Effective Dirac Neutrino Mass Operator in the Standard Model with a Local Abelian Extension

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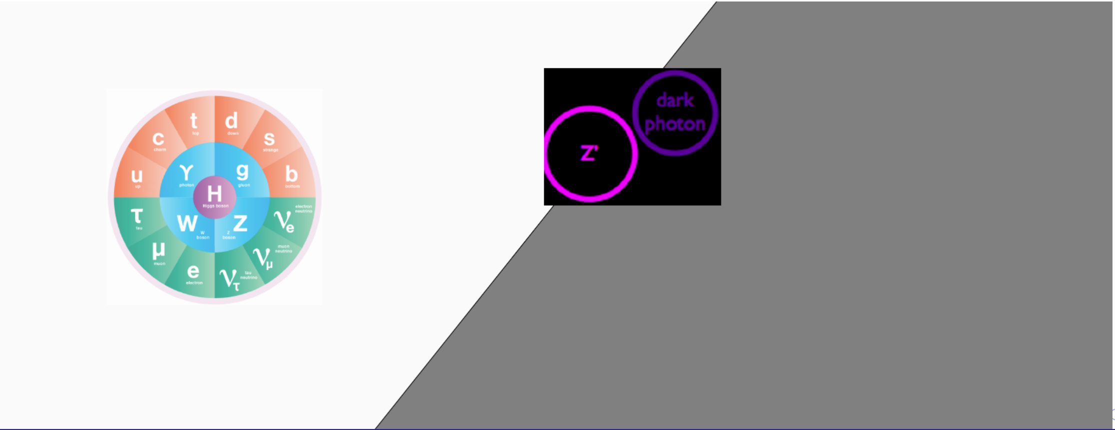
Content

Motivation



Anomaly conditions

The Diophante's equations:

$$\sum_\rho n_\rho = 0$$
$$\sum_\rho n_\rho^3 = 0$$

The we have:

- **Dark symmetries:** $m = 0$, $N_{\text{chiral}} = N$, D-Charges: $n_1, \cdots, n_N$ and dark photon.
- **Active symmetries:** $m \neq 0$, $N_{\text{chiral}} = N - 3$, X-Charges: $n_1, \cdots, n_{N_{\text{chiral}}}$ and heavy mediator boson.

Active Symmetries

For active symmetries, the X-Charges of Standard Model right-handed fermions can be written in terms of m, and afree parameter that is chosen to be the X-Charge of the Standard doublet Lepton such that:

$$
\begin{aligned}
u =& \frac{4L}{3} - m \\
u =& m - \frac{2L}{3} \\
Q =& -\frac{L}{3} \\
e =& m - 2L \\
h =& L - m
\end{aligned}
$$

Effective Dirac neutrino mass operator

$$\mathcal{L}_{\text{eff}} = h_\nu^{\alpha i}\, (\nu_{R\alpha})^\dagger\, \epsilon_{ab}\, L_i^a\, H^b \left(\frac{S^*}{\Lambda}\right)^\delta + \text{h.c.}, \qquad \text{with } i = 1, 2, 3\,,$$

and $\delta = 1, 2, \ldots$ for dimension 5 (D-5) or 6 (D-6) operators, etc. Here $h_\nu^{\alpha i}$ correspond to dimensionless induced couplings, $\nu_{R\alpha}$ are at least two RHNs ($\alpha = 1, 2, \ldots$) with the same D or X-charge ν, L_i are the lepton doublets with X-charge $-L$, H is the SM Higgs doublet with X-charge $h = L - m$, S is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry with D or X-charge

$$s = -(\nu + m)/\delta\,,$$

Diracness of non-zero DM and Dirac neutrinos masses from $U(1)_X$

Starting from the extended dataset with the solutions with N integers to the Diophantine equations, we apply the following steps

- Check that the solution has two (three) repeated integers to be identified as ν and fix $N_\nu = 2$ ($N_\nu = 3$).
- For $\delta = 1, 2, \ldots$ and all the possible combinations for m and ν in the solution, including $m = 0$, find the s value compatible with the effective Dirac neutrino mass operator of D-4 $+ \delta$ according to the effective Lagrangian.

