



STANDARD MODEL PREDICTION UNCERTAINTIES

The 31st International Symposium on Lepton Photon Interactions at High Energies



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Nationalfonds

Xuan Chen
Melbourne, Australia
17 July, 2023



Standard Model Prediction Uncertainties

► SM has a wide range of theoretical uncertainties

$$a_\mu = 116591810(43) \times 10^{-11}$$

Phys. Reports 887 (2020) 1-116

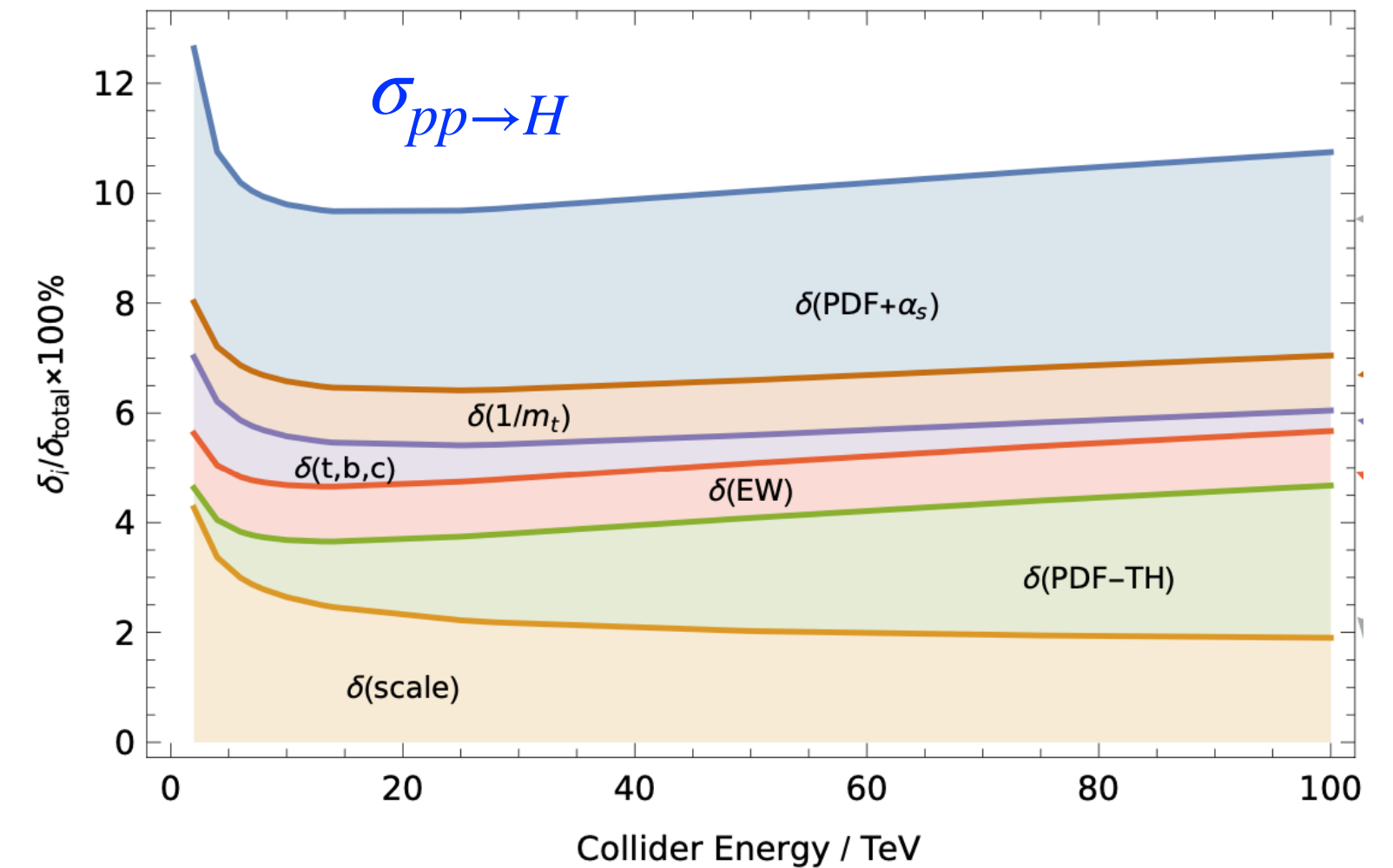
$$a_e = 1159652180.252(95) \times 10^{-12}$$

Nature (London) 588, 61 (2020)

$$\alpha^{-1} = 137.035999166(15)$$

Phys. Rev. Lett. 130, 071801 (2023)

0.1/billion ~ 10/cent
Uncertainties



F. Dulat, A. Lazopoulos, B. Mistlberger 2018

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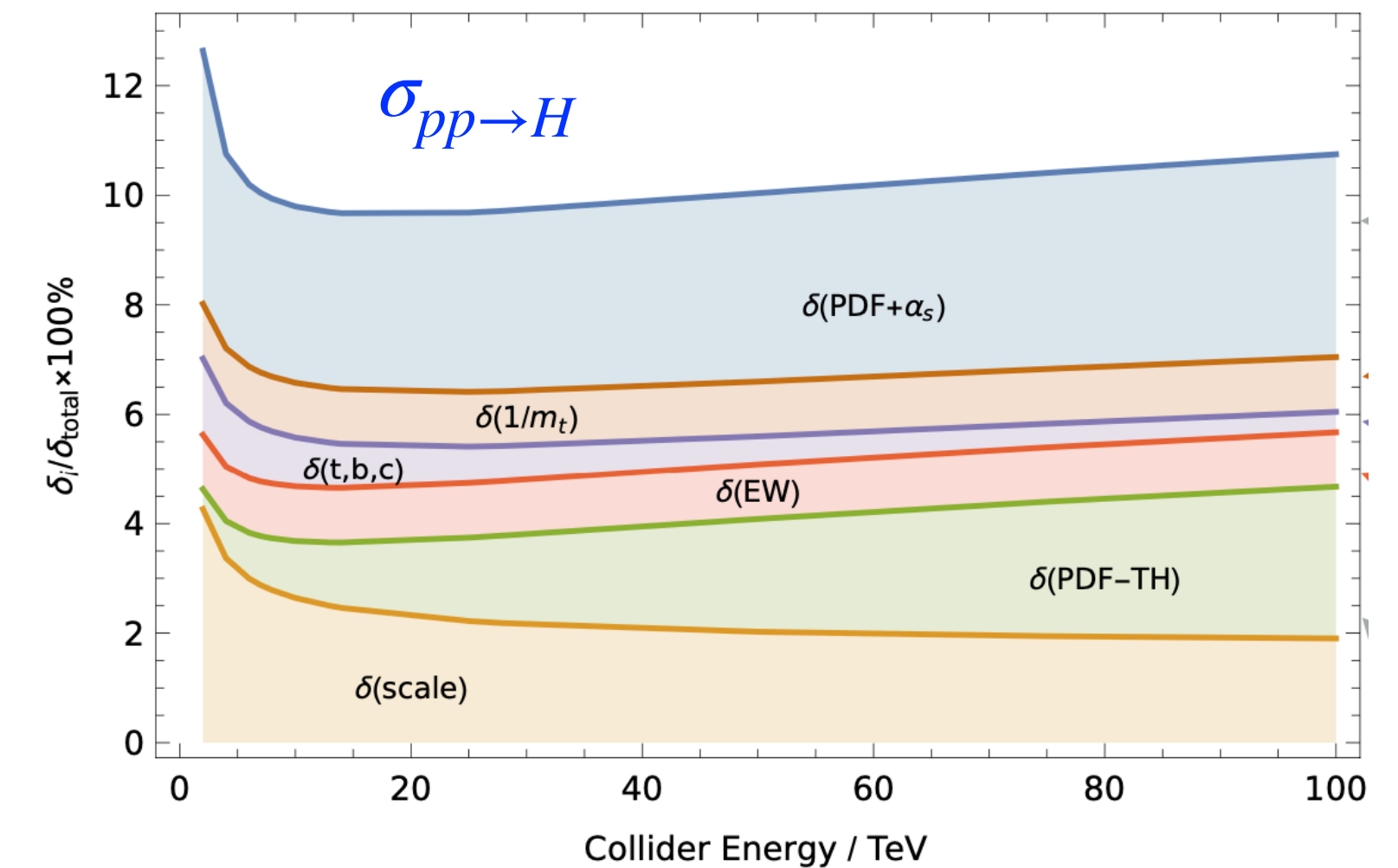
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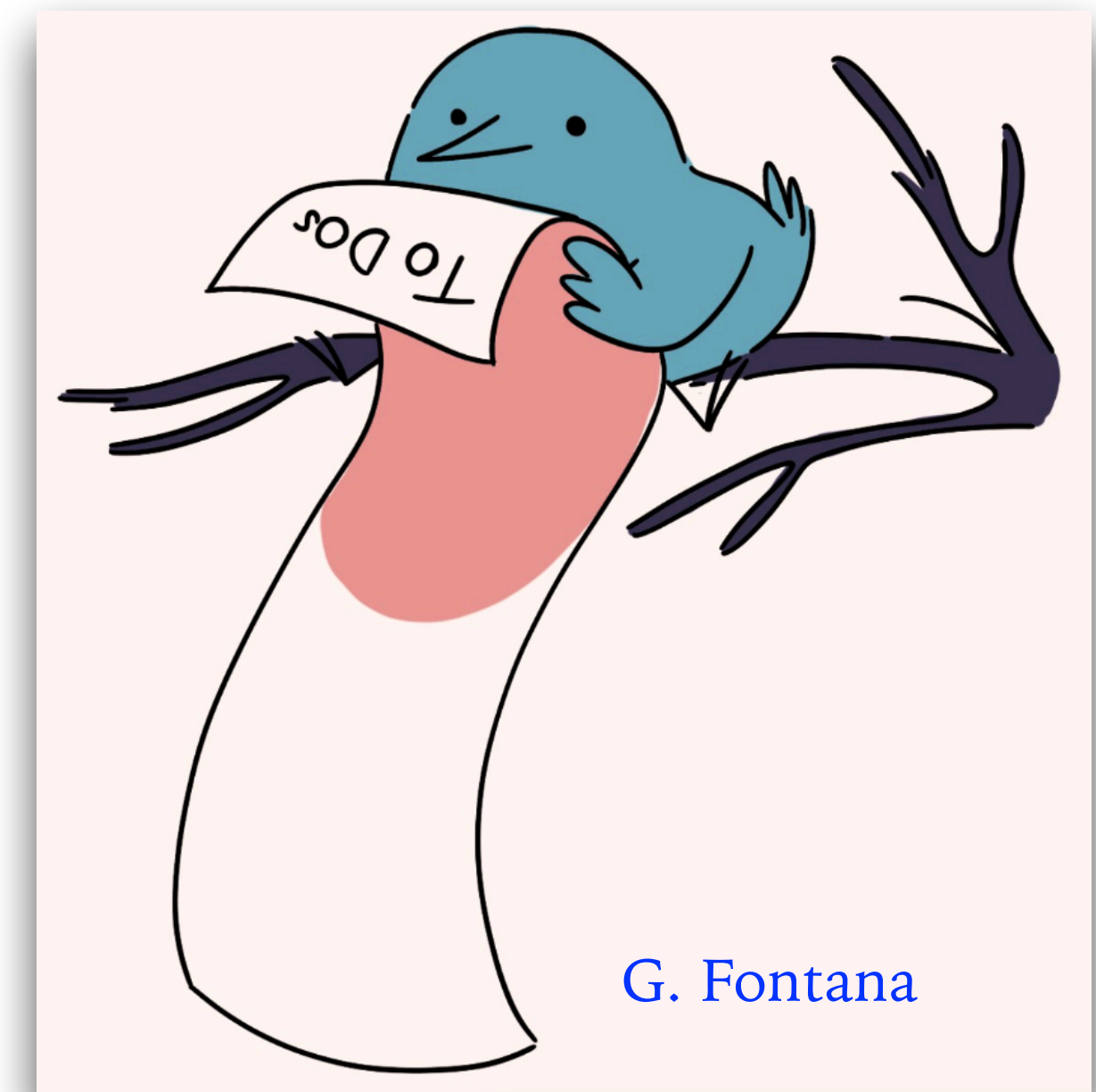
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Uncertainties



F. Dulat, A. Lazopoulos, B. Mistlberger 2018

► Motivations of scrutinisation:

- To exercise our understanding of the Standard Model
- To establish new sector of the Standard Model (Higgs)
- To maximise sensitivity to new physics in measurements



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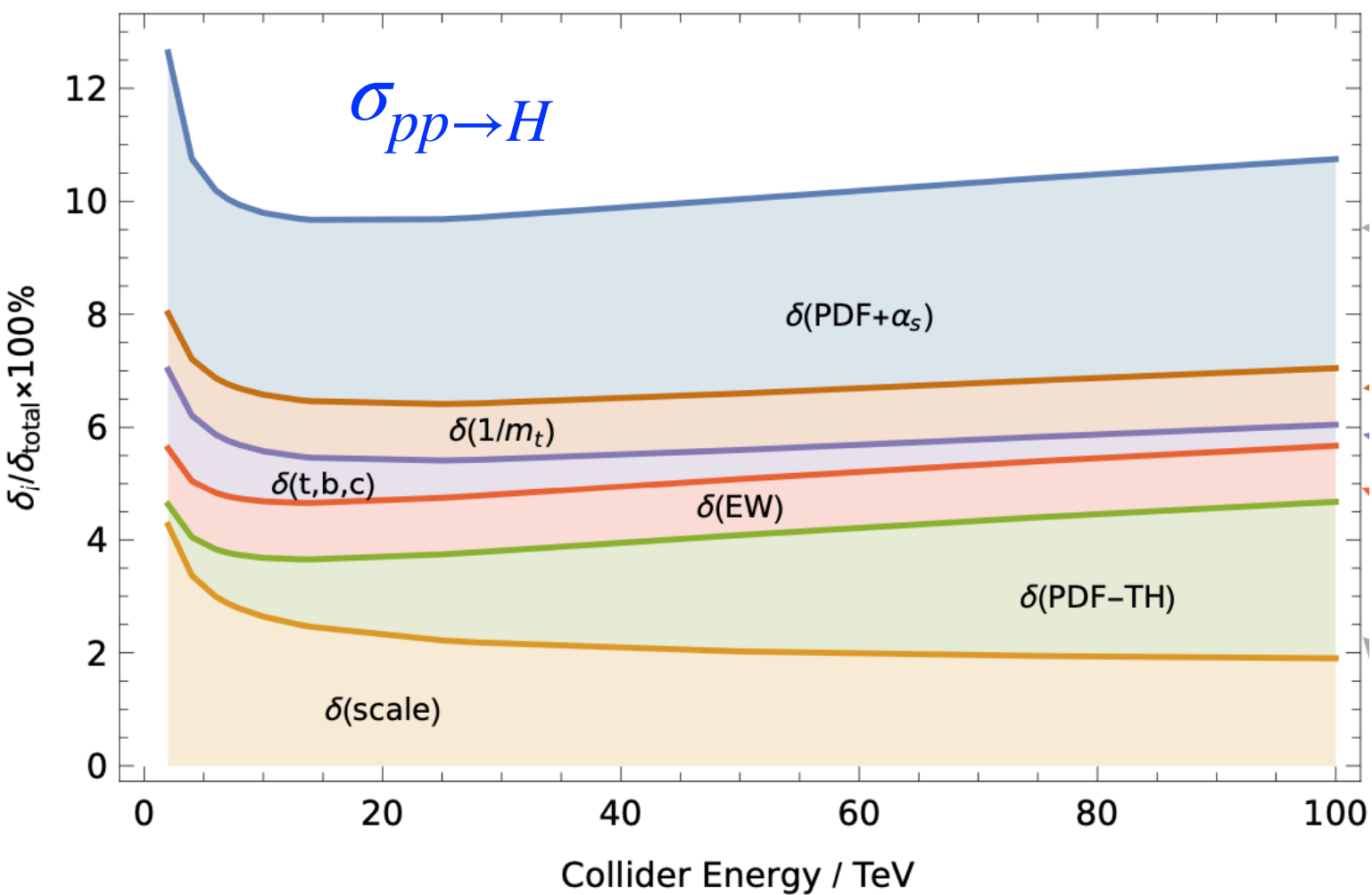
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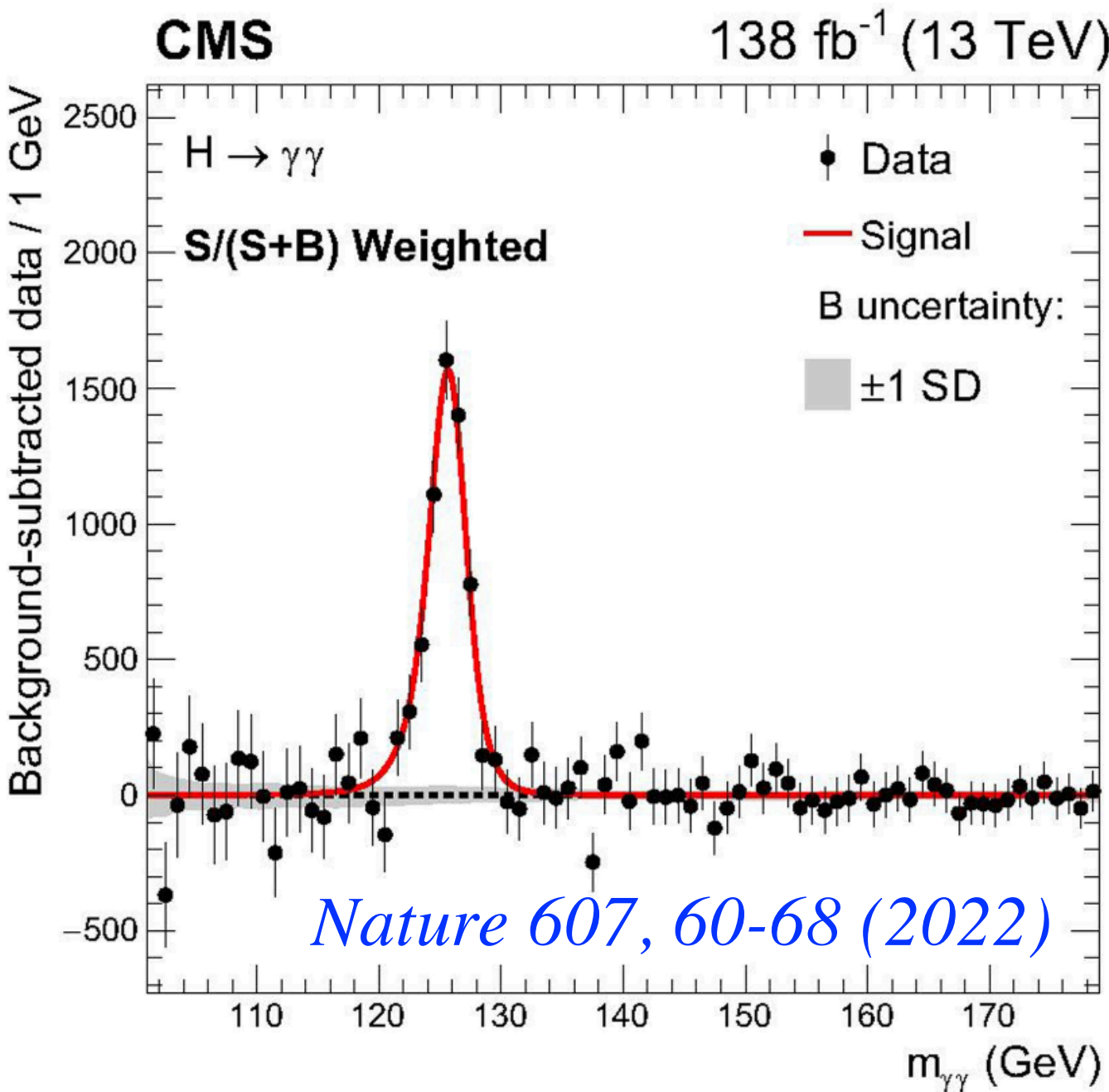
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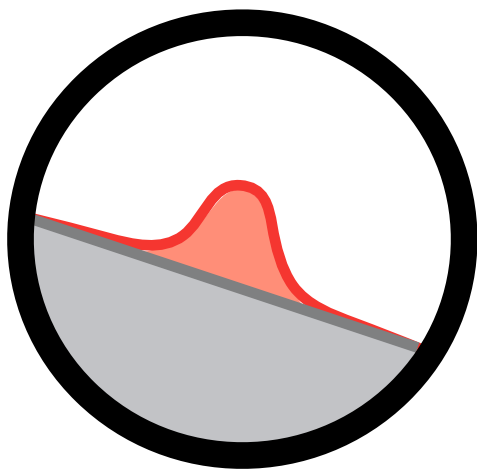


F. Dulat, A. Lazopoulos, B. Mistlberger 2018

► Direct discovery for new channels and new resonants



Nature 607, 60-68 (2022)



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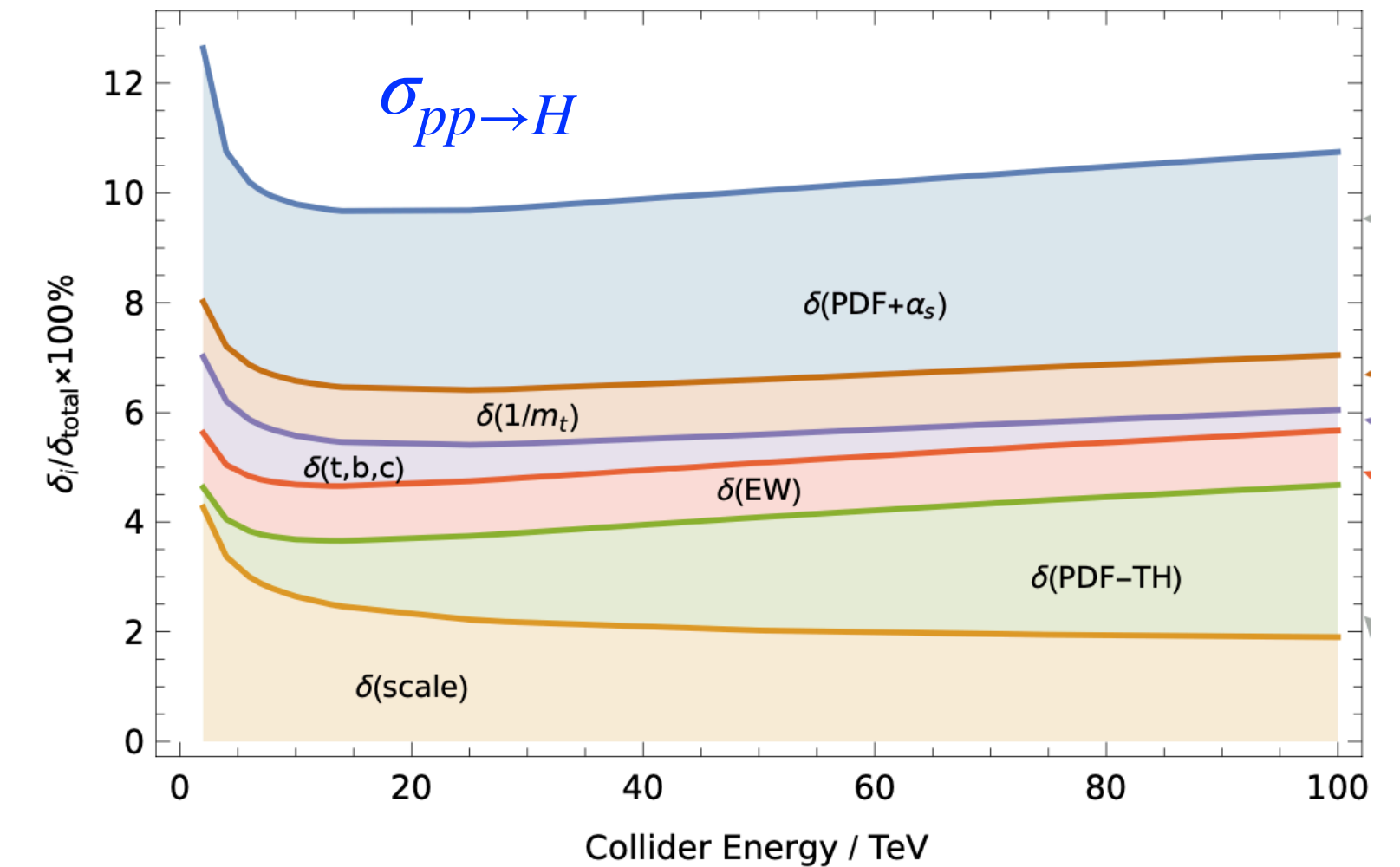
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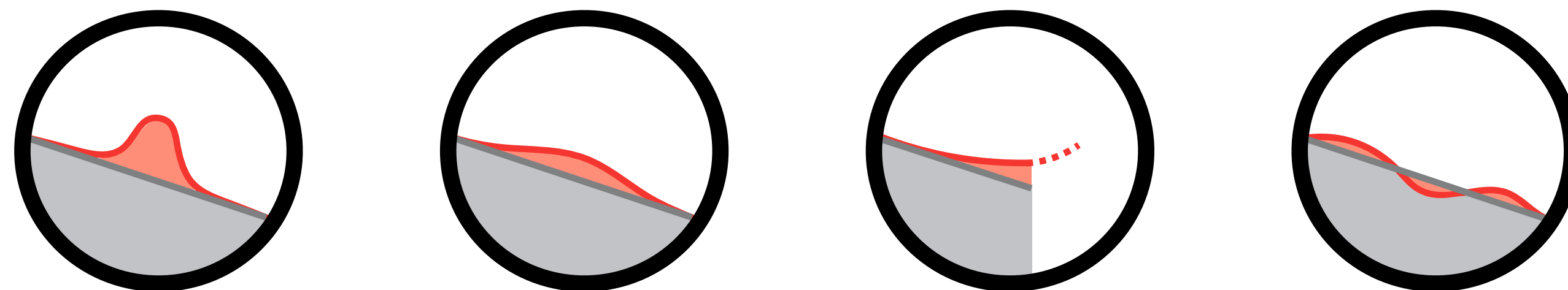
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Uncertainties



F. Dulat, A. Lazopoulos, B. Mistlberger 2018

- Direct discovery for new channels and new resonants
- Indirect discovery with high precision
 - Wide resonance, Prepeak uptrend, Shape distortion



$$E - T_{SM} \propto \frac{1}{\Lambda_{BSM}^2}$$

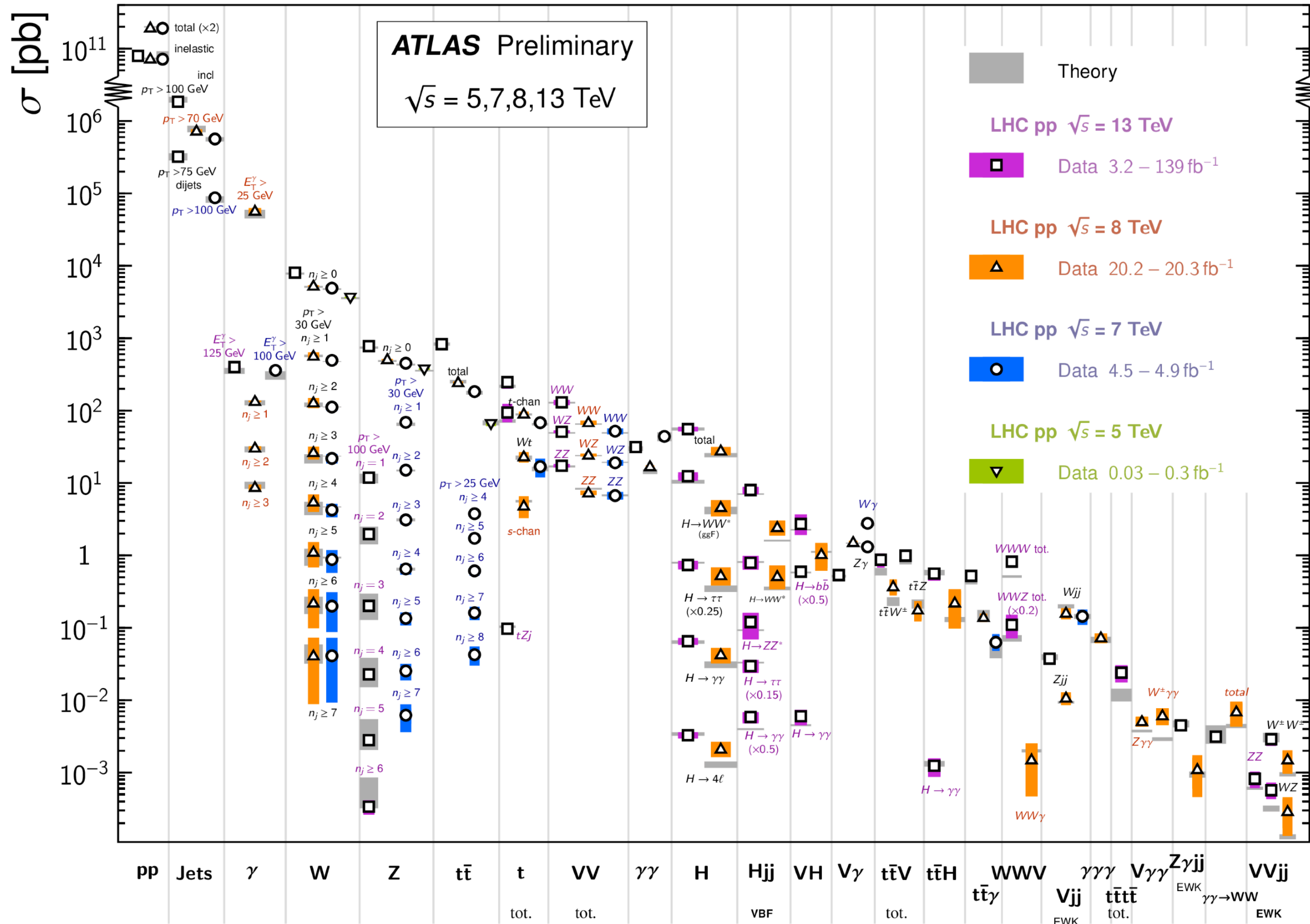
↓

$$(E \pm \delta E) - (T_{SM} \pm \delta T_{SM}) \propto \frac{1}{\Lambda_{BSM}^2}$$

Standard Model Production Cross Section Measurements

Status: February 2022

Precision (Rate)



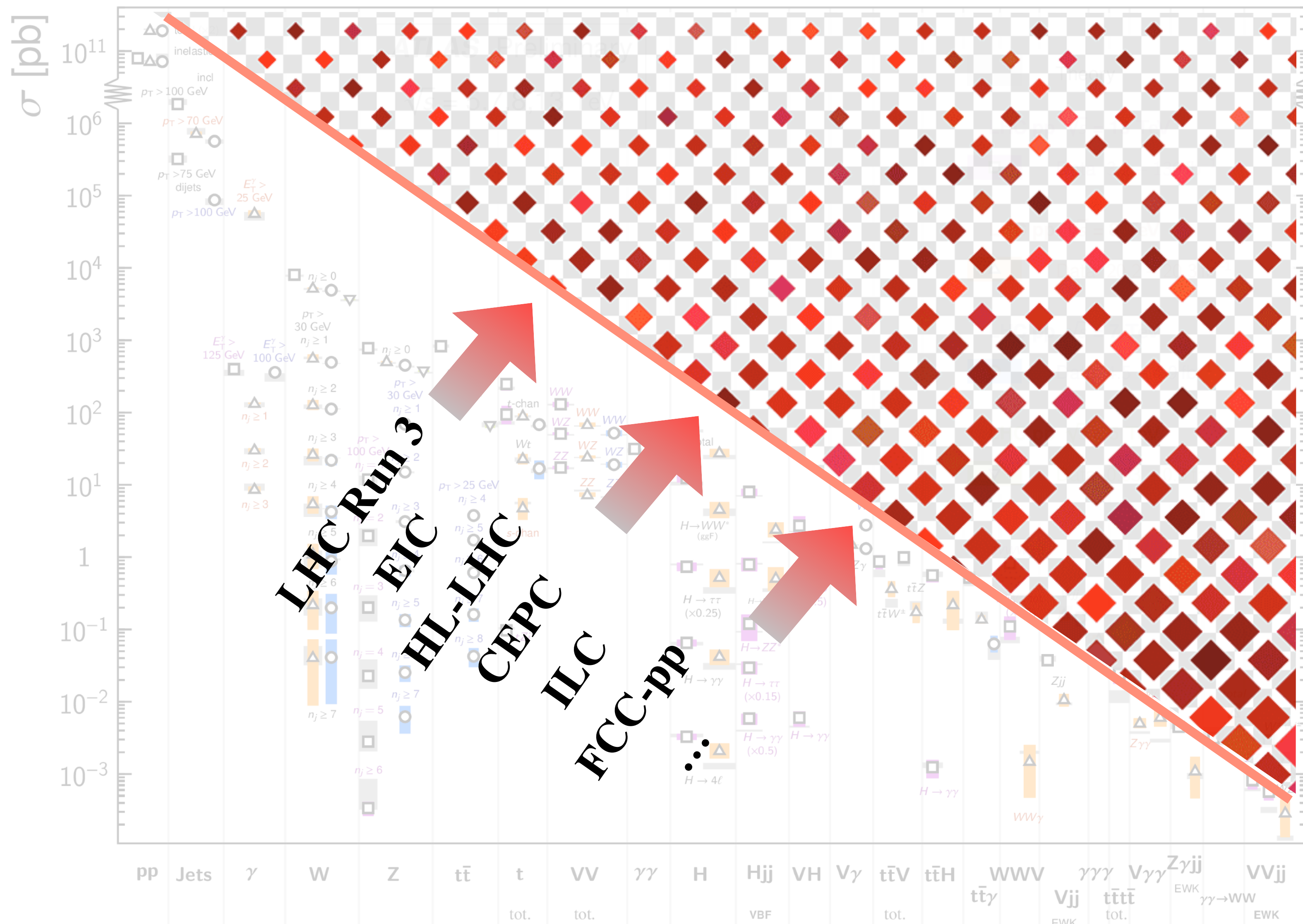
Energy (Multiplicity)

Standard Model Prediction Uncertainties

Standard Model Production Cross Section Measurements

Status: February 2022

Precision

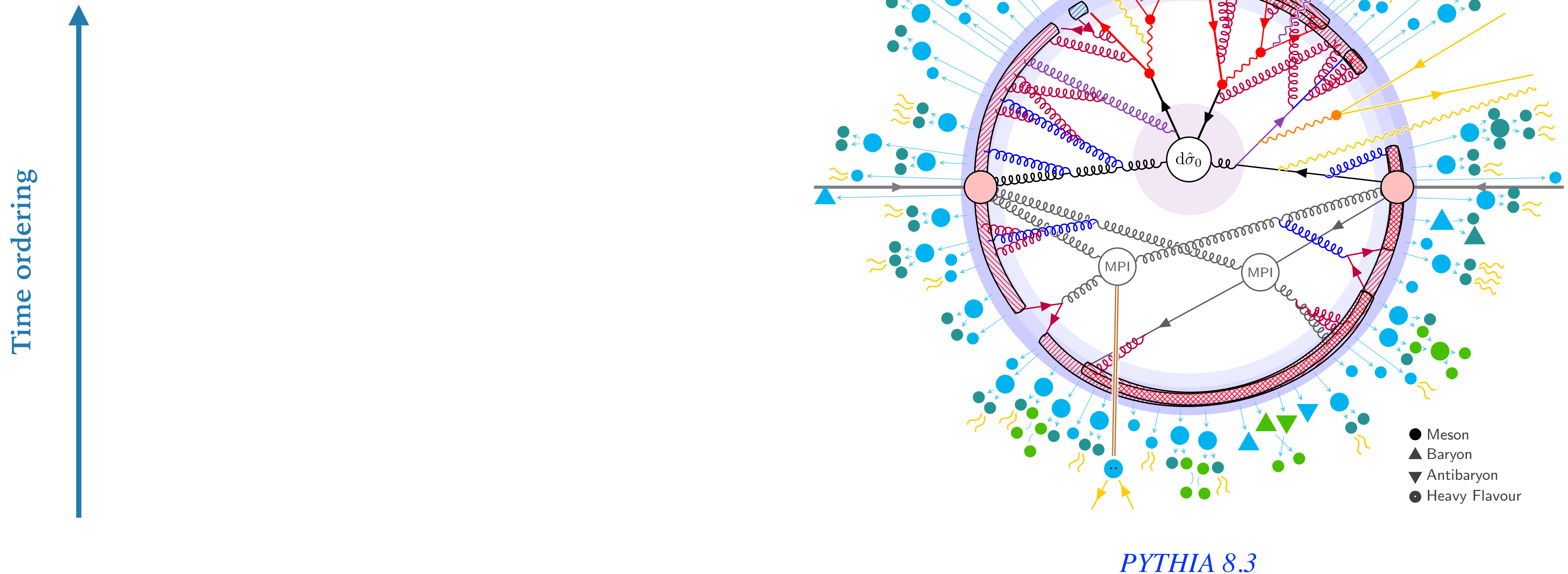


Energy

Standard Model Prediction Uncertainties

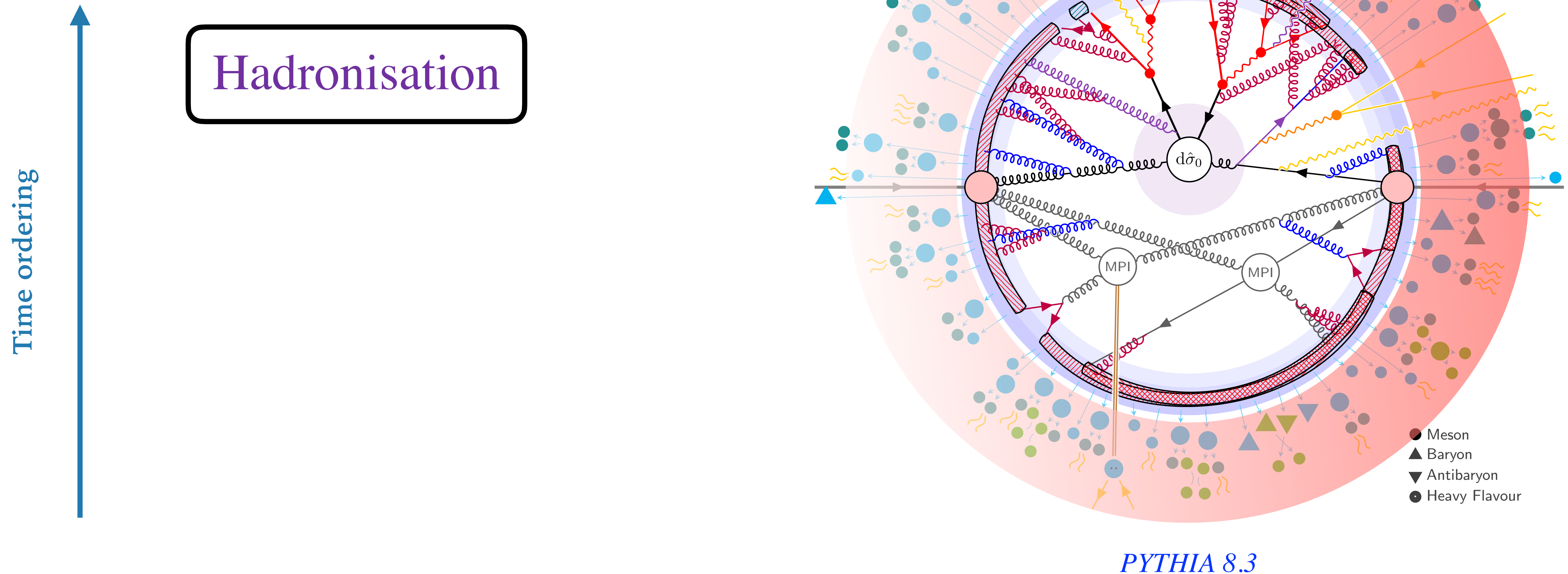
Collider Event in Theorist's Eye

- The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:



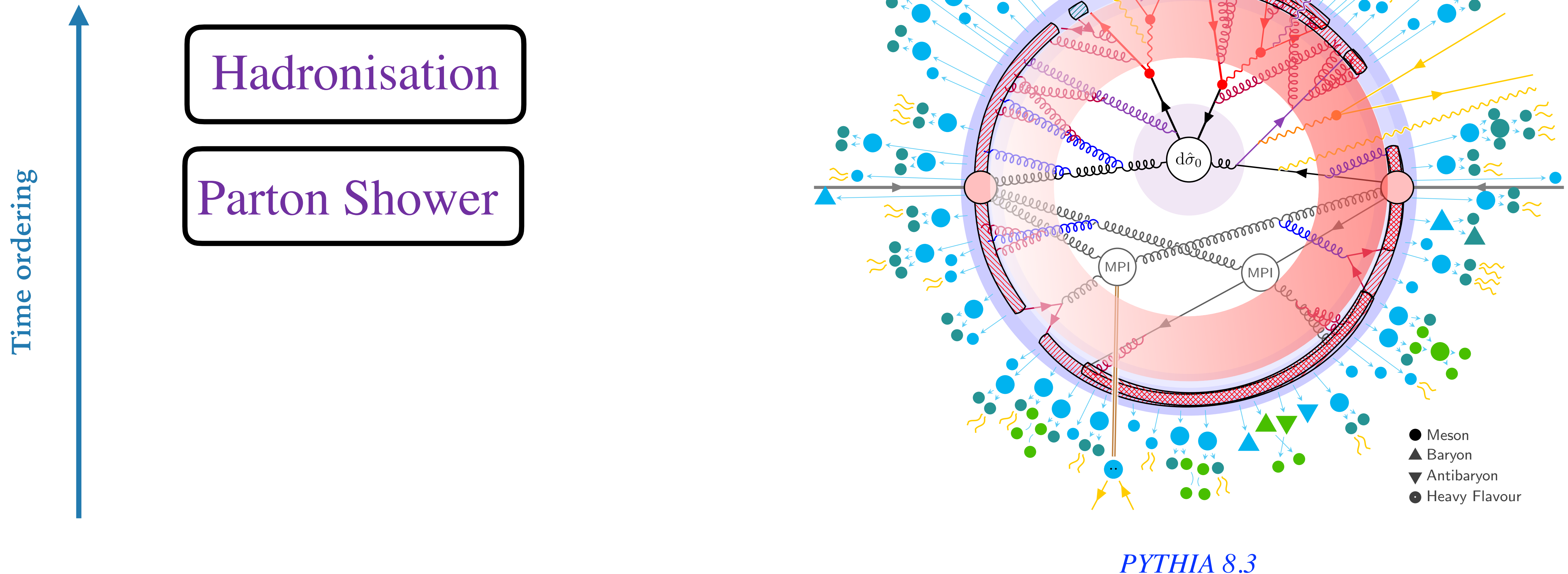
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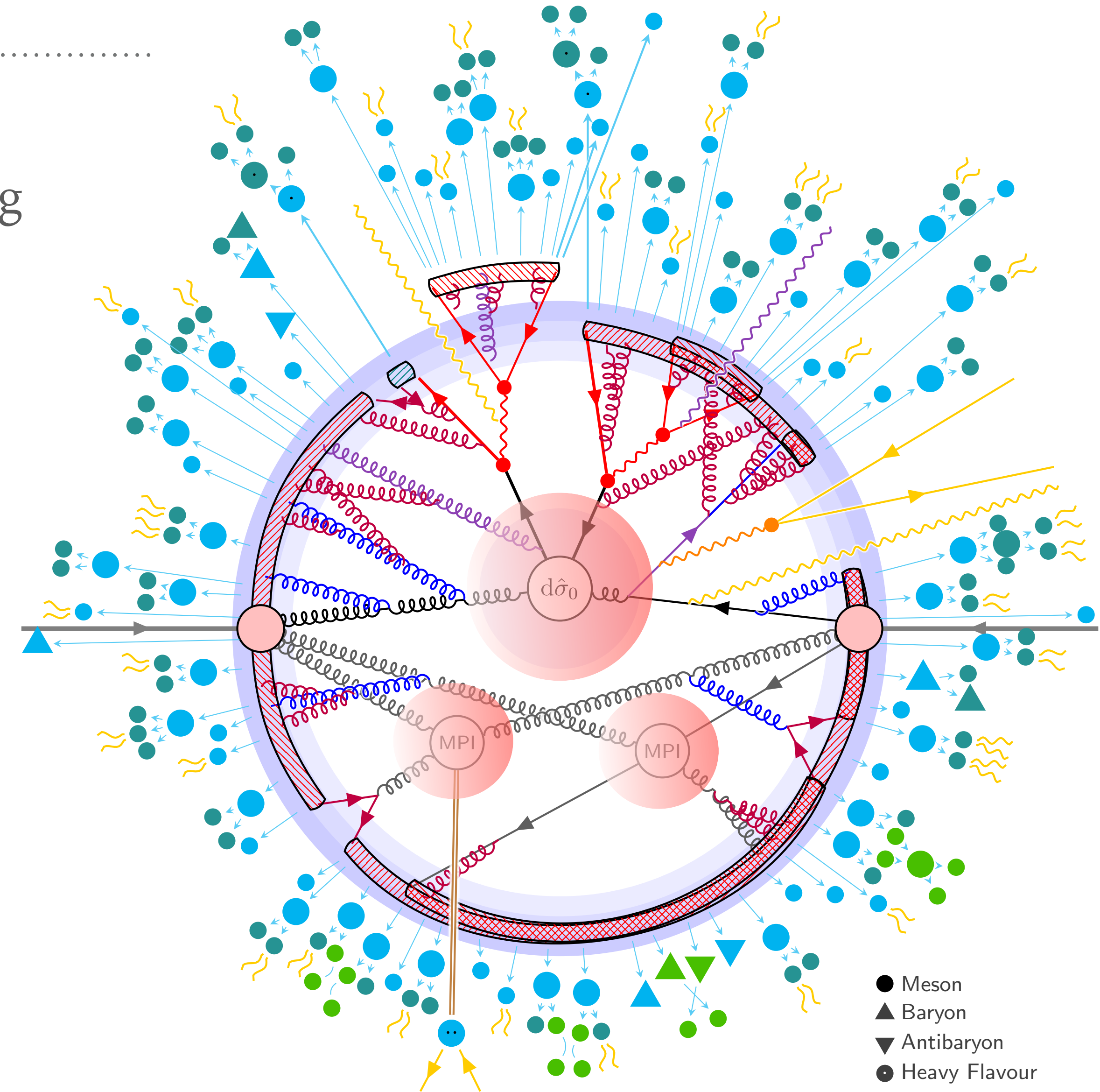
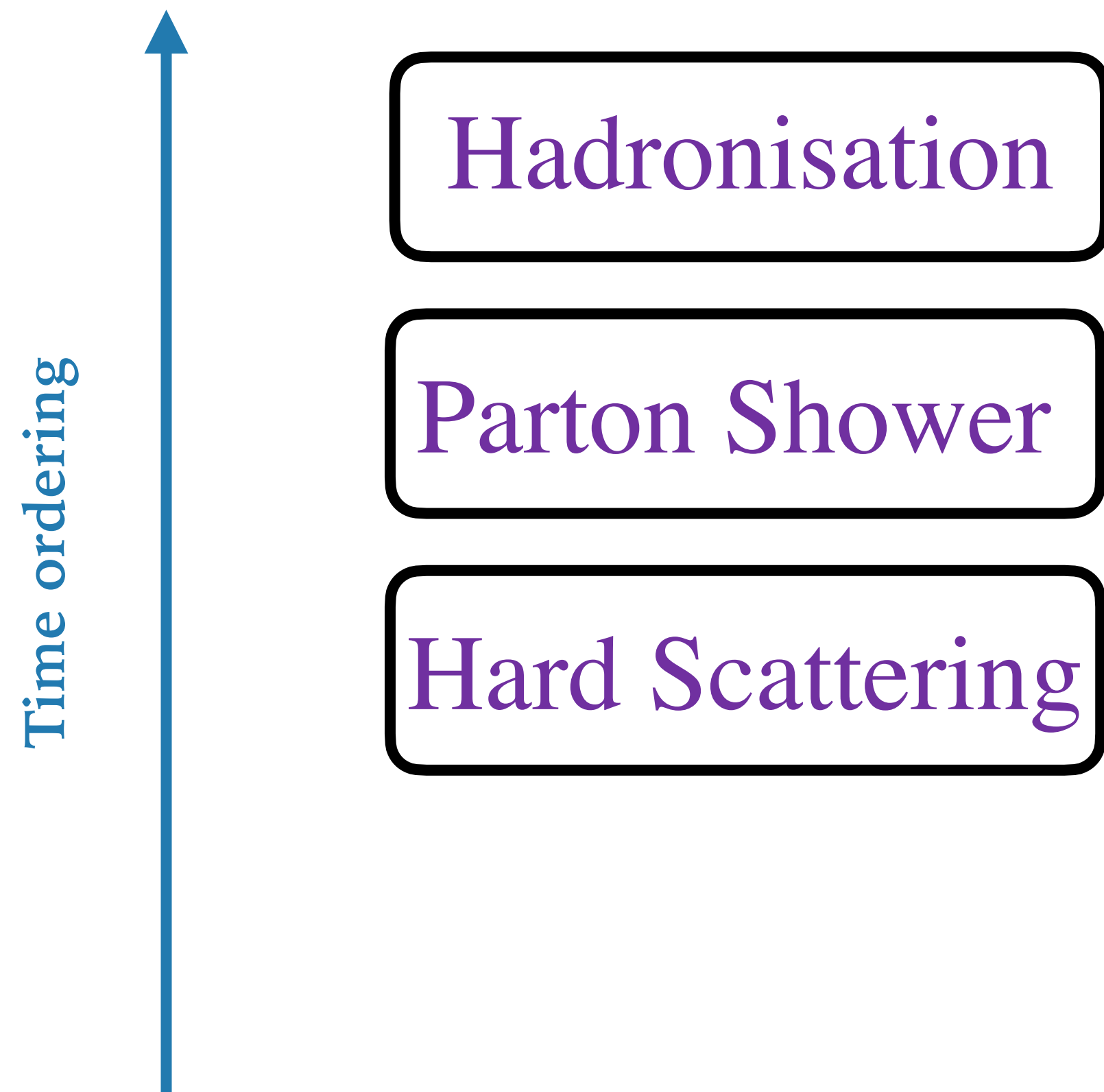
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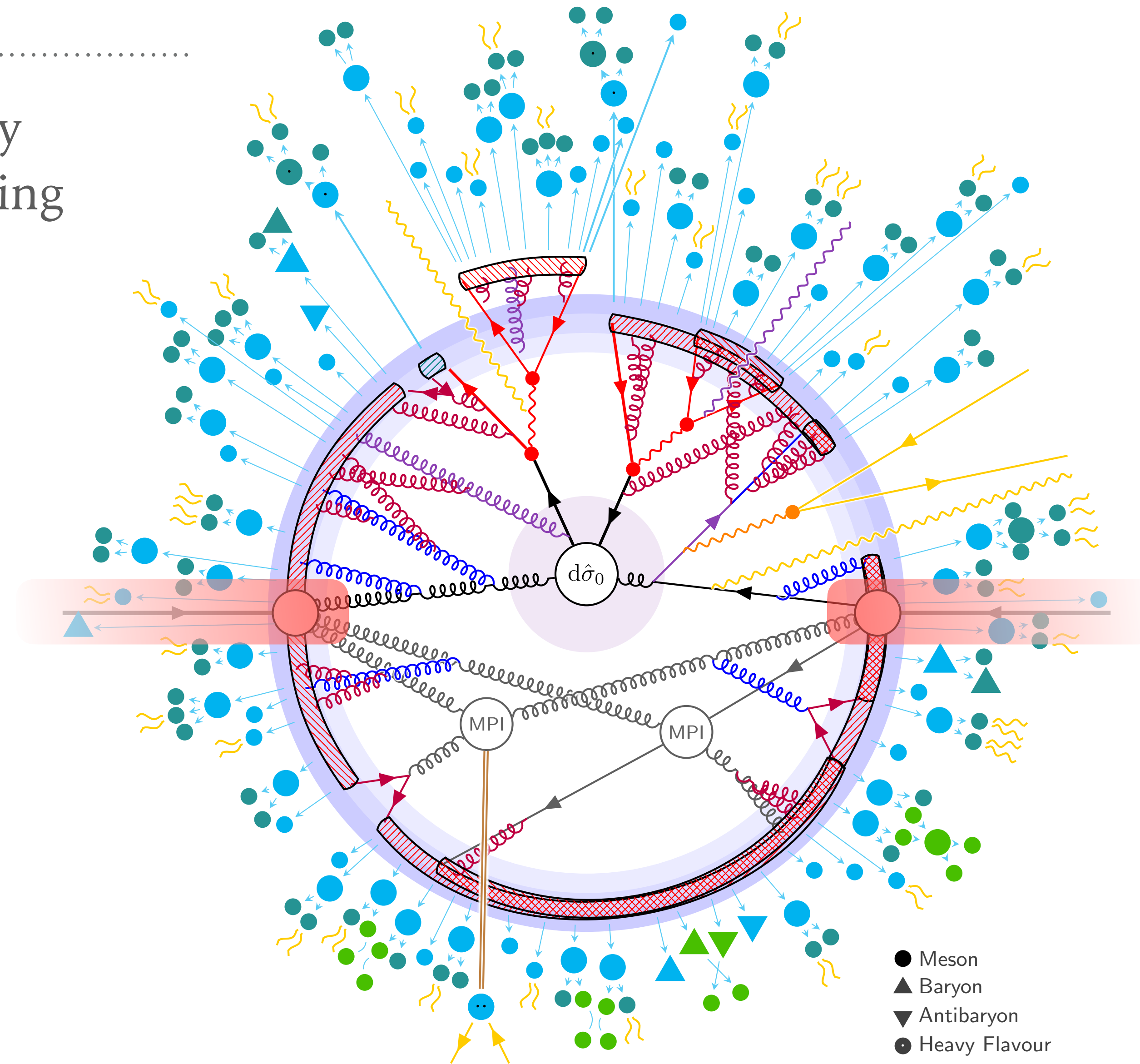
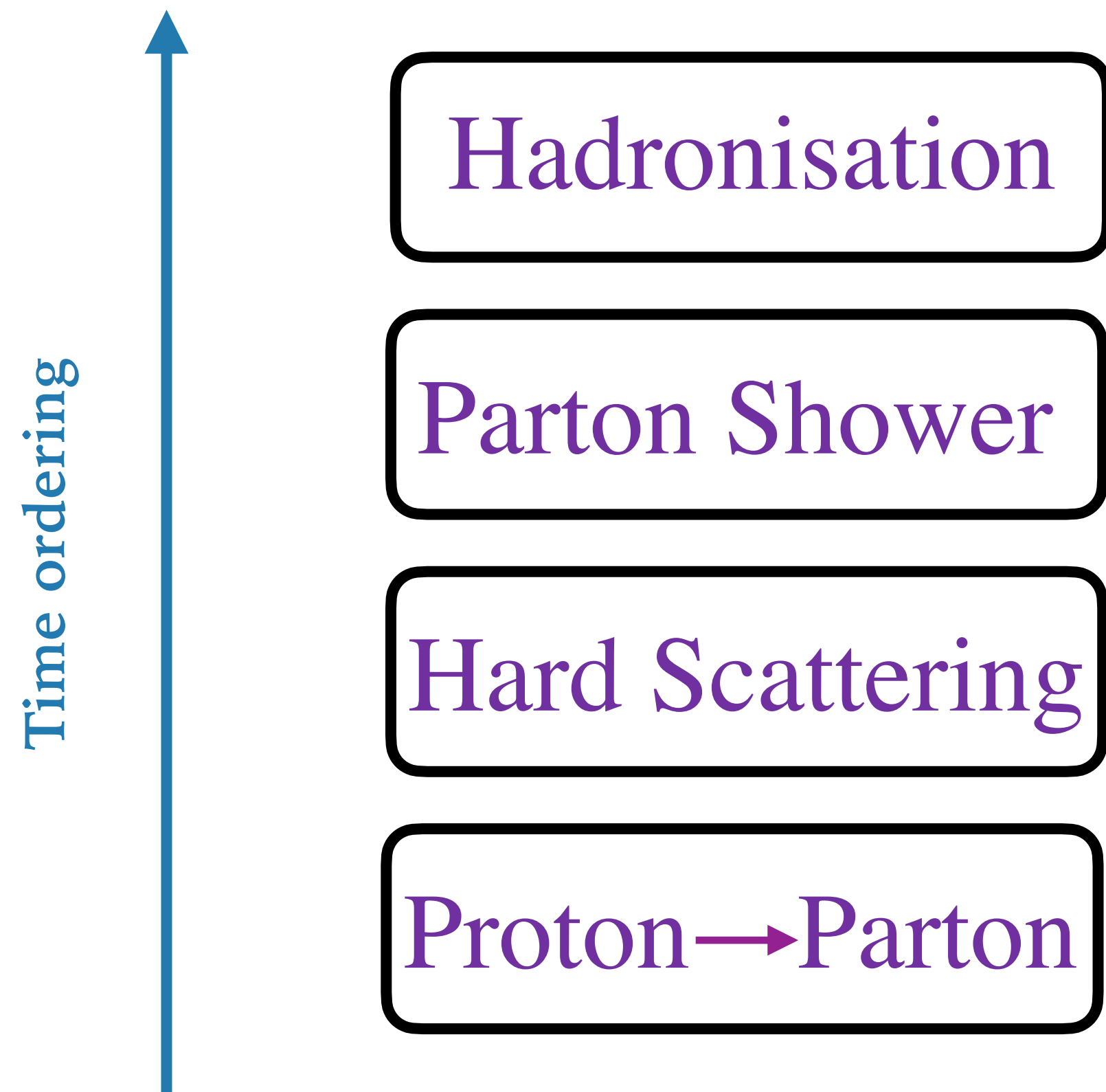
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PYTHIA 8.3

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PYTHIA 8.3

Theory Tools for Precision Predictions

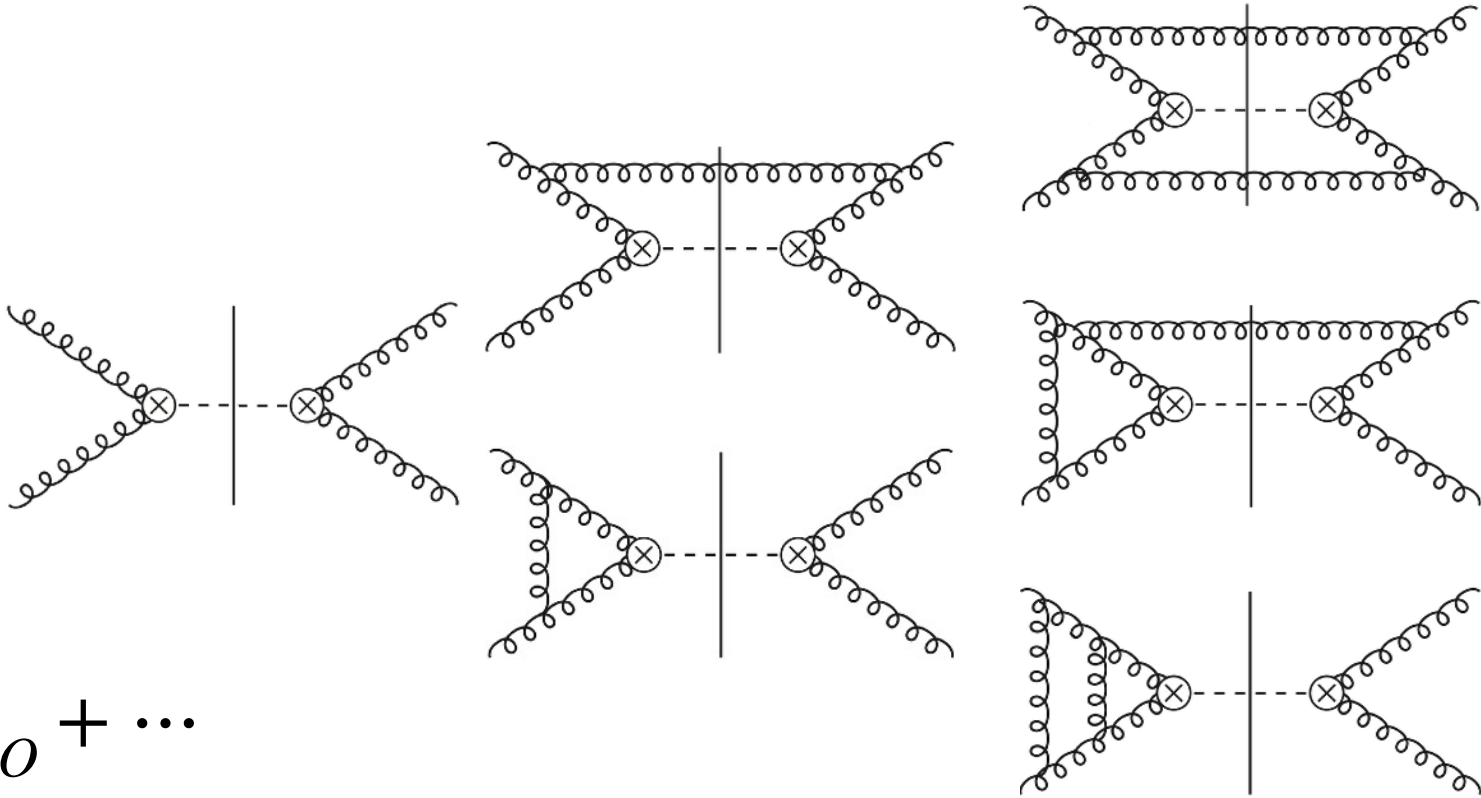
General Tools (perturbative-QFT)

$$m|_{\geq 1} \rightarrow n|_{\geq 1}$$

Hard Scattering

Parton Shower

$$Q^2 \frac{d\alpha_S}{dQ^2} = \beta(\alpha_S) = -\alpha_S^2 (b_0 + b_1 \alpha_S + \dots) \quad \hat{\sigma} = \hat{\sigma}_{LO}^{(0,0)} + \left(\frac{\alpha}{2\pi}\right) \hat{\sigma}_{NLO}^{(0,1)} + \left(\frac{\alpha_S}{2\pi}\right) \hat{\sigma}_{NLO}^{(1,0)} + \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{\sigma}_{NNLO}^{(2,0)} + \dots$$



Theory Tools for Precision Predictions

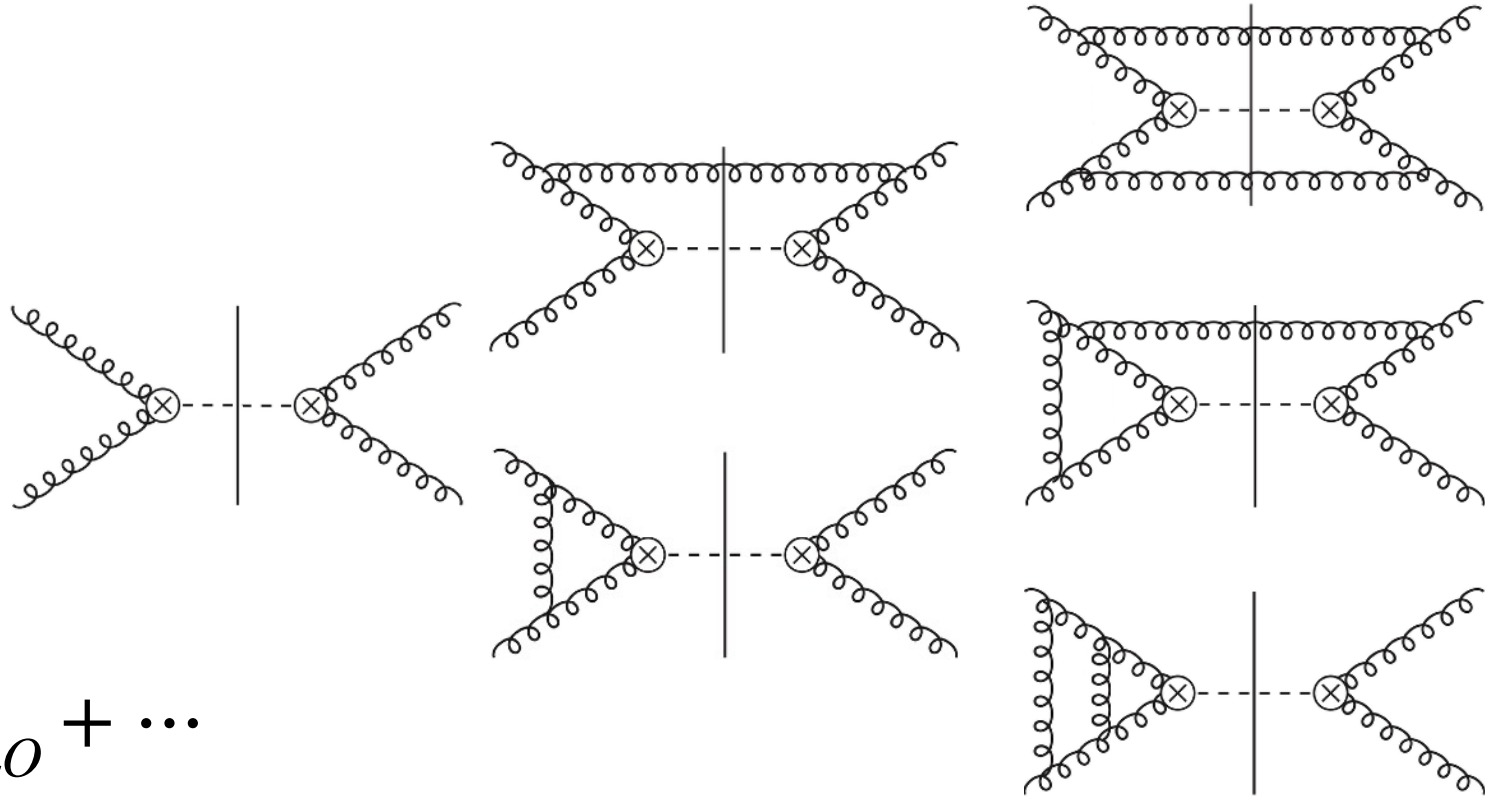
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Special Tools (non-perturbative-QFT)

m_q

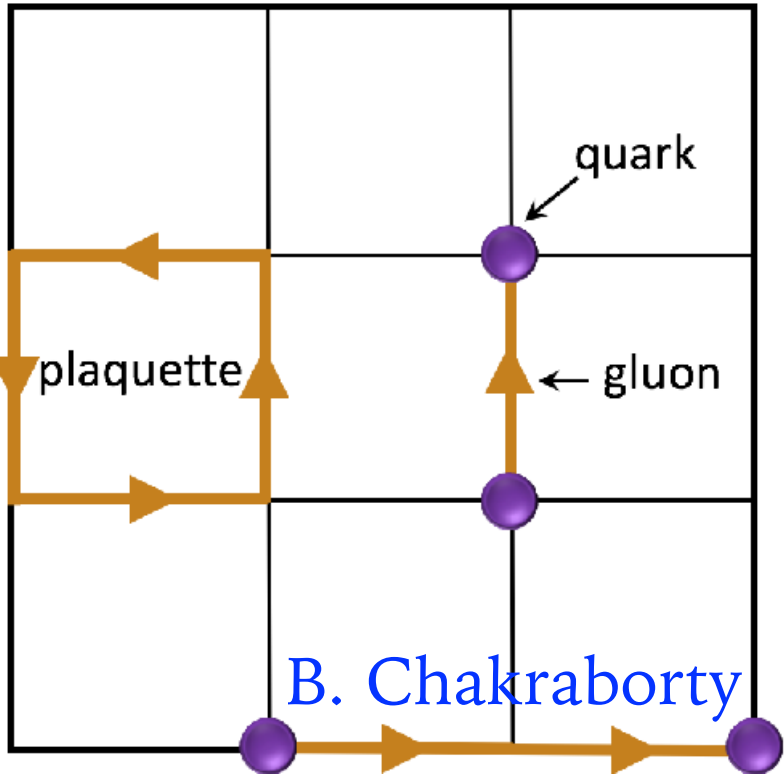
Proton → Parton

α_μ^{HVP}

α_S

CKM

$(\Lambda/Q)^n$



Theory Tools for Precision Predictions

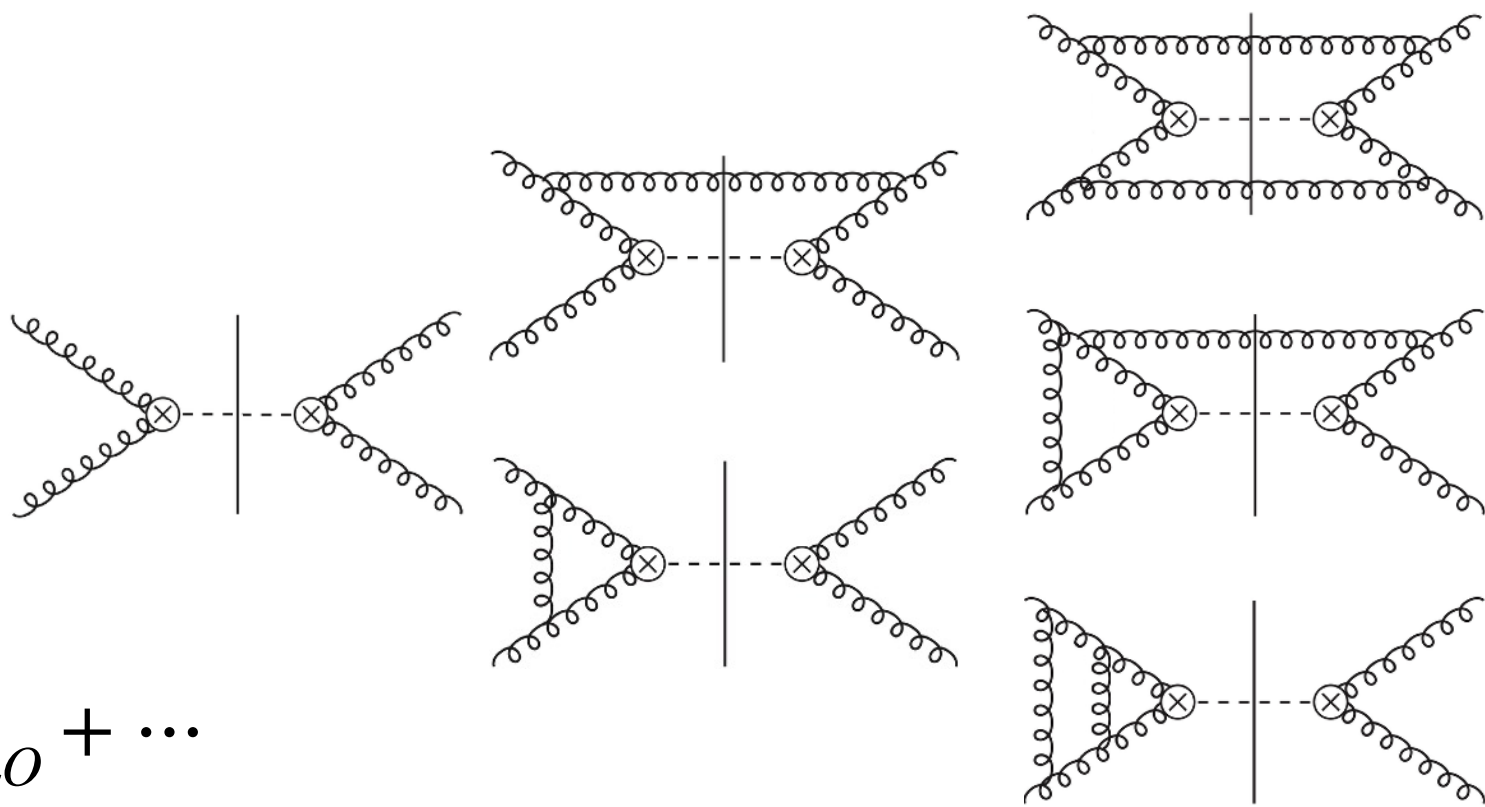
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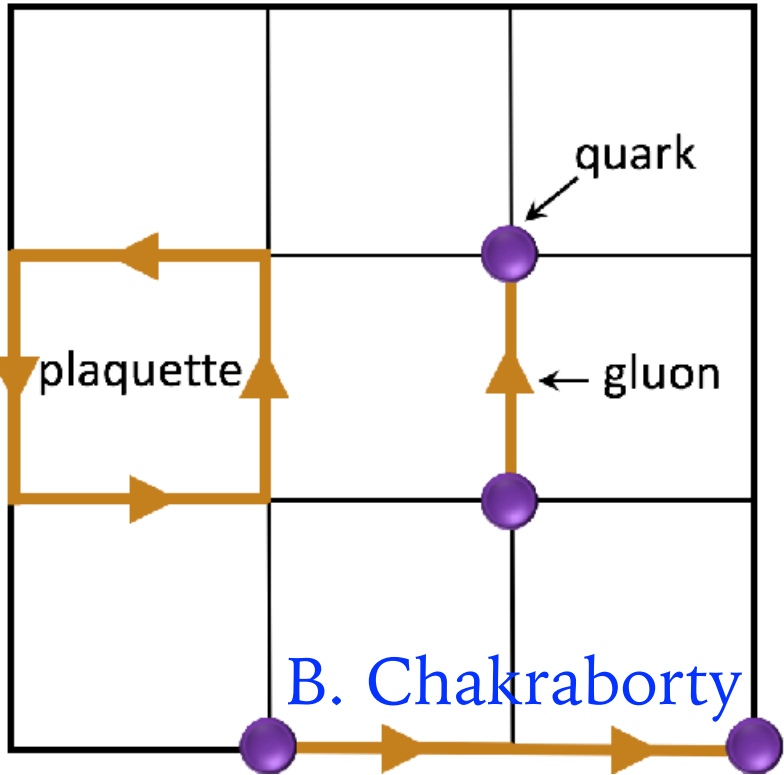
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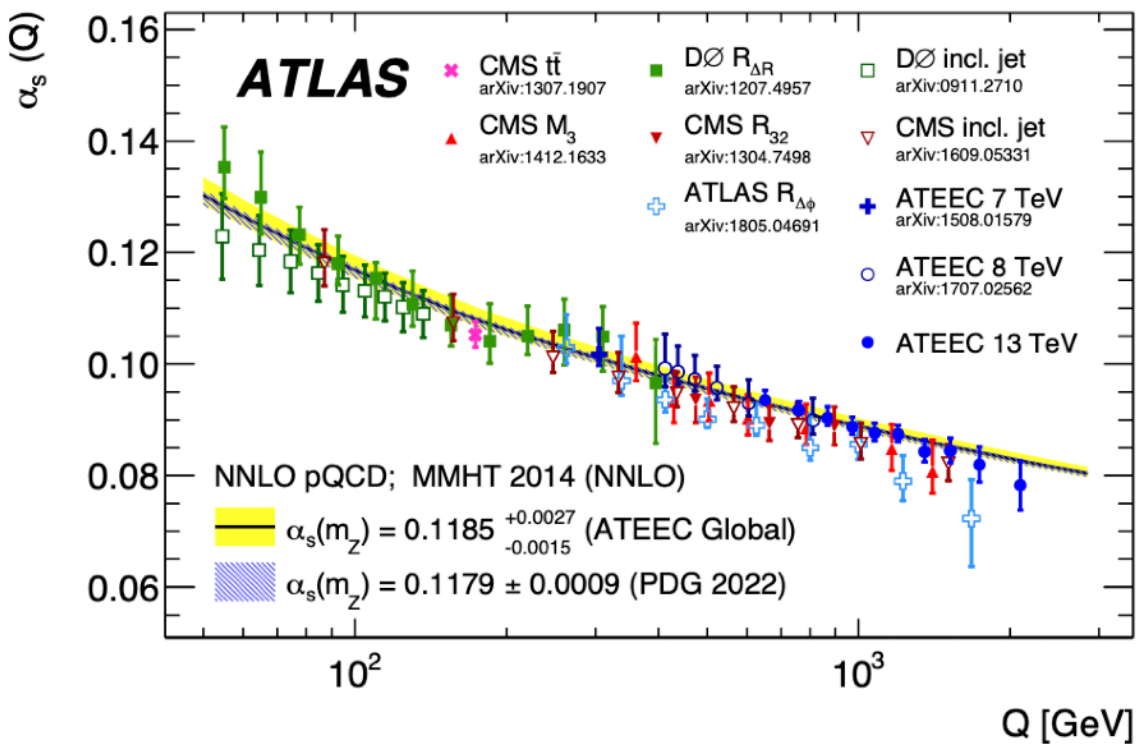
m_q	Proton \rightarrow Parton	α_μ^{HVP}
α_S	CKM	$(\Lambda/Q)^n$



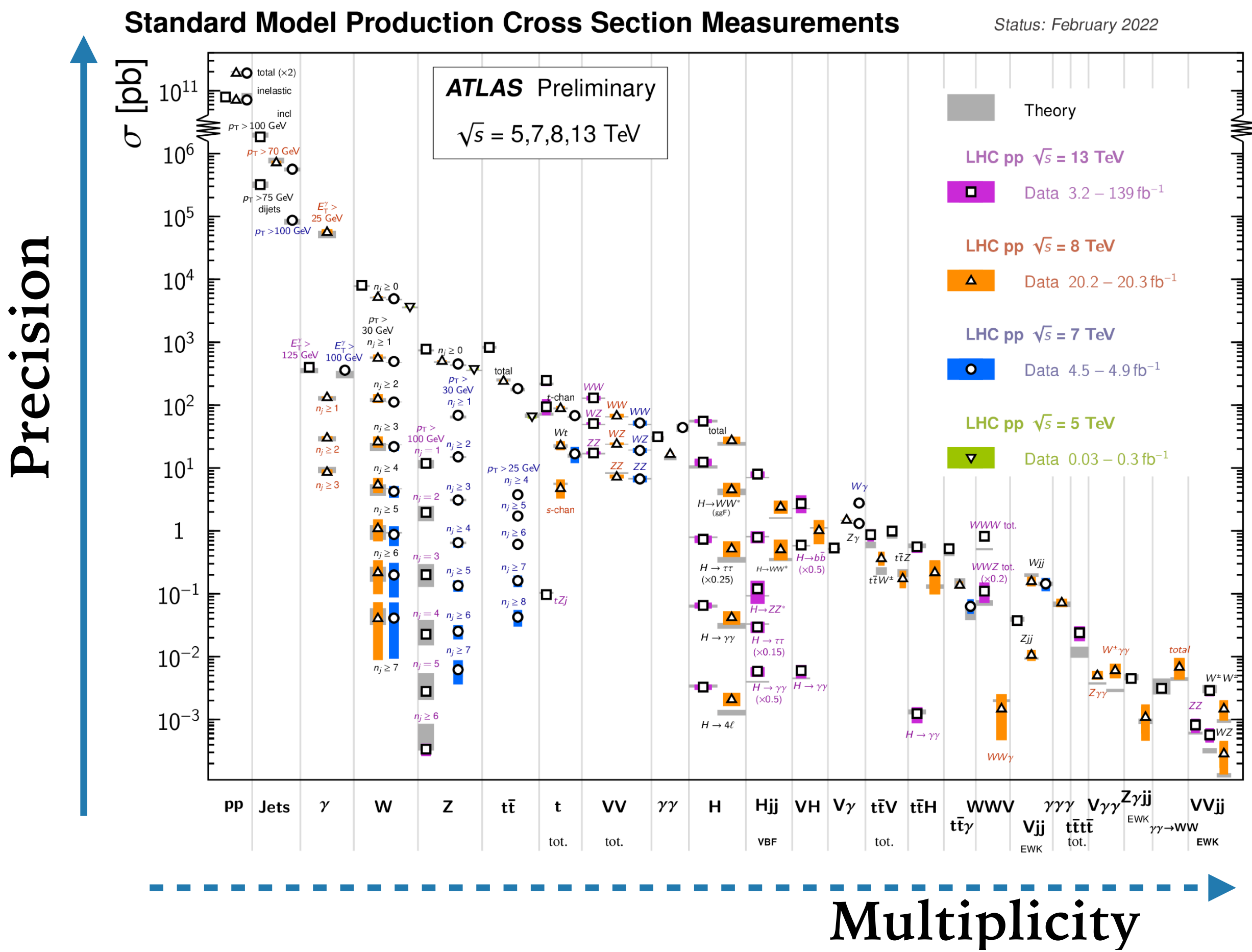
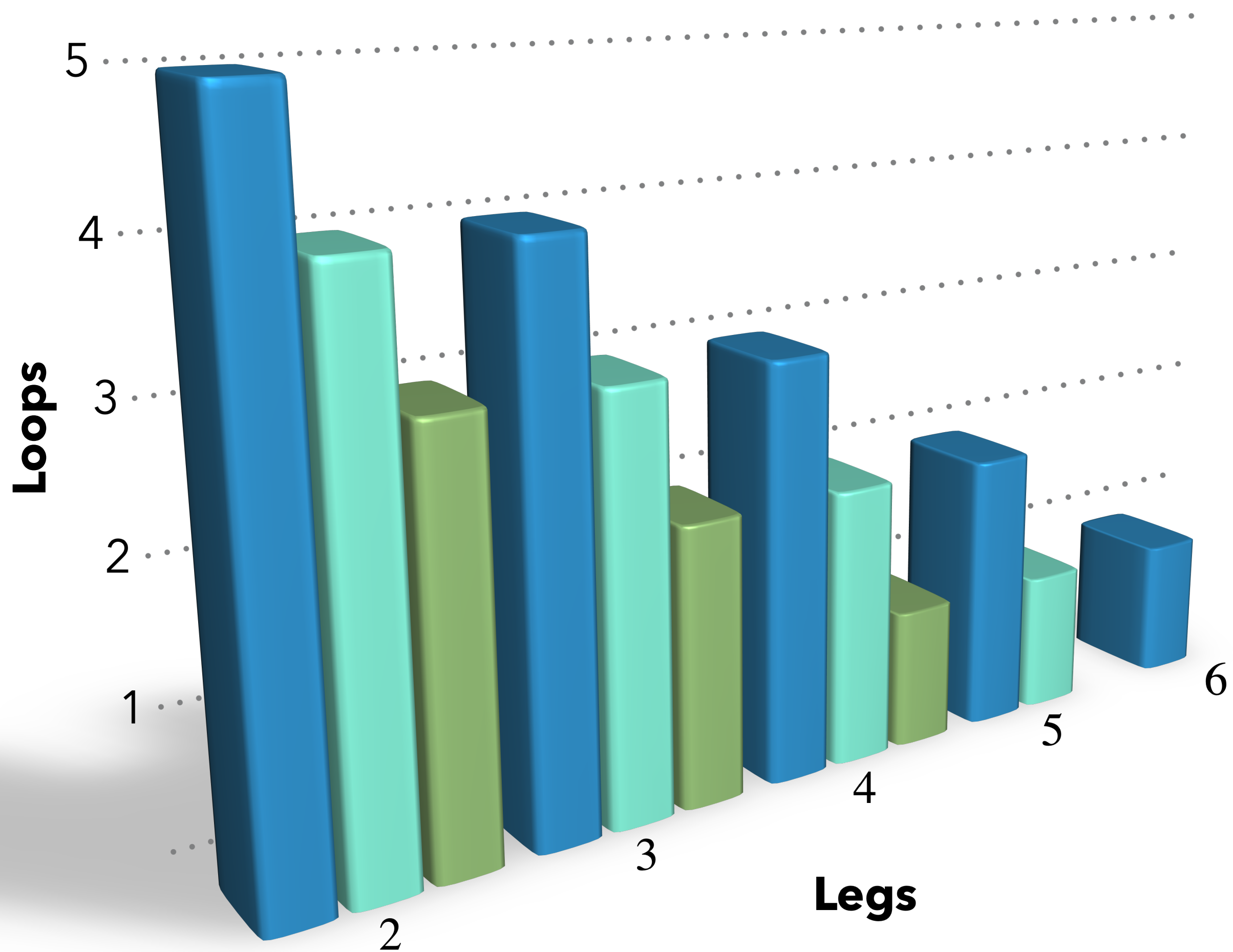
Dedicate Tools (fitting)

Theory + Experiment
To fit NP model

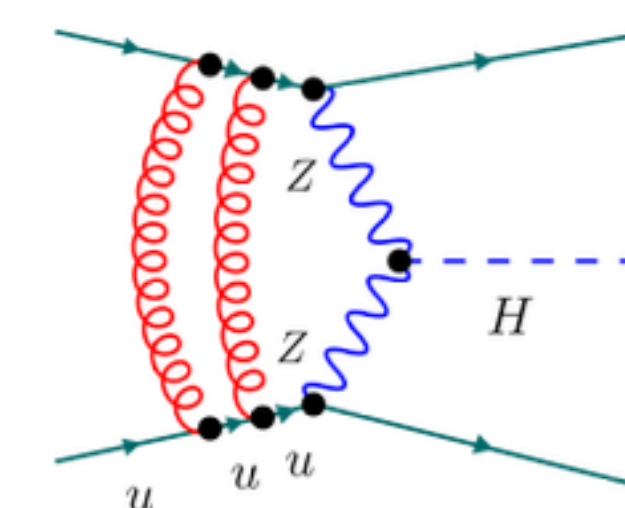
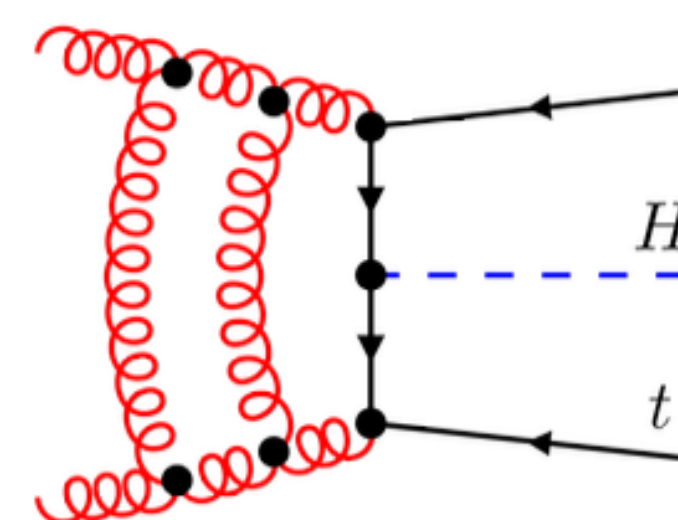
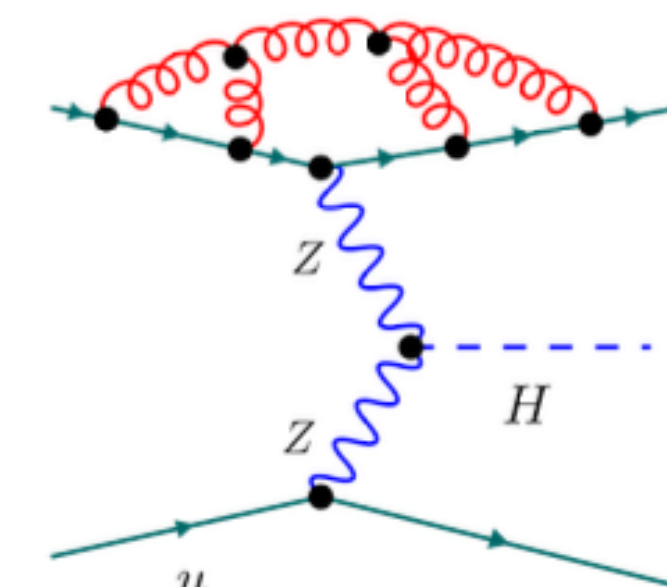
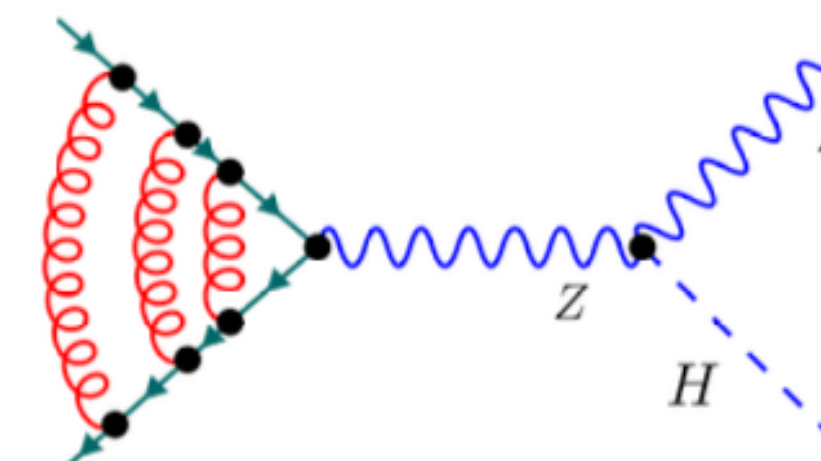
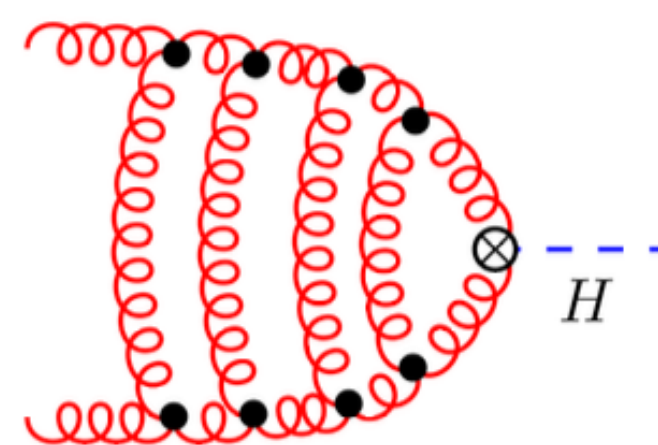
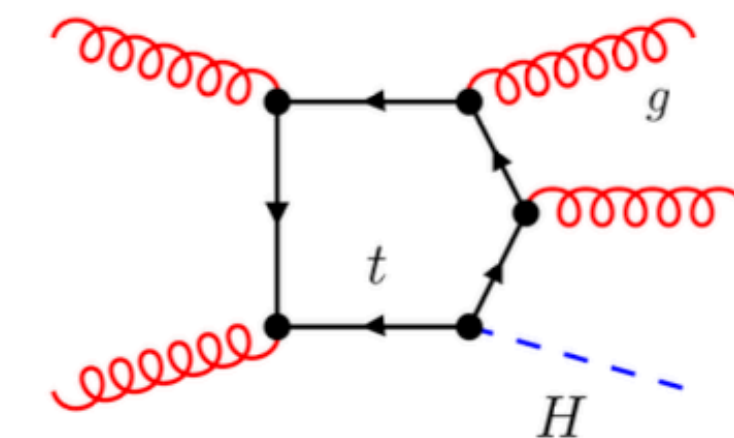
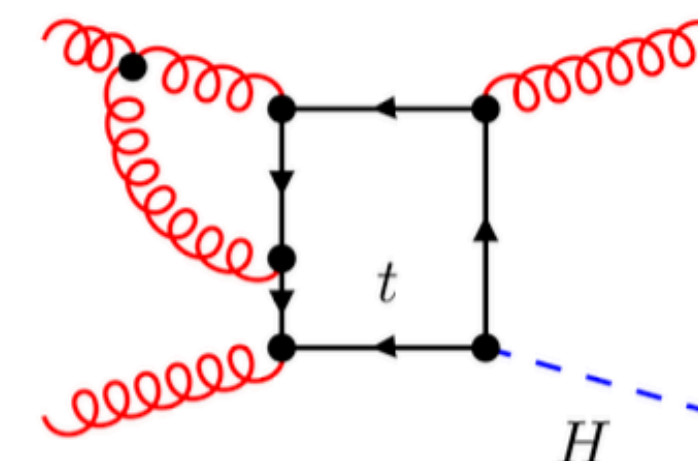
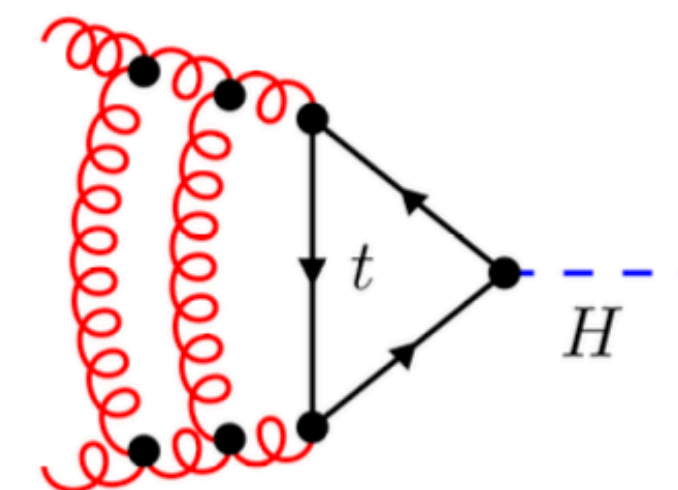
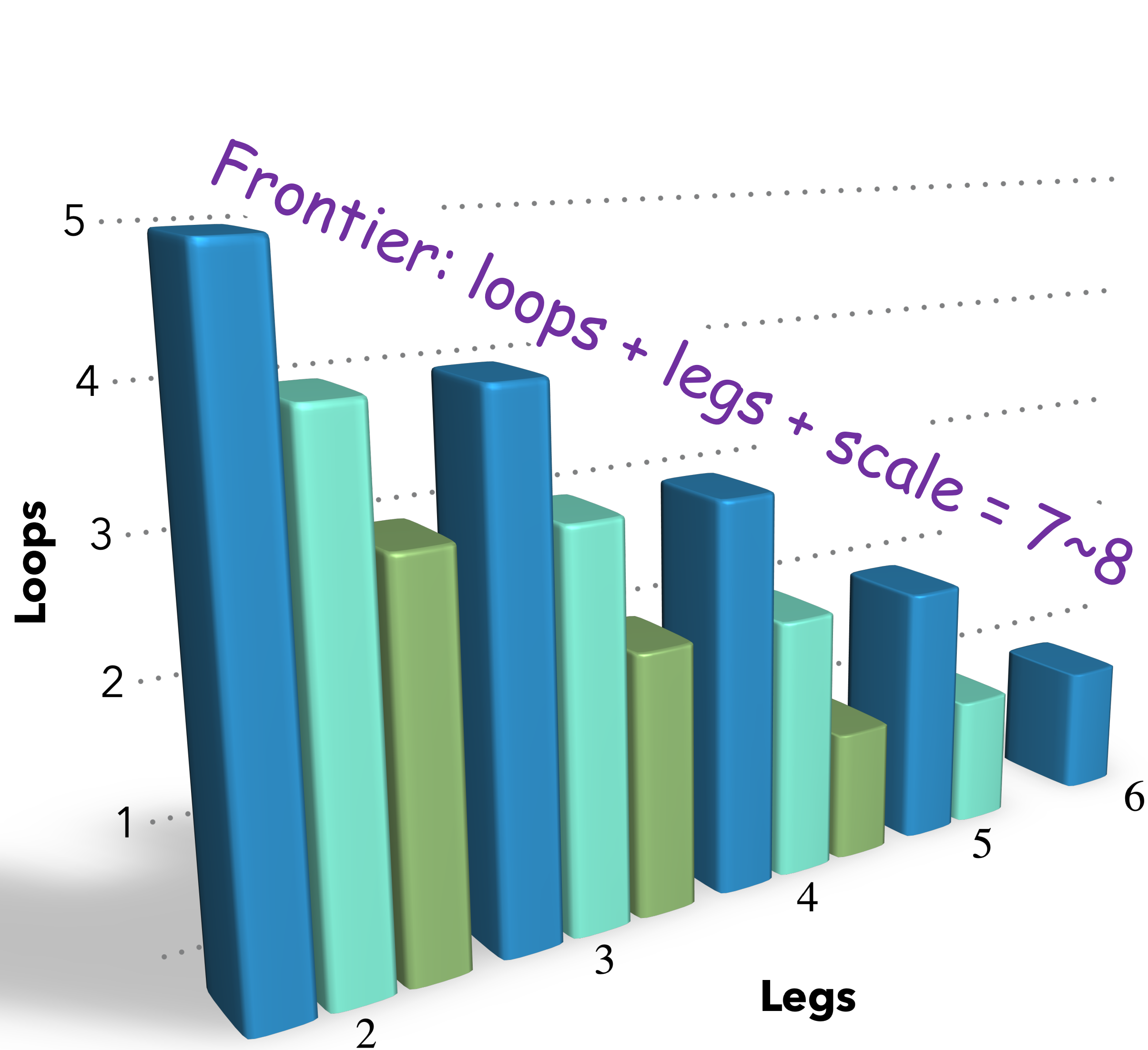
α_S	Proton \rightarrow Parton
Hadronisation	Fragmentation



