Intrinsic quantum mechanics behind the Standard Model? - Predictions in baryon and Higgs sectors

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Abstract

I introduce quantum mechanics on an intrinsic configuration space for baryons, the Lie group U(3), which carries the three gauge groups of the standard model of particle physics as subgroups SU(3), SU(2) and U(1). The strong and electroweak interactions become related via the Higgs mechanism. I namely settle the electroweak energy scale by the neutron to proton decay where both sectors are involved through quark flavour changes. Predictions of neutral pentaquark resonances reachable at LHCb follow in the baryon sector as does an accurate expression in the electroweak sector for the Higgs mass (yielding 125.104(14) GeV) and predictions on the couplings of the Higgs to itself and to the gauge bosons with signal strengths deviating by the presence of the up down quark mixing matrix element. The intrinsic view means that quantum fields are generated by the momentum form on intrinsic wavefunctions and local gauge transformations in laboratory space equate translations in the intrinsic configuration space which may be likened to a generalised spin space. Further insight is gained for the Cabibbo and Weinberg angles expressed in traces of u and d flavour quark generators.

Key references: EPL **102** (2013) 42002, Int. J. Mod. Phys. A **30** (2015) 1550078, EPL **124** (2018) 31001, EPL **125** (2019) 41001, EPL **133** (2021) 31001. See also arXiv:2007.02936.

Intrinsic space for baryon mass states

I describe baryon mass eigenstates as intrinsic configurations on the Lie group U(3) shaped by a Hamiltonian structure (reinterpreted Kogut-Susskind).

$$\frac{\hbar c}{a}\left[-\frac{1}{2}\Delta+\frac{1}{2}\mathrm{Tr}\,\chi^2\right]\Psi(u)=E\Psi(u) \qquad (1)$$

Here u is the configuration variable

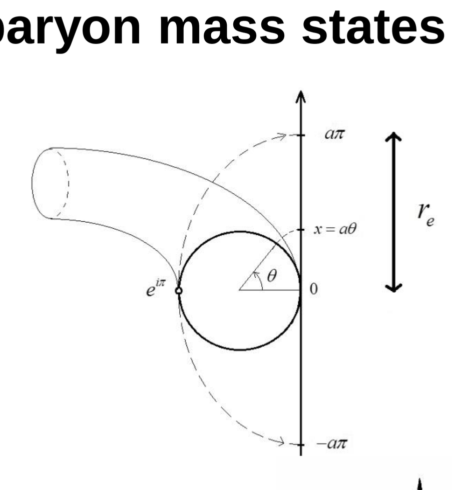
$$u=e^{i(\theta_j T_j+(\alpha_j S_j+\beta_j M_j)/\hbar)}\equiv e^{i\chi},\quad j=1,2,3$$

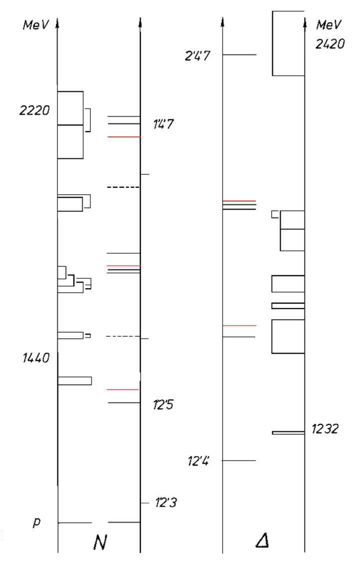
of an entire baryonic entity. The length scale a is set by the classical electron radius in projection to lab space

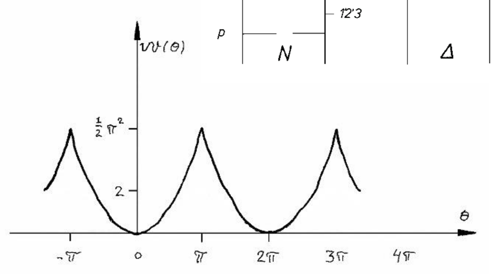
$$\pi a=r_e\equiv\frac{e^2}{4\pi\varepsilon_0 m_e c^2},\qquad \theta_j=x_j/a,\qquad iT_j=\frac{\partial}{\partial\theta_j}$$