Diracness of non-zero DM and Dirac neutrinos masses from $U(1)_\mathcal{X}$ II

- Interpret the integers in the solution which are different from m and ν as the D-charges for $m = 0$ or X-charges for $m \neq 0$ of a set of singlet chiral fermions: ψ_i, $i = 1, \ldots, N_{\text{chiral}} - N_\nu$. Then select the solutions for which the condition

$$|n_i + n_j| = |s| \tag{1}$$

which guarantees that all the singlet chiral fermions, ψ_i, acquire masses after the spontaneous symmetry breaking of the gauge Abelian symmetry through $\langle S \rangle$.

Unconditional stability

There is *unconditional* stability when two remnant symmetries satisfy that $\mathbb{Z}_{|s|} \cong \mathbb{Z}_p \otimes \mathbb{Z}_q$ with p and q coprimes and $|s| = pq$. For the two DM candidates associated to the set of chiral fields ψ_i and χ_j, we consider below the following two possibilities for $|s|$

- $\mathbb{Z}_6 \cong \mathbb{Z}_2 \otimes \mathbb{Z}_3$: solutions with at least a set of chiral fields with $\psi_i \sim \left[\omega_6^2 \vee \omega_6^4\right]$ under $\mathbb{Z}_6$, and at least a set of chiral fields with $\chi_i \sim \omega_6^3$ under $\mathbb{Z}_6$,
- $\mathbb{Z}_{14} \cong \mathbb{Z}_2 \otimes \mathbb{Z}_7$: solutions with at least a set of chiral fields with $\psi_i \sim \left[\omega_{14}^2 \vee \omega_{14}^6 \vee \omega_{14}^8 \vee \omega_{14}^{10} \vee \omega_{14}^{12}\right]$ under $\mathbb{Z}_{14}$ and at least a set of chiral fields with $\chi_i \sim \omega_{14}^7$ under $\mathbb{Z}_{14}$,

where $\omega_{|s|} = e^{i\,2\pi/|s|}$.

48 type of representative solutions

Solution	N	N_{chiral}	m	ν	δ	s	N_D	N_M	G_D	G_M
$(1,-2,-3,5,5,-6)$	6	6	0	5	1	-5	2	0	1	0
$(3,3,3,-5,-5,-7,8)$	7	4	3	-5	2	1	1	0	1	0
$(1,-2,3,4,6,-7,-7,-7,9)$	9	9	0	-7	1	7	3	0	1	0
$(1,1,-4,-5,9,9,9,-10,-10)$	9	9	0	9	1	-9	3	0	2	0
$(1,2,-6,-6,-6,8,9,9,-11)$	9	6	-6	9	1	-3	2	0	1	0
$(1,-3,8,8,8,-12,-12,-17,19)$	9	6	8	-12	2	2	2	1	1	1
$(8,8,8,-12,-12,15,-17,-23,25)$	9	6	8	-12	2	2	2	0	1	0
$(1,-2,-2,3,3,-4,-4,6,6,-7)$	10	10	0	6	1	$\mathbf{-6}$	3	2	2	2
$(1,-2,-2,3,4,-5,-5,7,7,-8)$	10	10	0	-5	1	5	4	0	2	0
$(1,-2,-2,3,5,-6,-6,8,8,-9)$	10	10	0	-6	1	$\mathbf{6}$	4	0	2	0
$(2,2,3,4,4,-5,-6,-6,-7,9)$	10	10	0	2	1	-2	4	2	2	2
$(1,1,5,5,5,-6,-6,-6,-9,10)$	10	10	0	1	1	-1	4	0	3	0
$(2,2,4,4,-7,-7,-9,-9,10,10)$	10	10	0	10	2	-5	3	0	2	0
$(1,2,2,-3,6,6,-8,-8,-9,11)$	10	10	0	-8	1	8	4	1	2	1
$(1,-2,-3,5,6,-8,-9,11,11,-12)$	10	10	0	11	1	-11	4	0	1	0