Perturbative QFT for Precision Predictions

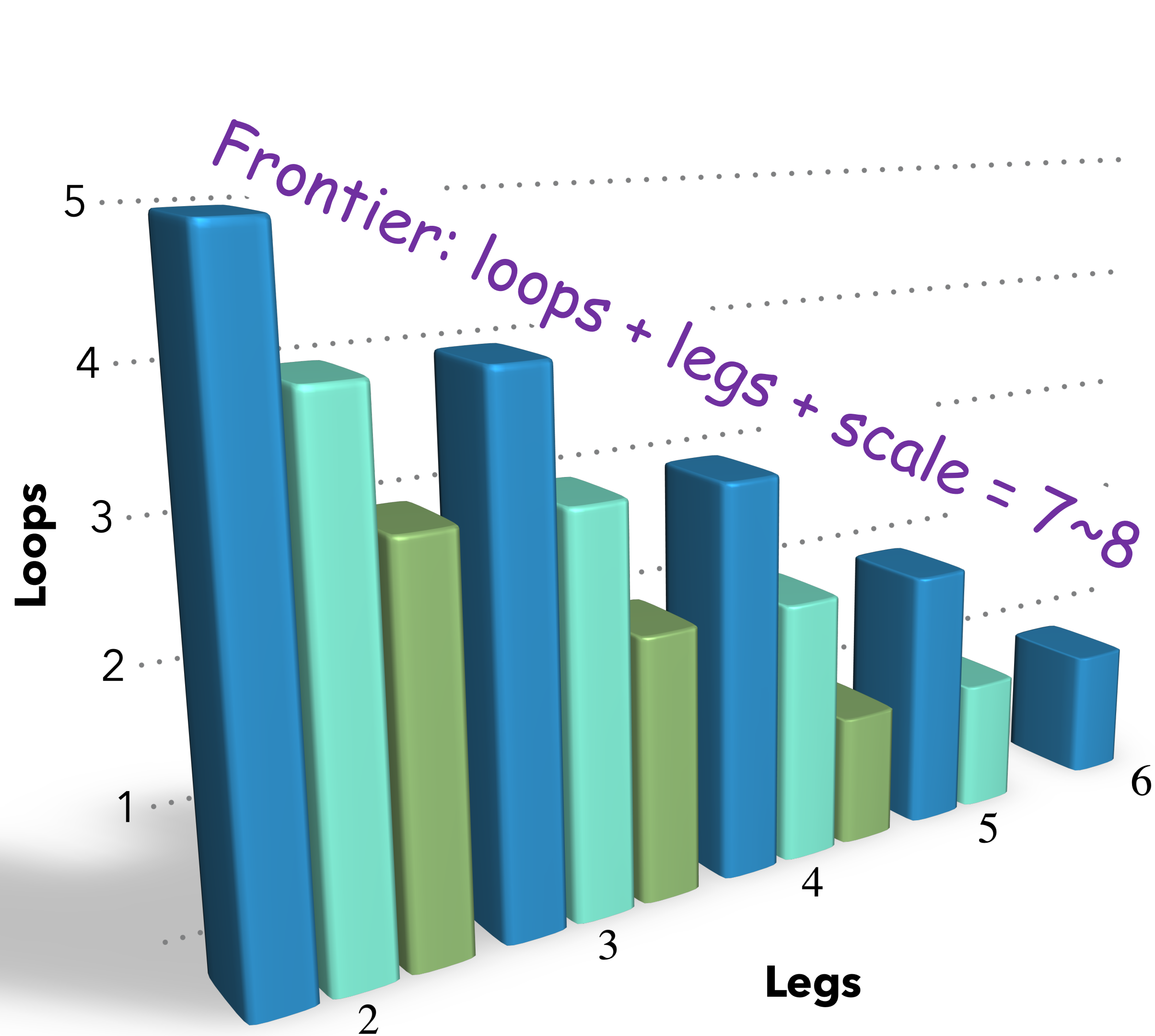


Perturbative QFT for Precision Predictions



Diagrams: FeynGame
Comput.Phys.Comm.
256 (2020) 107465

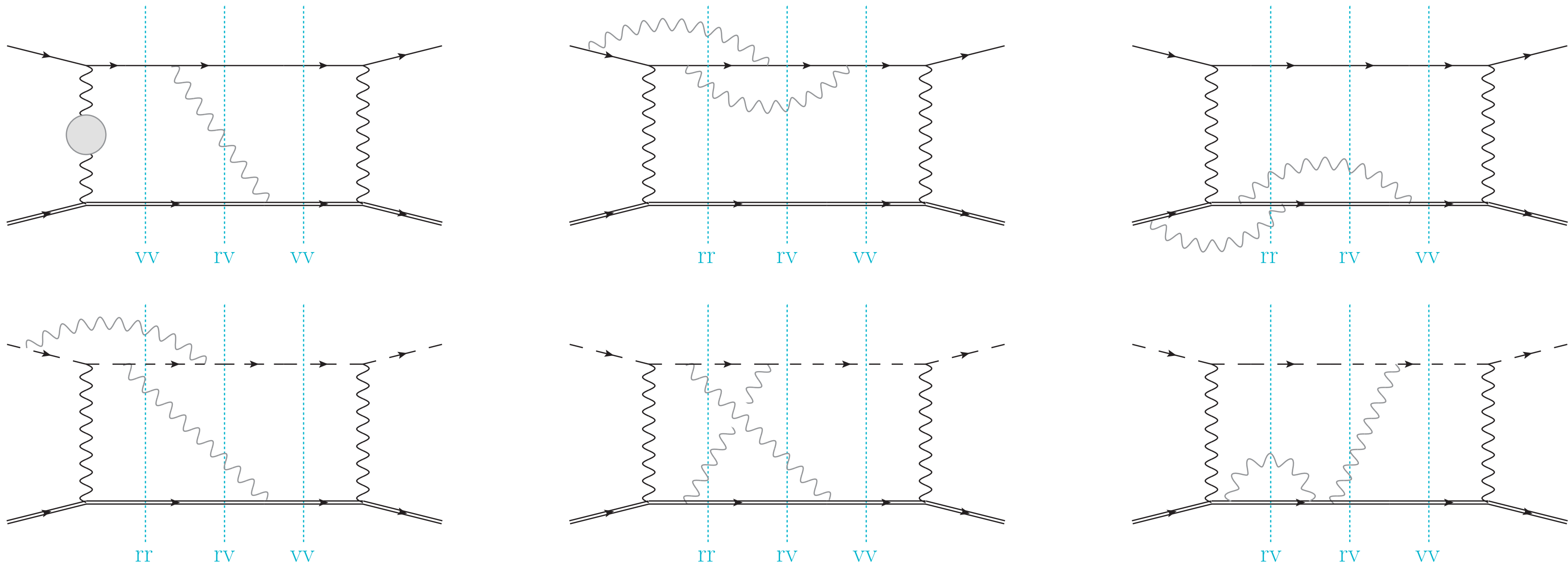
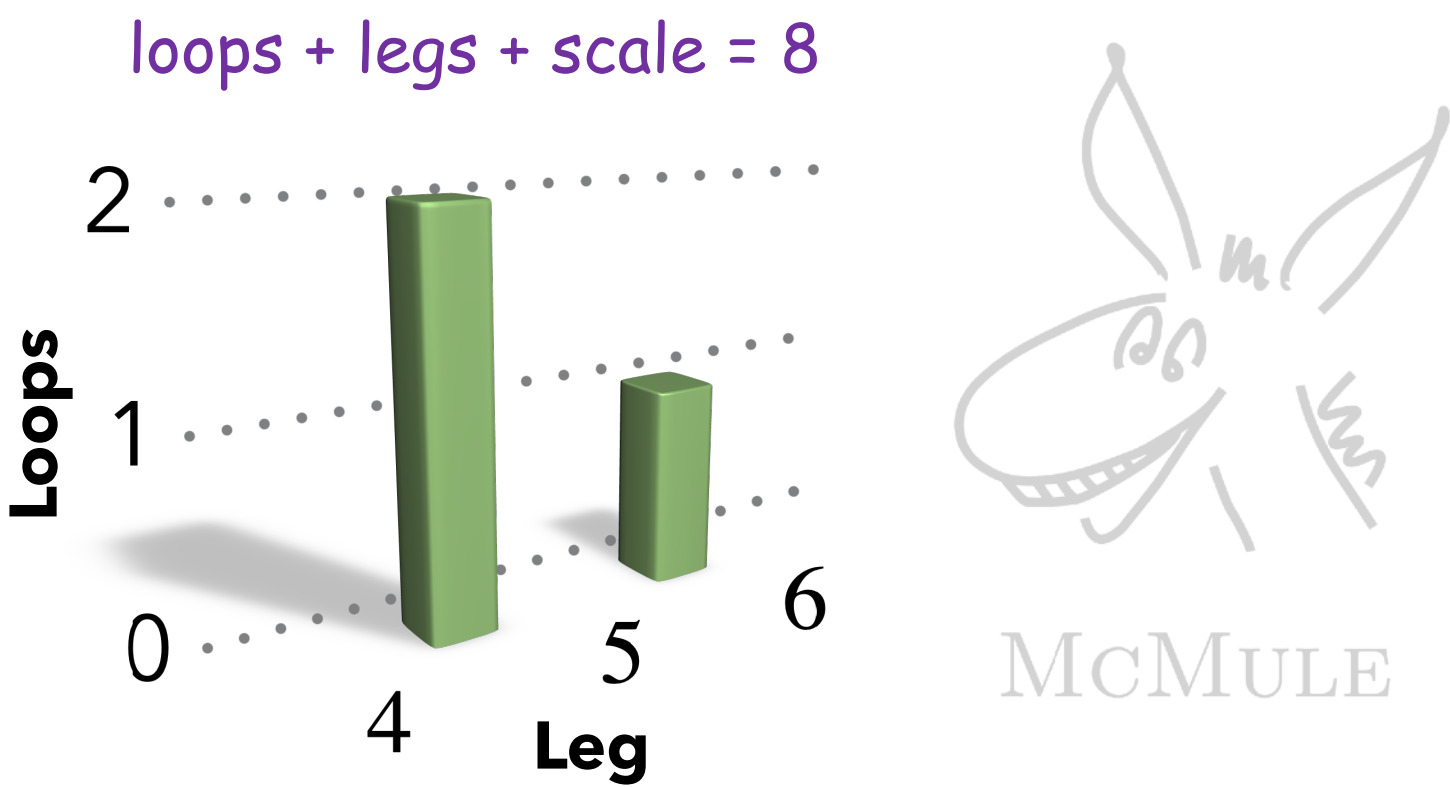
Perturbative QFT for Precision Predictions



- Generalised polylogarithms
- Riemann zeta values
- Elliptic functions
- ...
- Unitarity
- Generalised Unitarity
- Recursion
- Twistors
- Differential equations
- Integrand/Integral
- Sector decomposition
- Numerical unitarity
- Finite field
- Auxiliary mass flow
- Neural network amplitude
- ...

Perturbative QFT for Precision Predictions

$e\mu \rightarrow e\mu$ @NNLO QED



- Complete NNLO photon corrections via McMule framework
 - Full m_e and m_μ dependence of RR, RV and factorisable VV (top).
 - m_e effects in mixed VV (bottom) estimated via massification.
 - IR divergence handled by FKS² subtraction method.
 - Fully differential MC tool for MUonE experiment.
 - Key input to extract $\Delta\alpha_{\text{had}}(Q^2)$ for $Q^2 < 0$.
 - Alternative dispersive approach from R-ratio to calculate a_μ^{HVP} .

A. Broggio, T. Engel, A. Ferroglia et. al. JHEP 01 (2023) 112

MUonE Fiducial	$\sigma/\mu\text{b}$		$\delta K^{(i)}/\%$	
	S1	S2	S1	S2
σ_0	106.44356	106.44356		
$\sigma_1 \begin{cases} - \\ + \end{cases}$	106.99038(3)	102.86304(3)	0.51372(3)	-3.36377(3)
	107.41847(3)	103.18338(3)	0.91589(3)	-3.06283(3)
$\sigma_2 \begin{cases} - \\ + \end{cases}$	106.97977(3)	102.88154(3)	-0.00992(4)	0.01799(4)
	107.41832(3)	103.19386(3)	-0.00013(4)	0.01016(4)

Perturbative QFT for Precision Predictions

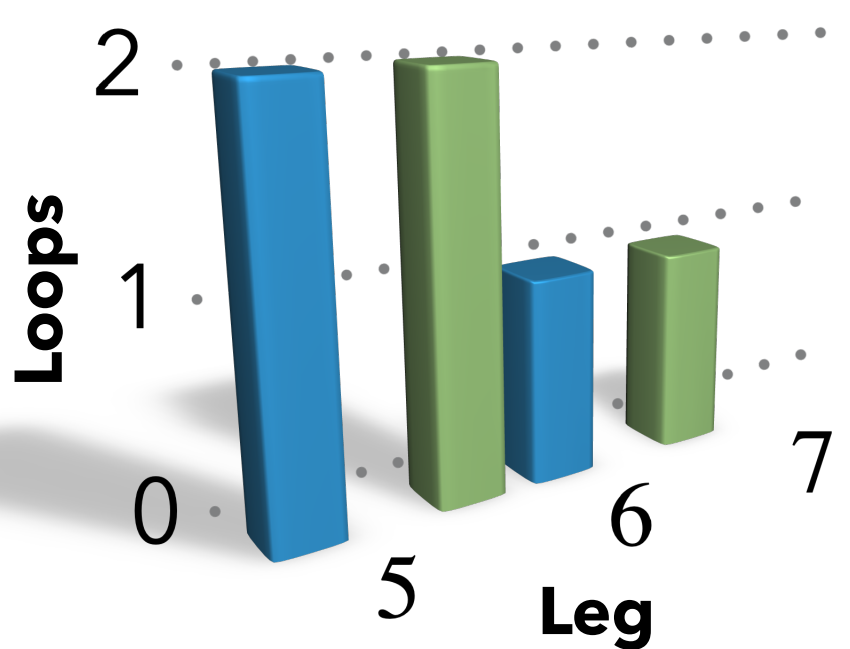
$pp \rightarrow t\bar{t}W, \gamma JJ, t\bar{t}H$ @NNLO QCD

- Rapid progress of NNLO QCD corrections to $2 \rightarrow 3$ scattering at the LHC
 - Automation of tree and 1-loop scattering ME with OpenLoops.
 - Processes dependent calculation/approximation for 2-loop-5-leg ME:
 - Complete analytical amplitudes for $\gamma q\bar{q}gg, \gamma q\bar{q}Q\bar{Q}$ at 2-loop
 - Eikonal or massification approximation to estimate $Vt\bar{t}gg, Vt\bar{t}q\bar{q}$ @ 2-loop
 - Mature machinery of NNLO subtraction methods for event generator:
 - STRIPPER (Sector-improved), MATRIX (qT-slicing)

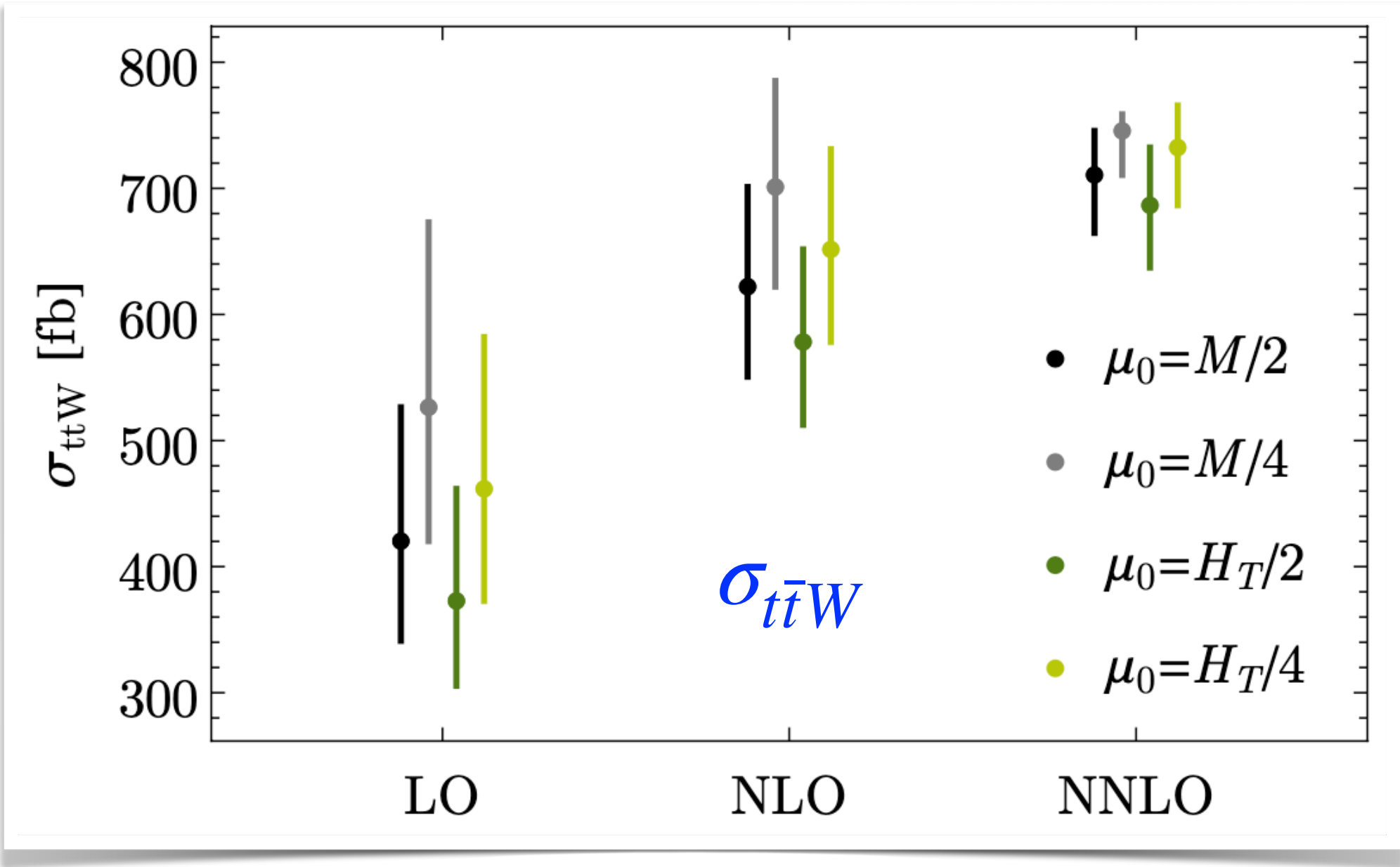
loops + legs + scale = 7~8

Also available @ NNLO

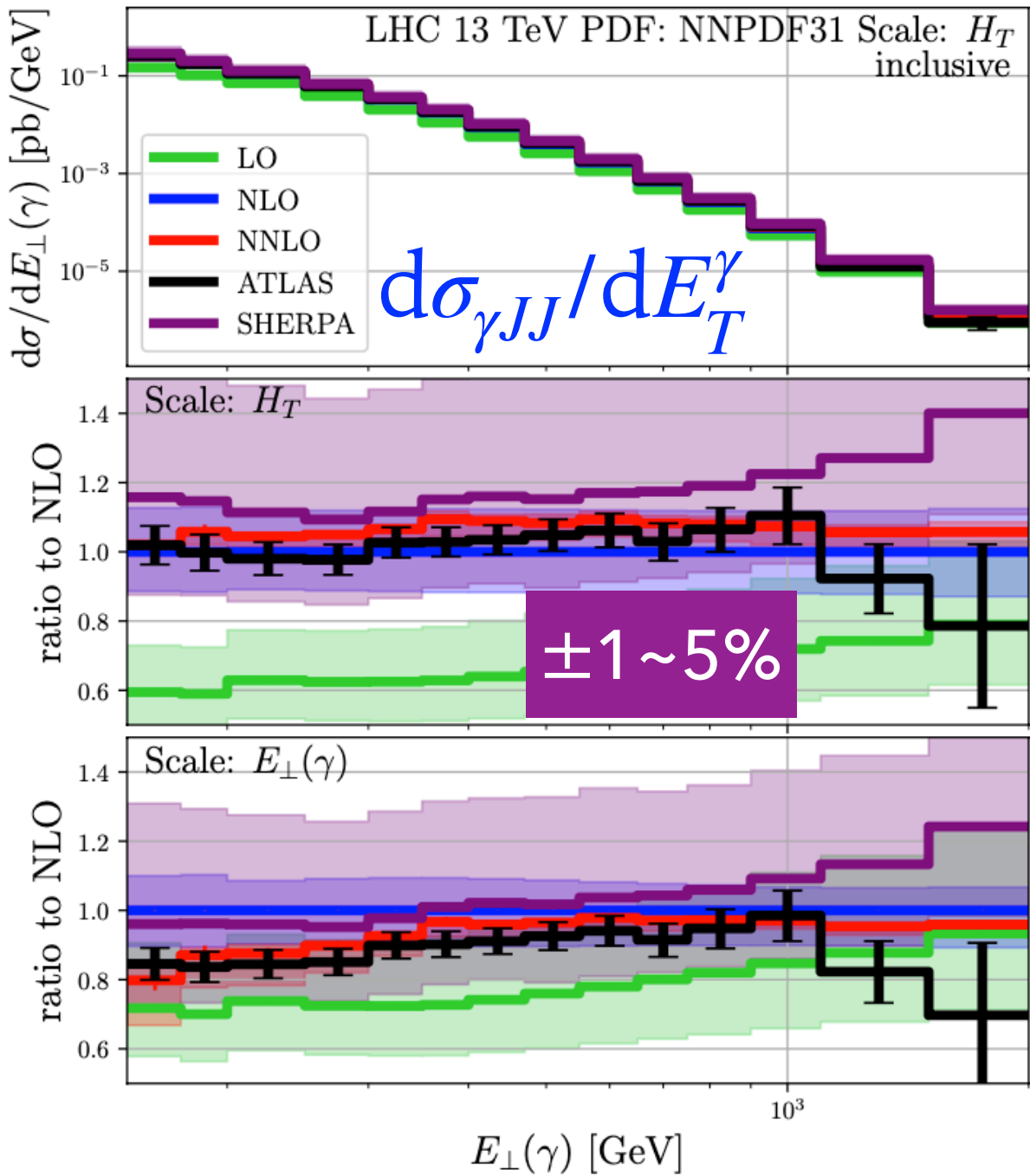
- $pp \rightarrow \gamma\gamma J$ ✓
- $pp \rightarrow JJJ$ ✓
- $pp \rightarrow \gamma\gamma\gamma$ ✓
- $pp \rightarrow Wb\bar{b}$ ✓



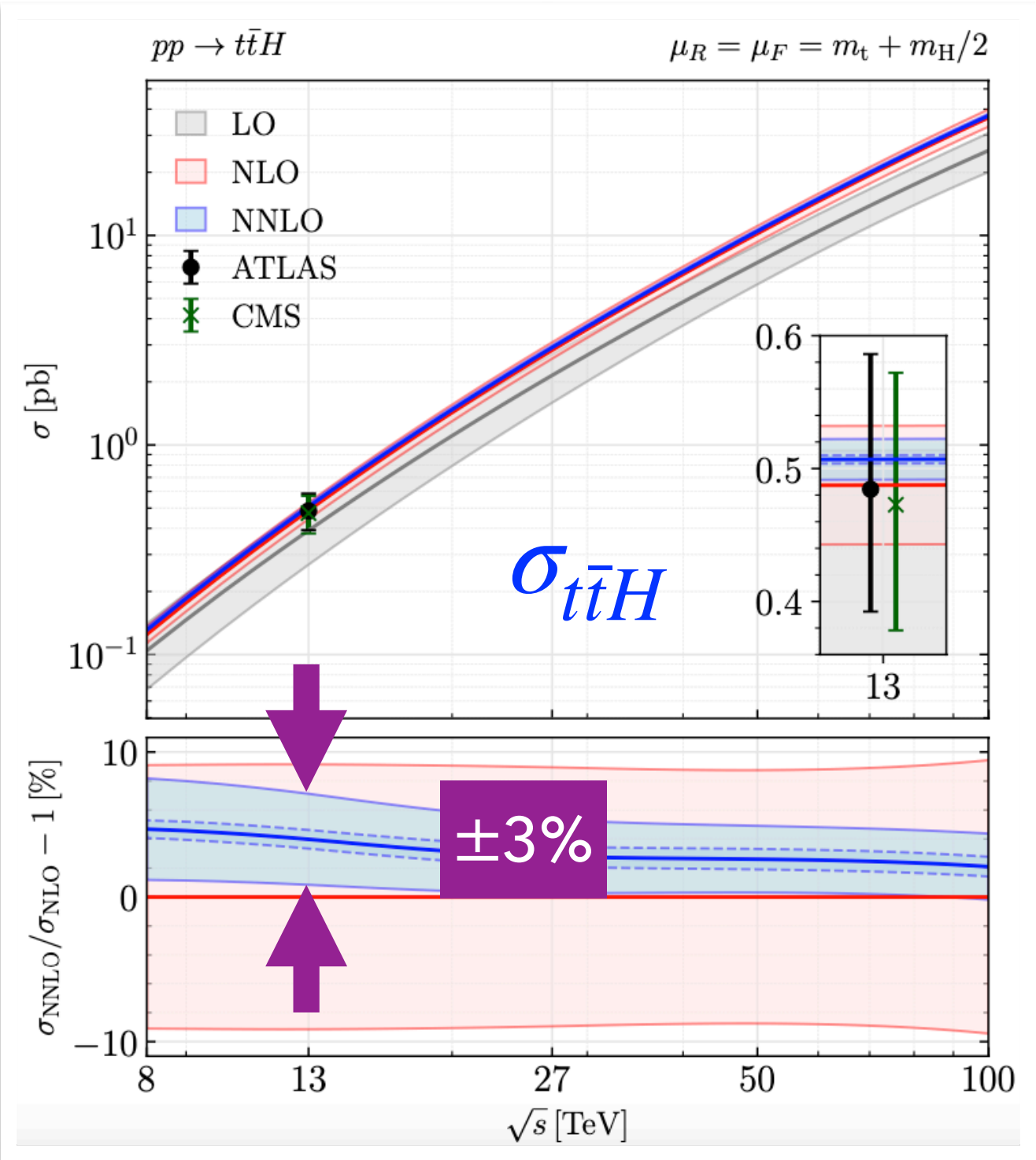
L. Buonocore et. al. 2306.16311



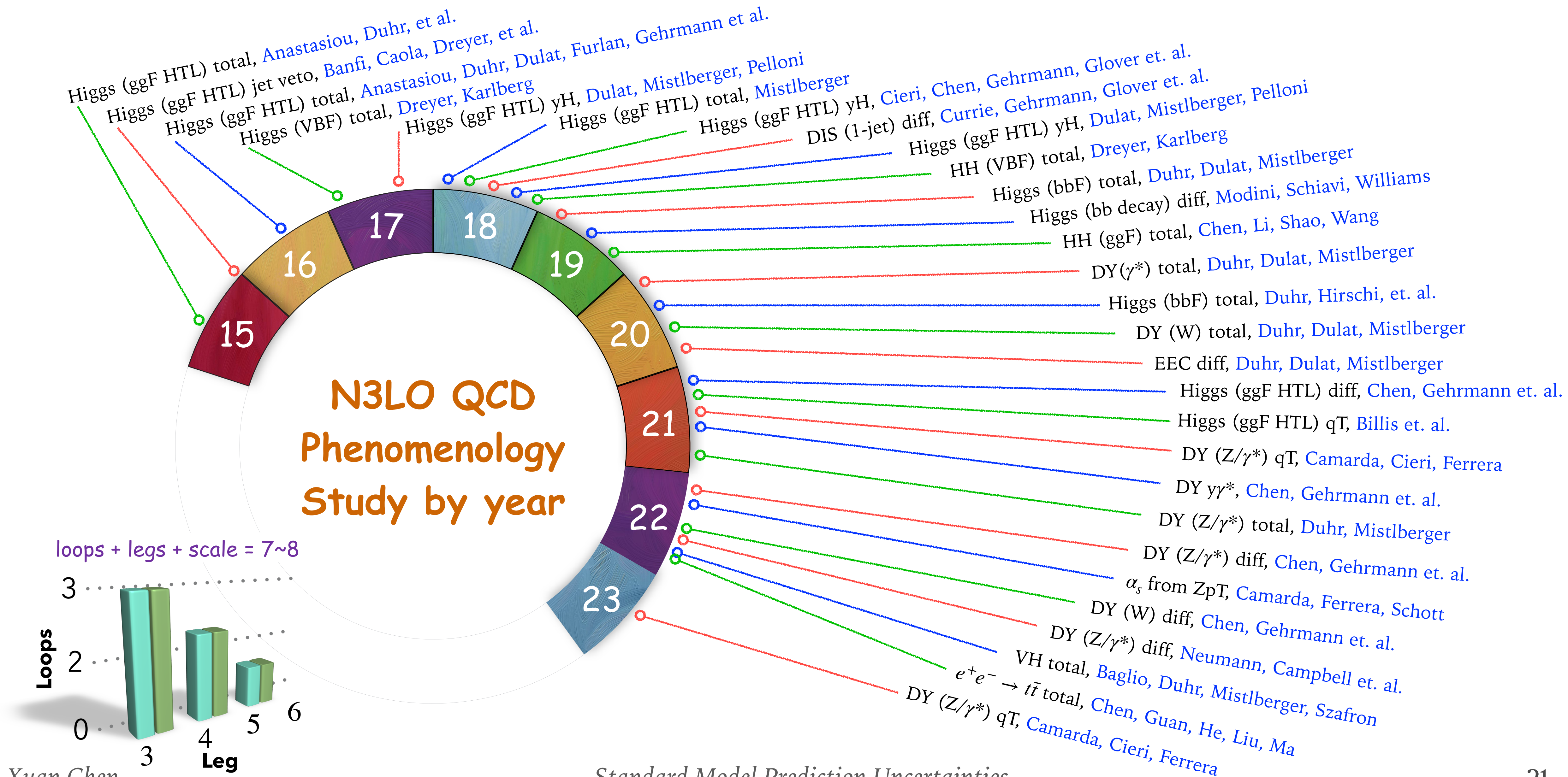
S. Badger, M Czakon et. al. 2304.06682



S. Catani et. al. PRL 130, 111902 (2023)



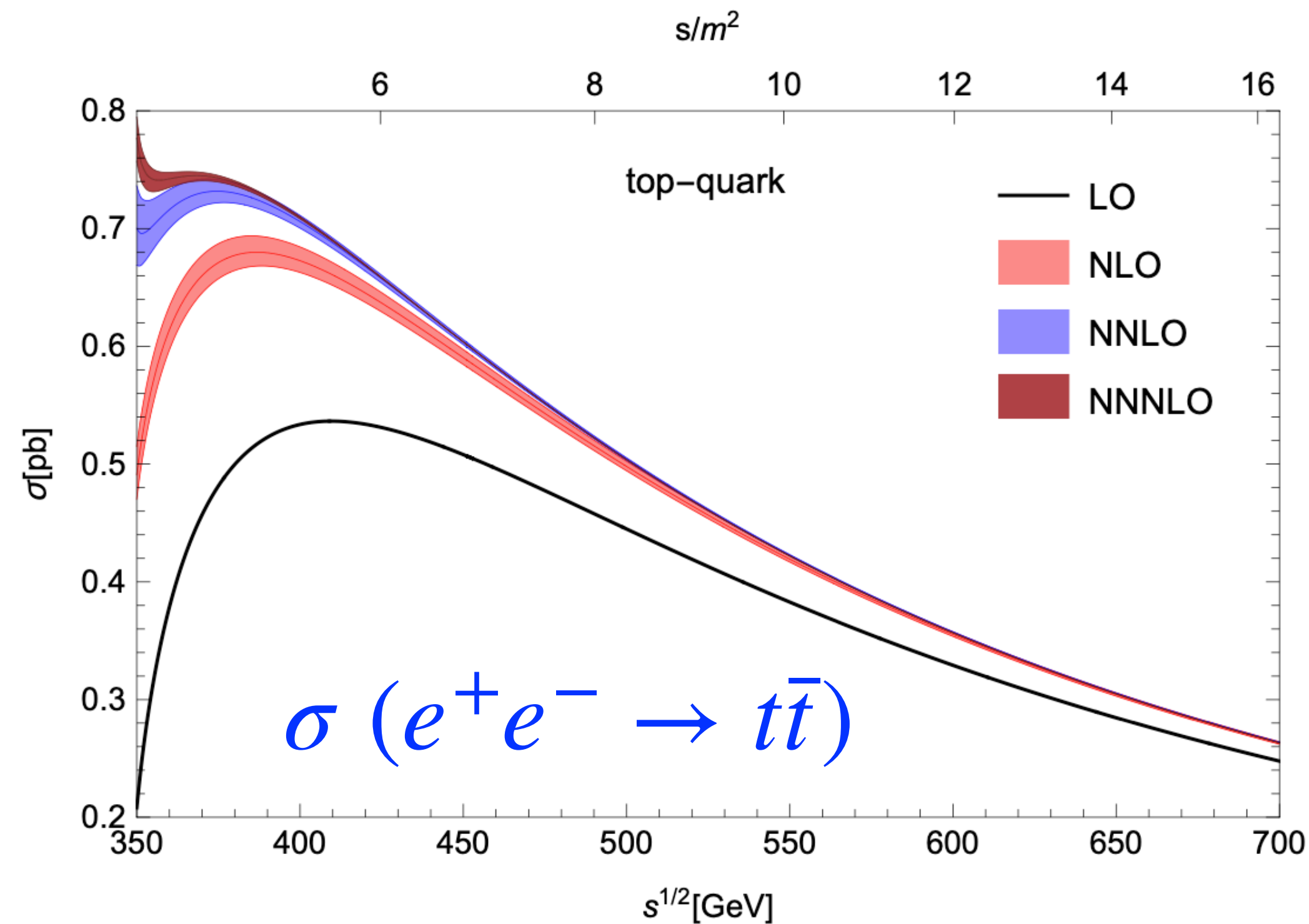
Perturbative QFT for Precision Predictions



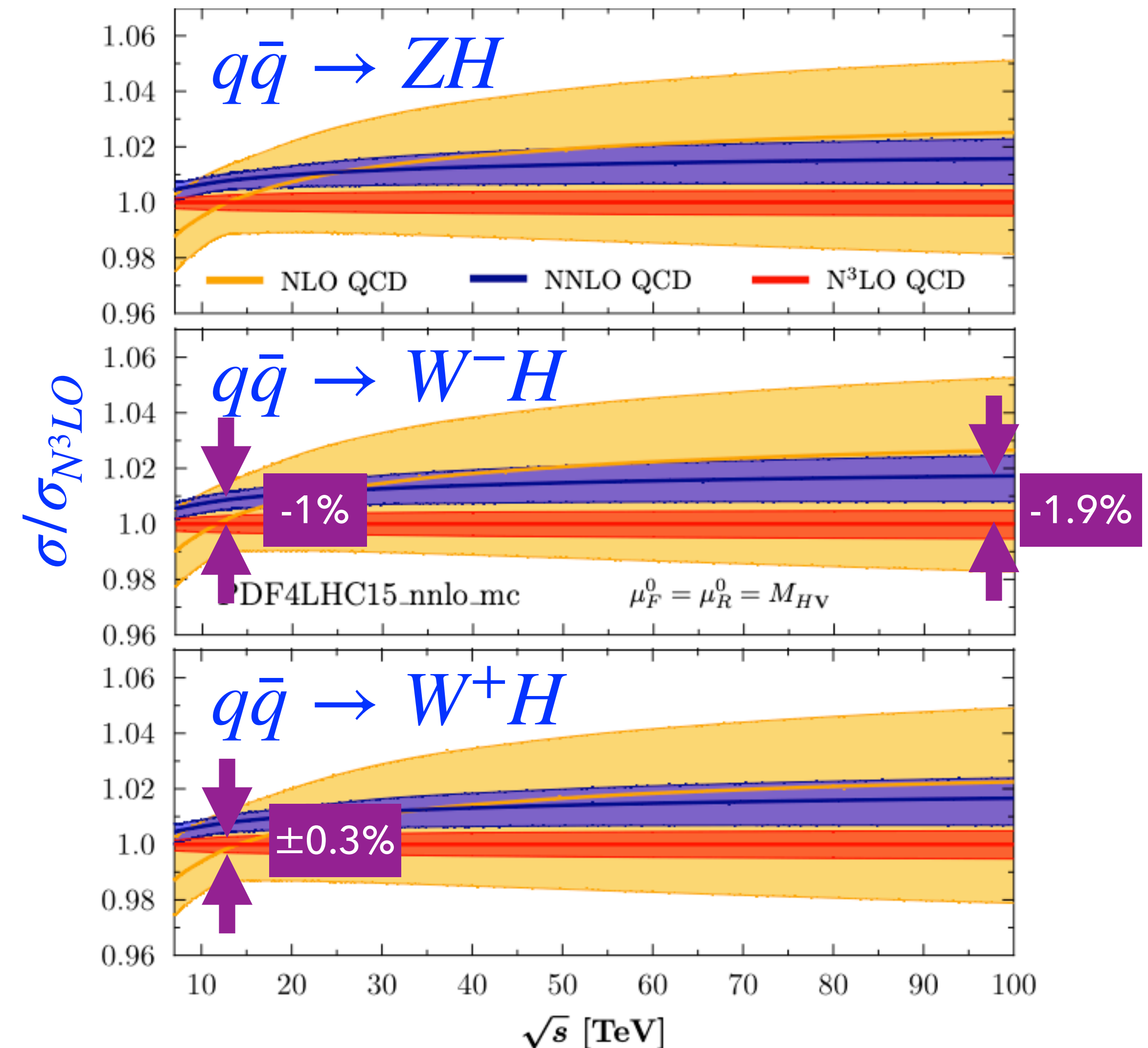
Perturbative QFT for Precision Predictions

$2 \rightarrow 2$ @N3LO QCD

- Total cross section for pp and epem collider
 - ME from $2 \rightarrow 3$ @ NNLO + ME @ 3-loop.
 - Use reverse unitarity for IR pole cancellation.
 - Different perturbative-series convergent behaviour



X. Chen, X. Guan, C.-Q. He, X. Liu, Y.-Q. Ma 2209.14259



J. Baglio, C. Duhr, B. Mistlberger, R. Szafron JHEP 12 (2022) 066

Perturbative QFT for Precision Predictions

2 → 1 @ N3LO (+ N3LL) QCD

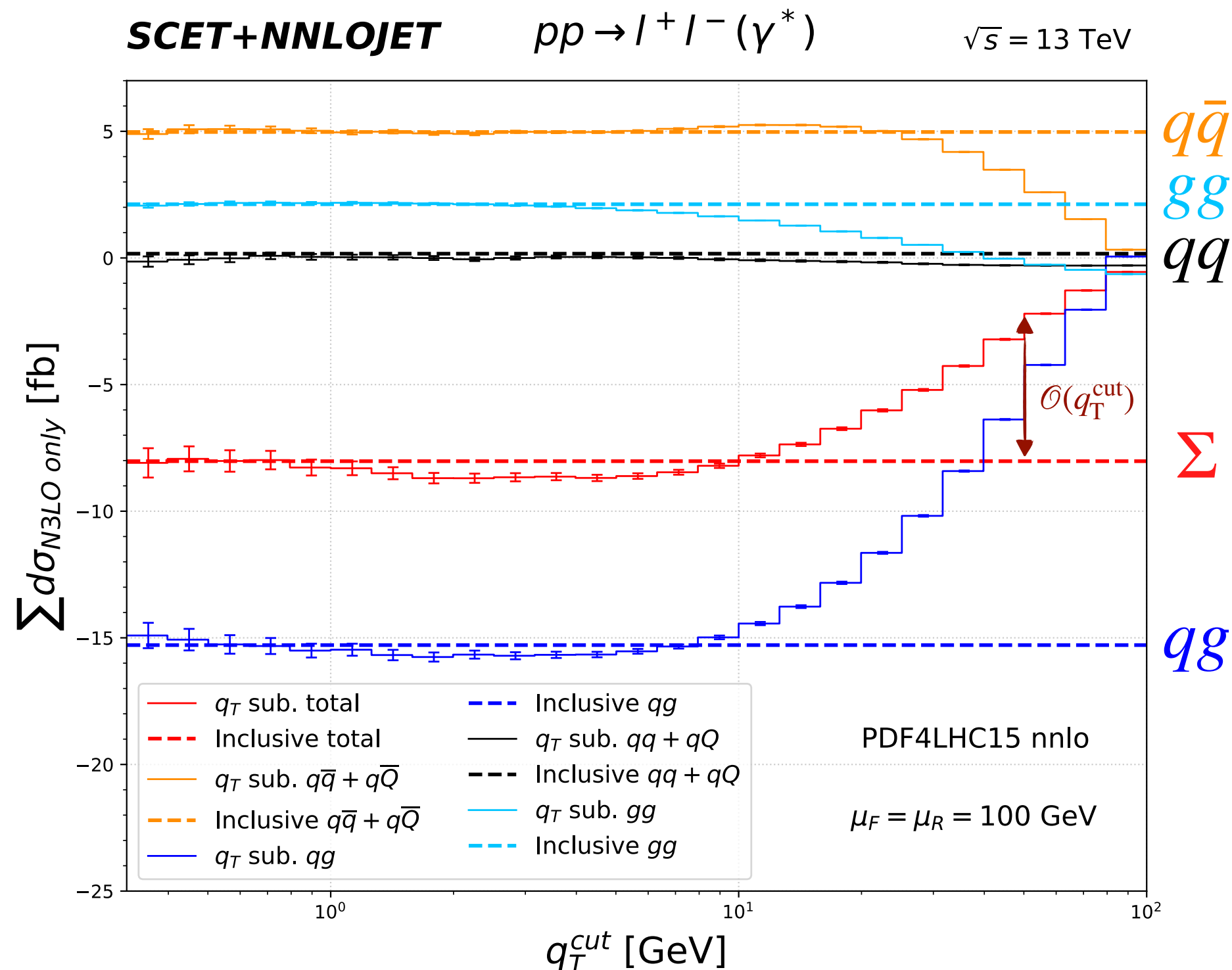
- Fully differential N3LO correction in event generator

- Recycle $pp \rightarrow V + J$ @ NNLO with τ_{cut} slicing

$$d\sigma_{N^kLO}^F = \mathcal{H}_{N^kLO}^F \otimes d\sigma_{LO}^F \Big|_{\delta(\tau)} + [d\sigma_{N^{k-1}LO}^{F+jet} - d\sigma_{N^kLO}^{FCT}]_{\tau > \tau_{\text{cut}}} + \mathcal{O}(\tau_{\text{cut}}^2/Q^2)$$

- Fiducial power correction removed via MC recoil technique.

