Astrophysical uncertainties on dark matter direct detection results

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- Dependence of the event rate on astrophysical inputs and signals.
- The standard halo model (and halo modelling).
- Observations.
- Numerical simulations.
- Consequences.
- How to handle the uncertainties?

Dependence of the event rate on astrophysical inputs and signals

Differential event rate for elastic scattering:

(assuming spin-independent coupling and f_p=f_n)

$$\frac{\mathrm{d}R}{\mathrm{d}E}(E,t) = \frac{\sigma_p \rho_\chi}{\mu_{p\chi}^2 m_\chi} A^2 F^2(E) \int_{v_{\min}}^{\infty} \frac{f(v,t)}{v} \, \mathrm{d}v$$

$$v_{\min} = \left(\frac{E(m_A + m_\chi)^2}{2 m_A m_\chi^2} \right)^{1/2}$$

f(v,t) is speed distribution in lab frame

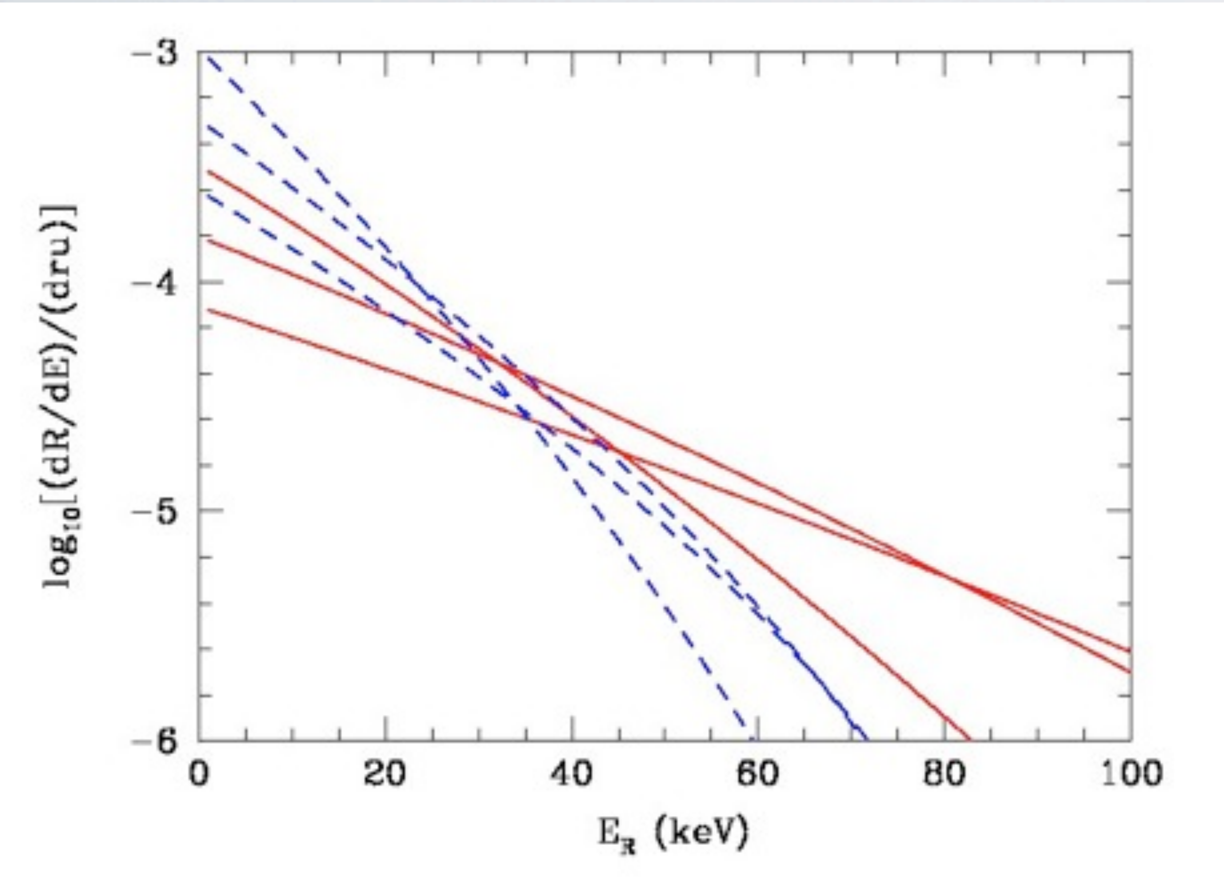
Realisation that uncertainties in f(v) will affect signals goes right the way back to the early direct detection papers in the 1980s (e.g. Drukier, Freese & Spergel).

Assuming (for now) the standard halo model with an isotropic gaussian speed distribution:

$$f(\mathbf{v}) \propto \exp\left(-\frac{|\mathbf{v}|^2}{2\sigma^2}\right) \qquad \sigma = \frac{v_c}{\sqrt{2}}$$

i) energy spectra

Lewin & Smith



Differential event rate:
Ge and Xe m_χ = 50, 100, 200 GeV

Energy spectrum has characteristic energy which depends on the WIMP mass, target mass and velocity dispersion:

$$E_R = \frac{2\mu_{A\chi}^2 v_c^2}{m_A}$$

$$\propto m_\chi^2 \qquad m_\chi \ll m_A$$

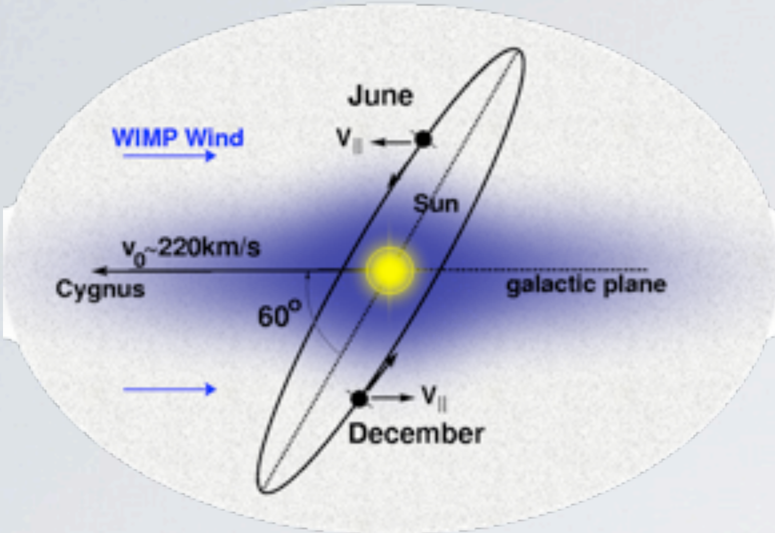
$$\sim \text{const} \qquad m_\chi \gg m_A$$

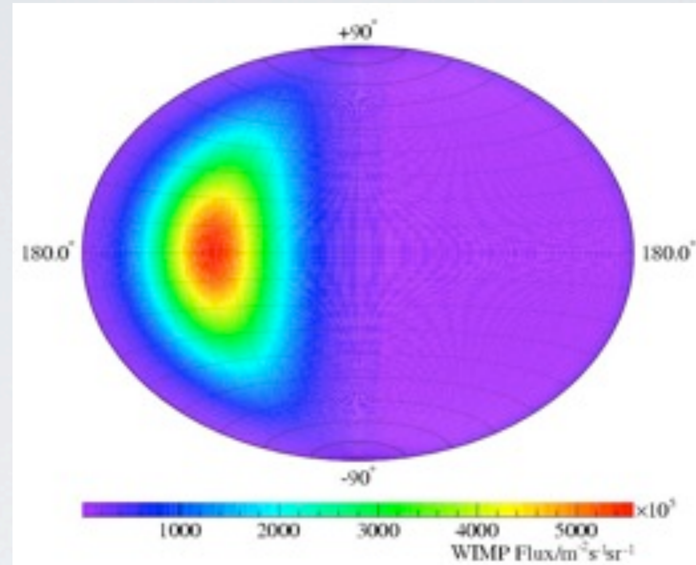
Check WIMP origin of signal via consistency of energy spectra for different target nuclei.

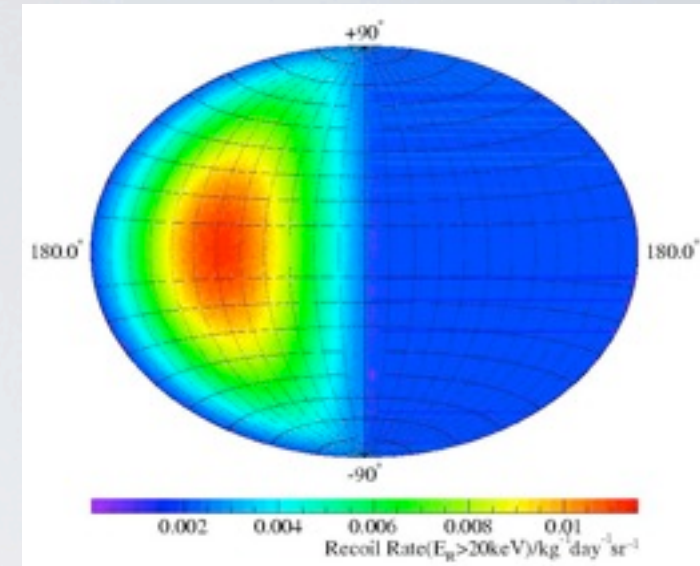
Measure the WIMP mass from a energy spectrum in a single expt. (if you know the WIMP velocity dispersion and provided WIMP mass is not too large or too small).

i) directional dependence

Spergel







Sheffield DM group

WIMP flux

Recoil rate
(above 20 keV, 100 GeV WIMP)

Recoil rate largest in direction opposite to direction of Solar motion.

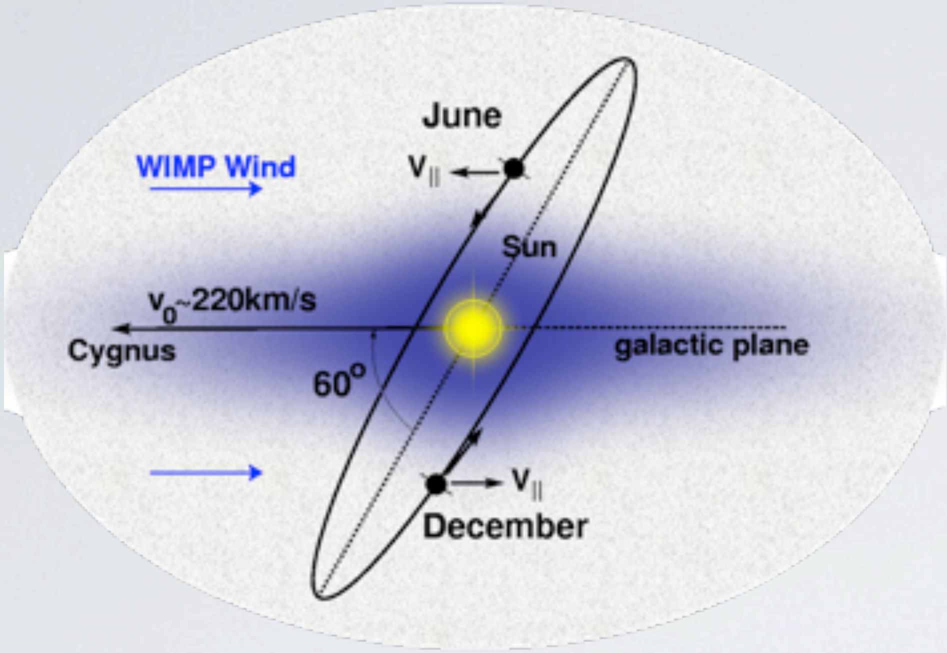
Ratio of rates in rear and forward directions is large.

