Asymmetric dark matter

Part I - Models

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Part II by Hai-Bo Yu

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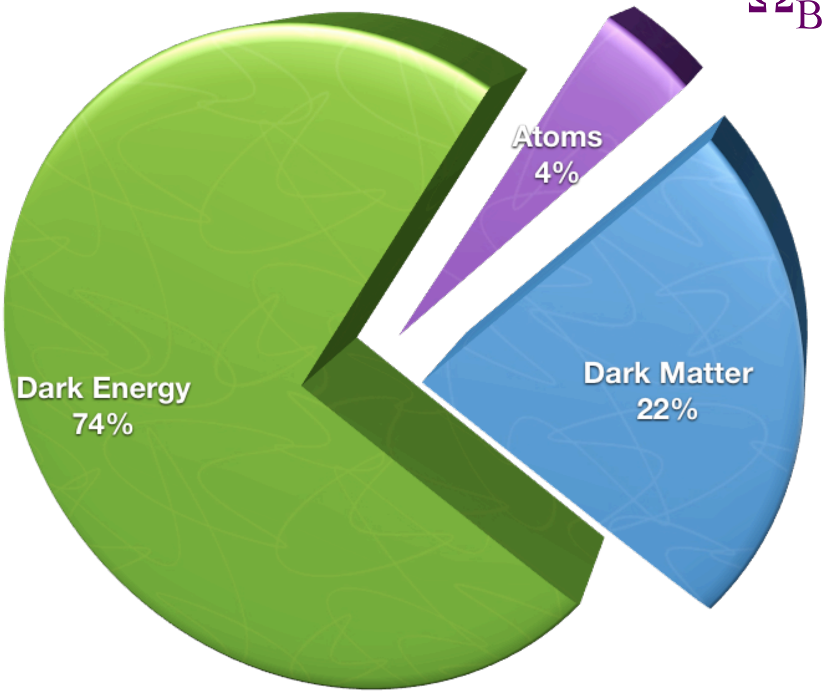
Outline

- Motivate asymmetric dark matter genesis
- Overview of baryogenesis and ADM-genesis frameworks
- ADM from Technicolor - New Strong Dynamics

What is the world made of?



$\Omega_B \sim 0.05$

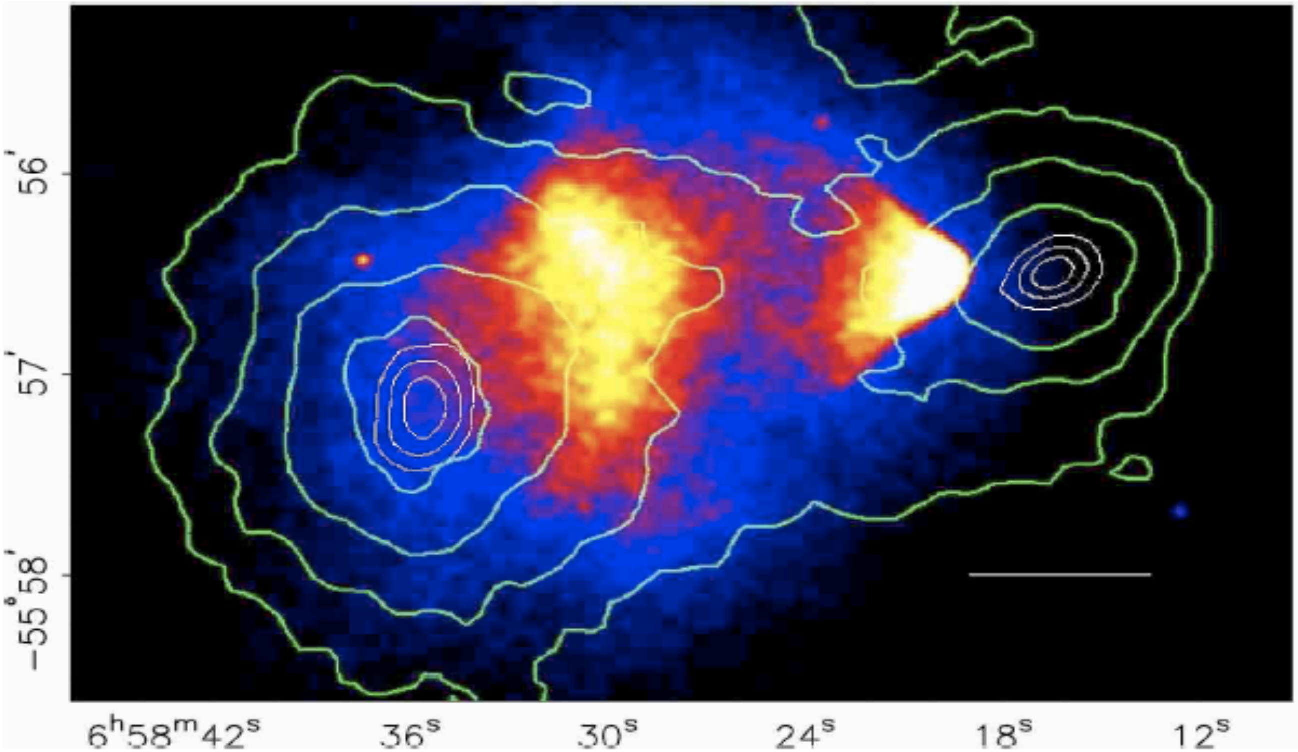
Baryons (but *no* antibaryons)

$\Omega_{DM}/\Omega_B \approx 5$

Mass scale	Particle	Symmetry/ Quantum #	Stability	Production	Abundance
Λ_{QCD}	**Baryons**	U(1) Baryon number	$\tau > 10^{33}$ yr (dim-6 OK)	'freeze-out' from thermal equilibrium Asymmetry	$\Omega_B \sim 10^{-10}$ *cf.* observed $\Omega_B \sim 0.05$

What is the strength or scale of DM (self-interactions) Λ_χ ?

$\sigma_{\chi\chi} \leq 10^{-24}$ cm^2/GeV from the 'Bullet cluster'