- Small p_T resummation at N3LL and partial N4LL

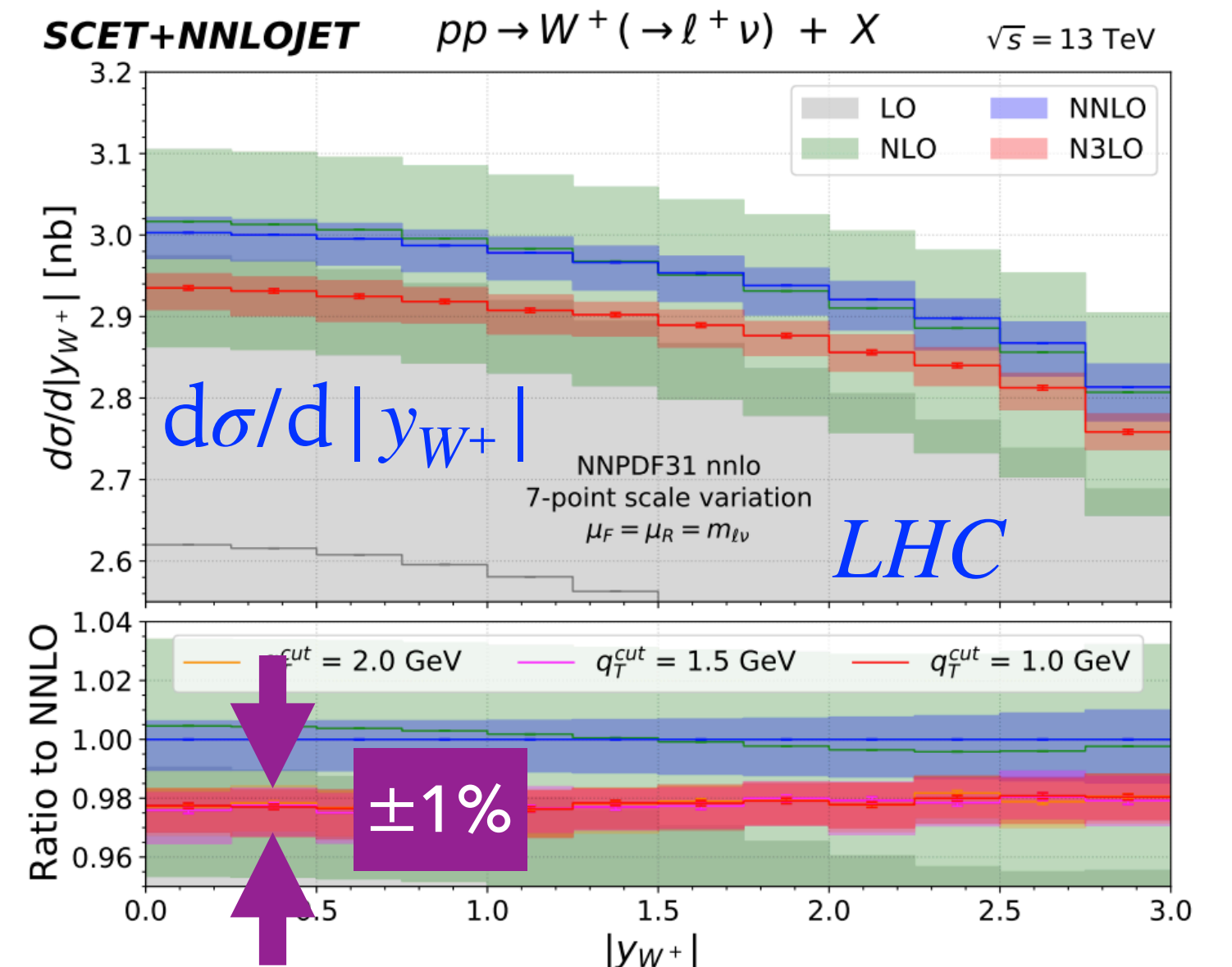
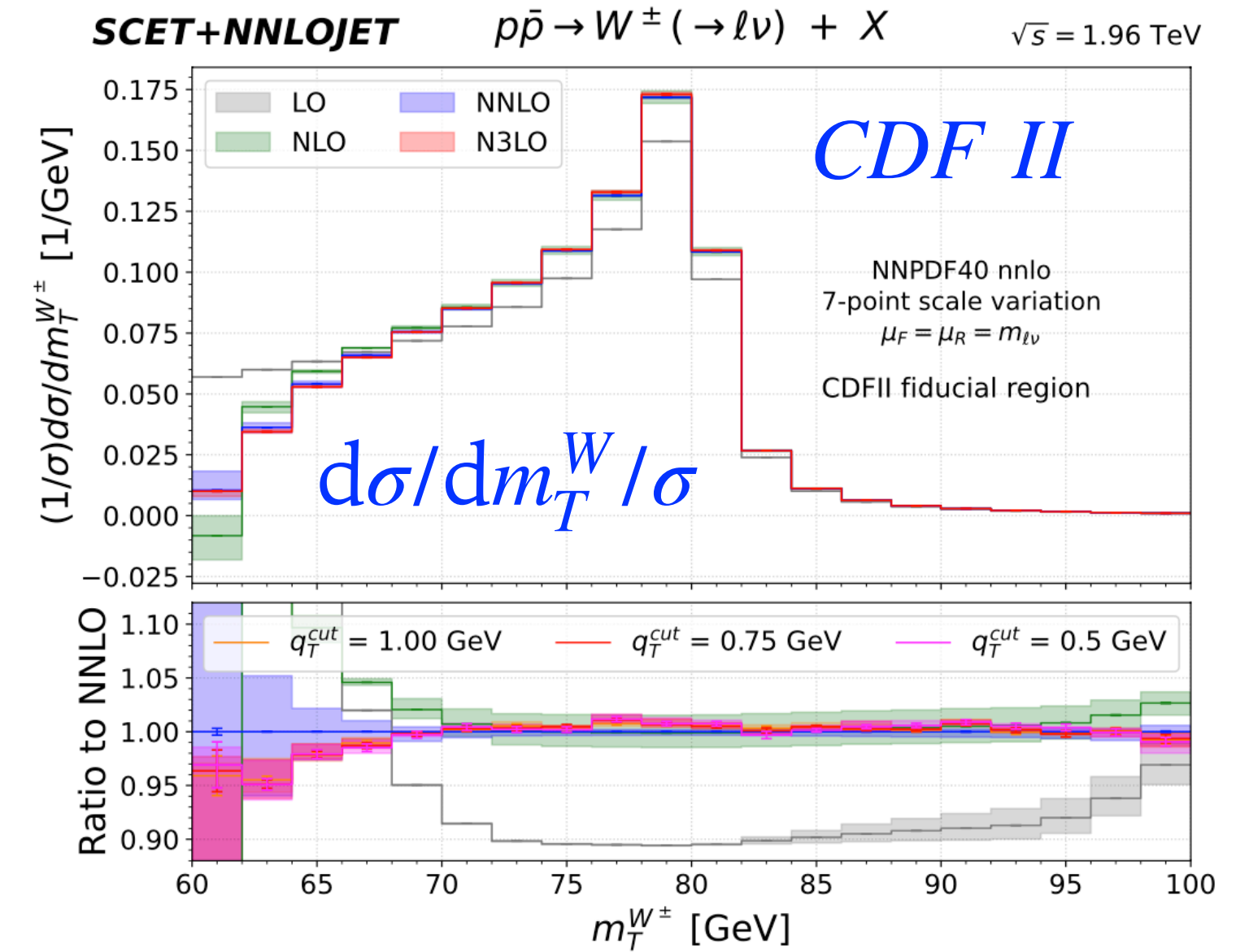


- Validation of inclusive total cross section for $q_T^{\text{cut}} < 1 \text{ GeV}$.

C. Duhr, F. Dulat, B. Mistlberger.
PRL. 125, 172001 (2020)

- Separated in parton channels

- Foundation of numerical Monte Carlo setup for differential predictions.



Perturbative QFT for Precision Predictions

$2 \rightarrow 1$ @ N3LO (+ N3LL) QCD

- Fully differential N3LO correction in event generator

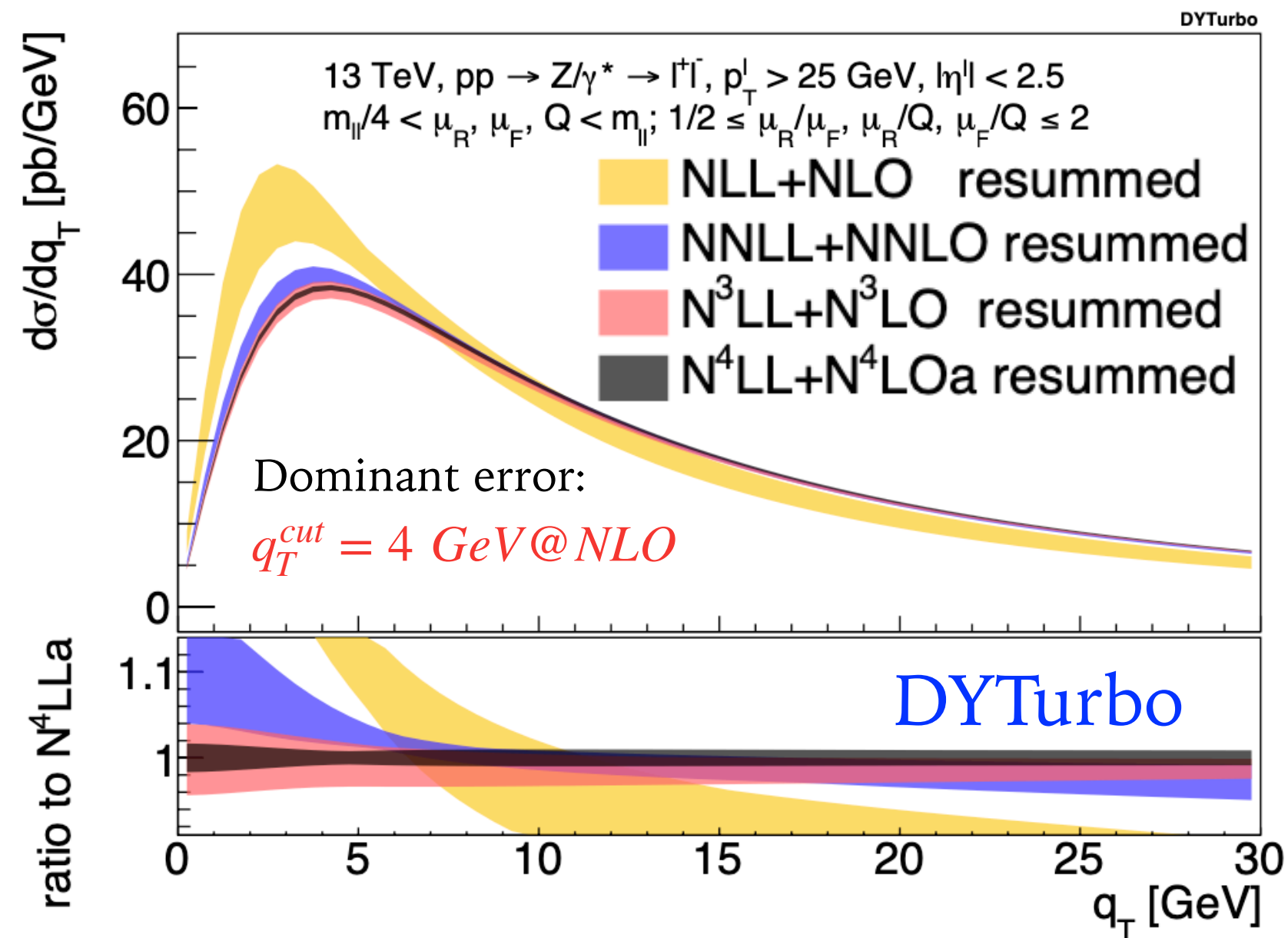
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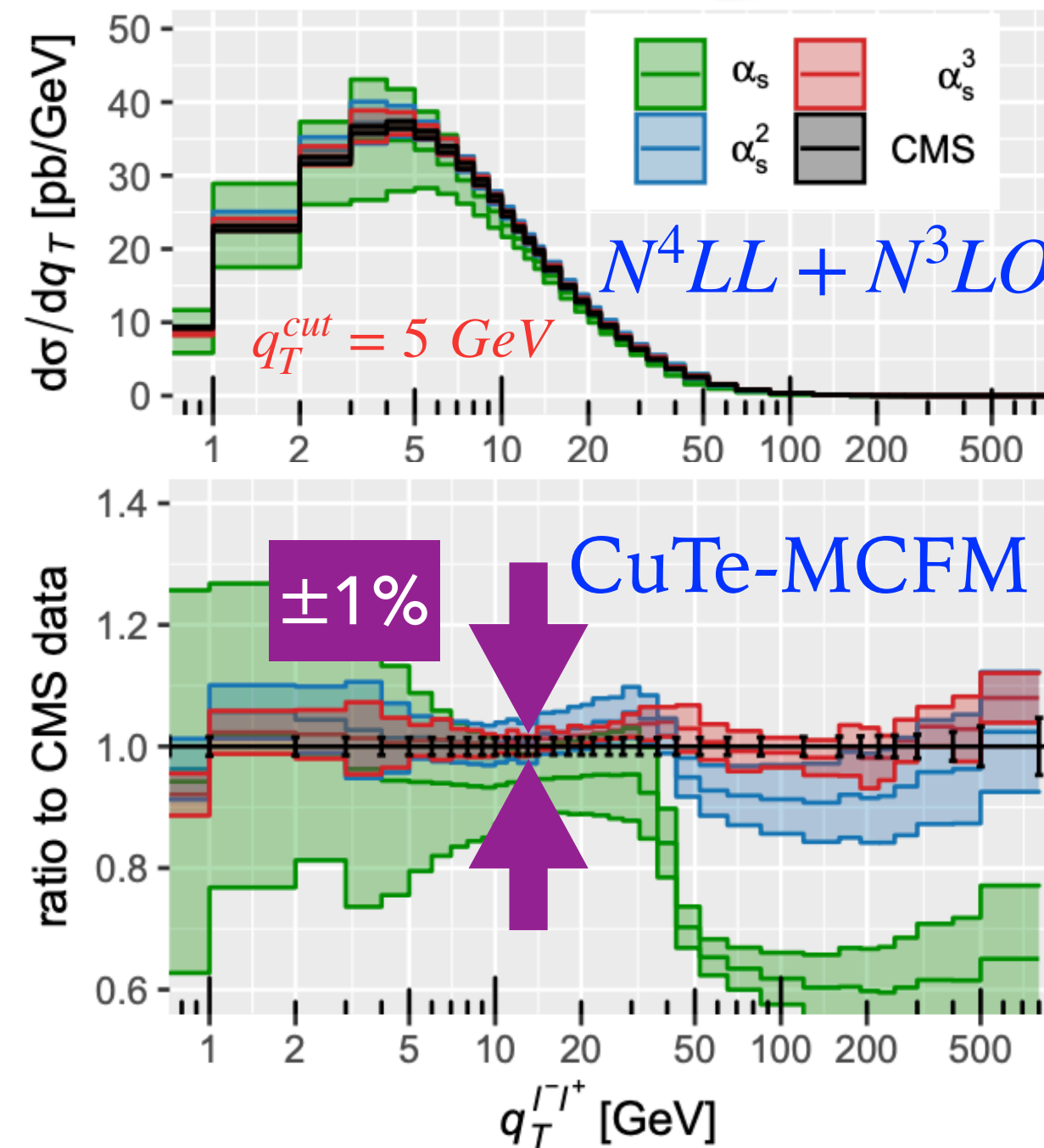
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- Small p_T resummation at N3LL and partial N4LL

$$d\sigma/dp_T^Z$$

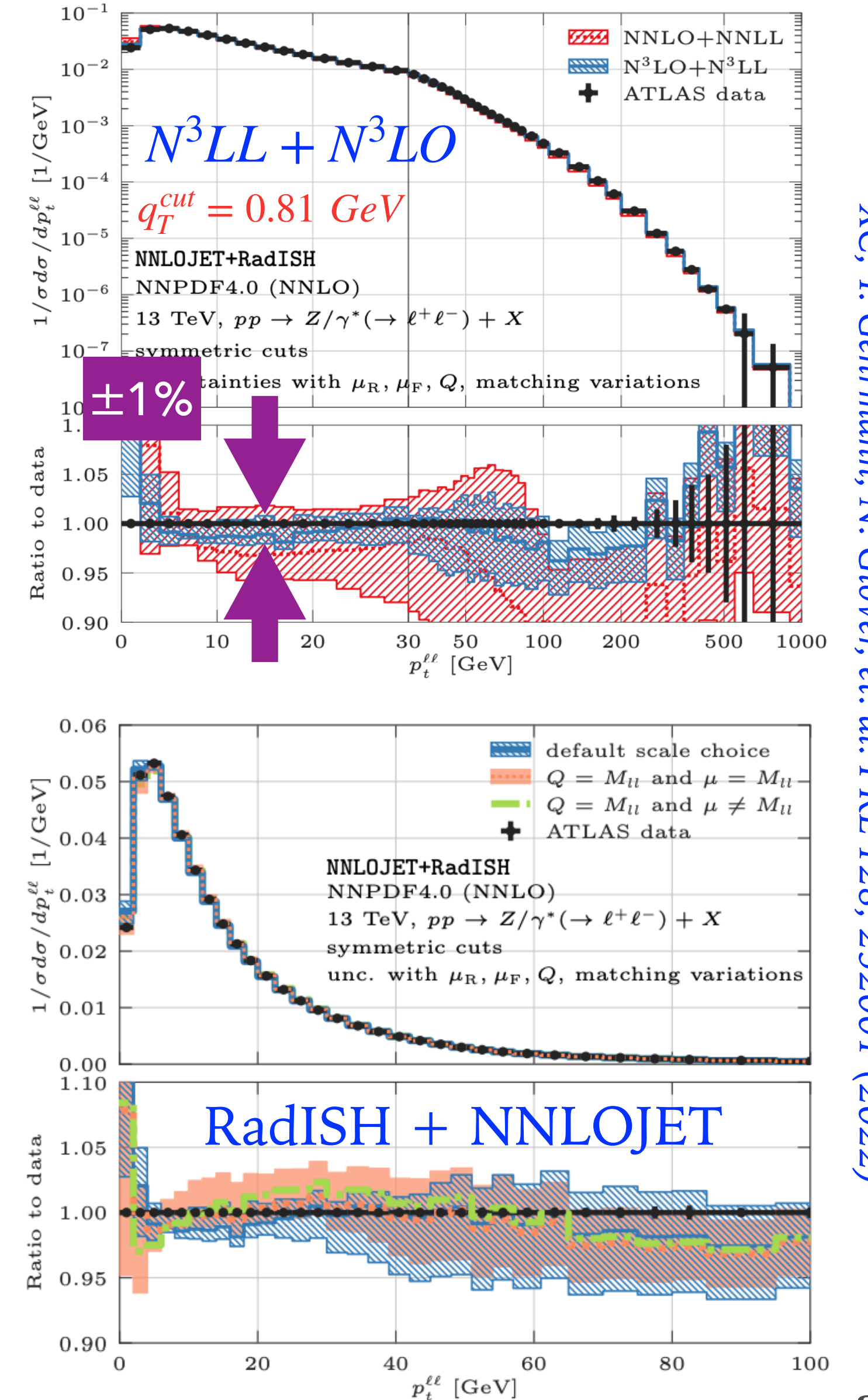


S. Camarda, L. Cieri, G. Ferrera 2303.12781



T. Neumann, J. Campbell PRD 107, L011506 (2023)

Standard Model Prediction Uncertainties



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

Perturbative QFT for Precision Predictions

$2 \rightarrow 1$ @ N3LO (+ N3LL) QCD

- Fully differential N3LO correction in event generator

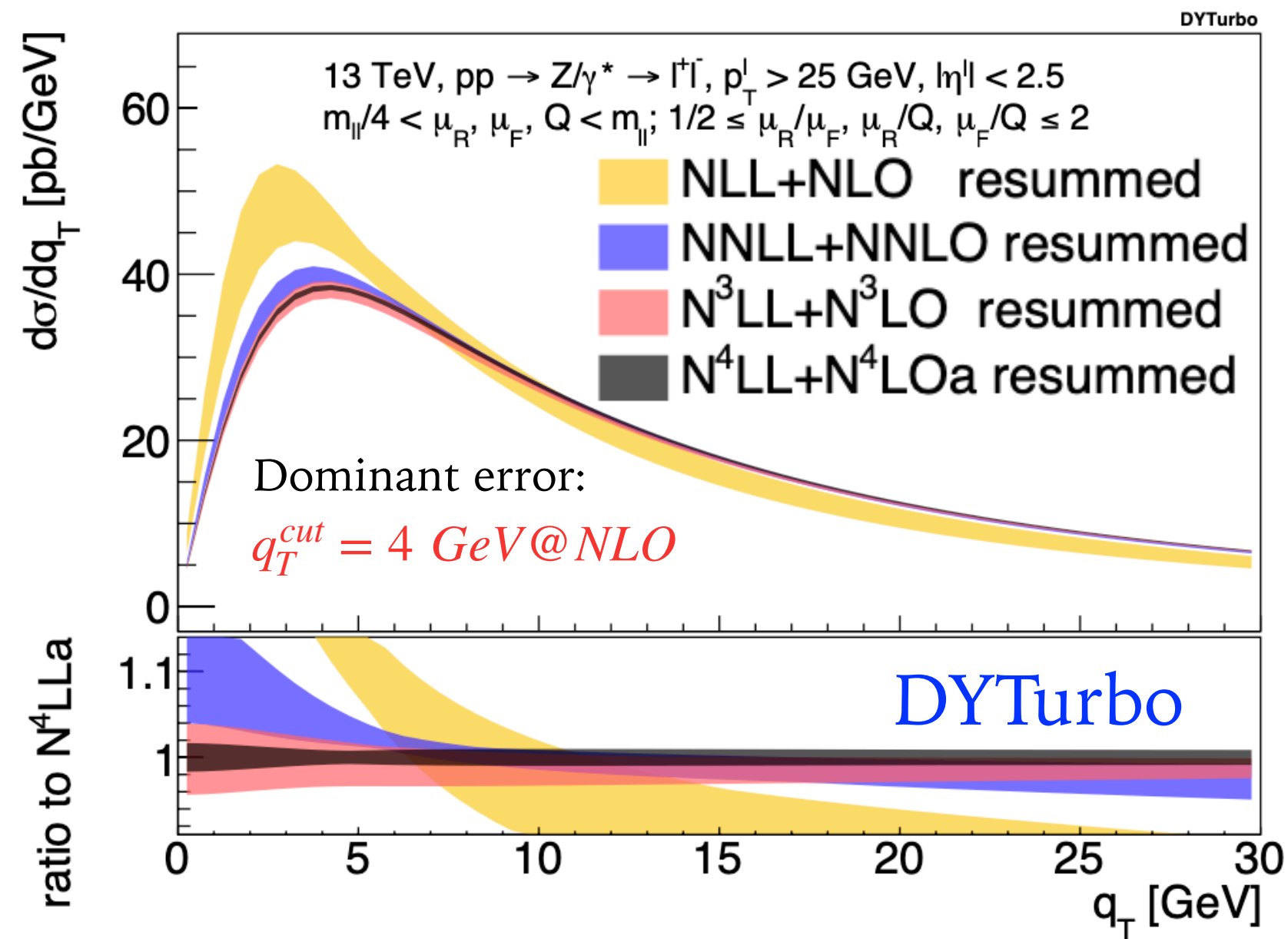
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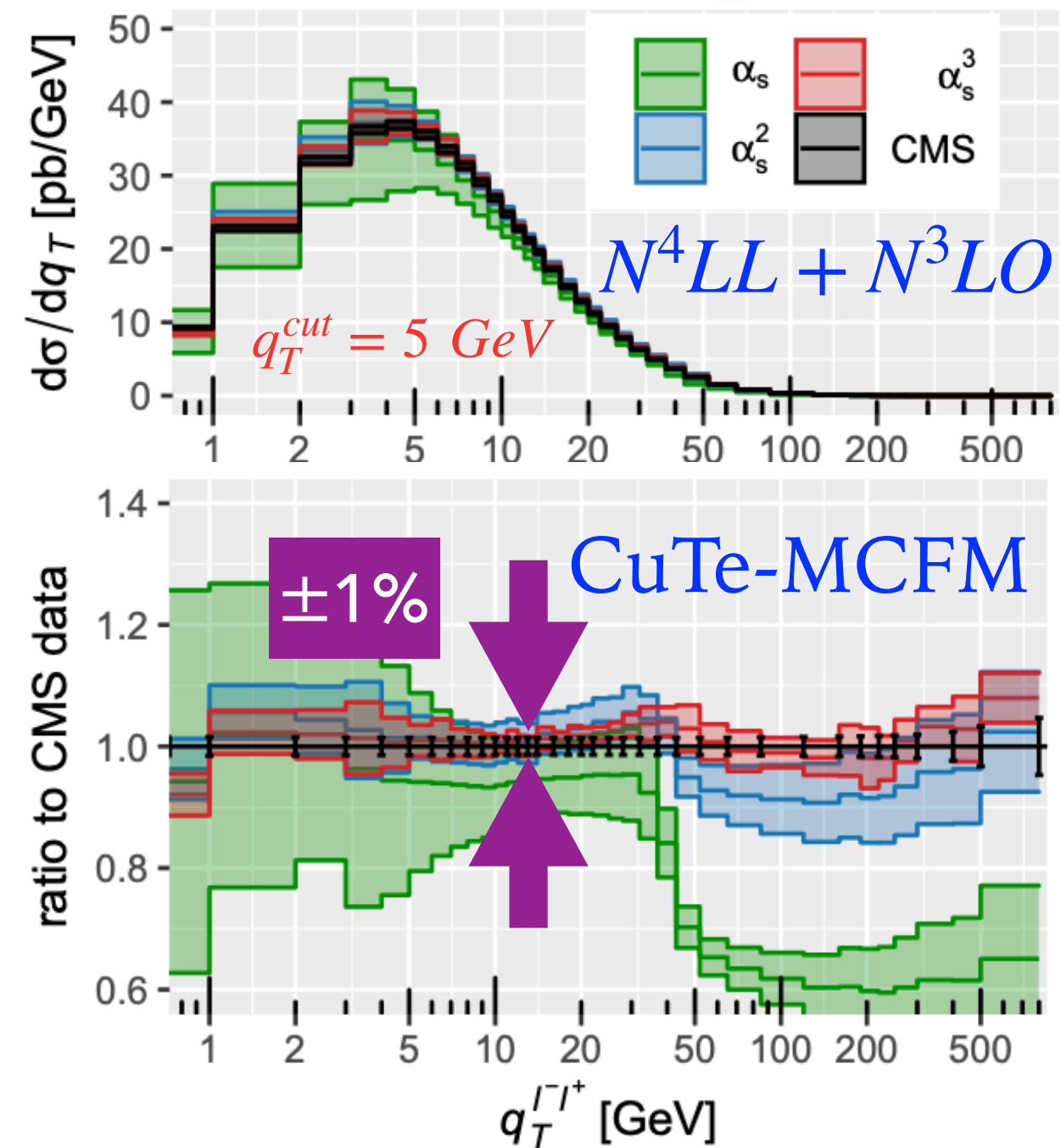
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$$d\sigma/dp_T^Z$$

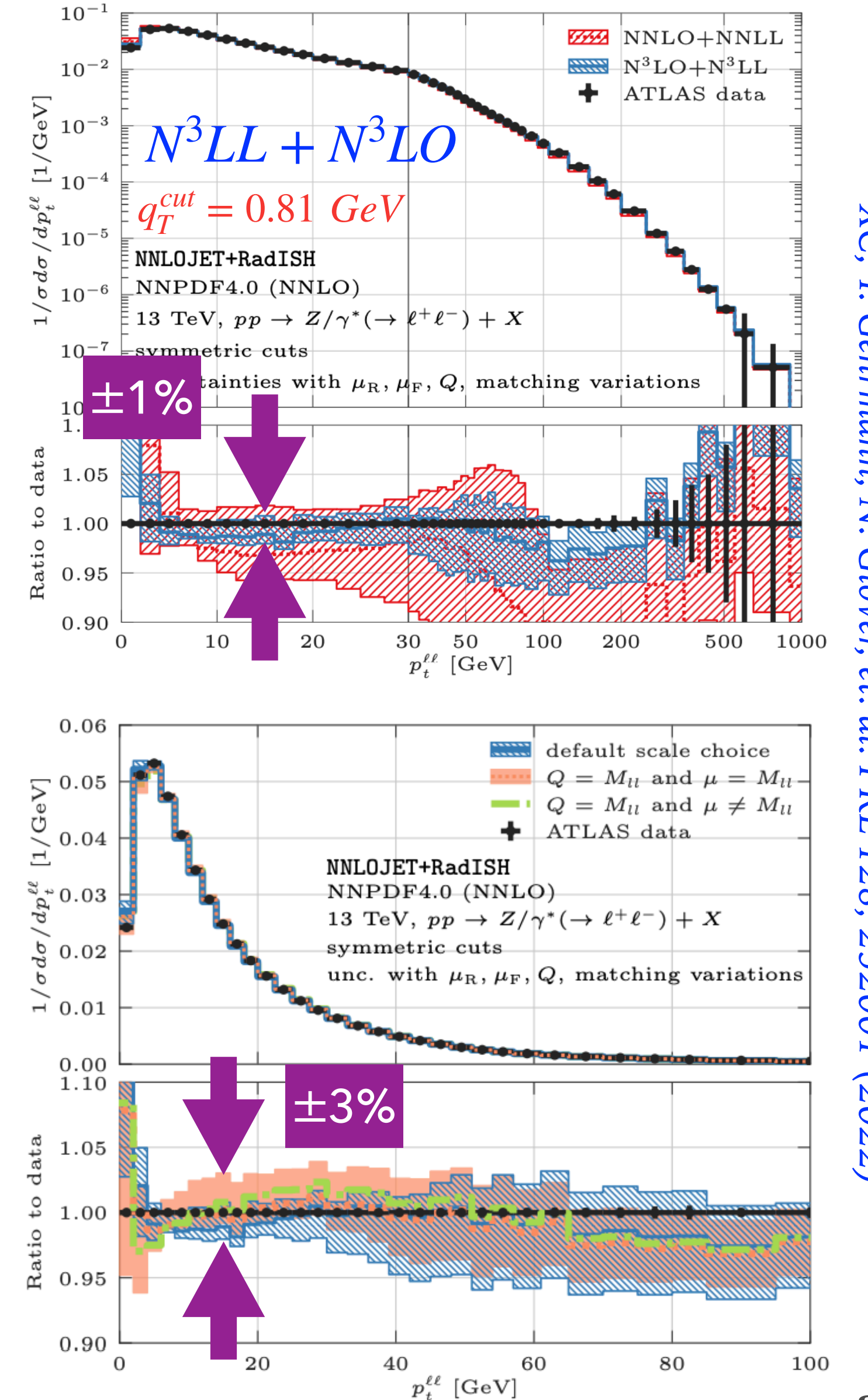


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Perturbative QFT for Precision Predictions

$2 \rightarrow 1$ @ N3LO (+ N3LL) QCD

- Fully differential N3LO correction in event generator

- Recycle $pp \rightarrow V + J$ @ NNLO with τ_{cut} slicing

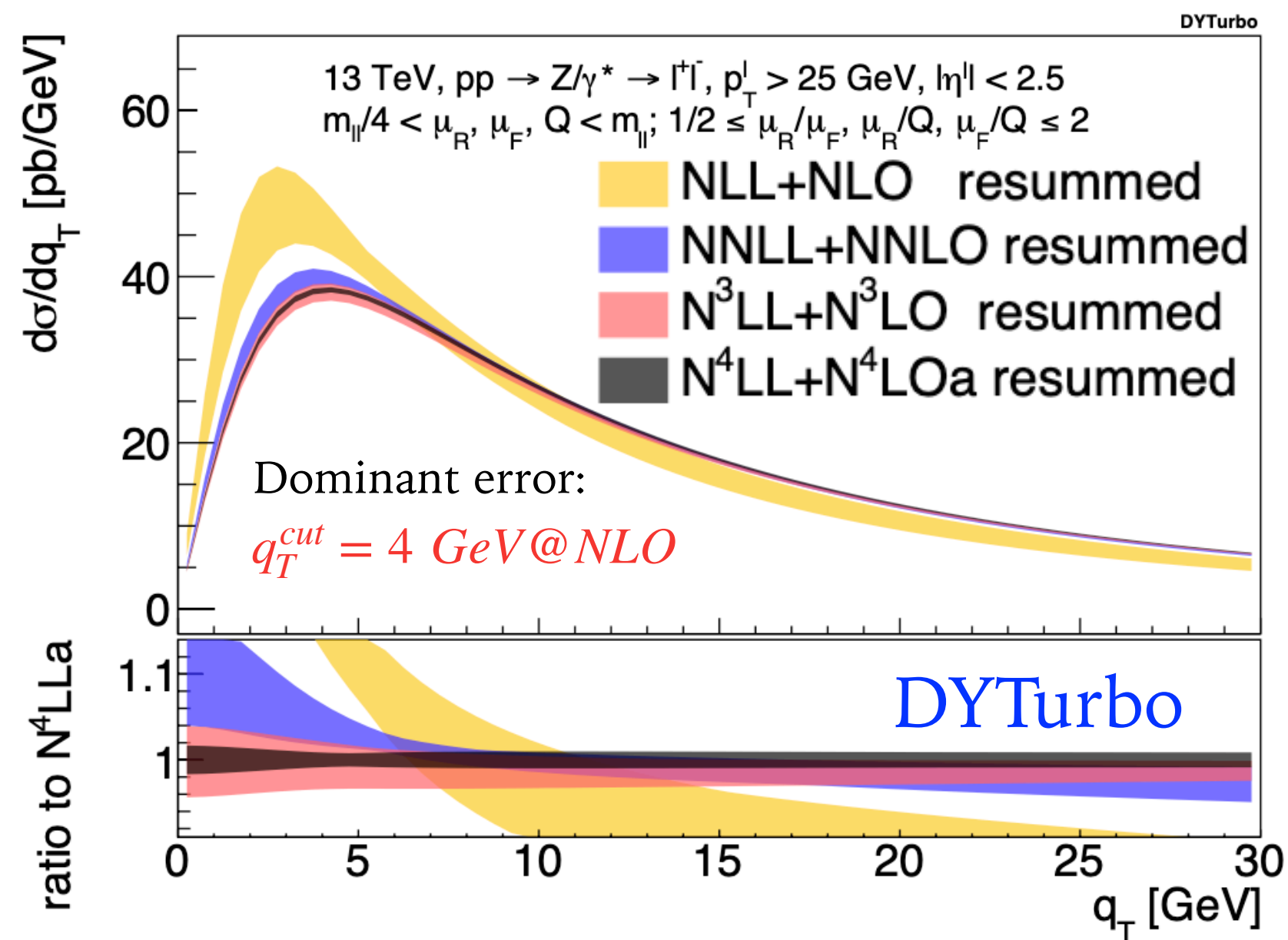
$$d\sigma_{N^kLO}^F = \mathcal{H}_{N^kLO}^F \otimes d\sigma_{LO}^F \Big|_{\delta(\tau)} + [d\sigma_{N^{k-1}LO}^{F+jet} - d\sigma_{N^kLO}^{FCT}]_{\tau > \tau_{\text{cut}}} + \mathcal{O}(\tau_{\text{cut}}^2/Q^2)$$

- Fiducial power correction removed via MC recoil technique.

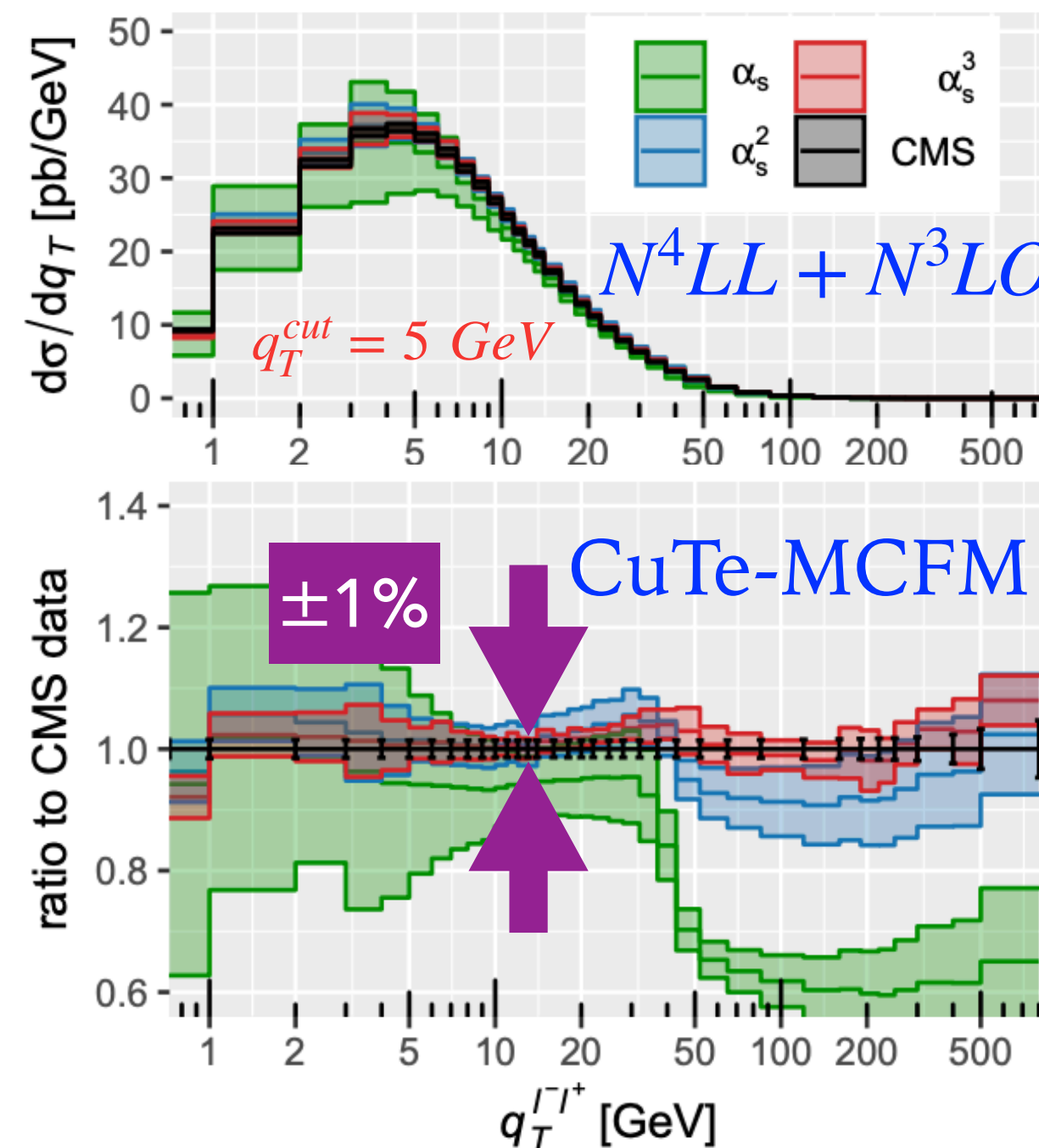
- Small p_T resummation at N3LL and partial N4LL



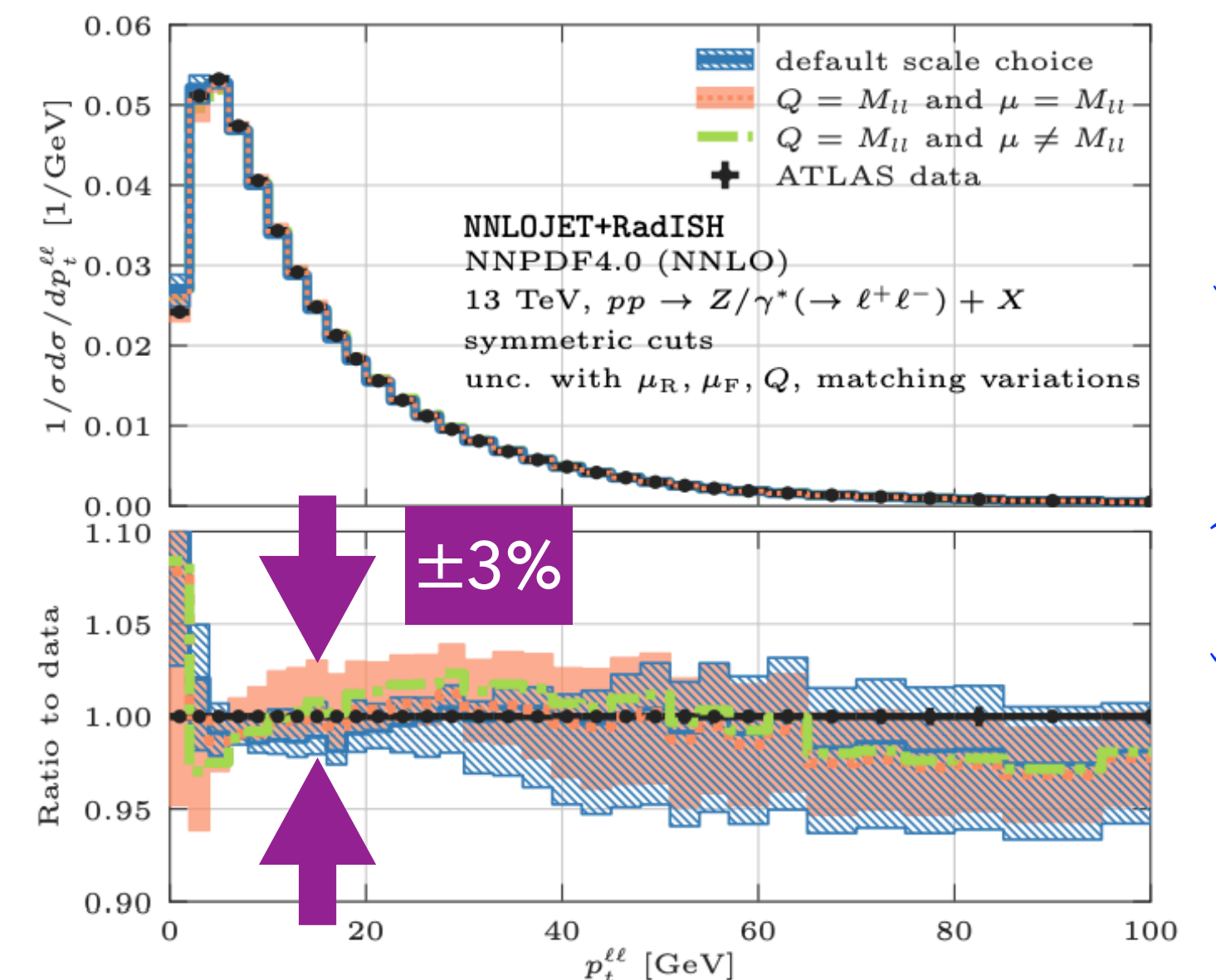
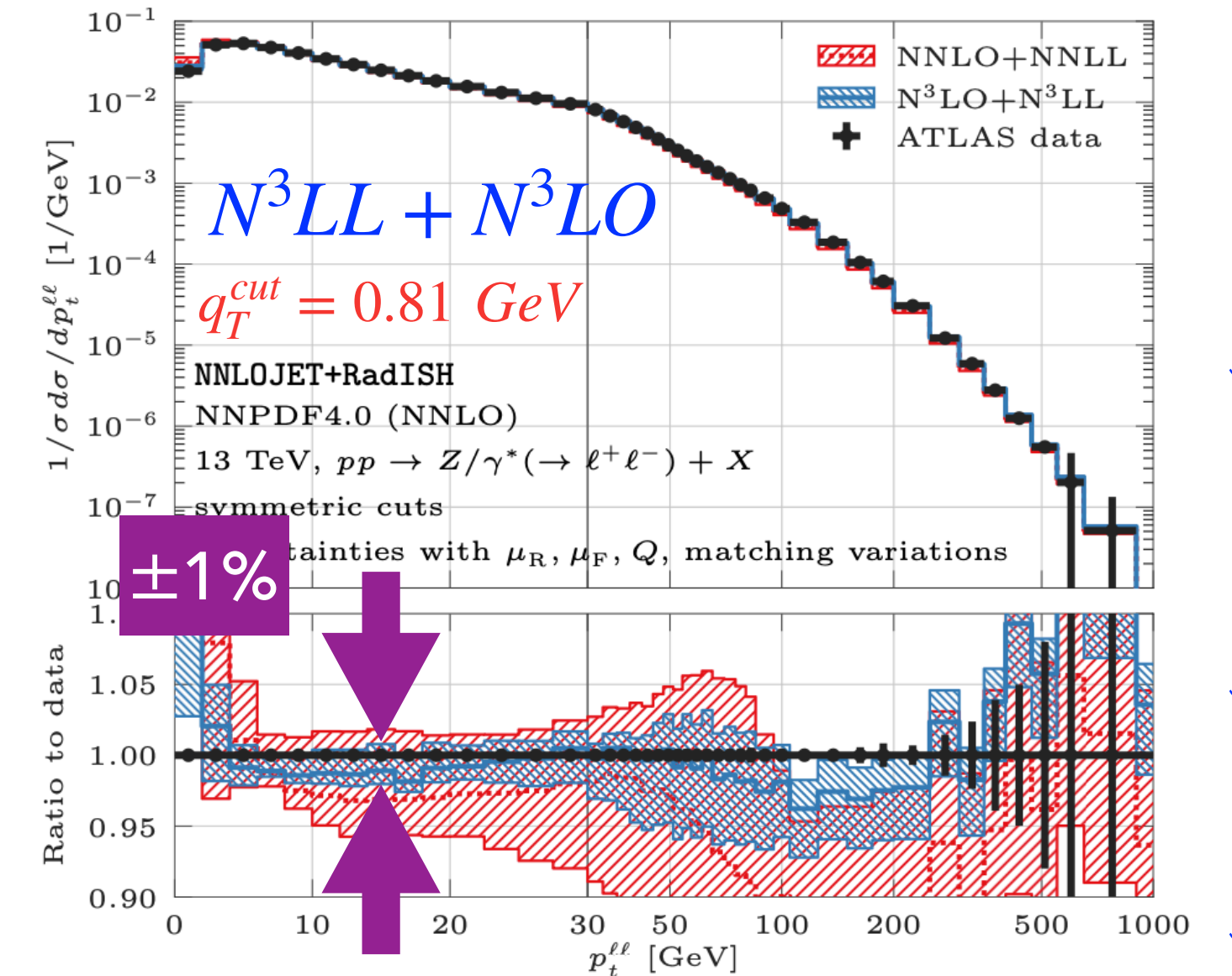
G. Fontana



