

Factorisation and resummation with massive particles

Matteo Cacciari
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NLO initial state conditions
for heavy quarks perturbative
fragmentation functions

$$\left\{ \begin{array}{l} D_Q^Q = \delta(1-x) + \frac{\alpha_s C_F}{2\pi} \left[\frac{1+x^2}{1-x} \left(\log \frac{\mu_0^2}{m^2} - 2 \log(1-x) - 1 \right) \right]_+ \\ D_g^Q = \frac{\alpha_s T_f}{2\pi} (x^2 + (1-x)^2) \log \frac{\mu_0^2}{m^2} \\ D_{q, \bar{q}}^Q = 0 \end{array} \right.$$

(B. Hebe
P. Nason
NP B361 (1991)
626)



From one of my first talks

Circa 1994

NLO evolution

$$\frac{\partial D_i^Q(z)}{\partial \log \mu^2} = \frac{\alpha_s(\mu)}{2\pi} \sum_j \int_z^1 P_{ij}(x) D_j^Q\left(\frac{z}{x}\right) \frac{dx}{x}$$

evolution kernels:

- Altarelli-Parisi $O(\alpha_s)$
- Curci-Furmanski-Petronzio $O(\alpha_s^2)$

Nuclear Physics B361 (1991) 626–644
North-Holland

THE FRAGMENTATION FUNCTION FOR HEAVY QUARKS IN QCD

B. MELE

CERN, Geneva, Switzerland

P. NASON

INFN, Gruppo Collegato di Parma, Parma, Italy

Received 13 February 1991
(Revised 26 March 1991)

I keep going back to this paper regularly, even today

Heavy quark fragmentation

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An erratum almost 30 years later

Available online at www.sciencedirect.com

  **ScienceDirect** 

www.elsevier.com/locate/nucphysb

Nuclear Physics B 921 (2017) 841–842

Corrigendum

Corrigendum to “The fragmentation function for heavy quarks in QCD” [Nucl. Phys. B 361 (1991) 626–644]

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Available online 24 May 2017

... and apparently I'm not the only one
(no worry, just some typos)

I keep going back to this paper regularly, even today

Heavy quark fragmentation

Collinear resummation

$$\sigma_N(Q) = \hat{\sigma}_N(Q, \mu) \exp \left\{ P_N^{(0)} t + \frac{1}{4\pi^2 b_0} (\alpha_S(\mu_0) - \alpha_S(\mu)) \left(P_N^{(1)} - \frac{2\pi b_1}{b_0} P_N^{(0)} \right) \right\} \hat{D}_N^{[1]}(\mu_0, m)$$

with universal, and calculable in pQCD, 'initial conditions, at $\mu \sim m$

$$\hat{D}^{[1]}(x, \mu, m) = \delta(1-x) + \frac{\alpha_S C_F}{2\pi} \left[\frac{1+x^2}{1-x} \left(\log \frac{\mu^2}{m^2} - 2 \log(1-x) - 1 \right) \right]_+$$

Soft-gluon resummation

$$D(x, Q^2) = \frac{C_F}{\pi} \int_x^1 dz \int_{(1-z)m^2}^{Q^2} \frac{dV}{V} \left[\frac{\alpha_S(V(1-z))}{1-z} \right]_+ D\left(\frac{x}{z}, V\right) + \delta(1-x)$$

Of course, the above derivation cannot be considered as a solid proof. Eq. (5.6), however, reproduces a leading singularity of the term of order α_S in the first iteration, which is in agreement with eq. (3.4). Furthermore, it certainly sums up correctly the leading logarithms arising in the region of virtualities larger than m . In the following we will therefore assume that it is correct.

Heavy quark fragmentation

Soft-gluon resummation

$$D_N(Q^2) = D_N^{(S, \text{ev})}(Q^2) D_N^{(S, \text{in})}$$

$$D_N^{(S, \text{ev})}(Q^2) = \exp \left\{ \frac{C_F}{\pi b_0} \left[\log \frac{Q^2}{N\Lambda^2} \log \log \frac{Q^2}{N\Lambda^2} - \log \frac{Q^2}{\Lambda^2} \log \log \frac{Q^2}{\Lambda^2} \right. \right. \\ \left. \left. - \log \frac{m^2}{N\Lambda^2} \log \log \frac{m^2}{N\Lambda^2} + \log \frac{m^2}{\Lambda^2} \log \log \frac{m^2}{\Lambda^2} \right] \right\},$$

$$D_N^{(S, \text{in})} = \exp \left\{ \frac{C_F}{\pi b_0} \left[\log \frac{m^2}{\Lambda^2 N} \log \log \frac{m^2}{\Lambda^2 N} \right. \right. \\ \left. \left. - \frac{1}{2} \log \frac{m^2}{\Lambda^2 N^2} \log \log \frac{m^2}{\Lambda^2 N^2} - \frac{1}{2} \log \frac{m^2}{\Lambda^2} \log \log \frac{m^2}{\Lambda^2} \right] \right\}$$



RECEIVED: *March 25, 1998*, ACCEPTED: *May 21, 1998*

The p_T spectrum in heavy-flavour hadroproduction

Matteo Cacciari*

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Mario Greco

*Dipartimento di Fisica E. Amaldi, Università di Roma Tre
and INFN, Laboratori Nazionali di Frascati, Italy*

Paolo Nason†

CERN, TH Division, Geneva, Switzerland

My first paper with Paolo

Turn collinear resummation into a phenomenology-ready public tool

My first exposure to Paolo's powers:

- coding
- analytical manipulation (with MACSYMA!)
- tricks for checks/debugging
- physical intuition for “what matters”

VOLUME 89, NUMBER 12

PHYSICAL REVIEW LETTERS

16 SEPTEMBER 2002

Is There a Significant Excess in Bottom Hadroproduction at the Tevatron?

Matteo Cacciari

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(Received 3 April 2002; published 28 August 2002)

Exactly twenty
years ago today

VOLUME 89, NUMBER 12

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[**Betteridge's law of headlines:**

“Any headline that ends in a question mark can be answered by the word *no*.”

https://en.wikipedia.org/wiki/Betteridge%27s_law_of_headlines]

VOLUME 89, NUMBER 12 PHYSICAL REVIEW LETTERS 16 SEPTEMBER 2002

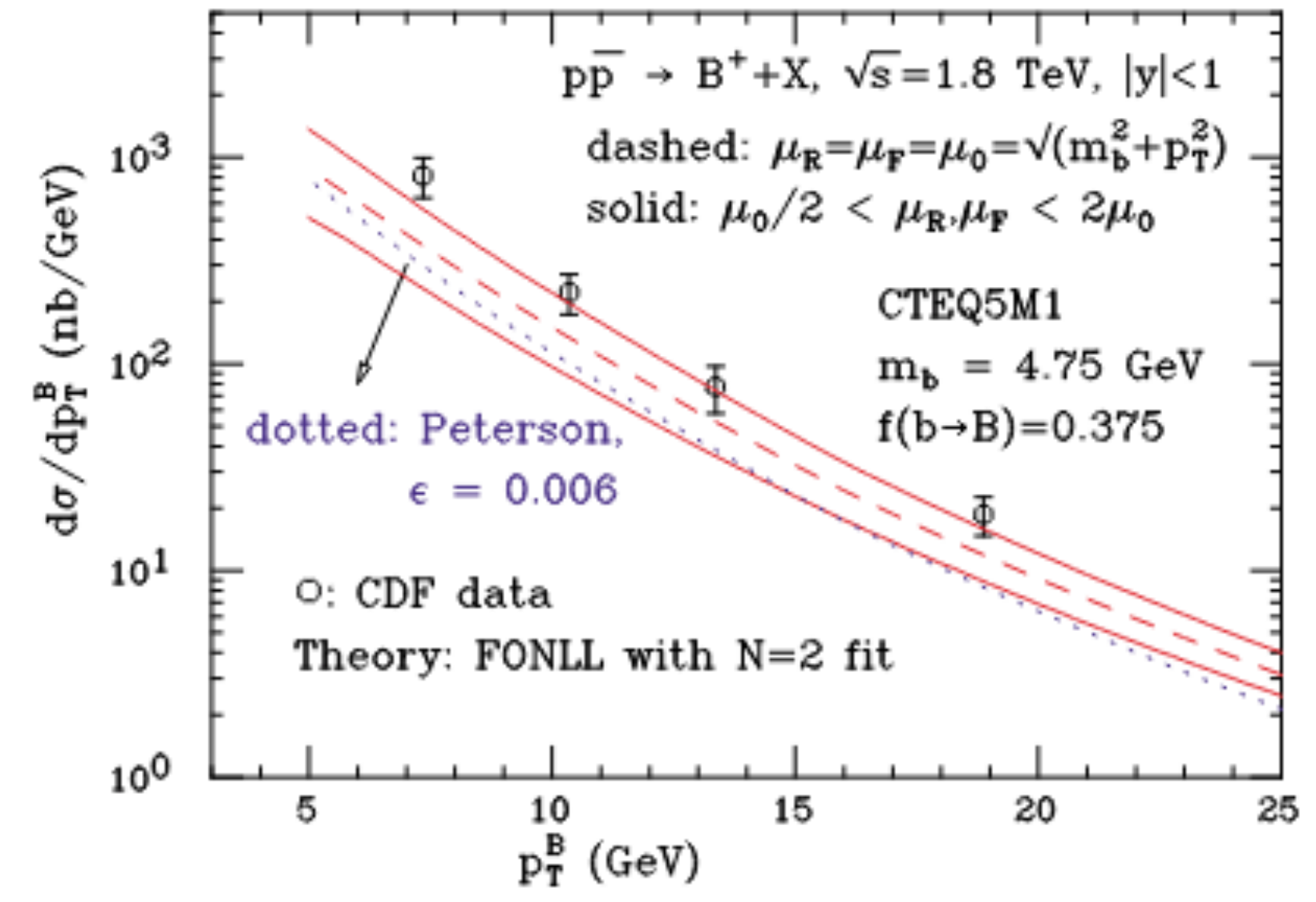
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A study of heavy flavoured meson fragmentation functions in e^+e^- annihilation

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A significant milestone (at the NLO+NLL level) of the approach bootstrapped by the 1991 Mele-Nason paper allowing for a maximally perturbative description of heavy quark production and resummations.