48 type of representative solutions

Solution	N	N_{chiral}	m	ν	δ	s	N_D	N_M	G_D	G_M
$(1,1,-3,4,4,-7,8,-10,-10,12)$	10	10	0	−10	2	5	4	0	2	0
$(1,1,-2,-2,-4,6,-10,11,12,-13)$	10	10	0	−2	1	2	3	2	1	2
$(3,4,4,4,4,-5,-8,-8,-11,13)$	10	10	0	−8	1	8	2	4	1	4
$(4,4,5,6,6,-9,-10,-10,-11,15)$	10	10	0	6	1	**−6**	4	0	2	0
$(1,-2,-4,7,7,-10,-12,14,14,-15)$	10	10	0	14	1	**−14**	3	2	1	2
$(1,2,2,-3,4,-6,12,-13,-14,15)$	10	10	0	2	1	−2	4	1	1	1
$(1,4,4,-7,8,8,-9,-12,-12,15)$	10	10	0	8	1	−8	4	2	2	2
$(1,2,2,-9,-9,16,16,17,-18,-18)$	10	10	0	−18	1	18	3	2	2	2
$(1,-3,-6,7,-10,11,-16,18,18,-20)$	10	10	0	18	2	−9	4	0	1	0
$(1,-4,5,-6,-6,10,-14,15,20,-21)$	10	10	0	−6	1	**6**	4	0	1	0
$(2,-3,-6,7,12,-14,-14,17,20,-21)$	10	10	0	−14	1	**14**	4	1	1	1
$(3,6,6,-7,8,8,-14,-14,-17,21)$	10	10	0	−14	1	**14**	4	1	2	1

48 type of representative solutions

Solution	N	N_{chiral}	m	ν	δ	s	N_D	N_M	G_D	G_M
$(8, 8, 9, 10, 10, -13, -18, -18, -27, 31)$	10	10	0	−18	1	18	4	1	2	1
$(1, 1, 1, -2, -2, -5, -5, 6, 6, 7, -8)$	11	8	1	−2	1	1	3	0	2	0
$(1, -2, -2, -2, -3, 4, 4, -5, 6, 7, -8)$	11	8	−2	4	1	−2	3	1	1	1
$(1, 1, 2, 2, 2, -4, -4, 7, -8, -9, 10)$	11	8	2	−4	1	2	2	2	1	2
$(2, 2, 2, -4, -4, -5, 7, -8, 9, 10, -11)$	11	8	2	−4	1	2	3	0	1	0
$(1, -2, -3, -3, -3, 5, 5, -7, 8, 10, -11)$	11	8	−3	5	2	−1	3	0	1	0
$(3, 3, 3, -4, -4, 7, 7, -8, -9, -9, 11)$	11	8	3	−9	2	3	3	0	2	0
$(1, 3, 5, -6, -6, -6, 8, -9, 12, 12, -14)$	11	8	−6	12	1	**−6**	3	1	1	1
$(1, -2, 6, 6, 6, -7, 8, -9, -12, -12, 15)$	11	8	6	−12	1	**6**	3	0	1	0
$(1, 3, 3, 6, 6, 6, -7, -10, -12, -12, 16)$	11	8	6	−12	1	**6**	2	2	1	2