S. Camarda, L. Cieri, G. Ferrera 2303.12781



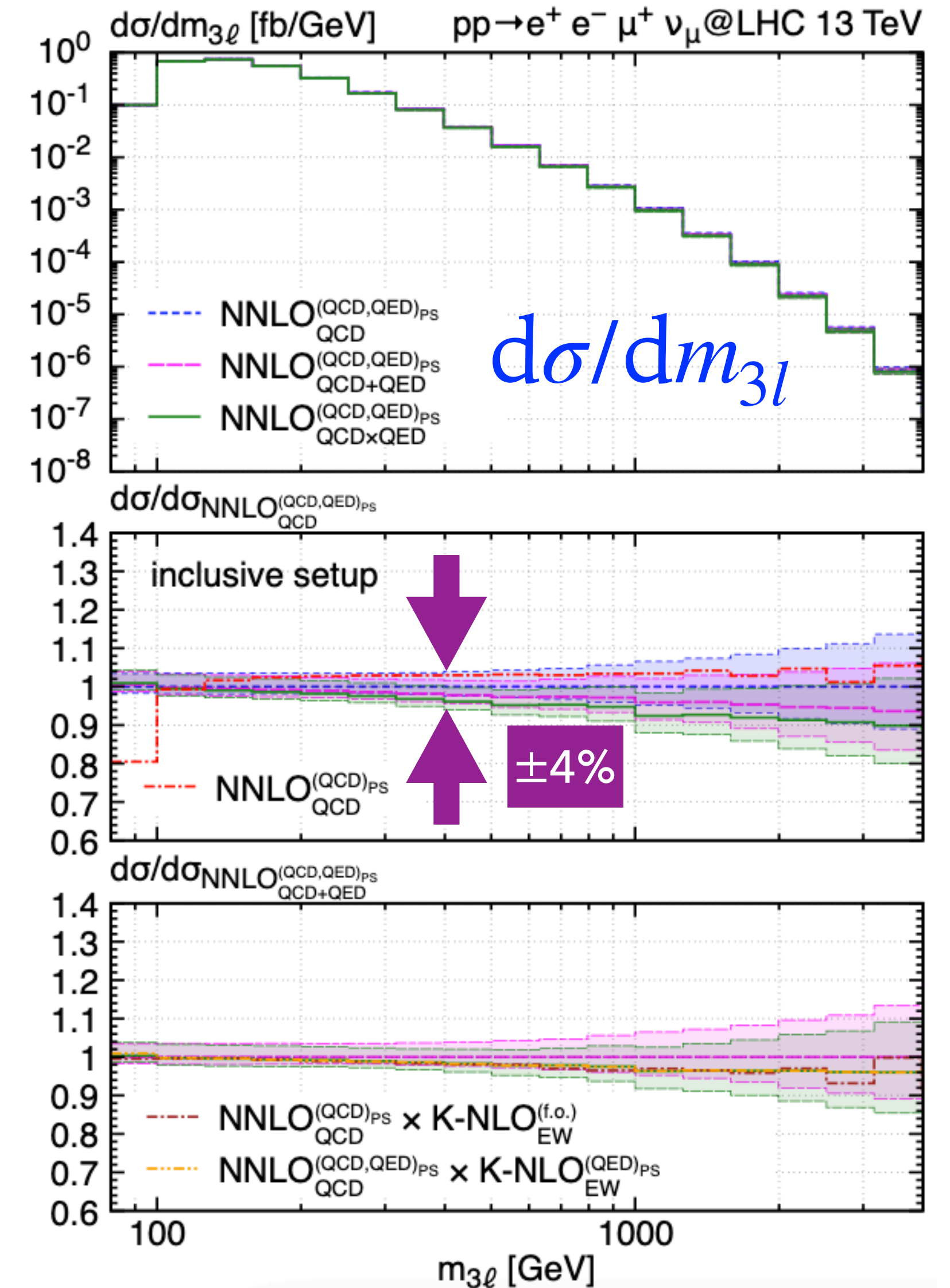
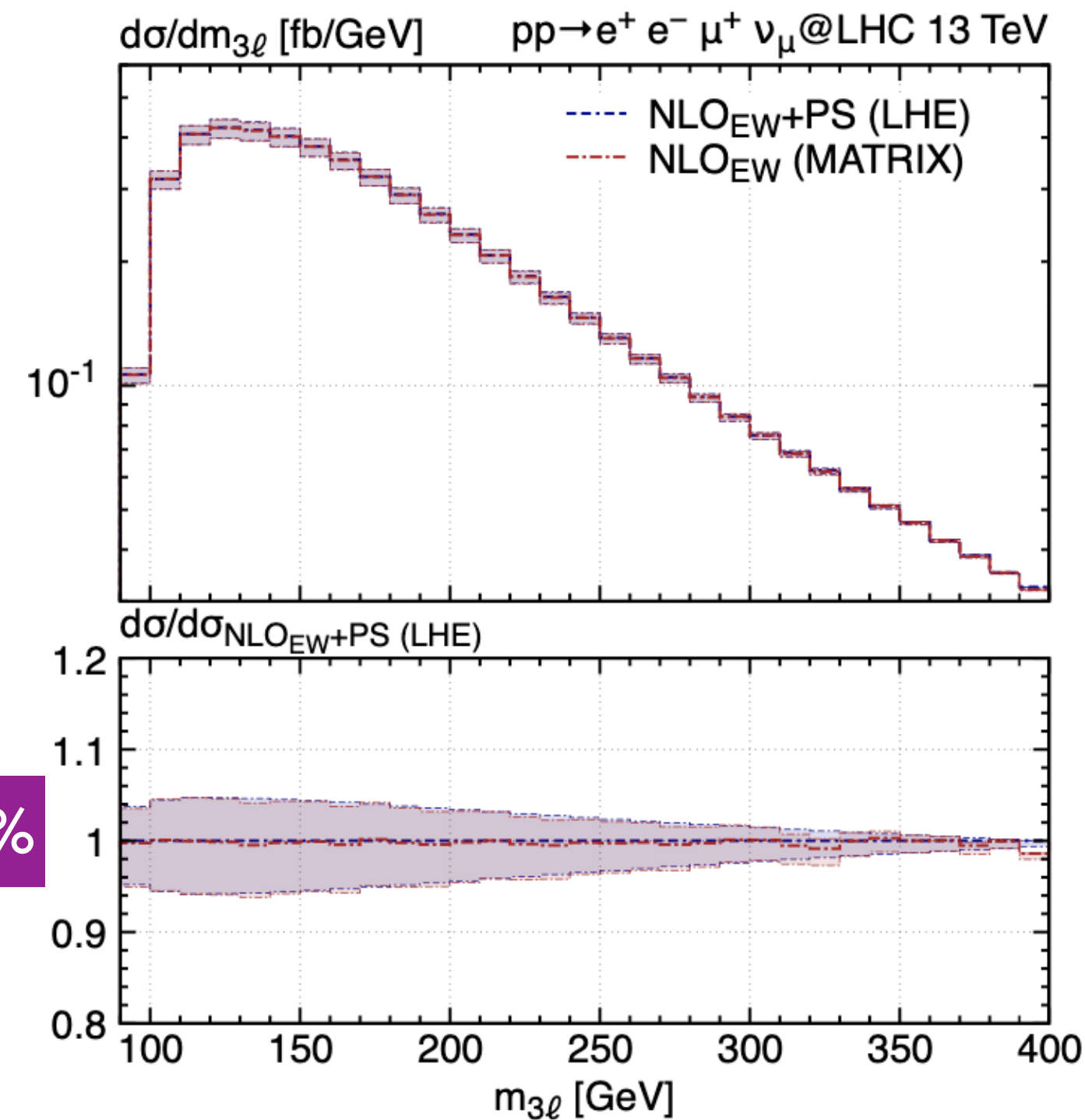
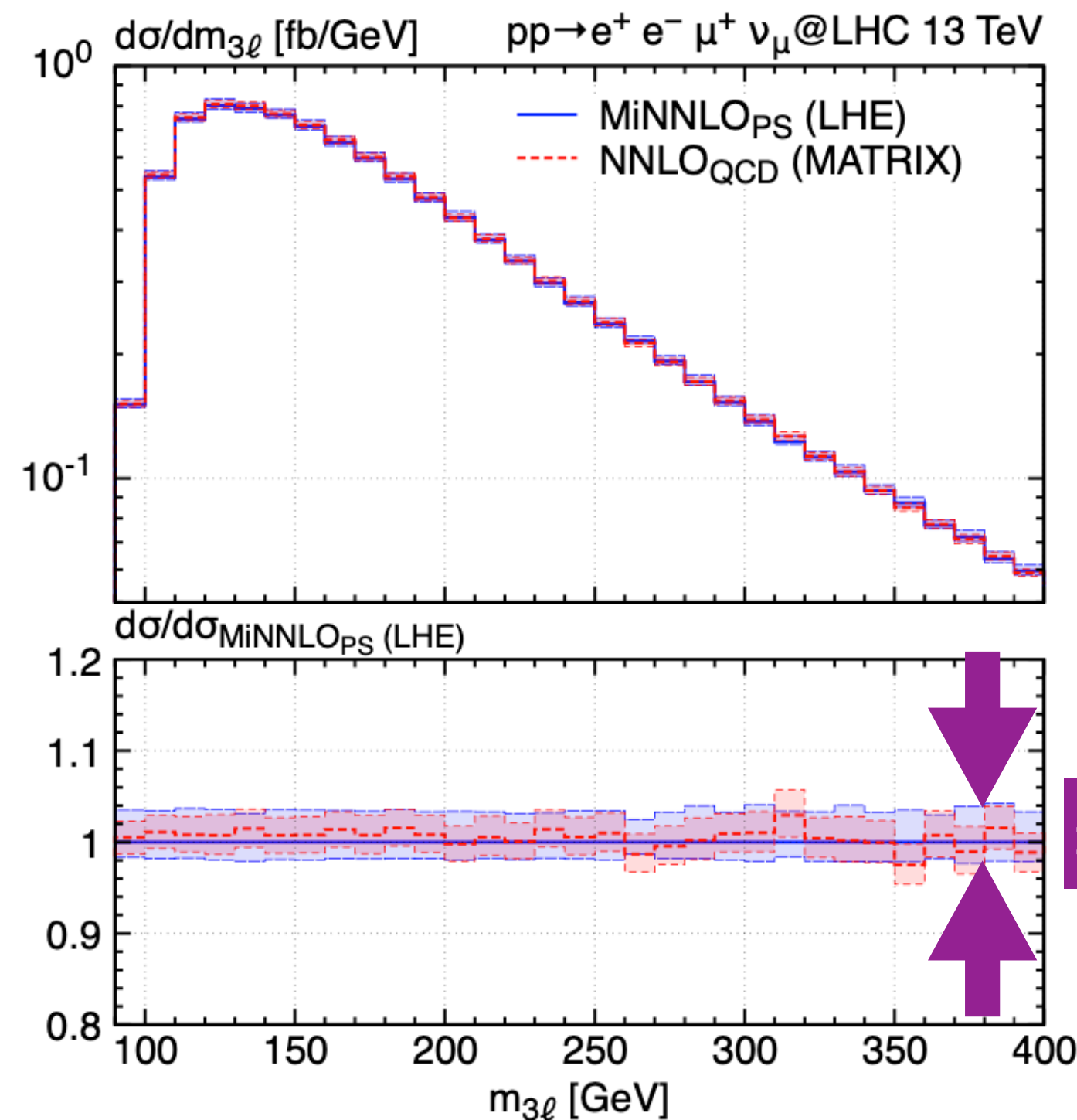
T. Neumann, J. Campbell PRD 107, L011506 (2023)



Perturbative QFT for Precision Predictions

State-of-the-art Parton Shower accuracy

- Standard parton showers are Leading Logarithmic (LL) accurate.
([SHERPA](#), [PYTHIA](#), [DIRE](#), [GENEVA](#), [HERWIG](#), [VINCIA](#) etc.)
- NNLO + LL PS established for $2 \rightarrow 2$ colour singlet and $t\bar{t}$.
- $pp \rightarrow W^\pm Z \rightarrow l^+ l^- l'^\pm \nu_l' + [\text{QCD, QED}]$ shower
J. M. Lindert, D. Lombardi, M. Wiesemann et. al. JHEP 11 (2022) 036



Perturbative QFT for Precision Predictions

State-of-the-art Parton Shower accuracy

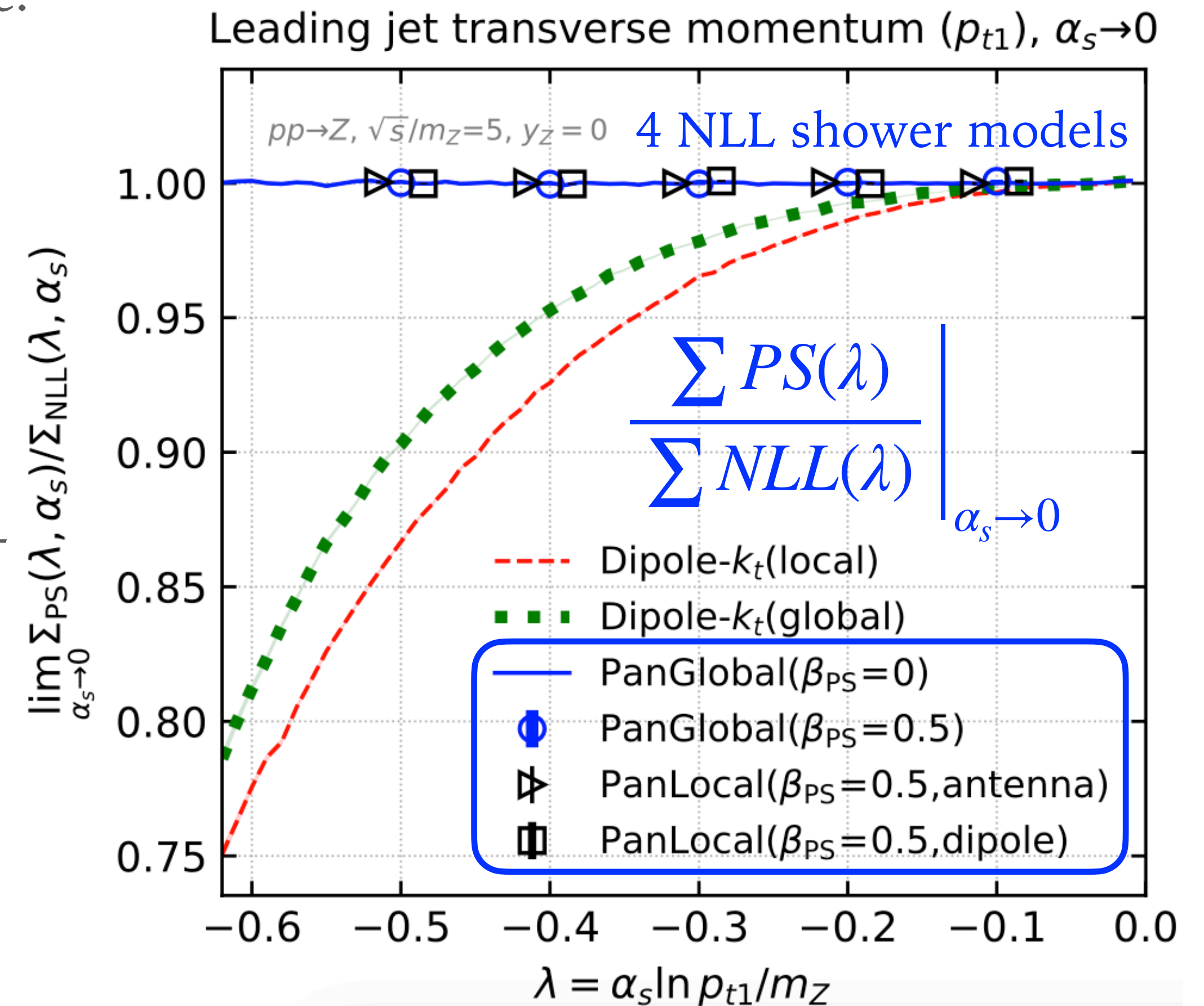
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J. M. Lindert, D. Lombardi, M. Wiesemann et. al. JHEP 11 (2022) 036
- Several groups working on new PS framework aiming for NLL:
 - [CVOLVER](#): Forshaw, Holguin, Plätzer
 - [DEDUCTOR](#): Nagy, Soper
 - [ALARIC](#): Assi, Herren, Höche, Krauss, Reichelt, Schönherr
 - [PANSCALES](#): van Beekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso, Soye, Verheyen, Halliwell, Medves, Dreyer, Scyboz, Karlberg, Monni, El-Menoufi
- Test of shower accuracy ([PANSCALES](#)):

$$\lim_{\alpha_s \rightarrow 0} \frac{\Sigma_{\text{PS}}(\lambda) - \Sigma_{\text{NLL}}(\lambda)}{\Sigma_{\text{NLL}}(\lambda)}, \quad \lambda = \alpha_s L$$

- [PANSCALES](#): VBFH (initial and final NLL shower)
 - First NLL shower uncertainty estimation at $\sim 10\%$
- [ALARIC](#): massive shower (final NLL shower)

Alaric Collaboration 2208.06057, B. Assi, S. Höche 2307.00728

$$pp \rightarrow Z + PS$$



More validations in:

PanScales Collaboration JHEP 11 (2022) 020

Perturbative QFT for Precision Predictions

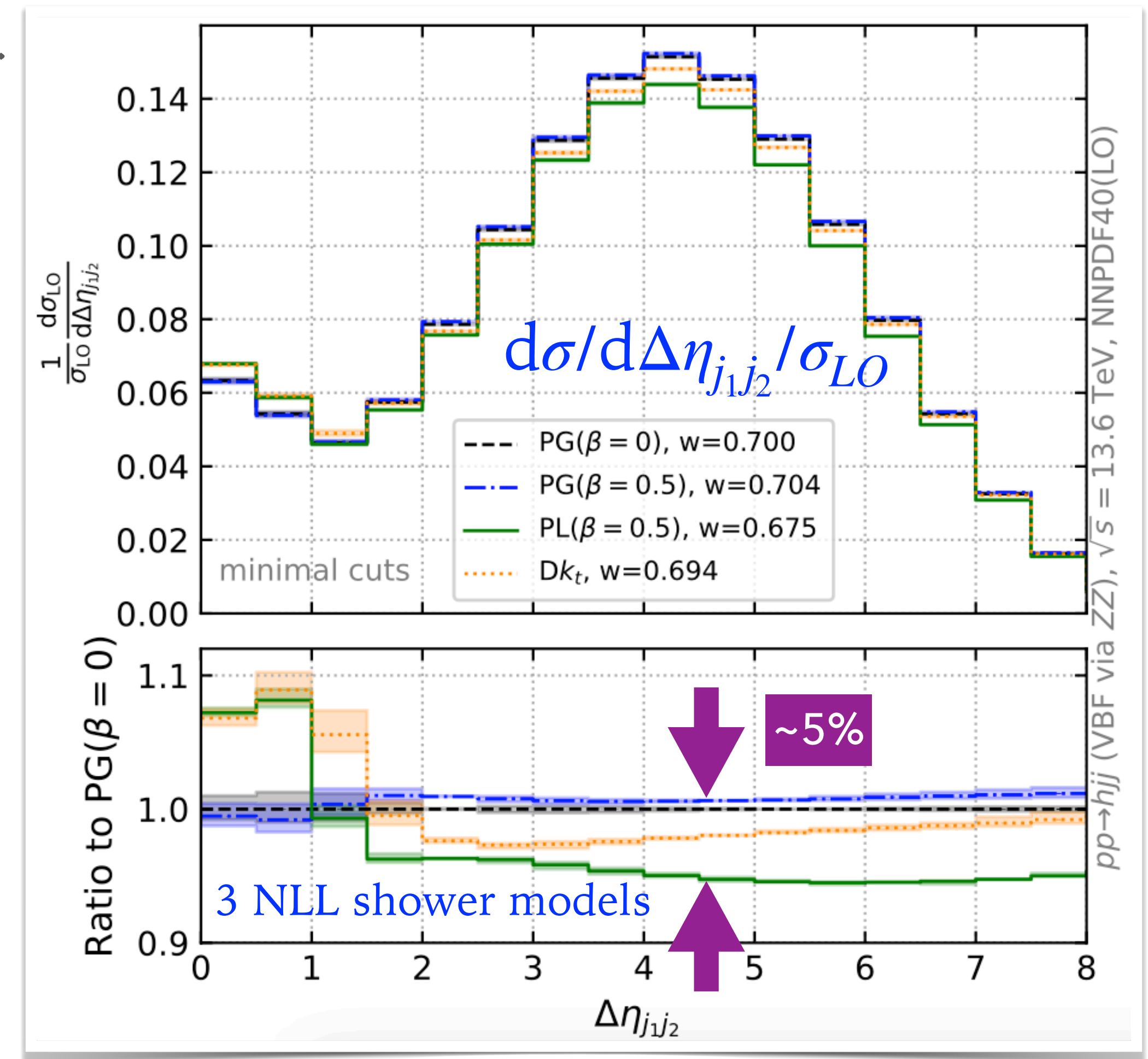
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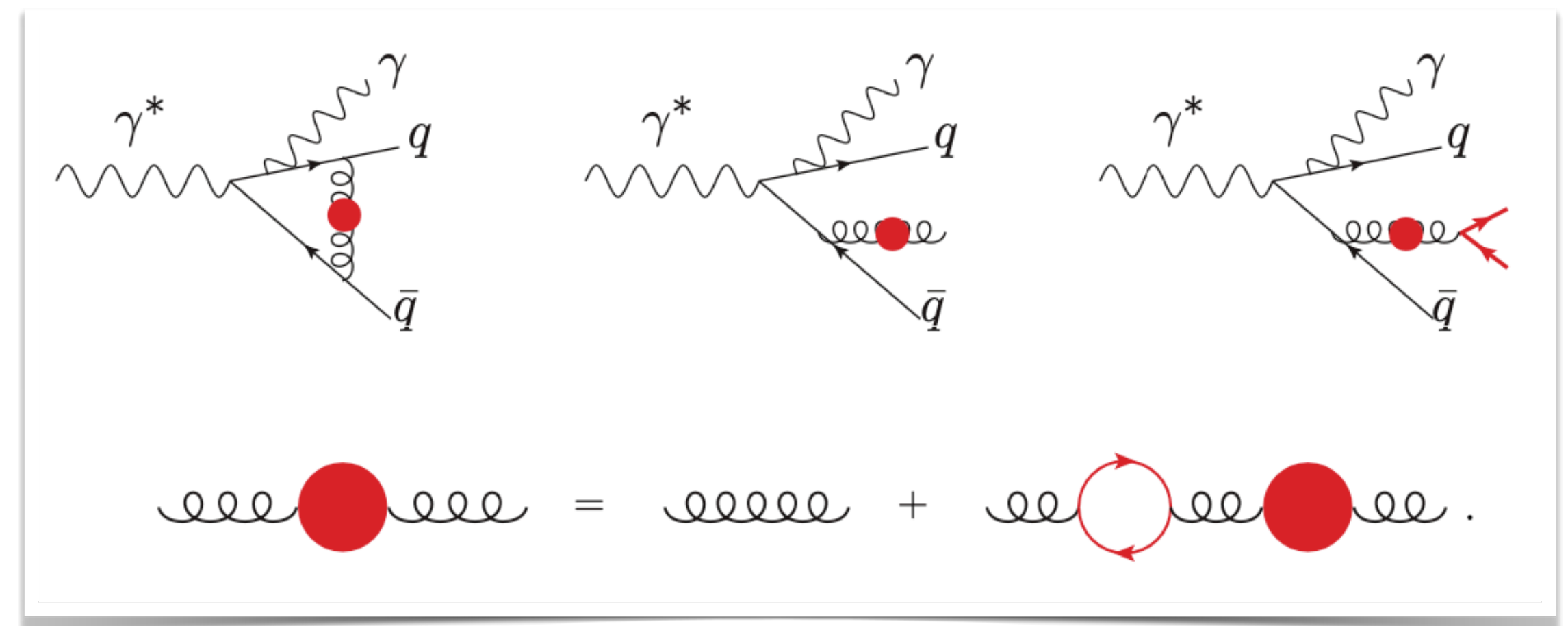
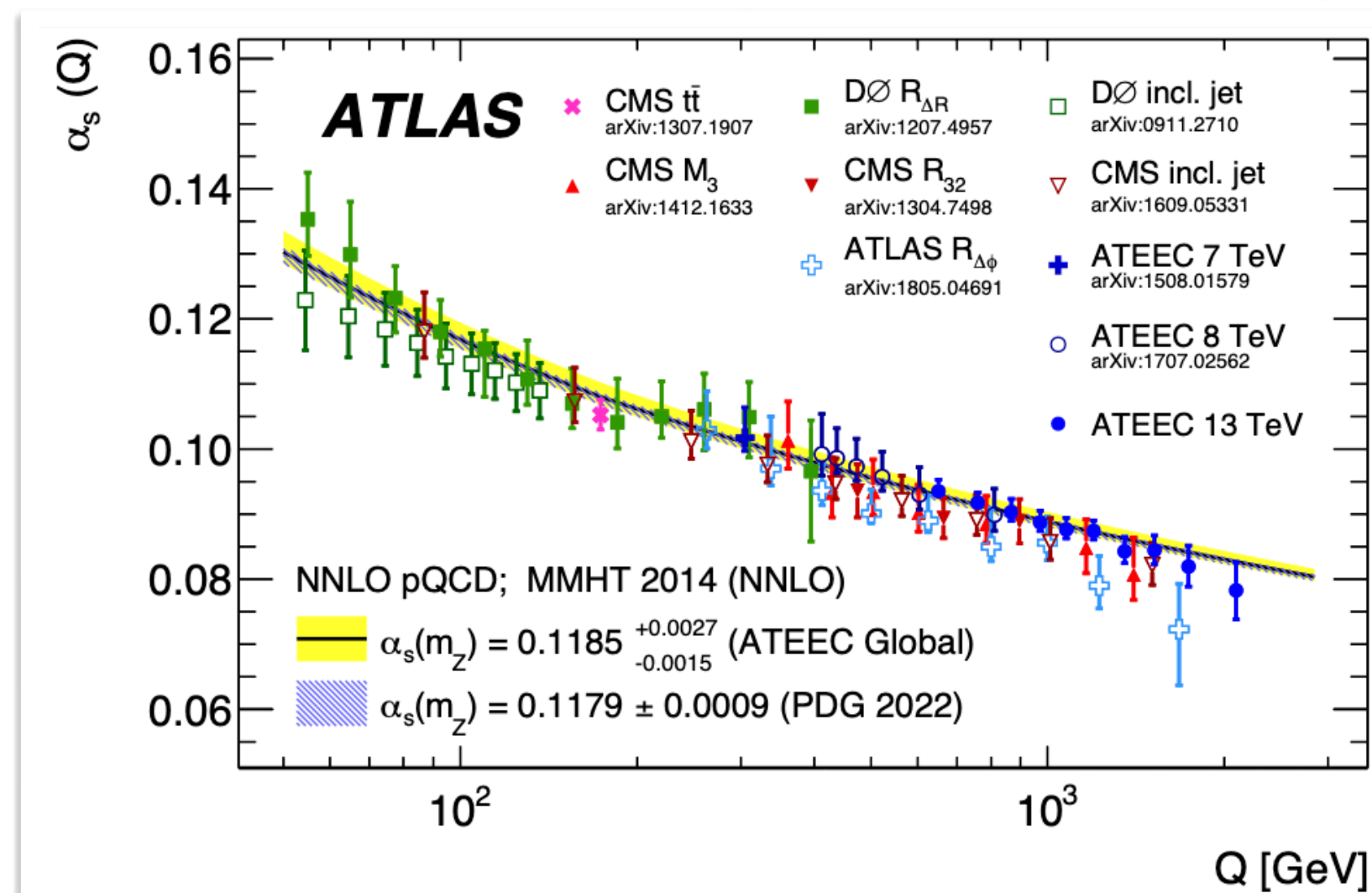
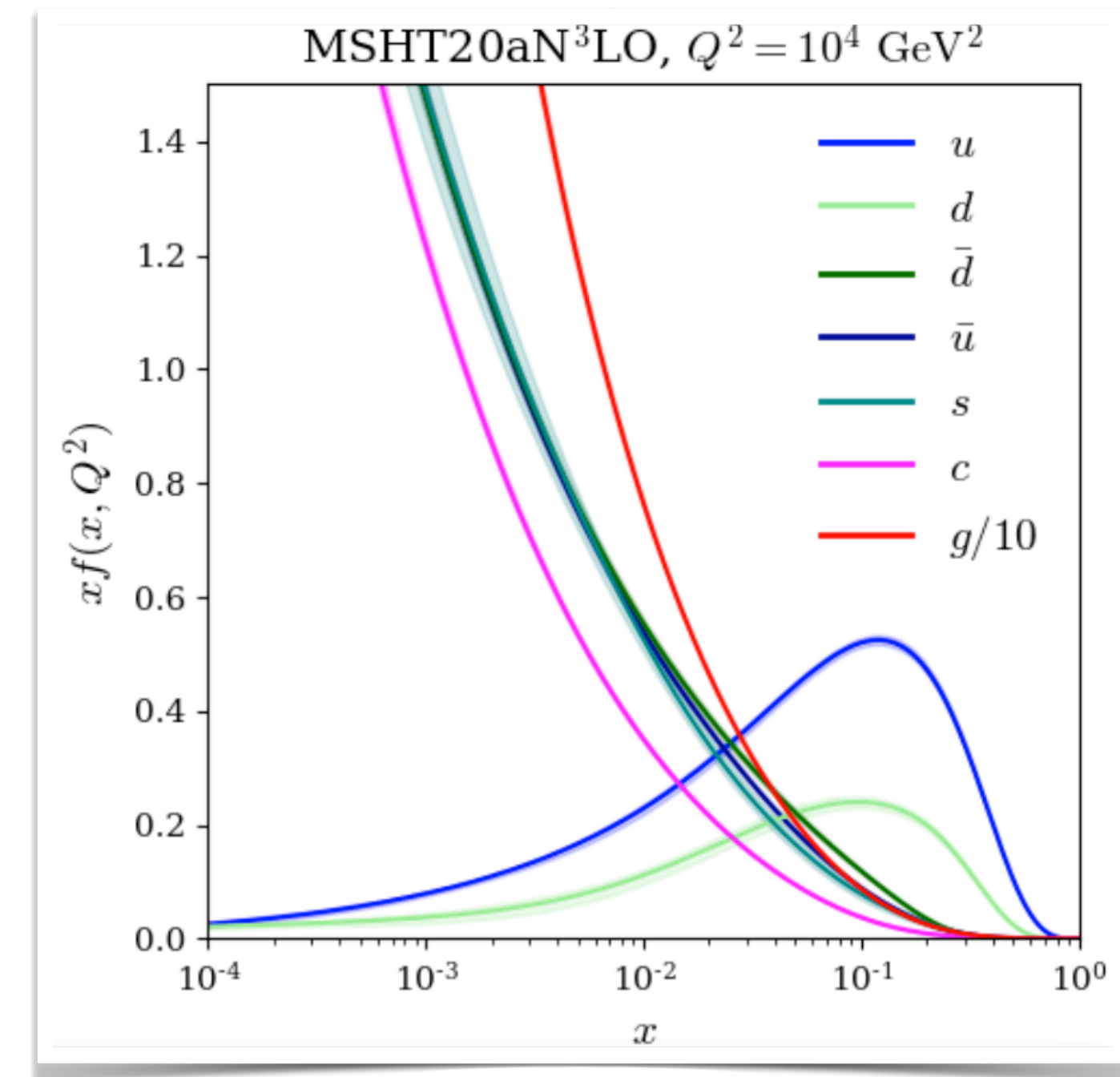
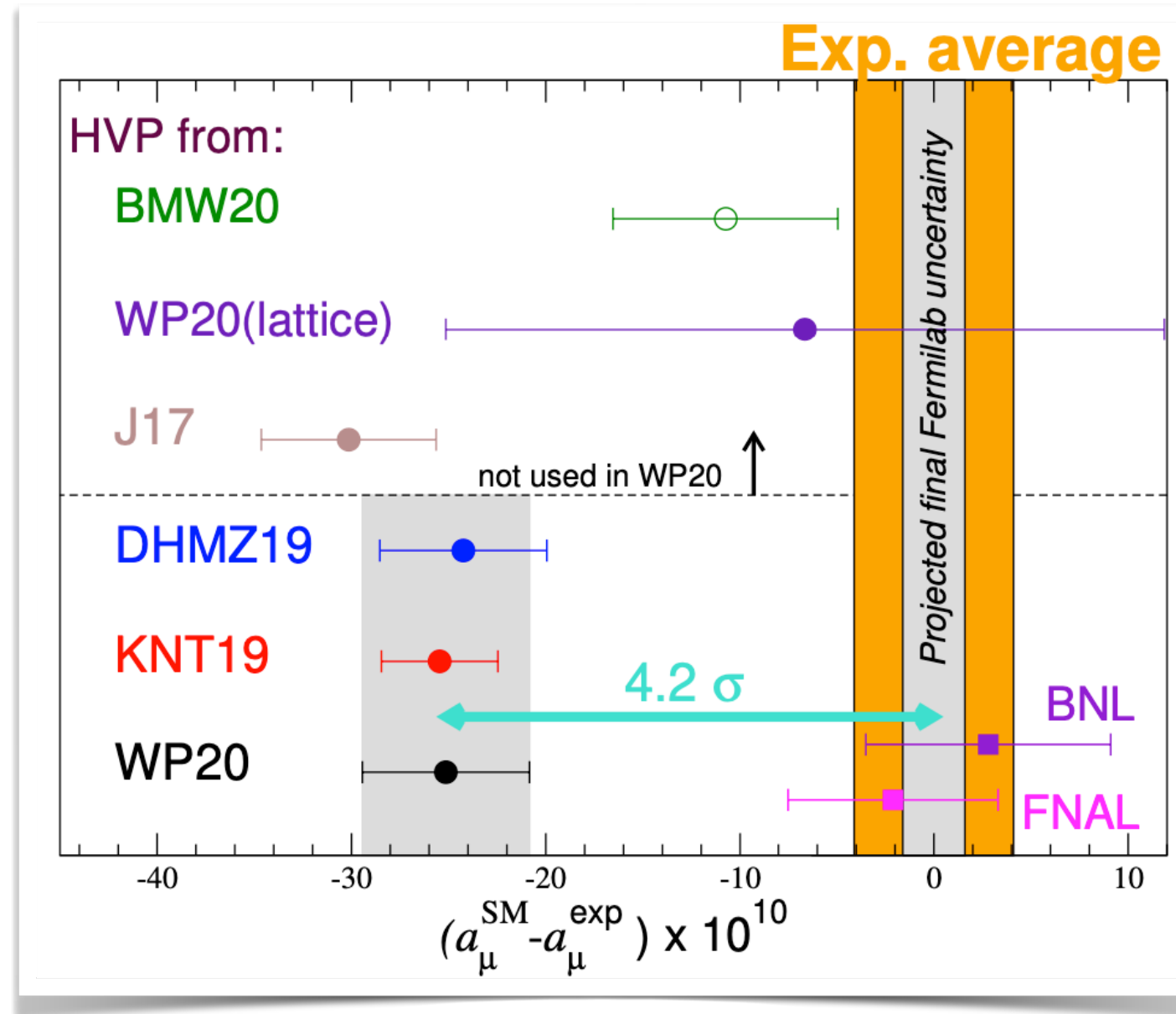
Alaric Collaboration 2208.06057, B. Assi, S. Höche 2307.00728

$$pp \rightarrow H \text{ (VBF)} + PS$$



M. van Beekveld, S. Ferrario Ravasio 2305.08645

Non-Perturbative QFT for Precision Predictions



Non-Perturbative QFT for Precision Predictions

a_μ^{HVP} Data driven vs. Lattice QCD

Data from SM White Paper Phys.Rept. 887 (2020)

SM contrib.	$a_\mu^{\text{contrib.}} \times 10^{10}$	
HVP-LO (e^+e^-)	693.1	± 4.0
HVP-NLO (e^+e^-)	-9.83	± 0.07
HVP-NNLO (e^+e^-)	1.24	± 0.01
HLbL-LO (pheno)	9.2	± 1.9
HLbL (lattice <i>usd</i>)	7.8	± 3.4
HLbL (pheno+lattice)	9.0	± 1.7
HLbL-NLO (pheno)	0.2	± 0.1
QED (5 loops)	11 658 471.8931	± 0.0104
EW (2 loops)	15.36	± 0.10
HVP (e^+e^- , LO + N(N)LO)	684.5	± 4.0
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SM Total	11 659 181.0	± 4.3

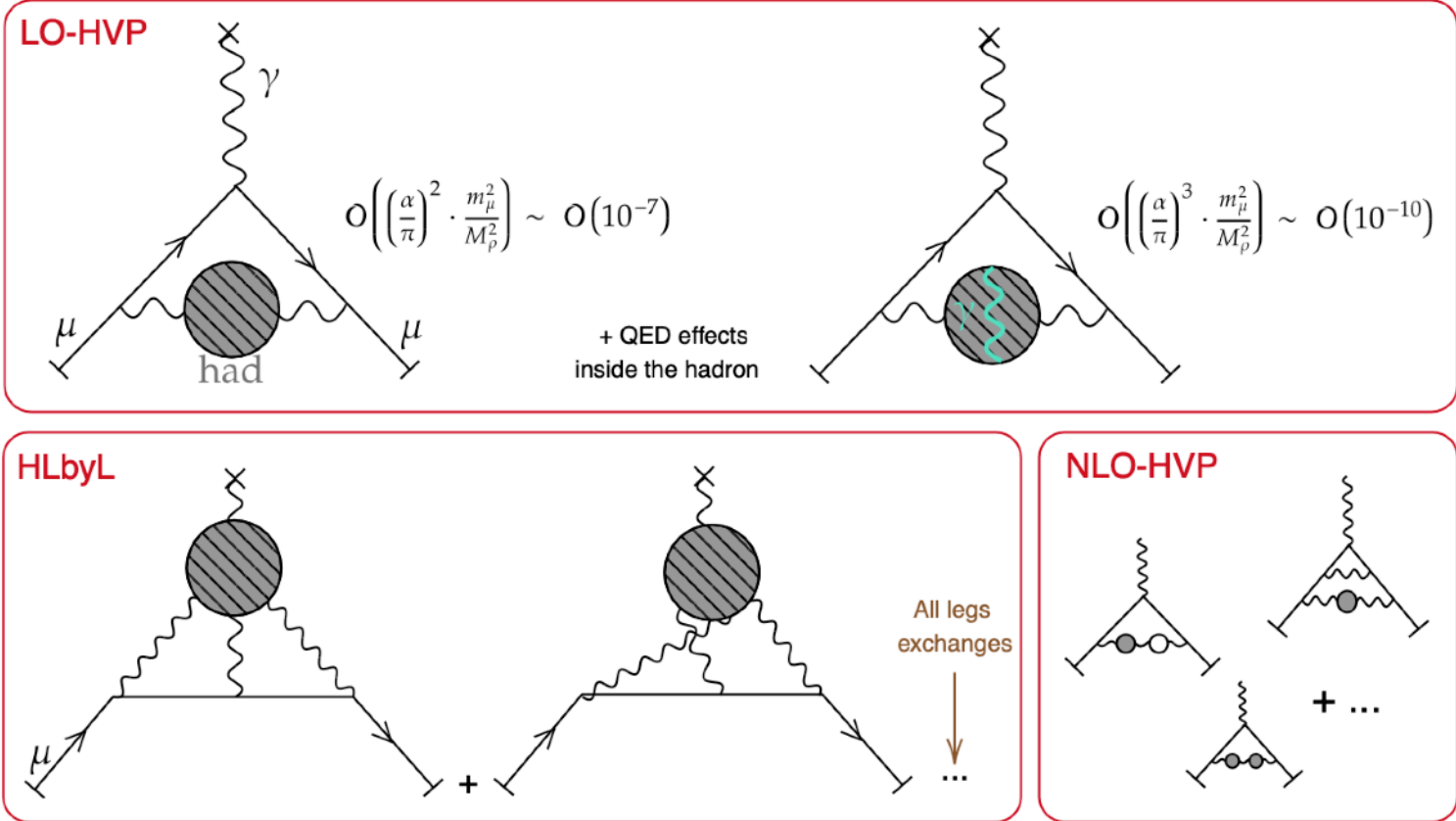


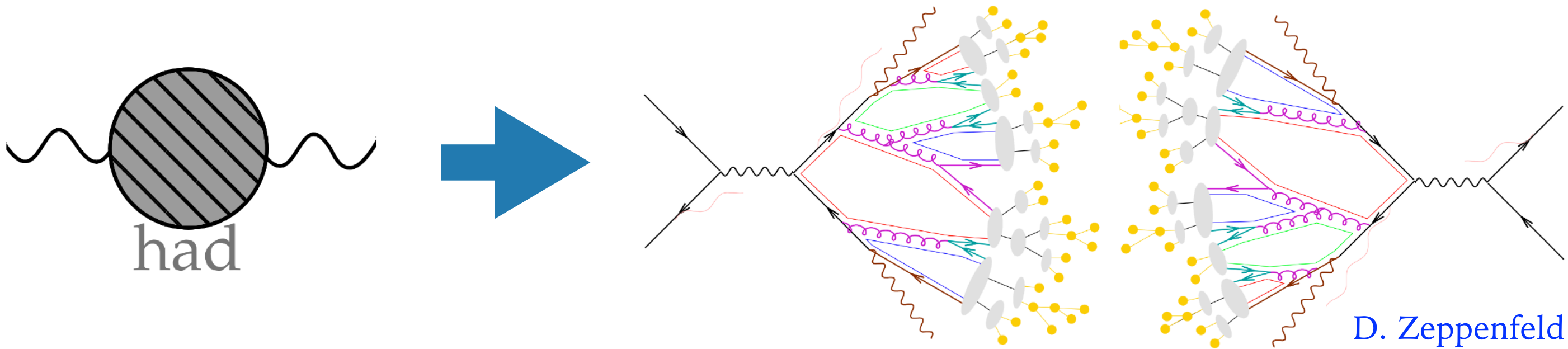
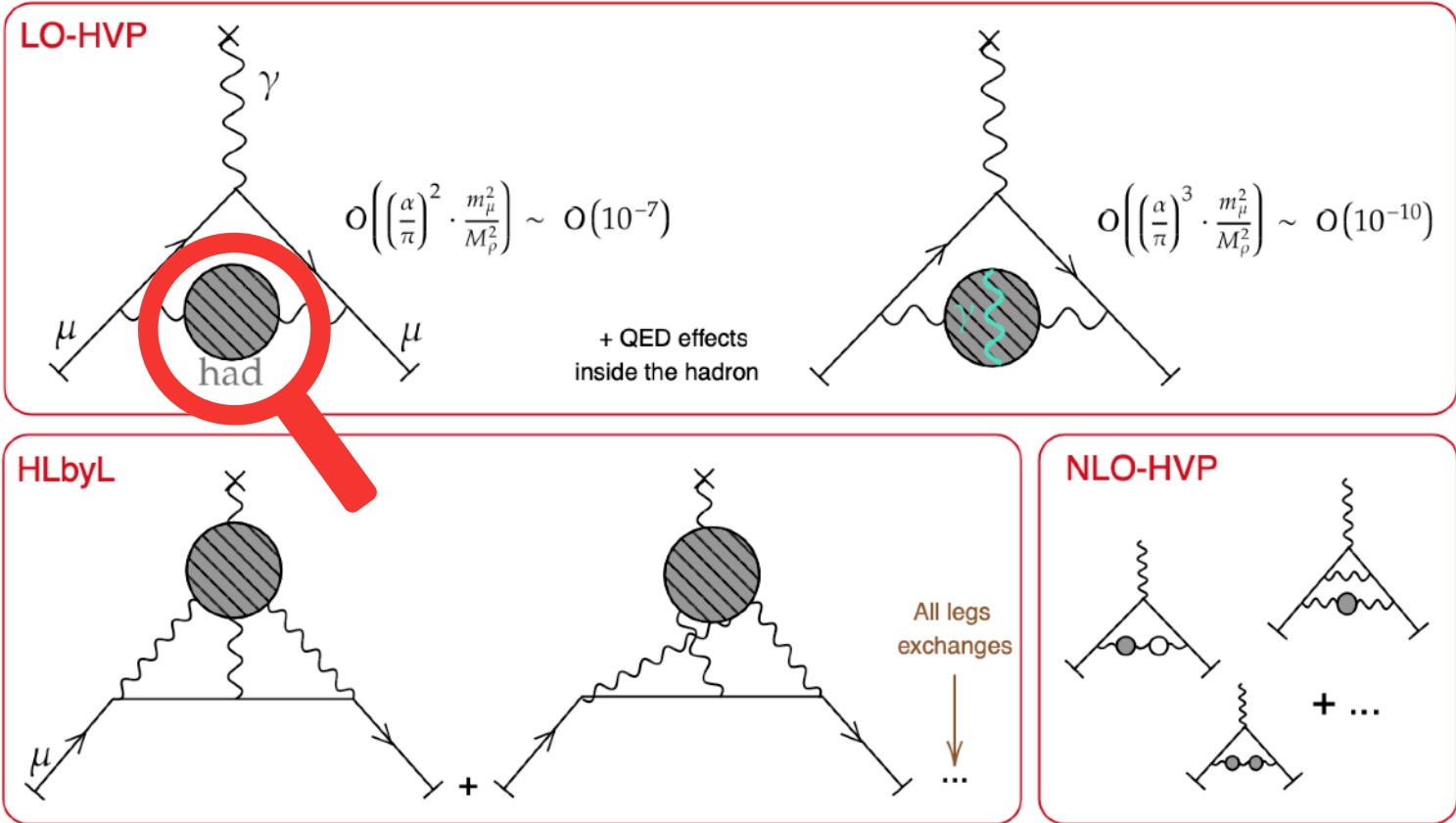
Table and diagram by L. Pareao at Zurich Workshop in June 2023