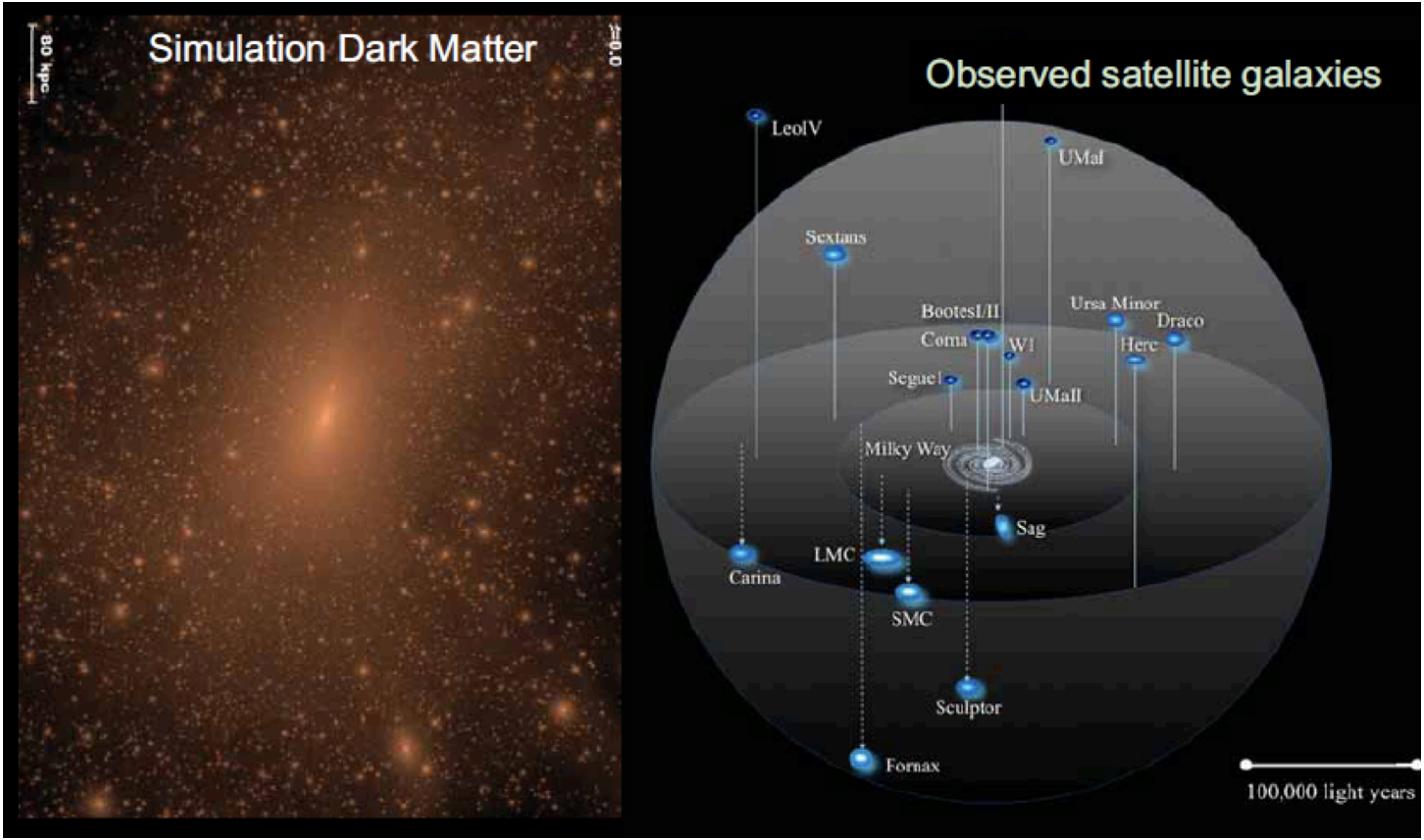


The **self-interaction** cross-section scaled up from QCD is $\sigma_{\chi\chi} \sim \sigma_{nn} (\Lambda_n/\Lambda_\chi)^2$, where $\sigma_{nn} \sim 10^{-23}$ cm^2, so in this case $\Lambda_\chi \geq 5$ GeV

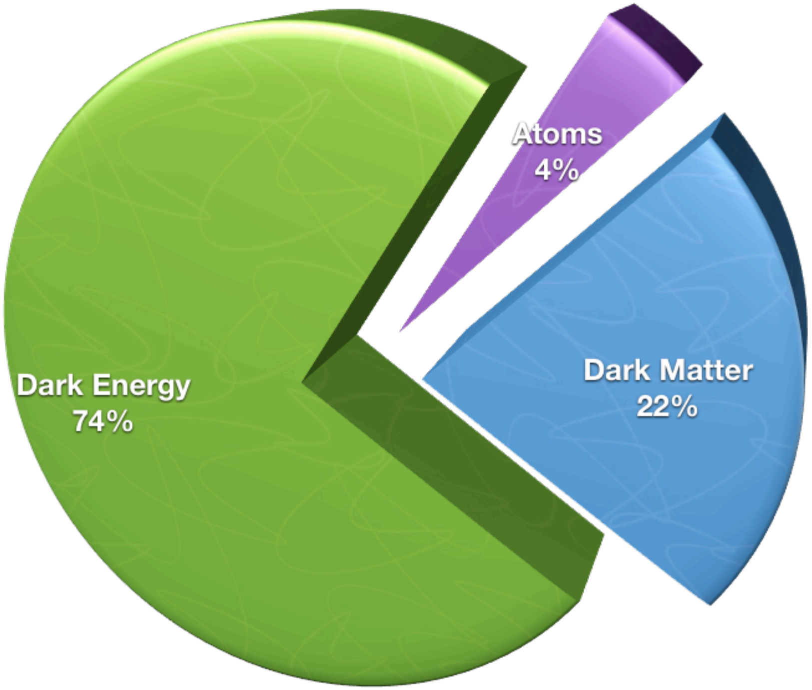
(However, see part-II of this talk by Hai-Bo Yu)

Amusingly $$\Lambda_\chi \sim \sqrt{f_\pi \, v_{\rm EW}} \approx 5GeV$$

Self-interacting dark matter has been invoked to reduce excessive substructure in simulations of *collisionless* dark matter (Spergel & Steinhardt '99)



(See also talk by M. Kuhlen this workshop)



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What should the world be made of?

What do we expect for the *symmetric* thermal relic abundance of baryons?

$$\dot{n} + 3Hn = -\langle\sigma v\rangle(n^2 - n_{\mathrm{T}}^2)$$

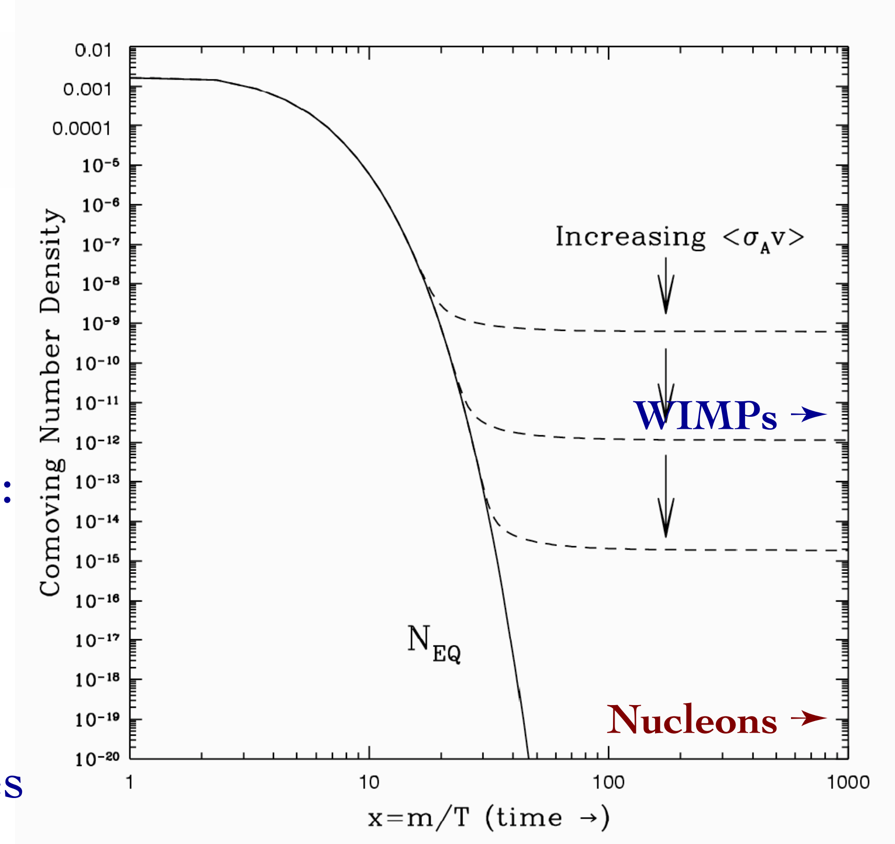
Chemical equilibrium is maintained as long as annihilation rate exceeds the Hubble expansion rate