48 type of representative solutions

Solution	N	N_{chiral}	m	ν	δ	s	N_D	N_M	G_D	G_M
$(1,-2,-2,-2,3,3,4,4,-5,-5,-5,6)$	12	9	−5	−2	1	7	3	0	2	0
$(1,1,-3,4,5,5,5,-6,-7,-7,-8,10)$	12	9	5	−7	1	2	3	2	1	2
$(1,1,1,-2,4,-7,-7,-7,8,9,9,-10)$	12	9	−7	9	1	−2	2	3	1	3
$(1,1,-3,-3,-5,-5,-5,7,7,7,9,-11)$	12	9	−5	7	1	−2	3	2	2	2
$(1,-3,-3,-3,4,6,7,9,-10,-10,-10,12)$	12	9	−3	−10	1	13	3	0	1	0
$(1,1,1,3,3,-5,7,7,-11,-11,-11,15)$	12	9	1	−11	1	10	3	1	2	1
$(1,1,1,3,5,5,-5,5,-9,-9,-13,15)$	12	9	5	−9	2	2	2	3	1	3
$(1,-2,-2,3,6,-10,-10,-10,13,14,14,-17)$	12	9	−10	14	1	−4	4	2	2	2
$(1,-3,9,-11,-13,-13,-13,15,15,15,21,-23)$	12	9	−13	15	1	−2	3	1	1	1

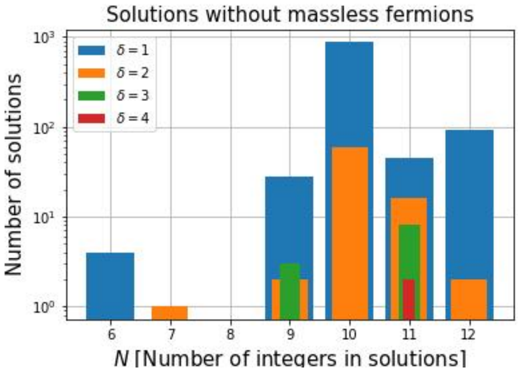


Figure: Distribution of solutions with N integers to the Diophantine equations which allow the effective Dirac neutrino operator at D-4 $+\,\delta$.

Multi-component dark matter I

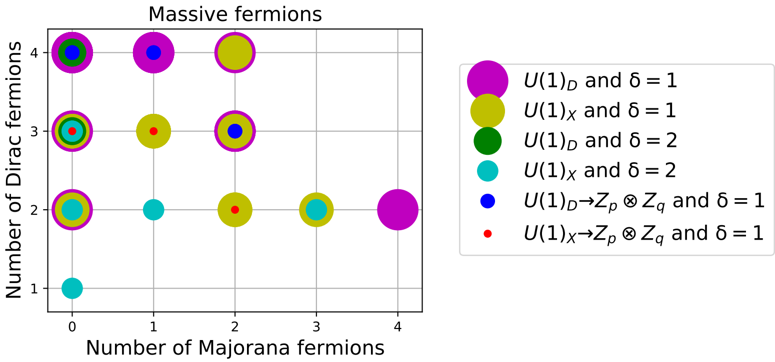


Figure: Number of massive Dirac and Majorana fermions.

Multi-component dark matter II

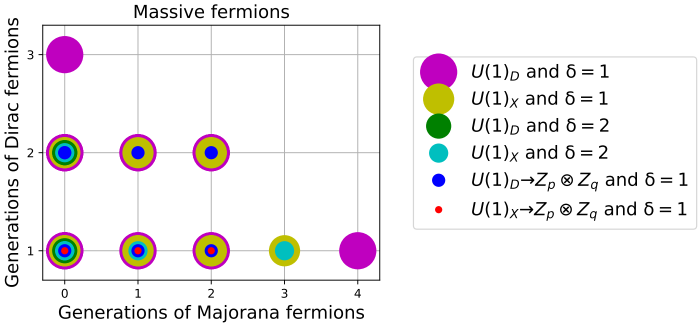


Figure: Same as Fig. 6 but for number generations of massive Dirac and Majorana fermions in each type of the 48 types of solutions of the full set of solutions in Fig. 2.

Solution: $(3, 3, 3, -5, -5, -7, 8)$

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$U(1)_{B-L}$
L_i	**2**	$-1/2$	$-L$	-1
e_{Ri}	**1**	-2	$3-2L$	-1
$\nu_{R\alpha}$	**1**	0	-5	$-5/3$
ψ_1	**1**	0	-7	$-7/3$
ψ_2	**1**	0	8	$8/3$
H	**2**	$1/2$	$L-3$	0
S	**1**	0	1	$1/3$
σ_1^+	**1**	$+2$	$2L$	2
σ_2^+	**1**	$+2$	$-(2-2L)$	$4/3$

Table: Charges for last solution. $i = 1, 2, 3$, $\alpha = 1, 2, 3$. Note that $\left(\omega_n^d\right)^* = \omega_n^{-d} = \omega_n^{n-d}$.