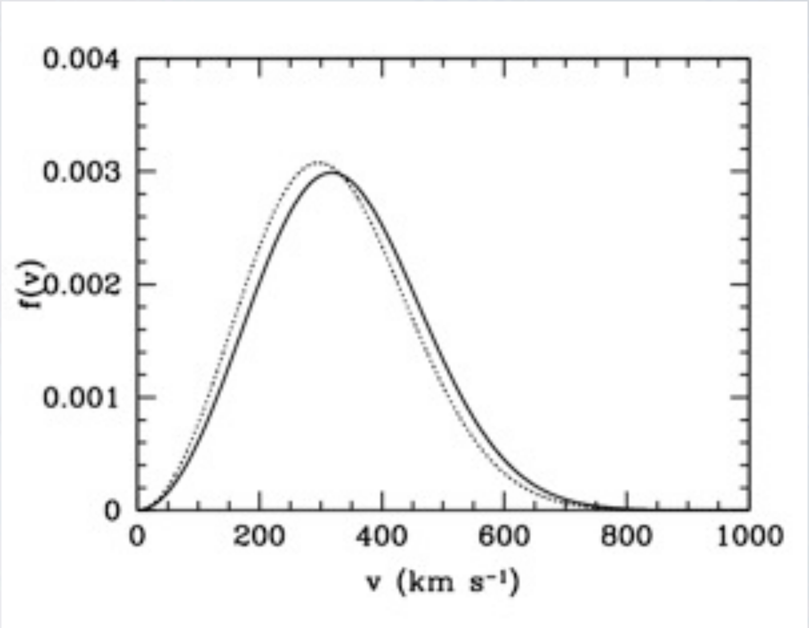
Potentially only O(10) events required to detect anisotropy [Copi & Krauss; Morgan, Green & Spooner] and O(30) to confirm median recoil direction [Green & Morgan; Billard et al.].

But need a detector which can measure recoil directions (e.g. DMTPC, DRIFT, MIMAC, NEWAGE).

iii) annual modulation of event rate

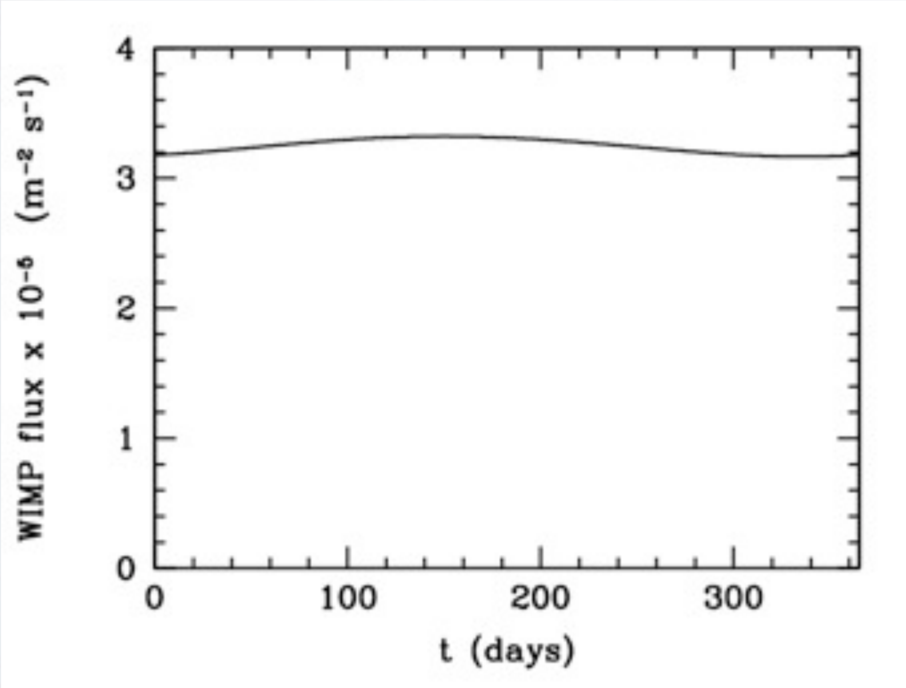
Drukier, Freese & Spergel



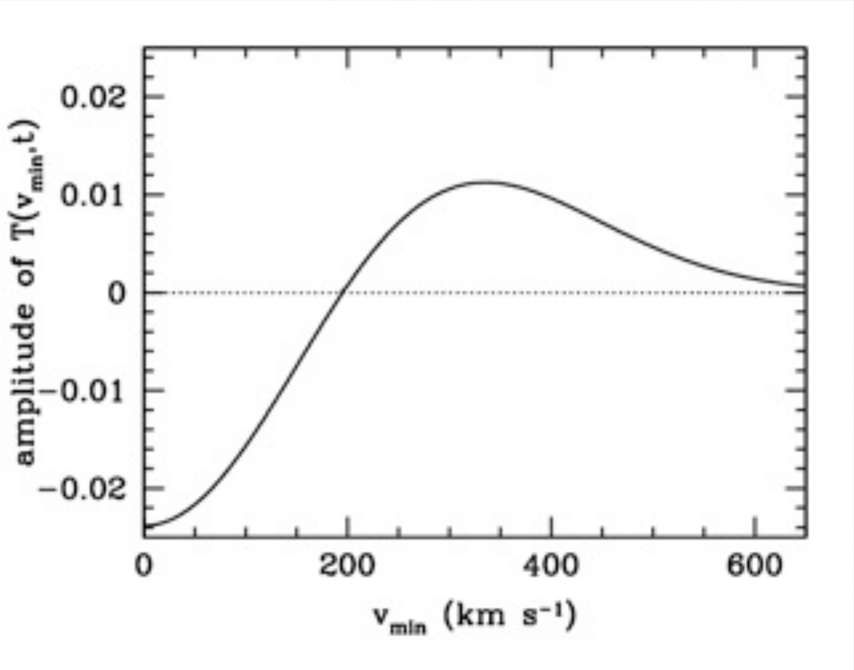


Maxwellian speed dist.

detector rest frame (summer and winter)



total WIMP flux



modulation amplitude

Signal <O(10%)

The standard halo model (and halo modelling)

Standard halo model:

isothermal sphere with isotropic Maxwellian velocity distribution in Galactic rest-frame:

$$f(\mathbf{v}) \propto \exp\left(-\frac{|\mathbf{v}|^2}{2\sigma^2}\right)$$

Solution of collisionless Boltzmann equation (assumes phase space distribution function has reached a steady state) for $\rho \propto r^{-2}$, assuming isotropy.

Formally extends to infinity, therefore velocity distribution is truncated by hand at escape velocity. (In a 'complete' model, c.f. Chaudhury et al. King/lowered isothermal sphere, manual truncation would not be required.)

standard parameter values:

local density		$\rho = 0.3\,\mathrm{GeV\,cm^{-3}}$	
local circular speed		$v_{\mathrm{c}} = 220\,\mathrm{km\,s^{-1}}$	
local escape speed	traditionally	$v_{\mathrm{esc}} = 650\,\mathrm{km\,s^{-1}}$	
	more recently	$v_{\mathrm{esc}} = 554\,\mathrm{km\,s^{-1}}$	[RAVE]

Halo modelling:

Phase space distribution function: $f(\mathbf{x}, \mathbf{v}, t)$

Steady-state phase space distribution of a collection of collisionless particles is given by the solution of the collisionless Boltzmann equation:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = 0$$

In Cartesian co-ordinates:

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{x}} + \frac{\partial \Phi}{\partial \mathbf{x}}.\frac{\partial f}{\partial \mathbf{v}} = 0$$

For a self-consistent system (where density distribution generates potential):

$$\nabla^2 \Phi = 4\pi G \int f \mathrm{d}^3\mathbf{v}$$

For spherical isotropic systems there's a unique relationship between **ρ**(r) and f (v) given by Eddington's equation:

$$f(\Phi) = \frac{1}{\sqrt{8}\pi^2}\left[\int_0^{\Phi} \frac{\mathrm{d}\Phi'}{\sqrt{\Phi - \Phi'}} \frac{\mathrm{d}^2\rho}{\mathrm{d}\Phi'^2} + \frac{1}{\sqrt{\Phi}}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\Phi}\right)_{\Phi=0}\right]$$

but if the system is triaxial and/or anisotropic this is not the case.

Multiplying the collisionless Boltzmann equation by the velocity components and integrating produces the Jeans equations:

in Cartesian co-ordinates: $$\frac{\partial(\rho\bar{v_j})}{\partial t}+\frac{\partial(\rho\overline{v_i v_j})}{\partial x_i}+\rho\frac{\partial\Phi}{\partial x_j}=0$$

Get three equations for six unknowns: $\overline{v_1^2},\ \overline{v_2^2},\ \overline{v_3^2},\ \overline{v_1 v_2},\ \overline{v_2 v_3},\ \overline{v_1 v_3}$

Therefore need to make assumptions about alignment of velocity ellipsoid (e.g. choose co-ordinates such that $\overline{v_i v_j}=0 \quad \text{if} \quad \mathrm{i}\neq\mathrm{j}$).

Often then assume that velocity dispersion is a multivariate gaussian in these co-ordinates:

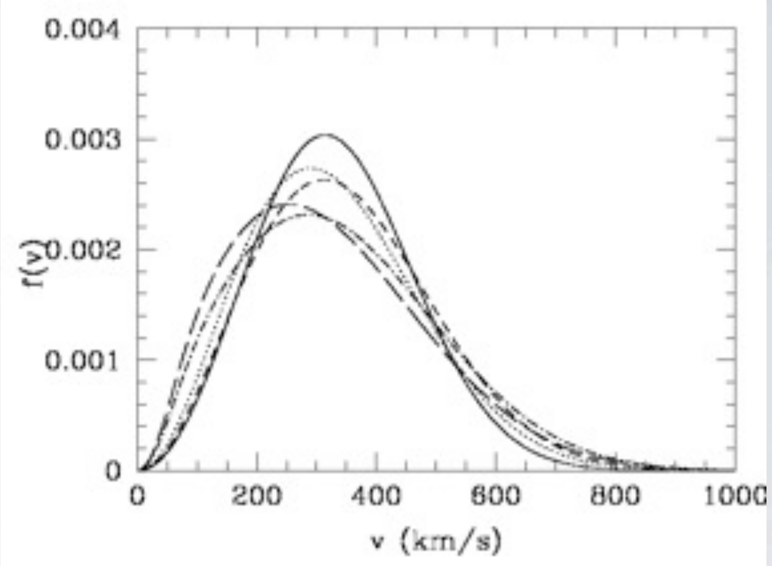
$$f(\mathbf{v})\propto\exp\left(-\frac{v_1^2}{2\sigma_1^2}-\frac{v_2^2}{2\sigma_2^2}-\frac{v_3^2}{2\sigma_3^2}\right)$$

examples:

logarithmic ellipsoidal model

Evans, Carollo, de Zeeuw

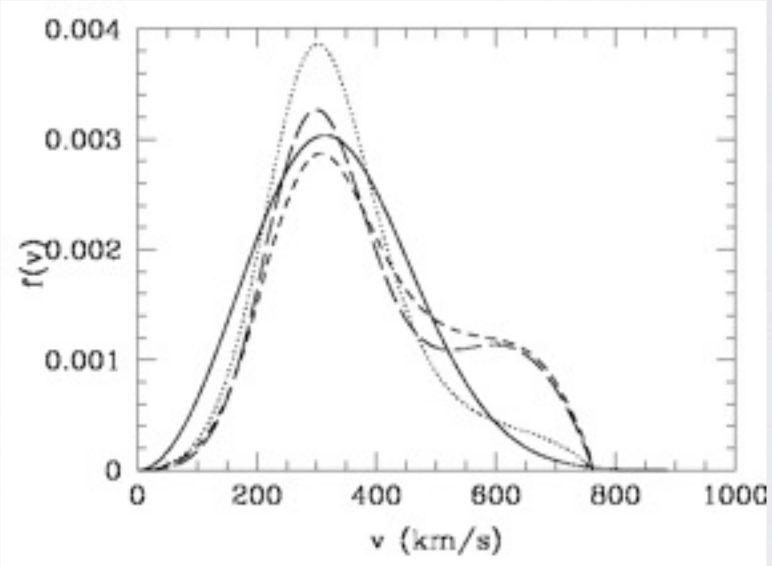
simplest triaxial generalisation of isothermal sphere,
f(v) multi-variate gaussian in cylindrical polars,
velocity dispersions depend on axis ratios and value of (constant) velocity anisotropy.