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Now including:

- Full mixings with gluons and light flavours in NLL DGLAP evolution
- NLL soft resummation, regularised in a sensible way to avoid the Landau pole
- Deconvolution of electromagnetic initial state radiation
- Analytical modelling of electroweak decays of D mesons
- Non-perturbative fragmentation functions fitted to data

→ good description of CLEO/BELLE and LEP data, up to one puzzling issue

CLEO/BELLE v. LEP

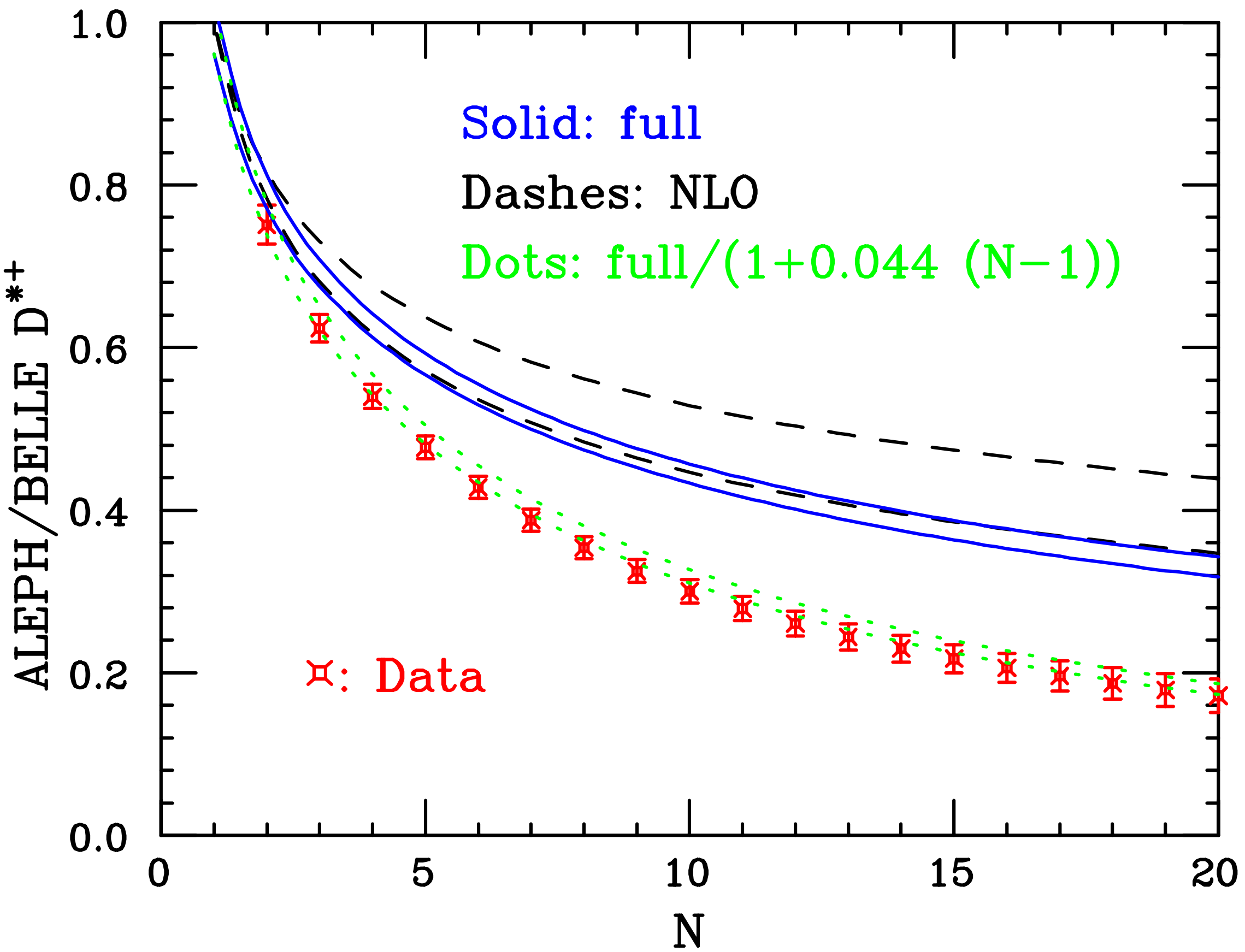
$$\frac{\sigma_Q(N, M_Z^2, m^2)}{\sigma_Q(N, M_\Upsilon^2, m^2)} = \frac{\bar{a}_q(N, M_Z^2, \mu_Z^2)}{1 + \alpha_s(\mu_Z^2)/\pi} E(N, \mu_Z^2, \mu_\Upsilon^2) \frac{1 + \alpha_s(\mu_\Upsilon^2)/\pi}{\bar{a}_q(N, M_\Upsilon^2, \mu_\Upsilon^2)}$$

The ratio between moments of LEP data (91 GeV) and CLEO/BELLE (10.6 GeV) for D mesons should be well predicted by pQCD:

- Effects at the heavy quark mass scale cancel out
- Power suppressed mass terms are small
- Non-perturbative effects cancel out
- Power corrections to the coefficient function are expected to scale like $1/Q^2$

However:

- Data lie outside of the perturbative band
- A $1/Q^2$ power correction would need a large 5 GeV² coefficient
- An unexpected $1/Q$ power correction would only need a 0.5 GeV coefficient

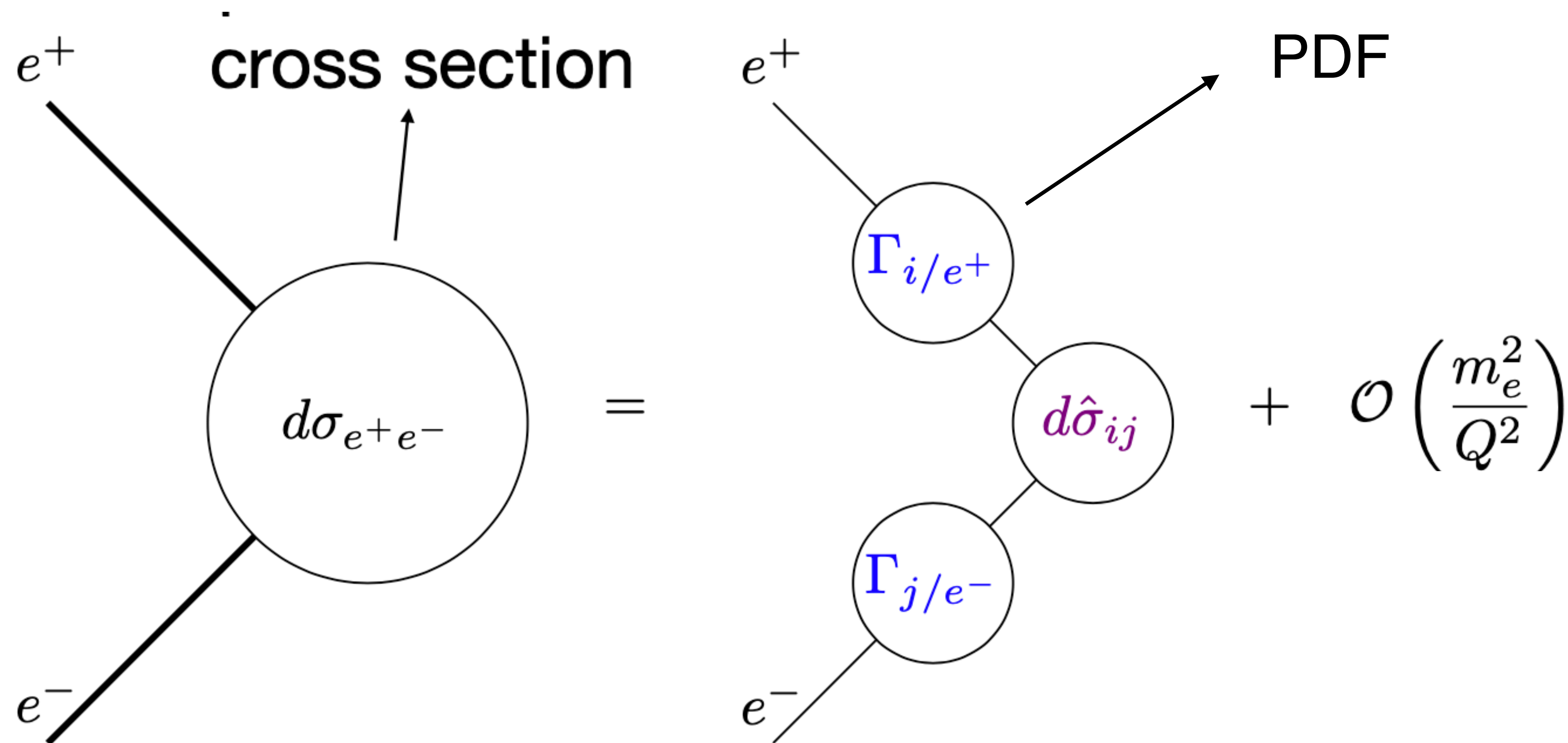


An update of this analysis to NNLL (collinear) + NNLL (soft) is in preparation, but it's not expected to change the picture

From fragmentation to electron PDFs

[S. Frixione, 1909.03886]

The Mele-Nason approach can be applied to electrons and photons, factorising an e^+e^- collision in the initial state and leading to calculable electron/photon PDF initial conditions



From fragmentation to electron PDFs

[S. Frixione, 1909.03886]