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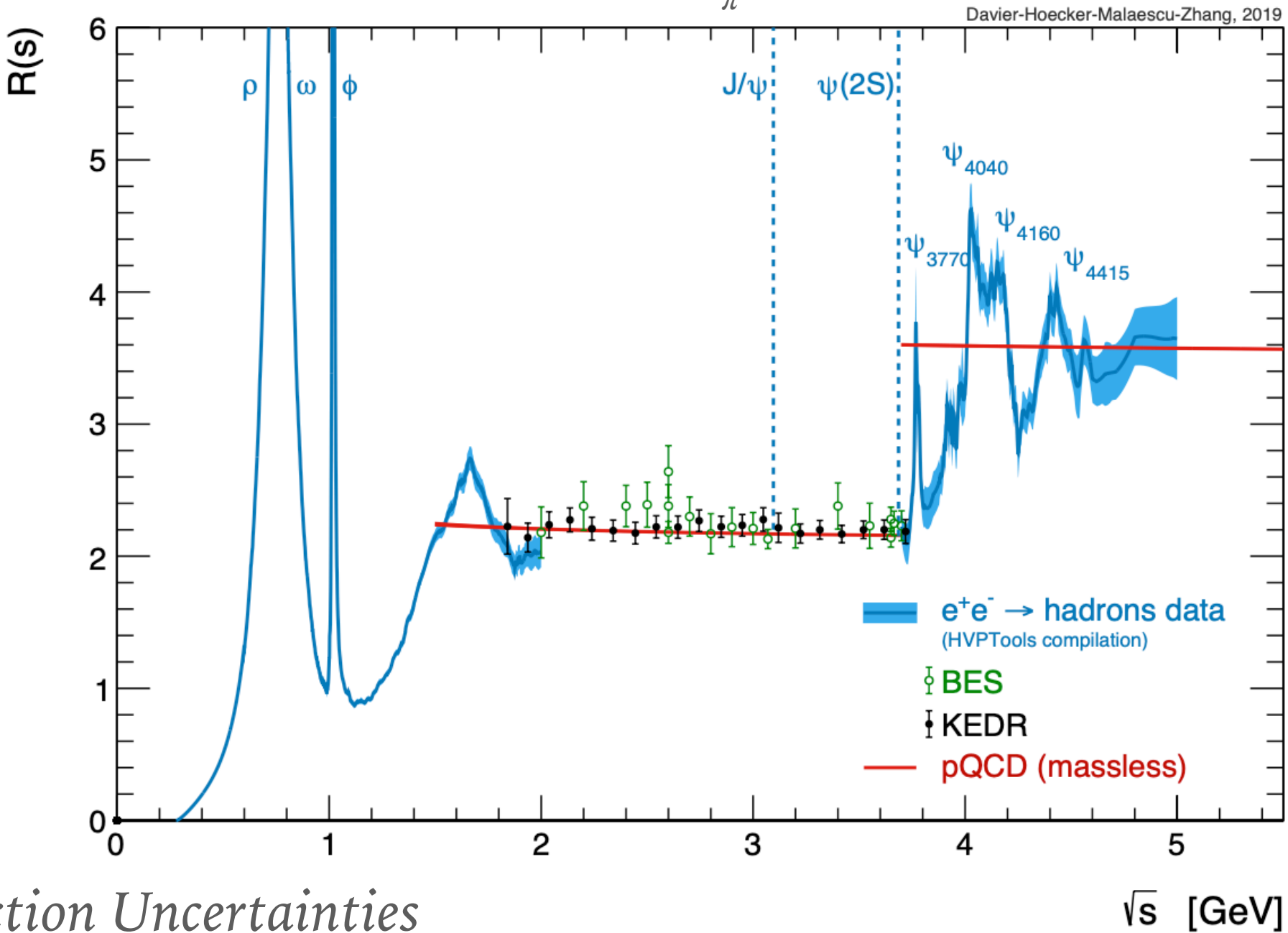
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- Perturbative QCD is not valid for $\Lambda = m_\mu \ll \Lambda_{QCD}$
- Use dispersive approach to include $e^+e^- \rightarrow$ Hadron data via R-ratio:

$$a_{\mu,DA}^{LO-HVP} = \frac{\alpha^2}{3\pi^3} \int_{m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$



M. Davier et. al. Eur.Phys.J.C 80 (2020) 5

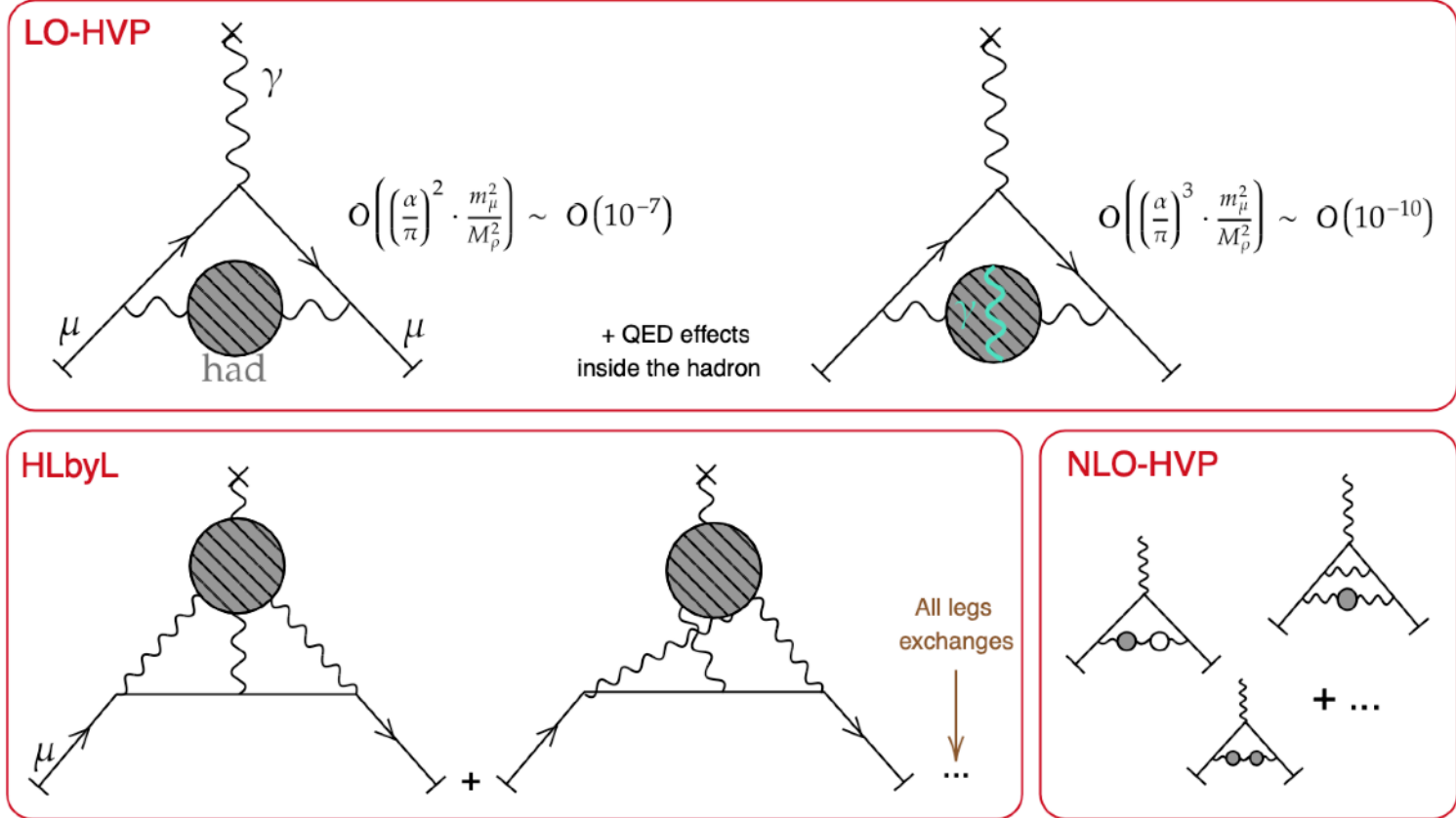
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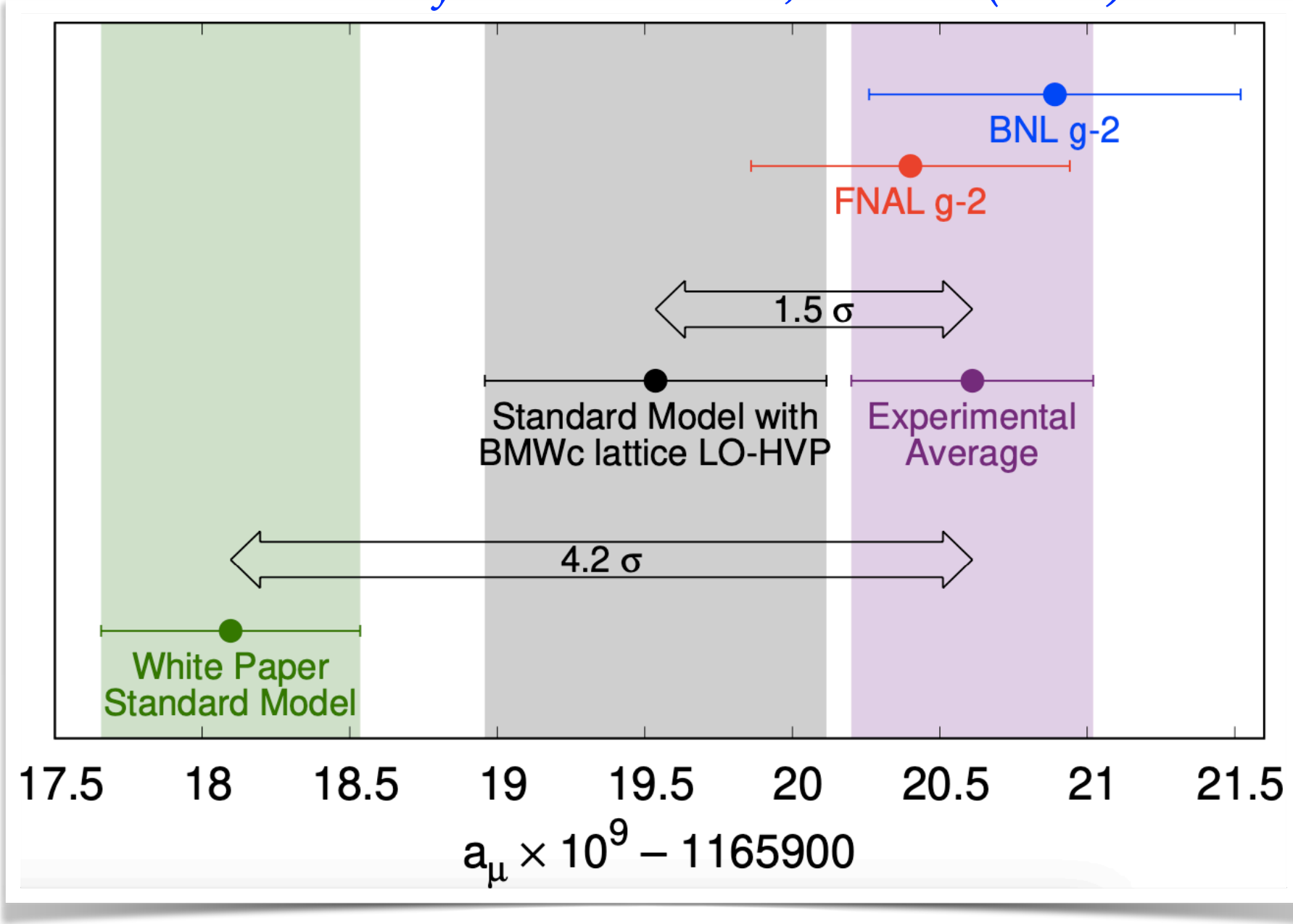
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FNAL Phys. Rev. Lett. 126, 141801 (2021)



L. Pareao at Zurich Workshop in June

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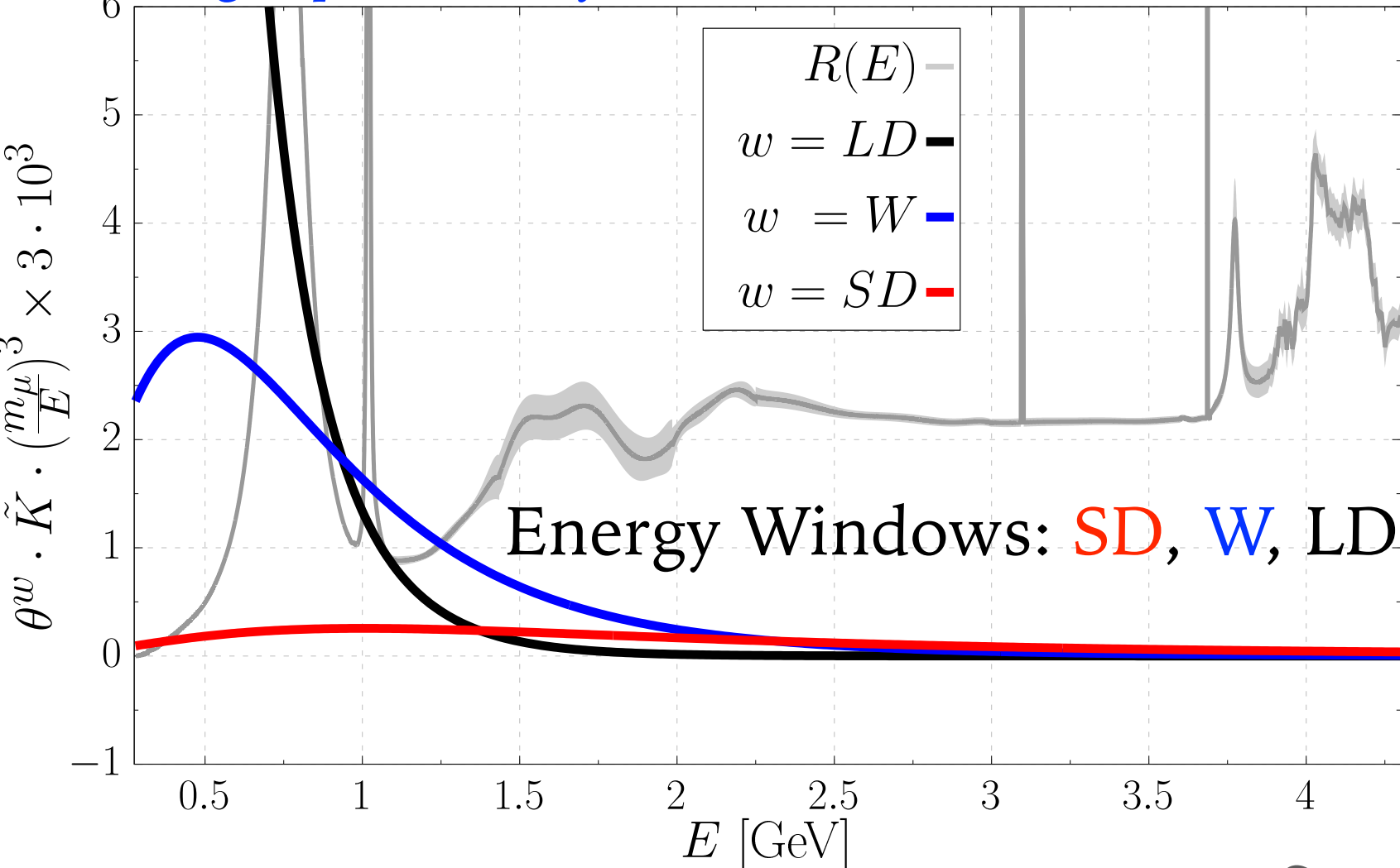
a_μ^{HVP} Data driven vs. Lattice QCD

$$a_{\mu,DA}^{LO-HVP} = \frac{\alpha^2}{3\pi^3} \int_{m_\pi^2}^\infty \frac{ds}{s} K(s) R(s) \qquad a_{\mu,LQCD}^{LO-HVP} = 2\alpha^2 \int_0^\infty t^2 dt K(m_\mu t) V(t)$$

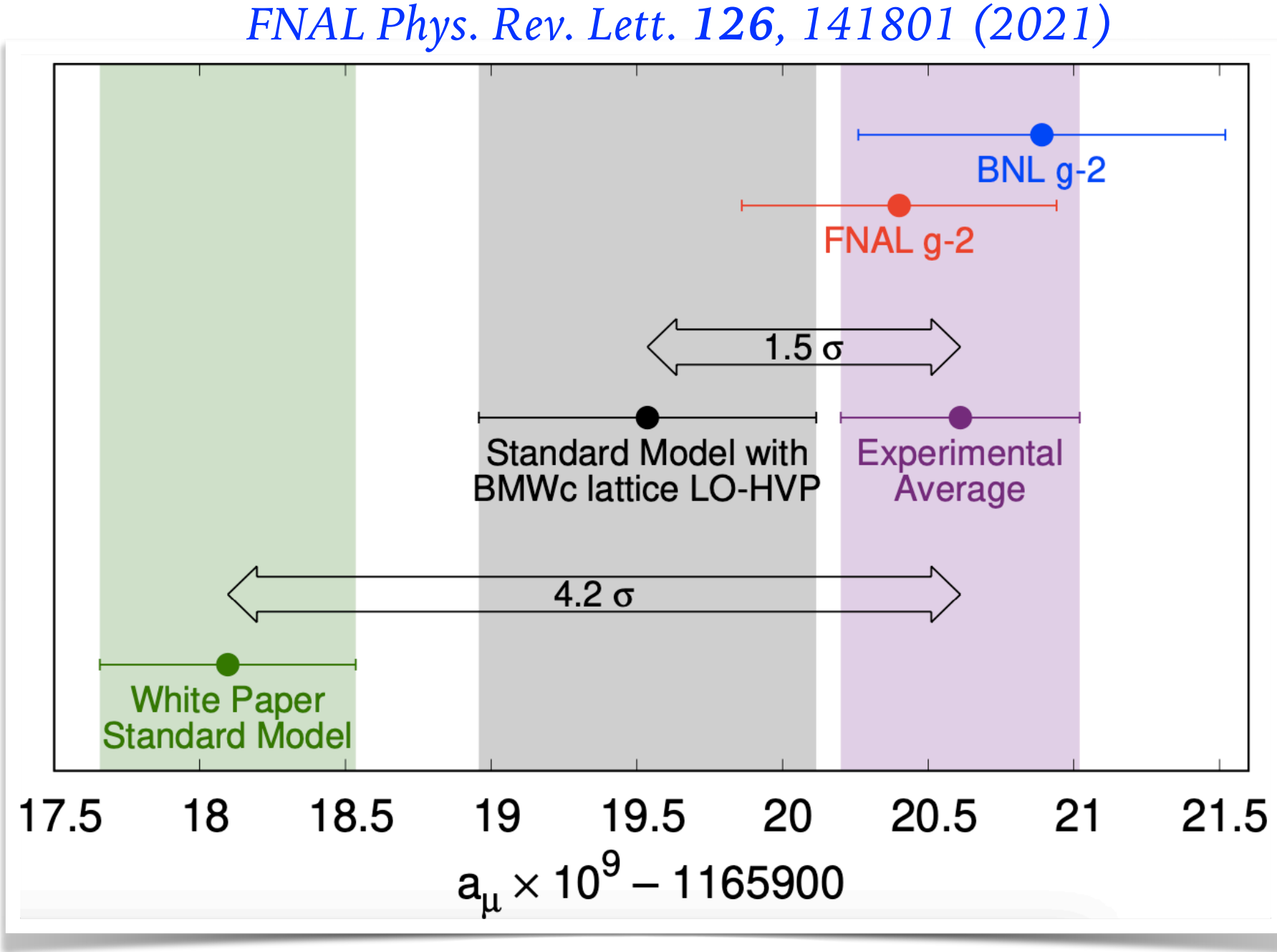
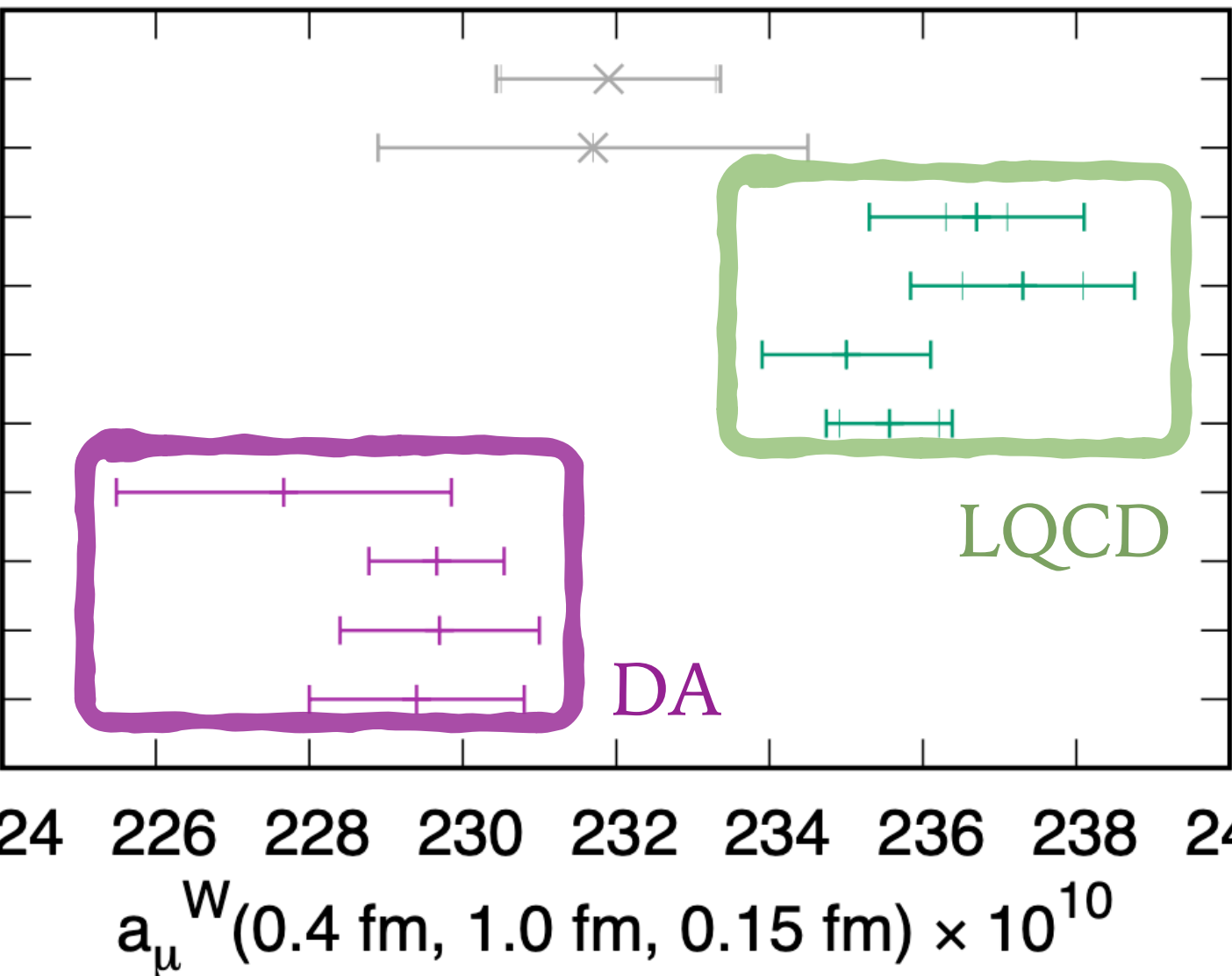
$$a_{\mu,LQCD}^{LO-HVP,\omega} = 2\alpha^2 \int_0^\infty t^2 dt K(m_\mu t) \Theta^\omega(t) V(t)$$

- Time \leftrightarrow Energy Window
 - $[0, t_0] \oplus [t_0, t_1] \oplus [t_1, +\infty]$ for SD, W, LD.
 - SD and W precisely predicted by Lattice QCD in continuum.
 - SD and W energy windows with precise e^+e^- EXP data.
 - a_μ^W (intermediate window) has 3.7σ tension for DA vs. LQCD

See also light quark result from FL-HPQCD-MILC collaboration, Phys.Rev.D 107 (2023) 11



- RBC/UKQCD 2018
- ETMC 2021
- BMW 2020
- Mainz 2022
- ETMC 2022
- RBC/UKQCD 2023
- RBC/UKQCD 2018/FJ
- Aubin et al. 2019/CL/KNT
- BMW 2020/KNT
- Colangelo et al. 2022



Parton Distributions and α_s

State-of-the-art Parton Distribution Functions

► Theory input

- Option A: solve proton wave function with Lattice QCD *Recent progress in D. Chakrabarti, P. Choudhary et. al. 2304.09908*

- Option B: collinear factorisation $f_a \rightarrow f_a(x, \mu)$ with p-QCD evolution of factorisation scale

$$\frac{d}{d\ln\mu^2} \begin{pmatrix} f_q \\ f_g \end{pmatrix} = \begin{pmatrix} P_{q \leftarrow q} & P_{q \leftarrow g} \\ P_{g \leftarrow q} & P_{g \leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} f_q \\ f_g \end{pmatrix}$$

DGLAP evolution with

$$P_{a \leftarrow b} = \underbrace{\frac{\alpha_s}{\pi} P_{a \leftarrow b}^{(0)}}_{1970's} + \underbrace{\frac{\alpha_s^2}{\pi^2} P_{a \leftarrow b}^{(1)}}_{1980} + \underbrace{\frac{\alpha_s^3}{\pi^3} P_{a \leftarrow b}^{(2)}}_{2004} + \dots$$

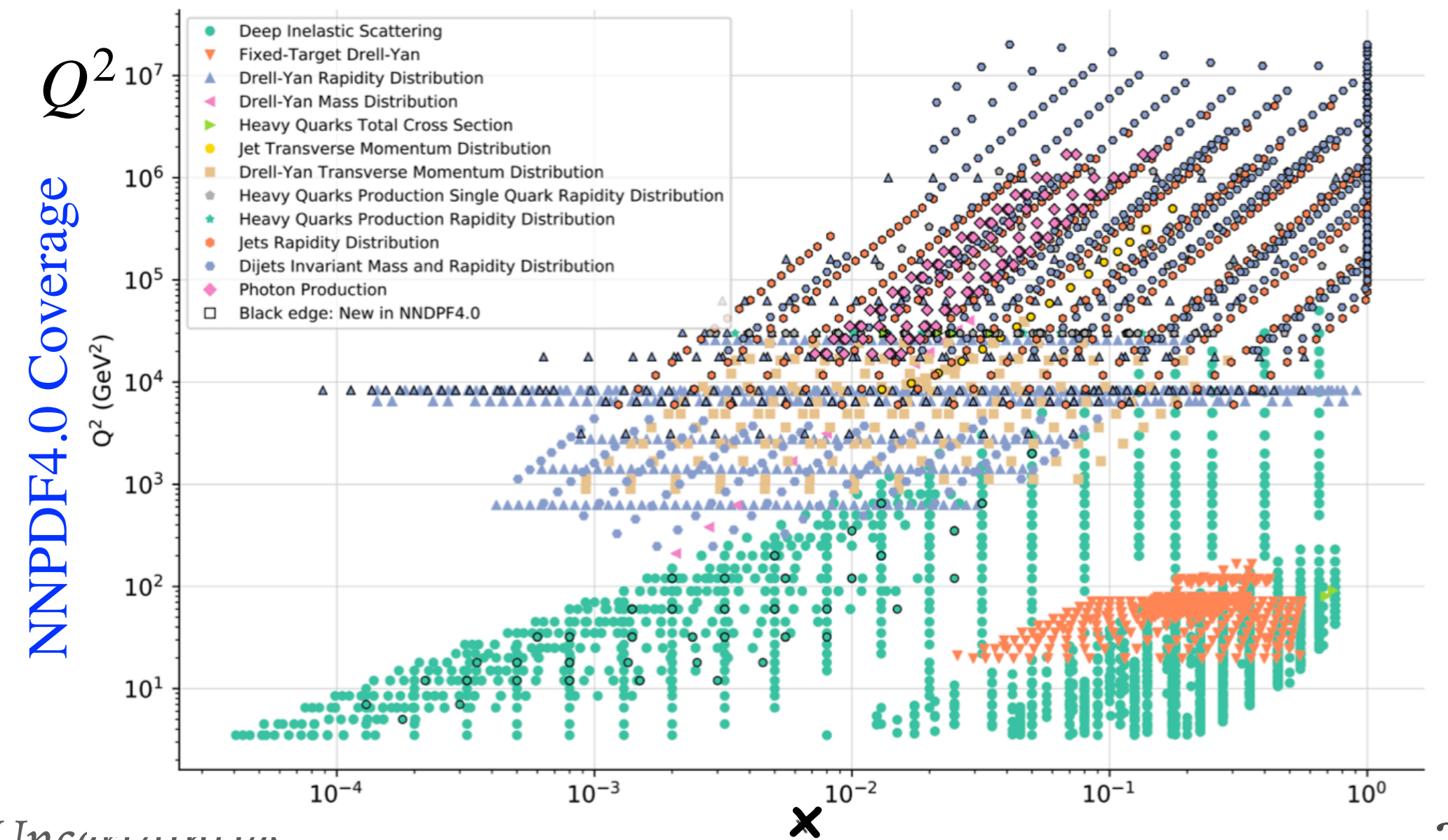
$$\gamma_{q \leftarrow q}^{(3)}(N) = - \int_0^1 dx x^{N-1} P_{q \leftarrow q}^{(3)}(x) \quad G. Falcioni, F. Herzog et. al. Phys.Lett.B 842 (2023)$$

$$\gamma_{q \leftarrow g}^{(3)}(N) = - \int_0^1 dx x^{N-1} P_{q \leftarrow g}^{(3)}(x) \quad G. Falcioni, F. Herzog, S. Moch, A. Vogt 2307.04158$$

For $N = 2, 4, \dots, 20$

► Experiment input

- All past and current measurements of DIS, DY, jets etc. provide fitting targets of $f_a(x, Q)$
- Differential and total cross sections provide sensitivity in different regions of $x \in [0, 1]$
- Various technology for fitting: functional form, neural network, fast evaluation grids etc.

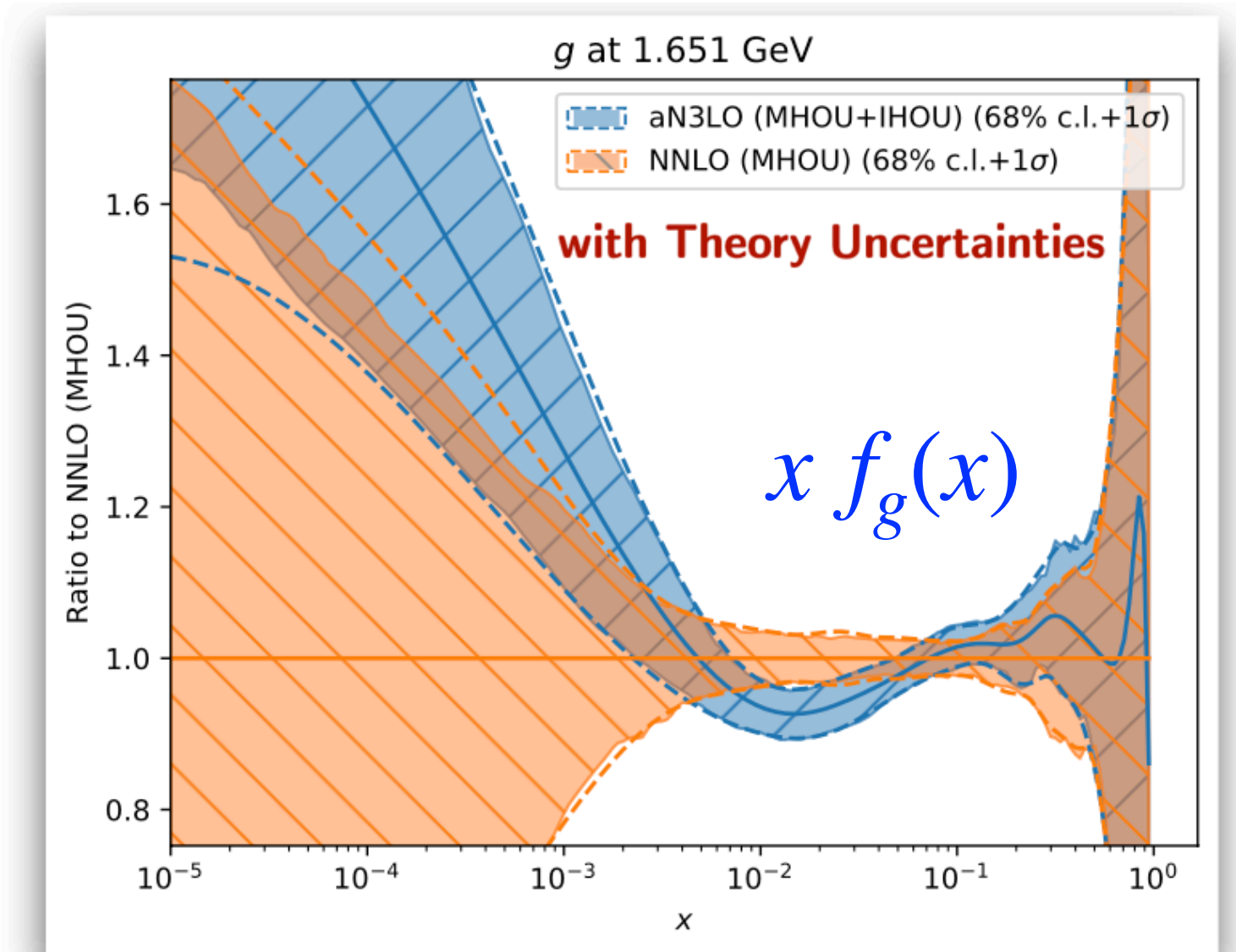
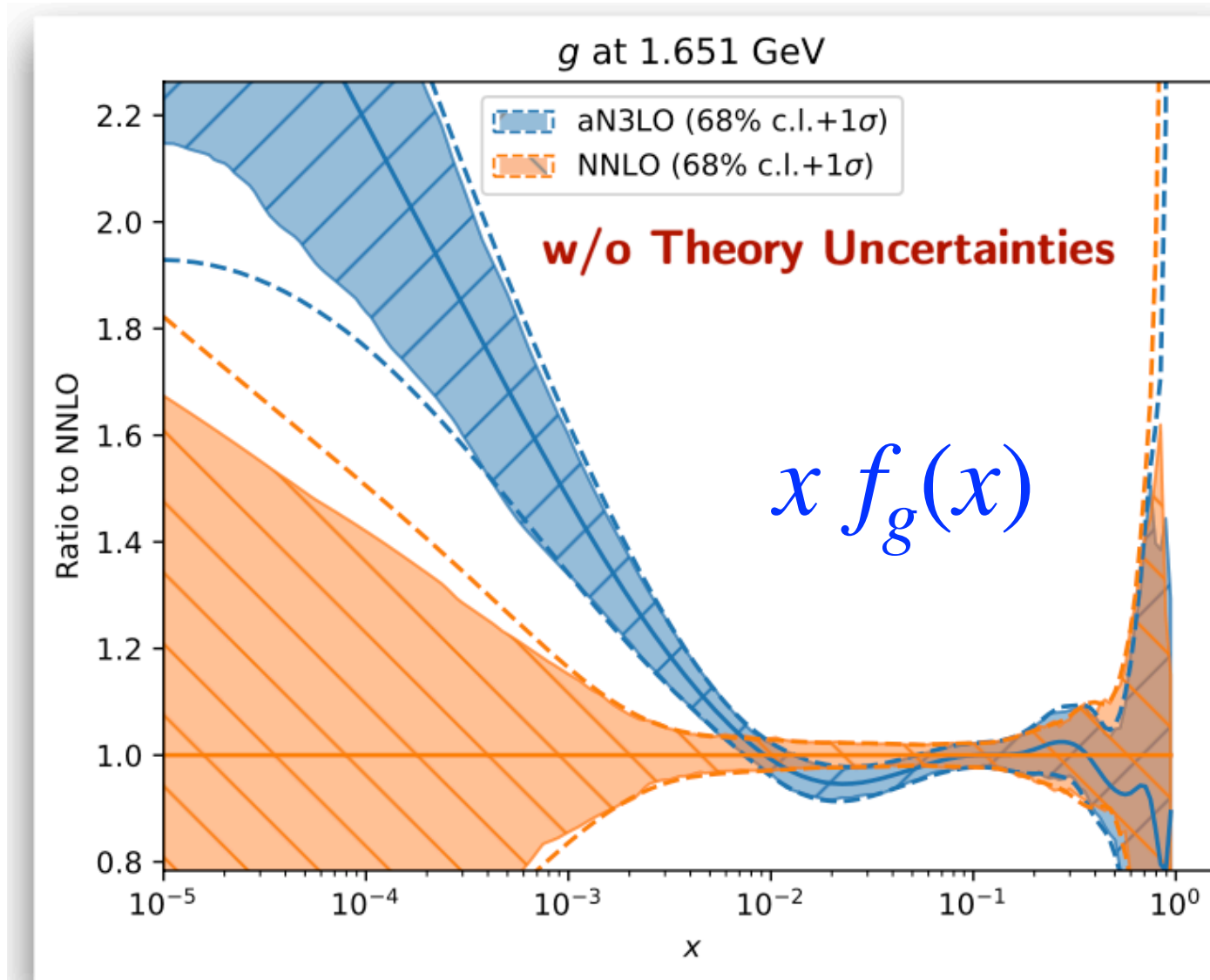
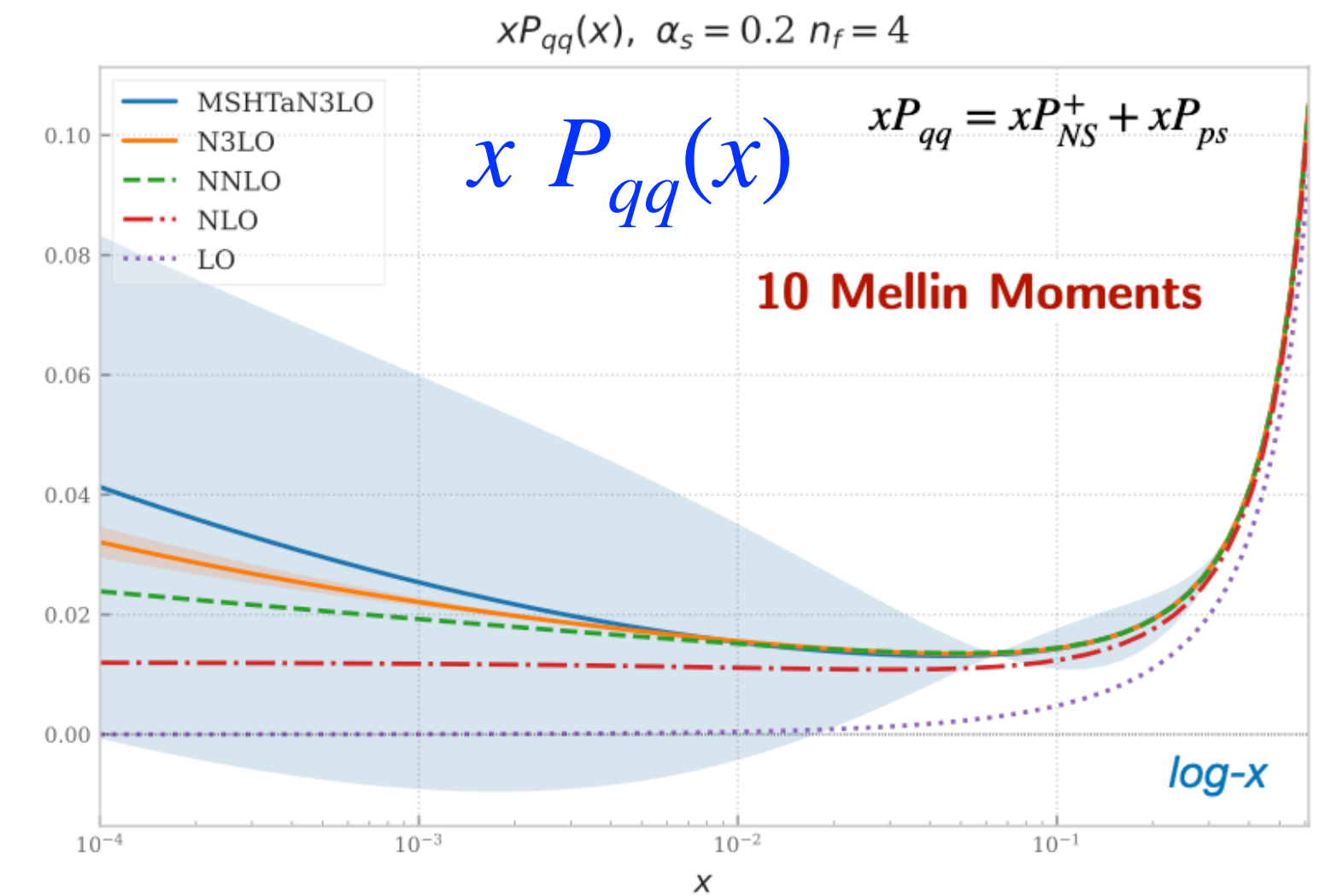
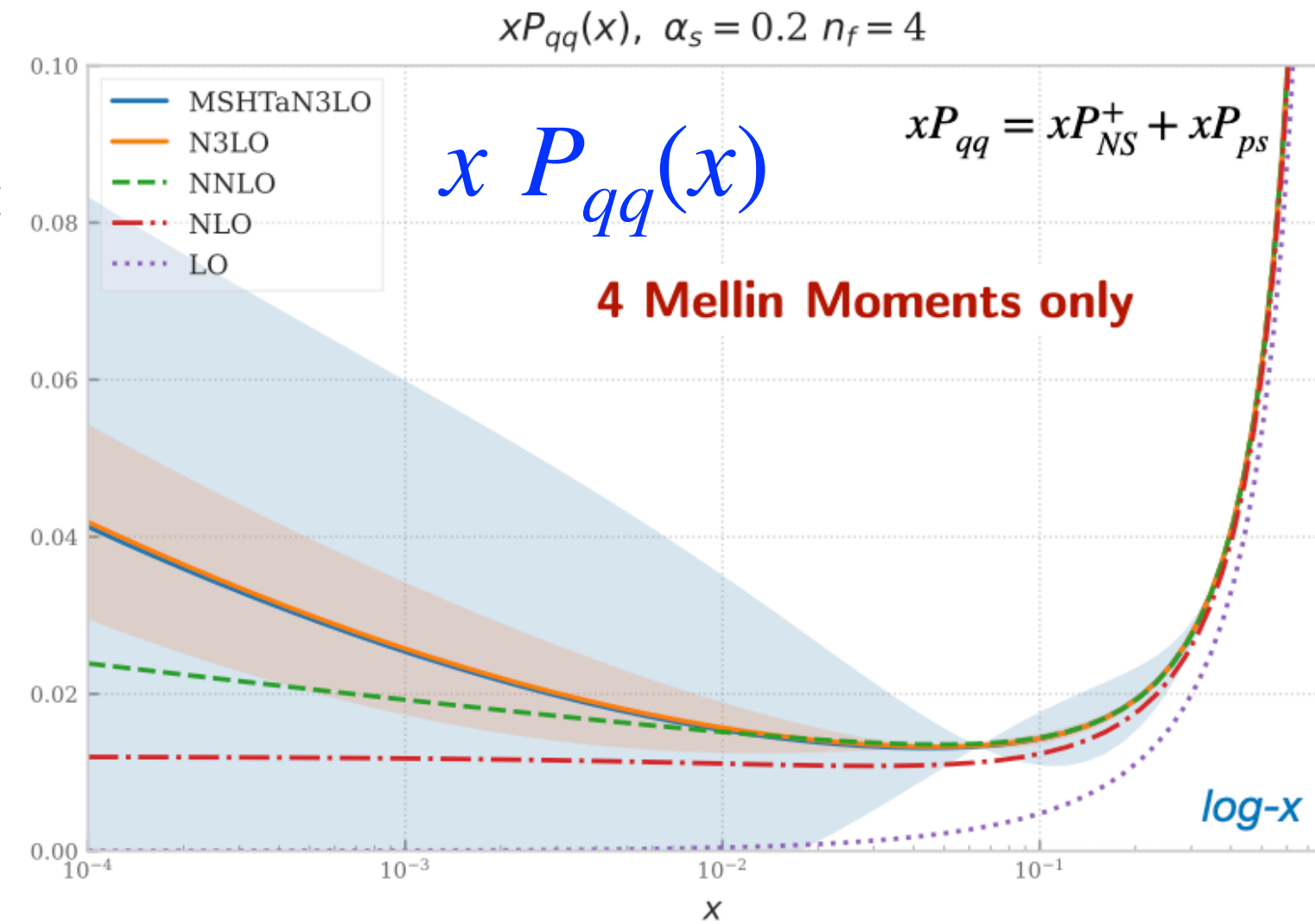


Parton Distributions and α_s

State-of-the-art Parton Distribution Functions

- Approximated N3LO PDF available:
MSHT20aN3LO *Eur.Phys.J.C* 83 (2023) 4
NNPDFa3LO *NNPDF preliminary*
- More precise 4-loop splitting functions affect small x region.
- Large correction at aN3LO at small x region outside 68% c.l. region.
- Missing Higher Order Uncertainty (MHOU) not included in standard NNLO PDF.
- Crucial to consider MHOU and IHOU to understand consistency between NNLO and N3LO PDF.

