Charge and heat transport coefficients of a hot and dense QCD matter in the presence of a weak magnetic field

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Introduction

- The noncentral events of heavy ion collisions produce extremely strong magnetic fields with strength varying between 10^{18} G ($1\ m_\pi^2$) to 10^{20} G.
- Depending on the electrical conductivity, the lifetime of magnetic field gets elongated and it may affect the properties of the partonic medium [1, 2].
- The lifetime of weak magnetic field might be larger. Thus, it could leave significant impacts on different transport properties of partonic medium.
- Weak magnetic field limit: $T^2 \gg |q_f B|$.
- Quasiparticle mass:

$$m_{fT}^2 = \frac{g^2T^2}{6}\left(1+\frac{\mu_f^2}{\pi^2T^2}\right),$$

where $g^2 = 4\pi\alpha_s$ and α_s = one-loop running coupling,

$$\alpha_s\left(\Lambda^2, eB\right) = \frac{\alpha_s\left(\Lambda^2\right)}{1+b_1\alpha_s\left(\Lambda^2\right)\ln\left(\frac{\Lambda^2}{\Lambda^2+eB}\right)}, \text{ with}$$

$$\alpha_s\left(\Lambda^2\right) = \frac{1}{b_1\ln\left(\frac{\Lambda^2}{\Lambda_{\overline{\rm MS}}^2}\right)},$$

where $b_1 = \frac{11N_c-2N_f}{12\pi}$, $\Lambda_{\overline{\rm MS}} = 0.176$ GeV and $\Lambda = 2\pi\sqrt{T^2+\mu_f^2/\pi^2}$.

Charge and heat transport coefficients [3]

- The spatial component of the induced current:

$$J^i = \sum_f g_f\int\frac{d^3{\rm p}}{(2\pi)^3\omega_f}p^i[q\delta f_f + \bar{q}\delta\bar{f}_f],$$

with $\delta f_f = f_f - f_f^0$, $\delta\bar{f}_f = \bar{f}_f - \bar{f}_f^{\,0}$.
- For a general configuration of electric and magnetic fields:

$$J^i = \sigma^{ij}E_j = \sigma_0\delta^{ij}E_j + \sigma_1\epsilon^{ijk}b_kE_j + \sigma_2 b^ib^jE_j,$$

with $\mathbf{b} = \frac{\mathbf{B}}{B}$.
- If electric and magnetic fields are perpendicular to each other:

$$J^i = \sigma^{ij}E_j = \left(\sigma_{\rm el}\delta^{ij} + \sigma_{\rm H}\epsilon^{ij}\right)E_j.$$

- Relativistic Boltzmann transport (RBT) equation in the relaxation time approximation (RTA):

$$p^\mu\frac{\partial f_f(x,p)}{\partial x^\mu} + F^\mu\frac{\partial f_f(x,p)}{\partial p^\mu} = -\frac{p_\nu u^\nu}{\tau_f}\delta f_f(x,p),$$

$$\tau_f = 1/\left[5.1T\alpha_s^2\log(1/\alpha_s)(1+0.12(2N_f+1))\right],$$

u^ν: four-velocity of fluid,
$F^\mu = qF^{\mu\nu}p_\nu = (p^0\mathbf{v}\cdot\mathbf{F}, p^0\mathbf{F})$,
$F^{\mu\nu}$: electromagnetic field strength tensor,
$\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$.
- The spatial component of heat flow:

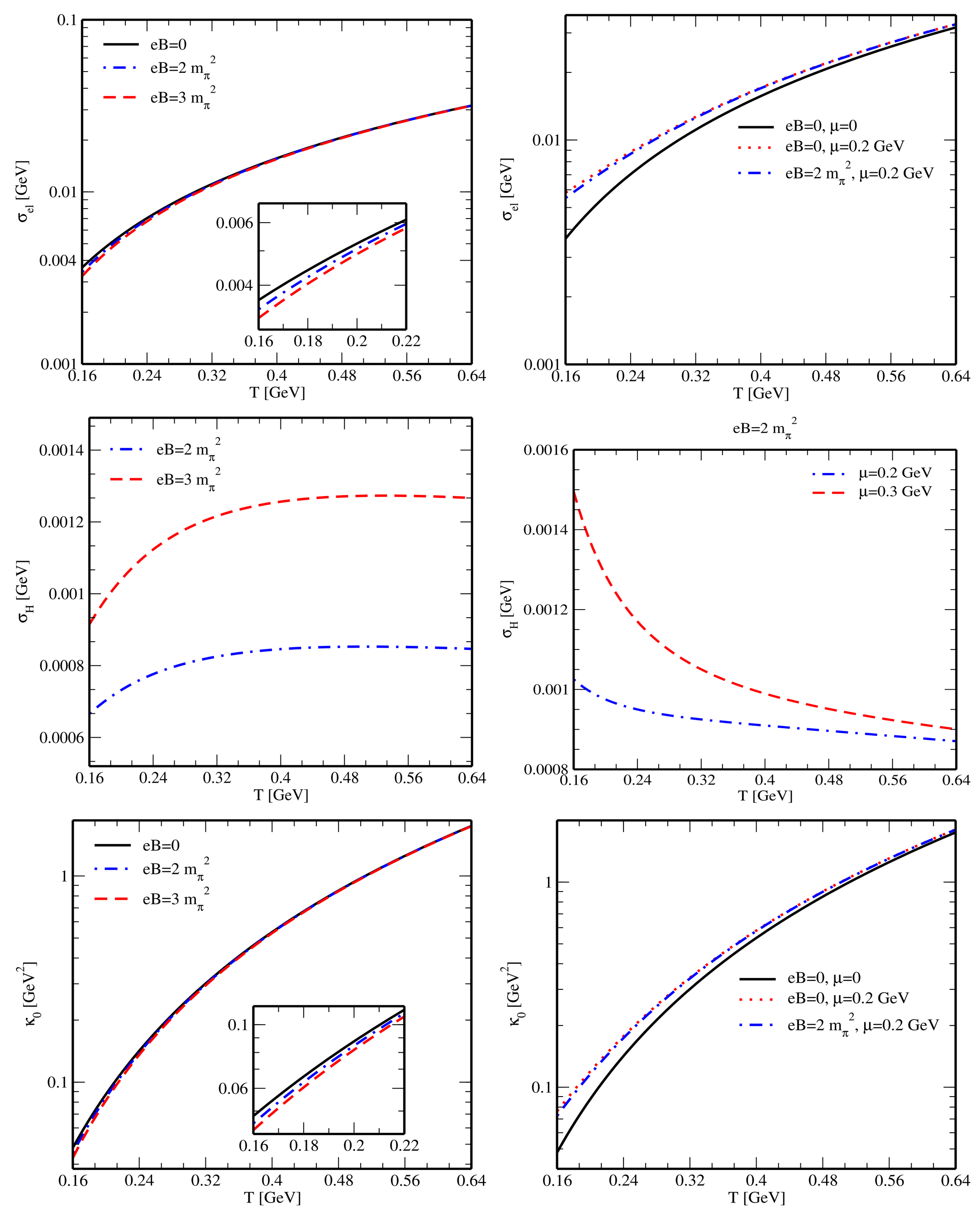
$$Q^i = \sum_f g_f\int\frac{d^3{\rm p}}{(2\pi)^3\omega_f}\frac{p^i}{}\left[(\omega_f - h_f)\delta f_f + (\omega_f - \bar{h}_f)\delta\bar{f}_f\right].$$

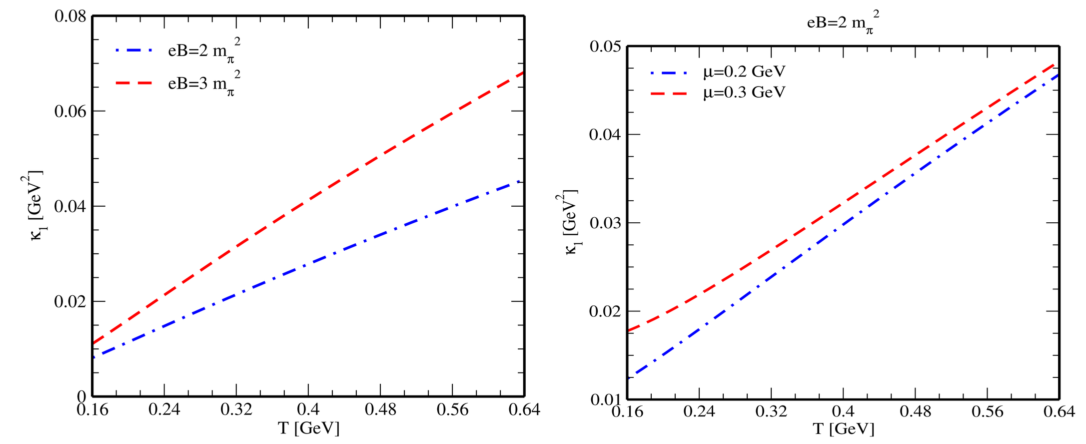
- The flow of heat in a medium:

$$Q^i = -\left(\kappa_0\delta^{ij} + \kappa_1\epsilon^{ijk}b_k + \kappa_2 b^ib^j\right)\left[\partial_j T - \frac{T}{\varepsilon+P}\partial_j P\right].$$

