Flavor violating ℓ_i decay into ℓ_j and a light gauge boson χ

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Outline

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Lepton Flavor Violation (LFV)

$$\left.\begin{array}{l} SM \\ m_\nu = 0 \end{array}\right\} \stackrel{\text{invariant}}{\Longrightarrow} \left[U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau\right]_{\text{global}}$$

- In the original SM with massless neutrinos ⇒ conservation of LF and LN.

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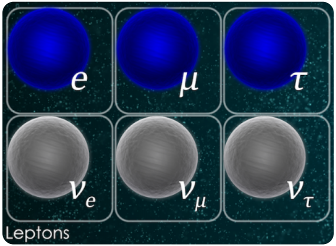

Q_e	$+1$	0	0
Q_μ	0	$+1$	0
Q_τ	0	0	$+1$

and negative for $\bar{\ell}$ and $\bar{\nu}_\ell$

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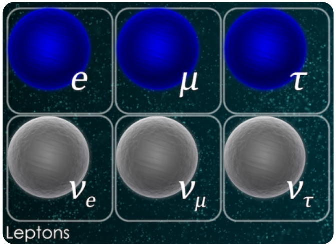

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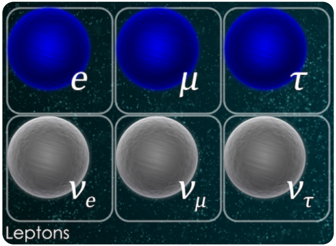

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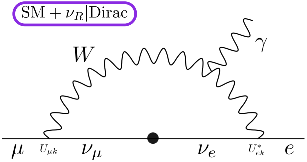
- Neutrino oscillations $\Rightarrow$ Neutrino masses are non-zero $\Rightarrow$ LFV.
- SM minimally extended with ν masses $\Rightarrow$ Unobservable cLFV (GIM-like suppression).

SM (Standard Model) , cLFV (charged LFV)

cLFV in $SM + \nu_R|$Dirac

$$\mathcal{B}r(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3} U_{\mu k}U^*_{ek}\frac{\Delta m^2_{\nu k}}{M^2_W}\right|^2 \sim 10^{-54}$$

T. P. Cheng & L. F. Li, ′77



- Strongly suppressed by a GIM-like mechanism and their tiny fraction on $\Delta m^2_\nu / M^2_W$.

Other SM Predictions:

$\mathcal{B}r(Z \to \ell\ell') \sim 10^{-54}$ J. I. Illana & T. Riemann, ′01
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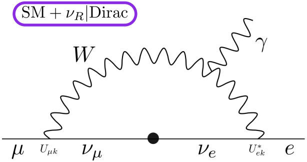

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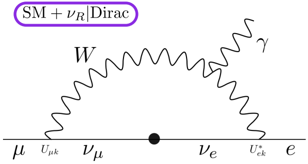

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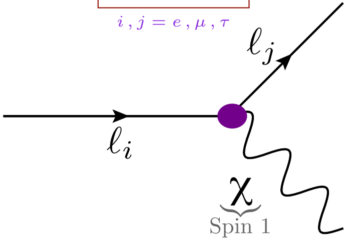
Our Focus

$$\boldsymbol{\ell_i \to \ell_j \chi}$$

$i\,,j = e\,,\mu\,,\tau$



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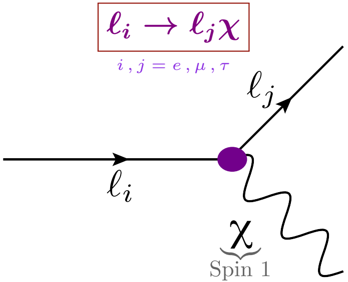

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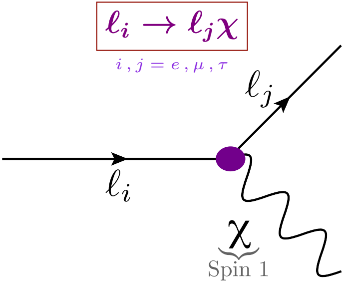

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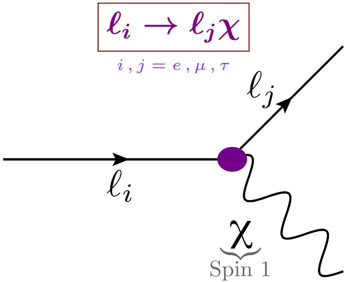

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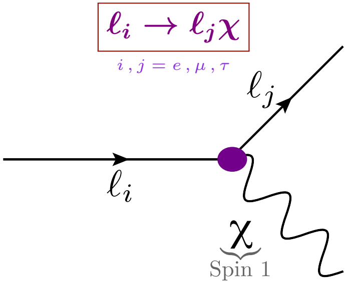

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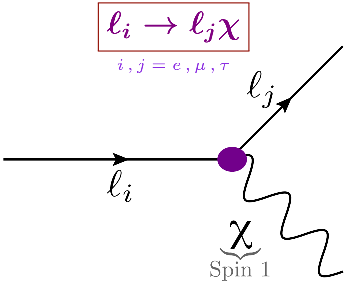

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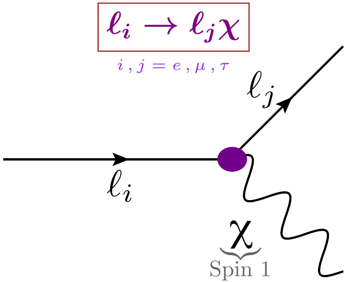

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- $\ell_i \to \ell_j\chi\cdots\chi$ could contribute significantly to the total decay width $\Rightarrow$ "hyperphoton catastrophe" L. B. Okun *et al.*, Phys.Lett.B 78 (1978) 597-600

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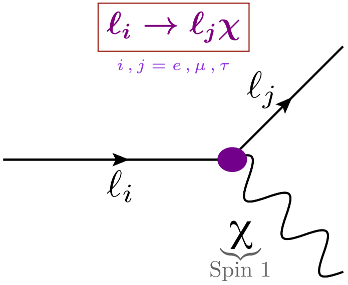

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We will show that in a renormalizable and gauge invariant theory the rate does not diverge when $m_\chi \to 0$.

Why effective LFV for $\ell_i \to \ell_j \chi$?

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- Effective Lagrangians offer the most general description of Physics that has not been resolved yet.
- Any χ can also be the mediator of SM-DM interactions.
- A spin 1 boson, like χ, could be a type of dark photon (therefore DM candidate) that mediates LFV.

Outline

Effective Theory $\ell_i(p_i) \to \ell_j(p_j)\chi(p_\chi)$

The transition amplitude is given by $M = \bar{u}(p_j)\Gamma^\alpha(p_i, p_j)u(p_i)\epsilon^*_\alpha(p_\chi)$

$$\Gamma^\alpha = \left(\gamma^\alpha - \frac{\not{p}_\chi p_\chi^\alpha}{p_\chi^2}\right) F_1(p_\chi^2) + i\frac{\sigma^{\alpha\beta} p_{\chi\beta}}{m_i + m_j} F_2(p_\chi^2) + \frac{2p_\chi^\alpha}{m_i + m_j} F_3(p_\chi^2) +$$
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- $F_i(p^2_\chi)$ and $G_i(p^2_\chi)$ dimensionless scalar form factors
- The Ward identities imply that $p^\alpha_\chi \cdot \epsilon^*_\alpha(p_\chi) = 0$

Effective Field Theory (EFT) $\ell_i \to \ell_j \chi$

The decay rate can be expressed in terms of four form factors

$$\Gamma(\ell_i \to \ell_j \chi) = \frac{\lambda^{1/2}[m_i^2, m_j^2, m_\chi^2]}{16\pi m_i} \left[\left(1 - \frac{m_j}{m_i}\right)^2 \left(1 - \frac{m_\chi^2}{(m_i - m_j)^2}\right) \left(2\left|F_1(m_\chi^2) - F_2(m_\chi^2)\right|^2 \right.\right.$$
$$\left. + \left|F_1(m_\chi^2)\frac{(m_i + m_j)}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{(m_i + m_j)}\right|^2\right) + \left(1 + \frac{m_j}{m_i}\right)^2 \left(1 - \frac{m_\chi^2}{(m_i + m_j)^2}\right)$$
$$\left.\left(2\left|G_1(m_\chi^2) - G_2(m_\chi^2)\frac{(m_i - m_j)}{(m_i + m_j)}\right|^2 + \left|G_1(m_\chi^2)\frac{(m_i - m_j)}{m_\chi} + G_2(m_\chi^2)\frac{m_\chi}{(m_i + m_j)}\right|^2\right)\right]$$

$\lambda[m_j^2, m_i^2, m_\chi^2]$ is the usual Källén function

$\lambda[x, y, z] \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$

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In an EFT approach, great care should be taken when considering decays into ultralight gauge bosons, since in a gauge invariant and renormalizable theory one generically expects the rate of $\ell_i \to \ell_j \chi$ to be finite

Outline

Tree Level Model

The particle content and the corresponding spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$ are

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}
spin	$1/2$	$1/2$	$1/2$	$1/2$	0	0	0	0
$SU(2)_L$	2	2	1	1	2	2	2	2
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	Y_{11}	Y_{11}	Y_{21}	Y_{21}
$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

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$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

$L_i = (\nu_{L_i}, e_{L_i})$ and e_{R_i}, $i = 1, 2$, denote the SM $SU(2)_L$ lepton doublets and singlets, respectively.

$\Downarrow$

We have restricted ourselves to the two generation case, although the extension to three generations is straightforward

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$SU(2)_L$	2	2	1	1	2	2	2	2
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	Y_{11}	Y_{11}	Y_{21}	Y_{21}
$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

- ϕ_{jk} complex scalar fields and doublets under $SU(2)_L$. We assume that the hypercharge $Y_{jk} = 1/2$ and charge under $U(1)_\chi$ $q_{\phi_{jk}} = q_{L_j} - q_{e_k}$.
- We also assume that ϕ_{jk} acquire a vacuum expectation value $\Rightarrow$ $\langle \phi_{jk} \rangle = v_{jk}$
- We need to allow for generation dependent charges under $U(1)_\chi$.

Tree Level Model

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}
spin	1/2	1/2	1/2	1/2	0	0	0	0
$SU(2)_L$	**2**	**2**	**1**	**1**	**2**	**2**	**2**	**2**
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	Y_{11}	Y_{11}	Y_{21}	Y_{21}
$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

The kinetic term is:

$$\mathcal{L}_{\text{kin}} = \sum_{j=1}^{2} (i\overline{L}_j \not{D} L_j + i\overline{e}_{R_j} \not{D} e_{R_j}) + \sum_{j,k=1}^{2} (D_\mu \phi_{jk})^\dagger (D^\mu \phi_{jk}) \ ,$$

where D_μ denotes the covariant derivative

$$D_\mu = \partial_\mu + igW_\mu^a T_a + ig' Y B_\mu + ig_\chi q \chi_\mu \quad \text{for the } SU(2)_L \text{ doublets} \ ,$$
$$D_\mu = \partial_\mu + ig' Y B_\mu + ig_\chi q \chi_\mu \quad \text{for the } SU(2)_L \text{ singlets} \ ,$$

with g, g' and g_χ the coupling constants of $SU(2)_L$, $U(1)_Y$ and $U(1)_\chi$ respectively.

Tree Level Model

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}
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$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	Y_{11}	Y_{11}	Y_{21}	Y_{21}
$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

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The Yukawa interaction term is:

$$-\mathcal{L}_{\text{Yuk}} = \sum_{j,k=1}^{2} y_{jk} \overline{L}_j \phi_{jk} e_{R_k} + \text{h.c.}$$

Tree Level Model

The non-zero $\langle \phi_{jk} \rangle = v_{jk}$ generate a mass for the χ boson:

$$\mathcal{L}_{\text{kin}} \supset \boxed{\sum_{j,k=1}^{2} (D_\mu \phi_{jk})^\dagger (D^\mu \phi_{jk})}$$

$$\Downarrow$$

$$m_\chi^2 = g_\chi^2 (q_{\phi_{11}}^2 v_{11}^2 + q_{\phi_{12}}^2 v_{12}^2 + q_{\phi_{21}}^2 v_{21}^2 + q_{\phi_{22}}^2 v_{22}^2) \, .$$

$\langle \phi_{jk} \rangle = v_{jk}$ generates a mass term for the charged leptons. From $\boxed{\mathcal{L}_{\text{Yuk}}}$ in the mass eigenstate basis, assuming that $m_\mu \gg m_e$

$$m_\mu^2 \simeq y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2 \, ,$$

$$m_e^2 \simeq \frac{(y_{11} v_{11} y_{22} v_{22} - y_{12} v_{12} y_{21} v_{21})^2}{y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2} \, ,$$

$$\sin 2\theta_L \simeq -2 \frac{(y_{11} v_{11} y_{21} v_{21} + y_{12} v_{12} y_{22} v_{22})}{y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2} \, ,$$

$$\sin 2\theta_R \simeq -2 \frac{(y_{11} v_{11} y_{12} v_{12} + y_{21} v_{21} y_{22} v_{22})}{y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2} \, .$$

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Tree Level Model

We recast the kinetic Lagrangian in terms of the mass eigenstates, and we find flavor violating terms of the form

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$$D_\mu \supset i g_\chi q \chi_\mu$$

$\Downarrow$ After basis change to the mass eigenstate basis

$$\Downarrow$$

$$-\mathcal{L}_{\text{kin}} \supset \overline{e_R} i g_{e\mu}^{RR} \gamma^\rho \chi_\rho \mu_R + \overline{e_L} i g_{e\mu}^{LL} \gamma^\rho \chi_\rho \mu_L + \text{h.c.}\,, \quad \text{with}$$

$$g_{e\mu}^{RR} = g_\chi (q_{e_1} - q_{e_2}) \sin\theta_R \cos\theta_R \,, \quad \text{and} \quad g_{e\mu}^{LL} = g_\chi (q_{L_1} - q_{L_2}) \sin\theta_L \cos\theta_L \,.$$

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The rate for $\mu \to e\chi$ then reads:

$$\Gamma(\mu \to e\chi) \simeq \frac{m_\mu}{32\pi} \left(\left| g_{e\mu}^{LL} \right|^2 + \left| g_{e\mu}^{RR} \right|^2 \right) \left(2 + \frac{m_\mu^2}{m_\chi^2} \right) \left(1 - \frac{m_\chi^2}{m_\mu^2} \right)^2 \,.$$

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Assuming $y_{22} \gg y_{11} \gg y_{12}, y_{21}$, $v_{ij} = v$, and $q_{\phi_{ij}} = Q$ the relevant parameters are:

$$m_\mu^2 \simeq y_{22}^2 v^2, \qquad m_e^2 \simeq y_{11}^2 v^2, \qquad m_\chi^2 \simeq 4 g_\chi^2 Q^2 v^2$$
$$\sin 2\theta_L \simeq -2\frac{y_{12}}{y_{22}}, \qquad \sin 2\theta_R \simeq -2\frac{y_{21}}{y_{22}} .$$

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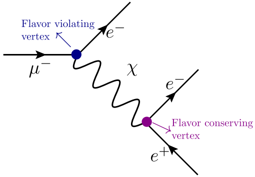
Therefore, the rate for $\mu \to e\chi$ in the limit $m_\chi \to 0$ is given by

$$\Gamma(\mu \to e\chi)\Big|_{m_\chi \to 0} \simeq \frac{m_\mu}{32\pi} \frac{g_\chi^2}{y_{22}^2} \left(2 + \frac{y_{22}^2}{4 g_\chi^2 Q^2}\right) \left(1 - \frac{4 g_\chi^2 Q^2}{y_{22}^2}\right)^2 \left[y_{12}^2 (q_{L_1} - q_{L_2})^2 + y_{21}^2 (q_{e_1} - q_{e_2})^2\right] .$$

The decay $\mu^- \to e^- e^+ e^-$ is generated in this model at tree-level via the exchange of a virtual χ.



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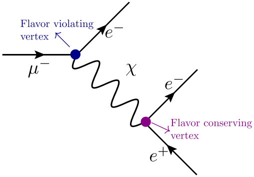

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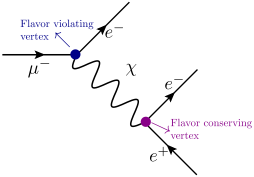
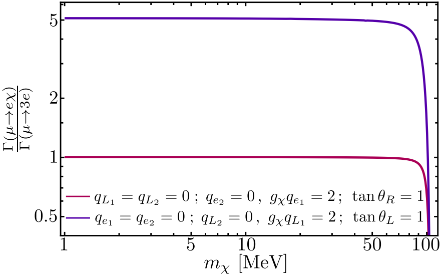


The ratio between $\Gamma(\mu \to e\chi)$ and $\Gamma(\mu \to 3e)$ is $\simeq 1$ or $\simeq 5$



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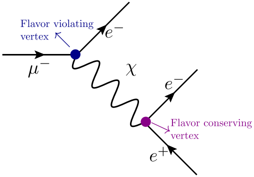

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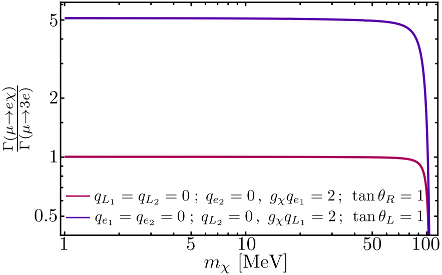
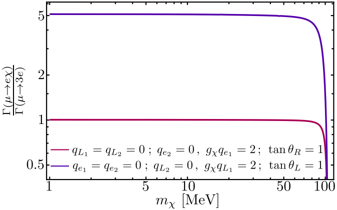


This result can be understood analytically employing the narrow width approximation (NWA)

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at tree level model



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$$\frac{1}{(x-m_\chi^2)^2+m_\chi^2\Gamma_\chi^2}\to\frac{\pi}{m_\chi\Gamma_\chi}\delta(x-m_\chi^2)$$

$$\chi\to e^-e^+\,,\bar{\nu}_{L_1}\nu_{L_1}\,,\bar{\nu}_{L_2}\nu_{L_2}$$

$$\Gamma_\chi=\frac{m_\chi}{24\pi}\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2\right)$$

$$\Gamma(\mu\to 3e)\simeq\frac{m_\mu}{32\pi}\frac{\left(|g_{e\mu}^{LL}|^2+|g_{e\mu}^{RR}|^2\right)\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2\right)}{|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2}\left(2+\frac{m_\mu^2}{m_\chi^2}\right)\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2$$
$$+\frac{m_\chi}{64\pi}\left(|g_{ee}^{LL}|^2|g_{e\mu}^{LL}|^2+|g_{ee}^{RR}|^2|g_{e\mu}^{RR}|^2\right)\frac{m_\chi}{m_\mu}\left(1-2\frac{m_\chi^2}{m_\mu^2}\right).$$

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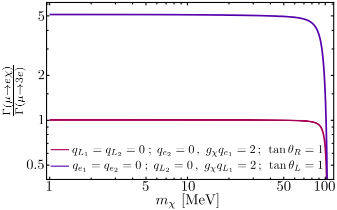

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$$\Gamma_\chi=\frac{m_\chi}{24\pi}\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2\right)$$

finite in the limit $m_\chi\to 0$

⇑

$$\Gamma(\mu\to 3e)=\frac{m_\mu}{32\pi}\frac{\left(|g_{e\mu}^{LL}|^2+|g_{e\mu}^{RR}|^2\right)\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2\right)}{|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2}\left(2+\boxed{\frac{m_\mu^2}{m_\chi^2}}\right)\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2$$

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⇒ subdominant contribution for $m_\chi\ll m_\mu$

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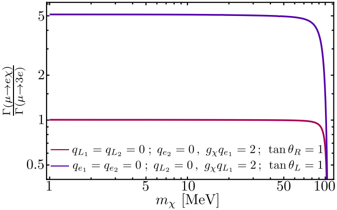

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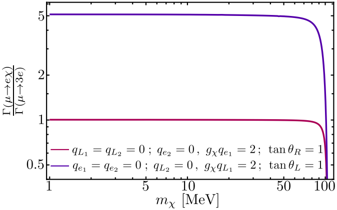
$$\Gamma(\mu\to 3e)\backsimeq\frac{m_\mu}{32\pi}\frac{\left(|g_{e\mu}^{LL}|^2+|g_{e\mu}^{RR}|^2\right)\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2\right)}{|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2}\left(2+\frac{m_\mu^2}{m_\chi^2}\right)\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2$$

$$\Gamma(\mu\to e\chi)\backsimeq\frac{m_\mu}{32\pi}\left(|g_{e\mu}^{LL}|^2+|g_{e\mu}^{RR}|^2\right)\left(2+\frac{m_\mu^2}{m_\chi^2}\right)\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2$$

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at tree level model



This result can be understood analytically employing the NWA

$$\frac{1}{(x-m_\chi^2)^2+m_\chi^2\Gamma_\chi^2}\to\frac{\pi}{m_\chi\Gamma_\chi}\delta(x-m_\chi^2)$$

$$\chi\to e^-e^+\,,\bar\nu_{L_1}\nu_{L_1}\,,\bar\nu_{L_2}\nu_{L_2}$$

$$\Gamma_\chi=\frac{m_\chi}{24\pi}\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2\right)$$

$$\Gamma(\mu\to 3e)\backsimeq\frac{m_\mu}{32\pi}\frac{\left(|g_{e\mu}^{LL}|^2+|g_{e\mu}^{RR}|^2\right)\left(|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2\right)}{|g_{ee}^{LL}|^2+|g_{ee}^{RR}|^2+|g_\chi q_{L_1}|^2+|g_\chi q_{L_2}|^2}\left(2+\frac{m_\mu^2}{m_\chi^2}\right)\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2$$

$$\Gamma(\mu\to e\chi)\backsimeq\frac{m_\mu}{32\pi}\left(|g_{e\mu}^{LL}|^2+|g_{e\mu}^{RR}|^2\right)\left(2+\frac{m_\mu^2}{m_\chi^2}\right)\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2$$

$$\boxed{\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}\simeq 1\quad\text{or}\quad 5}$$

Outline

1. Motivation
2. Effective Theory
3. Tree Level Model
4. One Loop Level Model
5. Conclusions

One Loop Level Model

Spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$ of the particles of the model

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ	ψ	η
spin	1/2	1/2	1/2	1/2	0	1/2	0
$SU(2)_L$	2	2	1	1	2	1	1
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	$+1/2$	Y_ψ	Y_η
$U(1)_\chi$	q_L	q_L	q_e	q_e	q_ϕ	q_ψ	q_η

- To violate the lepton flavor, we introduce a new Dirac fermion ψ and a new complex scalar η.
- We assume that $q_e = q_\psi + q_\eta$ and $Y_e = Y_\psi + Y_\eta$.
- We also assume that ϕ acquires a vacuum expectation value and has $q_\phi = q_L - q_e$, but η does not.

$L_i = (\nu_{L_i}, e_{L_i})$ and e_{R_i}, denote the SM $SU(2)_L$ lepton doublets and singlets, respectively.

One Loop Level Model

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ	ψ	η
spin	$1/2$	$1/2$	$1/2$	$1/2$	0	$1/2$	0
$SU(2)_L$	2	2	1	1	2	1	1
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	$+1/2$	Y_ψ	Y_η
$U(1)_\chi$	q_L	q_L	q_e	q_e	q_ϕ	q_ψ	q_η

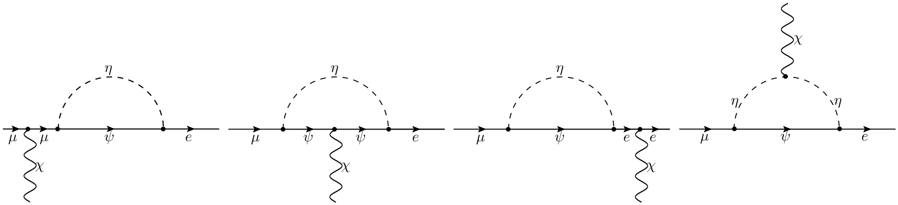
The interaction terms with the massive gauge boson χ in the mass eigenstates:

$$\mathcal{L} \supset -ig_\chi q_L \left(\bar{e}_L\gamma^\nu e_L + \bar{\mu}_L\gamma^\nu \mu_L + \bar{\nu}_{L1}\gamma^\nu \nu_{L1} + \bar{\nu}_{L2}\gamma^\nu \nu_{L2}\right)\chi_\nu - ig_\chi q_e \left(\bar{e}_R\gamma^\nu e_R + \bar{\mu}_R\gamma^\nu \mu_R\right)\chi_\nu$$
$$- ig_\chi q_\psi \overline{\psi}\gamma^\nu \psi \chi_\nu - iq_\eta g_\chi \left[\eta^*(\partial^\nu \eta) - (\partial^\nu \eta^*)\eta\right]\chi_\nu \,,$$

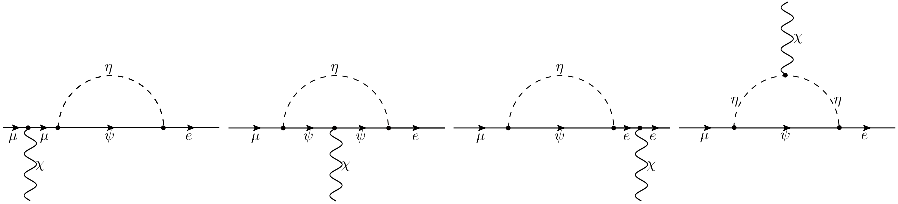
as well as a Yukawa coupling to the right-handed leptons:

$$\mathcal{L} \supset h_e \bar{e}_R \eta \psi + h_\mu \bar{\mu}_R \eta \psi + \text{h.c.}\,,$$

$\mu \to e\chi$ at the one loop level



$\mu \to e\chi$ at the one loop level

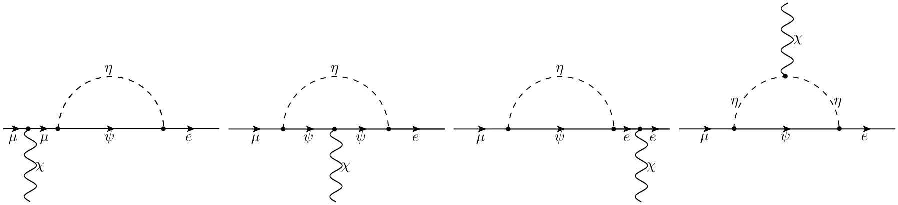


The form factors are finite and read:

$$F_1(m_\chi^2) = \quad G_1(m_\chi^2) = \frac{g_\chi y'_e y'_\mu}{384\pi^2} \boxed{\frac{m_\chi^2}{M_\eta^2}} \left[q_\eta \mathcal{F}_{1\eta}\left(\frac{M_\psi^2}{M_\eta^2}\right) + q_\psi \mathcal{F}_{1\psi}\left(\frac{M_\psi^2}{M_\eta^2}\right)\right],$$

$$F_2(m_\chi^2) = -G_2(m_\chi^2) = \frac{g_\chi y'_e y'_\mu}{384\pi^2} \boxed{\frac{m_\mu^2}{M_\eta^2}} \left[q_\eta \mathcal{F}_{2\eta}\left(\frac{M_\psi^2}{M_\eta^2}\right) + q_\psi \mathcal{F}_{2\psi}\left(\frac{M_\psi^2}{M_\eta^2}\right)\right],$$

$\mu \to e\chi$ at the one loop level



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where

$$\mathcal{F}_{1\eta}(x) = \frac{-2 + 9x - 18x^2 + x^3\,(11 - 6\ln x)}{3\,(1-x)^4}, \qquad \mathcal{F}_{2\eta}(x) = \frac{1 - 6x + 3x^2(1 - 2\ln x) + 2x^3}{(1-x)^4},$$

$$\mathcal{F}_{1\psi}(x) = \frac{16 - 45x + 36x^2 - 7x^3 + 6\,(2-3x)\ln x}{3\,(1-x)^4}, \quad \mathcal{F}_{2\psi}(x) = \frac{-2 - 3x(1 + 2\ln x) + 6x^2 - x^3}{(1-x)^4}.$$

$\mu \to e\chi$ at the one loop level

The decay rate reads for $\mu \to e\chi$:

$$\Gamma(\mu \to e\chi) \simeq \frac{m_\mu}{8\pi}\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2 \left[\left|F_1(m_\chi^2)\frac{m_\mu}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2\left|F_1(m_\chi^2) - F_2(m_\chi^2)\right|^2\right]$$

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The form factor F_1 (and G_1) is proportional to m_χ^2/M_η^2

$\mu \to e\chi$ at the one loop level

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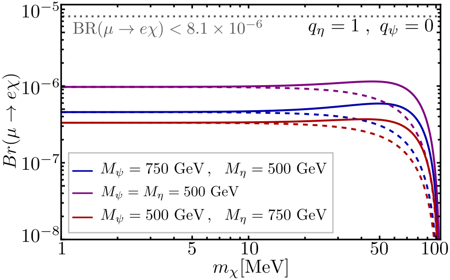
The form factor F_1 (and G_1) is proportional to m_χ^2/M_η^2

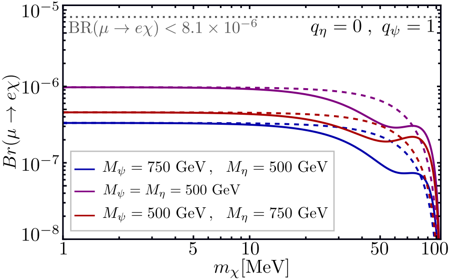
The factors $1/m_\chi$ from the emission of the longitudinal polarization cancel with the factors m_χ^2 implicit in the form factor F_1, yielding a finite rate for $\mu \to e\chi$ in the limit $m_\chi \to 0$.

We are assuming $M_\eta, M_\psi \gg m_\mu$, it follows that the rate in the limit $m_\chi \to 0$ will depend mostly on the form factors F_2 and G_2.

$\mu \to e\chi$ at the one loop level

Branching ratio of the process $\mu \to e\chi$ as a function of m_χ for the one loop model. For Simplicity, we took the Yukawa-type couplings equal to one.

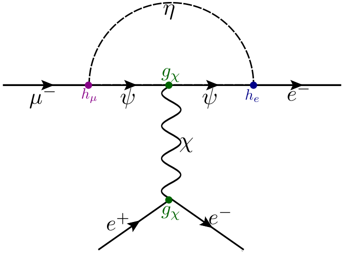




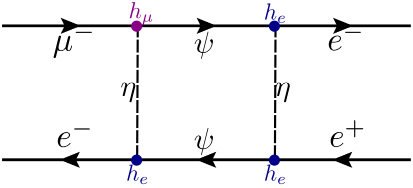
The solid lines show the full result, while the dashed lines assume $F_1 = G_1 = 0$. As apparent for the plot, while for $m_\chi \ll m_\mu$ the form factors F_1 and G_1 can be neglected, they modify the rate when $m_\chi / m_\mu \gtrsim 0.1$, especially close to the threshold.

$\mu^- \to e^- e^- e^+$ at the one loop level

The process $\mu^- \to e^- e^- e^+$ is generated through χ-penguin and through box diagrams.



$\Gamma_{\chi-\text{penguin}}(\mu \to 3e) \sim h_e^2 h_\mu^2 g_\chi^4$

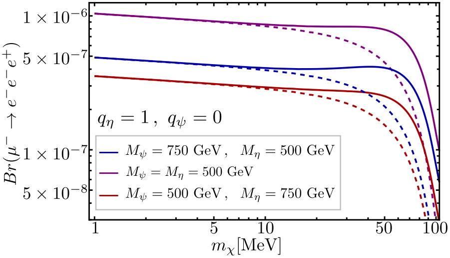


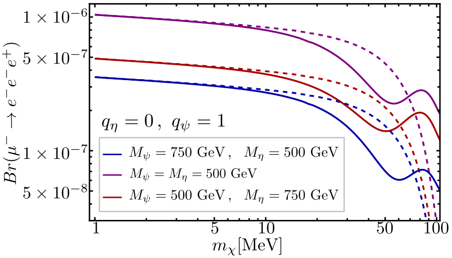
$\Gamma_{box}(\mu \to 3e) \sim h_e^6 h_\mu^2$

Assuming $h_e \ll g_\chi$, the decay will be dominated by the penguin diagrams.

$\mu^- \to e^- e^- e^+$ at the one loop level

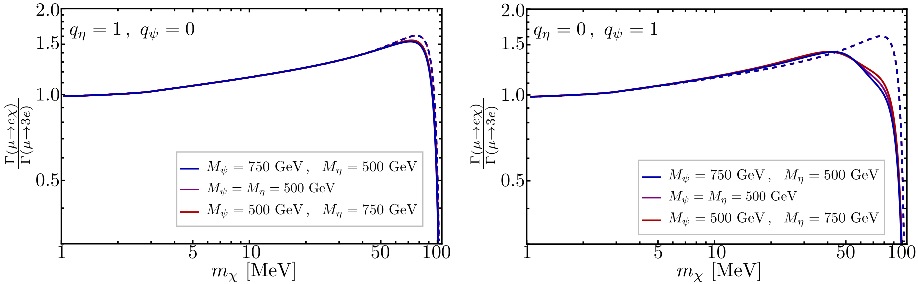
Branching ratio of the process $\mu^- \to e^- e^- e^+$ as a function of m_χ for the one loop model. For Simplicity, we took the Yukawa-type couplings equal to one.





The solid lines show the full result, while the dashed lines assume $F_1 = G_1 = 0$. From SINDRUM Collaboration we have the U.L. $\text{BR}(\mu^- \to e^- e^- e^+) \leq 1.0 \times 10^{-12}$.

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



The ratio is ~ 1, assuming $q_L = 1 + q_e$.

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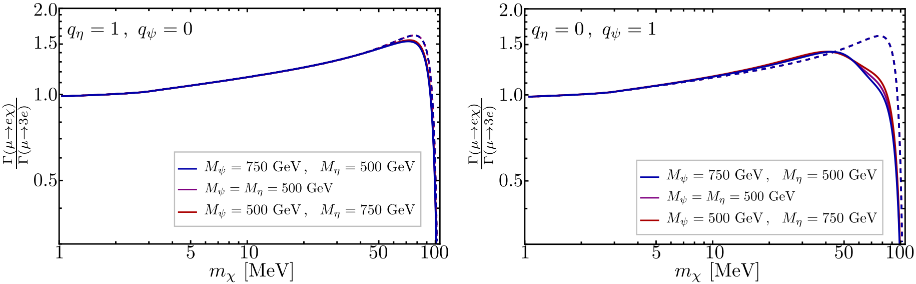
Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



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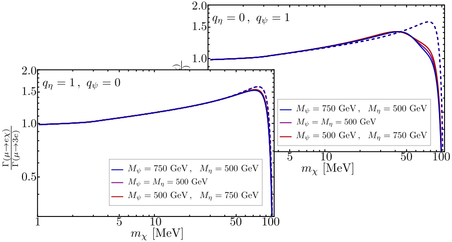
$\Downarrow$

This result can be understood analytically employing the narrow width approximation (NWA)

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



This result can be understood analytically employing the NWA.

