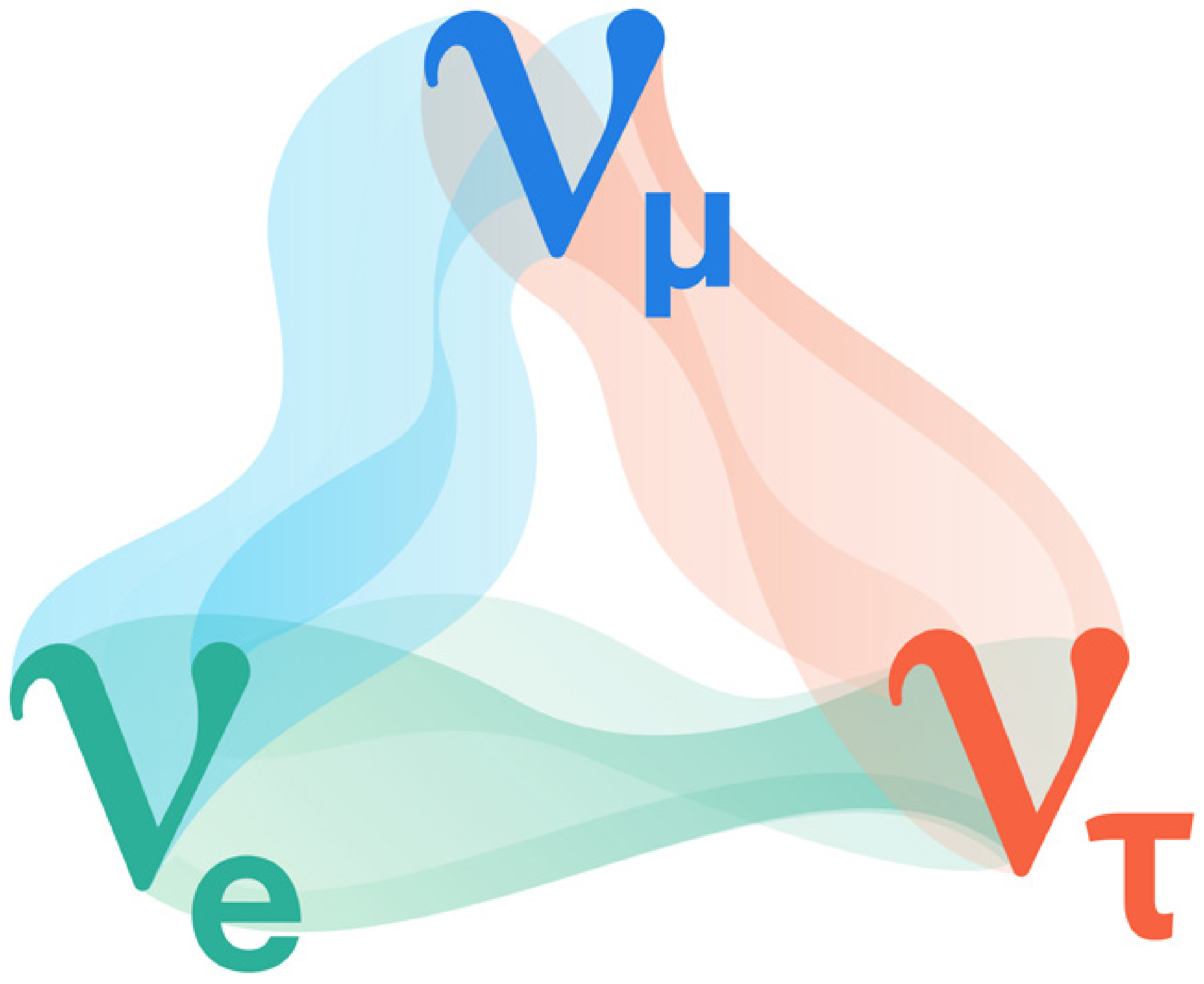
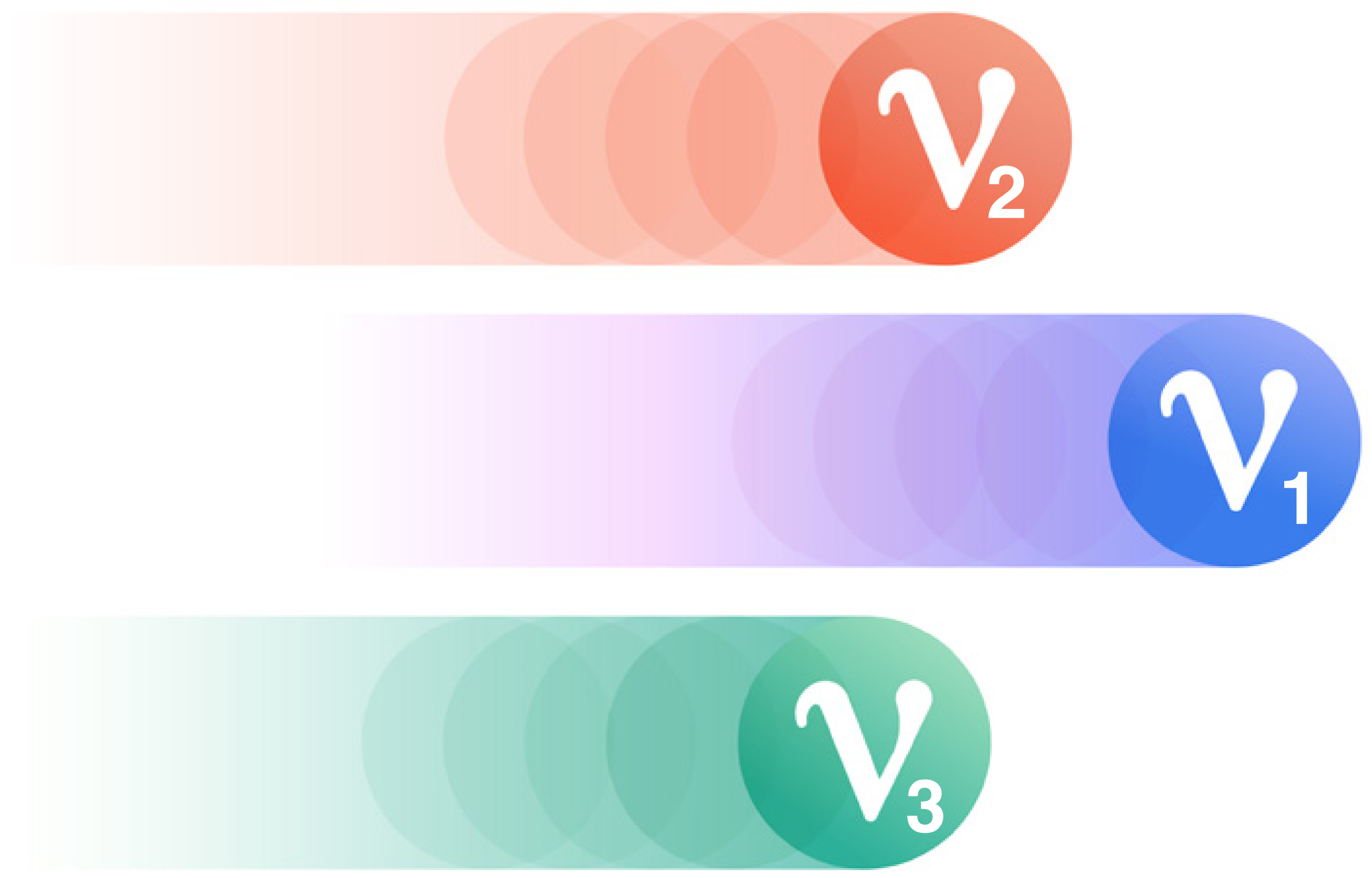
Theory Challenges in Neutrino Physics

Stephen Parke: Theory-Fermilab
linktr.ee/stephen.parke



$$= U$$



U_pdg

The 2023 **EPS High Energy and Particle Physics Prize** is awarded to

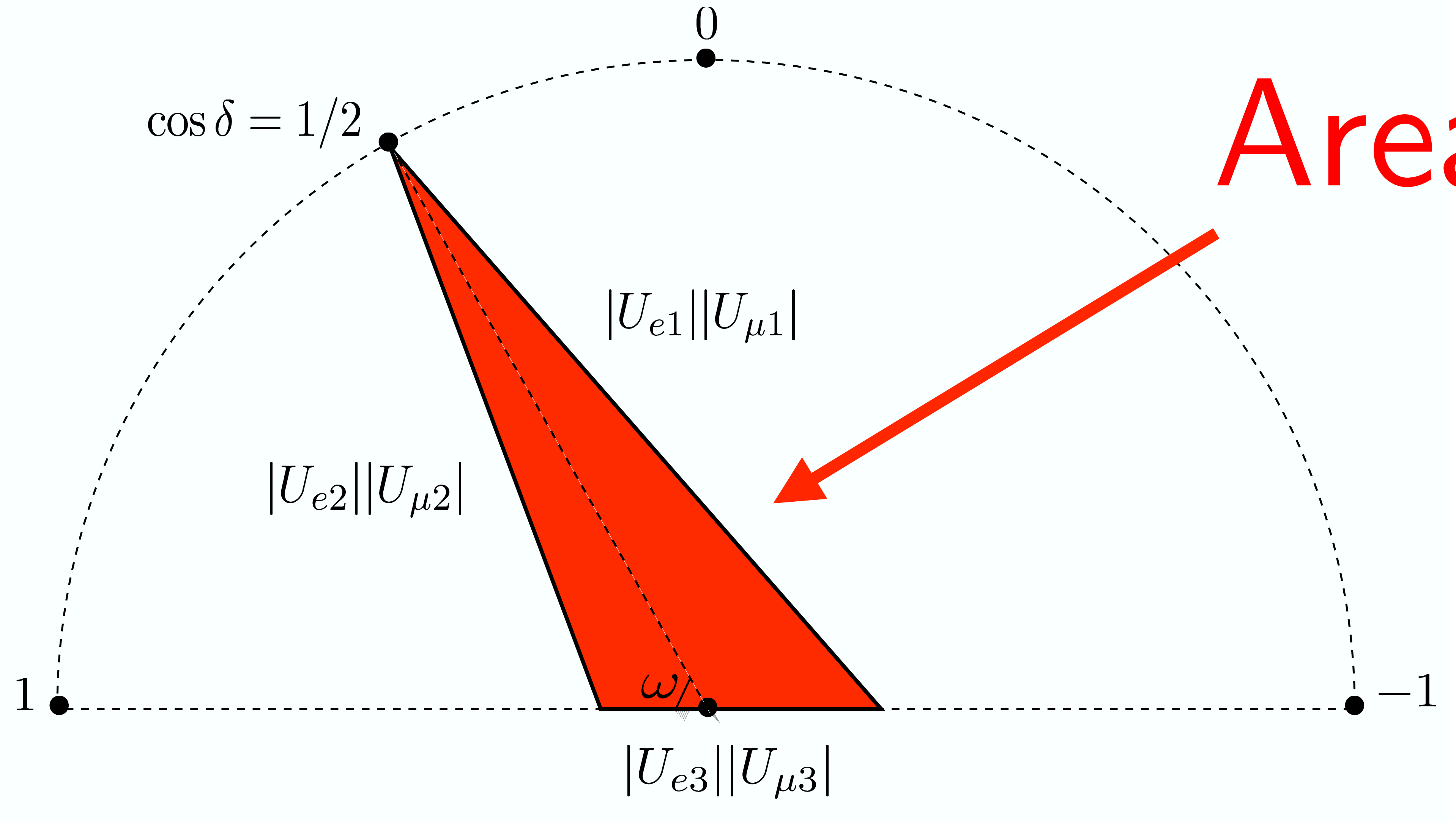
Cecilia Jarlskog for the discovery of an invariant measure of CP violation in both quark and lepton sectors; and …

