# Precision at low energy: the case of MUonE

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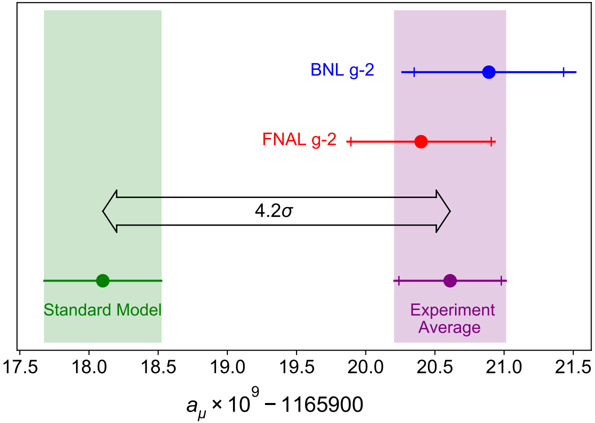
**A life in phenomenology**
**A conference in honour of Paolo Nason**

**Milano, September 15-16 2022**

# Starting point: muon $g - 2$

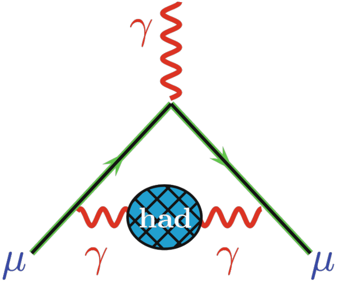



B. Abi et al., Phys. Rev. Lett. 126 (2021) 14, 141801 [arXiv:2104.03281[hep-ex]]

# Components of the theoretical prediction

$$
\begin{aligned}
a_\mu^{\mathrm{QED}} \times 10^{11} &= 116584718.931(104) \\
a_\mu^{\mathrm{EW}} \times 10^{11} &= 153.6(1.0) \\
a_\mu^{\mathrm{HLbL}} \times 10^{11} &= 92(18) \\
a_\mu^{\mathrm{HVP}} \times 10^{11} &= 6845(40) \\
\hline
a_\mu^{\mathrm{SM}} \times 10^{11} &= 116591810(43)
\end{aligned}
$$

T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166



# Standard approaches to $a_\mu^{\mathrm{HLO}}$

- dispersion relations, optical theorem and $e^+e^- \to hadrons$ data

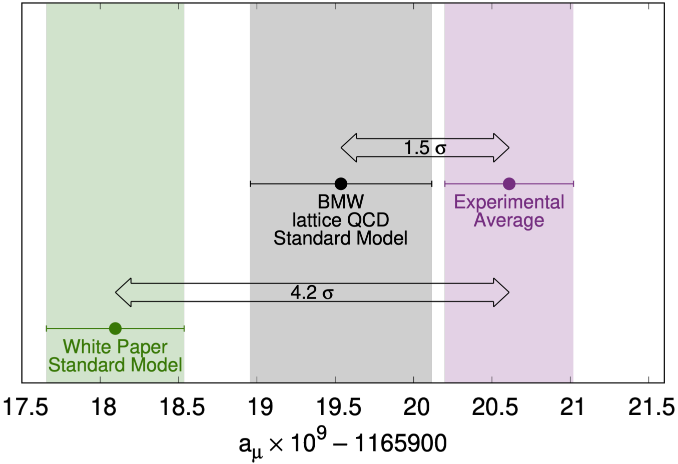
$$\boldsymbol{a_\mu^{\mathrm{HLO}}} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\; K(s)\;\sigma^0_{e^+e^-\to\text{had}}(s) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds\,\frac{K(s)R^{\text{had}}(s)}{s^2} =$$

$$= \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{m_\pi^2}^{E^2_{\text{cut}}} ds\,\frac{K(s)R^{\text{had}}_{\text{data}}(s)}{s^2} + \int_{E^2_{\text{cut}}}^{\infty} ds\,\frac{K(s)R^{\text{had}}_{\text{pQCD}}(s)}{s^2}\right]$$

$$K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}} \qquad R^{\text{had}}(s) = \frac{\sigma^0_{e^+e^-\to\text{had}}(s)}{\frac{4}{3}\pi\alpha^2/s}$$

- or first principles calculations with LQCD

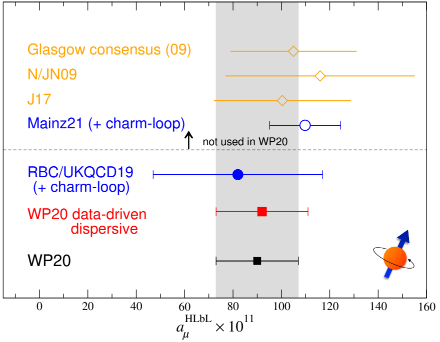
# An additional puzzle

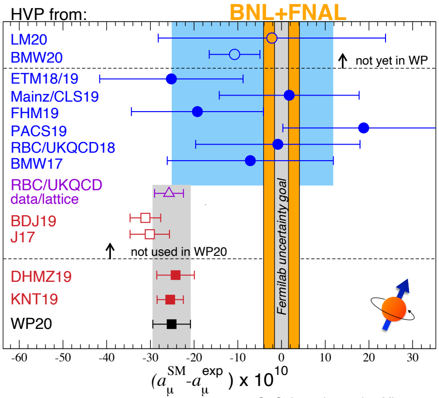



T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166

B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801.