The potential is half the squared geodetic distance from u to the 'origo' e

$$\frac{1}{2}d^2(e,u)=\frac{1}{2}\mathrm{Tr}\,\chi^2=\sum_{j=1}^{3}w(\theta_j), \qquad (2)$$







$$w(\theta)=\frac{1}{2}(\theta-q\cdot 2\pi)^2,\ \theta\in[(2q-1)\pi,(2q+1)\pi],\ q\in Z$$

The theory unfolded

The Laplacian in (1) contains off-toroidal derivatives which are represented by the off-diagonal Gell-Mann matrices. Three are grouped into spin operators $\boldsymbol{S}=(S_1,S_2,S_3)$. This interpretation is supported by their commutation relations as intrinsic angular momentum known in nuclear physics. The remaining three off-toroidal derivatives are grouped into $\boldsymbol{M}=(M_1,M_2,M_3)$, which is related to hypercharge and isospin. The Laplacian in polar decomposition thus reads

$$\Delta=\sum_{j=1}^{3}\frac{1}{J^2}\frac{\partial}{\partial\theta_j}J^2\frac{\partial}{\partial\theta_j}-\sum_{\substack{i<j\\ k\neq i,j}}\frac{(S_k^2+M_k^2)/\hbar^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)},\qquad J=\prod_{i<j}^{3}2\sin\tfrac{1}{2}(\theta_i-\theta_j)$$

The off-torus term is analogous to the centrifugal term in the usual treatment of the hydrogen atom. In a coordinate representation the off-toroidal generators are e.g.

$$S_1=a\theta_2 p_3-a\theta_3 p_2=\hbar\lambda_7,\qquad p_j=\frac{\hbar}{a}T_j,\qquad [M_i,M_j]=[S_i,S_j]=-i\hbar\varepsilon_{ijk}S_k$$

$$M_1/\hbar=\theta_2\theta_3+\frac{a^2}{\hbar^2}p_2p_3=\lambda_6,\qquad s(s+1)+m^2=\frac{4}{3}\left(n+\frac{3}{2}\right)^2-3-\frac{1}{3}y^2-4i_3^2,\ n=0,1,2,\cdots$$

With the periodic potential in (2) and with $\Psi(u)=\tau(\theta_1,\theta_2,\theta_3)\Upsilon(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\beta_3)$, I can integrate out the off-toroidal degrees of freedom to get for the measure-scaled toroidal wavefunction $R(\boldsymbol{\theta})=J(\boldsymbol{\theta})\tau(\boldsymbol{\theta})$

$$[-\Delta_e+W]R(\theta_1,\theta_2,\theta_3)=2\mathrm{E}\,R(\theta_1,\theta_2,\theta_3) \qquad (3)$$

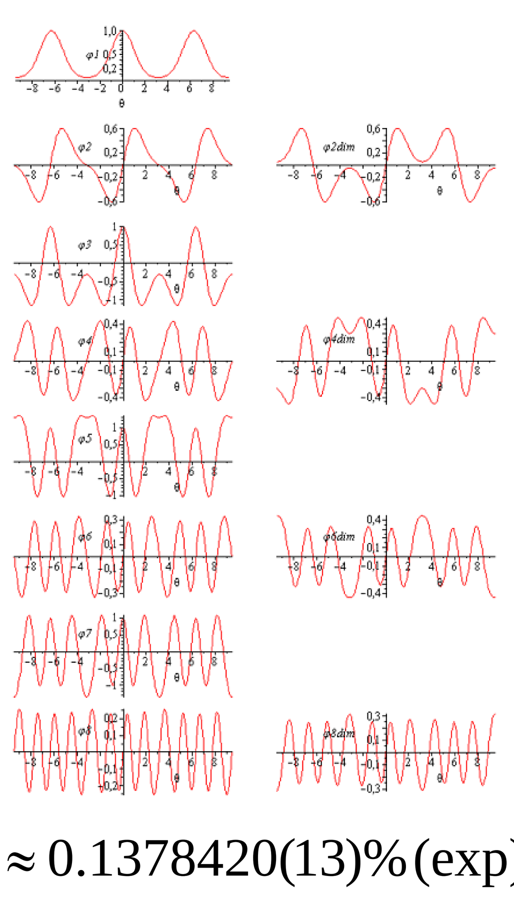
Here $\mathrm{E}=\frac{E}{\Lambda}$, $\Lambda\equiv\frac{\hbar c}{a}\approx 214\ \mathrm{MeV}$, $\Delta_e=\sum_{j=1}^{3}\frac{\partial^2}{\partial\theta_j^2}$ and

$$W=-2+\frac{1}{3}(s(s+1)+m^2)\sum_{i<j}^{3}\frac{1}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}+2\sum_{j=1}^{3}w(\theta_j)$$