Osipkov-Merritt

spherically symmetric,
radially dependent velocity anisotropy:

$$\beta(r) = 1 - \frac{\overline{v_{\rm t}^2}}{2\overline{v_{\rm r}^2}} \qquad\qquad \beta(r) = \frac{r^2}{r^2 + r_a^2}$$



These models (and the parameter values I've chosen) are somewhat out-dated given more recent results from numerical simulations, but they illustrate the point that the relationship between the 'observed' properties of DM halos and f(v) is non-trivial.

Observations

Local density:

General approach: use multiple data sets (rotation curve, velocity dispersions of halo stars, local surface mass density, total mass...) and model for the MW (luminous components and halo).

salucci errors:
slope of v_c(r) and ratio
of R_0 and scale length
of disk

Widrow et al. using spherical halo mode $\rho_0 = (0.3 \pm 0.05)\,\mathrm{GeV\,cm^{-3}}$

Catena & Ullio using NFW & Einasto pro $\rho_0 = (0.39 \pm 0.03)\,\mathrm{GeV\,cm^{-3}}$

Weber and de Boer using range of halo density profiles: $\rho_0 = (0.2 - 0.4)\,\mathrm{GeV\,cm^{-3}}$

Salucci et al.'model independent' method (eqn of centrifugal eqm): $\rho_0 = (0.43 \pm 0.11 \pm 0.10)\,\mathrm{GeV\,cm^{-3}}$

Garbari et al.'minimal assumption' method (solve Jeans-Poisson eqns): $\rho_0 = 0.11^{+0.34}_{-0.27}\,\mathrm{GeV\,cm^{-3}}$

Pato et al. DM density in stellar disc of simulated halos is ~ 20% larger than the shell average determined by observations.

Summary: recent determinations have ~10% statistical errors, but systematic uncertainties from modelling are still significantly larger.

Local circular speed:

IAU/Kerr & Linden-Bell compilation of measurements: $v_{\rm c} = (220 \pm 20)\,{\rm km\,s^{-1}}$

Reid et al. trigonometric parallaxes and proper motions of masers : $v_{\rm c} = (254 \pm 16)\,{\rm km\,s^{-1}}$

Bovy et al. masers (including phase modelling), Galactic center analyses & GD-1 stellar stream, assuming flat rotation curve: $v_{\rm c} = (236 \pm 11)\,{\rm km\,s^{-1}}$

McMillan & Binney masers, revised value of component of Sun's proper motion in direction of rotation, and allowing non-flat rotation curve: $v_{\rm c} = (200 - 280)\,{\rm km\,s^{-1}}$

Modelling uncertainties, larger than statistical uncertainties here too.

n.b. For the standard halo there's a one-to-one relationship between the circular speed and velocity dispersion, $\sqrt{2}\sigma = v_{\rm c}$, but in general the relationship depends on the density profile and velocity anisotropy:

$$\frac{1}{\rho}\frac{{\rm d}(\rho\sigma_r^2)}{{\rm d}r} + 2\frac{\beta\sigma_r^2}{r} = -\frac{v_c^2}{r}$$

Also for non-standard halos peak velocity, v_0, isn't equal to circular speed.

Values of v_c and ρ_0 are not independent, observations are sensitive to v_c/R_0 e.g. McMillan & Binney; McCabe

Local escape speed:

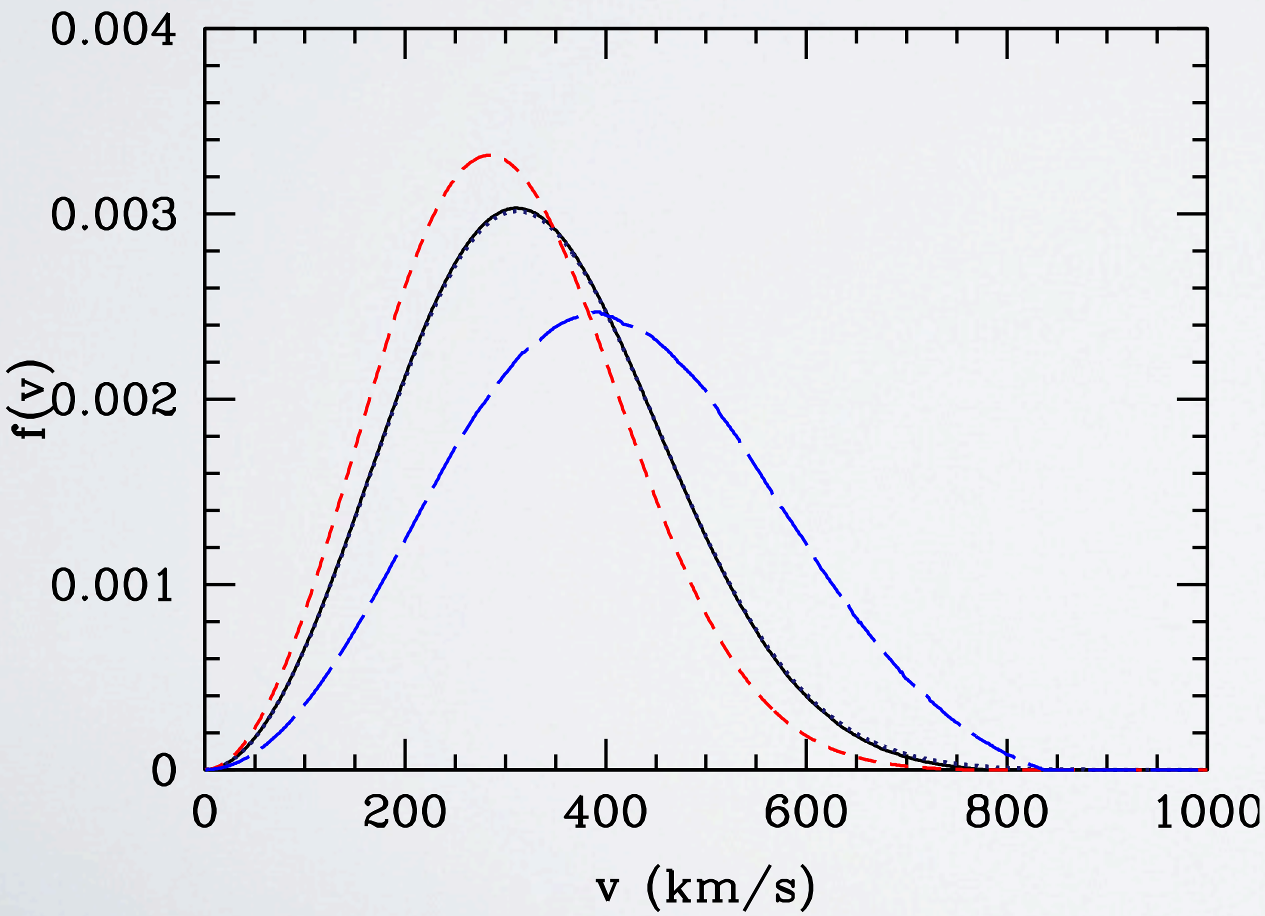
Smith et al: high velocity stars from the RAVE survey,

assume $f(|\mathbf{v}|) \propto (v_{\rm esc} - |\mathbf{v}|)^k$

with k in range 2.7 to 4.7 (motivated by numerical simulations):

$$498\,{\rm km\,s^{-1}} < v_{\rm esc} < 608\,{\rm km\,s^{-1}}$$

median likelihood: $v_{\rm esc} = 544\,{\rm km\,s^{-1}}$



Normalised average speed distribution in the lab, SH with varying dispersion

v_c = 220 km/s, v_{esc}=554 km/s

200 km/s

280 km/s

220 km/s, v_{esc}=650 km/s

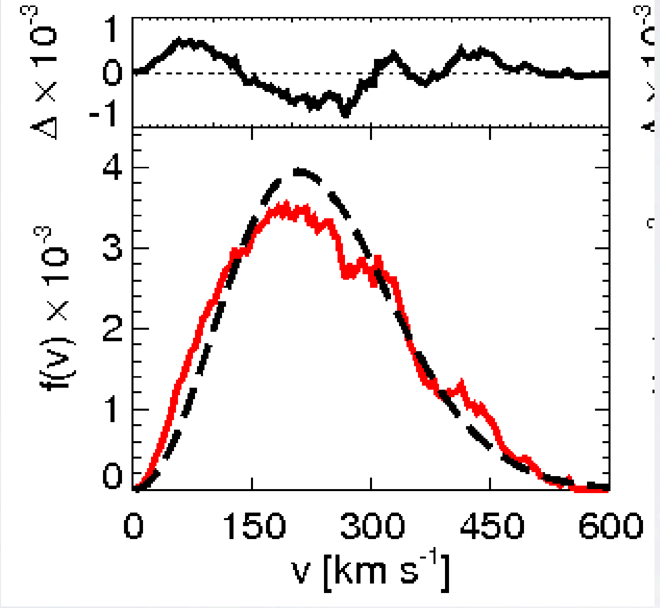
Simulations

Systematic deviations from multi-variate gaussian: more low speed particles, peak of distribution lower/flatter.

Stocastic high v features. Broad bumps which vary from halo to halo, (reflect formation history?). Also narrow spikes in some locations.