Jarlskog Invariant: 1985

$$J_{ij}^{\alpha\beta} \equiv \Im\{U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\} = J \sum_{k,\gamma} \epsilon_{ijk}\epsilon_{\alpha\beta\gamma} = 0, \pm 1$$

$$J_{pdg} = s_{23}c_{23}\ s_{13}c_{13}^2\ s_{12}c_{12}\ \sin\delta$$

$$\boldsymbol{J_l = (3.36 \pm 0.06)\sin\delta_{CP} \times 10^{-2}}$$



Area $= \frac{1}{2}\ |J|$

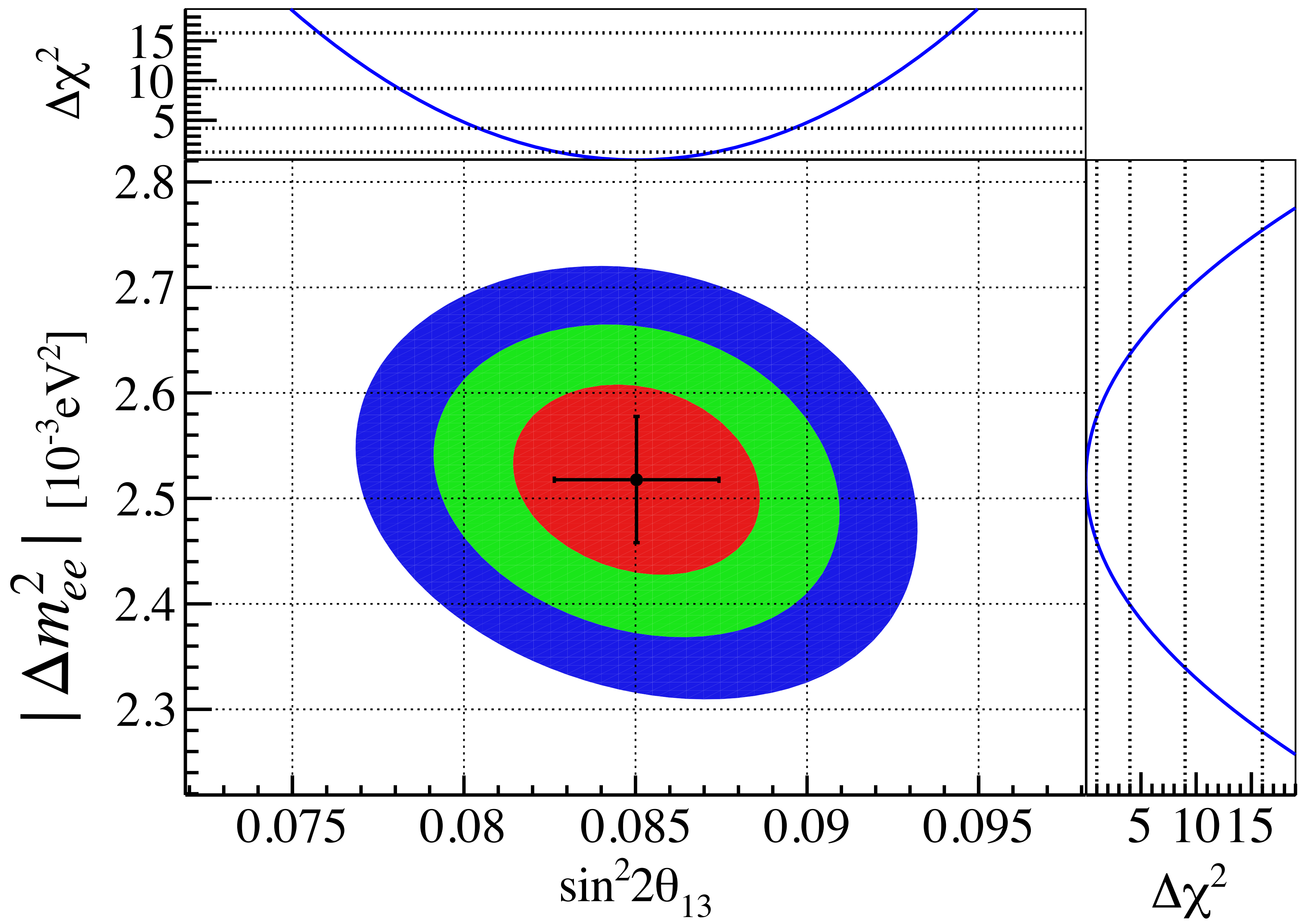
Quarks

$$\boldsymbol{J = (3.08 \pm 0.14) \times 10^{-5}}$$

also used in SMEFT

And the **Daya Bay and RENO collaborations** for the observation of short-baseline reactor electron-antineutrino disappearance, providing the first determination of the neutrino mixing angle Θ_{13}, which paves the way for the detection of CP violation in the lepton sector.

2211.14988



$$|U_{e3}|^2 = \sin^2\theta_{13} = 0.0215\ (\pm 2.8\%)$$

$$|\Delta m^2_{ee}| = 2.52\ (\pm 2.4\%) \times 10^{-3}\ \text{eV}^2$$

note: $$\frac{\Delta m^2_{21}}{|\Delta m^2_{ee}|} = 3.0\%$$

ν_e average of Δm^2_{31} and Δm^2_{32}

$$\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

Nunokawa, SP, Zukanovich hep/0503283

NO and IO orderings have same $|\Delta m^2_{ee}|$ within 2.4%

Neutrino Theory
Tasting Menu

My Selection

- Neutrino Flavor Puzzle
- Neutrino Oscillation Phenomenology
- Nuclear Theory for Neutrino Physics

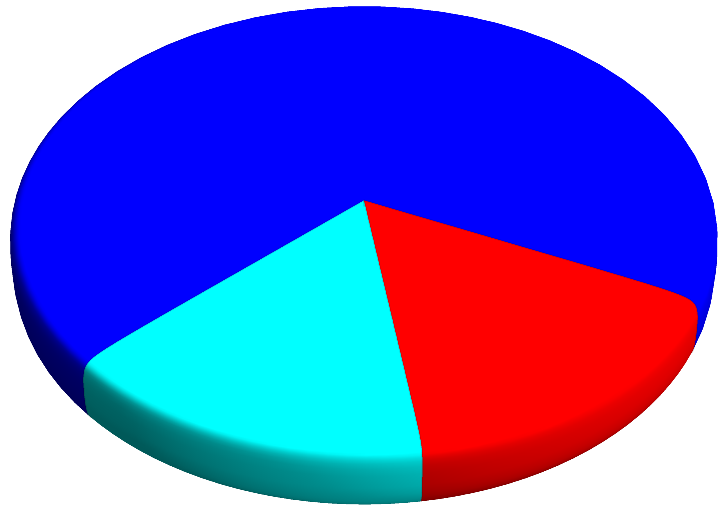
- Neutrino Flavor Puzzle

Neutrino Mass EigenStates or Propagation States:

$$\text{Propagator } \nu_j \rightarrow \nu_k \; = \boldsymbol{\delta_{jk}\, e^{-i\left(\frac{m_j^2 L}{2E_\nu}\right)}}$$

$\boldsymbol{\nu_1}$

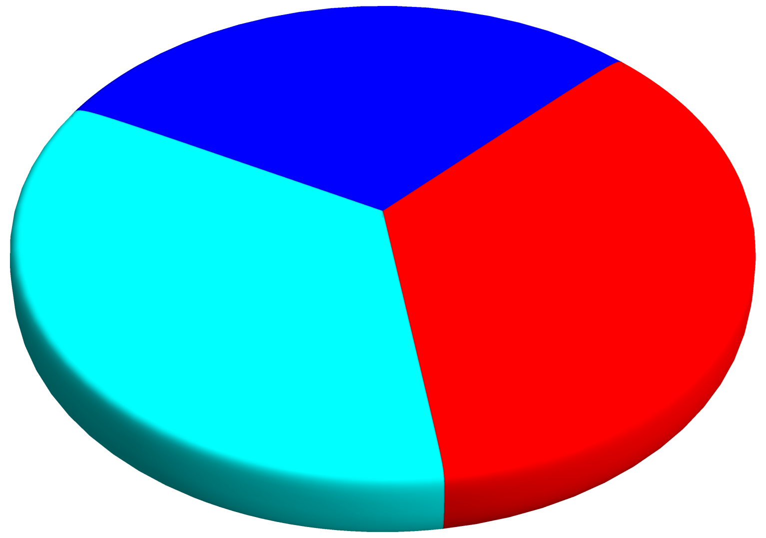
most ν_e **68%**



$\cos\delta, \theta_{23}$

$\boldsymbol{\nu_2}$

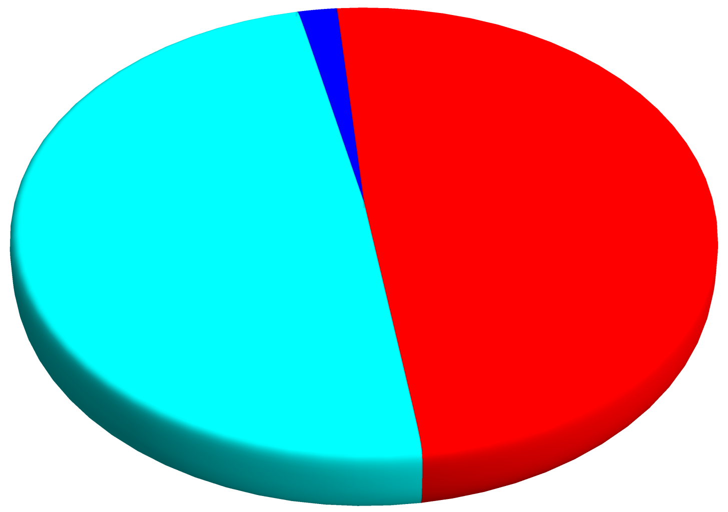
$\mathbf{30\,\%\,\nu_e}$



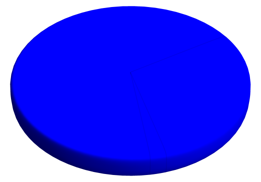
$\cos\delta, \theta_{23}$

$\boldsymbol{\nu_3}$

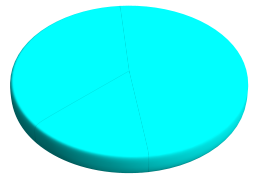
least ν_e **2%**



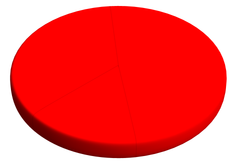
θ_{23}

$\boldsymbol{\nu_e} =$ 

Solar Exp, SNO
KamiLAND
Daya Bay, RENO, …

$\boldsymbol{\nu_\mu} =$ 

SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

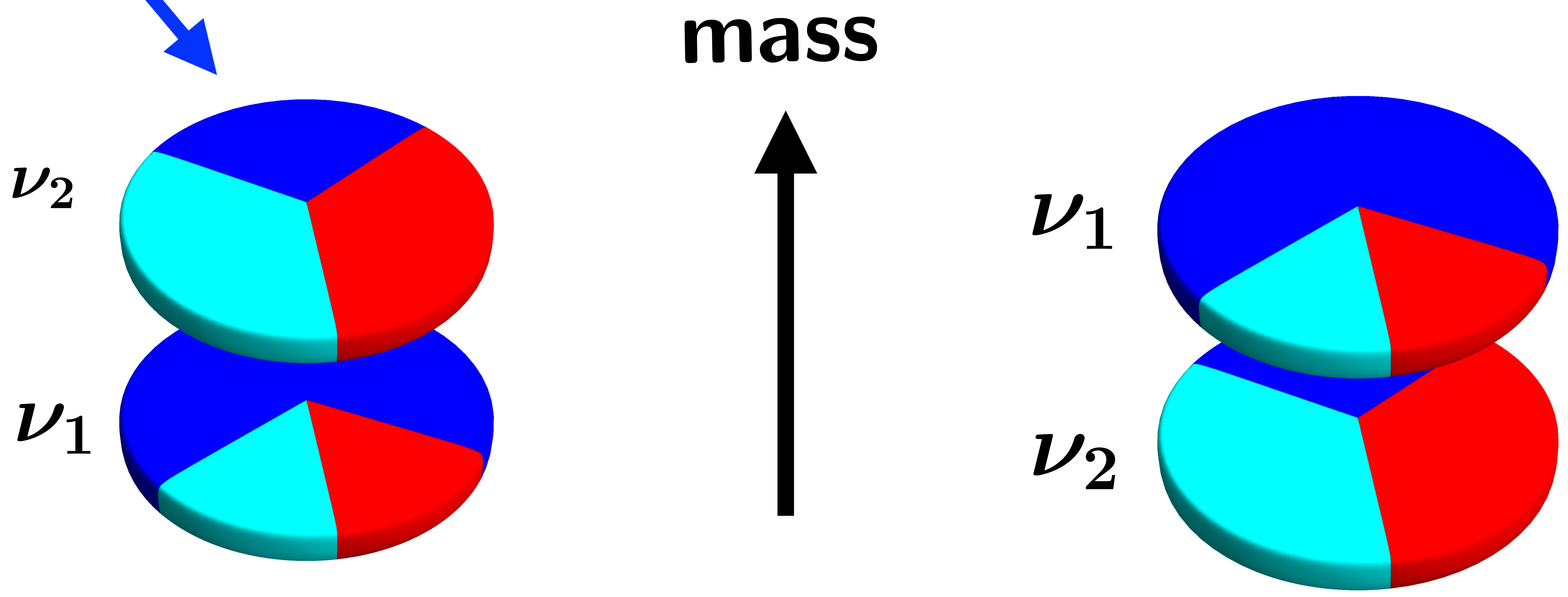
$\boldsymbol{\nu_\tau} =$ 

Unitarity
SK, Opera
ICECUBE

$$|U_{e2}|^2 \approx 0.3 \approx \frac{CC}{NC}$$

$\nu_1, \ \nu_2$ Mass Ordering:

