending the Anderson-Witting relaxation-time approximation. We argue that the present framework is compatible with conservation laws and derives first-order hydrodynamic equations in the landau frame. Further, we show that the transport coefficients, such as shear and bulk viscosity as well as charge and heat diffusion currents, have corrections due to the energy dependence of relaxation-time compared to what one obtains from the Anderson-Witting approximation of the collision term. The ratio of these transport coefficients are studied using a parametrized relaxation time, and several interesting scaling features are reported.

Motivation



Boltzmann Equation

$$p^\mu \partial_\mu f = \mathcal{C}[f] = \frac{(p\cdot u)}{\tau_R}\delta f = \text{Collision kernel}$$

$\tau_{R(\text{RTA})} \propto 1/\sigma_{(\text{cross section})} \propto 1/\alpha_{s(\text{coupling strength})} \propto E^{\ell}_{(\text{Energy})}$ Here $\tau_R(p) = (T/p) * \chi(p)$

TABLE I. Summary of the functional dependence of the departure from equilibrium on the theory and approximation considered.

Model	Physics	Formula
Relaxation time, $\tau_R \propto p$	Relaxation time grows with particle momentum.	$\chi(p) \propto p^2$
Relaxation time, $\tau_R = \text{const}$	Relaxation time independent of momentum.	$\chi(p) \propto p$
Scalar theory	Randomizing collisions which happen rarely.	$\chi(p) \propto p^2$
QCD soft scatt.	Soft $q \sim gT$ collisions lead to a random walk of hard particles.	$\chi(p) \propto p^2$
QCD hard satt.	Hard $q \sim \sqrt{pT}$ collisions lead to a random walk of hard particles.	$\chi(p) \propto \frac{p^2}{\log(p/T)}$
QCD rad. energy loss	Radiative energy controls the approach to equilibrium. In the LPM regime, $\hat{q}$ controls the radiation rate.	$\chi(p) \propto \frac{p^{3/2}}{\alpha_s\sqrt{\hat{q}}}$

Fig. 2: K Dusling,D. Moore,D.Teany,PHYSICAL REVIEW C 81, 034907 (2010)

Description of Frames

- u^μ is defined in hydro LRF and $u^{\mu *}$ is defined in thermodynamic LRF with $u_\mu u^\mu = 1$ and $u^*_\mu u^{\mu *} = 1$.
- $u^*_\mu \equiv u_\mu + \delta u_\mu \quad T^* \equiv T + \delta T \quad \mu^* \equiv \mu + \delta\mu \quad f^*_{\text{eq}} \equiv f_{\text{eq}} + \delta f^*$
- Boltzmann transport equation with Extended RTA is given by $p^\mu\partial_\mu f = \frac{-(u\cdot p)}{\tau_R(x,p)}(f - f^*_{\text{eq}}(u^*_\mu, T^*, \mu^*))$, $f^*_{\text{eq}} = (e^{-\beta^*(u^*\cdot p)-\alpha^*} \pm a)^{-1}$ a=0,1,-1 for MB,FD,BE
- $\tau_R(x,p) = \tau_{\text{eq}}(x)\tau_p(p)$ where $\tau_p = (\frac{u\cdot p}{T})^\ell$
- A order-by-order gradient expansion is followed here.
- We will consider dissipative function(δf) in hydrodynamics up to 1st order.
- $f = f_{\text{eq}} + \delta f_{(1)} \rightarrow f - f^*_{\text{eq}} = f_{\text{eq}} + \delta f - f^*_{\text{eq}} = f_{\text{eq}} + \delta f_{(1)} - f_{\text{eq}} - \delta f^* = \delta f_{(1)} - \delta f^*$