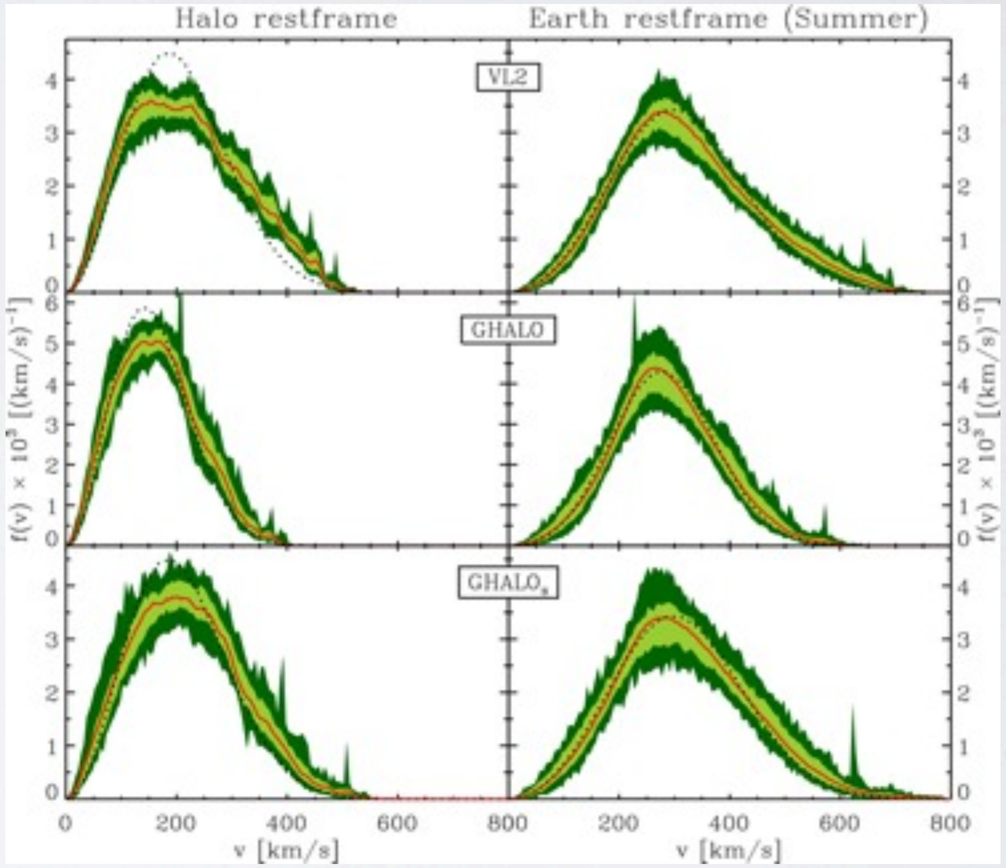
Deviations less pronounced in lab frame than Galactic rest frame.

Vogelsberger et al.



red lines: simulation data,
black lines: best fit multi-variate Gaussian

Kuhlen et al.



Hansen et al., Fairbairn & Schwetz and Kuhlen et al. have found similar results.

f(v) fit fairly well by modified Maxwellian or Tsallis distribution.

$$f(v_{\rm r}) = \frac{1}{N_{\rm r}} \exp\left[-\left(\frac{v_{\rm r}}{\bar{v}_{\rm r}}\right)^{\alpha_{\rm r}}\right] \qquad f(v_{\rm t}) = \frac{v_{\rm t}}{N_{\rm r}} \exp\left[-\left(\frac{v_{\rm t}}{\bar{v}_{\rm t}}\right)^{\alpha_{\rm t}}\right]$$

Lisanti et al.

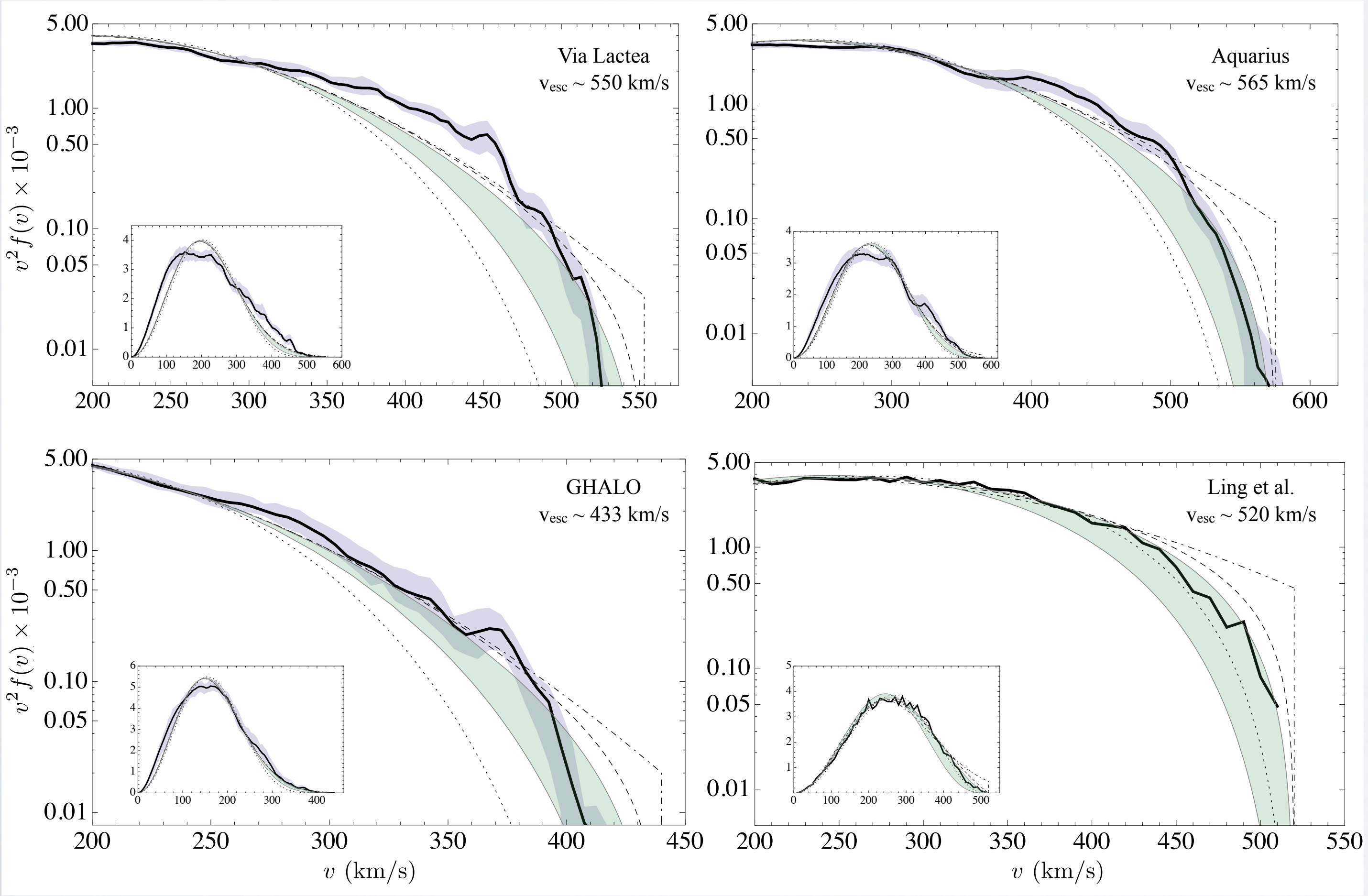
For a double power-law density distribution (NFW has $(\alpha, \gamma) = (1, 3)$)

$$\rho(r) = \frac{\rho_s}{(r/r_s)^\alpha (1 + (r/r_s))^{\gamma - \alpha)}}$$

the velocity distribution

$$f(v) \propto \left[\exp\left(\frac{v_{esc}^2 - v^2}{k v_0^2}\right) - 1\right]^k \Theta(v_{esc} - v)$$

is a good approx to numerical solutions of the Eddington equation (including a bulge and disk) and provides a better fit to the high speed tail of f(v) from simulations:



_._._._._._ standard Maxwellian

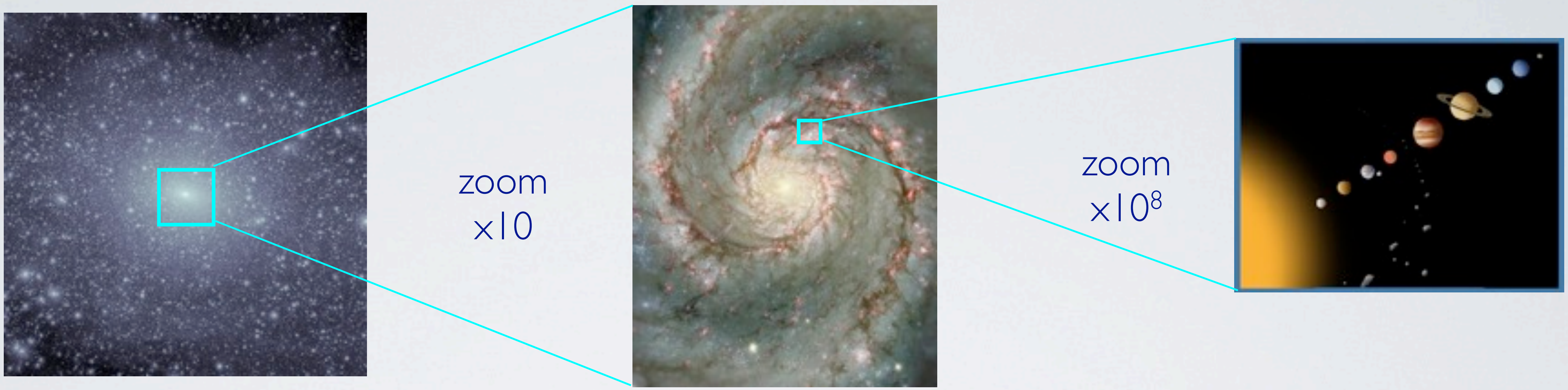
_ _ _ _ _ King

.... Tsallis

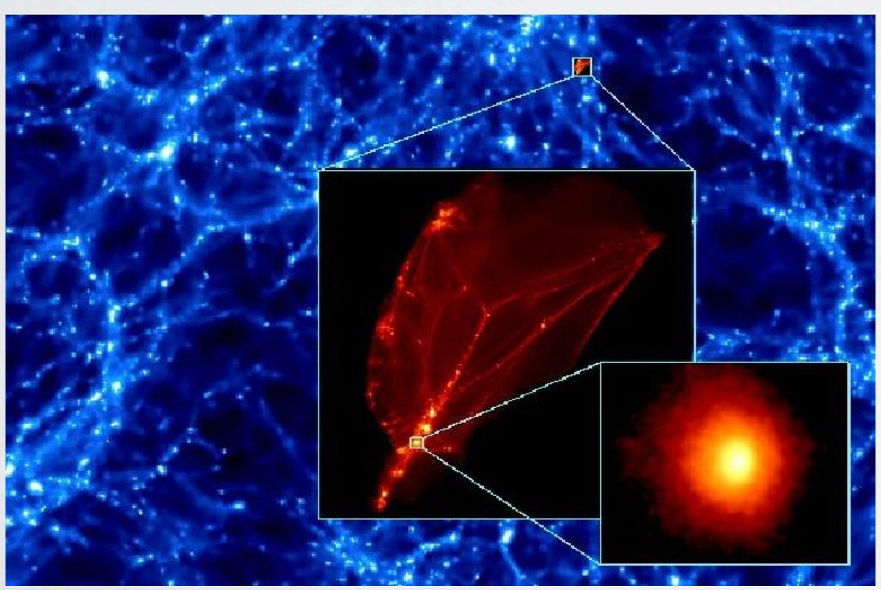
1.5 < k < 3.5 (corresponding to outer slope of density profile 3 < γ < 5)

Caveats:

a) scales resolved by simulations are many orders of magnitude larger than those probed by direct detection experiments



~300 kpc ~30 kpc ~0.3 mpc



microhalo simulation
[Diemand, Moore & Stadel]