The ‘initial conditions’, at scale $\mu_0 \sim$ electron mass

$$\Gamma_i^{[0]}(z, \mu_0^2) = \delta_{ie} - \delta(1-z),$$

$$\Gamma_{e^-}^{[1]}(z, \mu_0^2) = \left[\frac{1+z^2}{1-z} \left(\log \frac{\mu_0^2}{m^2} - 2 \log(1-z) - 1 \right) \right]_+ + K_{ee}(z),$$

$$\Gamma_\gamma^{[1]}(z, \mu_0^2) = \frac{1+(1-z)^2}{z} \left(\log \frac{\mu_0^2}{m^2} - 2 \log z - 1 \right) + K_{\gamma e}(z),$$

$$\Gamma_{e^+}^{[1]}(z, \mu_0^2) = 0,$$

[$K_{ee} = 0$ and $K_{\gamma e} = 0$ in the $\overline{\text{MS}}$ scheme, $K_{ee}^{(\Delta)}(z) = \left[\frac{1+z^2}{1-z} (2 \log(1-z) + 1) \right]_+$ and $K_{\gamma e}^{(\Delta)}(z) = \frac{1+(1-z)^2}{z} (2 \log z + 1)$ in the Δ scheme]

The DGLAP equations can be solved analytically to NLL, either recursively or to all orders in the soft limit

The analytical solution in the soft limit, which upgrades the Gribov-Lipatov solution to NLL, takes the form

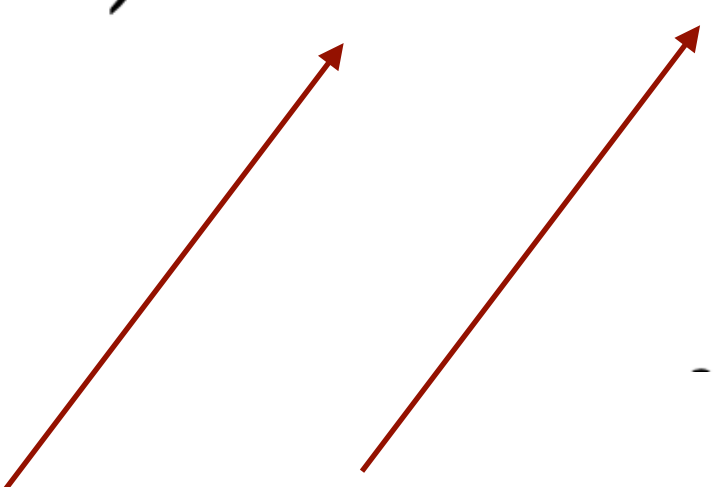
$$\Gamma_{e^-}^{\text{NLL}}(z, \mu^2) = \frac{e^{-\gamma_E \xi_1} e^{\hat{\xi}_1}}{\Gamma(1 + \xi_1)} \xi_1 (1 - z)^{-1 + \xi_1} h(z, \mu^2)$$

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$$\Gamma_{e^-}^{\text{NLL}}(z, \mu^2) = \frac{e^{-\gamma_E \xi_1} e^{\hat{\xi}_1}}{\Gamma(1 + \xi_1)} \xi_1 (1 - z)^{-1 + \xi_1} h(z, \mu^2)$$

$$h^{\overline{\text{MS}}}(z, \mu^2) = 1 + \frac{\alpha(\mu_0)}{\pi} \left[\left(\log \frac{\mu_0^2}{m^2} - 1 \right) \left(A(\xi_1) + \frac{3}{4} \right) - 2B(\xi_1) + \frac{7}{4} + \left(\log \frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1) \right) \log(1 - z) - \log^2(1 - z) \right] \overline{\text{MS}}$$

$$h^{\Delta}(z, \mu^2) = \frac{\alpha(\mu)}{\alpha(\mu_0)} + \frac{\alpha(\mu)}{\pi} \log \frac{\mu_0^2}{m^2} \left(A(\xi_1) + \log(1 - z) + \frac{3}{4} \right) \Delta$$


Large logarithmic terms in $\overline{\text{MS}}$, they are mostly absent in Δ scheme. Alternatively, they can be cured by soft-photon resummation (work in progress)

For those who like details

$$\begin{aligned}\xi_1 &= 2t - \frac{\alpha(\mu)}{4\pi^2 b_0} \left(1 - e^{-2\pi b_0 t}\right) \left(\frac{20}{9} n_F + \frac{4\pi b_1}{b_0}\right) \\ &= 2 \left[1 - \frac{\alpha(\mu)}{\pi} \left(\frac{5}{9} n_F + \frac{\pi b_1}{b_0}\right)\right] t \\ &\quad + \frac{\alpha(\mu)}{\pi} \left(\frac{10}{9} \pi b_0 n_F + 2b_1 \pi^2\right) t^2 + \mathcal{O}(t^3)\end{aligned}$$

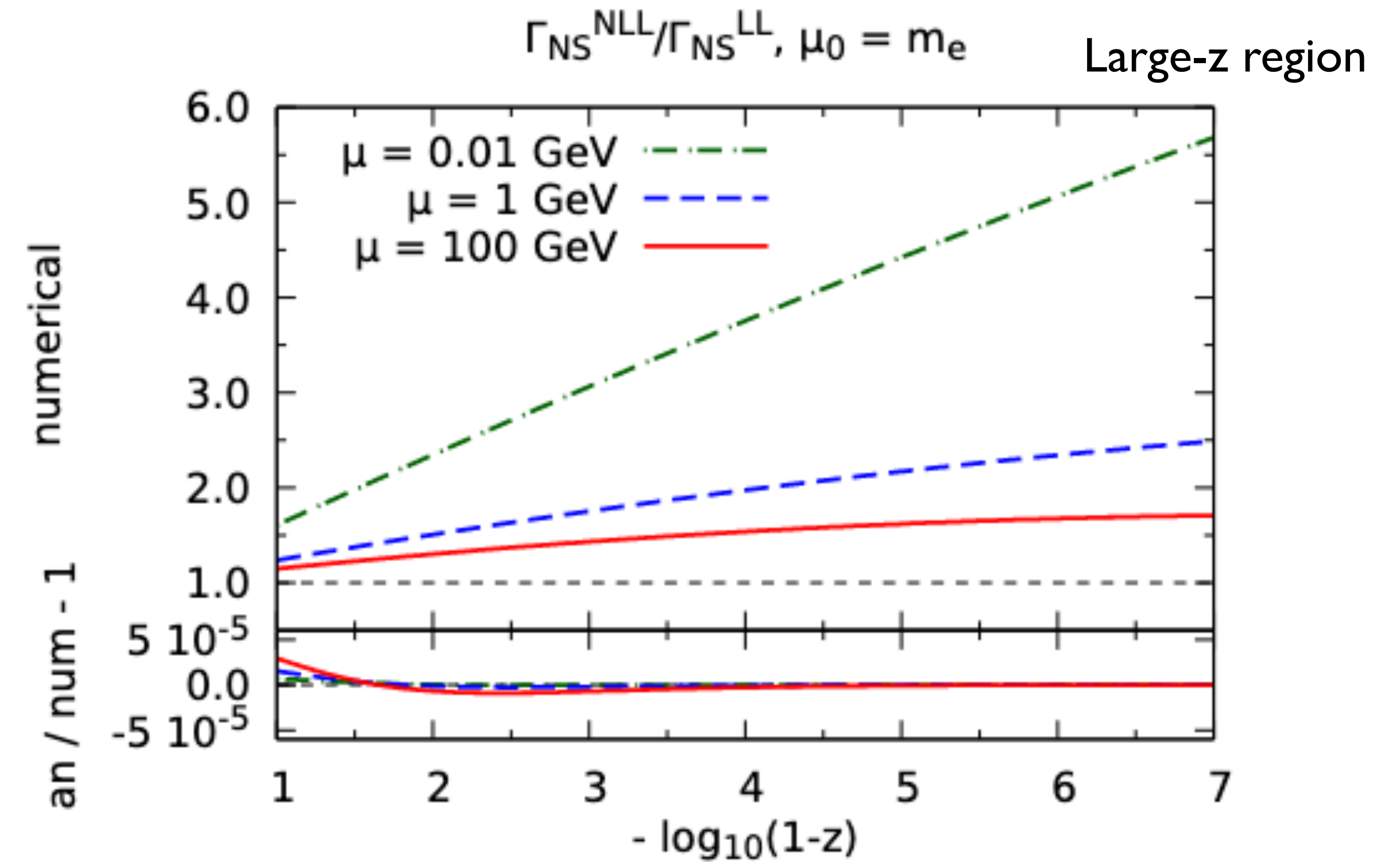
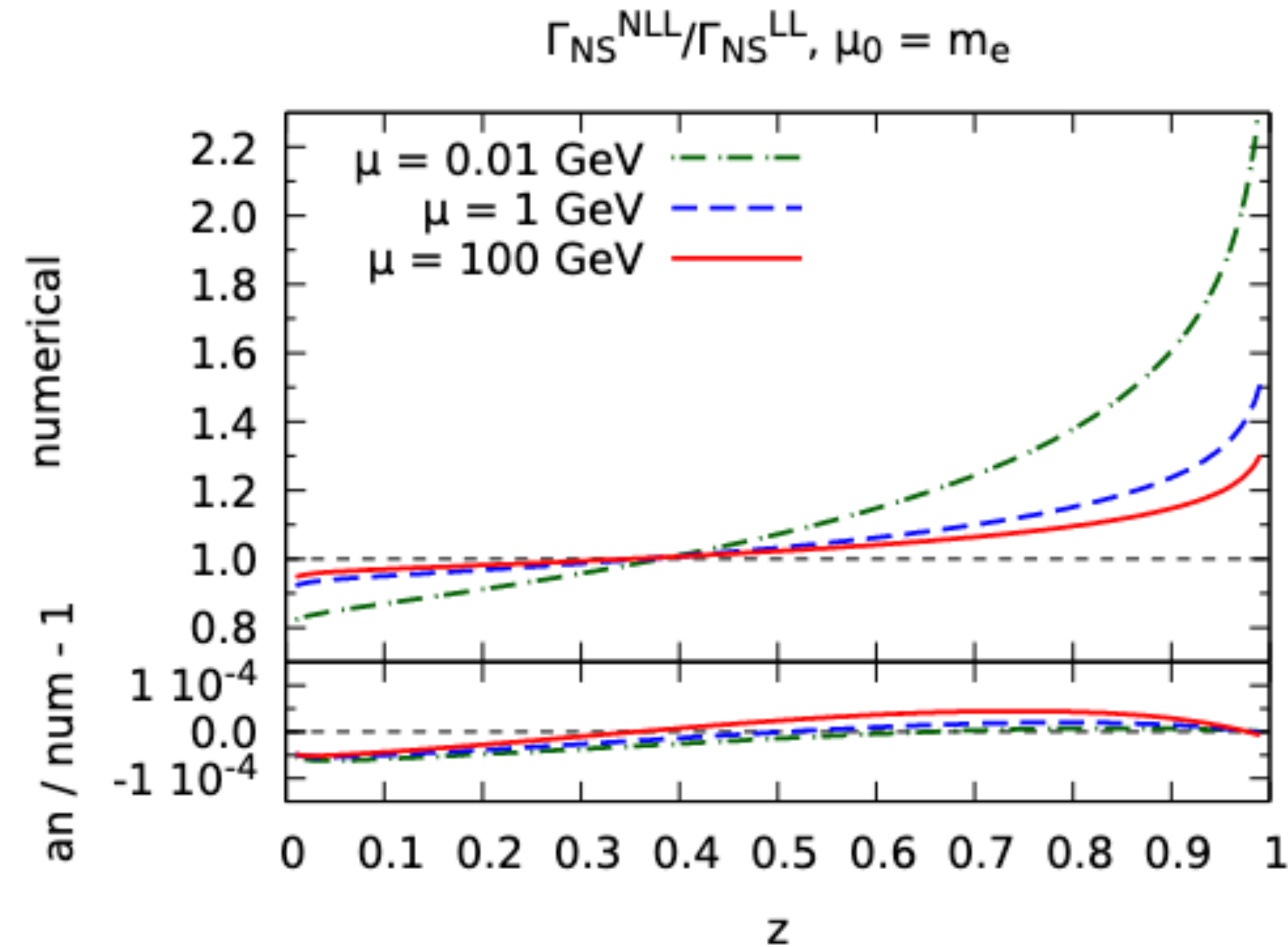
$$\lambda_1 = \frac{3}{8} - \frac{\pi^2}{2} + 6\zeta_3 - \frac{n_F}{18} (3 + 4\pi^2)$$