The constant term comes from the Laplacian by differentiating through J. Equation (3) can be solved by a Rayleigh-Ritz method with analytically calculated integrals to give high accuracy. The measure-scaled toroidal wavefunction R can also be expanded on Slater determinants constructed from 1D eigenstates (see figure)

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\theta^2}+w(\theta)\right]\varphi_i(\theta)=e_i\varphi_i(\theta)$$

$$R=\sum_{lmn}a_{lmn}R_{lmn},\qquad R_{lmn}(\boldsymbol{\theta})=\begin{vmatrix}\varphi_l(\theta_1)&\varphi_l(\theta_2)&\varphi_l(\theta_3)\\ \varphi_m(\theta_1)&\varphi_m(\theta_2)&\varphi_m(\theta_3)\\ \varphi_n(\theta_1)&\varphi_n(\theta_2)&\varphi_n(\theta_3)\end{vmatrix}$$



Introducing Bloch wave degrees of freedom, I get diminished energy levels by period doubling. Coupled period doublings represent transformation from a neutral state (neutron) to a charged state (proton).

$$n\rightarrow p\qquad R_{1'2'3}(\boldsymbol{\theta})=\begin{vmatrix}e^{-i\frac{1}{2}\theta_1}g_{1'}(\theta_1)&e^{-i\frac{1}{2}\theta_2}g_{1'}(\theta_2)&e^{-i\frac{1}{2}\theta_3}g_{1'}(\theta_3)\\ e^{i\frac{1}{2}\theta_1}g_{2'}(\theta_1)&e^{i\frac{1}{2}\theta_2}g_{2'}(\theta_2)&e^{i\frac{1}{2}\theta_3}g_{2'}(\theta_3)\\ \varphi_3(\theta_1)&\varphi_3(\theta_2)&\varphi_3(\theta_3)\end{vmatrix}$$

The resulting shift in eigenvalue is $\dfrac{m_n-m_p}{m_p}=0.13847(14)\%\approx 0.1378420(13)\%(\mathrm{exp})$

Conclusions

A baryon mass spectrum with essentially no missing resonance problem derives from intrinsic dynamics on the Lie group U(3). Neutral charge, neutral flavour singlets are predicted (pentaquarks at LHCb?). Quark and gluon fields derive from momentum forms on the intrinsic wavefunctions and yield e.g. the proton spin structure function.

Spontaneous period doublings in the intrinsic state happen by exchange of a quantum of action between the strong and electroweak sectors. The intrinsic potential thereby shapes the Higgs potential and an accurate expression for the Higgs mass is derived. Higgs to gauge bosons coupling values are predicted (testable at HL-LHC?).

Key predictions:

***accurate Higgs mass, Cabibbo and Weinberg angles, neutral pentaquarks**

$$v_{SM}=\sqrt{2}\varphi_0\sqrt{|V_{ud}|}=246.86(5)\,\mathrm{GeV},\ \varphi_0=\frac{2\pi}{\alpha}\Lambda,\ \Lambda\equiv\frac{\hbar c}{a}=\frac{\pi}{\alpha}m_e c^2$$

$$*m_H c^2=\frac{1}{\sqrt{2}}\frac{2\pi}{\alpha(m_W)}\frac{\pi}{\alpha_e}m_e c^2=125.104(14)\,\mathrm{GeV},\ \text{exp: }125.25(17)\ \mathrm{GeV}$$

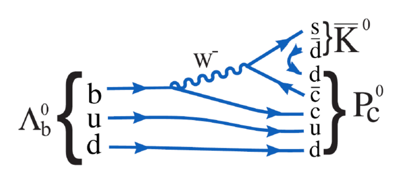
Excess couplings $\frac{\mu_{HWW}}{\mu_{HWW,SM}}=\frac{1}{|V_{ud}|}\approx 1.03$, $\frac{\mu_{HHHH}}{\mu_{HHHH;SM}}=V_{ud}^2\approx 1/1.06$, At HL-LHC ?

$$\sin\theta_C=\mathrm{Tr}\,T_u^\dagger T_s=-\frac{2}{9}\rightarrow\cos\theta_C=0.974996\cdots\approx|V_{ud}|=0.97370\pm 0.00014$$

$$\cos^2\theta_W=\mathrm{Tr}\,T_u^\dagger T_d=\frac{7}{9}=0.7777\cdots,\ \frac{m_W^2}{m_Z^2}=\left(\frac{80.379(12)GeV}{91.1876(21)GeV}\right)^2=0.7771(3)$$