'Freeze-out' occurs when annihilation rate:

$$\Gamma = n\sigma v \sim m_N^{3/2} T^{3/2} \mathrm{e}^{-m_N/T} \frac{1}{m_\pi^2}$$

becomes comparable to the expansion rate

$$H \sim \frac{\sqrt{g}T^2}{M_{\mathrm{P}}}$$ where $g \Rightarrow$ # relativistic species



i.e. freeze-out occurs at $T \sim m_B/45$, with:

$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \sim 10^{-19}$$

However the observed ratio is 10^9 times *bigger* for baryons, and there are *no* antibaryons, so we must invoke an **initial asymmetry**:

$$\mathcal{N}_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-9}$$

What *should* the world be made of ?

Mass scale	Particle	Symmetry/ Quantum #	Stability	Production	Abundance
Λ_{QCD}	**Nucleons**	Baryon number	$\tau > 10^{33}$ yr (dim-6 OK)	~~‘freeze-out’ from thermal equilibrium~~ Asymmetric baryogenesis	$\Omega_B \sim 10^{-10}$ *cf.* observed $\Omega_B \sim 0.05$
$\Lambda_{Fermi} \sim G_F^{-1/2}$	Neutralino?	*R*-parity?	Violated? (matter parity adequate for p stability)	‘freeze-out’ from thermal equilibrium	$\Omega_{LSP} \sim 0.25$

The ‘WIMP miracle’ naturalised e.g. in (softly broken) **supersymmetry** :

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27}\mathrm{cm}^{-3}\mathrm{s}^{-1}}{\langle \sigma_{\mathrm{ann}} v \rangle_{T=T_\mathrm{f}}} \simeq 0.1 \qquad \text{since } \langle \sigma_{\mathrm{ann}} v \rangle \sim \frac{g_\chi^4}{16\pi^2 m_\chi^2} \approx 3 \times 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$$

In this framework it is puzzling why $\Omega_{\mathrm{DM}}/\Omega_{\mathrm{B}} \approx 5$?

Natural to speculate that DMgenesis χ may be linked to baryogenesis with an asymmetric relic density related to that of baryons:

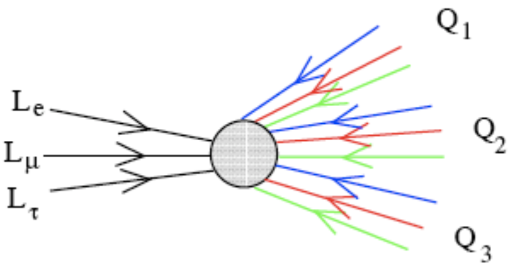
$$\Omega_\chi = (m_\chi \mathcal{N}_\chi / m_\mathrm{B} \mathcal{N}_\mathrm{B}) \Omega_B$$

__Sakharov conditions for baryogenesis:__

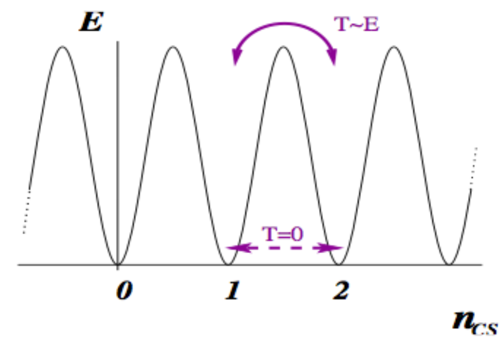
1. Baryon number violation
2. *C* and *CP* violation
3. Departure from thermal equilibrium

Classically baryon number can be violated by dim-6 operators in SM

When T>~ M_W Baryon number is also violated in the SM through sphaleron-mediated processes that preserve B – L, but violate B + L …



$$\partial_\mu j_i^\mu = \partial_\mu(\bar{\psi}^i \gamma^\mu \psi^i) = \frac{g^2}{8\pi} W^{a\mu\nu} \tilde{W}^a_{\mu\nu}$$



…*CP*-violation is *too weak* (and the electroweak phase transition is a 'cross-over')

The observed matter-antimatter asymmetry *requires* BSM physics

The matter we *know* originated *non*-thermally in the early universe

The same or a similar mechanism could generate the DM density

(See list of references in part-II of this talk by Hai-Bo Yu)

General features of ADM

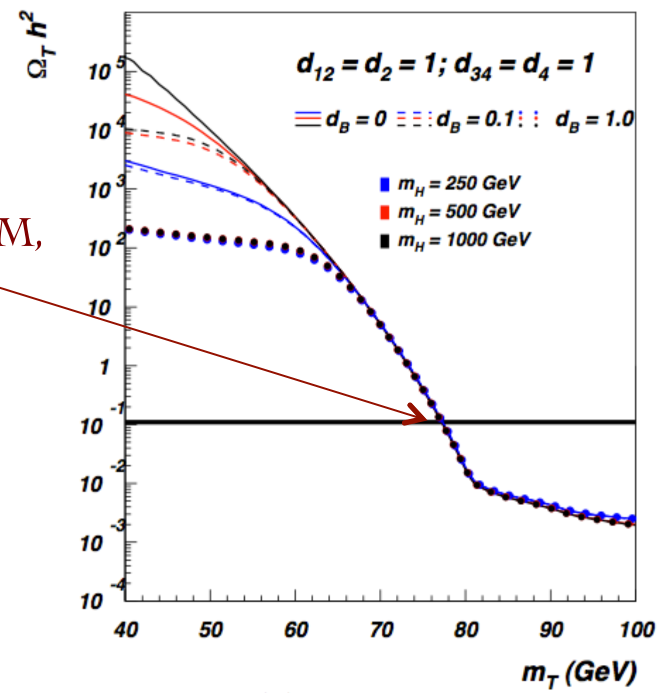
- ADM is a complex field, either Dirac fermion or complex scalar
- Requires asymmetry generating or transferring operator and
- large annihilation cross-section to suppress symmetric relic density

(Griest & Seckel '87; Hooper, March-Russell & West '05; Belyaev, M.T.F, Sannino & Sarkar '10; Graesser, Shoemaker & Vecchi 11; Buckley 11)