–solar mass ordering



$$|\Delta m^2_{21}| = |m^2_2 - m^2_1| = 7.5 \times 10^{-5} \ \mathrm{eV}^2$$

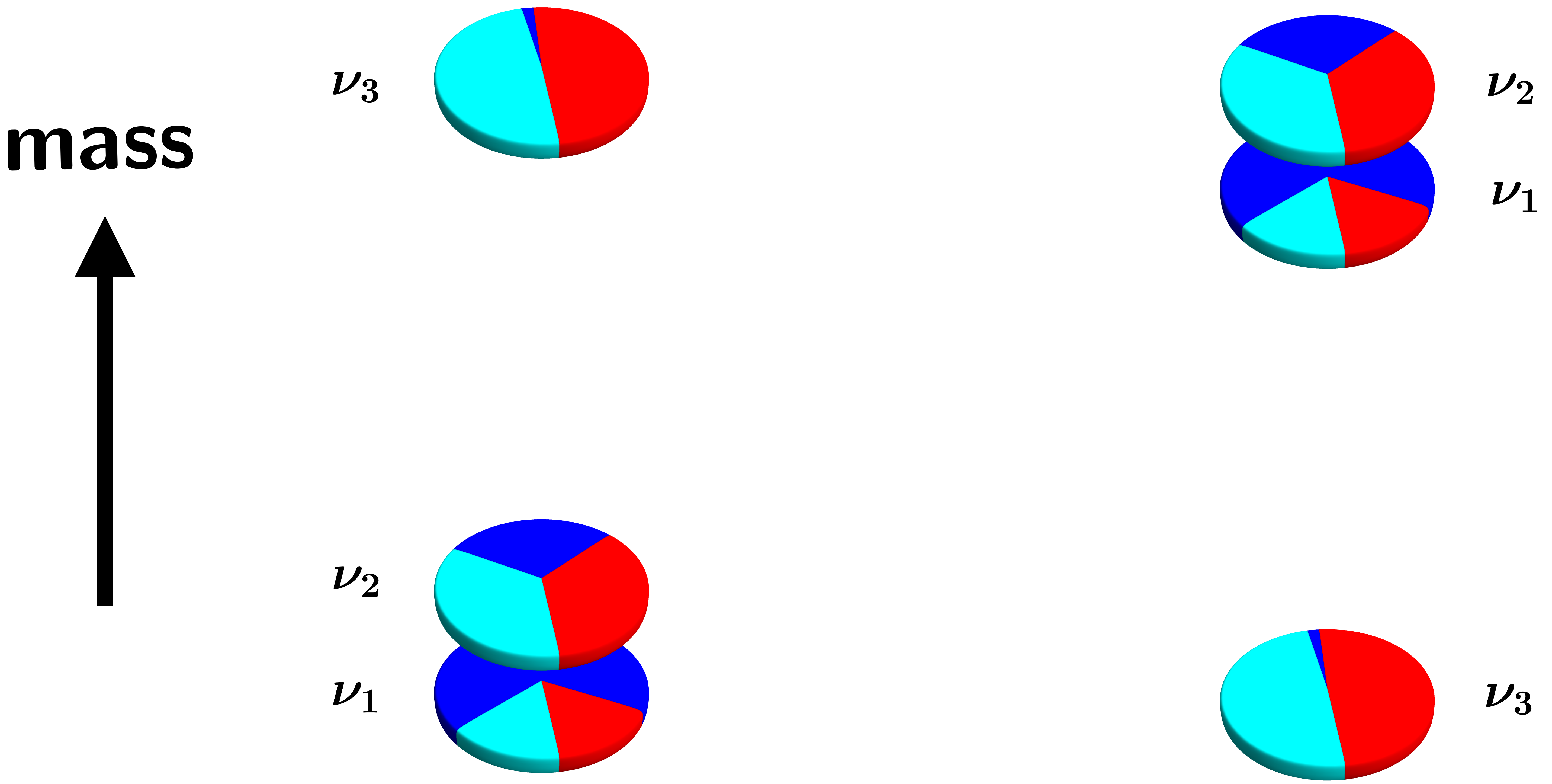
$$L/E = 15 \ \mathrm{km/MeV} = 15,000 \ \mathrm{km/GeV}$$

SNO $\quad m_2 > m_1$

$\nu_e =$ $\quad \nu_\mu =$ $\quad \nu_\tau =$

ν_3, ν_1/ν_2 Mass Ordering:

–atmospheric mass ordering

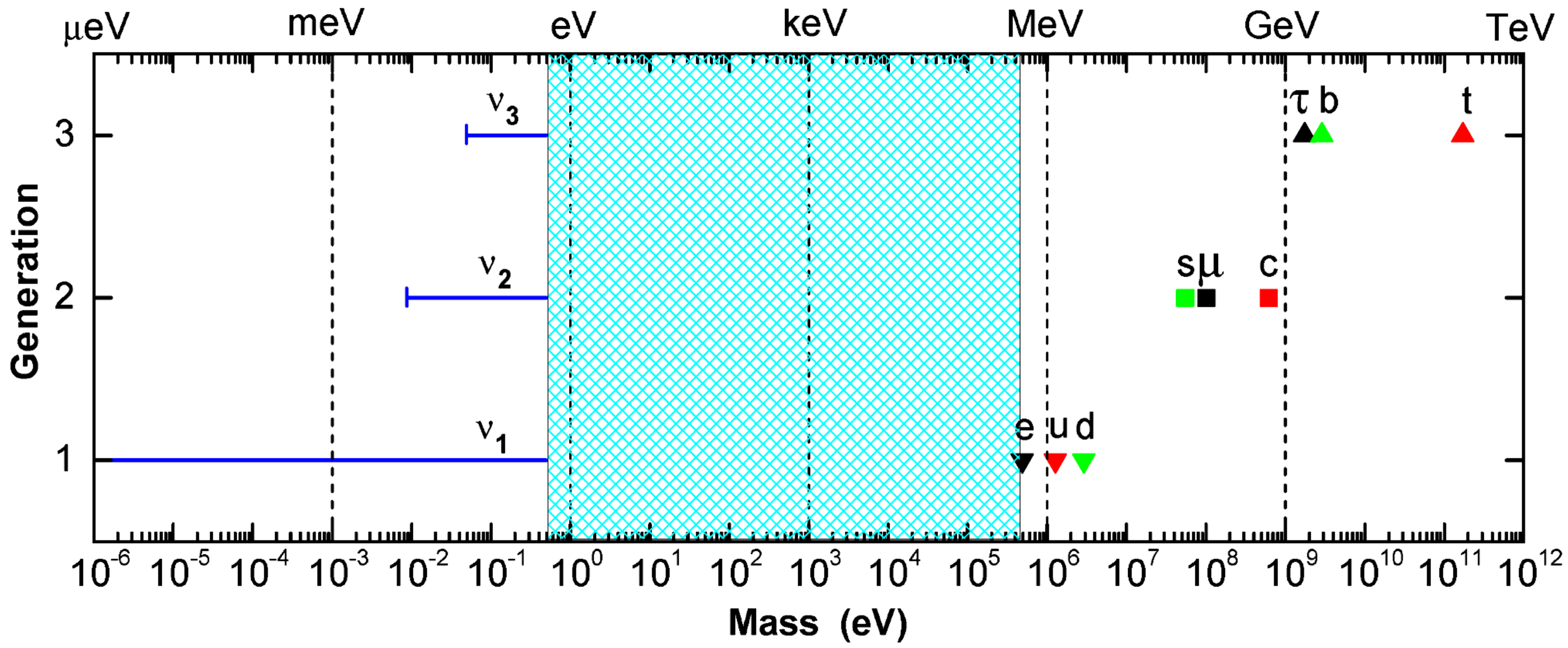


$$|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.5 \times 10^{-3}\ \mathrm{eV}^2 \qquad L/E = 0.5\ \mathrm{km/MeV} = 500\ \mathrm{km/GeV}$$

unknown: SK, T2K, NOvA, JUNO, ICECUBE, DUNE, KNO, …

$\nu_e =$ $\qquad \nu_\mu =$ $\qquad \nu_\tau =$

Two Big Challenges:



- Why are the Masses so Tiny ?

- Why is the Mixing Matrix so different

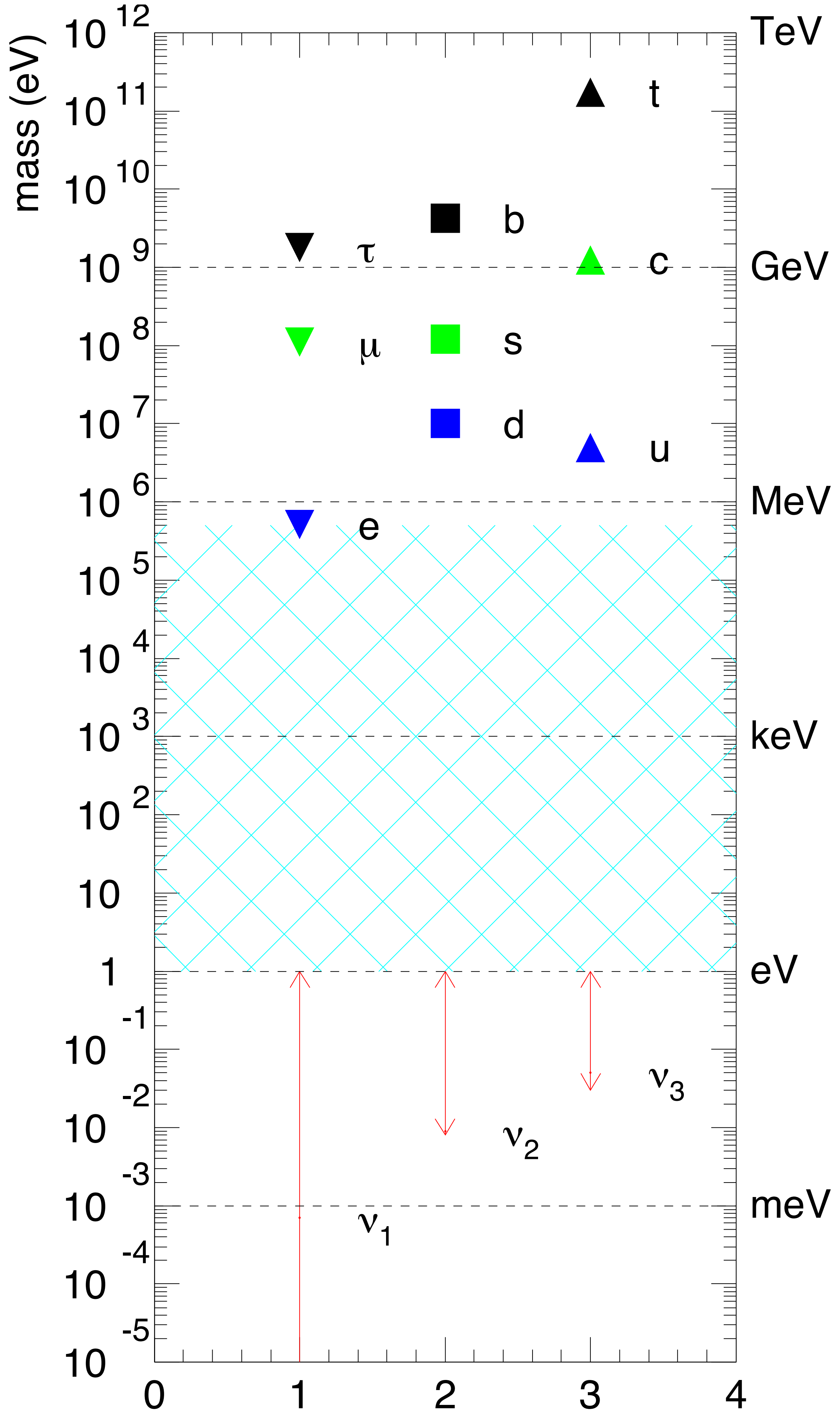
Neutrino

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

Why are the nu masses so Tiny ?