G. Magni (NNPDF) @ Les Houches 23



Parton Distributions and α_s

The running strong coupling

- Both non-perturbative and perturbative α_s determination depend on the beta-function.
- More and more precision predictions and measurements across 10^3 magnitude.

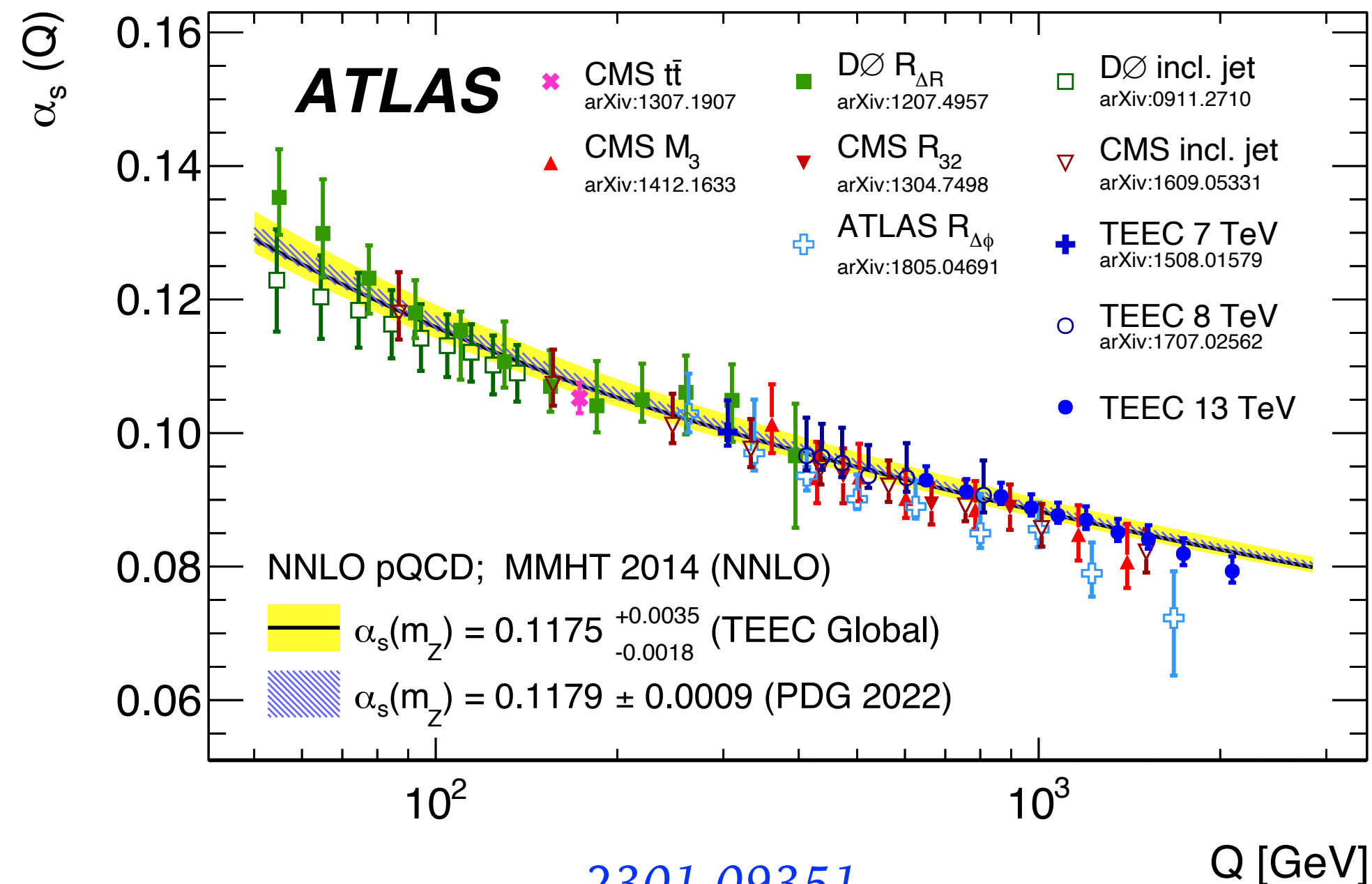
$$Q^2 \frac{d\alpha_s}{dQ^2} = \beta(\alpha_s) = -\alpha_s^2 \left(\underset{1973}{b_0} + \underset{1979}{b_1\alpha_s} + \underset{1993}{b_2\alpha_s^2} + \underset{1997}{b_3\alpha_s^3} + \underset{2017}{b_4\alpha_s^4} + \cdots \right)$$

Parton Distributions and α_s

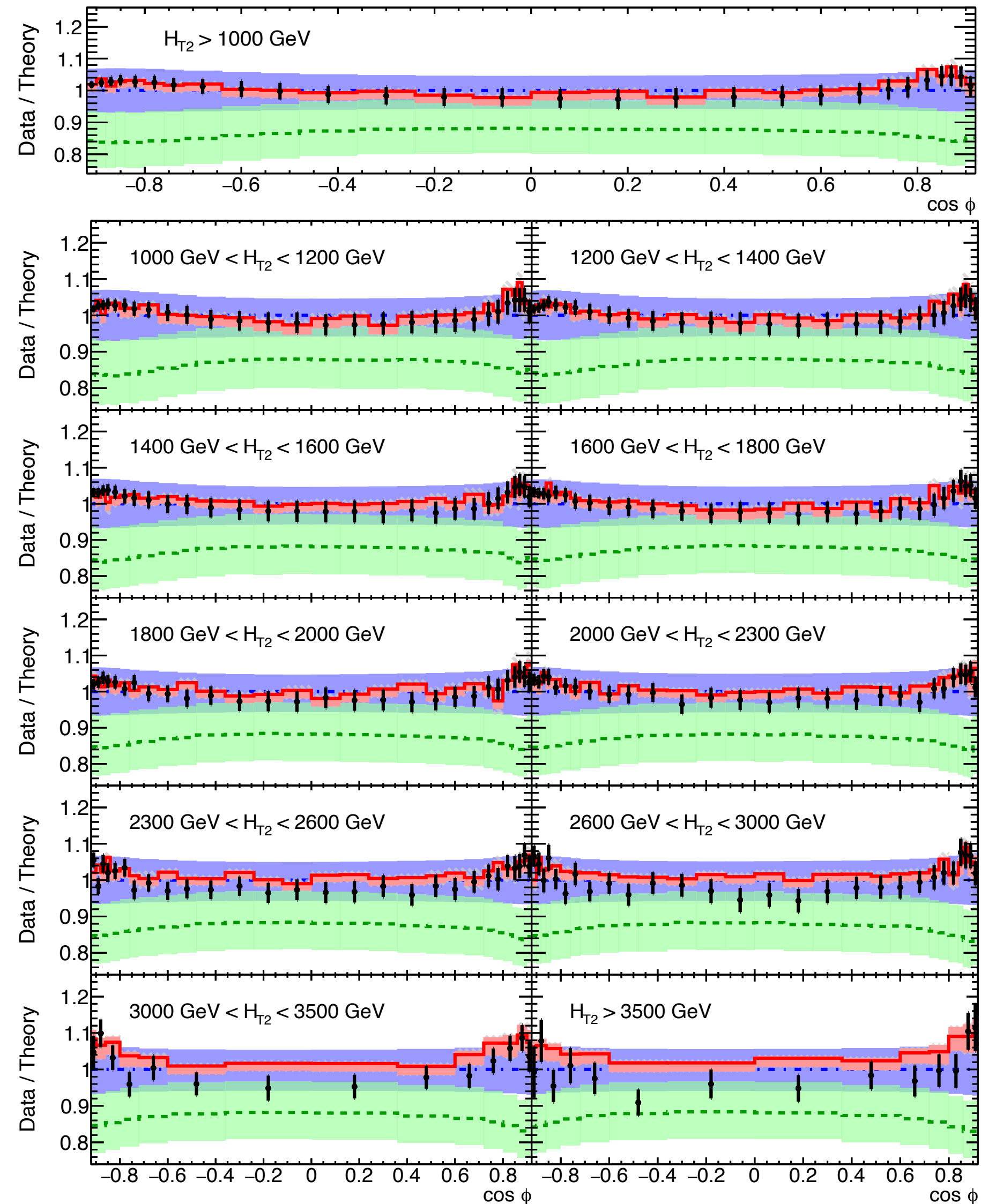
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$$\text{TEEC: } \frac{1}{\sigma} \frac{d\Sigma}{d\cos\phi} \equiv \frac{1}{N} \sum_{A=1}^N \sum_{ij} \frac{E_{Ti}^A E_{Tj}^A}{\left(\sum_k E_{Tk}^A\right)^2} \delta(\cos\phi - \cos\varphi_{ij})$$



2301.09351



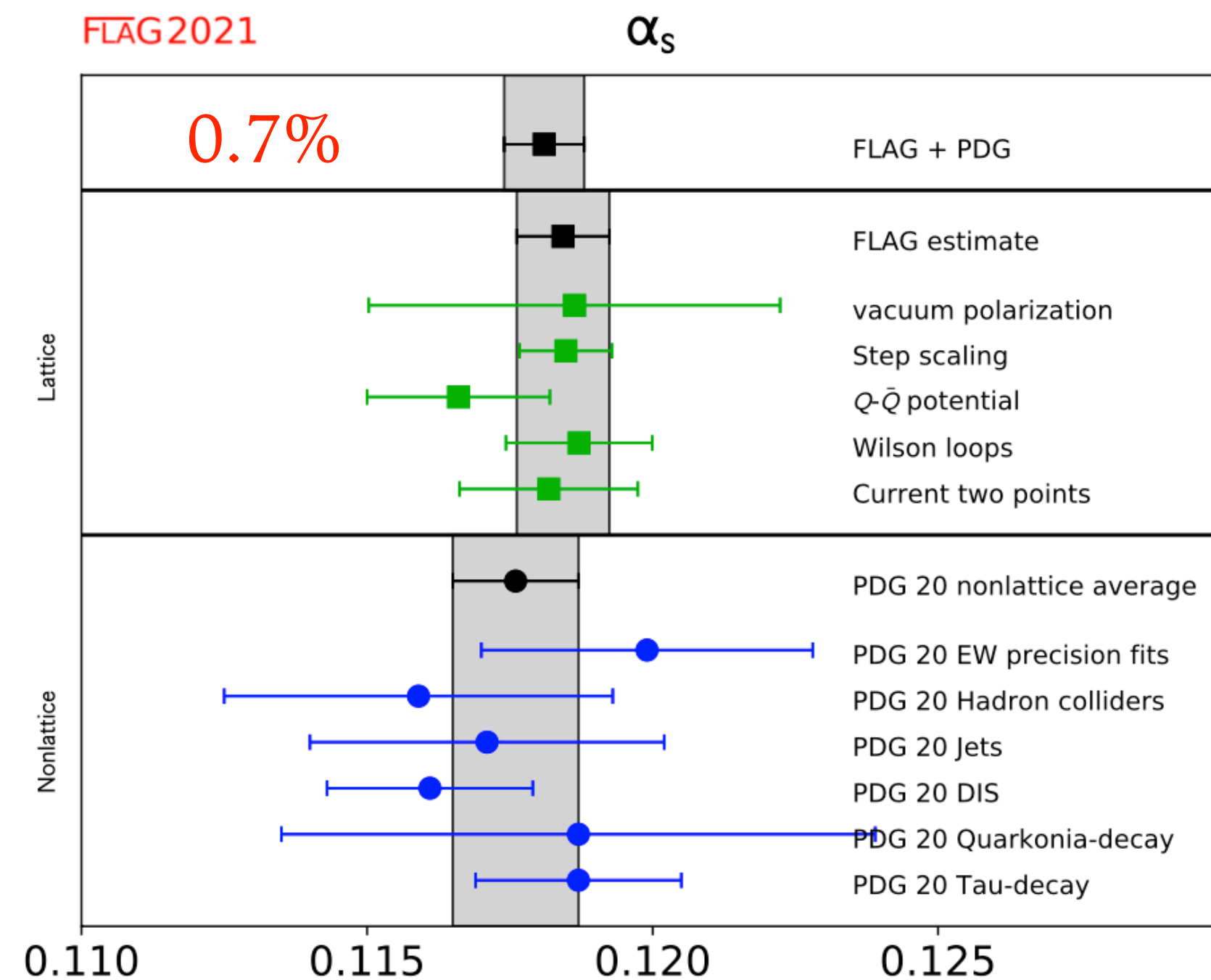
↓

*M. Alvarez, J. Cantero,
M. Czakon, J. Llorente,
A. Mitov, R. Poncelet
JHEP 03 (2023) 129*

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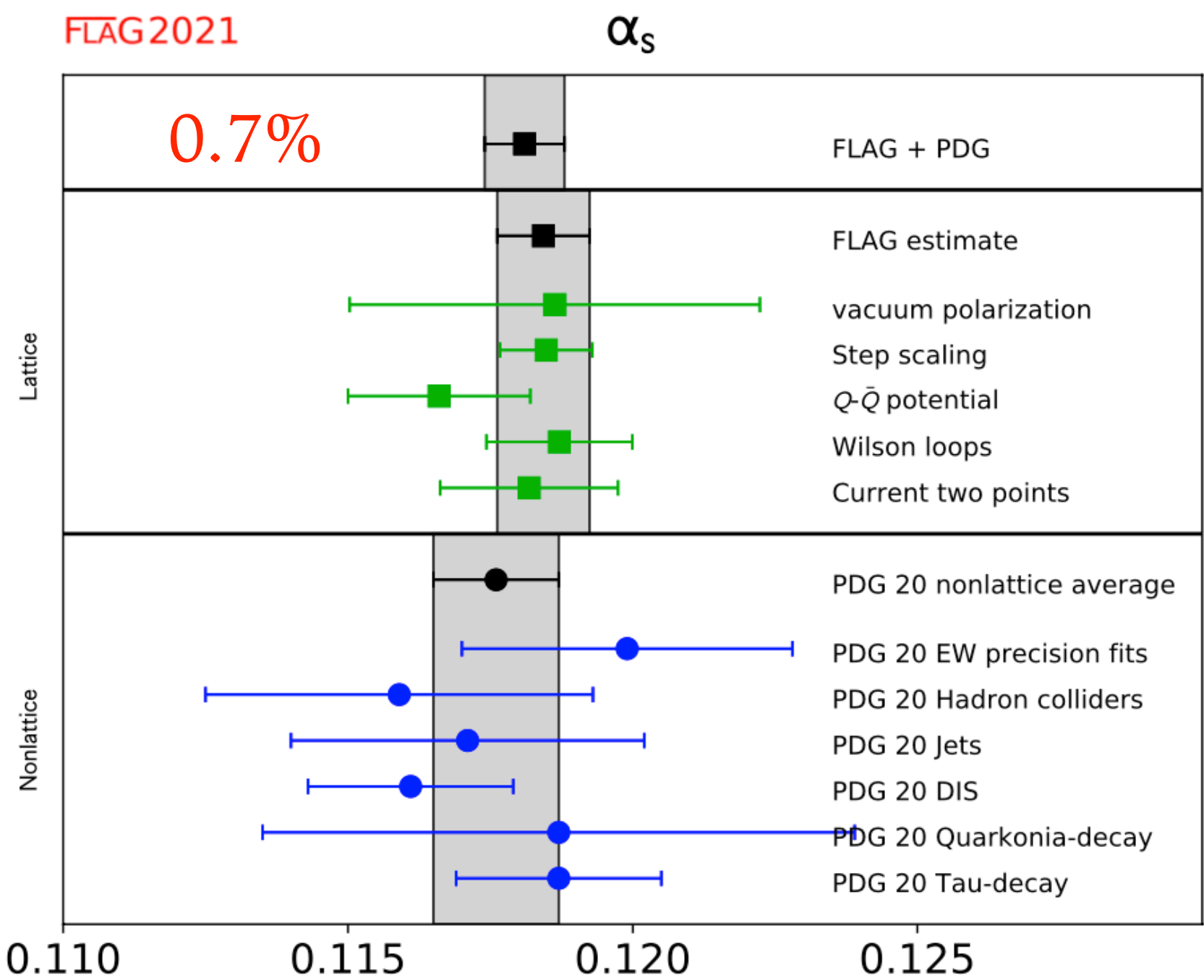
Flavour Lattice Averaging Group *Eur.Phys.J.C* 82 (2022) 10

Parton Distributions and α_s

The running strong coupling

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Error budget of ATLAS Z p_T 8 TeV		
Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088



ATLAS ATEEC

CMS jets

W, Z inclusive

$t\bar{t}$ inclusive

τ decays

$Q\bar{Q}$ bound states

PDF fits

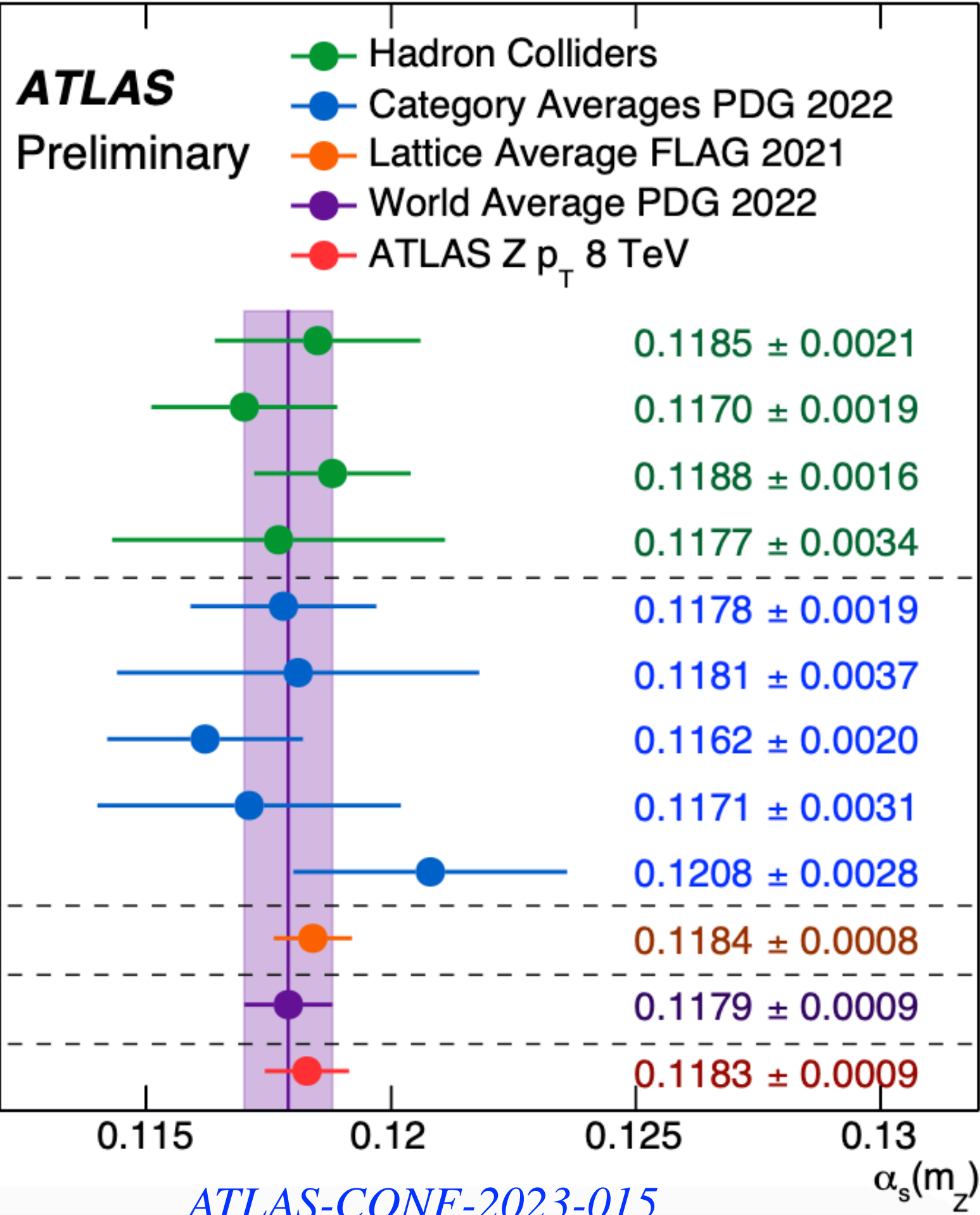
e^+e^- jets and shapes

Electroweak fit

Lattice

World average

ATLAS Z p_T 8 TeV



Parton Distributions and α_s

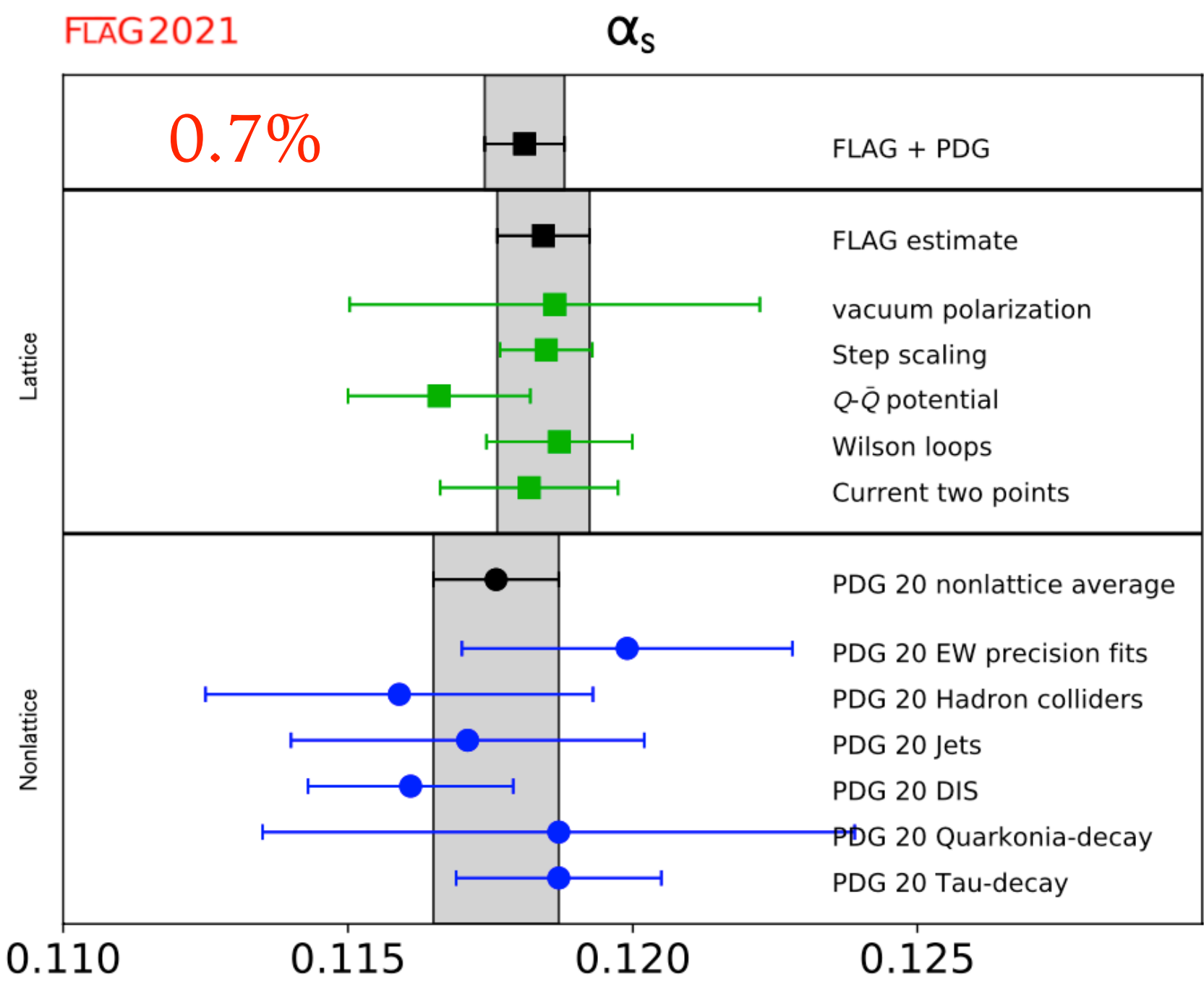
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Total	+0.00084	-0.00088

Missing: MHOU from aN3LOPDF; Dominant matching error; Systematic slicing error in DYTurbo and MCFM (double slicing);
→ Optimistic uncertainty estimation



ATLAS ATEEC

CMS jets

W, Z inclusive

$t\bar{t}$ inclusive

τ decays

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PDF fits

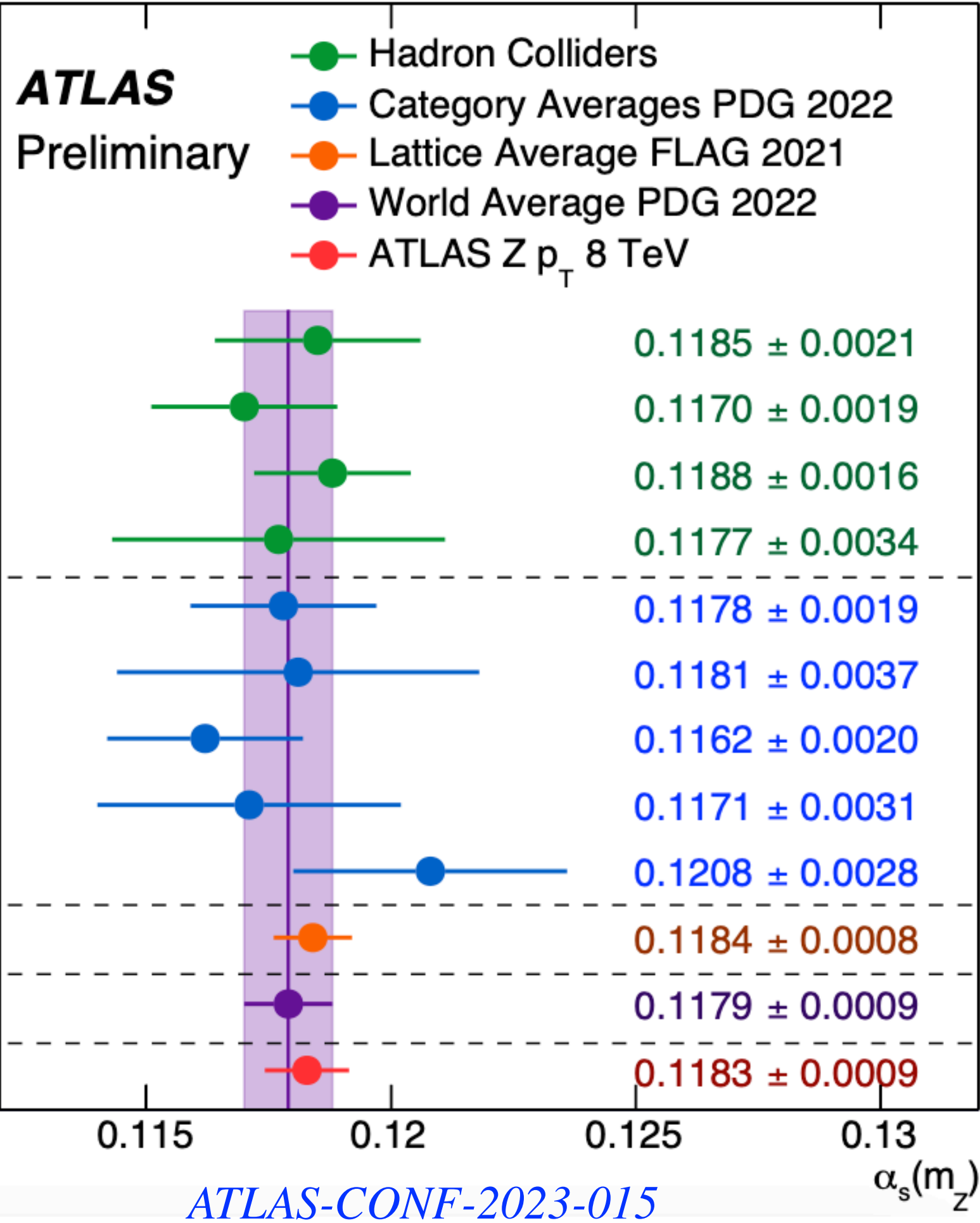
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Parton Distributions and α_s

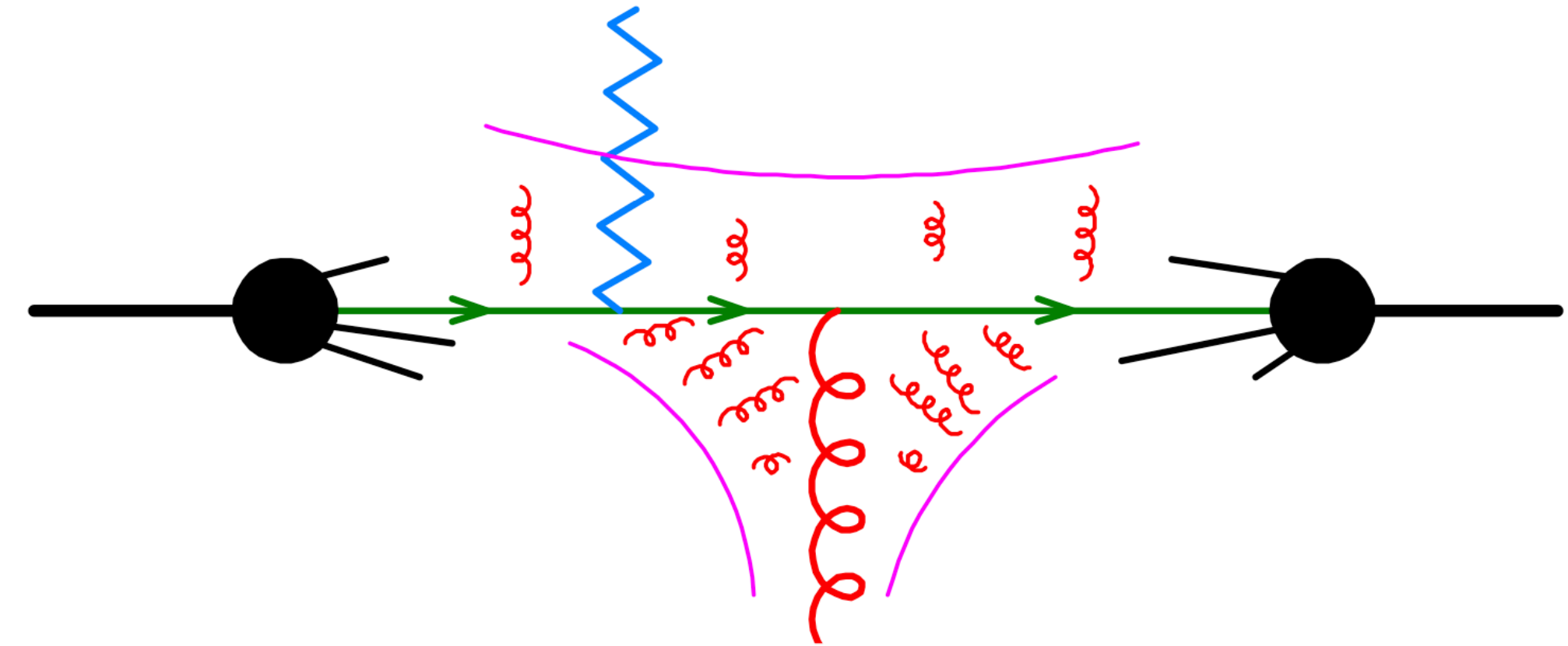
The running strong coupling

- Both non-perturbative and perturbative α_s determination depend on the beta-function.
- More and more precision predictions and measurements across 10^3 magnitude.
- To understand the NP power correction in collinear factorisation (hadron collider):
 - $n=2$ for inclusive DY, $n=1$ for hadronisation
 - What about Z/W at large p_T ?

$$\left(\frac{1 \text{ GeV}}{30 \text{ GeV}}\right)^n \approx 3\% \text{ (0.1\%)} \text{ for } n=1 \text{ (} n=2 \text{)}$$

- MC framework to estimate renormalon corrections:
[Ferraro Ravasio, Limatola, Nason JHEP 06 \(2021\) 018](#)
[Carla, Ferrario Ravasio, et. al. JHEP 01 \(2022\) 093, JHEP 12 \(2022\) 062](#)
- **Confirm** $n=2$ for p_T^Z at hadron colliders \rightarrow no need to update α_s fitting related to DY data.

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}(\hat{s}) \times [1 + \mathcal{O}(\Lambda/Q)^n]$$



$$\text{gluon self-energy} = \text{tree-level} + \text{loop correction}$$

- Linear NP corrections in $e^+e^- \rightarrow 3 \text{ jets}$ ease the tension in α_s fitting from C-parameter and thrust.

P. Nason, G. Zanderighi JHEP 06 (2023) 058

CONCLUSION AND OUTLOOK

- Reducing and understanding the Standard Model uncertainties is indispensable for future high energy experiment.
- It is about finding the shortest panel of a bucket rather than boosting the longest.
- Multiple solutions work together to test our understand of the Standard Model: perturbative and non-perturbative QFT, specialised fitting etc.
- There is rapid progress in the complexity of amplitudes, NNLO and N3LO phenomenology, parton shower framework, lattice QCD and machine learning technology etc.
- It is not only to predict a more precise number but to be confronted by conceptual problems that we previously ignored.

*[Apologies for the personal selection of topics, and
for the many interesting results not covered here]*

CONCLUSION AND OUTLOOK

- Reducing and understanding the Standard Model uncertainties is indispensable for future high energy experiment.
- It is about finding the shortest panel of a bucket rather than boosting the longest.
- Multiple solutions work together to test our understand of the Standard Model: perturbative and non-perturbative QFT, specialised fitting etc.
- There is rapid progress in the complexity of amplitudes, NNLO and N3LO phenomenology, parton shower framework, lattice QCD and machine learning technology etc.
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Thank You for Your Attention

BACK UP SLIDES

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^{3(4)}LL}$

FO	α_s^n	$P_{ab}^{(n)}(x)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu} (A(\alpha_s(\bar{\mu})) \ln \frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})))$						
$\frac{d\hat{\sigma}_{NLO}^V}{dq_T}$	1	✓	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1				
$\frac{d\hat{\sigma}_{NNLO}^V}{dq_T}$	2	✓	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1			
$\frac{d\hat{\sigma}_{N^3LO}^V}{dq_T}$	3	✓	$\ln^4(b^2 m_V^2)$	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1		
$\frac{d\hat{\sigma}_{N^4LO}^V}{dq_T}$	4	✗	$\ln^5(b^2 m_V^2)$	$\ln^4(b^2 m_V^2)$	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1	
...
$\frac{d\hat{\sigma}_{N^kLO}^V}{dq_T}$	K		$\ln^{k+1}(b^2 m_V^2)$	$\ln^k(b^2 m_V^2)$	$\ln^{k-1}(b^2 m_V^2)$	$\ln^{k-2}(b^2 m_V^2)$	$\ln^{k-3}(b^2 m_V^2)$
...
	Resum		LL	NLL	NNLL	N3LL	N4LL	...	$N^{k+1}LL$
	A		A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	...	A_{k+2}
	B			B1 ✓	B2 ✓	B3 ✓	B4 ✓	...	B_{k+1}

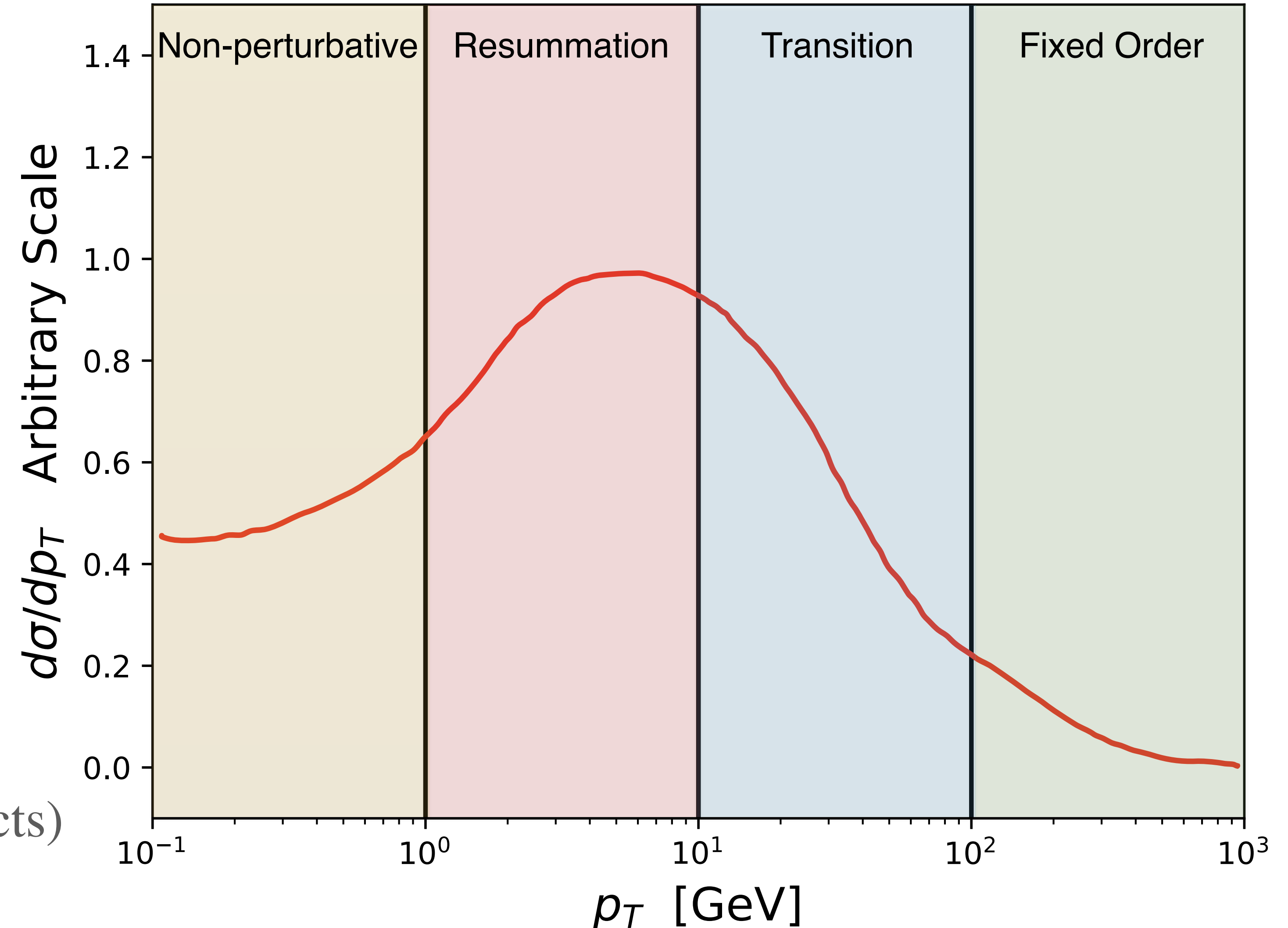
Predictions of Colourless p_T at Hadron Collider

p_T Spectrum = multi-scale problem

- Beyond QCD improved parton model
- pQCD describes the tail of spectrum
- Large logarithmic divergence

$$\ln \frac{p_T}{Q} \text{ as } p_T \rightarrow 1 \text{ GeV}$$

- Various LP resummation schemes
- Multiple solutions in transition region
- Non-perturbative effects ~ 1 GeV
(Short distance and long distance effects)



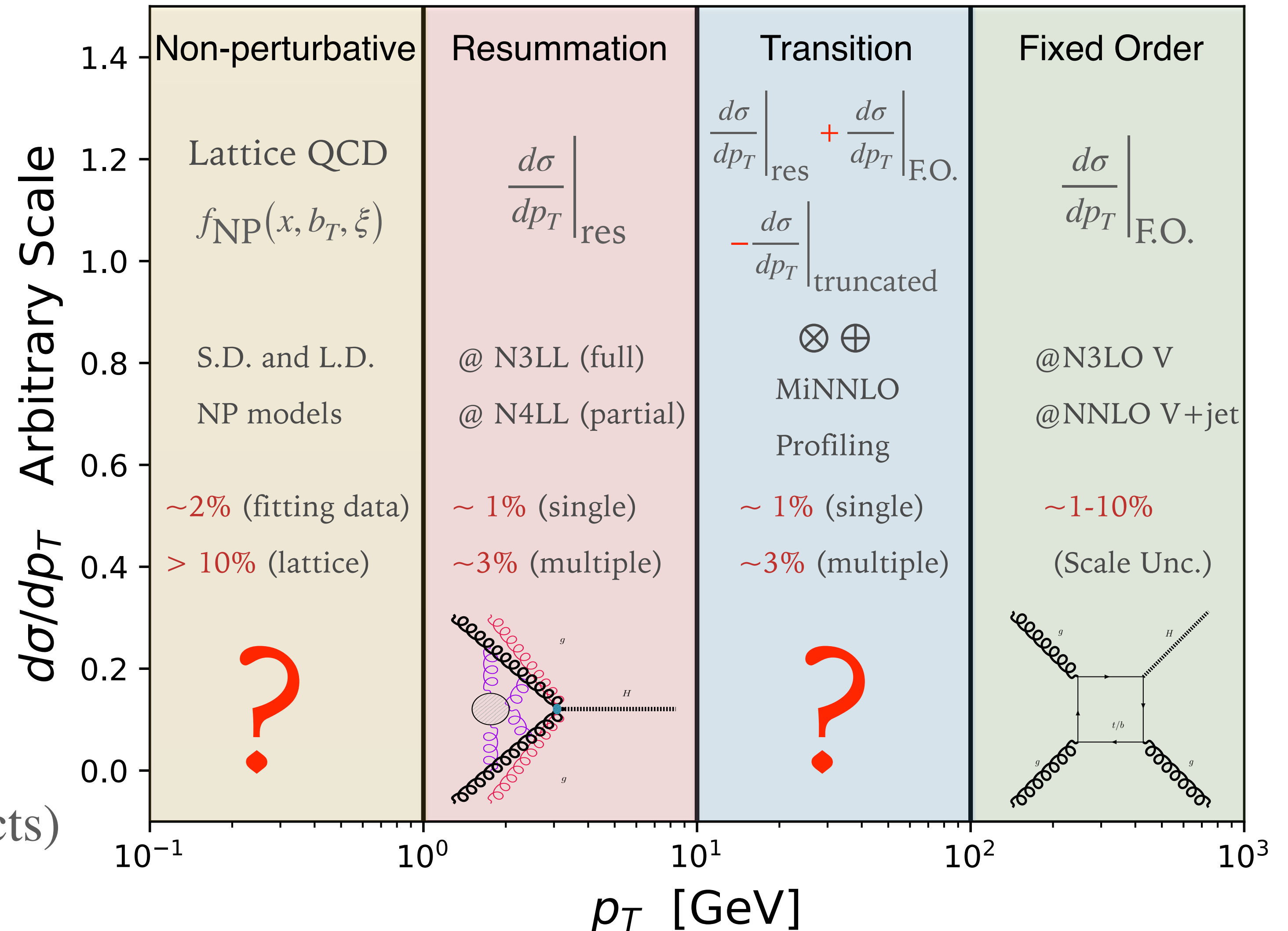
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Anatomy of differential cross sections $d\hat{\sigma}_{ab}$