Example: Symmetric relic density of composite scalar pNGB DM state ϕ

$$\mathcal{L} = \partial_\mu \phi^* \partial_\mu \phi - m_\phi^2 \phi^* \phi + \frac{d_1}{\Lambda} H \partial_\mu \phi^* \partial_\mu \phi + \frac{d_2}{\Lambda} m_\phi^2 H \phi^* \phi + \frac{d_3}{2\Lambda^2} H^2 \partial_\mu \phi^* \partial_\mu \phi + \frac{d_4}{2\Lambda^2} m_\phi^2 H^2 \phi^* \phi. \quad (2)$$

Overproduction of DM, below W threshold



If constituents of ϕ has weak charges it has additional interactions, lets call it T !

$$\mathcal{L}_B = ie\frac{d_B}{\Lambda^2} T^* \overleftrightarrow{\partial_\mu} T \, \partial_\nu F^{\mu\nu}$$

$$L_{WW,ZZ} = -\frac{T^* T}{2} \mathrm{Tr}\left[d_W \, W_\mu W^\mu + d_Z \, Z_\mu Z^\mu\right]$$

(M.T.F & Sannino '10; Belyaev, M.T.F, Sannino & Sarkar '10)

If ADM is light require new light *hidden sector* states

(e.g. Strassler & Zurek '06; Cohen, Phalen Pierce & Zurek '10; M.T.F, Sarkar & Schmidt-Hoberg '11)

Some frameworks for baryogenesis:

1. GUT scale baryogenesis (Yoshimura 78; Dimopoulos & Susskind '78)
   (Decay of GUT scale coloured states)
2. Electroweak baryogenesis (EWB) (Kuzmin, Rubakov & Shaposhnikov '85)
   (sphalerons & 1^{st} order phase transition)
3. Leptogenesis (Fukugita & Yanagida '86)
   (sphalerons, 1^{st} or 2^{nd} order phase transition)
4. Affleck-Dine baryogenesis (Affleck & Dine '84)
   (Decays of coherent scalar fields in SUSY)
5. Spontaneous baryogenesis (Cohen & Kaplan '87)
6. Cold baryogenesis (Krauss & Trodden '99)
7. … (Reviews in Dine & Kusenko '03; Cline '06; Schmidt '11)

DMgenesis can be linked to baryogenesis in *any of these frameworks*

Whatever the baryo/lepto-genesis mechanism, SM sphalerons may transfer an asymmetry if DM (constituents) carry weak charges

frameworks for DM-genesis linked to baryogenesis:

1. Via (e.g.) GUT scale baryogenesis

(Original technibaryon ADM from technicolor proposal) (Nussinov 85)

2. Via Electroweak baryogenesis (EWB) (Kaplan 92)

(requires 1st order EW phase transition)

3. Via sphalerons/leptogenesis (Barr, Chivukula & Farhi 90; Gudnason, Kouvaris & Sannino 05)

(sphalerons, 1st or 2nd order phase transition)

4. Via Affleck-Dine baryogenesis (e.g. Enqvist & McDonald '98)

5. Via spontaneous baryogenesis (March-Russell & McCullough '11)

6. Cold baryogenesis (Konstandin & Servant '11)

7. …

Common for 2., 3. is that DM carries $U(1)_X$ charge with $U(1)_X \times SU(2)_L^2$ anomaly

In 2. sphalerons produce the baryon/DM asymmetry – co-generation.

In 3. sphalerons transfer/process an existing asymmetry into DM – sharing.

Note that the DM or associated new physics sector may also change the EW phase-transition relevant for 2. (Grojean, Servant & Wells '04; Cline, Jarvinen & Sannino 08, Jarvinen, Ryttov & Sannino 09,…)

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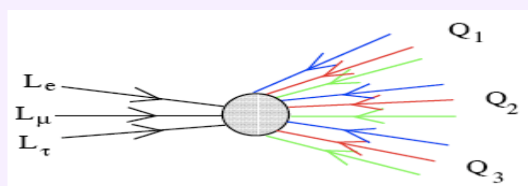
Asymmetry transfer/sharing:

- B or L asymmetry generated at a high scale, e.g. Leptogenesis.
- Below B-L is preserved, but transferred to DM
- $T_{\rm dec}$ Transfer operator decouples and asymmetry is 'frozen in'
- Symmetric component is annihilated away

Transfer via X and weak charges of DM (constituents)

$U(1)_X \times SU(2)_L^2$ anomaly, dim-4 (nonpert.) SM sphalerons, with $T_{\rm sph} \sim v_{\rm EW}$



(e.g. Barr, Chivukula & Farhi 90)

Light ADM via SM sphalerons challenging but possible

(M.T.F, Sarkar & Schmidt-Hoberg '11)

Transfer via induced L or B charges of DM

some higher dim. decay operator with model dependent $T_{\rm dec}$

$\mathcal{O}_{B-L}\mathcal{O}_X$ $\qquad$ $\Delta W_{\rm eff} = \frac{1}{M_i}\bar{X}^2 L_i H_u$

(e.g. Farrar, Zaharijas '05; D.E Kaplan, Luty & Zurek '09)

Asymmetry co-generation:

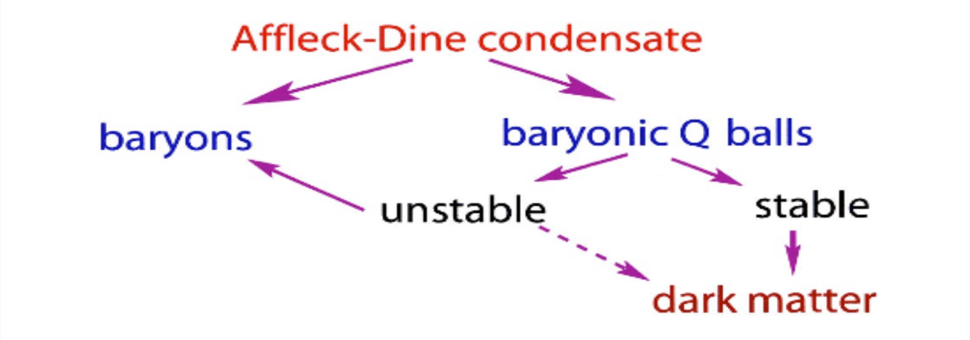
- B and DM asymmetries from same microphysics.
- Transfer operator decouples and either B-L & B-X or B-L+X is preserved

EWB co-generation via weak charges of DM (constituents)

(e.g Kaplan 92)

$U(1)_X \times SU(2)_L^2$ anomaly, sphalerons, 1^{st} order phase transition, $T_{\rm sph} \sim v_{\rm EW}$

Or via new decaying states (e.g. Affleck-Dine co-genesis)



(e.g. Enqvist & McDonald '98; Hall, March-Russell & West 10; Hook 11; Cheung & Zurek 11; March-Russell & McCullough '11; Graesser, Shoemaker & Vecchi '11)