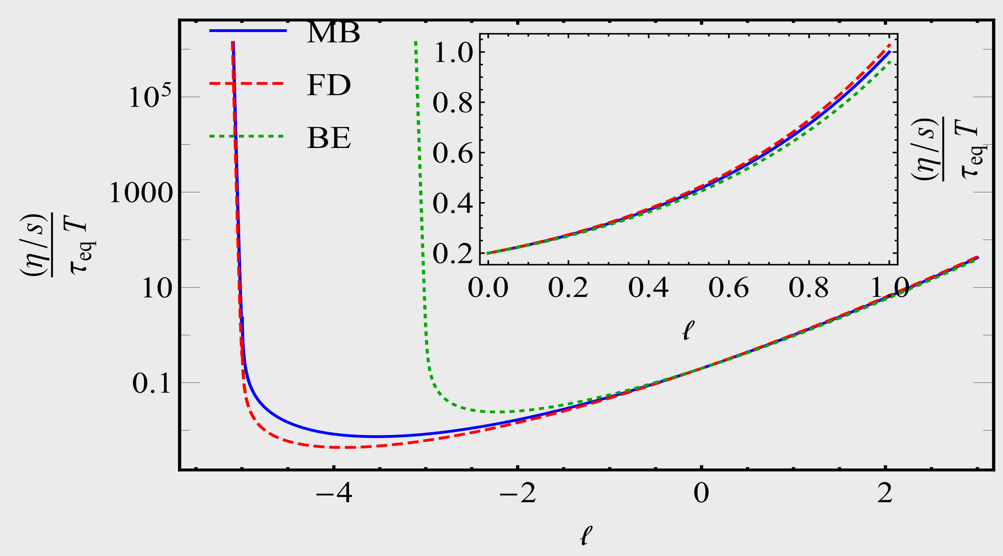
Aim

- Normally RTA is taken to be momentum independent i.e $\tau_R(x)$.
- Our aim is to consider momentum dependent AW RTA i.e $\tau_R(x,p)$ where
$$\tau_p = (\frac{u\cdot p}{T})^\ell$$

Constraints

- Two conservation laws must be obeyed for a dissipative fluid system.
- Energy-momentum conservation $(\partial_\mu T^{\mu\nu} = 0)$and Particle current conservation $(\partial_\mu N^\mu = 0)$ respectively.
- First moment of Boltzmann equation must vanish to satisfy the energy- momentum tensor conservation.
$$\int \mathrm{dP} p^\mu p^\nu \partial_\mu f = 0$$

Results for mass less and charge less case



$$\frac{\eta}{s\tau_{\text{eq}}T} = \frac{\Gamma(5+\ell)}{120}\left[\frac{\mathrm{Li}_{4+\ell}(-a)}{\mathrm{Li}_4(-a)}\right].$$

a=0,1,-1 for MB,FD BE stat respectively

Issue

- By using momentum dependent AW RTA(Extended RTA) in Boltzmann equation,it will lead to the violation of energy momentum conservation.
- $u_\mu u_\nu \delta T^{\mu\nu} = \int \mathrm{dP} p^\mu p^\nu \delta f^{(1)} \neq 0$
- Choice of landau frame for hydro need not correspond to the thermal equilibrium system