► State-of-the-art differential N3LO predictions

► Fully differential N3LO Drell-Yan production (via γ^*) (XC, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, H. X. Zhu 2021)

► Apply qt-slicing at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\text{LO}}^{\gamma^*} H_{q\bar{q}} \left[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \left. \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}(b_\perp, \mu) + (q \leftrightarrow \bar{q}) \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

► All factorised functions are recently known up to N3LO:

1) 3-loop hard function $H_{q\bar{q}}^{(3)}$ (T. Gehrmann, E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus 2010)

2) Transverse-momentum-dependent (TMD) soft function $S(b_\perp, \mu)$ at α_s^3 (Y. Li, H.X. Zhu 2016)

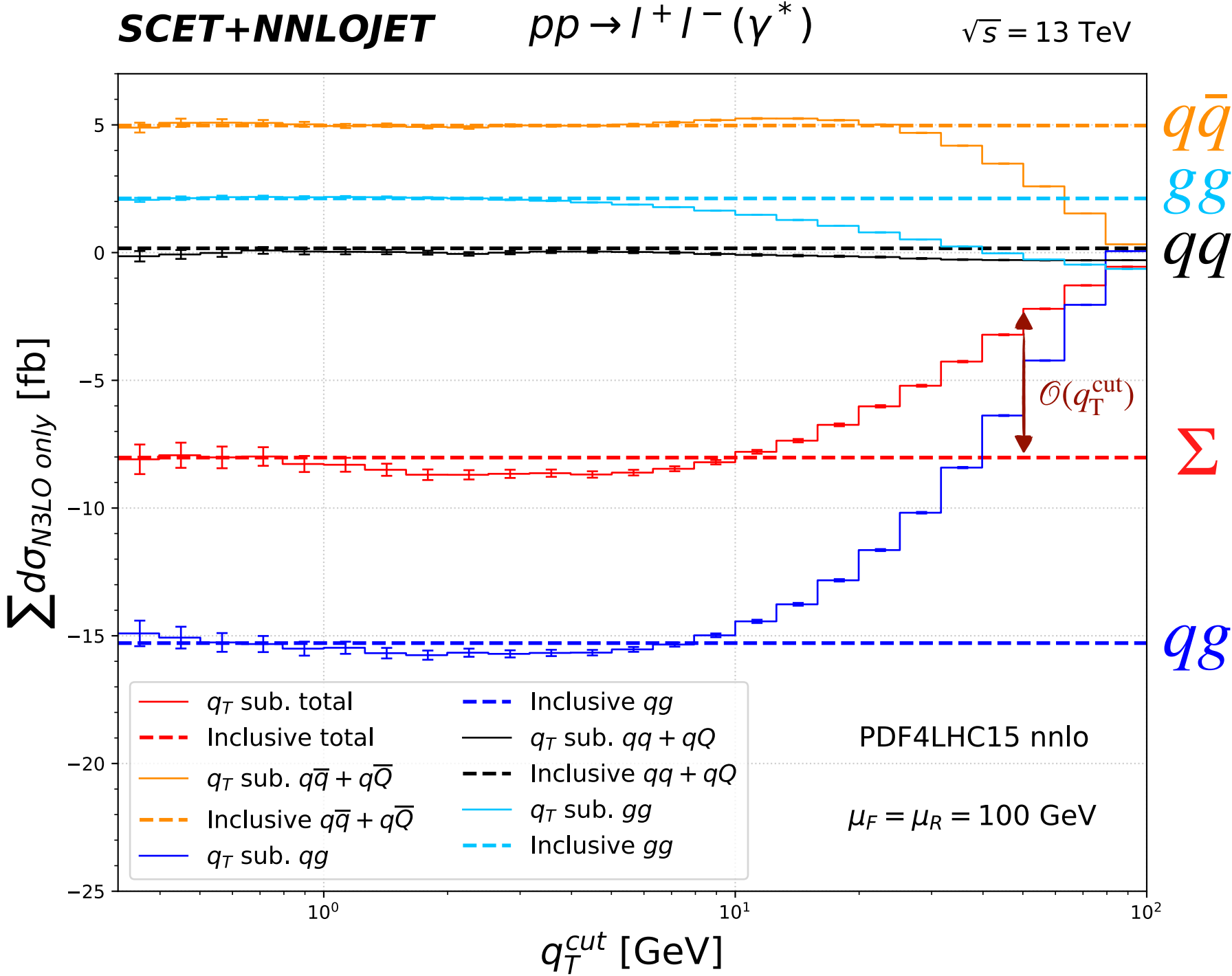
3) Matching kernel of TMD beam function I_{qk} at α_s^3 (M.-X. Luo, T.-Z. Yang, H. X. Zhu, Y. J. Zhu 2019, M. A. Ebert, B. Mistlberger, G. Vita 2020)

► Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathcal{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO} \Big|_{\delta(p_{T,\gamma^*})} + [d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^* CT}]_{p_{T,\gamma^*} > \textcolor{red}{qt}_{cut}} + \mathcal{O}(qt_{cut}^2/Q^2)$$

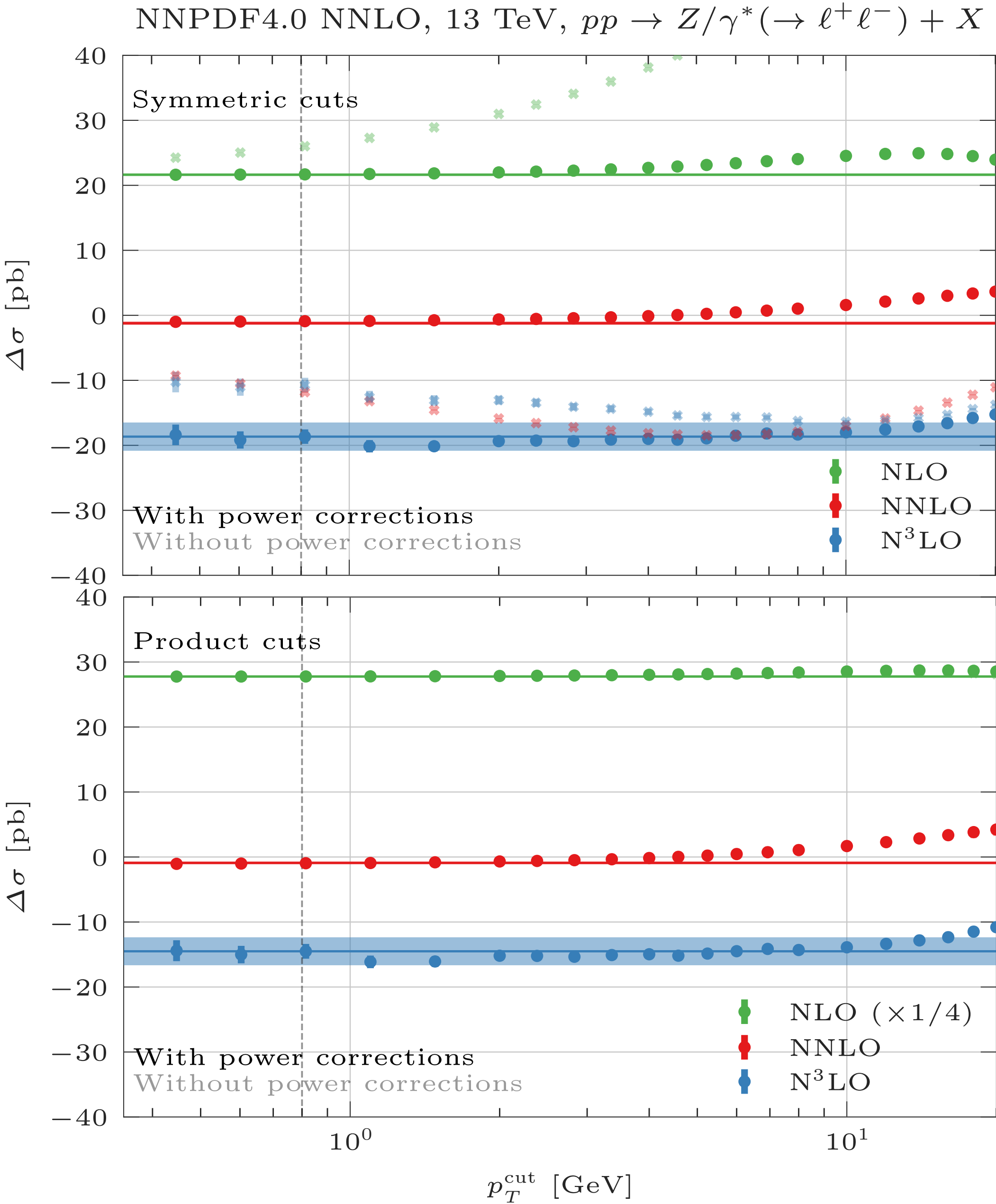
pp → γ*/Z @ N³LO

XC, T. Gehrmann, et. al. PRL. 128, 052001 (2022)



Fixed order	$\sigma_{pp \rightarrow \gamma^*}(\text{fb})$		
LO	$339.62^{+34.06}_{-37.48}$		
NLO	$391.25^{+10.84}_{-16.62}$		
NNLO	$390.09^{+3.06}_{-4.11}$		
N ³ LO	$382.08^{+2.64}_{-3.09}$ [14]		
N ³ LO only	$q_T^{\text{cut}} = 0.63$ GeV	$q_T^{\text{cut}} \rightarrow 0$ fit	[14]
qg	-15.32(32)	-15.34(54)	-15.29
$q\bar{q} + q\bar{Q}$	+5.06(12)	+5.05(12)	+4.97
gg	+2.17(6)	+2.19(6)	+2.12
$qq + qQ$	+0.09(13)	+0.09(17)	+0.17
Total	-7.98(36)	-8.01(58)	-8.03

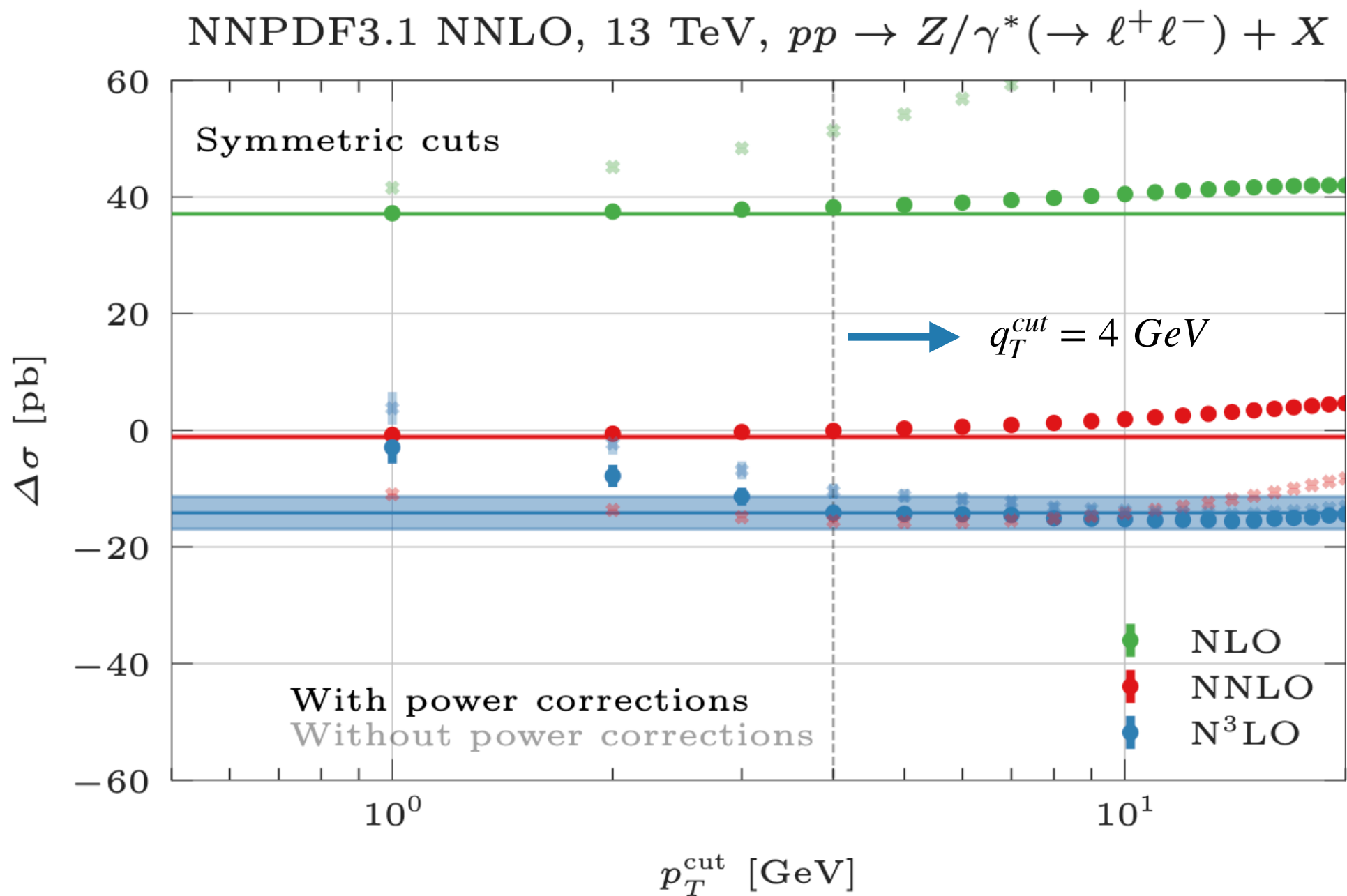
C. Duhr, F. Dulat, B. Mistlberger.
PRL. 125, 172001 (2020)



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

Precision Predictions at Hadron Collider

2 → 1 @ N3LO (+ N3LL) QCD



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

DYTurbo result with fiducial power correction

Order	N ³ LO
q_T subtr. ($q_T^{\text{cut}} = 4 \text{ GeV}$)	$747.1 \pm 0.7 \text{ pb}$
recoil q_T subtr.	$745.7 \pm 0.7 \text{ pb}$

S. Camarda, L. Cieri, G. Ferrera Eur.Phys.J.C 82 (2022) 6

- Solid horizontal lines: NLO, NNLO at 1 GeV, N3LO at 4 GeV with MC error.
- N3LO shows no plateau in 1905.05171
- Pale dots are values used by DYTurbo in 2103.04974 and 2303.12781 (taken from 1905.05171).
- Fiducial power corrections are not included.
- Leads to 30% difference of N3LO coefficients at $q_T^{\text{cut}} = 4 \text{ GeV}$.
- Solid dots are corrected values with fiducial power correction.
- Central value shifts **2 pb** starting from NLO (the dominant error).
- **$\pm 2.1 \text{ pb}$** uncertainty from MC and q_T^{cut} (estimated from [3,5] GeV region).
- Not included in DYTurbo update result with **$\pm 0.7 \text{ pb}$** uncertainty.

DYTurbo result without fiducial power correction cited in ATLAS α_s fitting

Order	NLO	NNLO	N ³ LO
$\sigma(pp \rightarrow Z/\gamma^* \rightarrow l^+l^-)$ [pb]	766.3 ± 1	757.4 ± 2	746.1 ± 2.5
Order	NLL+NLO	NNLL+NNLO	N ³ LL+N ³ LO
$\sigma(pp \rightarrow Z/\gamma^* \rightarrow l^+l^-)$ [pb]	773.7 ± 1	759.8 ± 2	749.6 ± 2.5

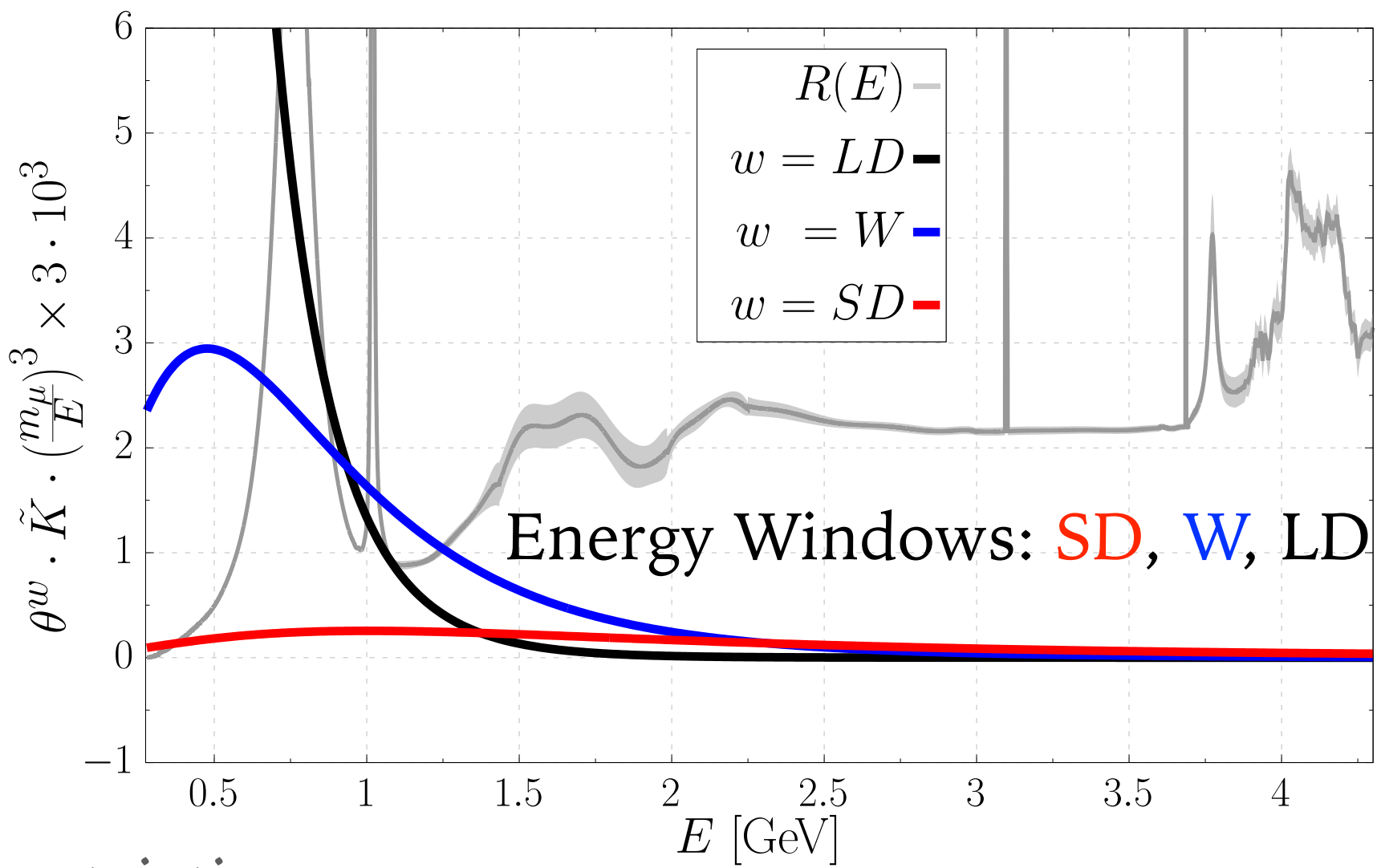
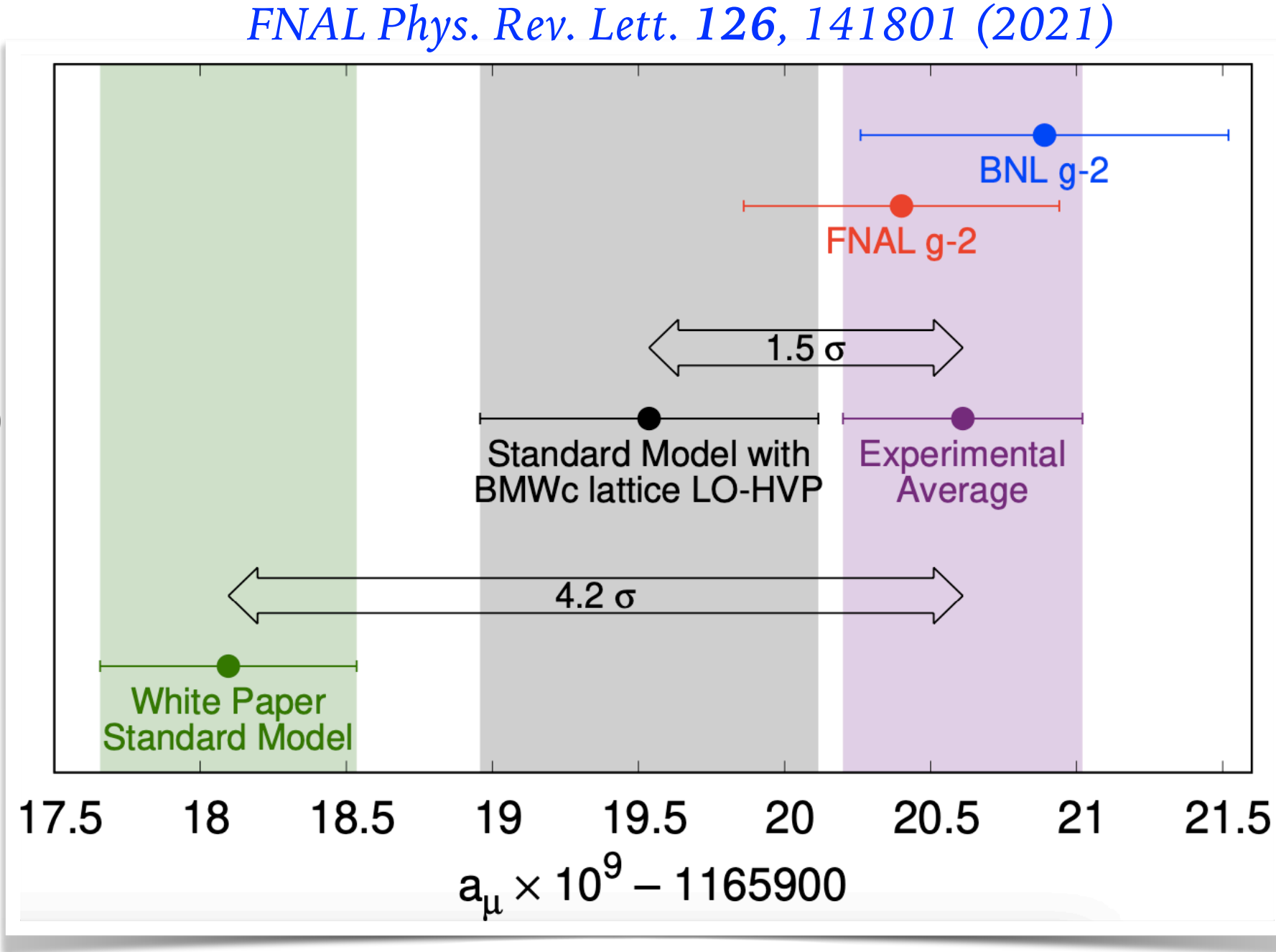
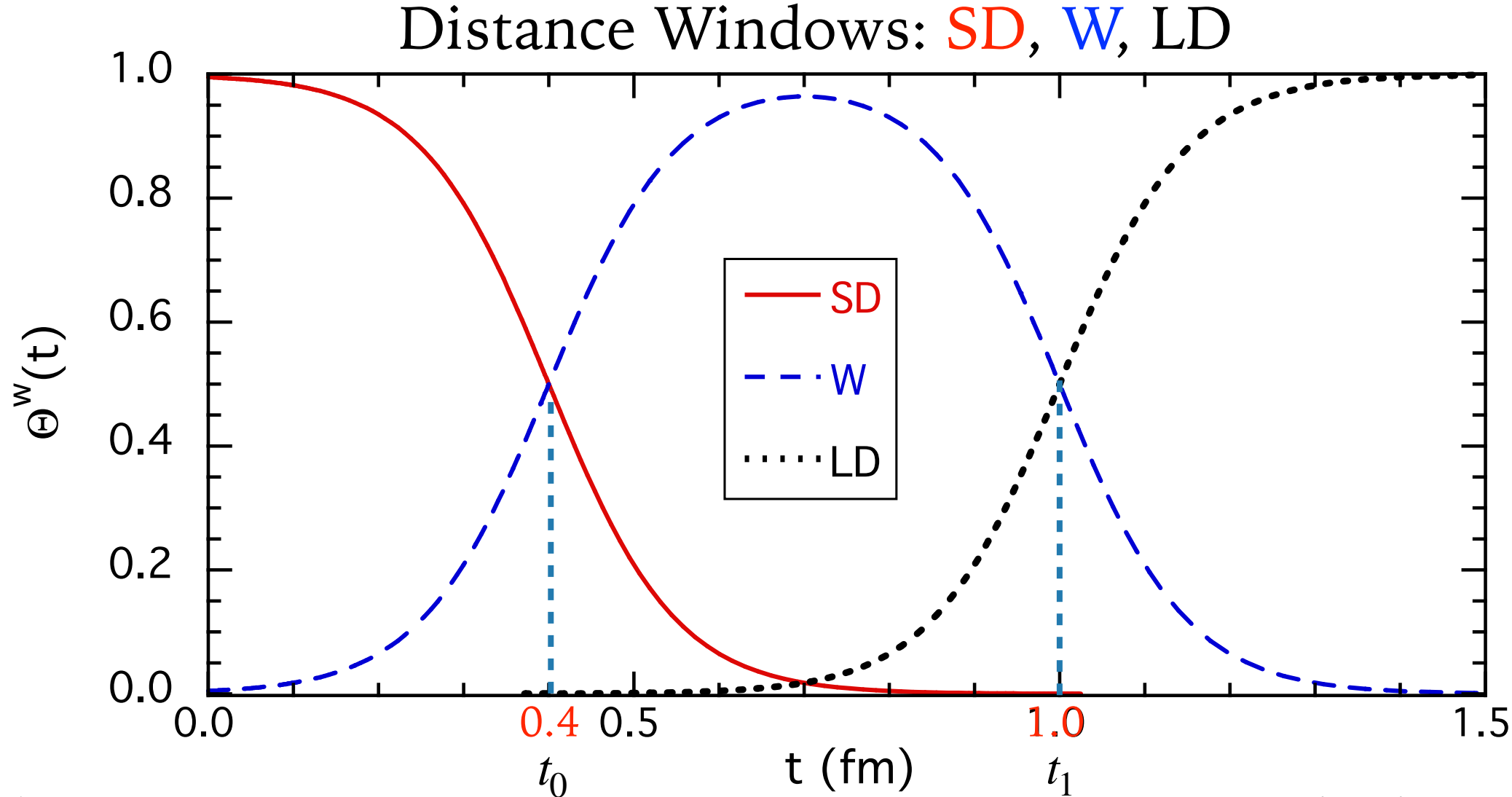
S. Camarda, L. Cieri, G. Ferrera Eur.Phys.J.C 82 (2022) 6

Non-Perturbative QFT for precision predictions

a_μ^{HVP} Data driven vs. Lattice QCD

$$a_{\mu,DA}^{LO-HVP} = \frac{\alpha^2}{3\pi^3} \int_{m_\pi^2}^\infty \frac{ds}{s} K(s) R(s) \qquad a_{\mu,LQCD}^{LO-HVP} = 2\alpha^2 \int_0^\infty t^2 dt K(m_\mu t) V(t)$$
$$a_{\mu,LQCD}^{LO-HVP,\omega} = 2\alpha^2 \int_0^\infty t^2 dt K(m_\mu t) \Theta^\omega(t) V(t)$$

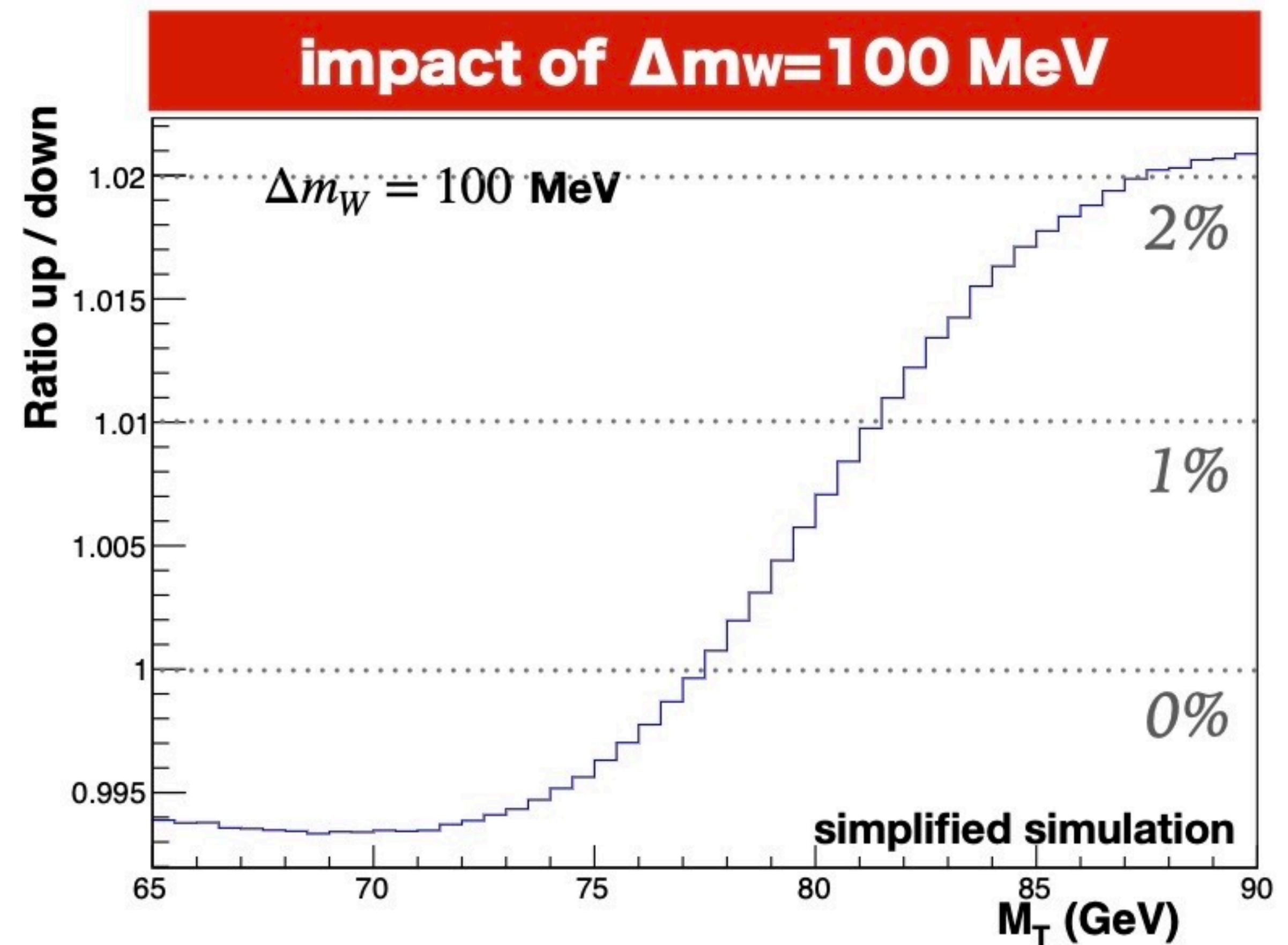
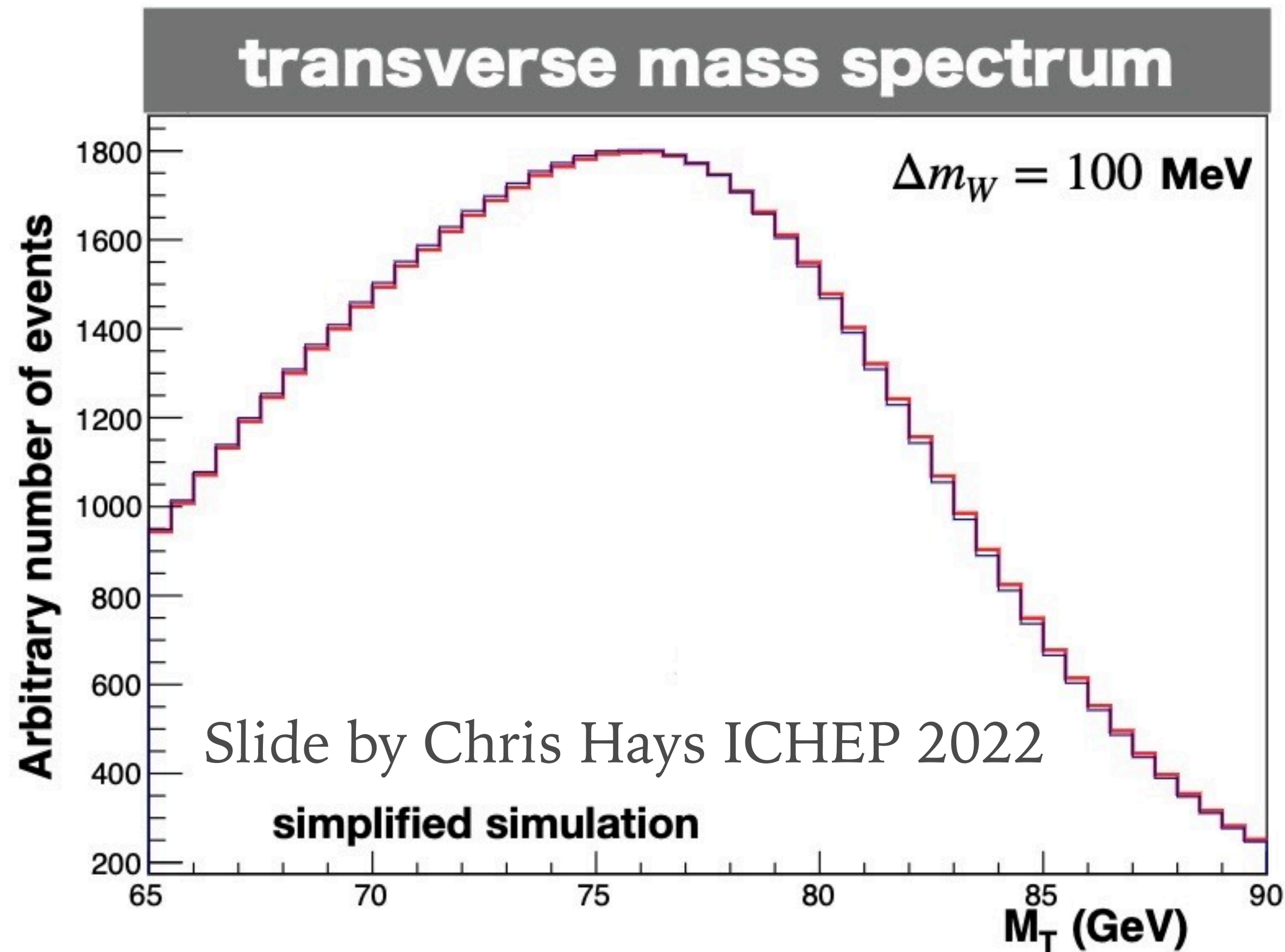
- Time ↔ Energy Window
 - $[0, t_0] \oplus [t_0, t_1] \oplus [t_1, +\infty]$ for SD, W, LD.
 - SD and W precisely predicted by Lattice QCD in continuum.
 - SD and W energy windows with precise e^+e^- EXP data.
 - a_μ^W (intermediate window) has 3.7 σ tension for DA vs. LQCD



L. Pavao at Zurich Workshop in June

W mass in CDFII measurement

► $d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV



$\Delta m_W = 100$ MeV ~ 0.5 -2% change in $d\sigma/dm_T^W \longrightarrow \Delta m_W = 10$ MeV $\sim 0.1\%$ precision in $d\sigma/dm_T^W$

Precision predictions in CDF II

► CDF II use ResBos to generate theory templates

► NLO+NNLL accuracy for W/Z production

Balazs, Brock, Landry, Nadolsky and Yuan '97 to '03

► CSS factorisation and resummation of p_T in b space:

$$\frac{d\sigma}{dQ^2 d^2\vec{p}_T dy d\cos\theta d\phi} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{p}_T \cdot \vec{b}} e^{-S(b)} \times C \otimes f(x_1, \mu) C \otimes f(x_2, \mu) + Y(Q, \vec{p}_T, x_1, x_2, \mu_R, \mu_F)$$

Collins, Soper and Sterman '85

► Non-perturbative effects at $\alpha_s(\Lambda)$ and large b :

$$S(b) = S_{\text{NP}} S_{\text{Pert}}, \quad \text{Collins and Soper '77}$$

$$S_{\text{Pert}}(b) = \int_{C_1^2/(b^*)^2}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \left(\frac{C_2^2 Q^2}{\bar{\mu}^2} \right) A(\bar{\mu}, C_1) + B(\bar{\mu}, C_1, C_2) \right]$$

$$S_{\text{NP}} = \left[-g_1 - g_2 \ln \left(\frac{Q}{2Q_0} \right) - g_1 g_3 \ln(100x_1 x_2) \right] b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan '02

► Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit '03	CDFII fit	Global fit '03	CDFII fit
p_T^Z/p_T^W			Global fit '03	

Global fit by Brock, Landry, Nadolsky and Yuan '03

$$m_T^W \sim 0.7 \text{ MeV}, p_T^l \sim 2.3 \text{ MeV}, p_T^\nu \sim 0.9 \text{ MeV}$$

CDF supplementary materials '22

► Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini '09 '11

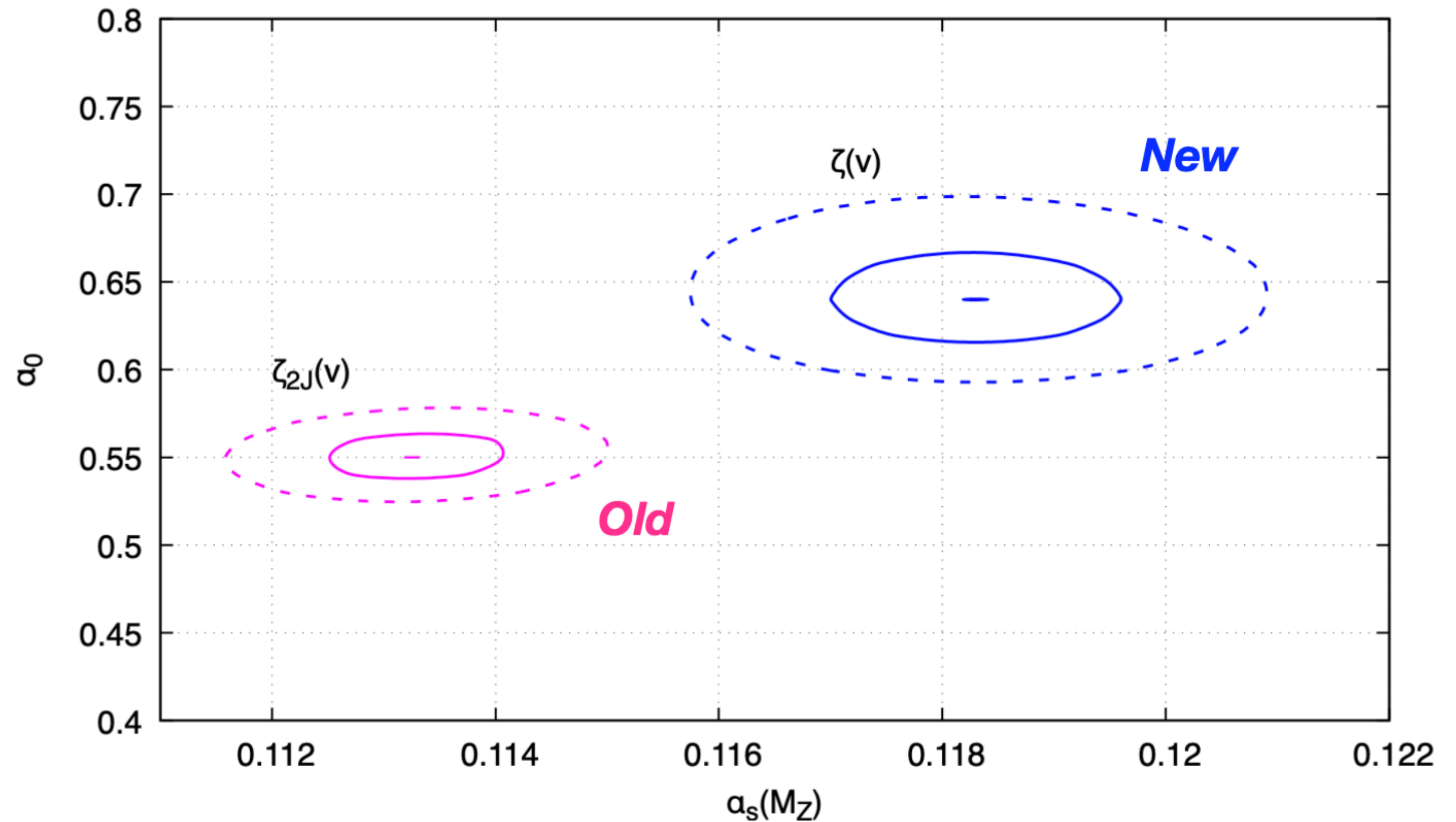
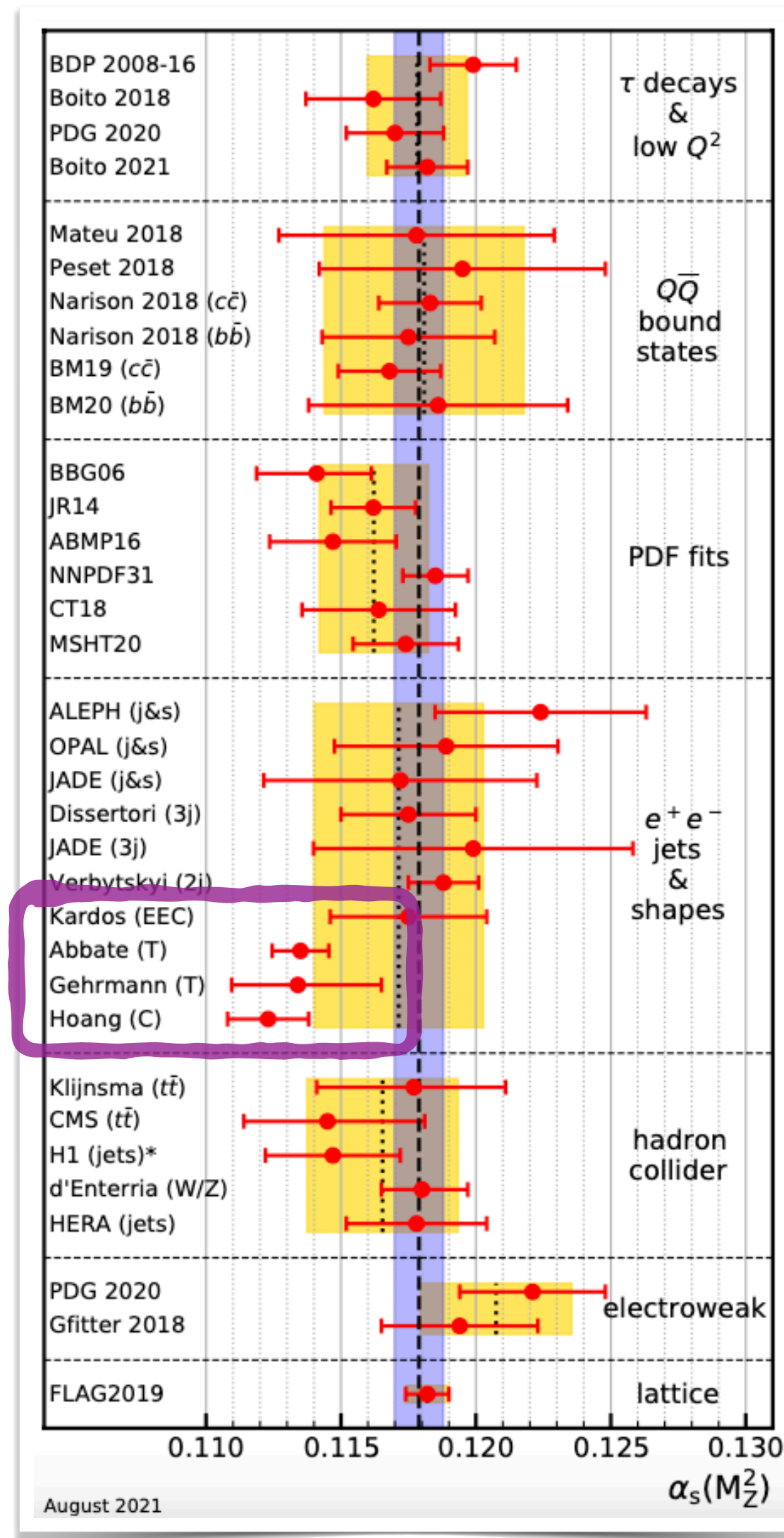
$$m_T^W \sim 3.5 \text{ MeV}, p_T^l \sim 10.1 \text{ MeV}, p_T^\nu \sim 3.9 \text{ MeV}$$

Not included in final result CDF sm'22

α_s Fitting With NP Corrections

- Linear NP corrections in $e^+e^- \rightarrow 3$ jets ease the tension in α_s fitting from C-parameter and thrust.

J. Huston, K. Rabbertz, G. Zanderighi PDG 2021



P. Nason, G. Zanderighi JHEP 06 (2023) 058