$$\begin{aligned}\hat{\xi}_1 &= \frac{3}{2} t + \frac{\alpha(\mu)}{4\pi^2 b_0} \left(1 - e^{-2\pi b_0 t}\right) \left(\lambda_1 - \frac{3\pi b_1}{b_0}\right) \\ &= \frac{3}{2} \left[1 + \frac{\alpha(\mu)}{\pi} \left(\frac{\lambda_1}{3} - \frac{\pi b_1}{b_0}\right)\right] t \\ &\quad - \frac{\alpha(\mu)}{\pi} \left(\frac{\pi b_0}{2} \lambda_1 - \frac{3}{2} \pi^2 b_1\right) t^2 + \mathcal{O}(t^3)\end{aligned}$$

$$A(\kappa) = \sum_{k=1}^{\infty} \frac{1}{k} \frac{\Gamma(1 - \kappa + k)}{k! \Gamma(1 - \kappa)} = -\gamma_E - \psi_0(\kappa),$$

$$\begin{aligned}B(\kappa) &= -\sum_{k=1}^{\infty} \frac{1}{k^2} \frac{\Gamma(1 - \kappa + k)}{k! \Gamma(1 - \kappa)} \\ &= \frac{1}{2} \gamma_E^2 + \frac{\pi^2}{12} + \gamma_E \psi_0(\kappa) + \frac{1}{2} \psi_0(\kappa)^2 - \frac{1}{2} \psi_1(\kappa),\end{aligned}$$

A physically meaningless comparison, and a useful one



NLL PDFs differ significantly from the LL ones. However, the difference will be largely compensated by convolution with the subtracted fixed-order cross section

More usefully, numerical and analytical solutions are seen to agree to a high level of precision

In order to do phenomenology with NLL electron PDFs one must address several points:

- Logarithmic artefacts in $\overline{\text{MS}}$ scheme, leading to large spurious corrections
- Evolution through more than one lepton and quark family, like in real life
- Use of different α_{em} renormalisation schemes

This is done in [Bertone, MC, Frixione, Stagnitto, Zaro, Zhao, 2207.03265], where we study

- Analytical solution for evolution through thresholds
- Full numerical solution
- Two factorisation schemes: $\overline{\text{MS}}$ and Δ (DIS-like, no logarithmic artefacts)
- Three renormalisation schemes for α_{em} : $\overline{\text{MS}}$ (running α), $\alpha(M_Z)$ and G_μ (fixed α)

A public code is available at <https://github.com/gstagnit/eMELA>

Again for those who like details

The analytical solution, in the large- z limit, with multiple families of leptons or quarks can be written simply by replacing the ξ_1 and $\hat{\xi}_1$ parameters in the single-family solution by

$$\begin{aligned} \xi_1^{(k)}/e_e^2 &= \frac{\alpha_R}{\pi} \left(\log \frac{\mu^2}{\bar{m}_k^2} + \sum_{i=1}^{k-1} \log \frac{m_{i+1}^2}{\bar{m}_i^2} \right) \\ &+ \frac{\alpha_R^2}{2\pi} \left\{ b_0^{(k)} \left(\log^2 \frac{\mu^2}{m_{k+1}^2} - \log^2 \frac{\bar{m}_k^2}{m_{k+1}^2} \right) - \sum_{i=1}^{k-1} b_0^{(i)} \log^2 \frac{\bar{m}_i^2}{m_{i+1}^2} \right. \\ &\quad \left. - \frac{1}{\pi} \left(\frac{10}{9} C^{(2,k)} - D^{(k)} \right) \log \frac{\mu^2}{\bar{m}_k^2} \right. \\ &\quad \left. - \frac{1}{\pi} \sum_{i=1}^{k-1} \left(\frac{10}{9} C^{(2,i)} - D^{(i)} \right) \log \frac{m_{i+1}^2}{\bar{m}_i^2} \right\}, \\ \hat{\xi}_1^{(k)}/e_e^2 &= \frac{3\alpha_R}{4\pi} \left(\log \frac{\mu^2}{\bar{m}_k^2} + \sum_{i=1}^{k-1} \log \frac{m_{i+1}^2}{\bar{m}_i^2} \right) \\ &+ \frac{3\alpha_R^2}{8\pi} \left\{ b_0^{(k)} \left(\log^2 \frac{\mu^2}{m_{k+1}^2} - \log^2 \frac{\bar{m}_k^2}{m_{k+1}^2} \right) - \sum_{i=1}^{k-1} b_0^{(i)} \log^2 \frac{\bar{m}_i^2}{m_{i+1}^2} \right\} \\ &+ \left(\frac{\alpha_R}{2\pi} \right)^2 \left\{ \lambda_1^{(k)} \log \frac{\mu^2}{\bar{m}_k^2} + \sum_{i=1}^{k-1} \lambda_1^{(i)} \log \frac{m_{i+1}^2}{\bar{m}_i^2} \right. \\ &\quad \left. + \frac{3}{2} \left(D^{(k)} \log \frac{\mu^2}{\bar{m}_k^2} + \sum_{i=1}^{k-1} D^{(i)} \log \frac{m_{i+1}^2}{\bar{m}_i^2} \right) \right\}. \end{aligned}$$

with
$$\lambda_1^{(k)} = e_e^2 \left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta_3 \right) - \frac{C^{(2,k)}}{18} (3 + 4\pi^2)$$

$$D^{(k)} = 2\pi \sum_{i=k+1}^M b_0^{(i)} \log \frac{m_i^2}{m_{i+1}^2} + 2\pi \Delta_{\overline{\text{MS}} \rightarrow R}$$

Study of physical cross sections, MG5_aMC framework, NLO EW + NLL em

$$\sigma(\tau_{min}) = \int d\sigma \Theta(\tau_{min} \leq M_{p\bar{p}}^2/s), \quad p = q, t, W^+ \quad \text{at } \sqrt{s} = 500 \text{ GeV}$$

- ▶ $e^+e^- \rightarrow q\bar{q}(\gamma)$ [pure QED, with real and virtual radiation limited to initial state]
- ▶ $e^+e^- \rightarrow W^+W^-(X)$ [full EW]
- ▶ $e^+e^- \rightarrow t\bar{t}(X)$ [full EW] and $e^+e^- \rightarrow t\bar{t}(X)$ [pure QED]

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$$\sigma(\tau_{min}) = \int d\sigma \Theta(\tau_{min} \leq M_{p\bar{p}}^2/s), \quad p = q, t, W^+ \quad \text{at } \sqrt{s} = 500 \text{ GeV}$$