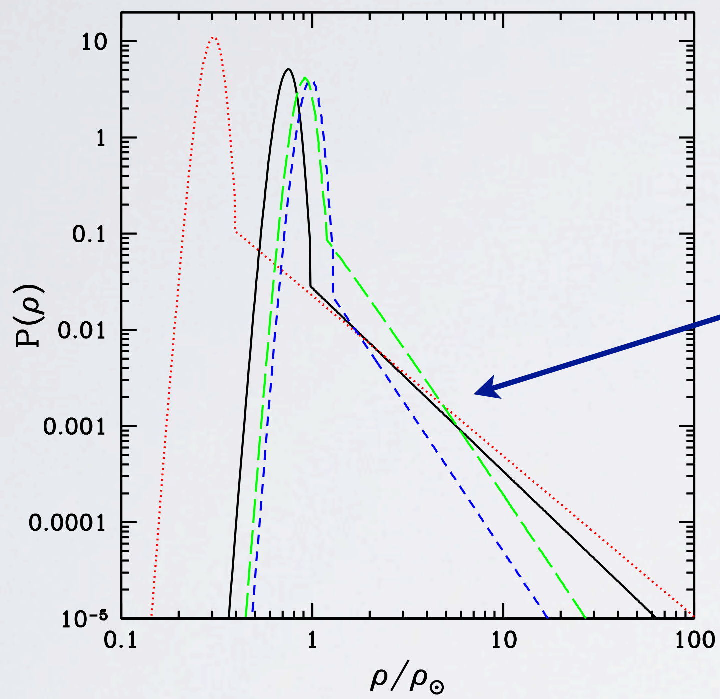
The first WIMP microhalos to form have
$M \sim 10^{-6} M_\odot$ [Green, Hofmann & Schwarz]

c.f. resolution of best Milky way simulations:

$$M \sim 10^5 M_\odot$$

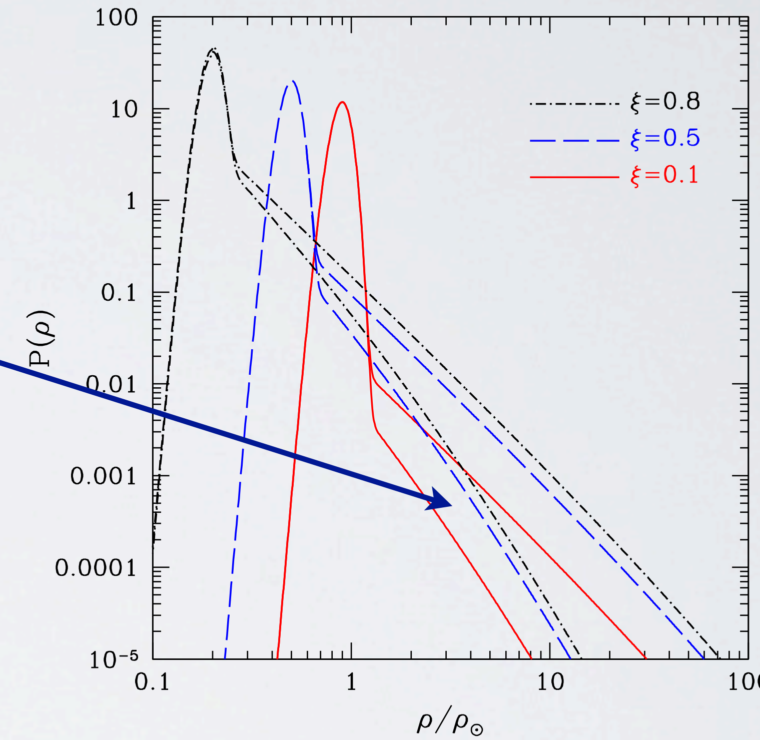
variations in ultra-local DM density?

Kamionkowski & Koushiappas: calculate probability distribution for local dark matter density, taking into account sub-halos.



analytic calc
(some frac of small dense halos survive)

power-law tail
from sub-halos



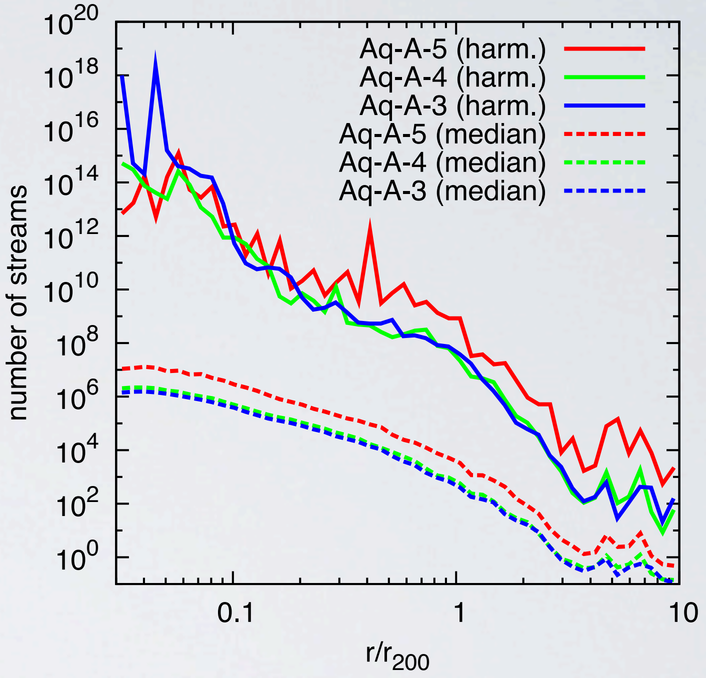
simulation inspired calc

fine structure in ultra-local DM velocity distribution?

Vogelsberger & White:

Follow the fine-grained phase-space distribution, in particular in streams, in Aquarius simulations of Milky Way like halos.

Find ultra-local DM distribution consists of a huge number of streams.



number of streams as a function of radius
calculated using harmonic mean/median stream density

Schneider, Krauss & Moore: reach similar conclusions from simulations of the evolution of microhalos, including tidal disruption and heating from encounters with stars.

But some features may be present Afshordi, Mohayee & Bertschinger; Fantin, Merrifield & Green; Lisanti & Spergel.

ii) effect of baryons on DM speed distribution?

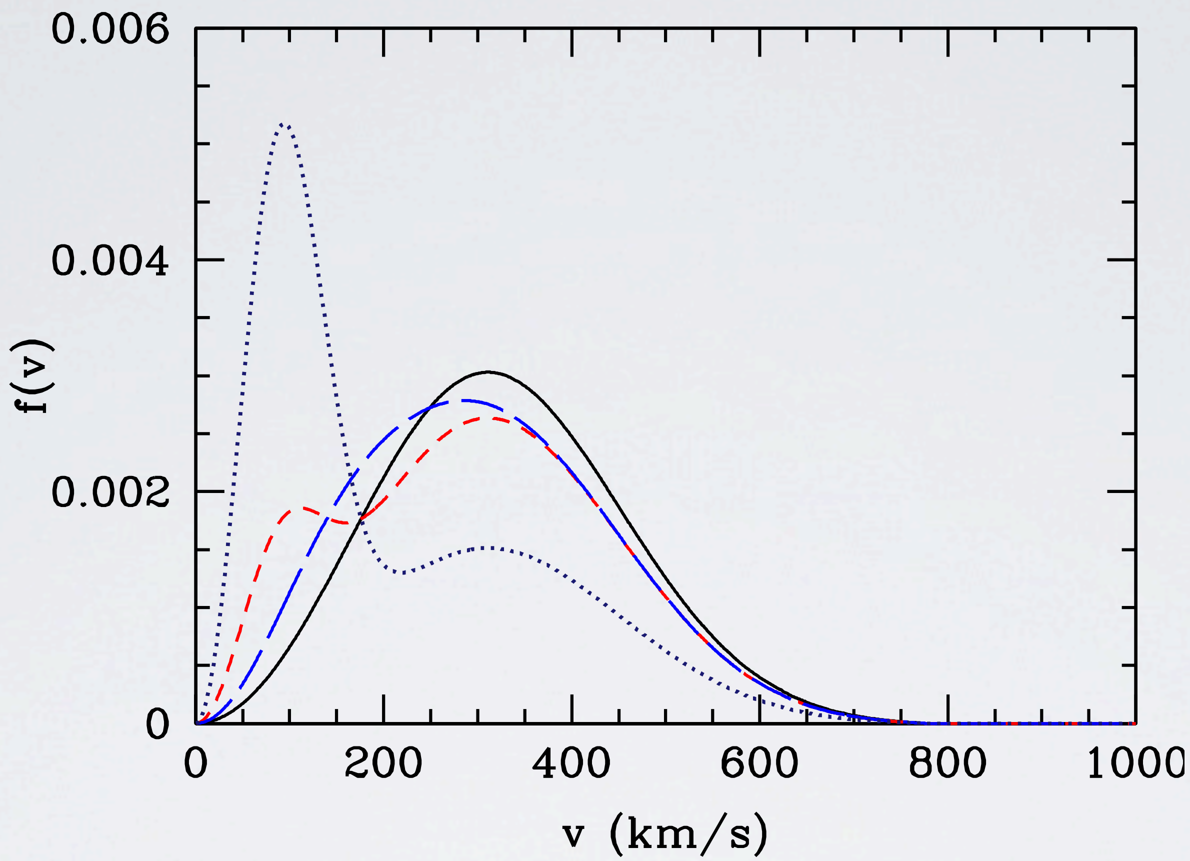
Sub-halos merging at $z<1$ preferentially dragged towards disc, where they're destroyed leading to the formation of a co-rotating dark disc. Read et al., Bruch et al., Ling et al.

n.b. producing simulated halos which match the properties of the Milky Way is an outstanding(?) challenge.

Detailed properties (& existence?) of dark disc are very uncertain.

Purcell, Bullock and Kaplinghat argue that to be consistent with the observed properties of thick disc, MW's merger history must be quiescent compared with typical ΛCDM merger histories, hence the DD density must be relatively low.

Bidin et al. measure surface density with 2-4 kpc of Galactic plane (using kinematics of thick disc stars), consistent with visible mass.