ADM models with 'Hybrid' asymmetry generation

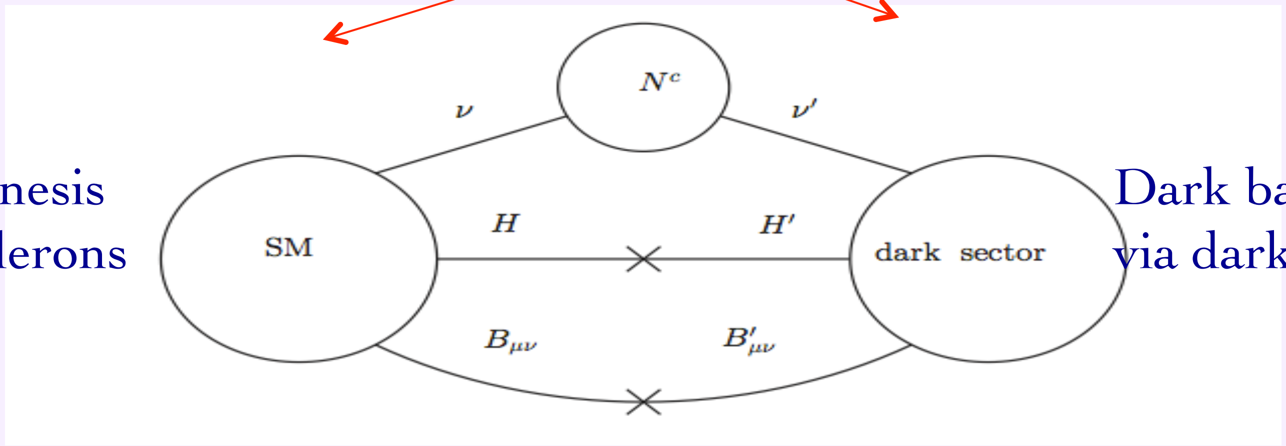
Asymmetry co-generation + transfer:

- L and L' (dark lepton number) co-generated at a high scale
- L transferred to B, and L' transferred to B' (dark baryon number B') by sphalerons and dark sphalerons

(e.g. An et al 09; Buckley & Randall '10)

Leptogenesis

Baryogenesis via sphalerons



Dark baryogenesis via dark sphalerons

- L, L'=DM charge X generated at a high scale
- L to B (X) by sphalerons

(e.g. Falkowski, Ruderman & Volansky '11)

ADM from Technicolor or New Strong Dynamics

Technicolour is a 'natural' framwork for ADM: (Nussinov 85, Barr, Chivukula and Farhi 90)

- Natural model of EWSB
- Composite neutral technibaryons charged under global $U(1)_{\rm TB}$ part of unbroken chiral symmetries
- TB has weak charged constituents there is $U(1)_{\rm TB} \times SU(2)_L^2$ anomaly
- Large annihilation cross-section so no symmetric cross-section (in general)

Mass scale	Particle	Symmetry/ Quantum #	Stability	Production	Abundance
Λ_{QCD}	**Nucleons**	Baryon number $U(1)_B$	$\tau > 10^{33}$ yr (dim-6 OK)	~~'Freeze-out' from thermal equilibrium~~ Asymmetric baryogenesis	$\Omega_B \sim 10^{-10}$ *cf.* observed $\Omega_B \sim 0.05$
$\Lambda_{Fermi} \sim G_F^{-1/2}$	Neutralino? Technibaryon?	R-parity? Technibaryon number $U(1)_{\rm TB}$	violated? $\tau \sim 10^{18}$ yr e^+ excess?!	'Freeze-out' from thermal equilibrium Asymmetric (like the *observed* baryons)	$\Omega_{LSP} \sim 0.25$ $\Omega_{TB} \sim 0.25$

Technicolor (New Strong Dynamics)

1. The SM gauge group is augmented:

$$G_{SM} \to SU(3)_{\rm c} \times SU(2)_{\rm W} \times U(1)_{\rm Y} \times G_{\rm TC} \ .$$

2. The Higgs sector of the SM is replaced:

$$Q^a_{\rm L} = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_{\rm L}, \quad Q^a_R = (U^a_{\rm R}, D^a_{\rm R}),$$
$$a = 1, \dots d(\mathcal{R}_{\rm TC})$$

$$\mathcal{L}_{Higgs} \to -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{Q}_{\rm L}\gamma_\mu D^\mu Q_{\rm L} + i\bar{Q}_{\rm R}\gamma_\mu D^\mu Q_{\rm R} + \dots$$

Lightest Technibaryon stable due to unbroken global $U(1)_{\rm TB}$ symmetry

Minimal chiral symmetries: 3 GB's + Custodial + DM.

$$SU_L(2) \times SU_R(2) \times U_{TB}(1) \to SU_V(2) \times U_{TB}(1) \ .$$

However, $U(1)_{\rm TB}$is anomalous due to the EW anomaly, when the technifermions are charged under SU(2)_EW

$$\partial_\mu J^\mu_{TB} = \frac{1}{2\sqrt{2}} \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} W^{\mu\nu} W^{\rho\sigma} \ , \quad \text{and} \quad J^\mu_{TB} = \frac{1}{2\sqrt{2}} \left(\bar{U}\gamma^\mu U + \bar{D}\gamma^\mu D\right)$$

Technibaryon ADM

(Nussinov 85)

The ADM relic density is simply related to the baryon relic density

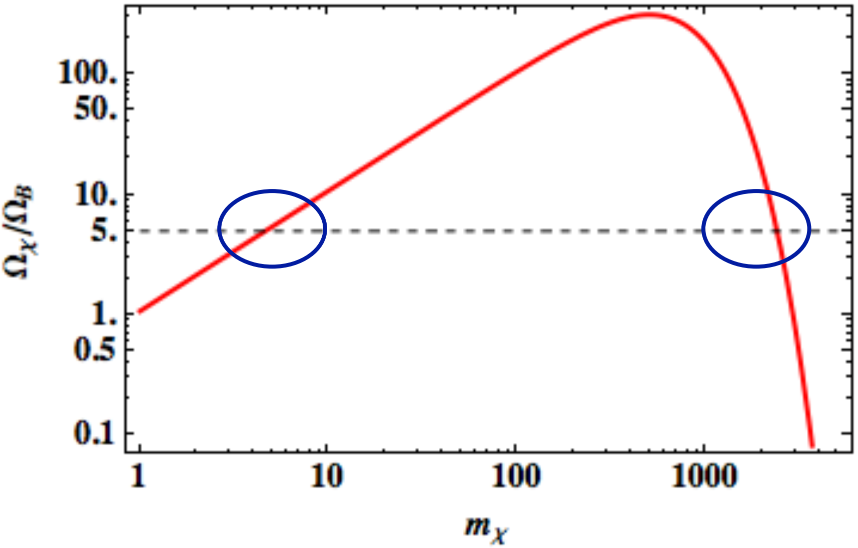
$$\Omega_\chi = (m_\chi \mathcal{N}_\chi / m_B \mathcal{N}_B)\Omega_B$$

Nussinov thought of TeV scale technibaryon when relic density was less well known so $\mathcal{N}_\chi \sim \mathcal{N}_B$, $m_\chi \sim 1$ TeV was close enough…

Now similar asymmetries (such as from GUTs or from sphalerons) $\mathcal{N}_\chi \sim \mathcal{N}_B \Rightarrow m_\chi \sim 5$ GeV

In fact. there are 'two' solutions via Sphalerons (e.g. staring from Leptogenesis):

(Barr, Chivukula and Farhi 90)



$$\frac{\Omega_\chi}{\Omega_B} \sim \frac{m_\chi}{m_B} \times \begin{cases} 1 & \text{for } m_\chi \sim GeV \\ \left(\frac{m_\chi}{T_{\rm Sph}}\right)^{3/2} e^{-m_\chi/T_{\rm Sph}} & \text{for } m_\chi \sim TeV \end{cases}$$

Really Ω_χ/Ω_B depends on L/B (and possible new L') so more general masses are possible.

