CP-Violating Invariants in the SMEFT

Q. Bonnefoy, Emanuele Gendy, C. Grojean, J. T. Ruderman: 2112.03889

In the Standard Model

In the Standard Model, CP breaking in the weak sector is a **collective effect**. Its order parameter is a flavor-reparametrization invariant, the **Jarlskog Invariant**:

$$J_4 = \mathrm{ImTr}\left([Y_u^\dagger Y_u, Y_d^\dagger Y_d]^3\right) =$$
$$= (m_t^2 - m_c^2)(m_t^2 - m_u^2)\times$$
$$\times\,(m_c^2 - m_u^2)(m_b^2 - m_s^2)\times$$
$$\times\,(m_b^2 - m_d^2)(m_s^2 - m_d^2)\times$$
$$\times\, s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin(\delta_{\mathrm{CKM}}) \sim$$
$$\sim \mathcal{O}(\lambda^{36})$$

CP in the Standard Model is conserved **iff** $J_4 = 0$

In the SMEFT

In the SMEFT, new coefficients can introduce new sources of CP violation

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \sum_{n\geq 5} \frac{c_n}{\Lambda^{n-4}}\mathcal{O}^{(n)}$$

The coefficients can contain phases. But some phases may be reabsorbed through flavor transformation. Which phases are physical?

More precisely

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Rightarrow |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right)$$

Conserves CP iff $J_4 = 0$

Conserves CP iff $J_4 = 0$ & ???=0

CPV in the SMEFT is better described via flavor-invariants

Focus at dimension 6 on CP-violating invariants **linear** in the Wilson coefficients. For example

$$\mathcal{O}_{HQ}^{(1)} = C_{HQ,mn}^{(1)}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\bar{Q}_m\gamma^\mu Q_n \longrightarrow$$

$$L_1^{HQ(1)} = \mathrm{ImTr}(Y_u Y_u^\dagger Y_d Y_d^\dagger C_{HQ}^{(1)})$$
$$L_2^{HQ(1)} = \mathrm{ImTr}((Y_u Y_u^\dagger)^2 (Y_d Y_d^\dagger)^2 C_{HQ}^{(1)})$$
$$L_3^{HQ(1)} = \mathrm{ImTr}(Y_u Y_u^\dagger Y_d Y_d^\dagger (Y_u Y_u^\dagger)^2 (Y_d Y_d^\dagger)^2 C_{HQ}^{(1)})$$

Here CP is conserved **iff** $J_4 = L_1^{HQ(1)} = L_2^{HQ(1)} = L_3^{HQ(1)} = 0$

The invariants help us in counting the number of new sources of CP violation that can affect observables at $\mathcal{O}(1/\Lambda^2)$.

In the end we find **699** independent CP-violating invariants. **That's a lot!**. We need a way to organize them in a hierarchical way

Hierarchical invariants

Use the Wolfenstein parametrization and expand in λ

$$Y_u = \mathrm{diag}(a_u\lambda^8,\ a_c\lambda^4, a_t\lambda^0)$$
$$Y_d = V_{\mathrm{CKM}}\mathrm{diag}(a_d\lambda^7,\ a_c\lambda^4, a_b\lambda^3)$$
$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

with $\lambda \approx 0.2$, $a_i = \mathcal{O}(1)$

For the invariants of $C_{HQ}^{(1)}$ at first non-zero order

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} A a_b^2 a_t^2 \mathrm{Im}C_{HQ,23}^{(1)}\lambda^8 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(\lambda^9)$$

Less flavor suppressed than J_4!

Clearly the suppression depends on the flavor assumption on the coefficients

Flavor assumptions

We can study different assumptions on the flavor structure of the SMEFT operators. Each of them can modify the number of independent new phases we can access at each order in λ, and the number of those with less flavor suppression than J_4.

Three possible choices are flavor anarchy, Minimal Flavor Violation or the $U(2)^5$ flavor symmetry. For the modified Yukawa operator, which has 9 new phases, we get something like: