Taming hadronisation corrections for collider observables

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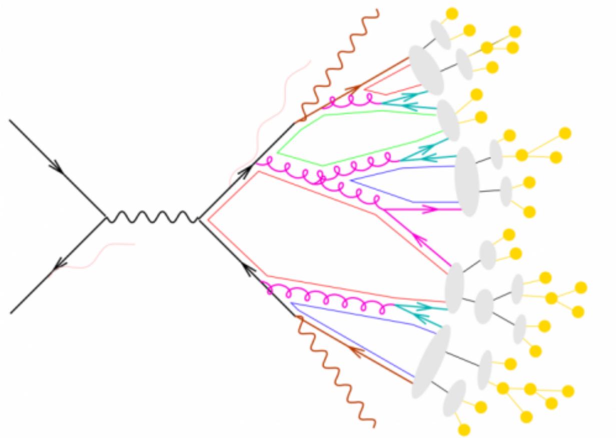
University of Oxford

A life in phenomenology: a conference in honour of Paolo Nason,
University of Milano – Bicocca,
15th September 2022

"*All-orders behaviour and renormalons in top-mass observables*", SFR, Nason, Oleari
"*Infrared renormalons in kinematic distributions for hadron collider processes,*" SFR, Limatola, Nason
"*On linear power corrections in certain collider observables*", Caola, SFR, Limatola, Melnikov, Nason
"*Linear power corrections to e^+e^- shape variables in the 3-jet region*", Caola, SFR, Limatola, Melnikov, Nason, Ozcelik

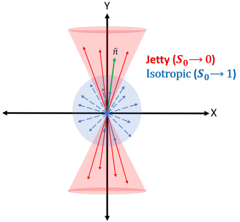
A typical collider event



Ingredients to describe a lepton collision

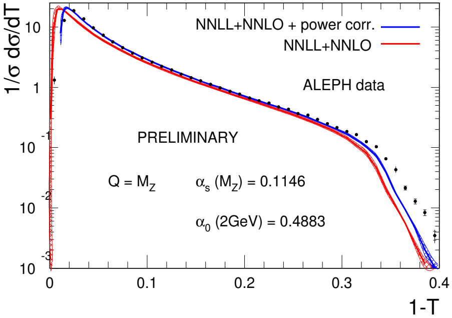
- Hard process ($Q \sim 100$ GeV): fixed order expansion in the strong coupling $\alpha_s(Q)$
- multiple soft and/or collinear emissions, with $Q > k_\perp > \Lambda$, with Λ ~1 GeV. Tools: **analytic resummation** (more accurate) or **parton shower algorithms** (more flexible)
- Hadronization corrections: **phenomenological models** (Lund or cluster) from **MC** event generators, or **analytic models**

Hadronization models for shape observables



- State-of the art most precise calculations (NNLO, NNLL, N^{3}LL, ...) are not interfaced to parton showers: e.g. **Event shapes**!
- Event shapes to perform **precise measurements of α_s**.

[Monni, '12]



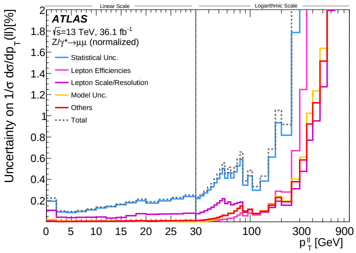
- **Non-perturbative linear power corrections** $\propto \mathbf{1/Q}$ must be provided in order to fit the data!
- **Analytic models:** shift the peturbative prediction by a **constant amount** $\propto 1/Q$

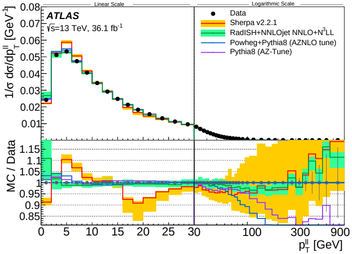
$$\Sigma(O) \to \Sigma(\; O - \underbrace{\mathcal{N}}_{\text{universal}} \; \underbrace{\Delta \mathbf{O}}_{\text{Independent of } O(\Phi)} \;)$$

We need to control linear NP corrections if we want percent or permille precision at Q ≈100 GeV!

Linear power corrections in collider observables

- We need to know when to expect linear power corrections!

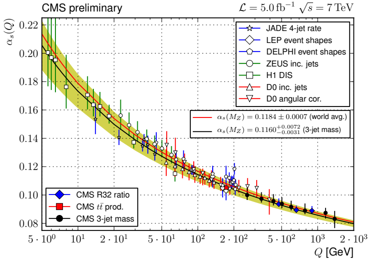




If present in the **transverse momentum of gauge bosons**, we will never match the experimental precision!

Estimating power corrections

- Several sources of non-perturbative corrections, e.g. the **Landau pole Λ** in the QCD coupling constant



$$\alpha_S(Q) = \frac{1}{2b_0 \log\left(\frac{Q}{\Lambda}\right)}; \quad b_0 = \frac{11C_A}{12\pi} - \frac{n_l T_R}{3\pi} > 0$$

which leads to an **instrinsic ambiguity** when integrating over the soft momenta.

$$\int_0^Q dk\, k^{p-1} \alpha_S(k) = \boxed{Q^p \times \frac{p}{2\,b_0} \sum_{n=0}^{\infty} \left(\frac{2\,b_0}{p}\,\alpha_S(Q)\right)^{n+1} n!}$$

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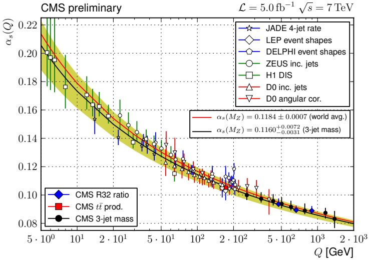

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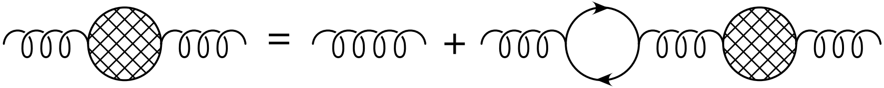
- The ambiguity has to cancel with contributions arising from physics beyond perturbation theory: estimate of **non-perturbative effects**. The smallest term in the series is

$$Q^p \sqrt{\frac{\alpha_s(Q) p\pi}{b_0}} e^{-\frac{p}{2b_0 \alpha_s}} = \boxed{\sqrt{\frac{\alpha_s(Q) p\pi}{b_0}} \Lambda^p}$$

- Ambiguity related to the appearance of the Landau pole can be studied in the large number of flavour n_f limit, which allows to perform all-orders computations exactly.



$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2,\epsilon)-\Pi_{\rm ct}}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} = \alpha_{\rm S}(\mu)\left(-\frac{n_f \mathrm{T_R}}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right] + \mathcal{O}(\epsilon)$$

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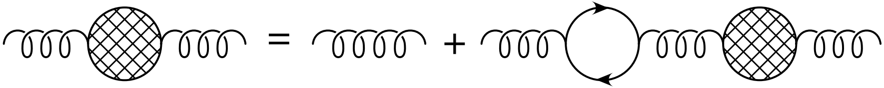

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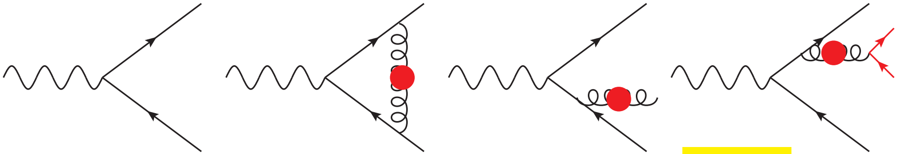
- **naive non-abelianization** at the end of the computation (large b_0)

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} \to \alpha_{\rm S}(\mu)\underbrace{\left(\frac{11\mathrm{C_A}}{12\pi}-\frac{n_l \mathrm{T_R}}{3\pi}\right)}_{b_0}\left[\log\left(\frac{|k^2|}{\mu^2}\right)-i\pi\theta(k^2)-C\right]$$

Large-b_0 approximation for realistic collider processes



$\alpha_{s,\mathrm{eff}}(\lambda)$ [Beneke, '98]

$$O = \int \mathrm{d}\Phi \, \frac{\mathrm{d}\sigma(\Phi)}{\mathrm{d}\Phi} O(\Phi) = O_{\mathrm{LO}} - \frac{1}{\pi b_0} \int_0^\infty \mathrm{d}\lambda \, \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\frac{T(\lambda)}{\alpha_s(\mu)} \right] \overbrace{\arctan\left[\pi \, b_0 \, \alpha_s\left(\lambda e^{-C/2}\right)\right]}^{\alpha_{s,\mathrm{eff}}(\lambda)}$$

- λ can be thought as a gluon **mass** / **virtuality**
- $T(\lambda) = \int \mathrm{d}\Phi_b \, V_\lambda(\Phi_b) O(\Phi_b) + \frac{\lambda^2}{\pi b_0} \int \mathrm{d}\Phi_{q\bar{q}} \, R_{q\bar{q}}(\Phi_{q\bar{q}}) O(\Phi_{q\bar{q}}) \delta(m_{q\bar{q}}^2 - \lambda^2)$
- $T(\lambda) \xrightarrow{\lambda \to 0} O_{\mathrm{NLO}}$

[S.F.R, Nason, Oleari '19]

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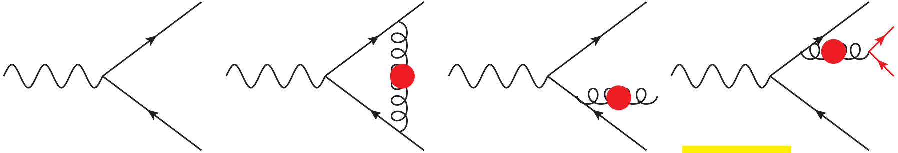

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- If $\boxed{\frac{1}{\alpha_s(\mu)} \frac{dT(\lambda)}{d\lambda}\Big|_{\lambda=0} = -A \neq 0}$, the low-λ contribution leads to ($a \equiv b_0\alpha_s$)

$$\underbrace{\frac{1}{\pi b_0}\arctan(\pi a) + \alpha_s \int_0^1 dz \frac{\pi a z\cos(\pi z/2) - \sin(\pi z/2)}{1+(z\pi a)^2}}_{\text{analytic}} + \underbrace{\frac{1}{\pi b_0} PV \int_0^\infty dt \frac{\exp\left(-\frac{t}{2a}\right)}{1-t}}_{\text{Borel sum + PV for pole}} - 2\underbrace{\frac{1}{2b_0}\exp\left(-\frac{1}{2a}\right)}_{\text{ambiguity} = \frac{\Lambda}{2b_0\mu}},$$

Assessing power corrections from first principles

- **Linear power corrections** in **transverse momentum** of the Z boson [Salam and Slade, JHEP **11** (2021), 220] can limit the **ultimate theoretical precision achievable**

$$\frac{\Lambda}{p_\perp} = \frac{1\text{GeV}}{30\text{GeV}} = \mathbf{3\%}$$

Current **theoretical** err $\approx$ 3%
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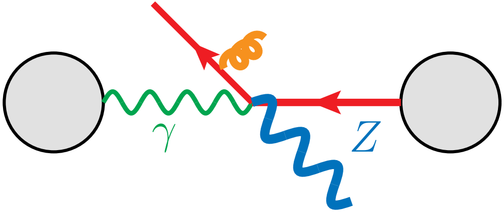

Within our framework, we found no evidence of linear **non-perturbative** power corrections [SFR, Limatola, Nason, JHEP **06** (2021), 018]

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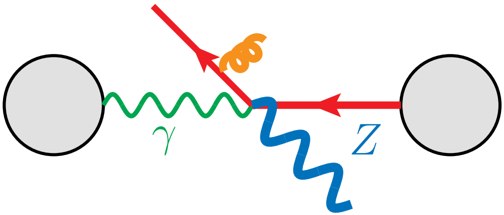

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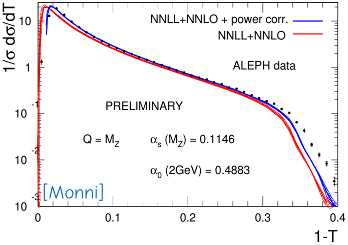
When the ultimate theoretical precision is not spoilt by linear power corrections?

When the observable is **inclusive** with respect to QCD radiation
[Caola, SFR, Limatola, Melnikov, Nason, JHEP **01** (2022), 093]

Hadronization corrections and α_s detarmination from event shapes



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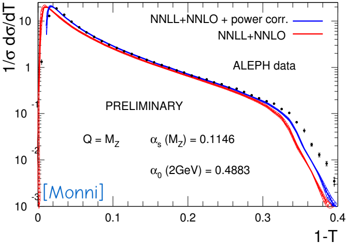

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$$\alpha_s = \mathbf{0.1179(10)} \text{ world average}$$

$$\alpha_s = \mathbf{0.1135(10)} \text{ from } \textbf{Thrust}$$

[Abbate et al., Phys. Rev. D **86** (2012), 094002]

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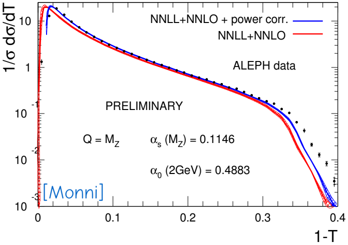

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- ...and assumed to be valid also for $\boxed{V>0}$

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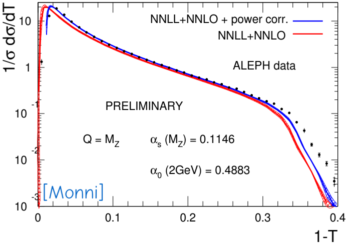

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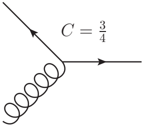
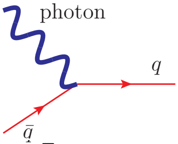
- . . . and assumed to be valid also for $\boxed{V > 0}$
- But for the **C-parameter** [Luisoni, Monni, Salam, Eur. Phys. J. C **81** (2021) no.2, 158]

$$\frac{\text{Linear power correction at } C = 0.75}{\text{Linear power correction at } C = 0} \approx 0.48$$



Revisiting NP corrections for event shapes



- Since we **cannot deal with gluons**, to study NP corrections away from the two-jet limit in the large-n_f limit we consider the process $\mathbf{Z} \to \mathbf{q\bar{q}\gamma}$.
- For many event shapes such as **thrust** and **C-parameter**, collinear contributions are exponentially suppressed, so the leading **soft** approximation is sufficient to compute $T(\lambda)$

$$O_{n+1}(p_1, \ldots, p_n, k) \approx O_n(\tilde{p}_1, \ldots, \tilde{p}_n) + \underbrace{\frac{k_\perp}{Q} f_n(\varphi, \eta, \{\tilde{p}_i\})}_{\Delta O}, \quad \lim_{\eta \to \pm\infty} f_n(\varphi, \eta, \{\tilde{p}_i\}) \propto \exp^{-|\eta|} = 0$$

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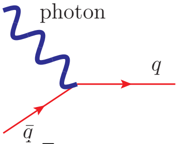

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- With a **smooth mapping** $\Phi_{n+1} \to \tilde{\Phi}_n$ the calculation largely simplifies

$$k = z_1 \tilde{p}_1 + z_2 \tilde{p}_2 + k_\perp$$

$$p_{1,2} \approx (1 - z_{1,2}) \tilde{p}_{1,2}$$

$$\Rightarrow \quad \frac{d\Sigma(O < o)}{d\lambda} = \int d\Phi\, \delta(O(\Phi) - o) \frac{d\sigma}{d\Phi} \left[\mathcal{M} \frac{2 C_F \alpha_s}{\pi} \int \frac{dk_\perp}{k_\perp} dy \frac{d\varphi}{2\pi} \Delta O(k_\perp, \varphi, \eta; \Phi)\, \delta(k_\perp - \lambda) \right]$$

[Caola, S.F.R., Limatola, Melnikov, Nason, '21, +Ozcelik, '22]

where $\mathcal{M}$ is the **Milan factor** [Dokshitzer, Lucenti, Marchesini, Salam, '98] computed in the 2-jet limit!

Revisiting NP corrections for event shapes: realistic QCD

We need to convert our simplified **abelian** calculation.

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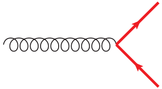
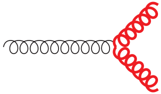
- The Milan factor $\mathcal{M}$ takes into account the difference between the emission of a **soft gluon** of $k_\perp = \lambda$, and the emission of an off-shell gluon decaying in a **pair of quarks** with $m_{q\bar{q}} = \lambda$.



- It is customary in the literature to also include the effect of $\mathbf{g} \to \mathbf{gg}$ splittings.



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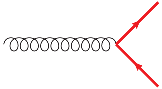

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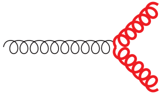
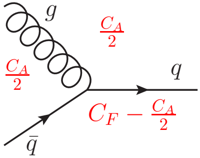
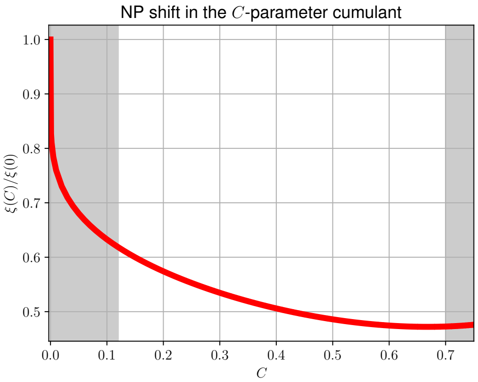


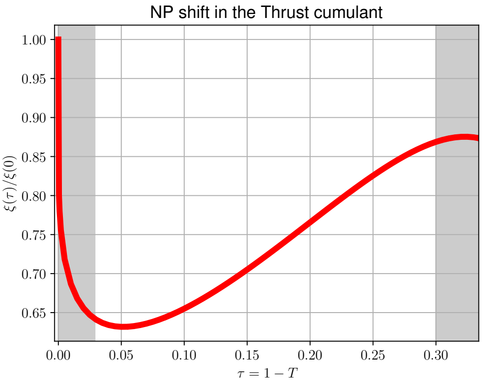


- We assume the same formulae for more complex final states, with C_F replaced by the proper color factor for each dipole

Results

Non-negligible kinematic dependence! [Caola, S.F.R., Limatola, Melnikov, Nason, Ozcelik, '22]





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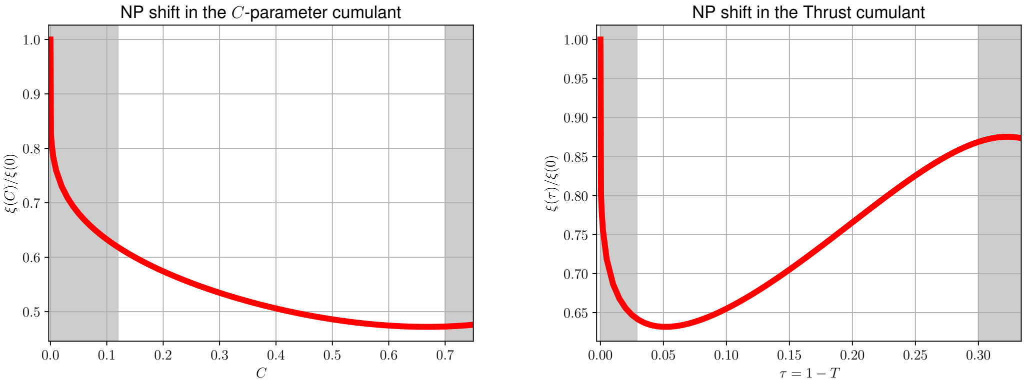
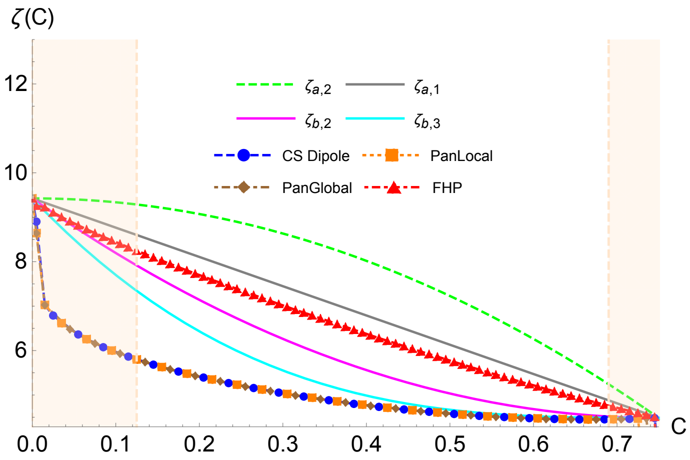


- Why people did not obtain these results before?
- Do we believe in these results?

Why the results are new



- Different **kinematic mapping** prescription to build the phase space for an additional soft parton, lead to different results away from the two jet limit.
- We identified which ones are **correct**: those who are analytic in the **soft limit**.

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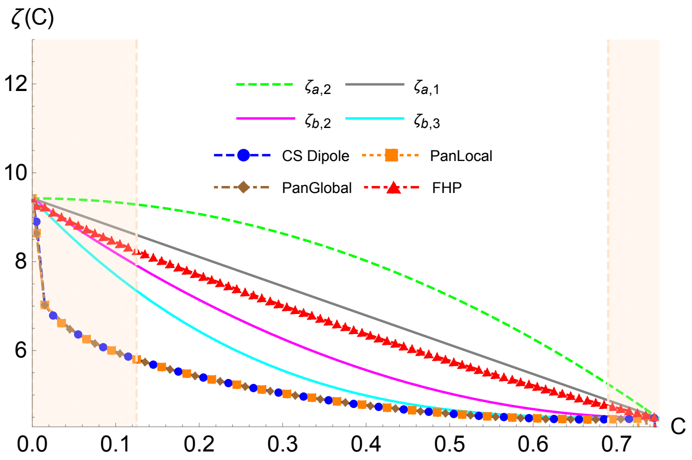
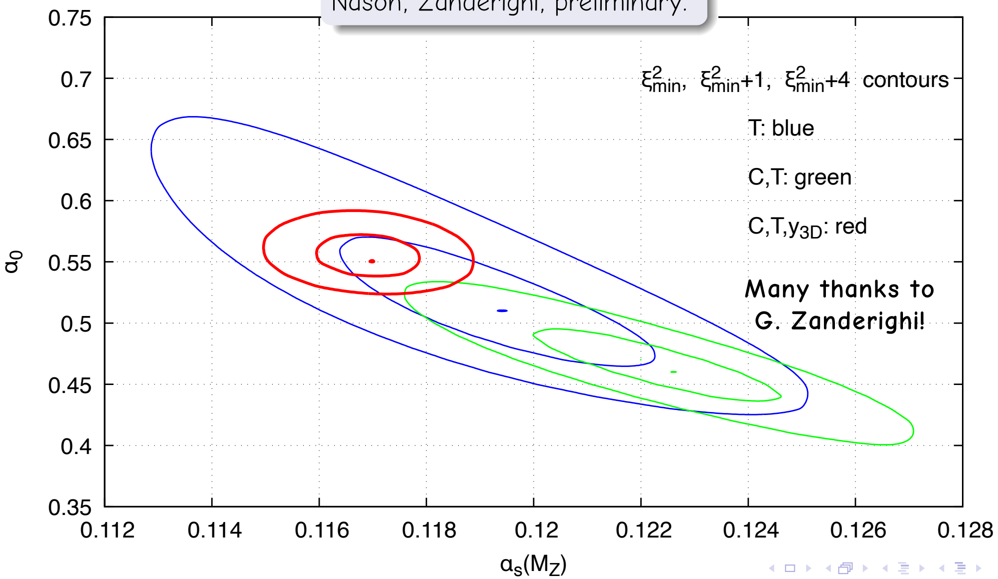


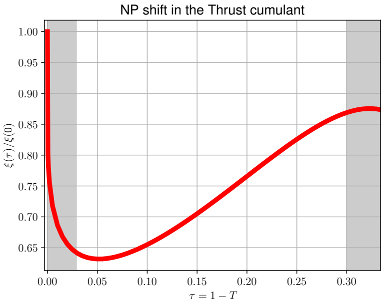
- Different **kinematic mapping** prescription to build the phase space for an additional soft parton, lead to different results away from the two jet limit.
- We identified which ones are **correct**: those who are analytic in the **soft limit**.
- Our results coincide with the results from [Luisoni, Monni, Salam '20] obtained using the smooth **PS** mappings (Catani-Seymour/Antenna/PanScales)
- Is it a fundamental constraint we want for a **Parton Shower** generator?

Preliminary fits of α_s

Nason, Zanderighi, preliminary.



Not yet the end of the story



- Power corrections very quickly drops away from the strict two jet limit;
- Our estimate on non-power corrections assumes the **fixed-order calculation** as perturbative baseline
- For $\tau \leq 0.05$ the cross section is overly dominated by singular terms / we need **resummation** also the evaluate non-perturbative corrections.

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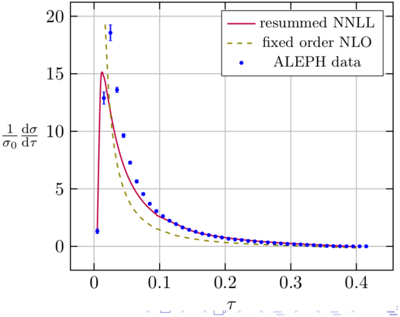
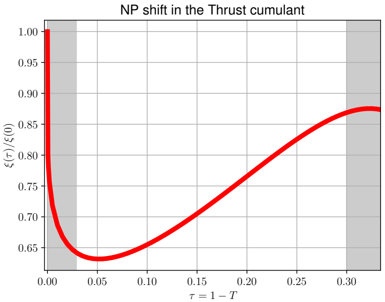


Not yet the end of the story



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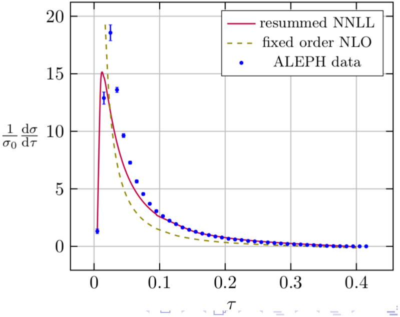
How do we reconcile the two pictures?

The transition between the two-jet and three jet behaviour should be more smooth, as it happens when a fixed order calculation is combined with resummation.



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- With this framework we could investigate any infrared safe observable (for processes without gluons at LO ...)
- We showed **inclusive observables** are free from linear power corrections
- More insights on the calculation of hadronisation corrections for **event shapes**
- ...although some freedom is taken during the **"non-abelianization"** phase, and we do not have a definitive recipe!

Large-n_f approximation in action: the top pole mass

- Due to the **confinement**, asymptotic states are ill defined in QCD: the **pole mass** has a $\mathcal{O}(\Lambda)$ ambiguity.

$$m_p = \overline{m}(\overline{m}) \sum_{i=0} c_i \alpha^i \text{ till } \mathcal{O}(\alpha_S^4)$$ [Marquard, Smirnov[2], Steinhauser '15]

$$m_c = 1.270 + 0.212 + 0.205 + 0.289 + 0.529 + \ldots \text{ GeV}$$

$$m_b = 4.180 + 0.398 + 0.198 + 0.144 + 0.135 + \ldots \text{ GeV}$$

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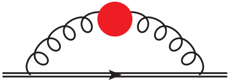

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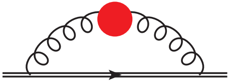

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- Asymptotic formula known [Beneke, Braun '94]

$$c_{n+1} \to N\,\overline{m}\,(2b_0)^n \frac{\Gamma(1+n+b)}{\Gamma(1+b)} \left(1 + \sum_{k=1}^{\infty} \frac{s_k}{n}\right) \quad \text{with } b = \frac{b_1}{2b_0^2},\ s_i = s_i(b_0, b_1, \ldots)$$

- Fitting N from the exact relation [Beneke, Marquard, Nason, Steinhauser, '16]

$$\Delta m_t = \underbrace{7.577 + 1.617 + 0.501 + 0.197}_{\text{exact}} + 0.112 + 0.079 + 0.066 + \mathbf{0.064} + 0.071 + \ldots \text{ GeV}$$

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…$\ldots)$

WWWARNING: ultimate top pole-mass ambiguity?

- Minimal term 64 MeV at $n = 8 \rightarrow$ ambiguity 70 MeV?
- Bottom and charm mass effects are important at higher orders: 110 MeV [Beneke, Marquard, Nason, Steinhauser, '16]
- Hoang, Lepenik and Preisser ('17) find an ambiguity $250\,\text{MeV} \approx \Lambda_3$.
- The most precise mass measurements have an uncertainty of $\approx$ 500 MeV: a precise estimate of such ambiguity is not crucial yet.
- **Having a simplified method to assess the presence of linear corrections for arbitrarily complicated infrared safe observables is usueful!**

- Fitting N from the exact relation [Beneke, Marquard, Nason, Steinhauser, '16]

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Single-top production and decay: reconstructed-top mass

Modulo **finite top width** effects, the **physical renormalon** which affects the observable definition, largely cancels when using the **pole mass**

		$\langle M \rangle = \sum_{i=0}^{\infty} c_i \alpha_s^i$; [MeV]	
i	$m_p - \overline{m}(\mu)$	pole, $R = 1.5$	$\overline{\rm MS}$, $R = 1.5$
3	+430	+ 14(1)	+438(1)
4	+171	−6(1)	+163(1)
5	+89	−10(1)	+79(1)
6	+60	−11(1)	+49(1)
7	+47	−11(1)	+35(1)
8	+44	−12(1)	+31(1)
9	+46	−15(1)	+31(1)
10	+55	−19(1)	+36(1)

Single-top production and decay: leptonic observables

A finite Γ_t changes significantly $T'(\lambda)$ for $\langle \mathbf{E_W} \rangle$

Γ_t	slope (pole)	slope ($\overline{\mathrm{MS}}$)
NWA	$0.53\,(2)$	$0.46\,(2)$
10 GeV	$0.058\,(8)$	$0.004\,(8)$
20 GeV	$0.061\,(2)$	$0.001\,(2)$

E_W (in the lab frame) in the $\overline{\mathrm{MS}}$ scheme has a linear renormalon only in NWA. (Top frame: λ^2 because of OPE)

$c_i \alpha_S^i$ [MeV]	pole	$\overline{\mathrm{MS}}$
$i = 4$	$-94\,(6)$	$-78\,(6)$
$i = 5$	$-44\,(5)$	$-35\,(5)$
$i = 6$	$-22\,(4)$	$-17\,(4)$
$i = 7$	$-13\,(4)$	$-8(4)$
$i = 8$	$-9\,(4)$	$-4\,(4)$
$i = 9$	$-7\,(4)$	$-2\,(4)$
$i = 10$	$-6\,(5)$	$-1\,(5)$
$i = 11$	$-7\,(6)$	$0\,(6)$
$i = 12$	$-9\,(9)$	$1\,(9)$

$\Gamma_t = 1.33$ GeV To see the linear renormalon screening provided by Γ_t in the $\overline{\mathrm{MS}}$ scheme, you need to be sensitive to $\Gamma_t = m_t e^{1-i}$, *i.e.* $i \approx 6$.

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WWWARNING!

Leptonic observables can have a linear power correction even with a well-defined renormalisation scheme!

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