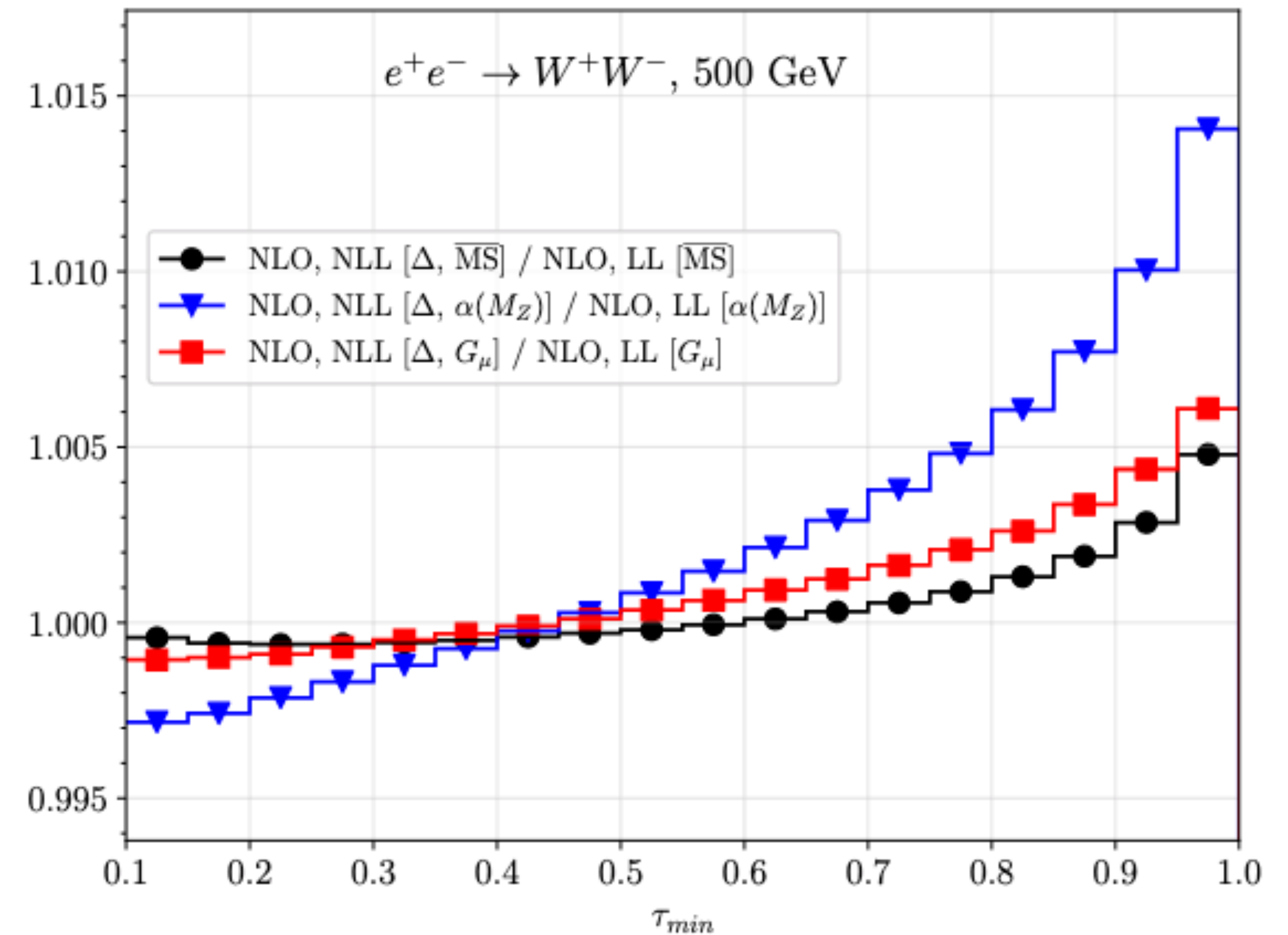
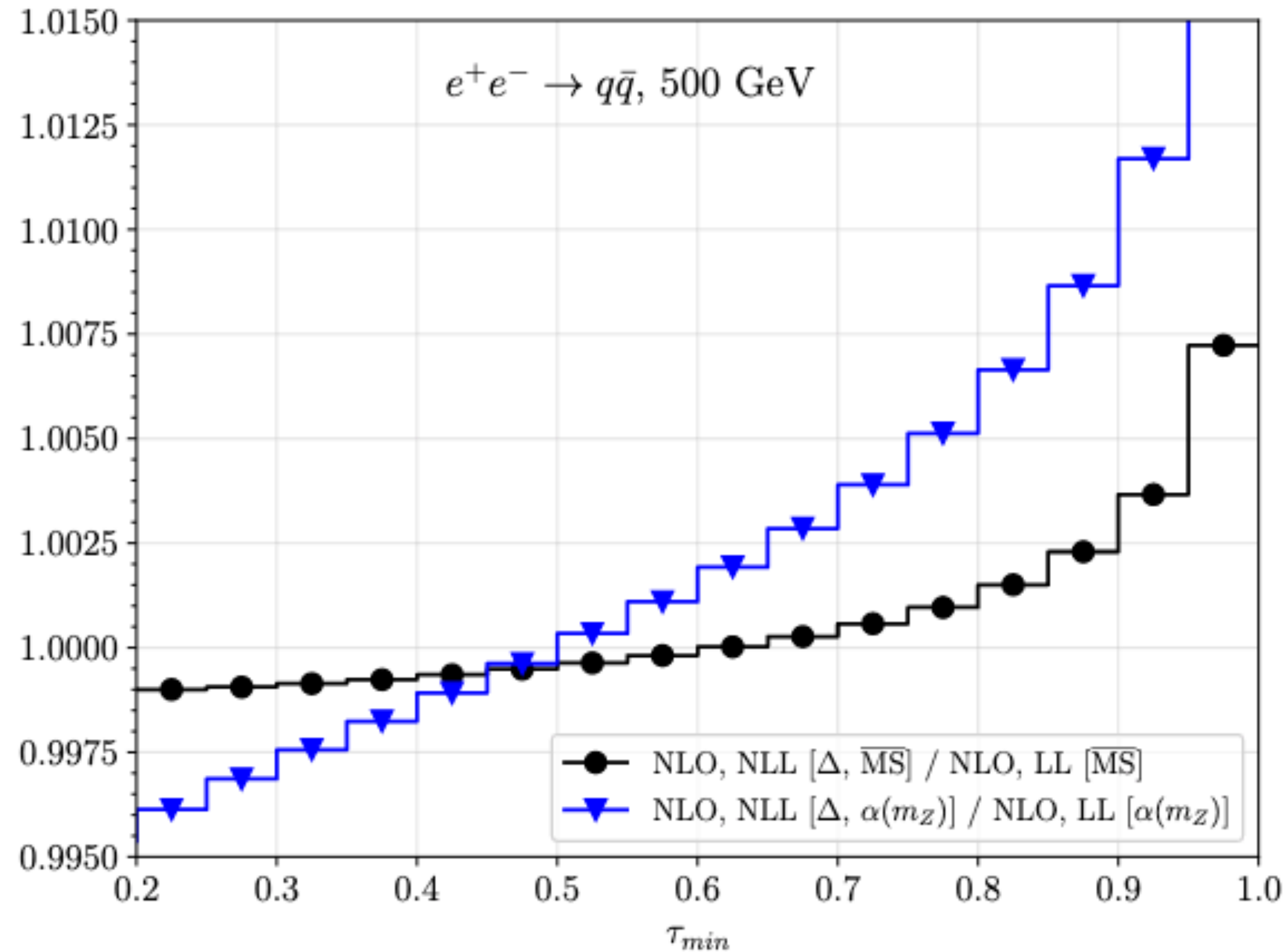
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Notation in plots: **NLO|LO** , **NLL** [fact. sch., ren. sch.] | **LL** [ren. sch.]

Short
distance
xsect

ePDF
accuracy

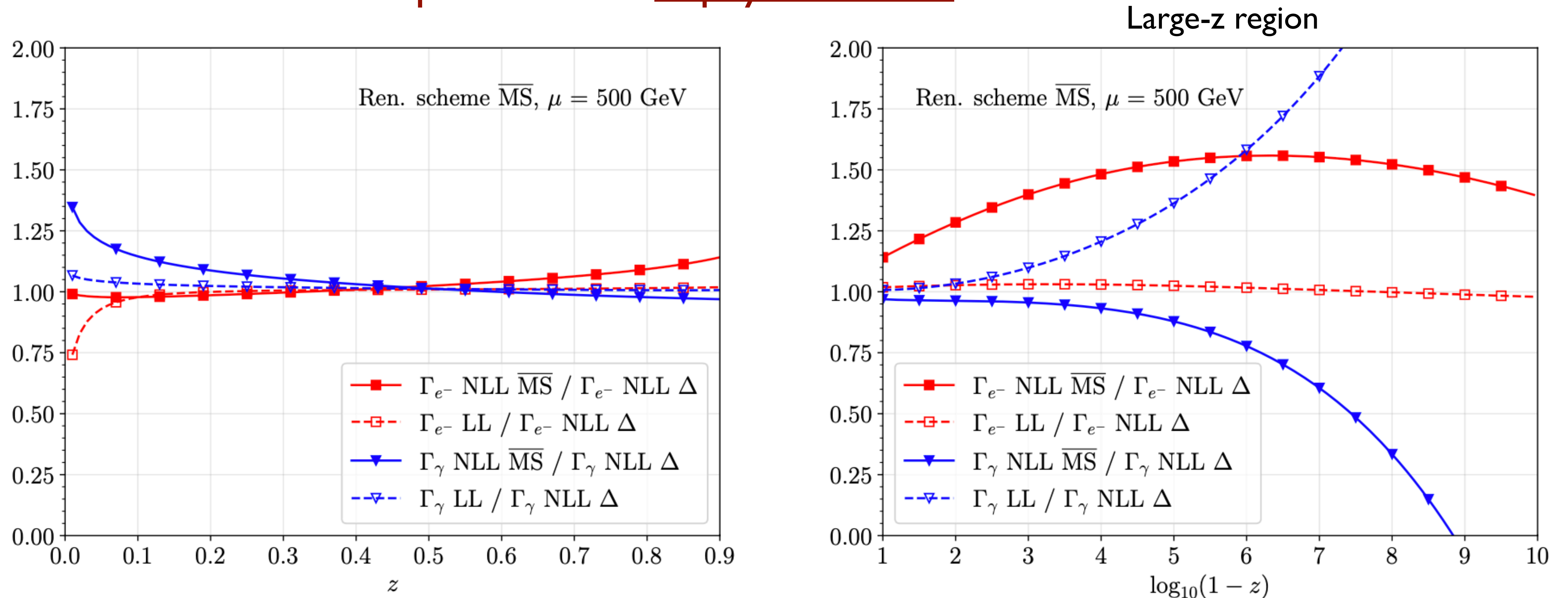
Impact of NLL resummation



- A few per mille, potentially as large as 1% (larger than precision target of future machines)
- Non negligible factorisation and renormalisation scheme dependence
- Non negligible observable dependence, no constant pattern: K-factor approach impossible

e^+e^- processes at NLO+NLL

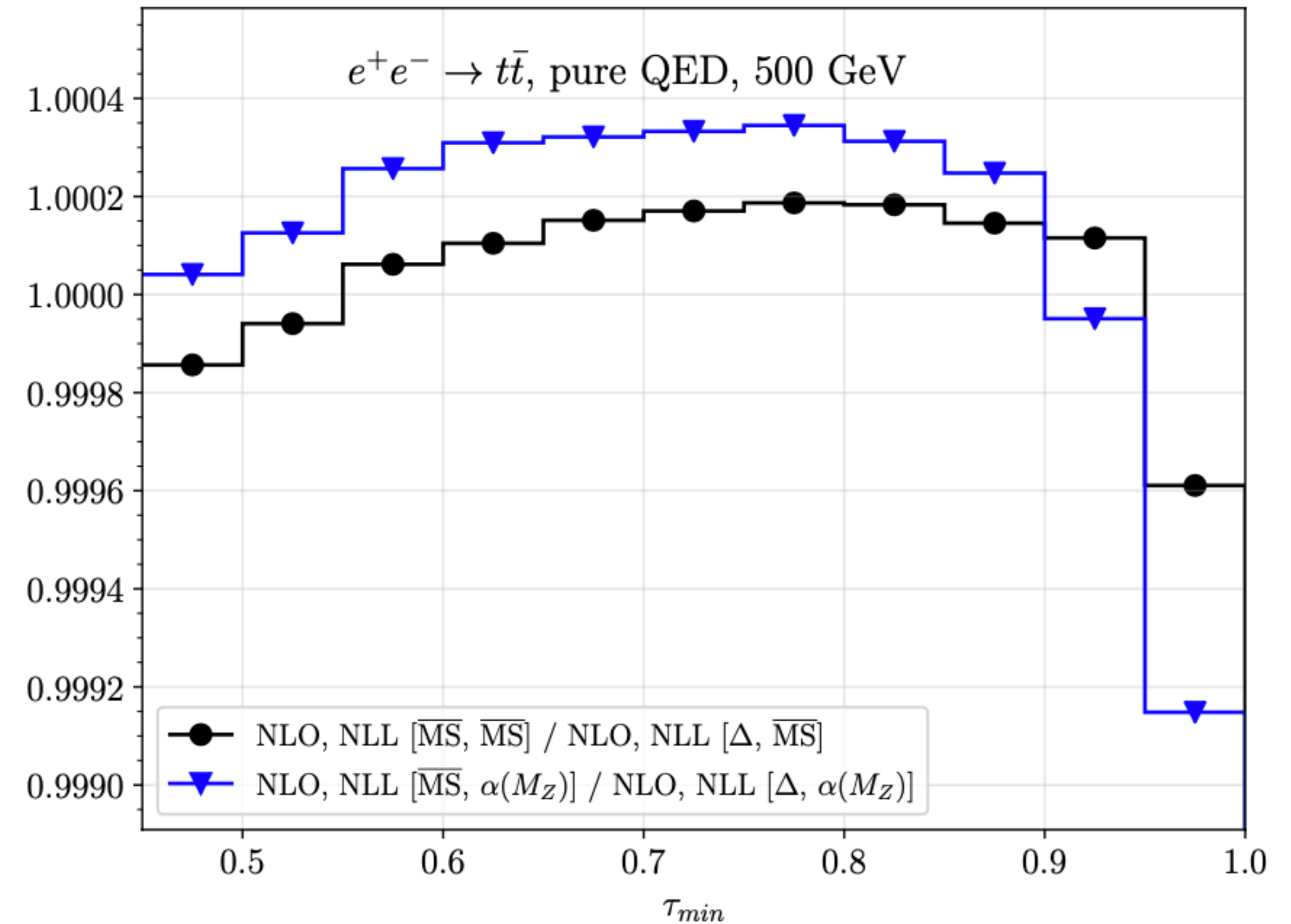
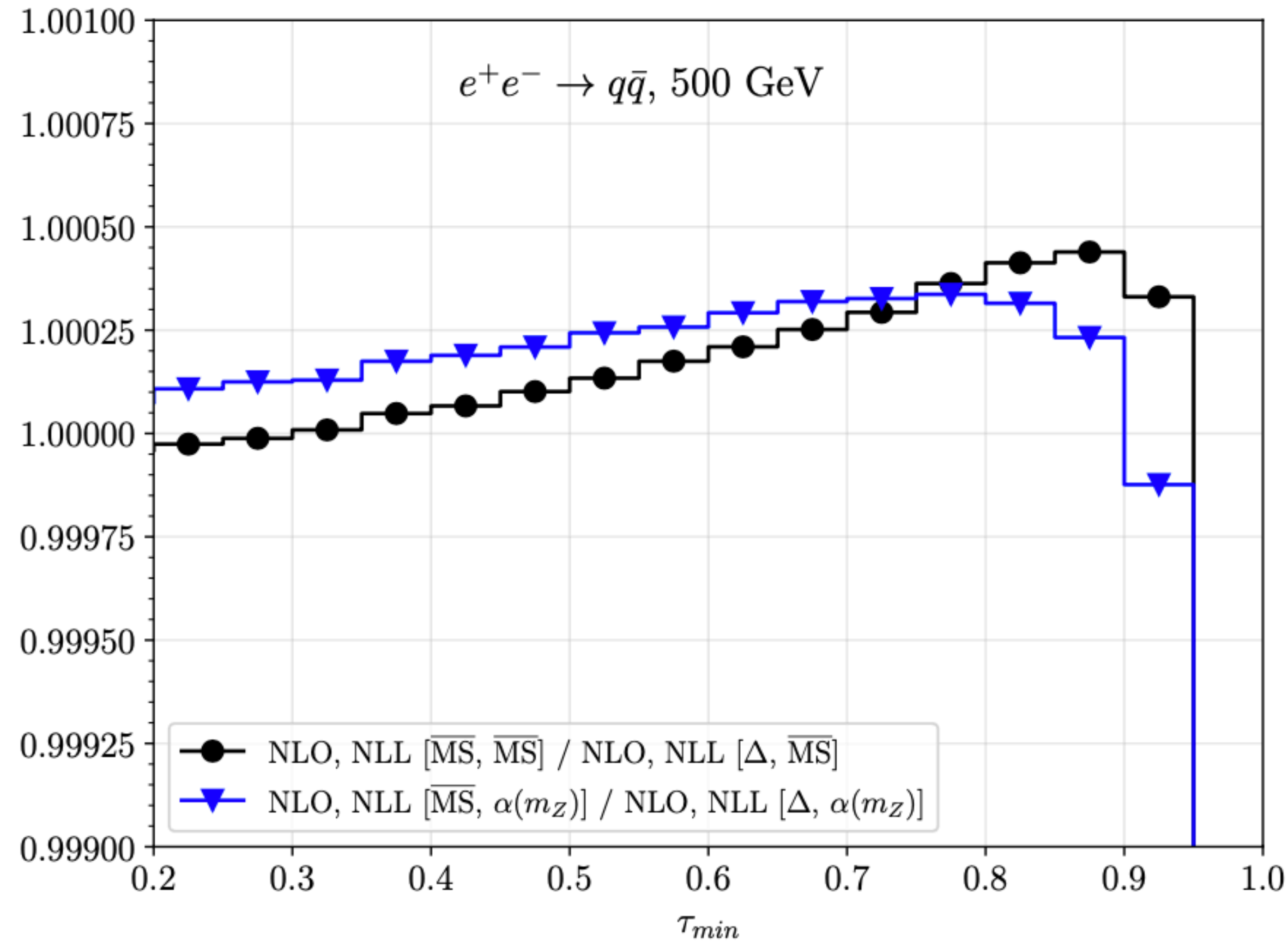
Factorisation scheme dependence of unphysical ePDF



- Sizable dependence (up to 10s of percent)
- Δ scheme mimics LL electron PDF at percent level, especially at large z