Seesaw Mass Matrix:

$$(\nu_L,\ \nu_R,\ \bar{\nu}_L,\ \bar{\nu}_R)$$

Note: $\nu_L \underset{\mathrm{CPT}}{\Longleftrightarrow} \bar{\nu}_R$ and $\nu_R \underset{\mathrm{CPT}}{\Longleftrightarrow} \bar{\nu}_L$

$$\begin{pmatrix} \nu_L \text{ to } \bar{\nu}_R & \nu_L \text{ to } \nu_R \\ \bar{\nu}_L \text{ to } \bar{\nu}_R & \bar{\nu}_L \text{ to } \nu_R \end{pmatrix} = \begin{pmatrix} 0 & D \\ D^* & M \end{pmatrix}$$

Eigenvalues & Eigenvectors:

Light Majorana Neutrino (mass $\frac{D^2}{M}$) $\quad \nu = (\nu_L,\ \bar{\nu}_R) + \frac{D}{M}(\bar{\nu}_L,\ \nu_R)$

Heavy Neutral Majorana Lepton (mass M) $\quad N = (\nu_R,\ \bar{\nu}_L) - \frac{D}{M}(\bar{\nu}_R,\ \nu_L)$

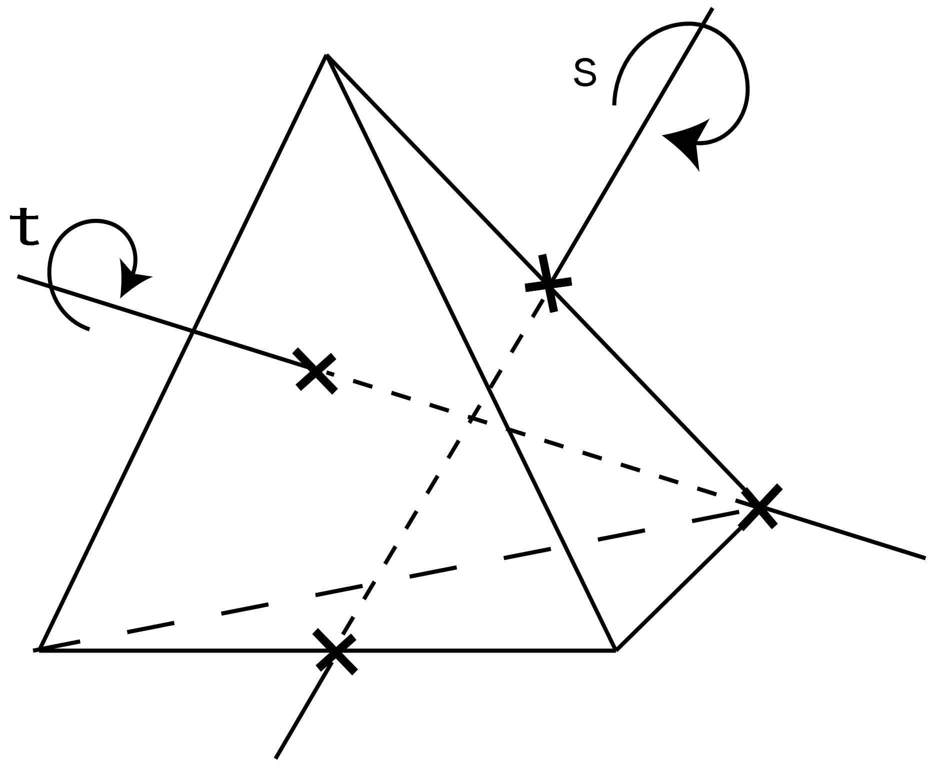
This is our BEST explanation of why Neutrino Masses are so SMALL $(\sum m_{\nu_i} < \mathcal{O}(m_e/10^6))$.

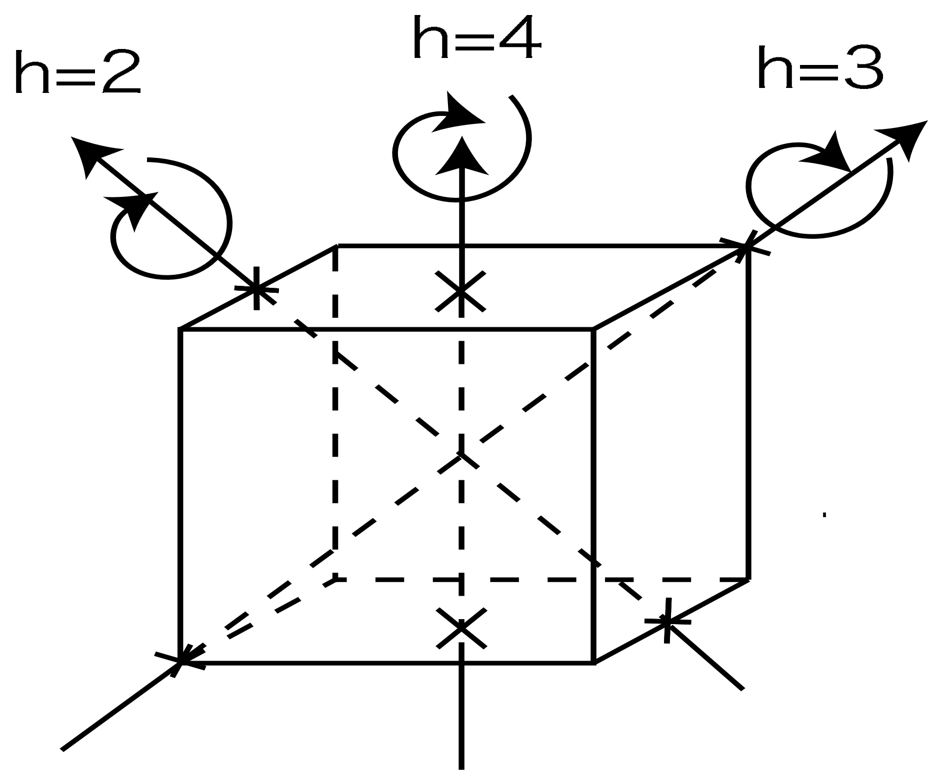
and

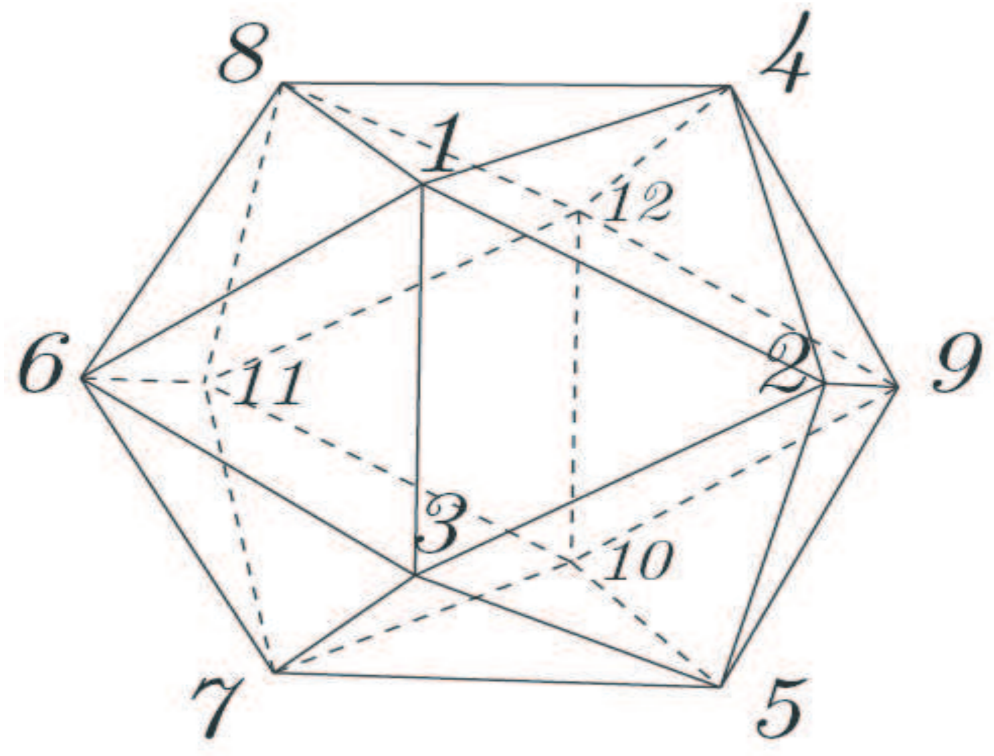
the Heavy Majorana Lepton could be responsible for Leptogenesis .

What about UV completion ?

Symmetries in the PMNS matrix:

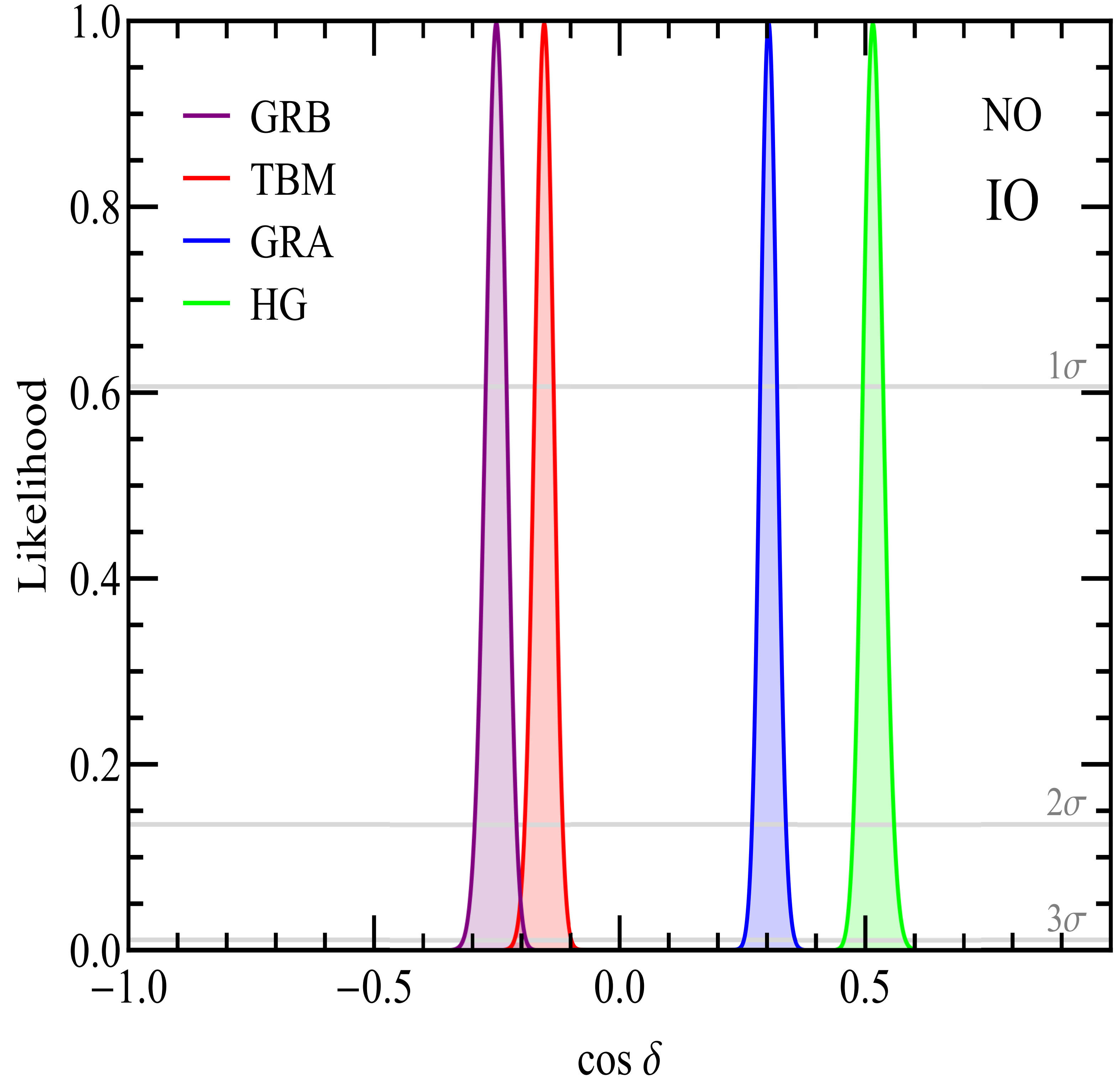






$$A_4, \; S_4, \; A_5$$

Updated from I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



Intergration of Seesaw
and
Symmetries Challenging !

Leptogenesis !

Hagedorn: Wed. 9 am
Phenomenology of low-scale seesaw with flavour and CP symmetries
Wed. 9 am

- Neutrino Oscillation Phenomenology

Advanced Understanding of Neutrino Oscillation Phenomena

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_j V^*_{\alpha j}\ V_{\beta j} e^{-i\lambda_j L/(2E)} \right|^2$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \boxed{\Re(V_{\alpha i} V^*_{\beta i}\, V^*_{\alpha j} V_{\beta j})} \sin^2(\Delta_{ij})$$

CPC

$$-8 \sum_{i>j} \boxed{\Im(V_{\alpha i} V^*_{\beta i}\, V^*_{\alpha j} V_{\beta j})}\ \sin\Delta_{ij} \sin\Delta_{ik} \sin\Delta_{jk}\ ,$$

CPV

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

V is PMNS matrix

k is arbitrary, all choices are equivalent

i,j,k all different

The usual way of writing this term, as in the PDG,

$$2 \sum_{i>j} \Im(V_{\alpha i} V^*_{\beta i} V^*_{\alpha j} V_{\beta j}) \sin(2\Delta_{ij})$$

Three Neutrinos:

$$J \equiv \Im(V_{\alpha i}V^{\dagger}_{\beta i}V^{\dagger}_{\alpha j}V_{\beta j})\left(\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\right)\left(\sum_{k}\epsilon_{ijk}\right)$$

$$R_{ij} \equiv \Re(V_{\alpha i}V^{\dagger}_{\beta i}V^{\dagger}_{\alpha j}V_{\beta j})$$

Unitarity - 3 ids

$$J^2 = R_{12}R_{13} + R_{12}R_{23} + R_{13}R_{23}$$

Luo, Xing - 2306.16231

Neutrino Propogation in Medium:

U is PMNS matrix: $M^2 = \textbf{Diag}(m_1^1, m_2^2, \cdots, m_n^2)$

Interactions with medium

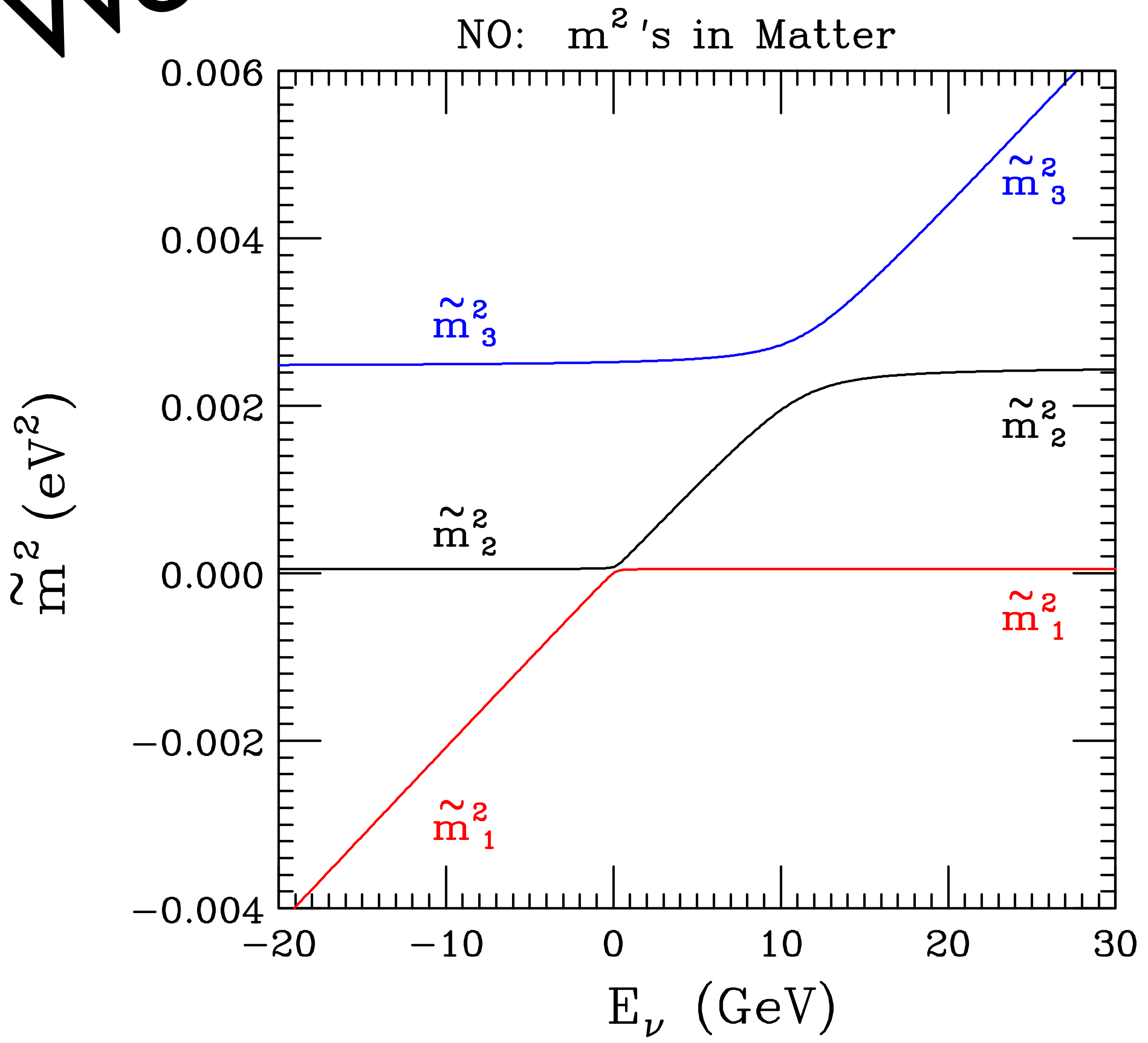
H in Flavor basis: $H = \frac{1}{2E}\, U M^2 U^\dagger + A$

Wolfenstein Matter Effect

For Neutrino Oscillations you need

the Eigenvalues ("masses")

and Eigenvectors ("PMNS matrix") of H.



Eigenvalues are given by solutions of

$$\textbf{Det}(\lambda I - H) = 0$$

Once you have the Eigenvalues, the Eigenvectors are easily obtained using:

i-th Eigenvector is given by $$V_{\alpha i}V^*_{\beta i} = \frac{\mathbf{Adj}(\lambda_i I - H)_{\alpha\beta}}{\Pi_j(\lambda_i - \lambda_j)}$$

Adjugate = transpose of cofactor matrix

Calculate $\mathbf{Adj}(H)$ (and $\mathbf{Det}(H)$) and replace m_j^2 with $(m_j^2 - \lambda)$

OR

LeVerrier-Faddeev algorithm $$\mathrm{Adj}[\lambda I - H] = A_1\lambda^{n-1} + A_2\lambda^{n-2} + \cdots + A_n$$

$A_1 = I$ then iterate $d_i = -(1/i)\mathrm{Tr}[HA_i]$ and $A_{i+1} = HA_i + d_i I$

each iteration requires one Trace and Matrix Multipication

$$\mathrm{Det}[\lambda I - H] = \lambda^n + d_1\lambda^{n-1} + \cdots + d_n$$

Abdullahi + Parke: 2212.12565

Three Neutrinos in Matter:

The Jarlskog in Matter

$$J_a \approx \frac{J_0}{S_\odot S_{atm}}$$

Two Resonant factors:

$$\mathcal{S}_\odot = \sqrt{(\cos 2\theta_{12} - \boxed{c_{13}^2} a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}}\,,$$

$$\mathcal{S}_{\rm atm} = \sqrt{(\cos 2\theta_{13} - a/\boxed{\Delta m_{ee}^2})^2 + \sin^2 2\theta_{13}}\,.$$

Resonances when (…) = 0

$$\Delta m_{ee}^2 \equiv c_{12}^2 \Delta m_{31}^2 + s_{12}^2 \Delta m_{32}^2$$

Accuracy better than 0.1%

Denton, Parke - 1902.07185

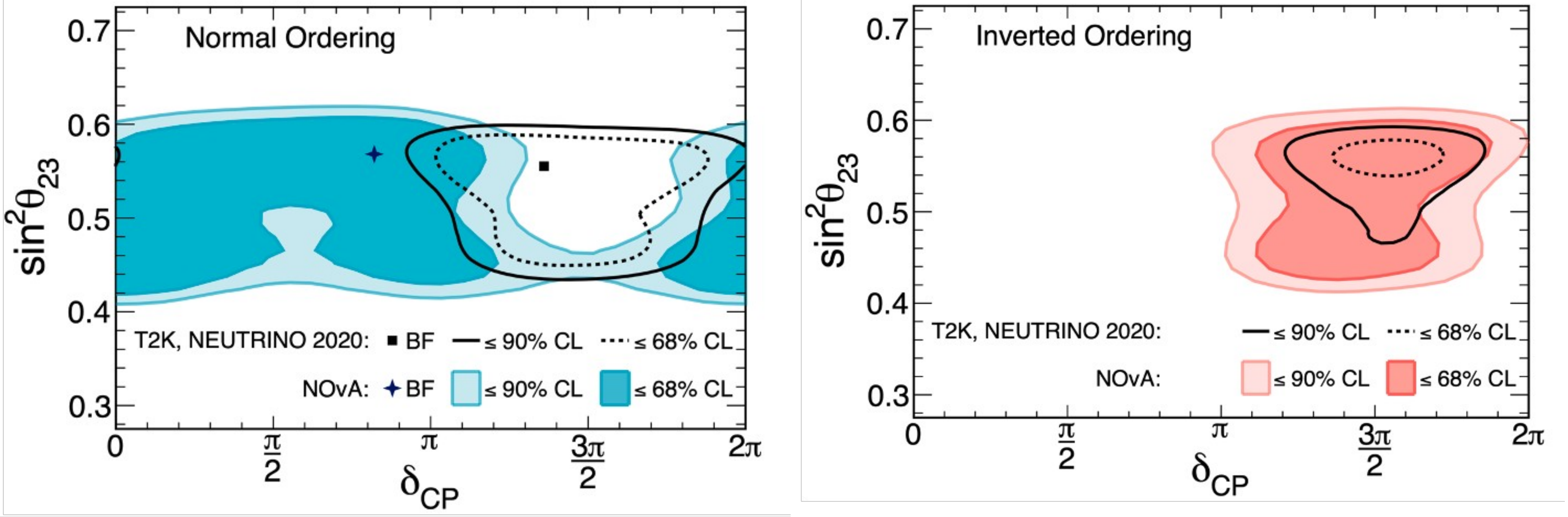
Wang-Zhou - 1908.07304

Determining the MO

- Current Status: T2K, NOvA, Daya Bay, SK
  - Appearance
  - Disappearance
  - Combined