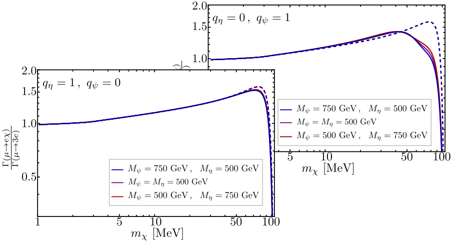
$$\frac{1}{(x-m_\chi^2)^2+m_\chi^2\Gamma_\chi^2} \to \frac{\pi}{m_\chi \Gamma_\chi}\delta(x-m_\chi^2)$$

$$\chi \to e^- e^+, \overline{\nu_{L_1}}\nu_{L_1}, \overline{\nu_{L_2}}\nu_{L_2}$$

$$\Gamma_\chi = \frac{g_\chi^2\ m_\chi}{24\pi}\left(|q_e|^2 + 3|q_L|^2\right)$$

$$\Gamma(\mu\to 3e) \simeq \frac{m_\mu}{4\pi}\frac{|q_e|^2+|q_L|^2}{|q_e|^2+3|q_L|^2}\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2\left[\left|F_1(m_\chi^2)\frac{m_\mu}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2|F_1(m_\chi^2)-F_2(m_\chi^2)|^2\right]$$
$$+\frac{g_\chi^2|q_e|^2}{16\pi}\frac{m_\chi^2}{m_\mu}\left(1-2\frac{m_\chi^2}{m_\mu^2}\right)\left(2\left(|F_1(m_\chi^2)|^2 - F_1(m_\chi^2)F_2(m_\chi^2)\right) + |F_2(m_\chi^2)|^2\frac{m_\chi^2}{m_\mu^2}\right)\,.$$

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



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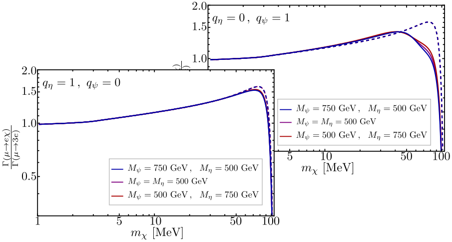
$F_1 \sim m_\chi^2 \Rightarrow$ finite in the limit $m_\chi \to 0$

$\Uparrow$

$$\Gamma(\mu\to 3e) \simeq \frac{m_\mu}{4\pi}\frac{|q_e|^2+|q_L|^2}{|q_e|^2+3|q_L|^2}\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2\left[\left|\boxed{F_1(m_\chi^2)\frac{m_\mu}{m_\chi}} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2|F_1(m_\chi^2)-F_2(m_\chi^2)|^2\right]$$
$$+\frac{g_\chi^2|q_e|^2}{16\pi}\frac{m_\chi^2}{m_\mu}\left(1-2\frac{m_\chi^2}{m_\mu^2}\right)\left(2\left(|F_1(m_\chi^2)|^2 - F_1(m_\chi^2)F_2(m_\chi^2)\right) + |F_2(m_\chi^2)|^2\frac{m_\chi^2}{m_\mu^2}\right).$$

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



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$\Uparrow$

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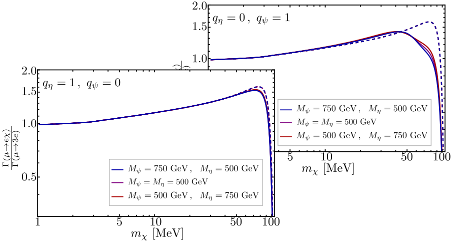
$$+\boxed{\frac{g_\chi^2|q_e|^2}{16\pi}\frac{m_\chi^2}{m_\mu}\left(1-2\frac{m_\chi^2}{m_\mu^2}\right)\left(2\left(|F_1(m_\chi^2)|^2 - F_1(m_\chi^2)F_2(m_\chi^2)\right) + |F_2(m_\chi^2)|^2\frac{m_\chi^2}{m_\mu^2}\right)}\,.$$

$\Rightarrow$ subdominant contribution for $m_\chi \ll m_\mu$.

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



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$$\Gamma(\mu\to e\chi) \simeq \frac{m_\mu}{8\pi}\left(1-\frac{m_\chi^2}{m_\mu^2}\right)^2\left[\left|F_1(m_\chi^2)\frac{m_\mu}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2|F_1(m_\chi^2)-F_2(m_\chi^2)|^2\right]$$

$$\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)} \sim 1$$

Outline

Conclusions

- To investigate the limit $m_\chi \ll m_\mu$ we have constructed two explicit renormalizable models where the decay $\mu \to e\chi$ is generated either at tree level or at the one-loop level. In both cases, we have found a finite rate for $\mu \to e\chi$ in the limit $m_\chi \to 0$.
- For the tree-level model we find that the decay is dominated by coupling terms proportional to γ^μ and $\gamma^5\gamma^\mu$.
- For the one-loop model the decay is mediated by interaction vertices proportional to γ^μ, $\gamma^5\gamma^\mu$, $\sigma^{\mu\nu}p_{\chi\nu}$ and$\gamma^5\sigma^{\mu\nu}p_{\chi\nu}$, although the latter two give the dominant contributions for $m_\chi \to 0$.
- Analogous conclusions apply to $\tau \to (\mu/e)\chi$ and $\tau \to 3e\,, 3\mu$.

Thank you!