e^+e^- processes at NLO+NLL

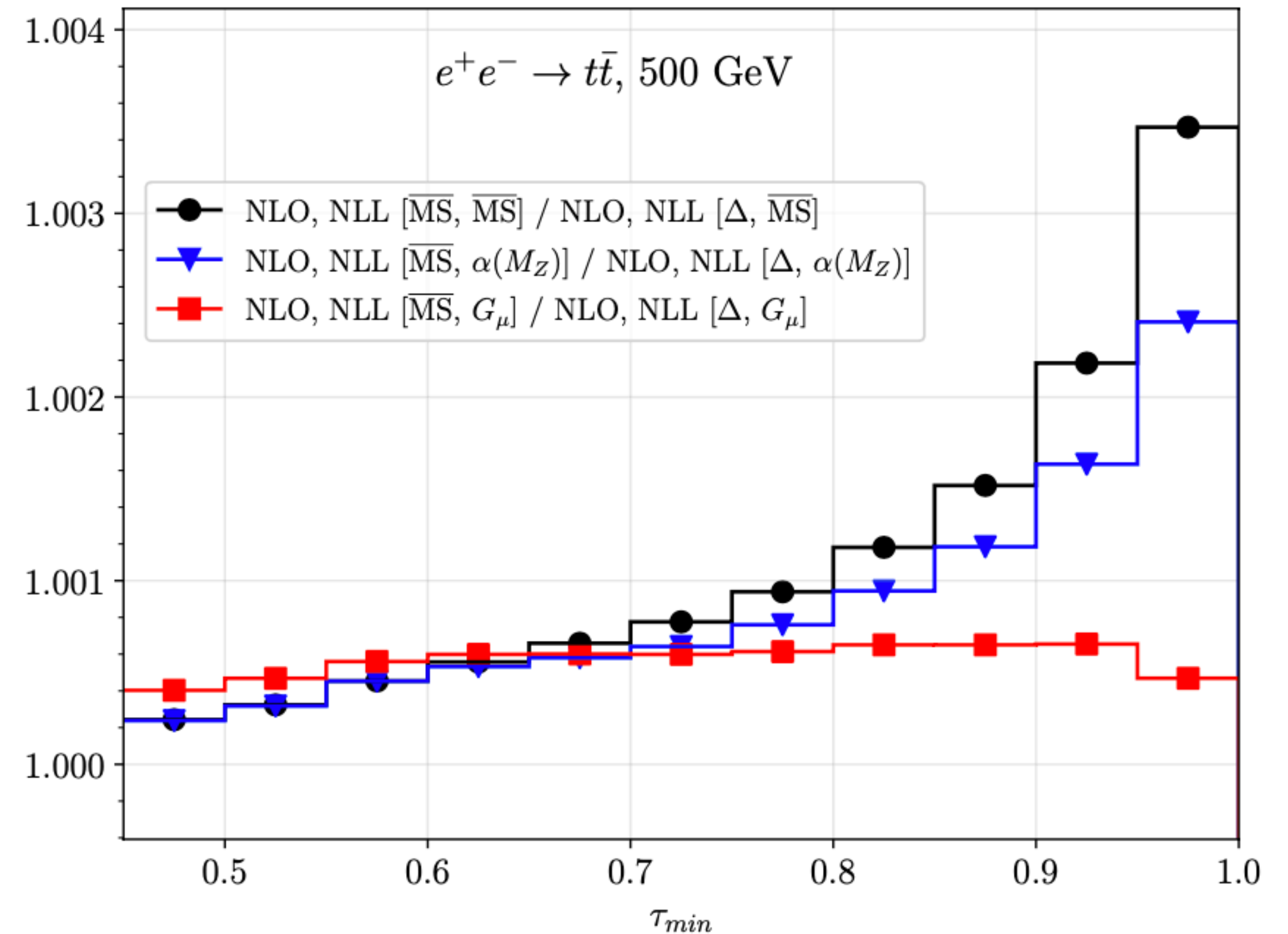
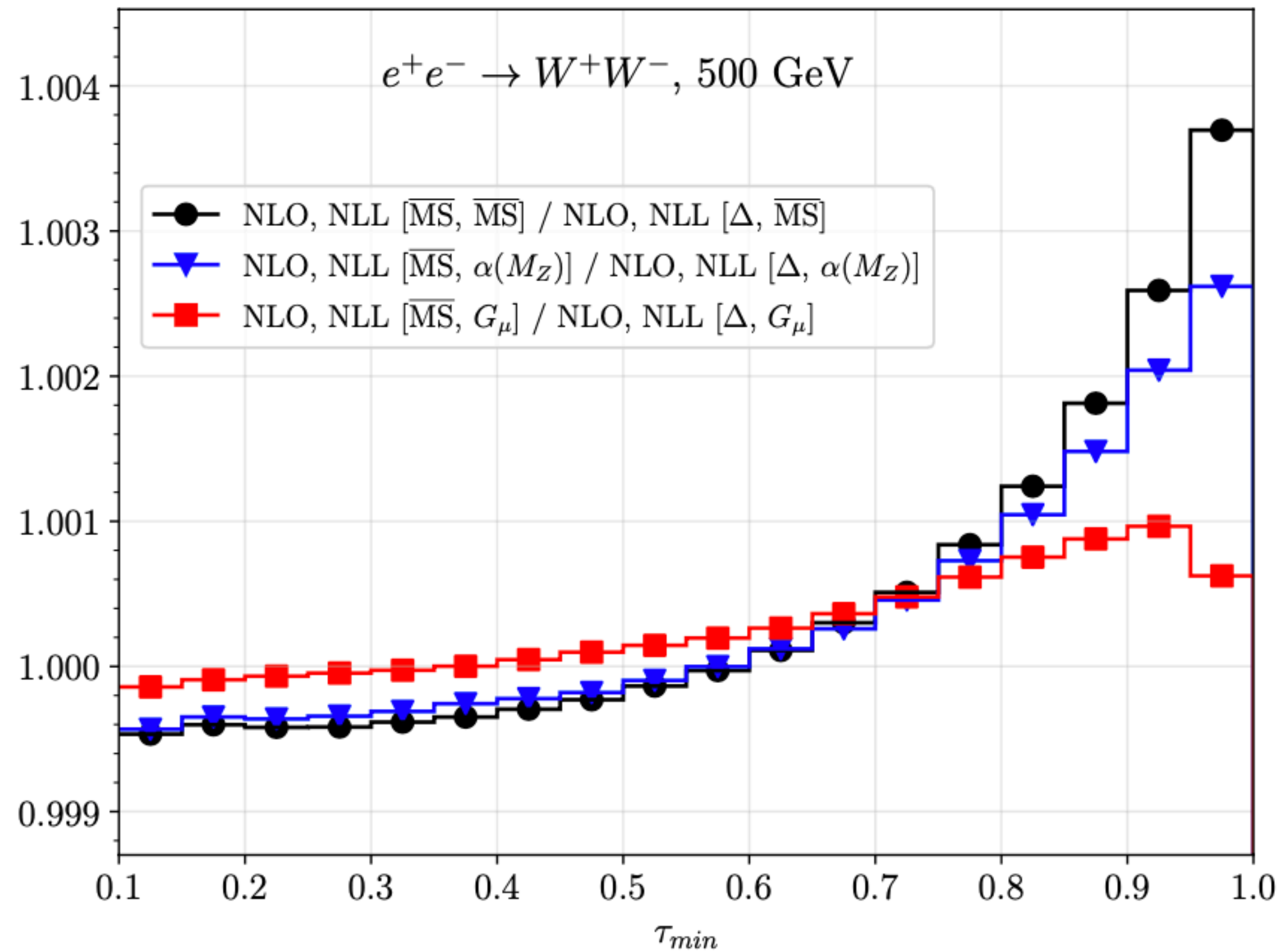
Factorisation scheme dependence of physical processes with small K-factors