(Barr, Chivukula and Farhi 90; Gudnasone, Kouvaris & Sannino '05)

Note BCF did not include charge neutrality of universe but this does not change qualitative picture

(Harvey & Turner 90)

pNGB Technibaryons in Minimal Walking Technicolor

(Sannino and collaborators '05;...)

Example of more general solutions termed: Techni-Interacting Massive Particles (TIMPs)

'iTIMP'

- $\mathcal{R}$ real
- $T^0 \sim U_L D_L$
- Iso-singlet GB
- $M_{T^0} \sim g\, F_\Pi$

(Gudnason, Kouvaris & Sannino 05; M.T.F & Sannino 09)

'TIMP'

- 4 of $SU(4)$
- $U_L D_L U_L D_L$
- SM singlet
- $M_T \sim N_{TC}^{3/2} F_\Pi$

(Barr, chivukula & Farhi 90)

'TIMP'

- $\mathcal{R}$ pseudo-real
- $T^0 \sim U_L D_L$
- SM singlet GB
- $M^2_{T^0} \sim -g^2\, F^2_\Pi$

(Ryttov and Sannino 09)

General analysis of signals and the annihilation cross-section

(Foadi, M.T.F & Sannino 08; Belyaev, M.T.F, Sarkar & Sannino 10, Del Nobile, Kouvars & Sannino '11)

'Minimal Dark Matter' in Technicolor theory space

(Cirelli, Fornengo Strumia '02)

Techni-Interacting Massive Particles

Minimal Technicolor Theory Space

Minimal Technicolor: 2 Dirac Flavors. No QCD charges.

$$Q_L = \left(U_L^{+1/2}, D_L^{-1/2}\right)^T \ , \quad U_R^{+1/2} \ , \ D_R^{-1/2} \ .$$

'Orthogonal TC'

- $\mathcal{R}$ real
- F of $SO(N)$
- $SU(4)/SO(4)$
- $3_\Pi \oplus 3 \oplus \bar{3}$

$$\begin{pmatrix} \Pi & T_i \\ T_i^* & \Pi^T \end{pmatrix}$$

'QCD TC'

- $\mathcal{R}$ complex
- F of $SU(N)$
- G_{GB}: $SU(2)$
- 3_Π

$$\Pi = \begin{pmatrix} \Pi^0 & \Pi^+ \\ \Pi^- & \Pi^0 \end{pmatrix}$$

'Symplectic TC'

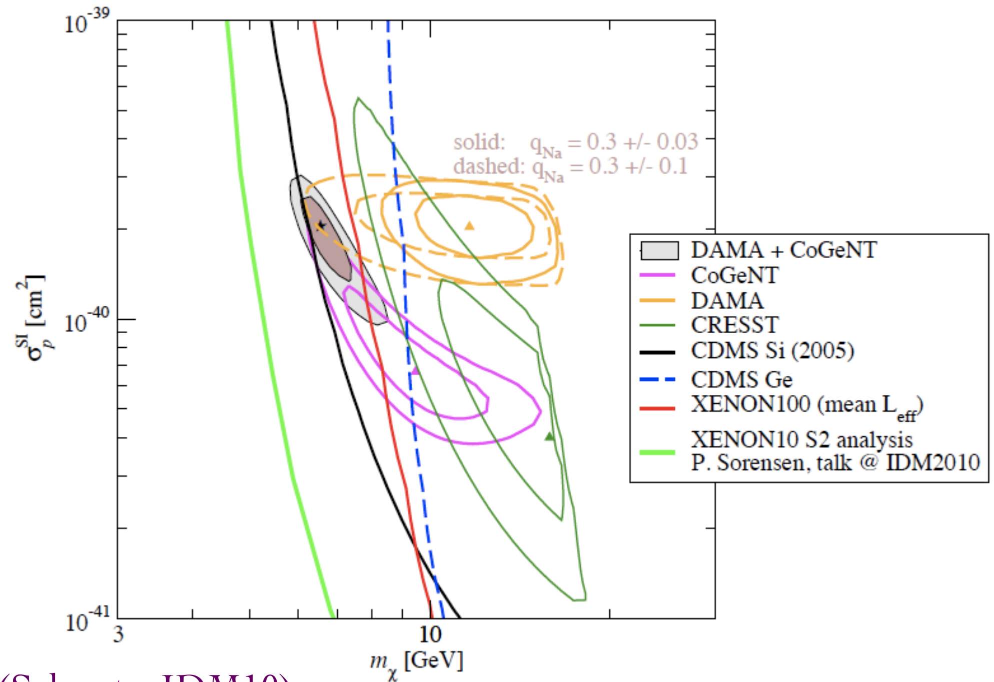
- $\mathcal{R}$ pseudo-real
- F of $Sp(2N)$
- $SU(4)/Sp(4)$
- $3_\Pi \oplus 1 \oplus \bar{1}$

$$\begin{pmatrix} \Pi & T_s \\ T_s^* & \Pi^T \end{pmatrix}$$

(M.T.F & Sannino 09)

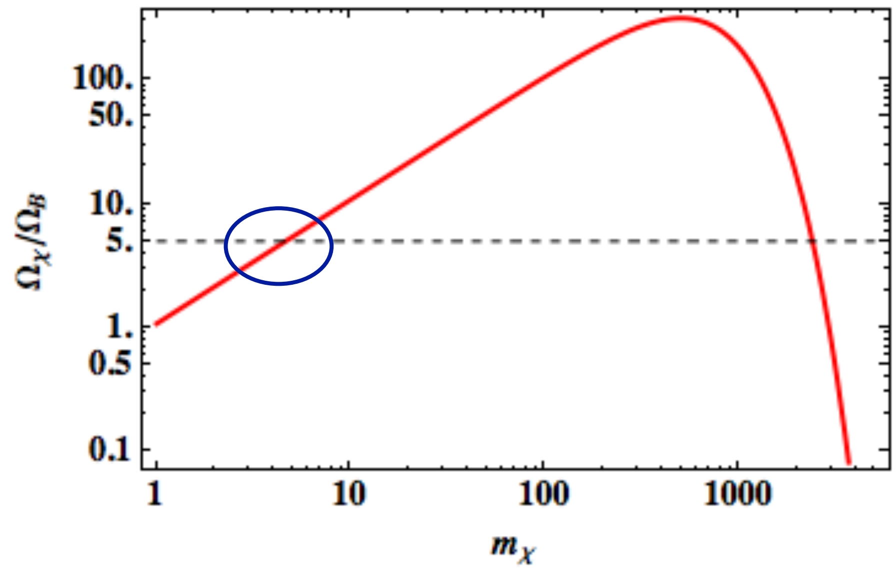
(Ryttov and Sannino 09)

GeV scale ADM



(Schwetz, IDM10)

$$\Omega_\chi = (m_\chi \mathcal{N}_\chi / m_B \mathcal{N}_B)\Omega_B$$



Recent surge of interest in GeV scale ADM after hints of light DM in direct detection experiments

(Talks this workshop by Collar, Schmidt-Hoberg, Zupan)

Can you get $\mathcal{N}_\chi \sim \mathcal{N}_B$ and light ADM via sphalerons in Technicolor (in agreement with constraints from e.g. W/Z measurements?)