T2K + NOvA COMBINED

https://doi.org/10.5281/zenodo.6683827

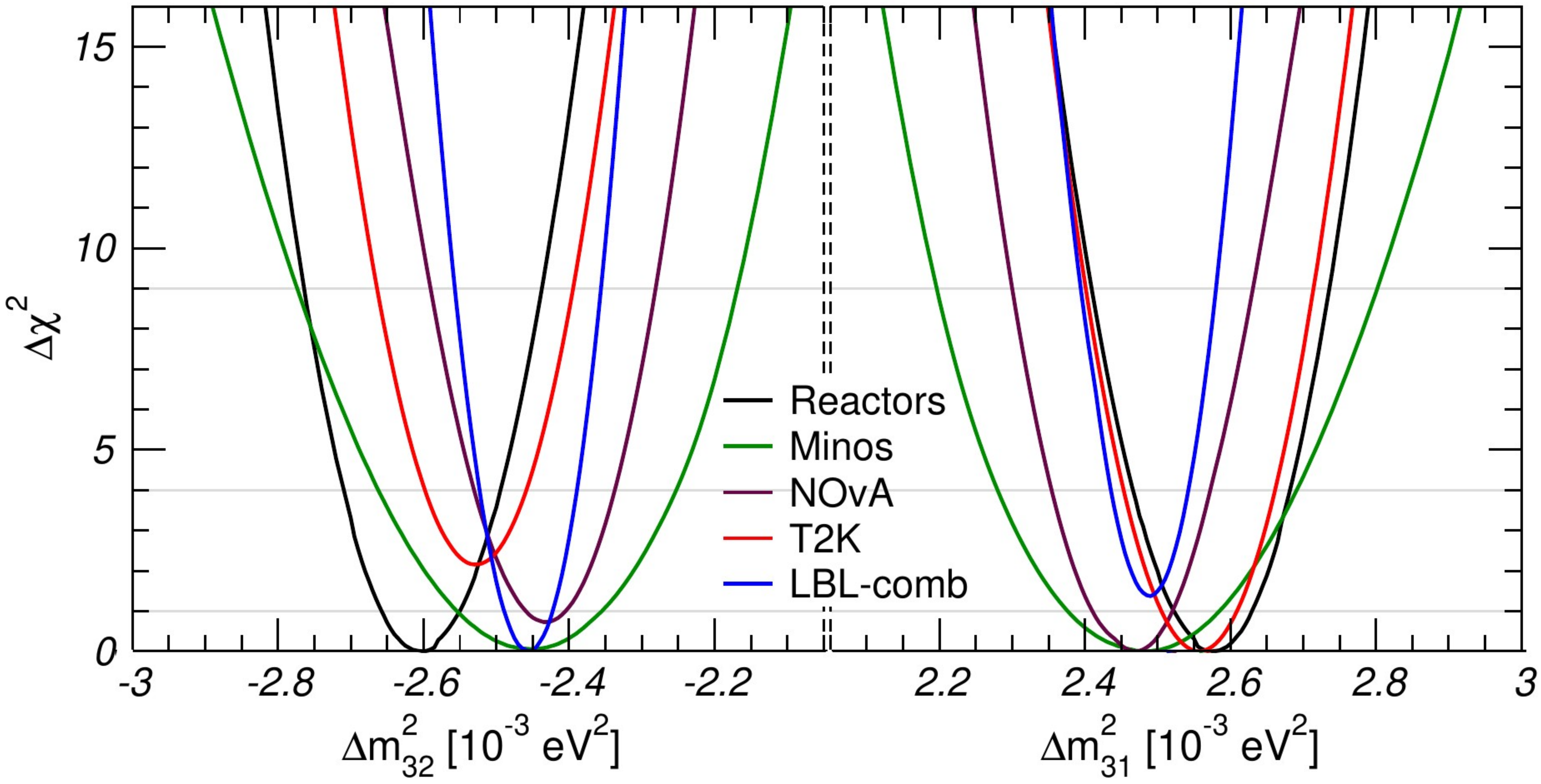


IO prefer by ~1.6 unit of $\Delta\chi^2$

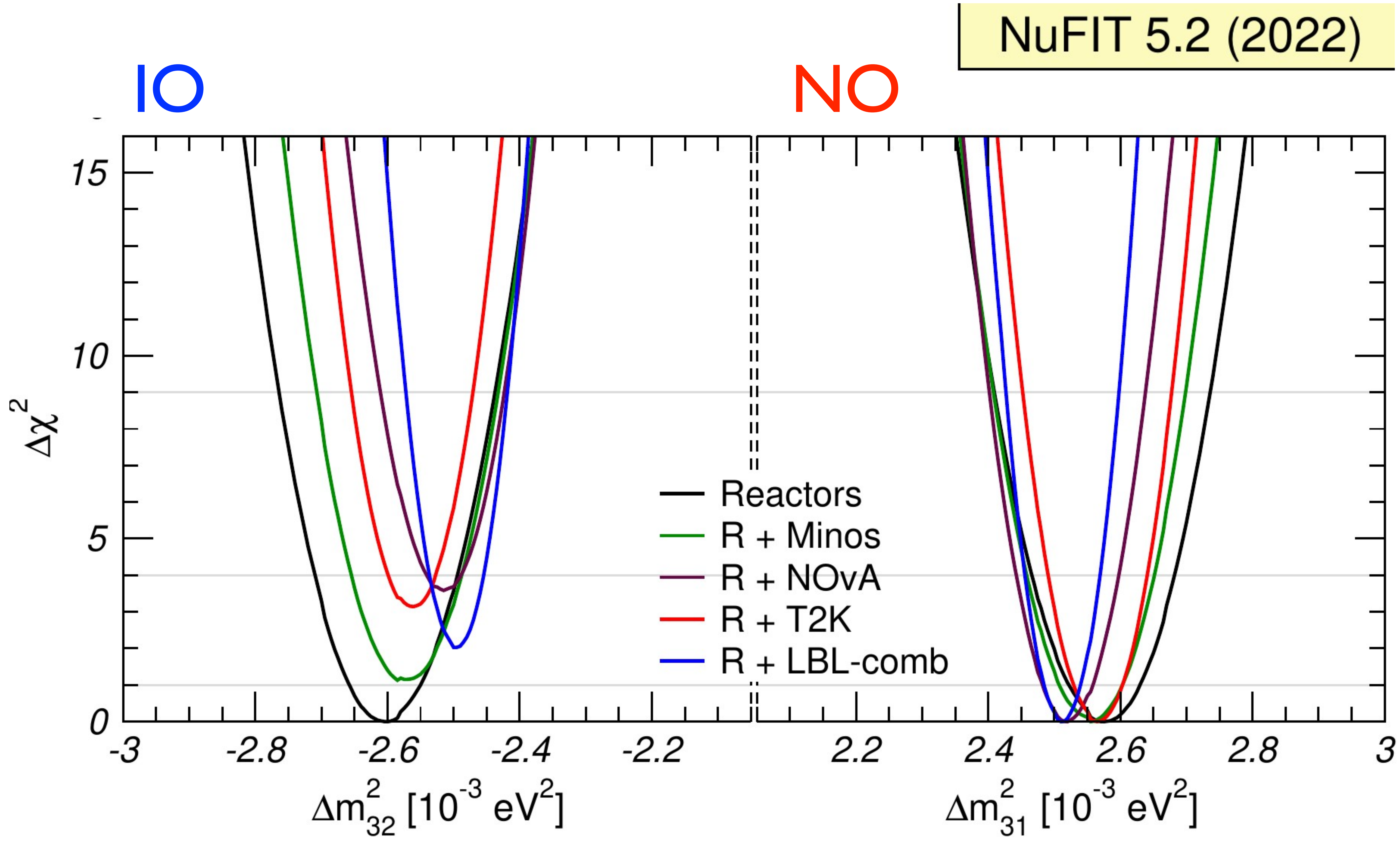
Kelly, Machado, SP, Perez, Zukanovich 2007.08526 plus other papers

Devi: Imprints of scalar mediated NSI on long baseline experiments
Mohanta: Vector leptoquark U_3: A possible solution ….. NOvA and T2K results on CP violation

IO NO NuFIT 5.2 (2022)



By construction $\Delta\chi^2_{min}$ for either (or both) NO or IO at zero

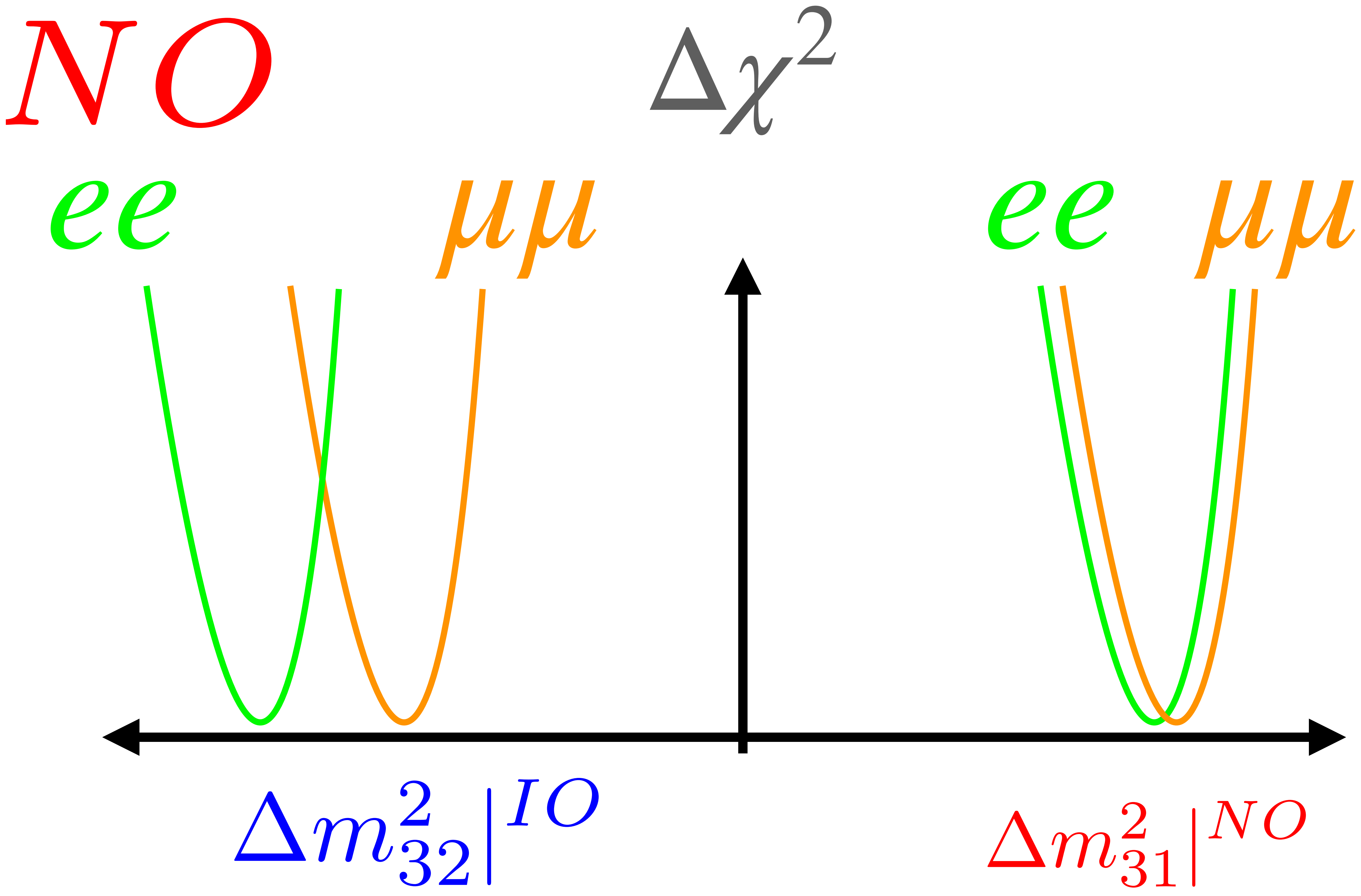


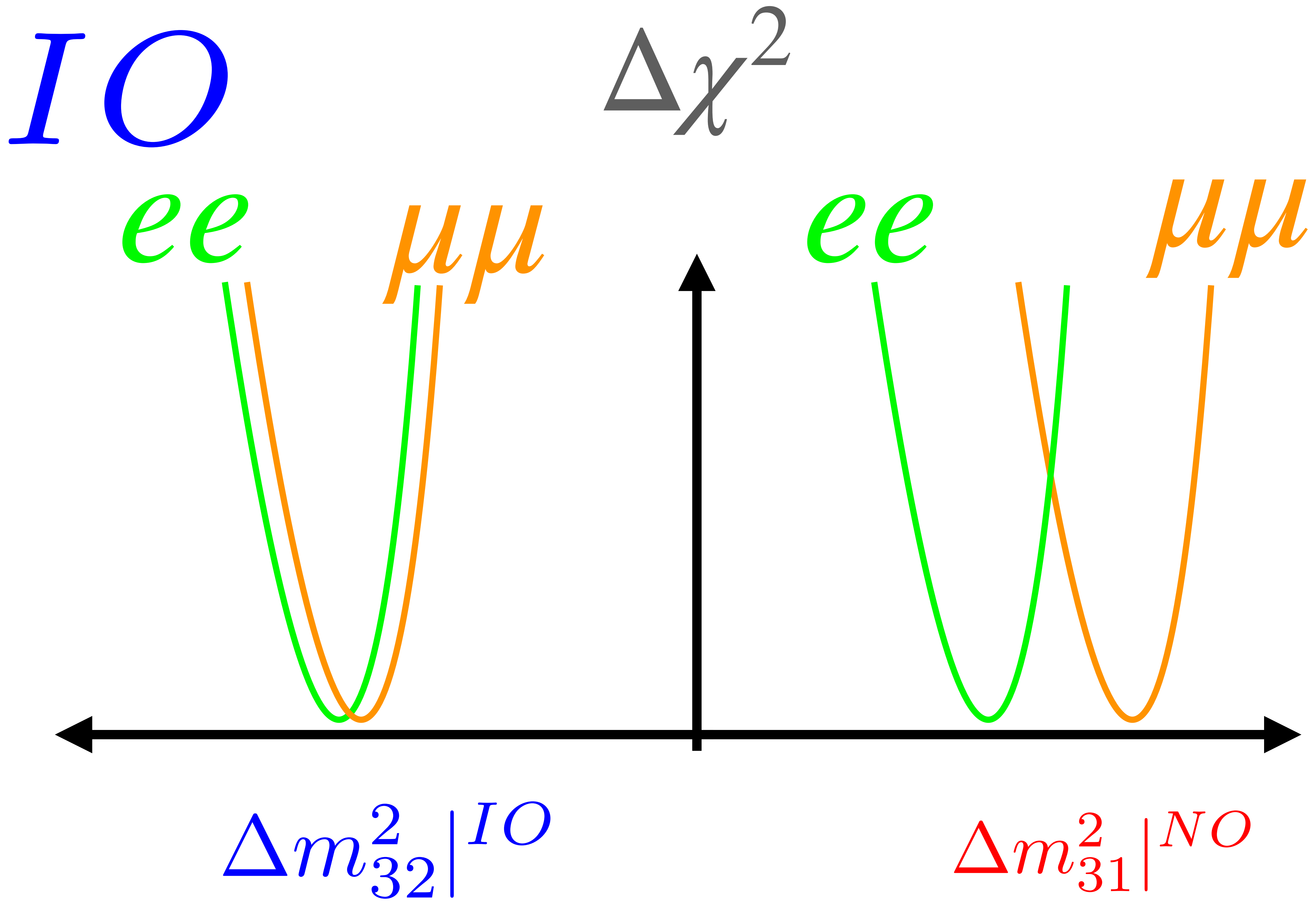
NuFIT 5.2 (2022)
IO
NO
15
10
5
0
Δχ²
-3
-2.8
-2.6
-2.4
-2.2
2.2
2.4
2.6
2.8
3
Δm²₃₂ [10⁻³ eV²]
Δm²₃₁ [10⁻³ eV²]
Reactors
R + Minos
R + NOvA
R + T2K
R + LBL-comb