SH

SH + high density $\rho_D = \rho_H$, low dispersion DD

SH + low density $\rho_D = 0.15\rho_H$, low dispersion DD

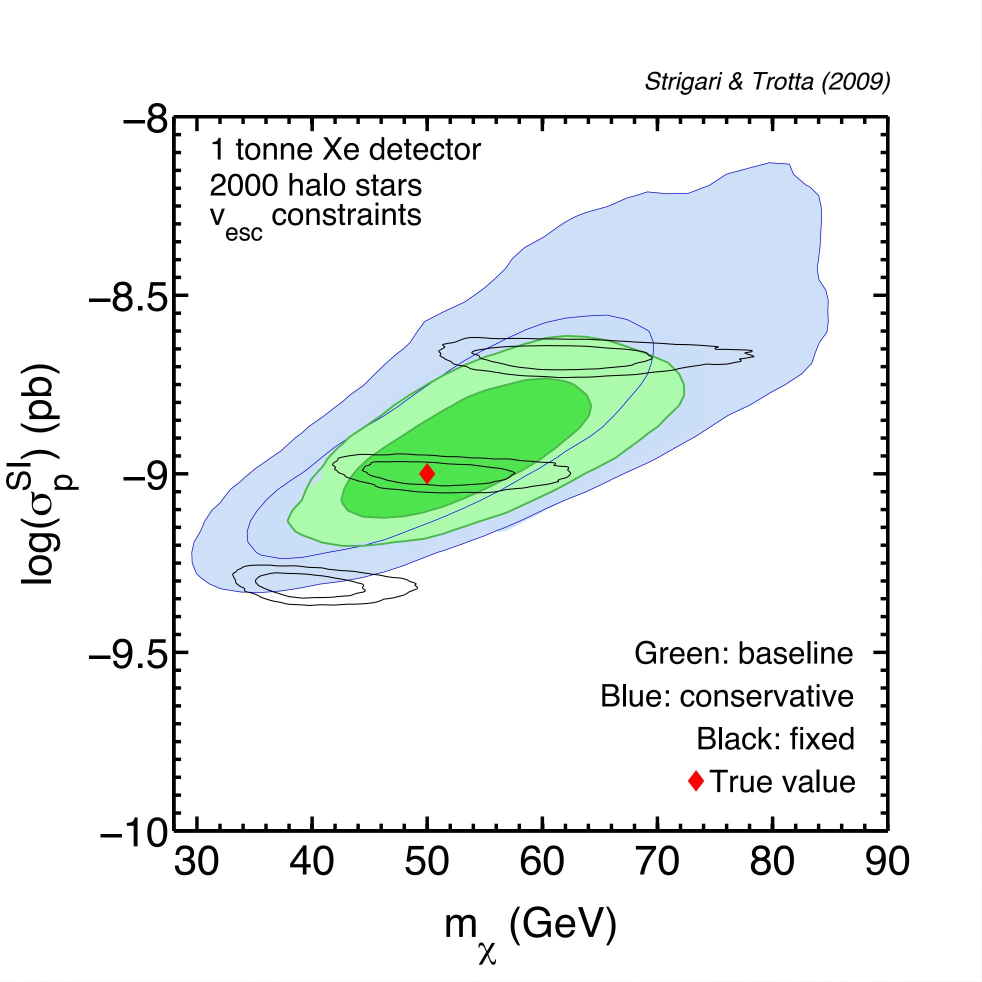
SH + low density, high dispersion DD

Consequences

Density:

Event rate proportional to product of $\boldsymbol{\sigma}$ and $\boldsymbol{\rho}$, therefore uncertainties in $\boldsymbol{\rho}$ translate directly into uncertainties in $\boldsymbol{\sigma}$, same for all DD experiments (but affects comparisons with e.g. collider constraints on $\boldsymbol{\sigma}$).

Strigari & Trotta uncertainty leads to bias in determination of WIMP mass:



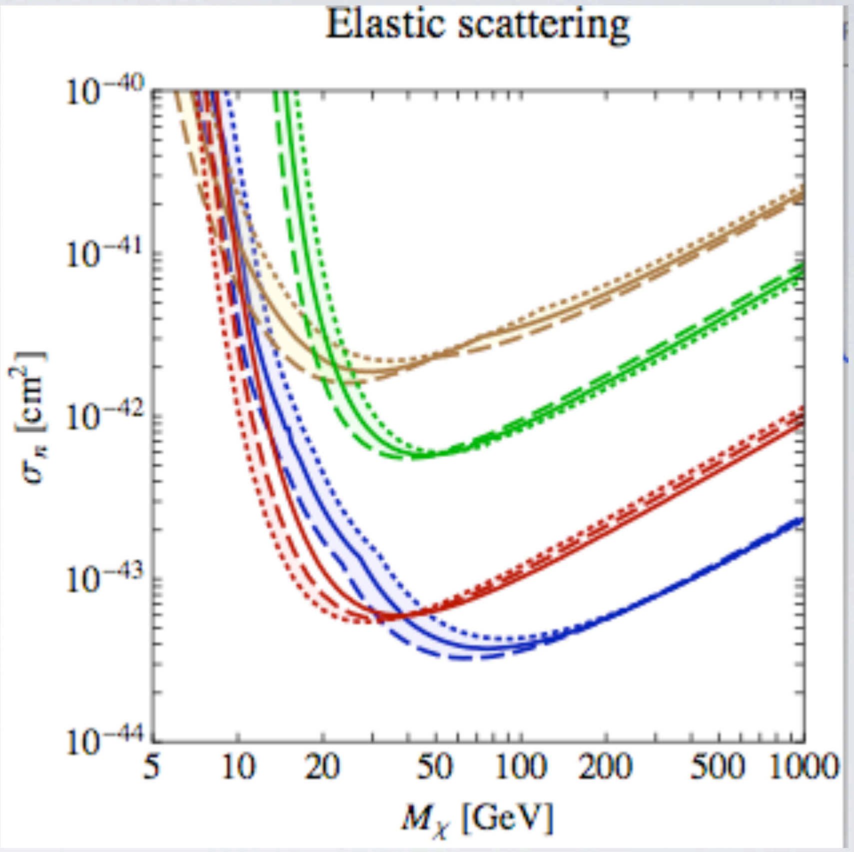
Velocity dispersion:

Shifts exclusion limits,
similar, but not identical, effect for all experiments

McCabe

....	v_0=195 km/s
———	v_0=220 km/s
- - - -	v_0=255 km/s

CDMSII Si, CDMSII Ge
CRESST, ZENON 10

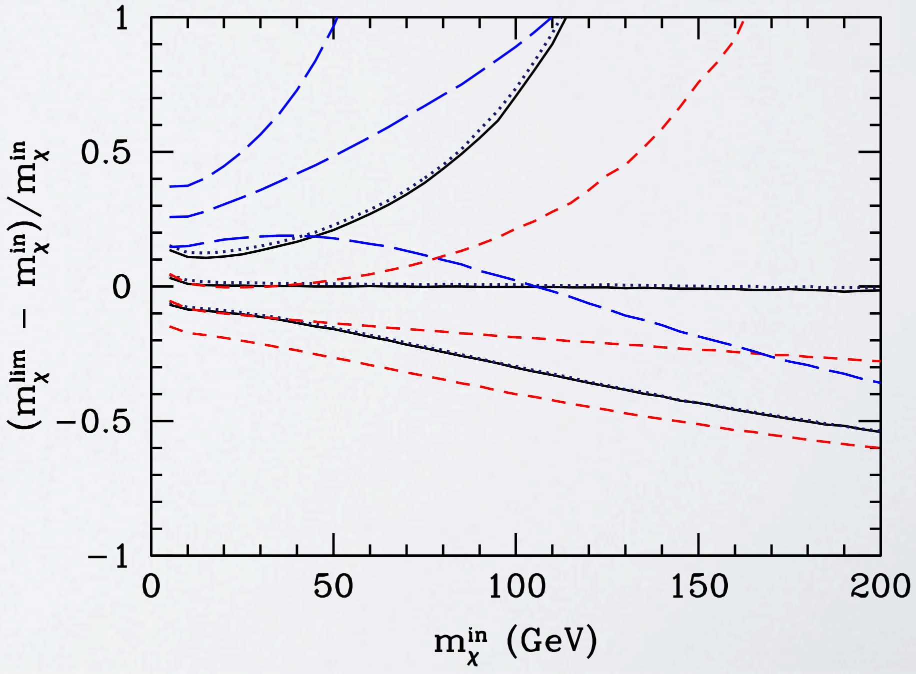


Bias in future WIMP mass determination:

$$\frac{\Delta m_\chi}{m_\chi} = [1 + (m_\chi/m_{\rm A})]\frac{\Delta v_{\rm c}}{v_{\rm c}}$$

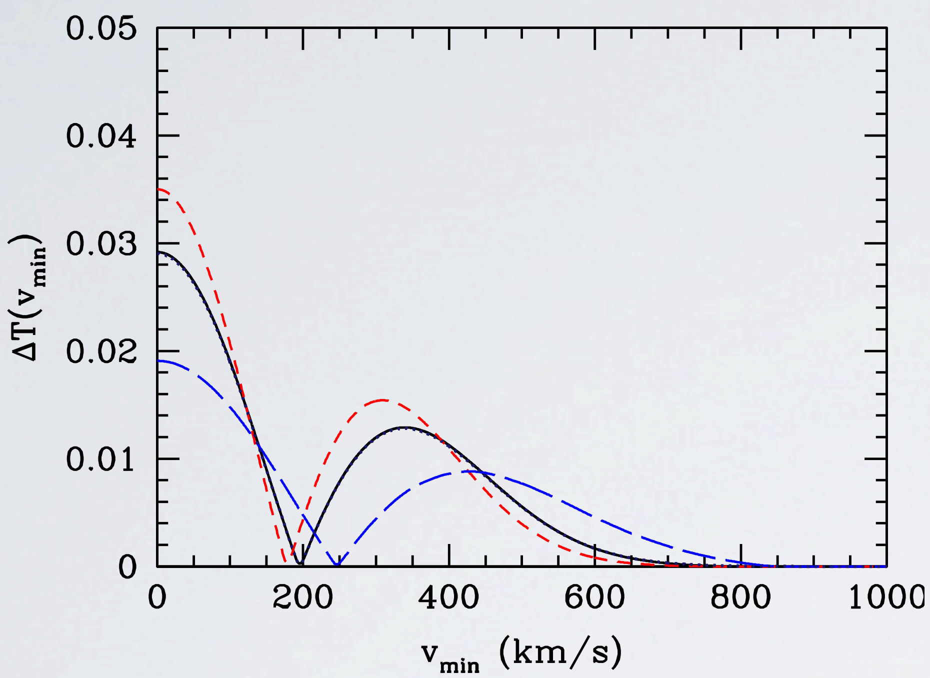
———	v_c = 220 km/s
- - - - - - - -	200 km/s
— — — —	280 km/s

fractional mass limits from a simulated ideal Ge experiment, σ = 10^{-8} pb $\mathcal{E} = 3 \times 10^4\,{\rm kg\,day}$



Significant change in amplitude of annual modulation:

(phase of annual modulation only changes by a few days in this case)



$$T(v_{\min}, t) = (220\,\mathrm{km\,s^{-1}}) \int_{v_{\min}}^{\infty} \frac{f(v,t)}{v}\,\mathrm{d}v$$

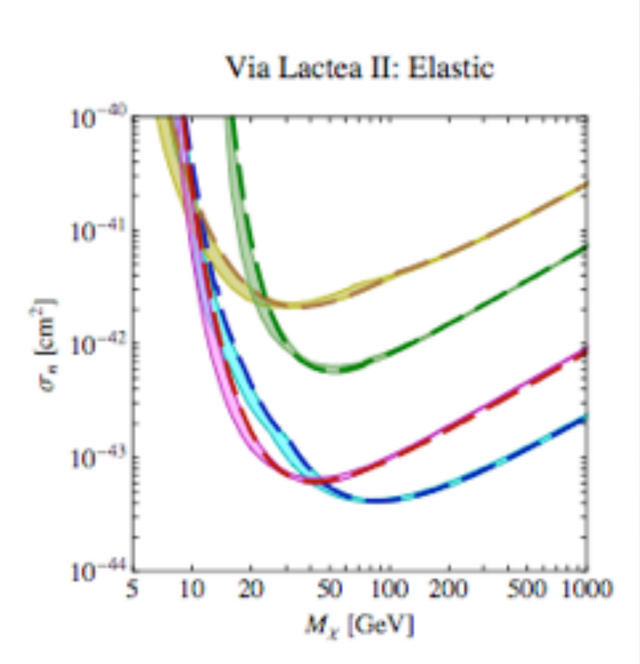
Shape of velocity distribution

Differential event rate is proportional to integral over speed distribution so exclusions limits are relatively insensitive to exact shape of velocity distribution:

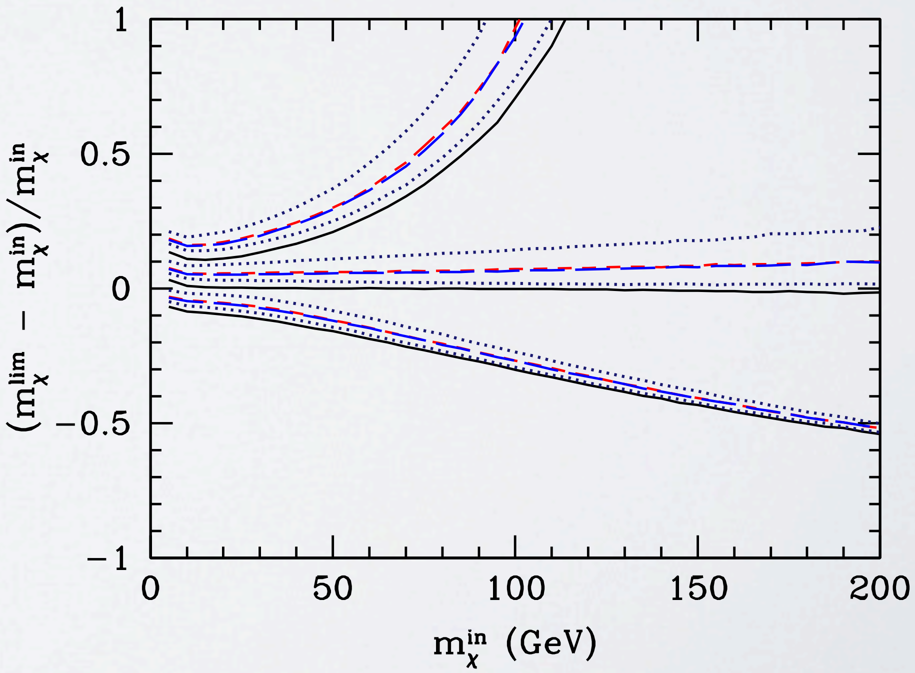
(smallish) change in shape/stochastic uncertainty in exclusion limits

McCabe

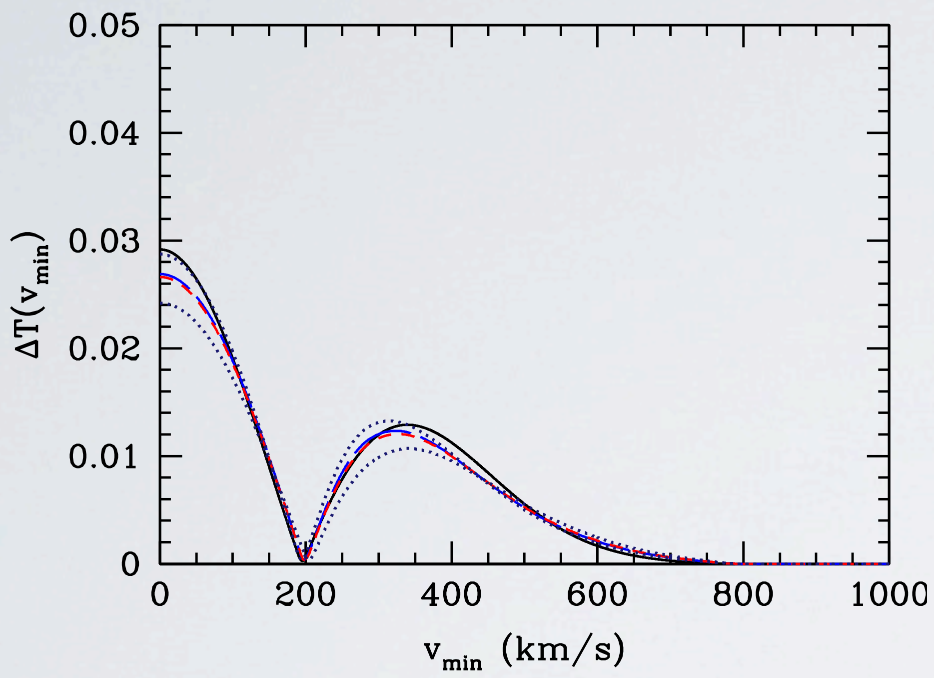
CDMSII Si, CDMSII Ge
CRESST, ZENON 10



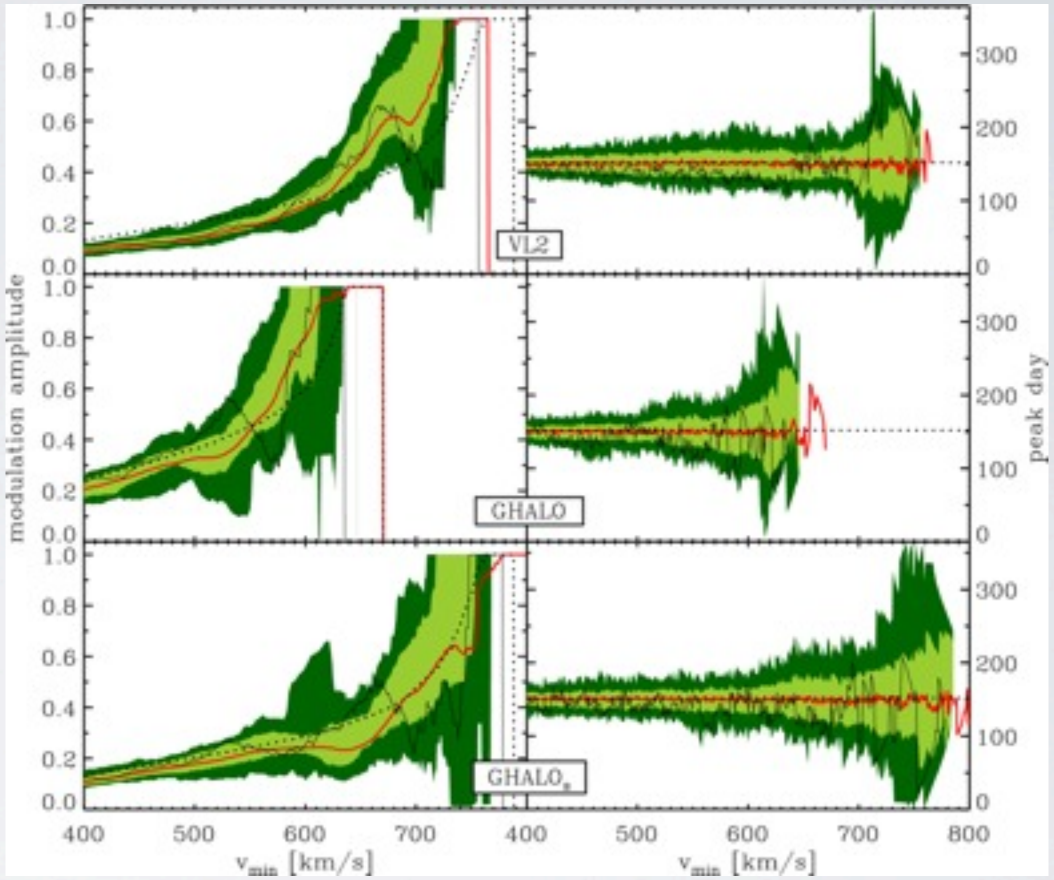
2-5% bias in future WIMP mass determination



More significant change in annual modulation, in particular from stochastic high v features (n.b. absolute size of signal and modulation small for large v_{min})



fractional

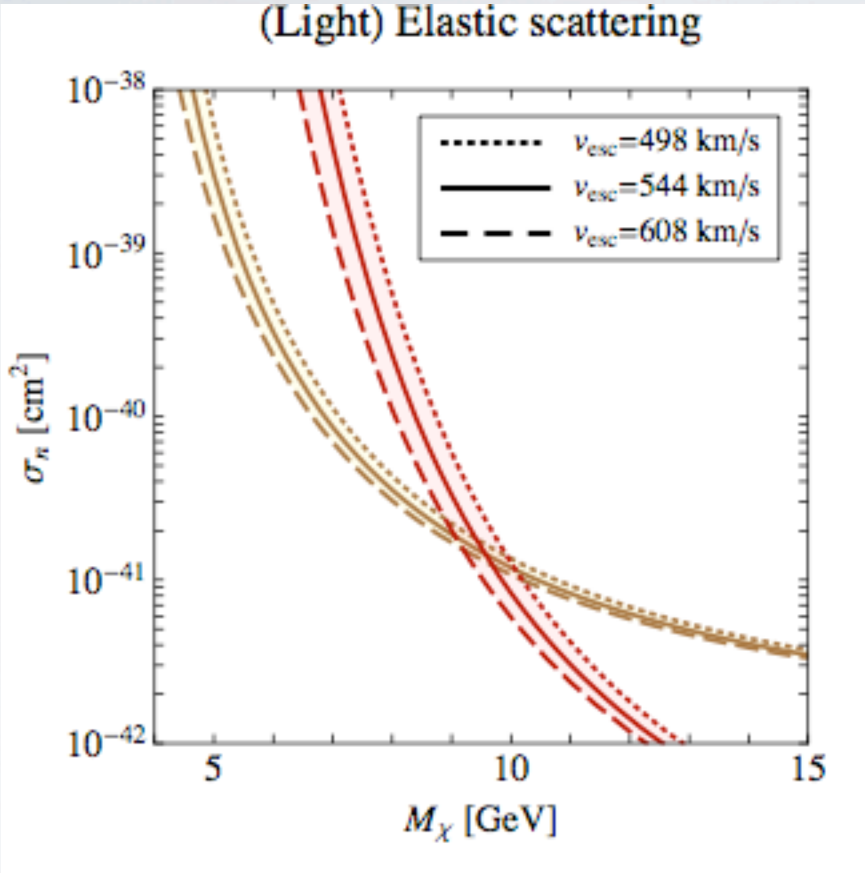


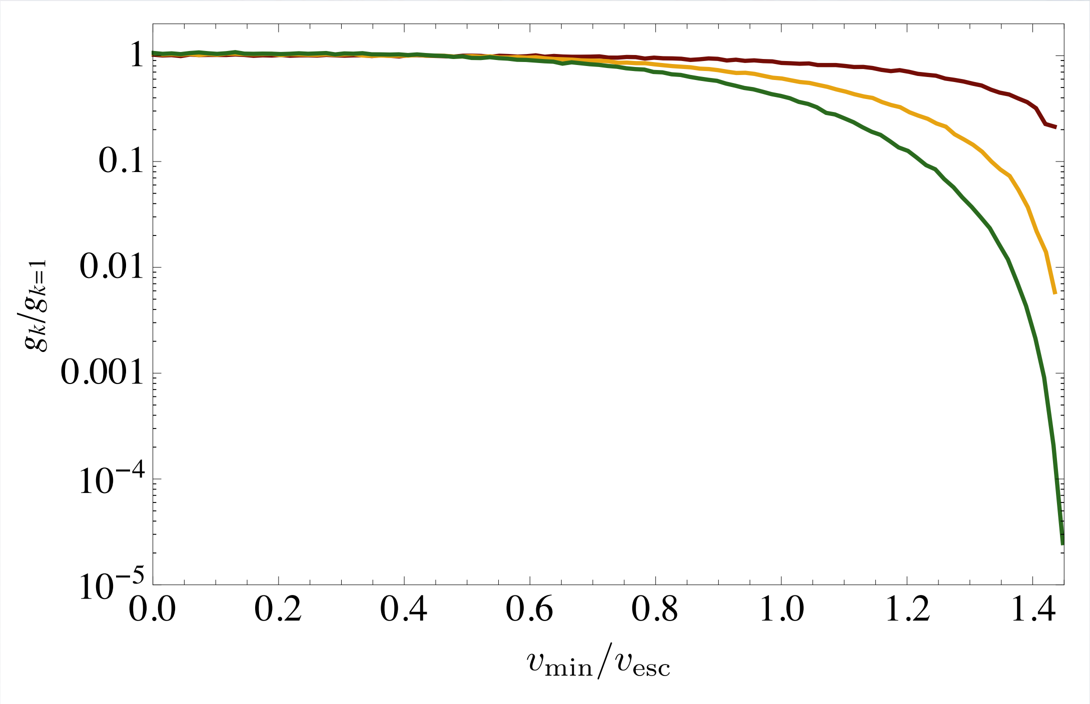
Kuhlen et al.