GeV scale ADM and EWSB from two sectors

Assume 2 sectors $\mathcal{S}_1 \sim \mathrm{TeV}$, $\mathcal{S}_2 \sim \mathrm{GeV}$

(M.T.F, Sarkar and Schmidt-Hoberg 11; M.T.F, Kahlhoefer, Sarkar & Schmidt-Hoberg 11 and work in progress)

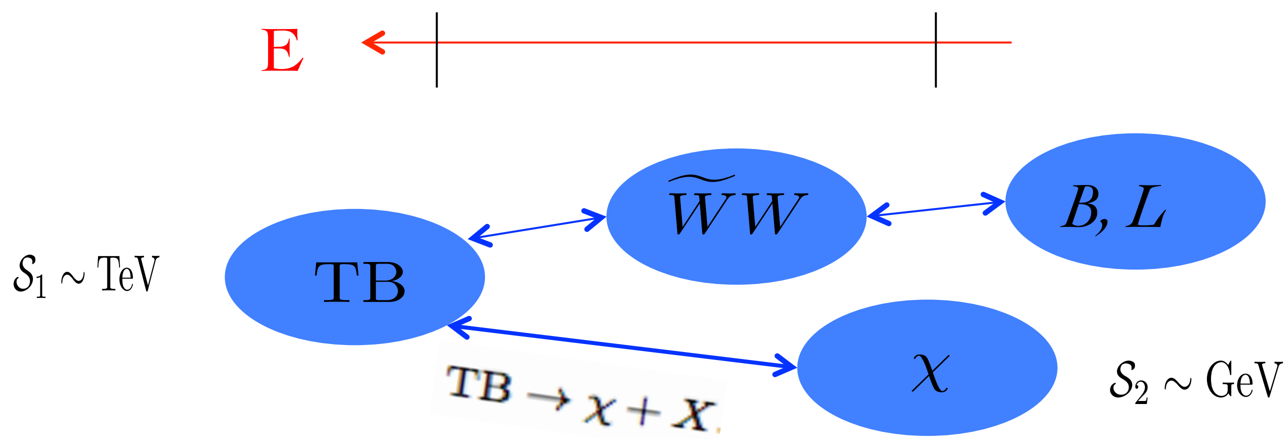
i) S_1 breaks EW symmetry dynamically at a scale Λ_1

composite spectrum includes $\mathcal{O}(\mathrm{TeV})$ mass particles TB carrying $U(1)_{\mathrm{TB}}$ charge

S_2 is SM singlet sector becoming strongly interacting at Λ_2 of $\mathcal{O}(\mathrm{GeV})$

composite spectrum includes $\mathcal{O}(\mathrm{GeV})$ mass particles χ carrying $U(1)_{\mathrm{TB}}$ charge

$S_{1,2}$ coupled via $U(1)_{\mathrm{TB}}$ preserving decay operator $\mathrm{TB} \to \chi + X$



S_1 States (constituents) carry weak charges and are connected to sphalerons

$\mathrm{TB} \to \chi + \mathrm{X}$ Is in equilibrium until $T \lesssim T_{\mathrm{sph}}$

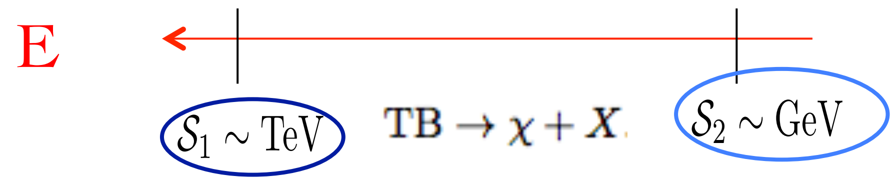
GeV scale ADM and EWSB from two scale strong dynamics

Assume new strong dynamics with weak charged fermions Q
and SM Singlet fermions $\lambda\ldots$ (Dietrich, Sannino & Tuominen 05; Sannino & Ryttov 08; Galloway, Evans, Luty & Tacchi '10)
motivated also by constraints from EW precision measurements

We identify composite states $\mathrm{TB} \sim \mathrm{Q}\cdots\mathrm{Q}$ and $\chi \sim \lambda\cdots\lambda$

$$Q^a_{\mathrm{L}} = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_{\mathrm{L}}, \; U^a_{\mathrm{R}},\; D^a_{\mathrm{R}},\; \lambda^b$$

$$a = 1,\ldots d(\mathcal{R}_1),\; b = 1,\ldots d(\mathcal{R}_2).$$



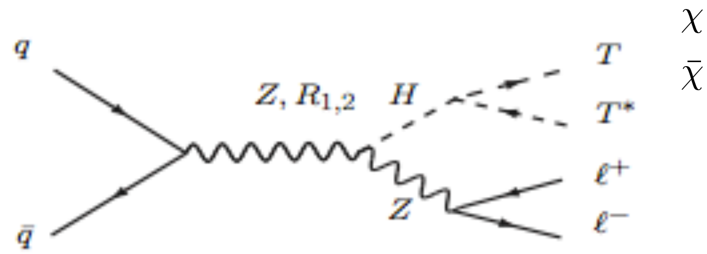
Scale separation can arise due do different Casimirs $\alpha_c = \dfrac{\pi}{3C_2(\mathcal{R})}$. (Raby, Dimopoulos & Susskind 80)