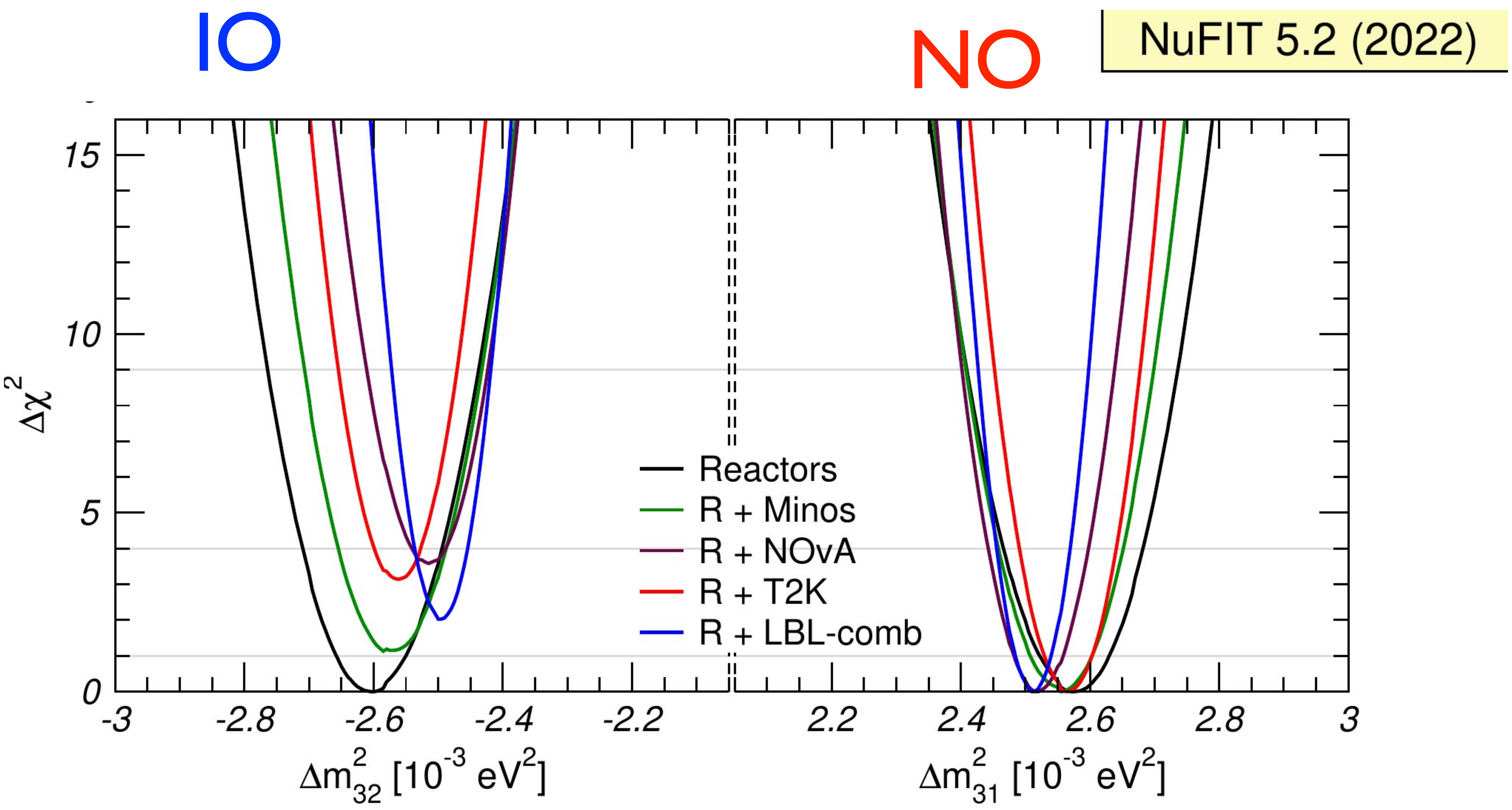
$$(\Delta m^2_{32}|^{IO}_{\mu dis} - \Delta m^2_{32}|^{IO}_{DB}) + (\Delta m^2_{31}|^{NO}_{\mu dis} - \Delta m^2_{31}|^{NO}_{DB}) = (2.4 - 0.9\cos\delta)\%\ \Delta m^2_{ee}$$

Nunokawa, SP, Zukanovich hep/0503283

	$\Delta m^2_{32}\|^{IO}_{\mu dis} - \Delta m^2_{32}\|^{IO}_{DB}$	$\Delta m^2_{31}\|^{NO}_{\mu dis} - \Delta m^2_{31}\|^{NO}_{DB}$
NO	$(2.4 - 0.9\cos\delta)\%$	≈ 0
IO	≈ 0	$(2.4 - 0.9\cos\delta)\%$







Hinting at NO and $\cos\delta \leq 0$

ν_μ disappearance at an L/E $\sim$ 500 km/GeV

$$\Delta m^2_{\mu\mu} \equiv \frac{|U_{\mu 1}|^2 \Delta m^2_{31} + |U_{\mu 2}|^2 \Delta m^2_{32}}{|U_{\mu 1}|^2 + |U_{\mu 2}|^2}$$

ν_μ average of Δm^2_{31} and Δm^2_{32}

$$\approx \Delta m^2_{ee} - (\cos 2\theta_{12} - \sin\theta_{13}\cos\delta)\Delta m^2_{21}$$

$(\sin 2\theta_{12} \tan\theta_{23} \approx 1)$

$|\Delta m^2_{ee}| > |\Delta m^2_{\mu\mu}|$ implies NO

$|\Delta m^2_{ee}| < |\Delta m^2_{\mu\mu}|$ implies IO

Nunokawa, SP, Zukanovich hep/0503283

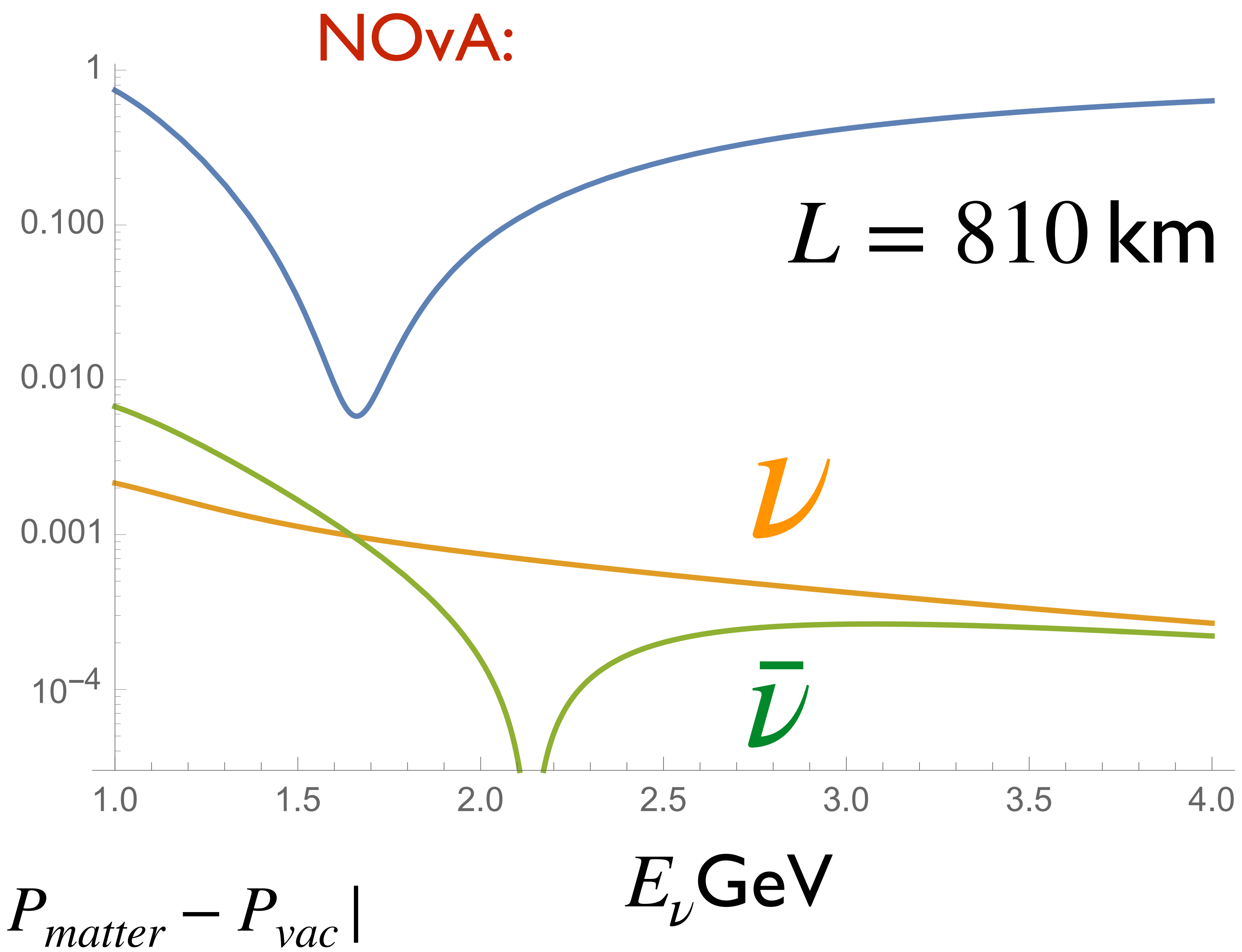
this is in vacuum, but for ν_μ disappearance matter effects are very small due to cancellations between $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ for θ_{13} effects:

$$|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = s^2_{23}c^2_{23} - s^2_{13}\cos 2\theta_{23} + s^4_{13}s^4_{23}$$

both s^2_{13} and $\cos 2\theta_{23}$ are small.