- If gradients of temperature and pressure are perpendicular to magnetic field:

$$Q^i = -\left(\kappa_0\delta^{ij} + \kappa_1\epsilon^{ij}\right)\left[\partial_j T - \frac{T}{\varepsilon+P}\partial_j P\right].$$



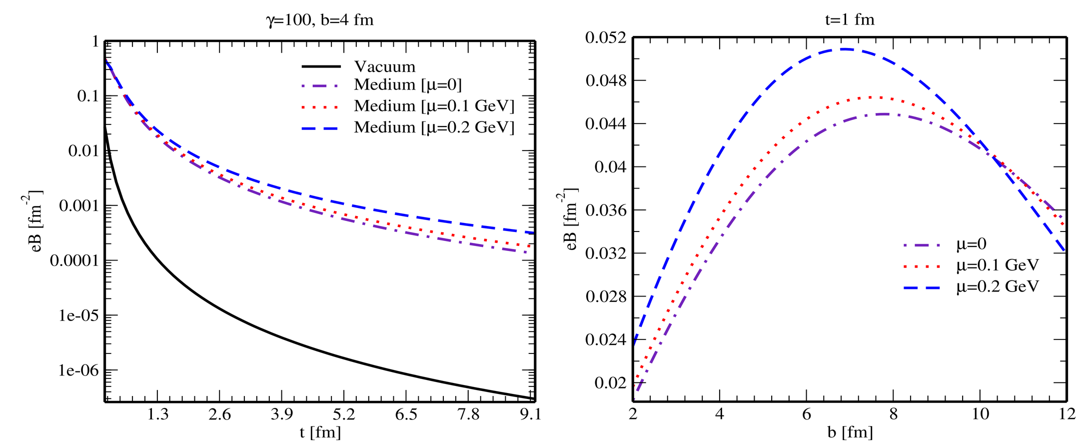


Lifetime of magnetic field [3]

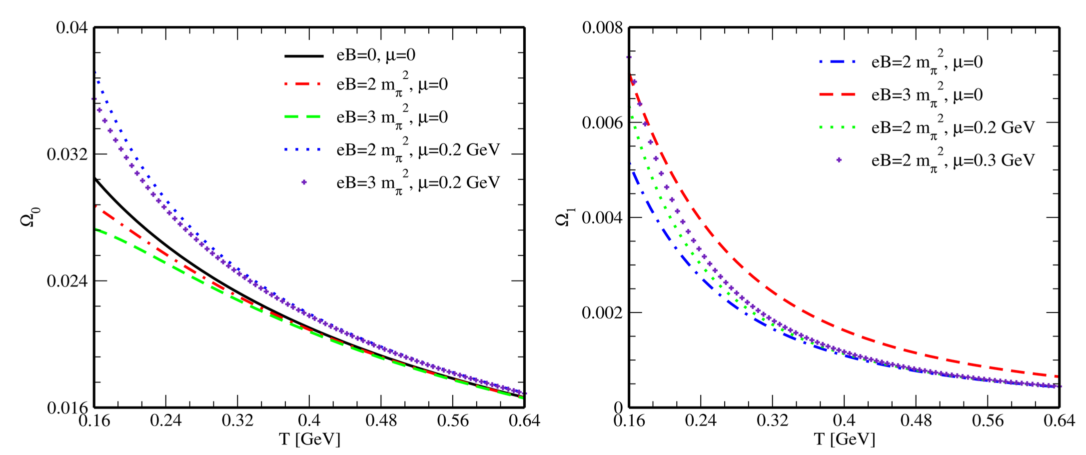
- The emergence of finite chemical potential in an electrically conducting medium increases the lifetime of magnetic field.

$$e\mathbf{B}_{\rm medium} = \frac{e^2b\sigma_{\rm el}}{8\pi(t-x)^2}e^{-\frac{b^2\sigma_{\rm el}}{4(t-x)}}\hat{\mathbf{z}},$$