Neutral charge, neutral flavour resonances N^0 to be sought at e.g. 1527, … 4231, 4501, 4655, 4726, 5101, 5146… MeV. Orange ~ open channel at LHCb.

$$\Lambda_b^0\rightarrow\overline{K}^0+P_c^0$$
$$\rightarrow\overline{K}^0+J/\psi+\Delta^0$$



Bloch wave degrees of freedom opened by Higgs mechanism

Spontaneous symmetry break in baryonic state.
Exchange quantum of action with electroweak sector:

$$\alpha\varphi_0 a=\hbar c,\qquad \Lambda=\frac{\hbar c}{a}$$
$$\varphi_0=\frac{2\pi}{\alpha(m_W)}\Lambda=\frac{v}{\sqrt{2}}$$
$$v_{SM}=v\sqrt{|V_{ud}|}$$

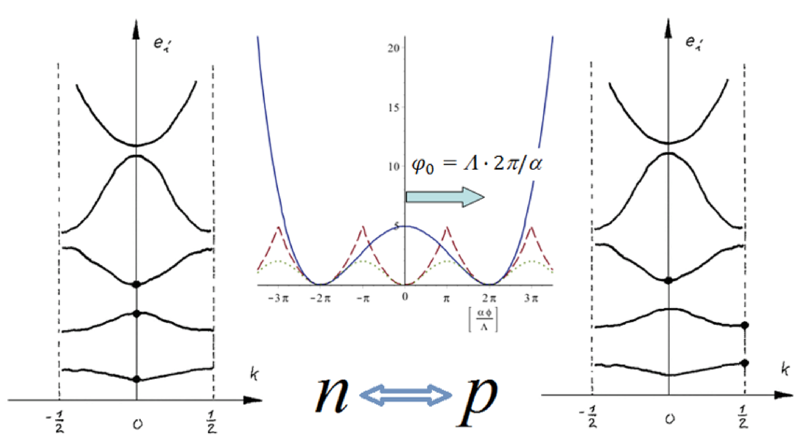
Intrinsic potential shapes Higgs potential

$$\frac{1}{2}(\theta\pm 2\pi)^2\rightarrow\frac{1}{8}(\phi^2-\phi_0^2)^2=V_H(\phi)$$

$\Lambda\theta\sim\alpha\varphi$ at $\theta=2\pi$ determines φ_0

$$V_H(\phi)=\frac{1}{2}\delta^2\phi_0^2-\frac{1}{2}\mu^2\phi^2+\frac{1}{4}\lambda^2\phi^4$$
$$\delta^2=\frac{1}{4}\varphi_0^2,\qquad \mu^2=\frac{1}{2}\varphi_0^2,\qquad \lambda^2=\frac{1}{2}$$

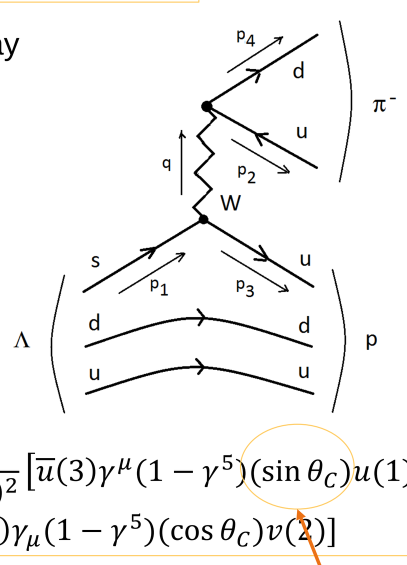
I interpret the period doublings in the wavefucntion as a topological origin for the creation of the proton charge in the neutron decay. The black dots in the figure show the Bloch wave number choices for the neutron (left) and the proton state (right).