Borsanyi, S. *et al.* *Nature* **593**, 51–55 (2021).




G. Colangelo *et al.* arXiv:2203.1581 (Snowmass 2021)

recent new developments e.g.

- Lattice 2022 (8-13 August 2022)
- Fifth Plenary Workshop of the Muon g-2 Theory Initiative (5-9 September 2022)

# A third independent determination more than welcome

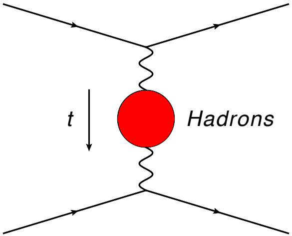
- ⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  *Measuring the leading hadronic contribution to the muon g-2 via $\mu e$ scattering*
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  *A new approach to evaluate the leading hadronic corrections to the muon g-2*
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

# Master formula

- Alternatively (exchanging $s$ and $x$ integrations in $a_\mu^{\rm HLO}$)

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$
$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

*e.g.* Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193




⇝ The hadronic VP correction to the running of $\alpha$ enters

⇝ Essentially the same formula used in lattice QCD calculation of $a_\mu^{\rm HLO}$

★ $\Delta\alpha_{\rm had}(t)$ (and $a_\mu^{\rm HLO}$) can be directly measured in a (single) experiment involving a space-like scattering process

**Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325**

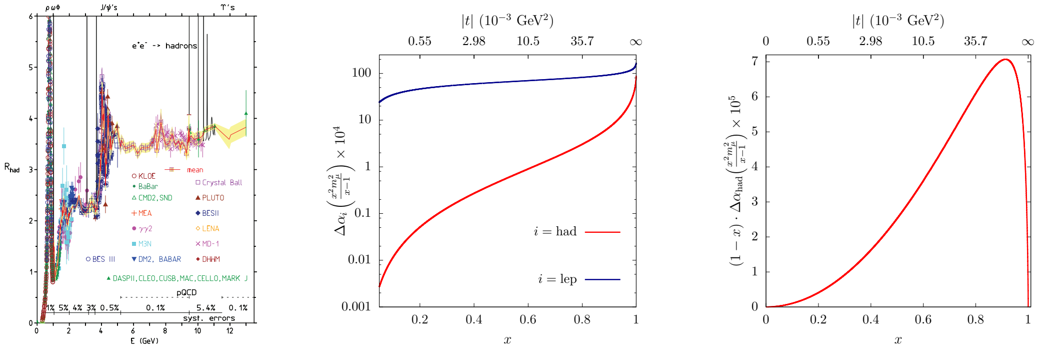
★ **Still a data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

- By modifying the kernel function $\frac{\alpha}{\pi}(1-x)$, also $a_\mu^{\rm HNLO}$ and $a_\mu^{\rm HNNLO}$ can be provided

Balzani, Laporta, Passera, **arXiv:2112.05704 [hep-ph]**

# From time-like to space-like evaluation of $a_\mu^{\mathrm{HLO}}$

**Time-like** $\mapsto$ **Space-like**




**Smooth function**

- $\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required
- $\mapsto$ **Space-like:** in principle, one single experiment, *it's a one-loop effect, very high accuracy needed*

# MUonE: $\mu e \to \mu e$ @CERN

**Abbiendi et al., EPJC 77 (2017) 3, 139**

**Abbiendi et al., *Letter of Intent: the MUonE project, CERN-SPSC-2019-026, SPSC-I-252 (2019)***

- Scattering $\mu$'s on $e$'s in a low $Z$ target looks like an ideal process (fixed target experiment)
- It is a pure $t$-channel process at tree level
- The M2 muon beam ($E_\mu \simeq 160$ GeV) is available at CERN
- $\sqrt{s} \simeq 0.4$ GeV and $-0.143 < t < 0$ GeV$^2$
- We can cover $87\%$ of the $a_\mu^{\text{HLO}}$ space-like integral (and extrapolate to $x \to 1$)
- With $\sim 3$ years of data taking, a statistical accuracy of $0.3\%$ on $a_\mu^{\text{HLO}}$ can be achieved

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\text{had}}$$

**$\Delta\alpha_{\text{had}}$ is a $0.1\%$ effect in this region $\to$ to measure it at $1\%$, $\sigma$ must be controlled at the $10^{-5}$ level**

# statistics and (main) systematic uncertainties

- **statistics**: CERN muon beam M2 ($E = 150$ GeV), $1.3 \cdot 10^7\ \mu/s$ with a target (Be/C) with total thickness of 60 cm $\Longrightarrow L \sim 1.5 \cdot 10^7$nb$^{-1}$ $\Longrightarrow$ statistical sensitivity $\sim 0.3$% on $a_\mu^{HLO} (\sim 20 \cdot 10^{-11})$ in about 3 yrs of data taking

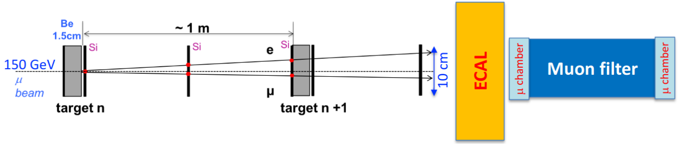
**Sistematics**

- (main) experimental sources
  - multiple scattering: $E_e$ in normalization region much lower than in signal region
    Effect $\sim 1/E \Longrightarrow$ it affects signal and normalization in different way
  - absolute $\mu$ beam energy scale, $5$ MeV $\Longrightarrow 10^{-5}$ effect
  - angular intrinsec resolution $(\sim 1\%)$
  - longitudinal alignment $(\sim 10 \mu m)$
- theoretical: higher order radiative corrections modify the shapes
  - order of magnitude estimate, barring infrared logs and setting $c_{i,j} \sim 10$
  - $c_{1,1} \left(\frac{\alpha}{\pi}\right) L \sim 0.2$ $\quad c_{1,0} \left(\frac{\alpha}{\pi}\right) \sim 2.5 \cdot 10^{-2}$
  - $c_{2,2} \left(\frac{\alpha}{\pi}\right)^2 L^2 \sim 5 \cdot 10^{-3}$ $\quad c_{2,1} \left(\frac{\alpha}{\pi}\right)^2 L \sim 5 \cdot 10^{-4}$ $\quad c_{2,0} \left(\frac{\alpha}{\pi}\right)^2 \sim 5 \cdot 10^{-5}$
  - $c_{3,3} \left(\frac{\alpha}{\pi}\right)^3 L^3 \sim 1.5 \cdot 10^{-4}$ $\quad c_{3,1} \left(\frac{\alpha}{\pi}\right)^3 L^2 \sim 1.5 \cdot 10^{-5}$ $\quad c_{3,0} \left(\frac{\alpha}{\pi}\right)^3 L \sim 1.5 \cdot 10^{-6}$
  - the most advanced technologies for NNLO calculations and higher order resummation and matching are needed

# On the experimental side

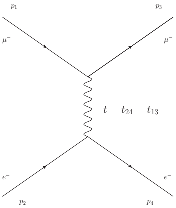
- a modular apparatus has been proposed (40 independent tracking stations)

G. Abbiendi et al., LoI CERN-SPSC-2019-026, SPSC-I-252, CERN




- whole acceptance covered with a $10 \times 10$ cm$^2$ silicon sensor
- thin targets equivalent to 60 cm
- ECal and Muon filter after last station, for PID and background rejection
- two Beam Tests already done at CERN (2017 and 2018)
  1. Multiple Scattering measurements — G. Abbiendi et al., arXiv:1905.11677
  2. selection of a clean sample of elastic events — G. Abbiendi et al., arXiv:2021.11111
- Further Beam Test in October 2022
- 3 weeks Test Run in 2023 (proof of concept of the experimental proposal)
- 10 stations before LHC LS3 (2026) with first measurements of $a_\mu^{\mathrm{HVP}}$ with $\sim 1\%$ accuracy

# First step towards precision: QED NLO



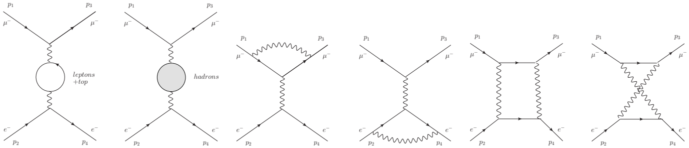

- analytical expression for tree level

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)}\left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2}\right]$$

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

(Van Nieuwenhuizen 1971, D'Ambrosio 1983, Kukhto et al. 1987, Bardin, Kalinovskaya 1997)

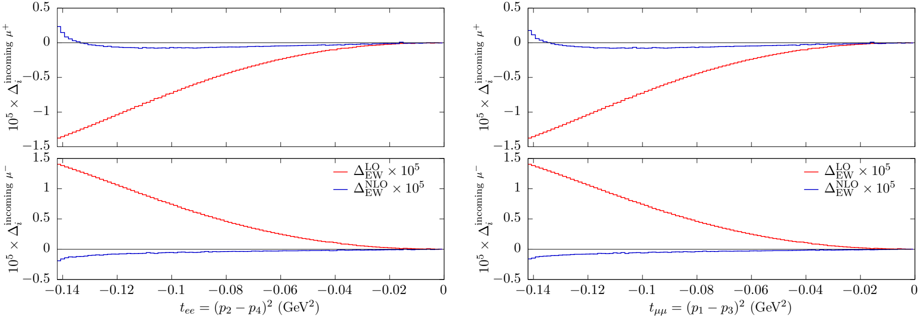
- NLO virtual diagrams




- and corresponding real emission diagrams
- **NLO matrix elements** calculated with finite $m_\mu$ and $m_e$ mass effects and a **Monte Carlo** program, MESMER, has been taylored to the fixed target kinematics

Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini, arXiv:1811.06743; JHEP 02 (2019) 155

# Weak interaction effects (LO and NLO)



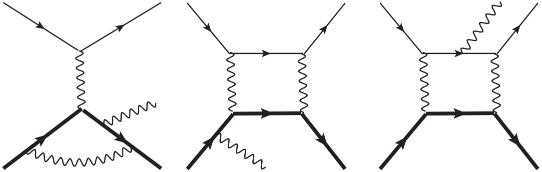

Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini, arXiv:1811.06743

- **tree-level $Z$-exchange important** at the $10^{-5}$ level
- purely weak RCs (in QED NLO units) at a few $10^{-6}$ level

# Second step, *photonic* radiative corrections at **NNLO**

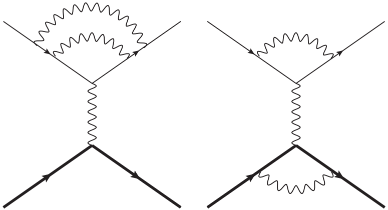
Carloni Calame et al., JHEP 11 (2020) 028

- | NLO virtual diagrams $|^2$ **calculated exactly**
- interference of LO $\mu e \to \mu e \gamma$ amplitude with



$+$ many others

- interference of LO $\mu e \to \mu e$ amplitude with

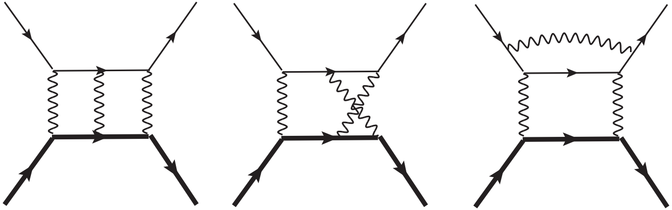


$+$ many others

2-loop QED vertex form factors borrowed from **Mastrolia and Remiddi, NPB 664 (2003) 341**

- interference of LO $\mu e \to \mu e$ amplitude with

**approximated à la YFS**



$+$ many others

⇝ NNLO double-virtual amplitudes where at least 2 photons connect the $e$ and $\mu$ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the infra-red divergent structure

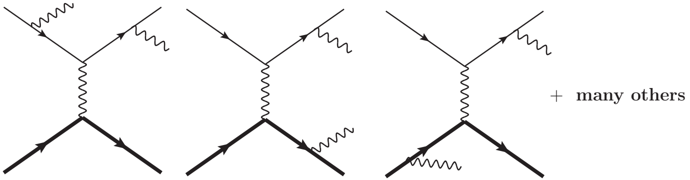
$$\widetilde{\mathcal{A}}^{\alpha^2} = \underbrace{\mathcal{A}_e^{\alpha^2} + \mathcal{A}_\mu^{\alpha^2} + \mathcal{A}_{e\mu,\,\mathrm{1L\times1L}}^{\alpha^2}}_{\text{exact}} + \underbrace{\frac{1}{2}Y_{e\mu}^2\mathcal{T} + Y_{e\mu}\left(Y_e+Y_\mu\right)\mathcal{T} + (Y_e+Y_\mu)\mathcal{A}_{e\mu}^{\alpha^1,\mathrm{R}} + Y_{e\mu}\mathcal{A}^{\alpha^1,\mathrm{R}}}_{\text{YFS approximated}}$$

- going beyond this requires the full two-loop virtual amplitudes

R. Bonciani *et al.*, PRL 128 (2022) 2

- squared absolute value of

**calculated exactly**



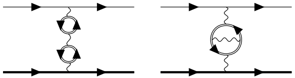

- also at NNLO we use a vanishingly small photon mass $\lambda$ and the “slicing method” to deal with IR divergences
- phase space integration and event generation is again performed with MC techniques allowing for fully exclusive event generation
- **we estimate the subset of amplitudes in YFS approximation to miss terms of order**

$$\left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(m_\mu^2/m_e^2\right) \simeq 5\times 10^{-4}$$

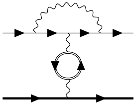
# NNLO Virtual leptonic pairs (vacuum polarization insertions)

E. Budassi et al., JHEP 11 (2021) 098

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes

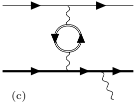


(a) (b)



(a)

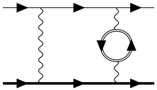
$+ \cdots$



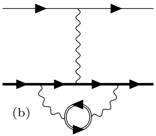
(c)

$+ \cdots$

- interfered with $\mu e \to \mu e$ amplitude



(a)



(b)

$+ \cdots$

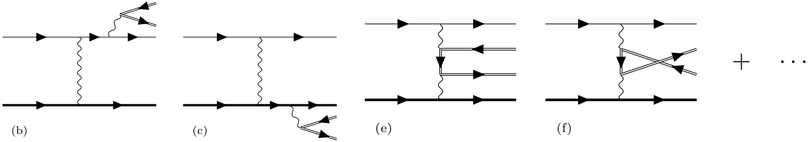
Here the 2-loop integral is evaluated with dispersion relation techniques

used in the past for Bhabha: Carloni Calame et al., JHEP 07 (2011) 126, and for hadr. corr. in MUonE: Fael & Passera, PRL 122 (2019) 19

$$\frac{g_{\mu\nu}}{q^2 + i\epsilon} \to g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{R_\ell(z)}{q^2 - z + i\epsilon} = g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_\ell^2}{2z}\right) \sqrt{1 - \frac{4m_\ell^2}{z}}$$

# Real pair emissions

- they also contribute at NNLO
- squared absolute vaule of



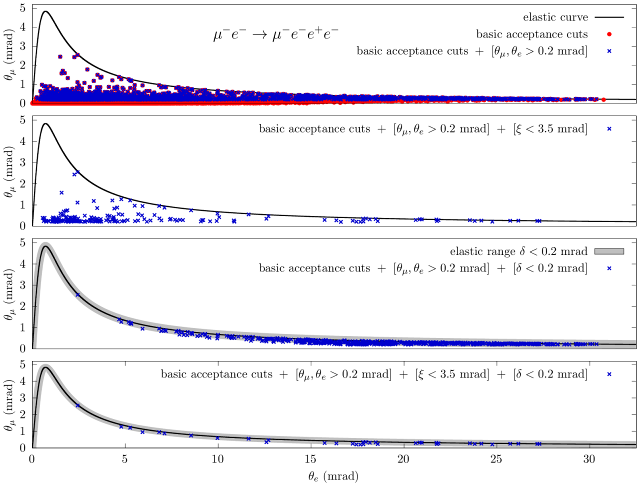

- the emission of an extra electron pair $\mu e \to \mu e\, e^+e^-$ is potentially a dramatically large (reducible) background, because of the presence of "peripheral" diagrams

⇝ A set of experimental cuts is needed to get rid of it.
In addition to basic cuts (exactly one muon-like and one electron-like, with $E \geq 1$ GeV, particle in the detector), we consider

1. $\theta_{\mu\text{-like}}, \theta_{e\text{-like}} \geq \theta_c = 0.2$ mrad
2. acoplanarity $\leq 3.5$ mrad
3. geometric distance from the elastic curve in the $[\theta_\mu, \theta_e]$ plane $< 0.2$ mrad

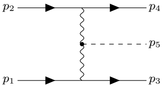
# Real $e^+e^-$ pairs

E. Budassi et al., JHEP 11 (2021) 098




⇝ only $0.007\%$ of $\mu e \to \mu e\, e^+e^-$ events survives the combination of the three cuts

- $\pi^0$ **production**




  - The process $\mu e \to \mu e \pi^0$ with $\pi^0 \to \gamma\gamma$ as possible background, using a phenomenological model for the $\gamma^\star\gamma^\star\pi^0$ effective vertex
  - ⇝ not an issue in the signal region

E. Budassi et al., PLB 829 (2022) 137138

  - ⇝ perhaps to be considered for NP searches in phase space region outside the signal one

- **robustness of the measurement against possible New Physics "contamination" has been studied**

A. Masiero, P. Paradisi and M. Passera, arXiv:2002.05418

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, arXiv:2002.04822

- **interesting proposals for New Physics searches at MUonE (new light mediators)**
  - invisibly decaying light $Z'$ in $\mu e \to \mu e Z'$

Asai et al., arXiv:2109.10093

  - long-lived mediators with displaced vertex signatures

Galon et al., arXiv:2202.08843

  - through scattering off the target nuclei $\mu N \to \mu N X$

Grilli di Cortona and E. Nardi, arXiv:2204.04227

# Summary

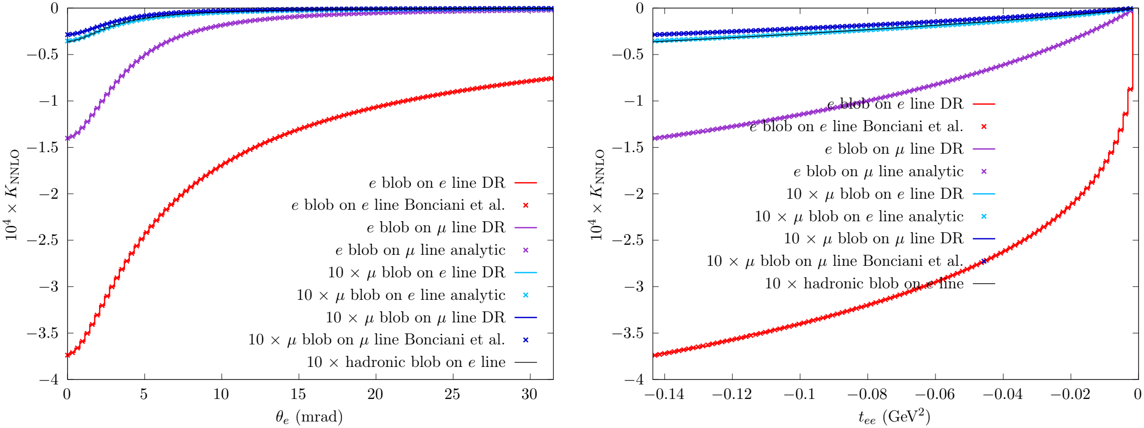
- Carloni Calame et al., PLB 746 (2015), 325
- Mastrolia et al., JHEP 11 (2017) 198
- Di Vita et al., JHEP 09 (2018) 016
- Alacevich et al., JHEP 02 (2019) 155
- Fael and Passera, PRL 122 (2019) 19, 192001
- Fael, JHEP 02 (2019) 027
- Carloni Calame et al., JHEP 11 (2020) 028
- Banerjee et al., SciPost Phys. 9 (2020), 027
- Banerjee et al., EPJC 80 (2020) 6, 591
- Budassi et al., JHEP 11 (2021) 098
- Balzani et al., **arXiv:2112.05704 [hep-ph]**
- Bonciani et al., PRL 128 (2022) 2, 022002
- Budassi et al., PLB 829 (2022) 137138

- A lively theory community is active to provide state-of-the-art calculations to match the required accuracy for meaningful data analysis
- Independent numerical codes (Monte Carlo generators and/or integrators) are developed and cross-checked to validate high-precision calculations. Chiefly
  - ✓ **Mesmer** in Pavia
    github.com/cm-cc/mesmer
  - ✓ **McMule** at PSI/IPPP
    gitlab.com/mule-tools/mcmule
- An international MUonE collaboration is growing

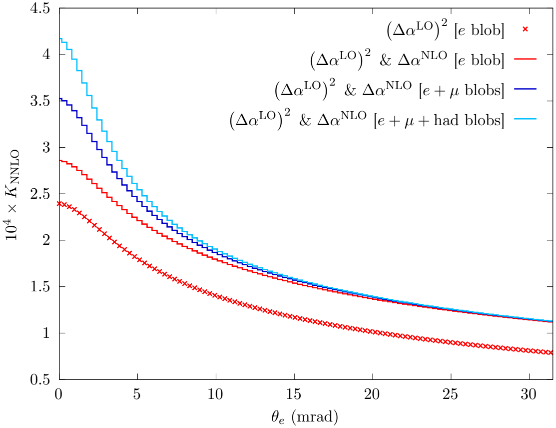
Thank you Paolo for your continuous support

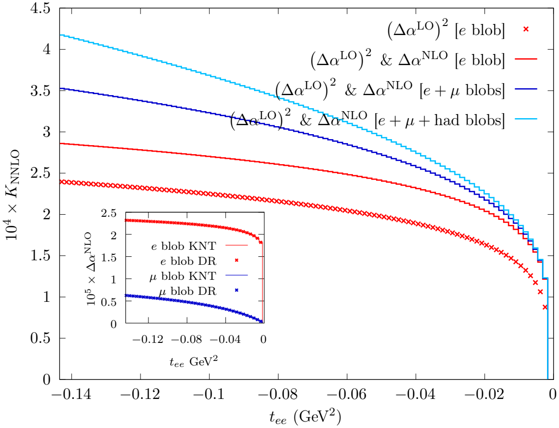
# SPARES

# Virtual leptonic (and hadronic NNLO) vertex corrections



# Virtual leptonic (and hadronic) NNLO VP corrections



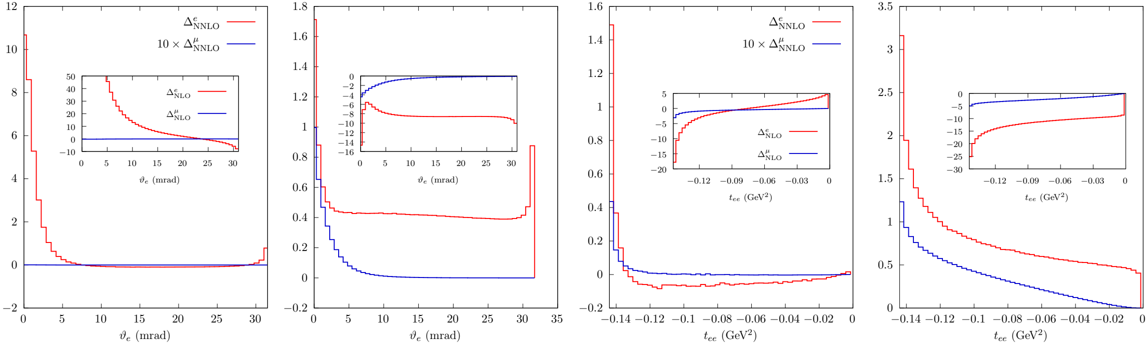
# NNLO results

JHEP 11 (2020) 028

- Showing

$$\Delta^i_{\text{NNLO}} \equiv 100 \times \frac{d\sigma^i_{\text{NNLO}} - d\sigma^i_{\text{NLO}}}{d\sigma_{\text{LO}}}$$