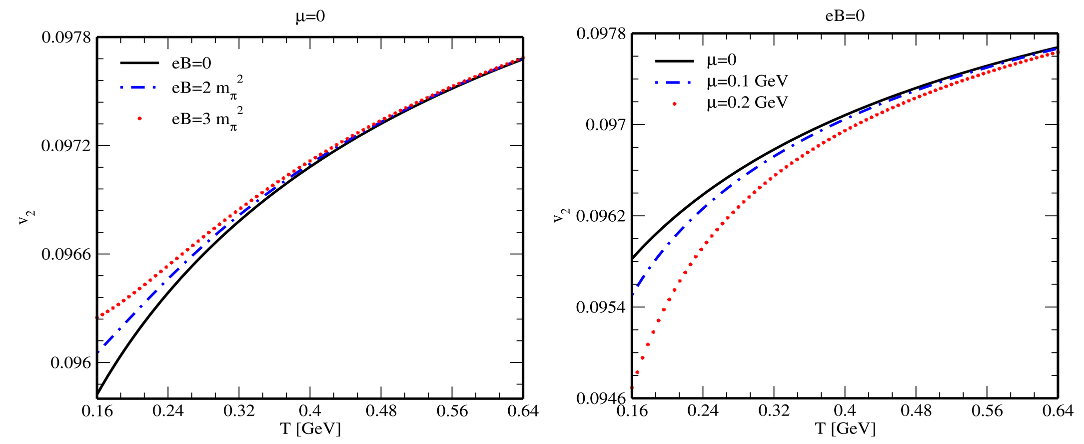
$$e\mathbf{B}_{\rm vacuum} = \frac{e^2b\gamma}{4\pi\left\{b^2+\gamma^2(t-x)^2\right\}^{3/2}}\hat{\mathbf{z}}.$$



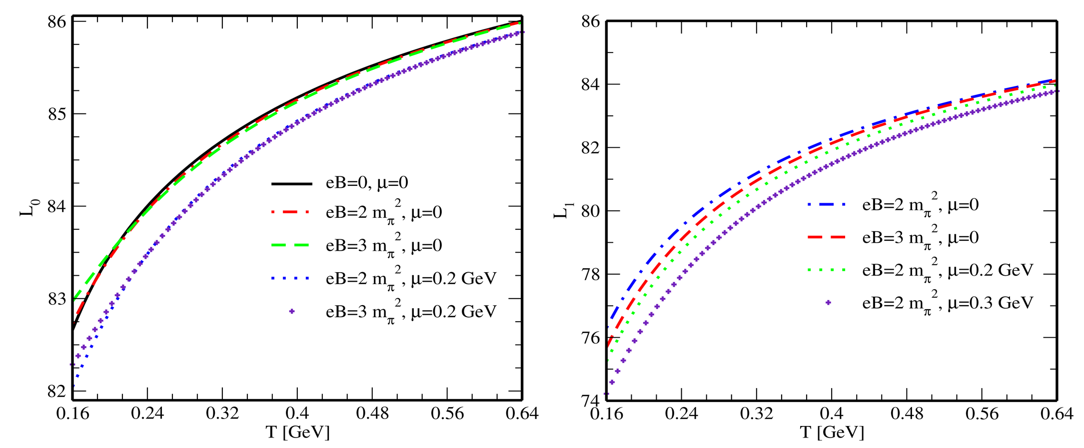
Knudsen number: $\Omega_{(0,1)} = \frac{\lambda}{L} = \frac{3\kappa_{(0,1)}}{LvC_V}$, elliptic flow: $v_2 = \frac{v_2^h}{1+\frac{\Omega}{\Omega_h}}$ and Wiedemann-Franz law: $\frac{\kappa_{(0,1)}}{\sigma_{\rm (el,H)}} = L_{(0,1)}T$ [3]



- In the presence of weak magnetic field and chemical potential, components of the Knudsen number remain below 1, so, the medium remains in the equilibrium state.



- The elliptic flow gets increased in a weak magnetic field, whereas chemical potential decreases its magnitude.



- The Wiedemann-Franz law gets violated for hot QCD matter at finite chemical potential and weak magnetic field.

Conclusions

- Effects of weak magnetic field on the charge and heat transport coefficients are more noticeable at low temperatures.
- Emergence of finite chemical potential enhances the charge and heat transports.
- The lifetime of magnetic field gets further increased at finite density.
- Furthermore, the Knudsen number, the Lorenz number and the elliptic flow also get significantly affected by the weak magnetic field and finite chemical potential.

References

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[2] S. Rath and B. K. Patra, Phys. Rev. D **100**, 016009 (2019).

[3] S. Rath and S. Dash, arXiv:2112.11802 [hep-ph].