Cabibbo and Weinberg angles + partons from flavour generators

Colour quark fields from momentum form
$$c_j=\mathrm{d}R_u(iT_j)$$
In general the momentum form acts as
$$\mathrm{d}R_u(Z)=\frac{\mathrm{d}}{\mathrm{d}t}R(ue^{tZ})\Big|_{t=0}$$
on any Z in the tangent space to the torus

Cabibbo angle in $\Lambda\rightarrow p$ decay



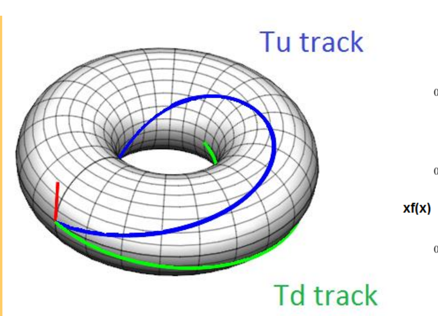
Flavour generators
$$T_u=\frac{2}{3}T_1-T_3$$
$$T_d=-\frac{1}{3}T_1-T_3$$
$$T_s=-\frac{1}{3}T_1$$

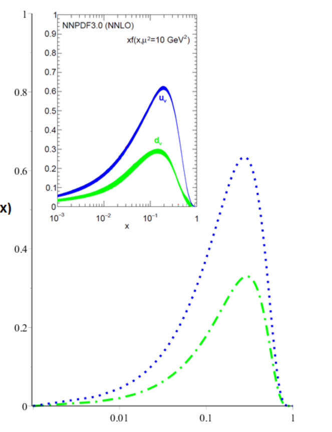
Colour states
$$c_r^\dagger=(1,0,0),\quad c_b^\dagger=(0,1,0),\quad c_g^\dagger=(0,0,1)$$

$$M=\frac{g_W^2}{8(M_W c)^2}[\overline{u}(3)\gamma^\mu(1-\gamma^5)(\sin\theta_C)u(1)]\cdot[\overline{u}(4)\gamma_\mu(1-\gamma^5)(\cos\theta_C)v(2)]$$

$n\rightarrow p$ − vertex $\sim g_W^2\rightarrow$
$g_0^2\mathrm{Tr}\,T_u^\dagger T_d=g^2$
$g_0^2=g^2+g'^2$

$$\sum_{j=r,b,g}[\overline{u}(3)(T_u c_j)^\dagger\gamma^\mu(1-\gamma^5)T_s c_j u(1)]=[\overline{u}(3)\gamma^\mu(1-\gamma^5)u(1)]\sum_{j=r,b,g}c_j^\dagger T_u^\dagger T_s c_j$$

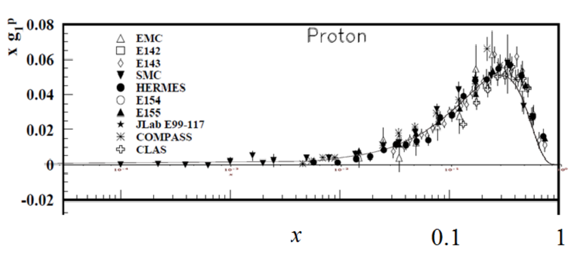




No fitting parameters !

$$f_T(x)dx=\left(\sum_{j=1}^{3}\mathrm{d}R_{u=\exp(\theta iT)}(iT_j)\right)^2 d\theta$$

Proton spin structure function
$$g_1^p(x)=\frac{1}{2}[e_u^2\tfrac{1}{3}f_{T_u}(x)+e_d^2\tfrac{1}{3}f_{T_d}(x)]$$



Unitary configuration and gauge invariance

Colour quark fields $\psi_j(u)=\mathrm{d}R_u(iT_j)=(uiT_j)[R]$ - from left-invariant coordinate fields $\partial_j|_u=uiT_j$
Field Hamiltonian $H=\int\psi^\dagger(-i\hbar c\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta mc^2)\psi d^3x$ (J.J. Sakurai) $\psi^\dagger=(\psi_1^*,\psi_2^*,\psi_3^*)$

Use left-invariance: $\psi'(u')^\dagger\psi'(u')=(u'iT_j[R])^\dagger(u'iT_j[R])=(iT_j[R])^\dagger(u')^\dagger u'(iT_j[R])=\psi(u)^\dagger\psi(u)$
→ invariant mass term for unitary configuration variable
Impose gauge transformation: $\psi\rightarrow\psi'=g(x)\psi$, $g(x)\in SU(3),\partial_\mu\rightarrow D_\mu=\partial_\mu+A_\mu$
$A'_\mu=g(x)A_\mu g^{-1}(x)+g(x)\partial_\mu g^{-1}(x)$, $A_\mu=ig_sA_\mu^k\lambda_k$,
choose $u=g(x)\rightarrow$ gauge invariance in lab from intrinsic left-invariance in configuration space

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