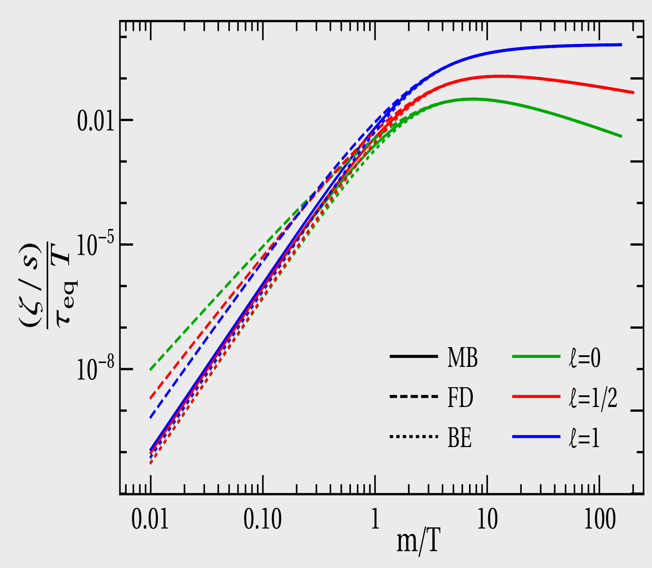
Approach

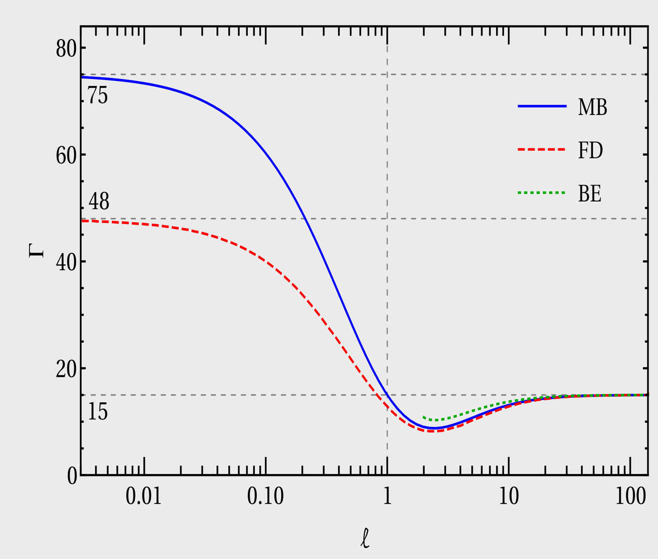
- Nonzero quantity of energy conservation can be compensated by the difference in by defining 2 different frames.
- The differences will be calculated using landau frame and matching condition.

$u_\mu T^{\mu\nu} = \epsilon_0 u^\nu, u_\mu u_\nu T^{\mu\nu} = \epsilon_0 , u_\mu N^\mu = n_0$

Phys. Rev. C 89, 014901 (2014), arXiv:1304.3753 [nucl-th]

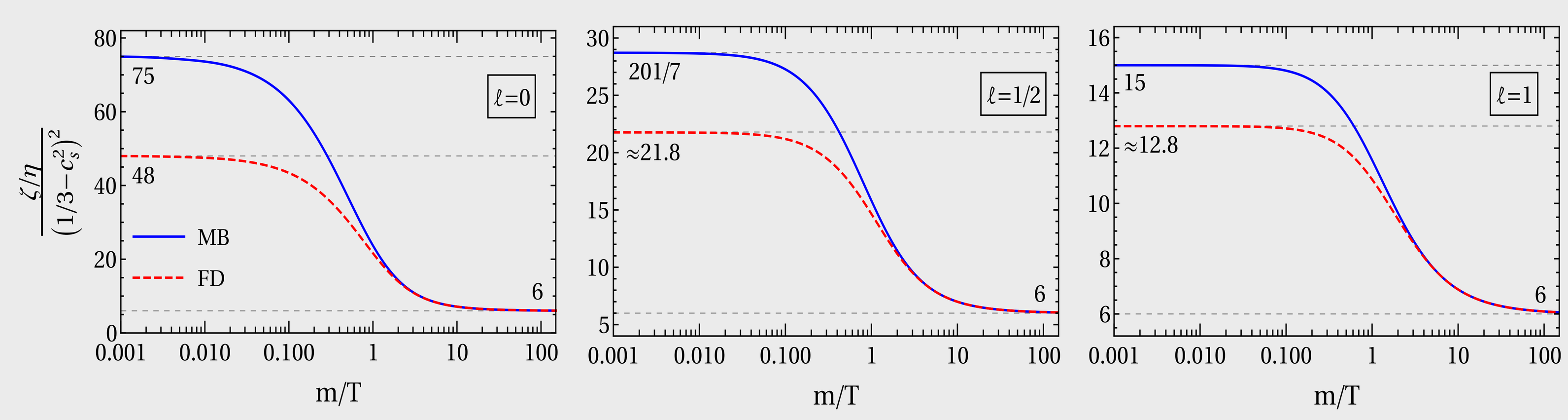
Results for massive and charge less case





$$\frac{\zeta/\eta}{(1/3 - c_s^2)^2} = \Gamma$$

$$\Gamma_{\text{MB}} = \frac{15\left(\ell^3 + 6\ell^2 - 13\ell + 30\right)}{(\ell+1)(\ell+2)(\ell+3)},$$



What's special?