Rear-front directional asymmetry is robust, but [Kuhlen et al.] peak/median recoil direction of high energy recoils may deviate somewhat from direction of solar motion.

Escape speed & shape of high v tail

Can have significant effect on event rates/exclusion limits for light WIMPs:





McCabe

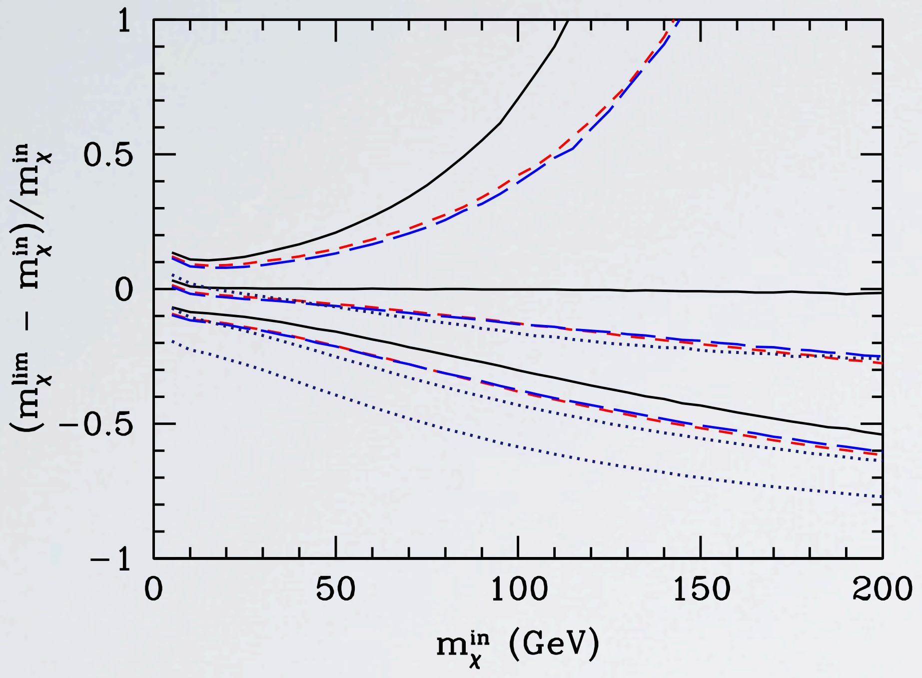
effect of varying v_{esc} on exclusion limits from CDMSII Si and ZENON 10

Lisanti et al.

fractional change in differential event rate compared to lowered isothermal sphere for k= 1.5, 2.5, 3.5.

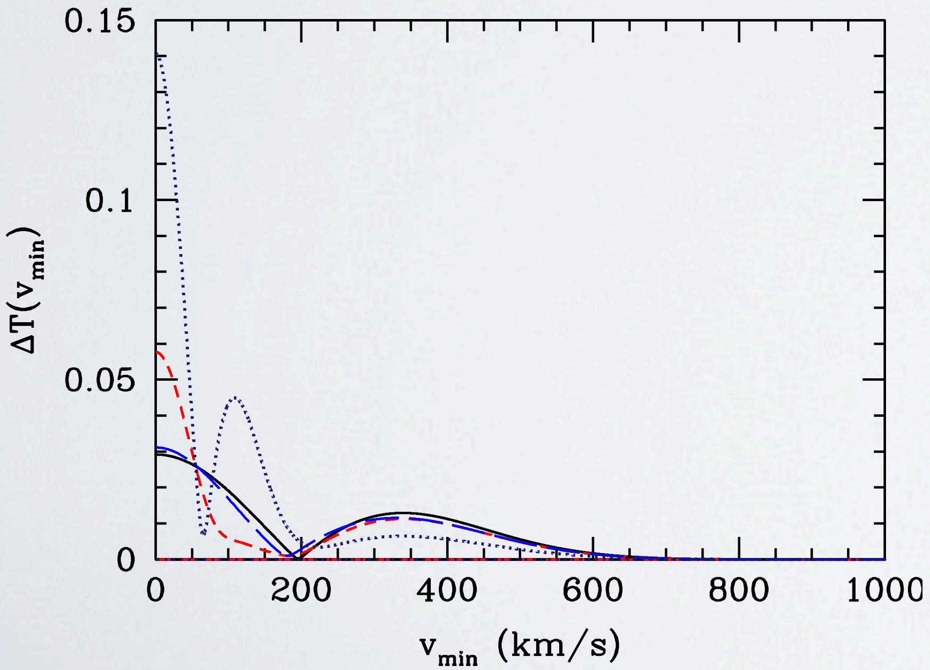
Dark disc

Could have a significant effect on mass determination and annual modulation, if density sufficiently high and/or velocity dispersion low.

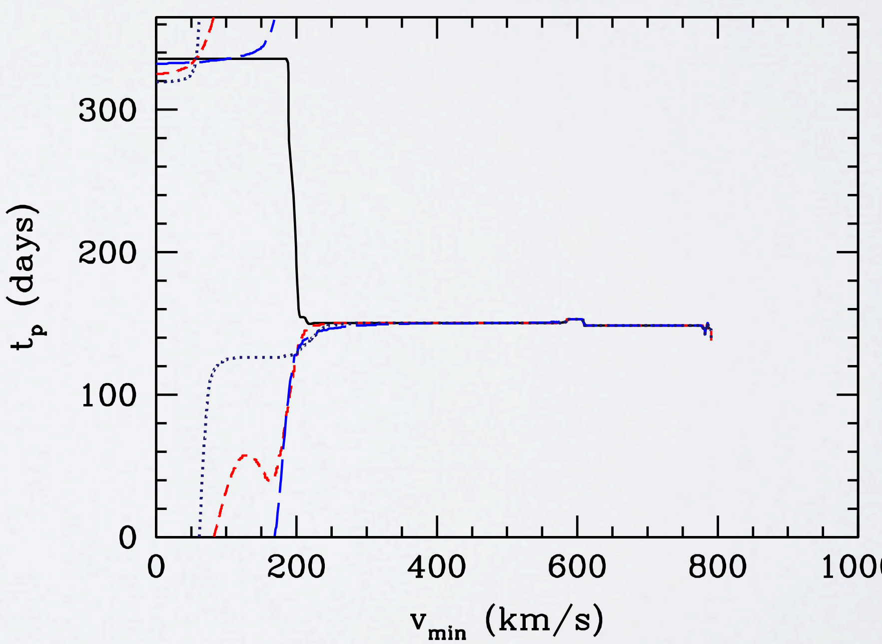


significant under-estimate of WIMP mass
(due to extra population of low v WIMPs)

amplitude of annual modulation



phase of annual modulation



at moderate v_{min} modulations from DD and halo out of phase, and t_p can vary significantly

How to handle the uncertainties?

astrophysics independent methods

Fox, Liu & Weiner; see also Fox, Kribs & Tait

Can write the differential event rate as

$$\frac{\mathrm{d}R}{\mathrm{d}E}(E) = \frac{\rho_0 \sigma_\mathrm{n}}{2 m_\chi \mu_{\mathrm{n}\chi}^2} \frac{C_\mathrm{T}}{f_\mathrm{n}^2} F^2(E) g(v_\mathrm{min}, t)$$

where $C_\mathrm{T} = \kappa (f_\mathrm{p} Z + f_\mathrm{n}(A - Z))^2$ and $g(v_\mathrm{min}, t) = \int_{v_\mathrm{min}}^{\infty} \frac{f(v,t)}{v} \mathrm{d}v$

and relate the differential event rates in 2 different expts independent of $g(v_{\mathrm{min}},t)$:

$$\frac{\mathrm{d}R_2}{\mathrm{d}E}(E_2) = \frac{C_{\mathrm{T}2}}{C_{\mathrm{T}1}} \frac{F_2^2(E_2)}{F_1^2\left(\frac{\mu_{A1\chi}^2 M_{A2}}{\mu_{A2\chi}^2 M_{A1}} E_2\right)} \frac{\mathrm{d}R_1}{\mathrm{d}E}\left(\frac{\mu_{A1\chi}^2 M_{A2}}{\mu_{A2\chi}^2 M_{A1}} E_2\right)$$

provided E_2 is in the range $[E_2^{\mathrm{low}}, E_2^{\mathrm{high}}] = \frac{\mu_{A2\chi}^2 M_{A1}}{\mu_{A1\chi}^2 M_{A2}} [E_1^{\mathrm{low}}, E_1^{\mathrm{high}}]$

i.e. provided the v_{min} range probed by the experiments overlaps.

Drees & Shan

With multiple experiments could measure WIMP mass (without assumptions for WIMP velocity distribution) by taking moments of differential event rate.

Astrophysics independent approaches are extremely useful for assessing compatibility of results from different experiments e.g. CoGeNT/DAMA/CDMS/Xenon, c.f. Fox, Kopp, Lisanti & Weiner; McCabe.

But: the nuclear form factor, F(E), complicates extracting $g(v_{min})$ from event rates.

different experiments probe different (WIMP mass dependent) ranges of v_{min}.

model/parameterise astrophysics

Strigari & Trotta

MCMC likelihood analysis of direct direction data

assume isotropic Maxwellian $f(v)$ characterised by v_0 and v_{esc}

use astronomical data (kinematics of MW halo stars and measurements of local escape speed)

marginalise over 3 or 7 parameter model for MW density distribution and anisotropy

Peter

combine data sets from different direct detection experiments

parameterise WIMP speed distribution

i) Maxwellian characterised by $v_{lag} = v_c + v_{LSR} + v_e$ & v_{rms}

ii) constant in 5 equal width bins

jointly constrain WIMP mass, cross-section and speed distribution parameters

See also Pato et al. who use k, $v_0 = v_c$ & v_{esc} as their $f(v)$ parameters.

What's the best way to parameterise the WIMP speed distribution?

Summary

- Direct detection signals, in particular the annual modulation, depend on the dark matter distribution:

uncertainty in local DM density → uncertainty in normalisation of event rate and hence cross-section (also biases mass determination)

uncertainty in WIMP velocity dispersion → uncertainty in characteristic scale of energy spectrum and hence WIMP mass

uncertainty in shape of WIMP velocity distribution → uncertainty in amplitude and phase of annual modulation signal and hence WIMP parameters

- Most important uncertainties:
  local DM density
  local DM velocity dispersion
  is there (a relatively high density) DD?
  if WIMPs are light, shape of high v tail of f(v)

- How to reduce/handle uncertainties? (postdoc position at Nottingham to work on this to be advertised v. soon)

- In long term could measure the ultra-local DM distribution (and probe the formation history of the MW?) using directional detection experiments.