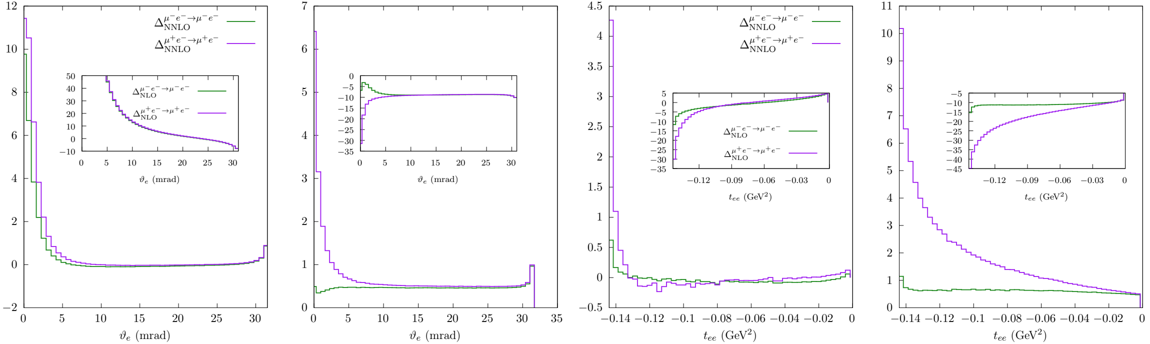
⇝ exact NNLO radiation from electron or muon leg, with or without acoplanarity cut



# (approximated) photonic NNLO results

JHEP 11 (2020) 028

⇝ full NNLO[1] radiation for incoming $\mu^+$ or $\mu^-$, with or without acoplanarity cut

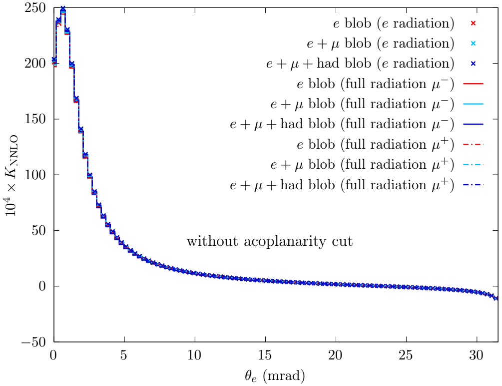


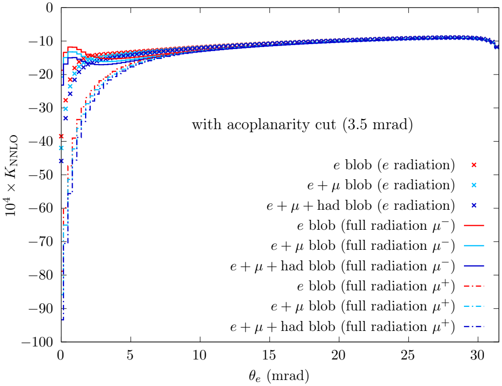

[1] of course with "double boxes" in YFS approximation

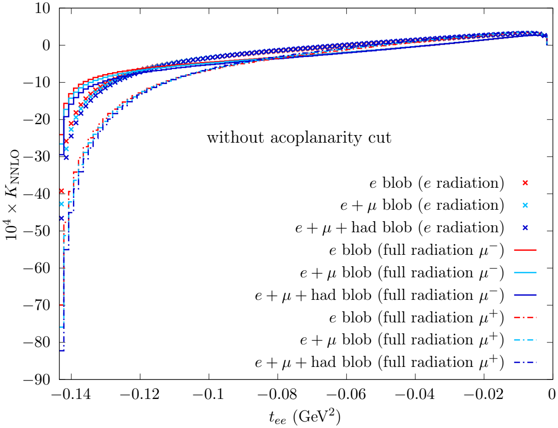
# Virtual pair effects

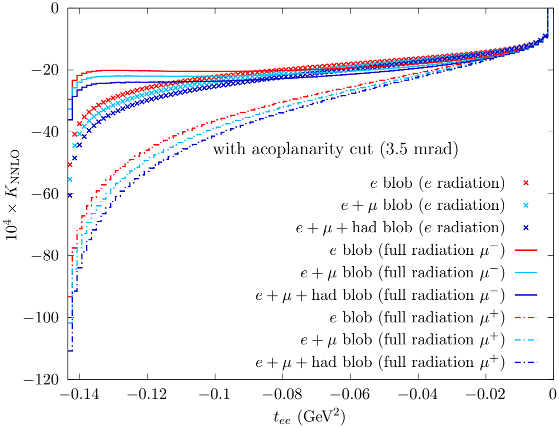
- Showing NNLO differential $K$-factors $\times 10^4$

$$K_{\text{NNLO}} \equiv \frac{d\sigma_i^{\alpha^2,\text{ virtual pairs}}}{d\sigma_{\text{LO}}}$$

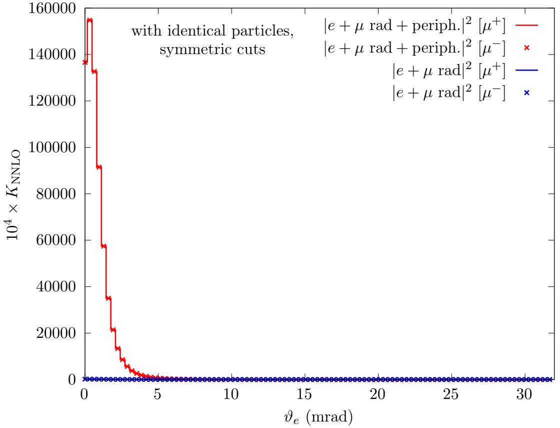


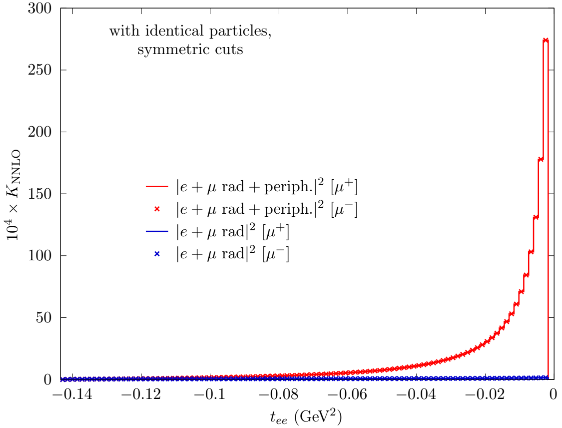



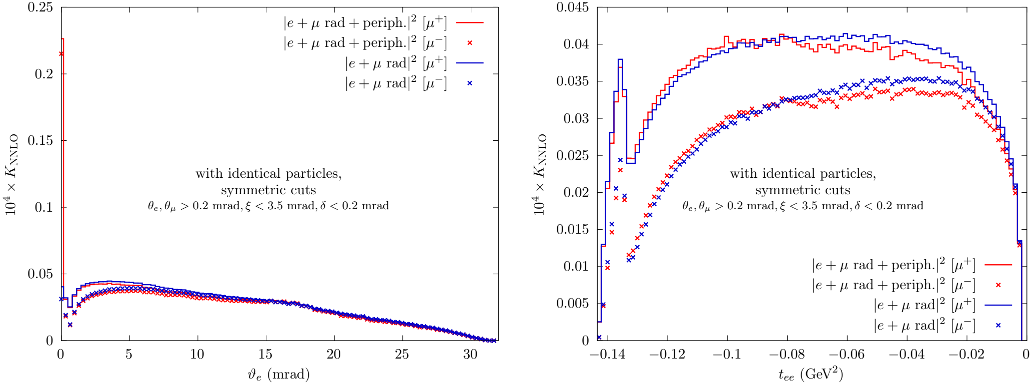

without acoplanarity cut
$10^4 \times K_{\rm NNLO}$
$t_{ee}$ (GeV$^2$)
$e$ blob ($e$ radiation)
$e+\mu$ blob ($e$ radiation)
$e+\mu+$had blob ($e$ radiation)
$e$ blob (full radiation $\mu^-$)
$e+\mu$ blob (full radiation $\mu^-$)
$e+\mu+$had blob (full radiation $\mu^-$)
$e$ blob (full radiation $\mu^+$)
$e+\mu$ blob (full radiation $\mu^+$)
$e+\mu+$had blob (full radiation $\mu^+$)


with acoplanarity cut (3.5 mrad)
$10^4 \times K_{\rm NNLO}$
$t_{ee}$ (GeV$^2$)
$e$ blob ($e$ radiation)
$e+\mu$ blob ($e$ radiation)
$e+\mu+$had blob ($e$ radiation)
$e$ blob (full radiation $\mu^-$)
$e+\mu$ blob (full radiation $\mu^-$)
$e+\mu+$had blob (full radiation $\mu^-$)
$e$ blob (full radiation $\mu^+$)
$e+\mu$ blob (full radiation $\mu^+$)
$e+\mu+$had blob (full radiation $\mu^+$)


with identical particles,
symmetric cuts
$|e+\mu \text{ rad} + \text{periph.}|^2$ $[\mu^+]$
$|e+\mu \text{ rad} + \text{periph.}|^2$ $[\mu^-]$
$|e+\mu \text{ rad}|^2$ $[\mu^+]$
$|e+\mu \text{ rad}|^2$ $[\mu^-]$
$10^4 \times K_{\text{NNLO}}$
$\vartheta_e$ (mrad)
160000
140000
120000
100000
80000
60000
40000
20000
0
0
5
10
15
20
25
30


with identical particles,
symmetric cuts
$|e+\mu \text{ rad} + \text{periph.}|^2$ $[\mu^+]$
$|e+\mu \text{ rad} + \text{periph.}|^2$ $[\mu^-]$
$|e+\mu \text{ rad}|^2$ $[\mu^+]$
$|e+\mu \text{ rad}|^2$ $[\mu^-]$
$10^4 \times K_{\text{NNLO}}$
$t_{ee}$ (GeV$^2$)
300
250
200
150
100
50
0
−0.14
−0.12
−0.1
−0.08
−0.06
−0.04
−0.02
0




⇝ $\mu e \to \mu e\ \mu^+\mu^-$ is always tiny, because of tiny available phase space