- Residual dependency at 10^{-4} level

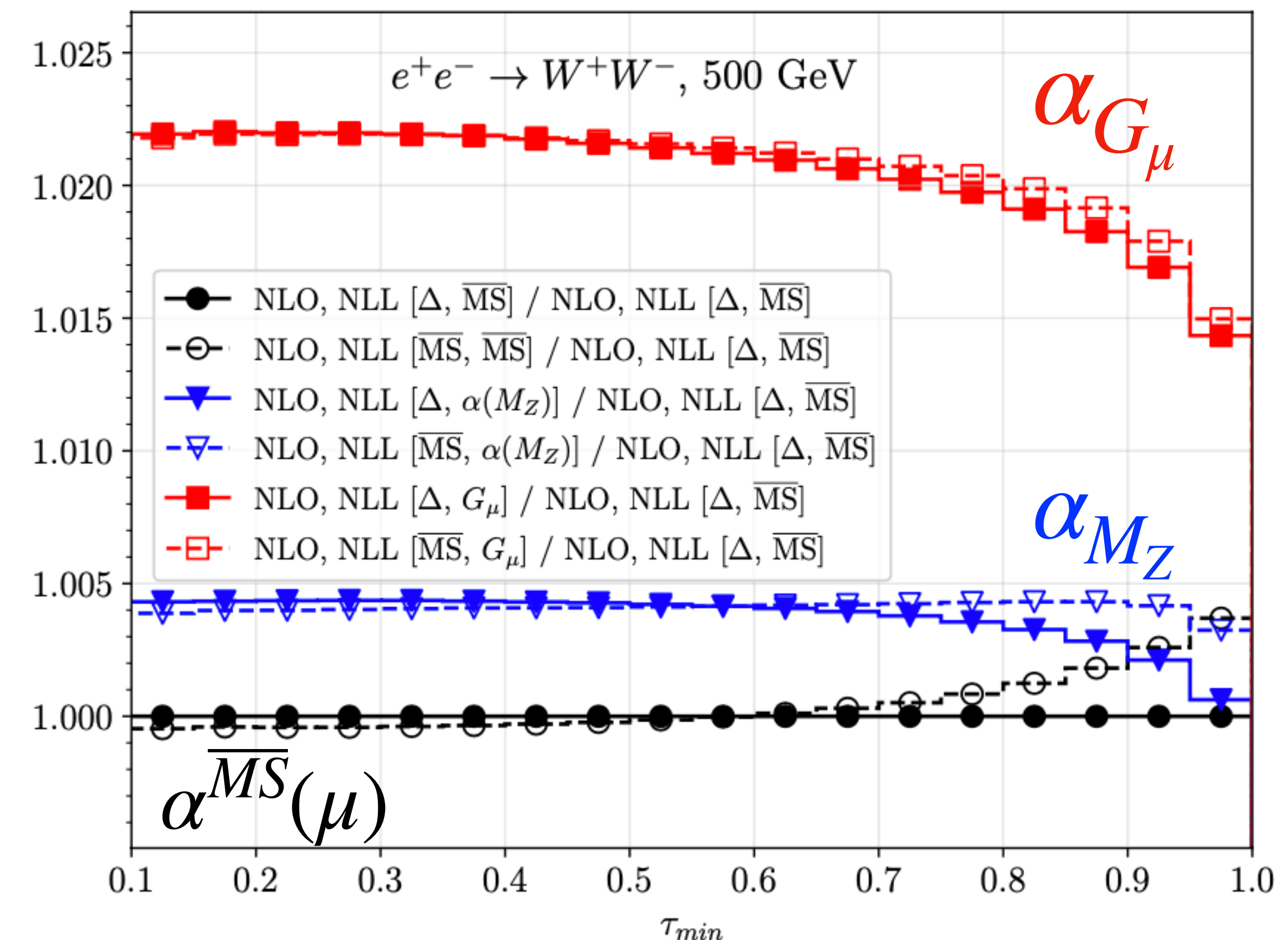
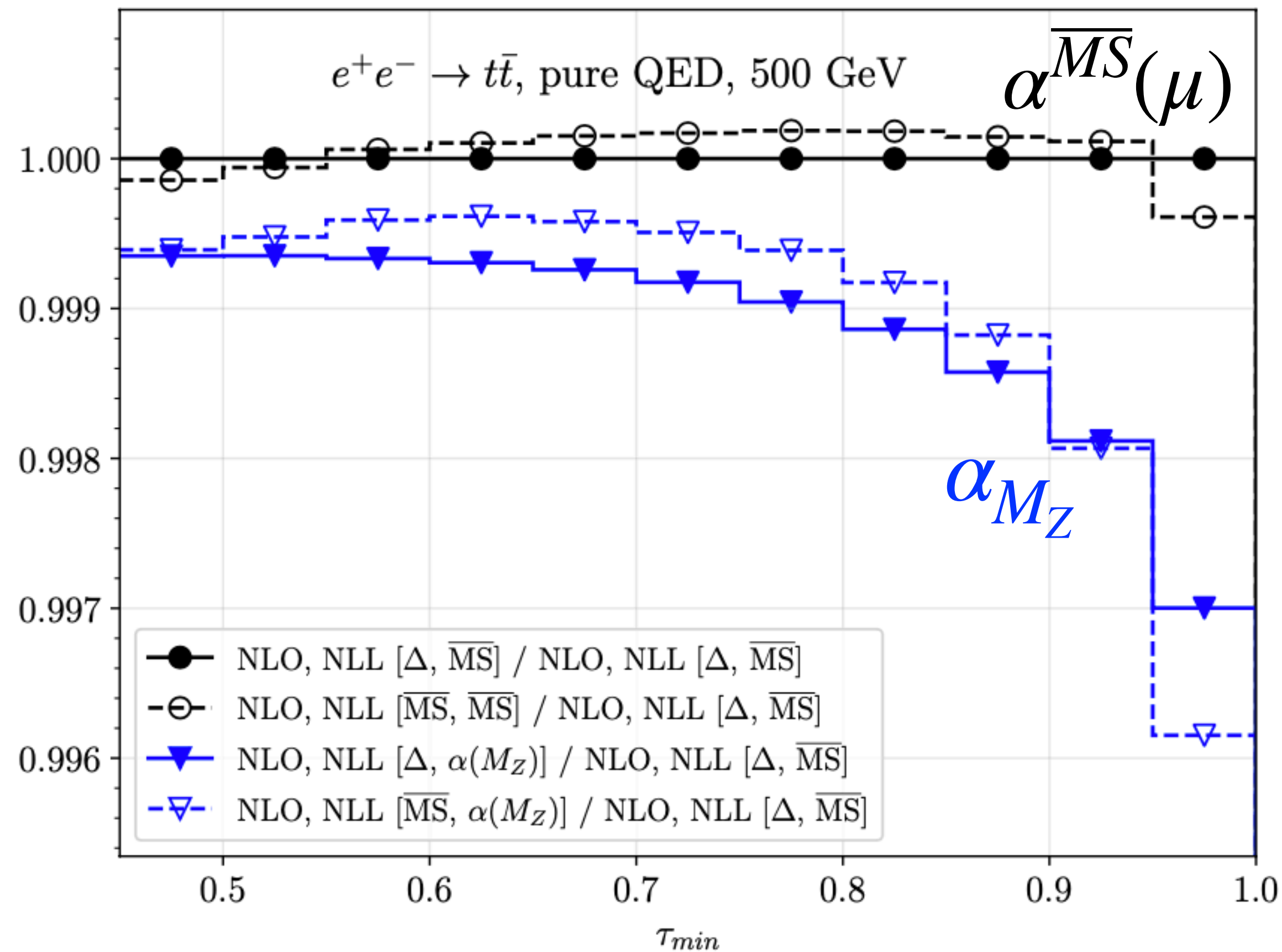
e^+e^- processes at NLO+NLL

Factorisation scheme dependence of physical processes with larger K-factors

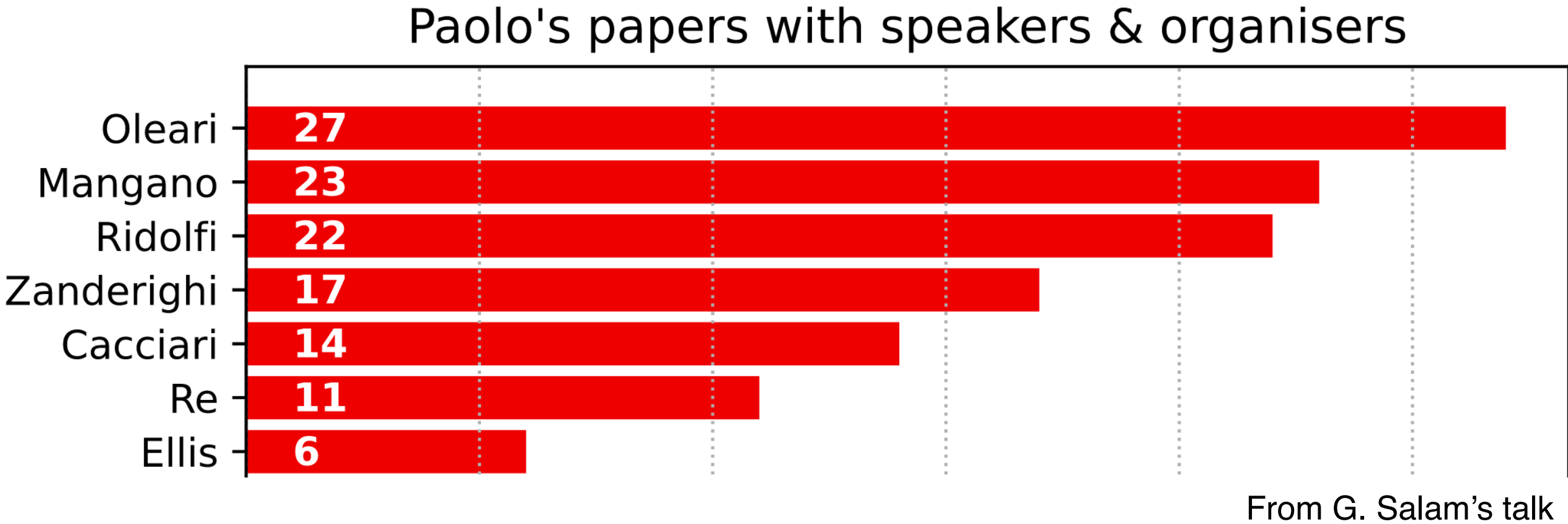


- Residual dependency now at per mille level, somewhat sensitive to renormalisation scheme

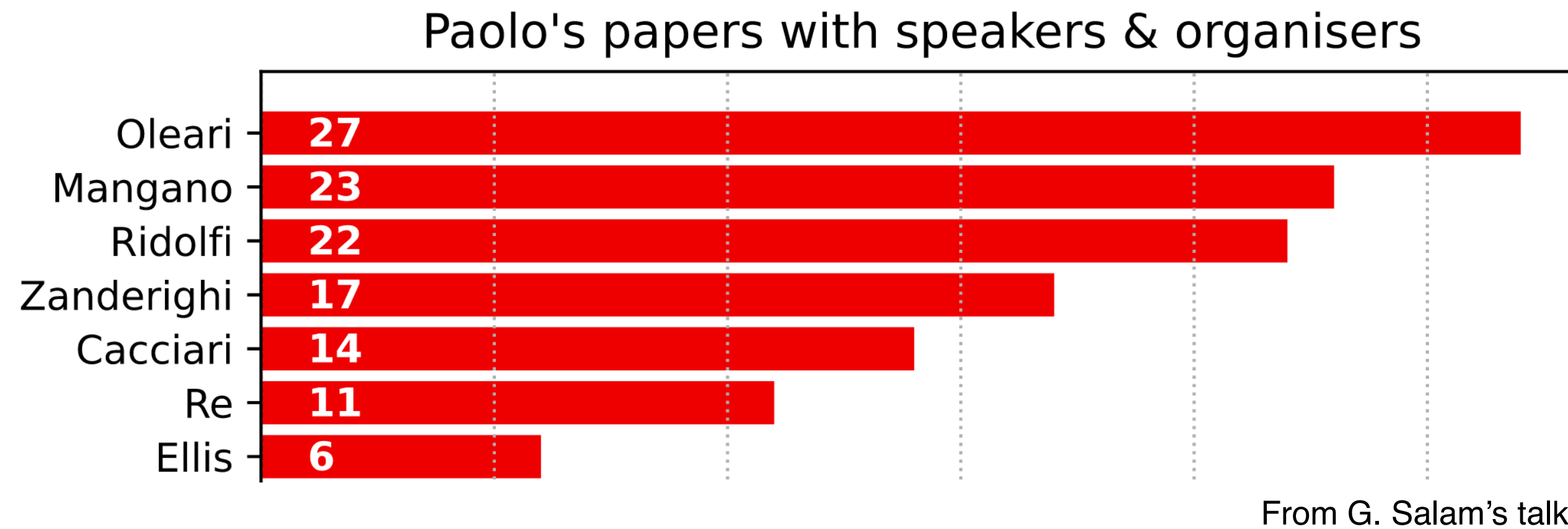
Renormalisation scheme dependence



- Effects generally larger than factorisation scheme
- Mostly a normalisation factor



I don't make the podium, but I still had plenty of opportunities to observe Paolo at work



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"There are two kinds of geniuses, the "ordinary" and the "magicians." An ordinary genius is a fellow that you and I would be just as good as, if we were only many times better. There is no mystery as to how his mind works. [...] It is different with the magicians. [...] Even after we understand what they have done, the process by which they have done it is completely dark. [...] — Mark Kac