- An Approach by changing the form of AW RTA is already available to deal ERTA,where,conservation equations are satisfied by compromising the simple form of RTA.Phys. Rev. Lett. 127,042301 (2021), arXiv:2103.07489 [nucl-th].
- In our case we kept RTA as usual, but compromise by satisfying the conservation equation order by order in gradient expansion.

Formalism setup

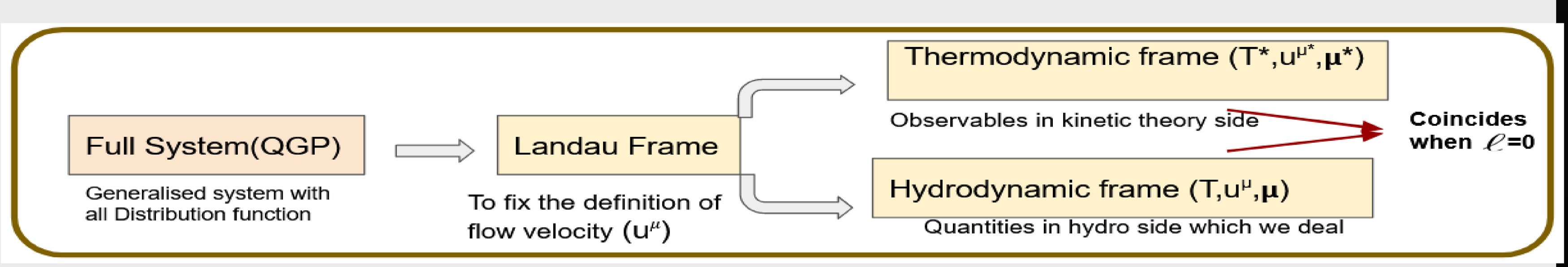
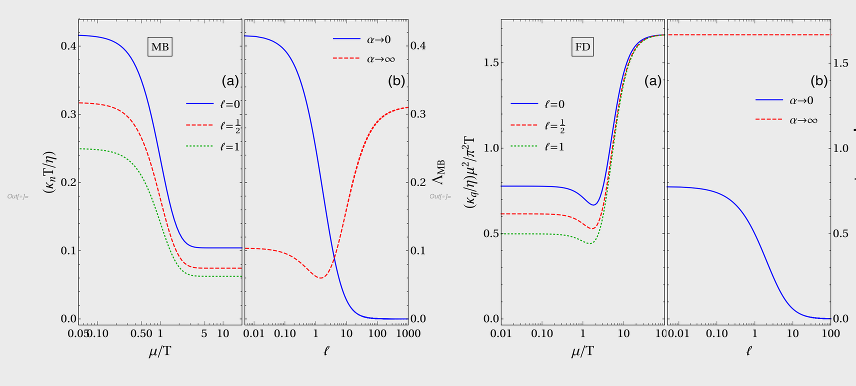


Fig. 3: Flow chart to Introduce different frames

Results for mass less and charged case



$$\Lambda_{\text{MB}} = \begin{cases} \frac{5}{(4+\ell)(3+\ell)} & \text{for } \alpha \to 0\,, \\ \frac{5(\ell^2-\ell+4)}{16\,(4+\ell)(3+\ell)} & \text{for } \alpha \to \infty\,. \end{cases}$$

$$\Lambda_{\text{FD}} = \begin{cases} \frac{196\,\pi^2\left(2^{1+\ell}-1\right)\zeta(2+\ell)}{45\,(2^{3+\ell}-1)(4+\ell)(3+\ell)\zeta(4+\ell)} & \alpha \to 0 \\ \frac{5}{3} & \alpha \to \infty \end{cases}$$

Summary and Conclusion

- A successful and well defined frame is developed to consider momentum dependent RTA. Ratios of the transport coefficient up to first order are studied.
- New and interesting features of transport coefficients for different statistics are revealed. arXiv:2112.14581v2[nucl-th]7Feb2022
- So many other questions are still needed to address (e.g other functional form of momentum dependent τ_R, $\frac{\zeta}{\eta}$ behaviour for $-\ell$, study in other frame of reference etc).