Dirac Zee Mechanism for Dirac Neutrino Masses

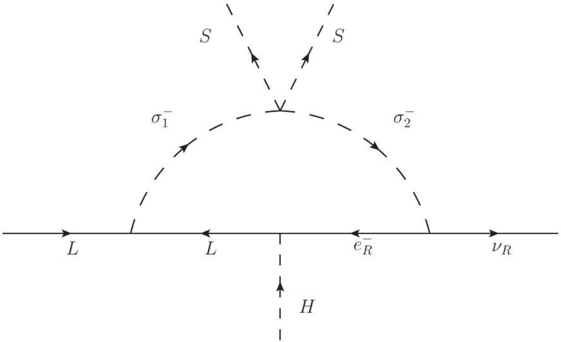


Figure: Diagram with the X-charge flux of the fields in the one-loop Dirac Zee model which realizes the effective Dirac neutrino mass operator at $d = 6$

Spin Independent Cross Section for Direct Detection

The corresponding spin-independent direct detection cross section of dark matter per nucleon is:

$$\sigma_{\psi N} = \frac{\mu_N^2}{\pi} \frac{g_X^4}{M_{Z'}^4} (\psi_1 - \psi_2)^2 L^2 , \tag{2}$$

where $\psi_1 = -7/3$, $\psi_2 = 8/3$ and $L = 1$, are the $\mathrm{U}(1)_{B-L}$ charges in and:

$$\mu_N = \frac{M_N M_\Psi}{M_N + M_\Psi} , \tag{3}$$

is the reduced mass.

The cross section can be written as

$$\sigma_{\psi N}^{\mathrm{SI}} \, (\mathrm{cm}^2) = 4.8 \times 10^{-47} \left(\frac{\mu_N}{1\mathrm{GeV}} \right)^2 \left(\frac{0.1}{g_X} \right)^4 \left(\frac{M_{Z'}}{4 \text{ TeV}} \right)^4 (\psi_1 - \psi_2)^2 L^2 \, \mathrm{cm}^2 .$$

Relic Abundance

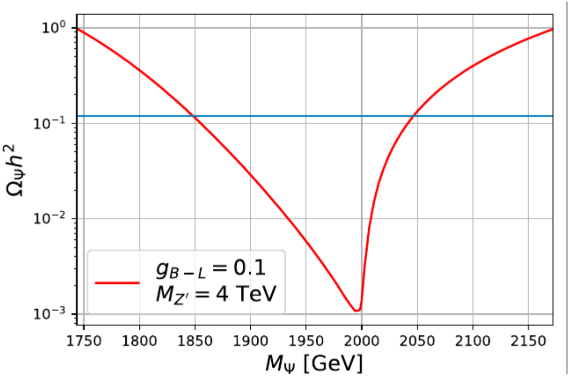


Figure: Dirac fermionic Dark matter relic density as a function of its mass for $U(1)_{B-L}$ with copuling $g_{B-L} = g_X = 0.1$, $M_{Z'} = 4\text{TeV}$, $m_S = 5\text{TeV}$.

Conclusions

We found around one thousand solutions to the anomaly free conditions of local Abelian extensions of the SM with N integers and N_{chiral} right handed SM-singlet fermions which include the ones forming the effective set of Dirac neutrinos and a dark sector with massive fermions. We classify the solutions in 48 types depending if the symmetry is dark ($N_{\text{chiral}} = N$) or active ($N_{\text{chiral}} = N - 3$), the dimension of the effective Dirac neutrino mass operator ($d = 4 + \delta$), the number of independent dark matter candidates, and the number of generations of each massive fermion.