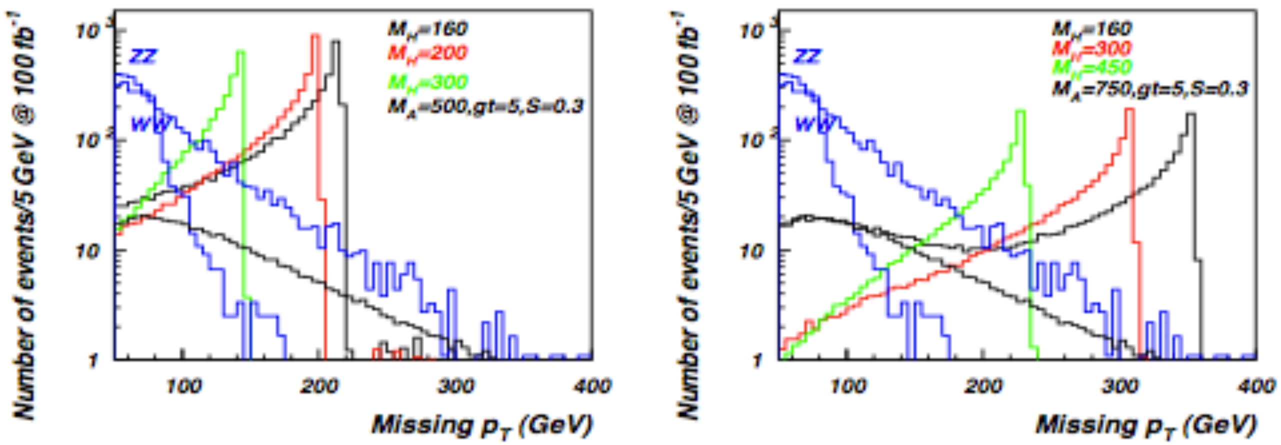
Can also arise/be enhanced in presence of 4-fermion interactions (M.T.F, Sarkar and Schmidt-Hoberg 11

Recent 1-loop exact susy comp of a 2-scale model (Antipin, Mojaza & Sannino '11)

Model provides natural EWSB, DM stability and (putative) light ADM
via weak sphalerons – decay operator can but need not arise in TC sector itself
Achieving scale separation is obviously a challenge

(i)TIMP missing energy signals





(Foadi, M.T.F and Sannino 08; Godbole, Guchait, Mazumdar, Moretti and Roy 03).

Summary

Asymmetric dark matter is motivated by the observed asymmetry of baryonic matter and the desire to explain why

$$\Omega_{\rm DM}/\Omega_{\rm B} \approx 5$$

- ADM natural in the framework of New Strong Dynamics such as Technicolour

Recently vigorous ADM model building activity in a large variety of frameworks

- Interesting alternative to WIMP paradigm…Experiment will tell

Appendices

Two scales from strong dynamics

(Marciano 80; Lane & Eichten 89)

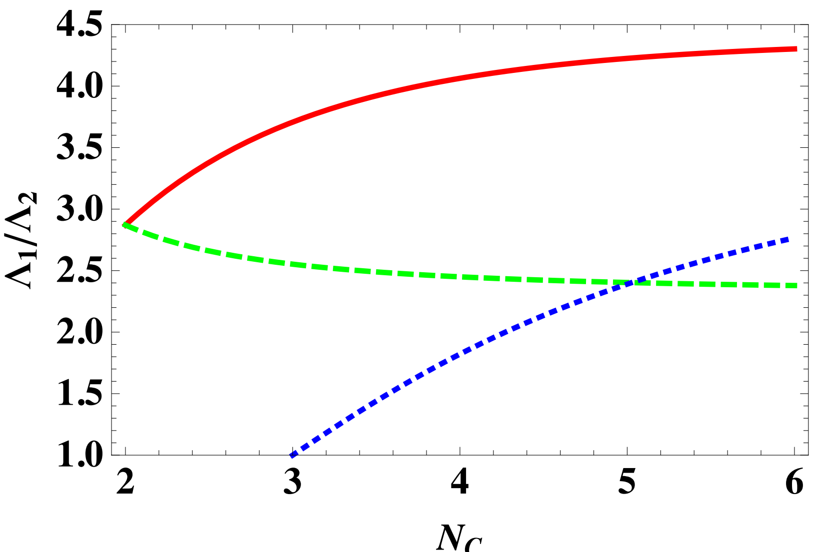
Critical coupling for ChiSB depends on the fermion rep…

$$\alpha_c = \frac{\pi}{3C_2(\mathcal{R})}, \qquad \alpha_* = -4\pi\frac{\beta_0}{\beta_1}.$$

(Raby, Dimopoulos & Susskind 80)

One-loop estimate of scale separation with 2 representations

$$\frac{\Lambda_1}{\Lambda_2} \simeq \exp\left[\frac{2\pi}{\beta_0(\mathcal{R}_2)}\left(\alpha_c(\mathcal{R}_1)^{-1} - \alpha_c(\mathcal{R}_2)^{-1}\right)\right]$$



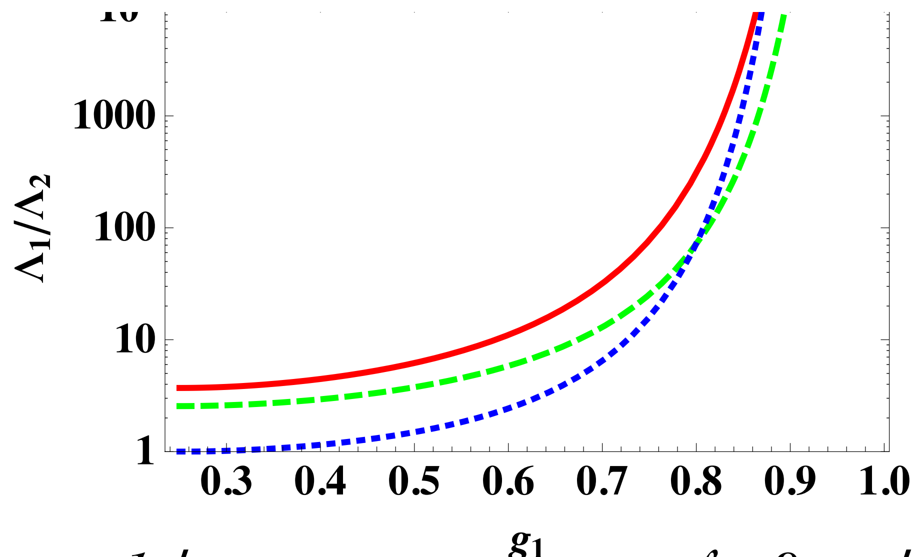
(M.T.F, Sarkar & Schmidt-Hoberg 11)

and in the NJL model on the 4-fermion coupling

(Appelquist et al 88, Kondo et al 89, Kurachi et al 07, Fukano & Sannino 09)

$$\mathcal{L} = \bar{Q}i\not{D}Q - \frac{1}{4}\mathrm{Tr}F^a_{\mu\nu}F^{a\mu\nu} + \frac{4\pi^2 g_1}{\Lambda^2 N_1 d(\mathcal{R}_1)}\left[(\bar{Q}Q)^2 + (\bar{Q}i\gamma_5 T^a Q)^2\right]$$

$$\alpha_c(\mathcal{R}_1, g_1) = \begin{cases} 4\left(\sqrt{g_1} - g_1\right) \times \alpha_c(\mathcal{R}_1) & \text{for } \frac{1}{4} < g_1 \leq 1, \\ \alpha_c(\mathcal{R}_1) & \text{for } 0 \leq g_1 < \frac{1}{4}. \end{cases}$$



Recent 1-loop exact susy comp of a 2-scale model

(Antipin, Mojaza & Sannino11)