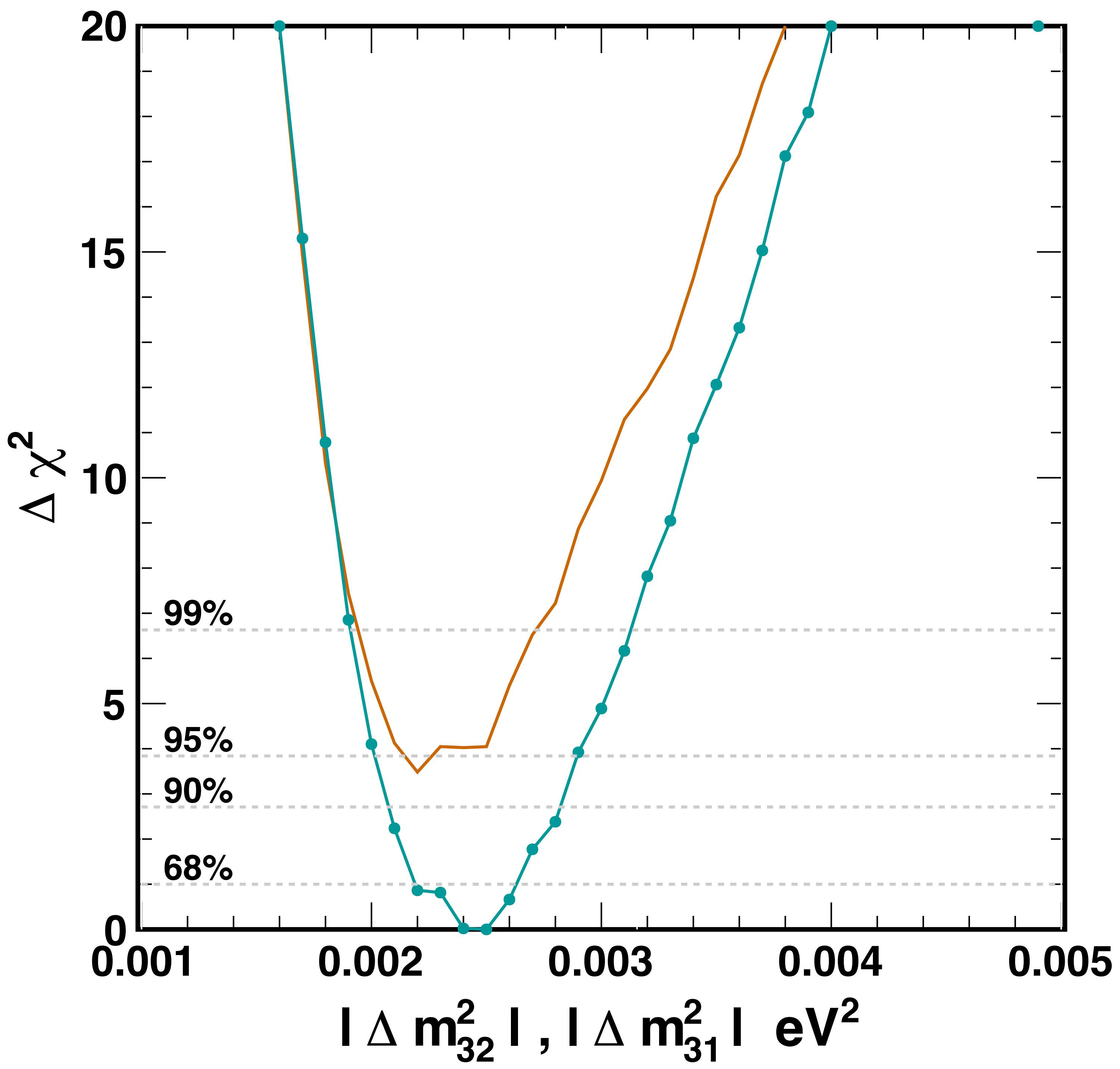
P

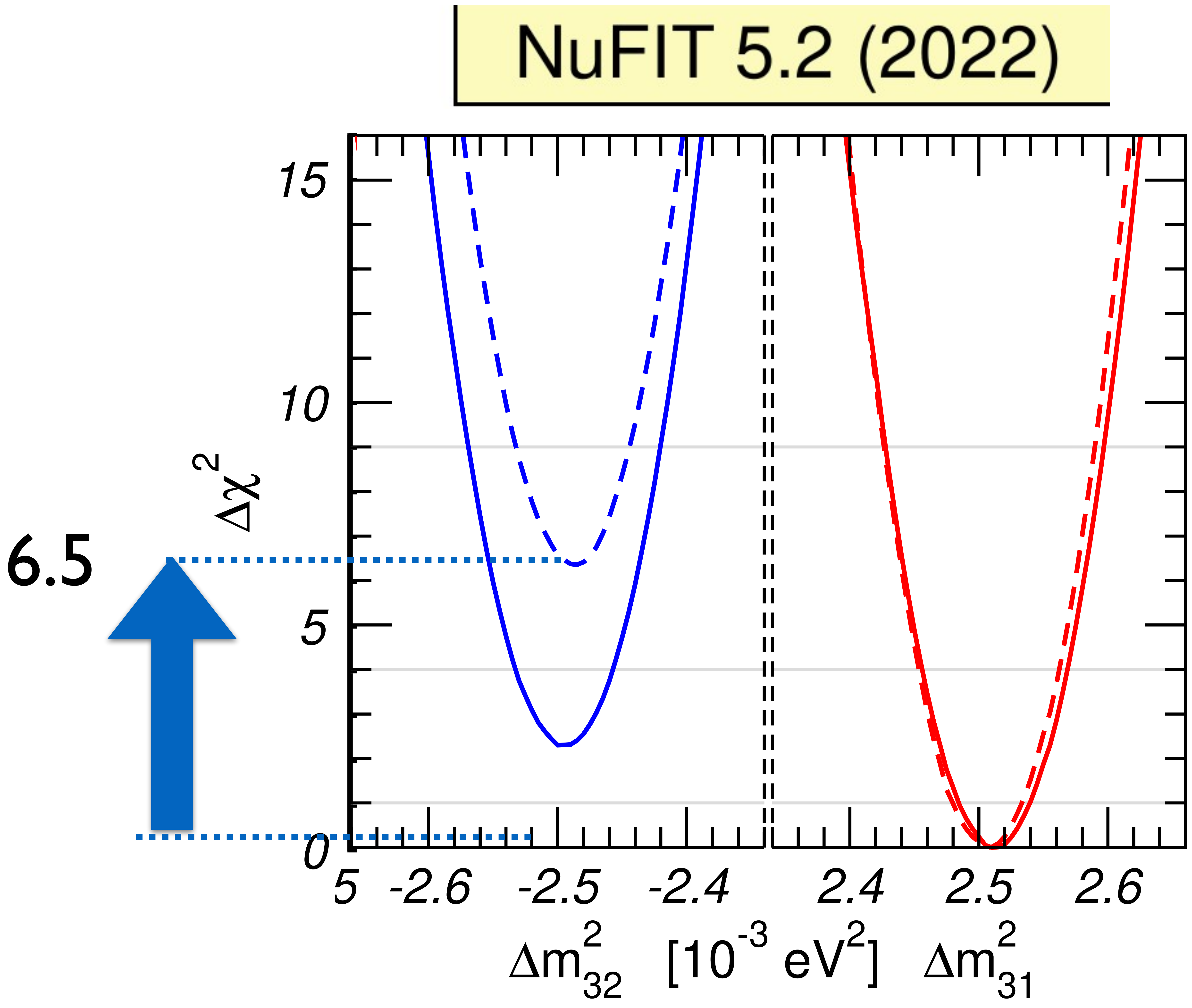
ΔP



$\Delta P \equiv |P_{matter} - P_{vac}|$

add SuperKamiokaNDE   arXiv:1710.09126



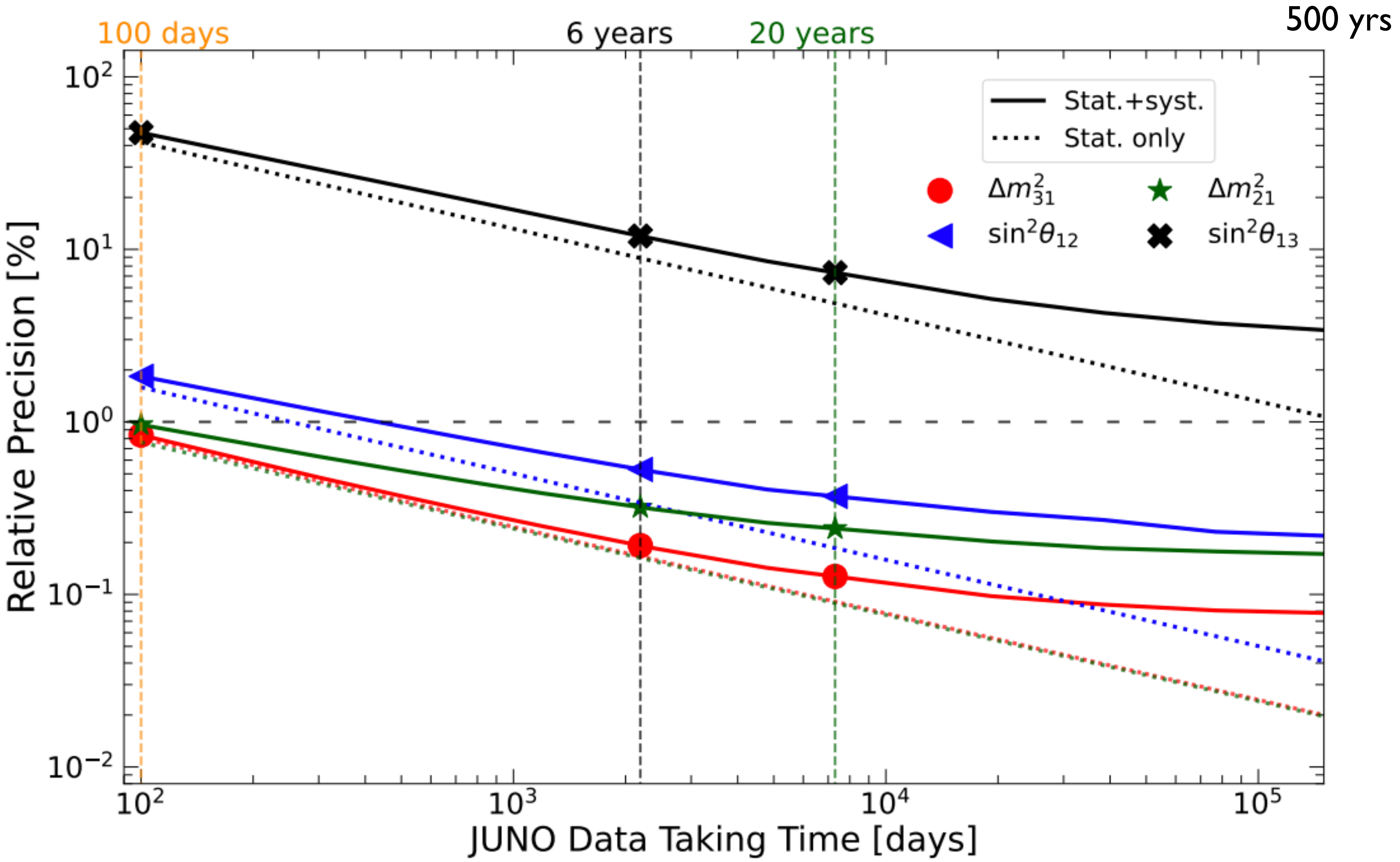


NO preference with $\Delta\chi \sim 4.0$

6.5 approx +4.0 (SK) -1.6 (App LBL) +4.1 (Dis LBL)

Time Evolution of JUNO measurements

	100 days	6 years	20 years
Δm^2_{31} ($\times 10^{-3}$ eV^2)	±0.021 (0.8%)	±0.0047 (0.2%)	±0.0029 (0.1%)
Δm^2_{21} ($\times 10^{-5}$ eV^2)	[illegible]	±0.024 (0.3%)	±0.017 (0.2%)
$\sin^2\theta_{12}$	±0.0058 (1.9%)	±0.0016 (0.5%)	±0.0010 (0.3%)
$\sin^2\theta_{13}$	±0.010 (47.9%)	±0.0026 (12.1%)	±0.0016 (7.3%)

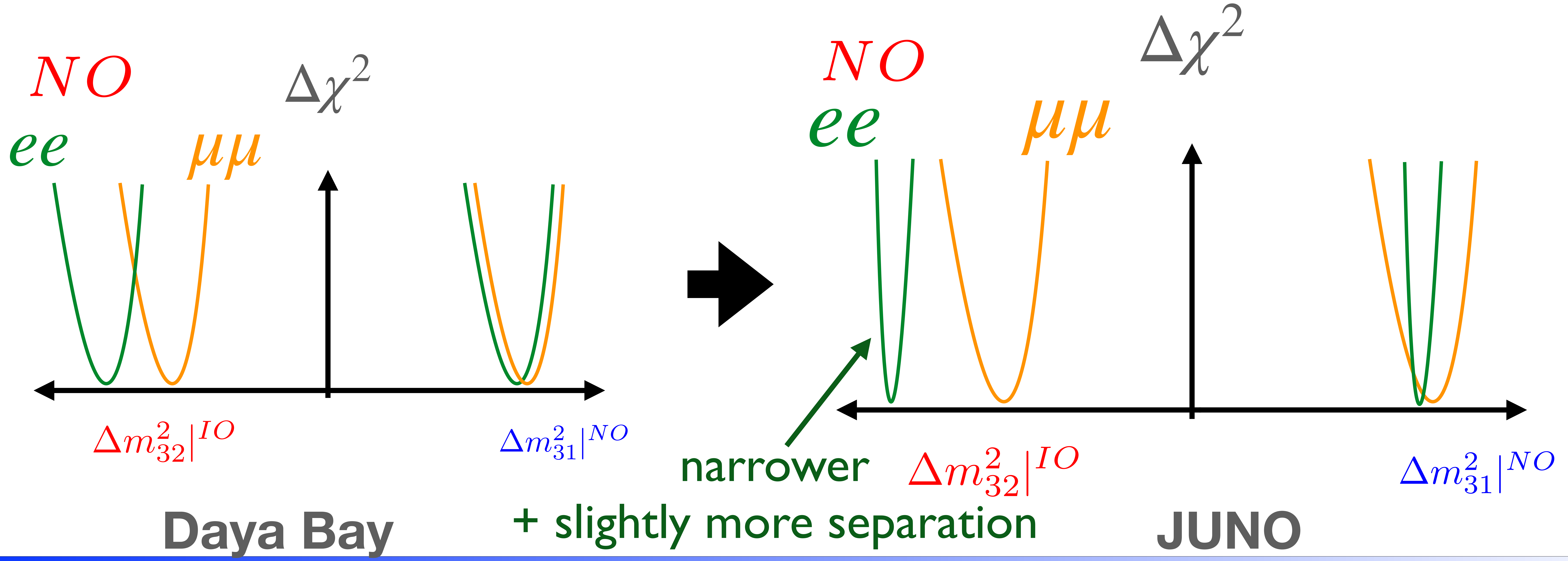


JUNO_update_2204.13249

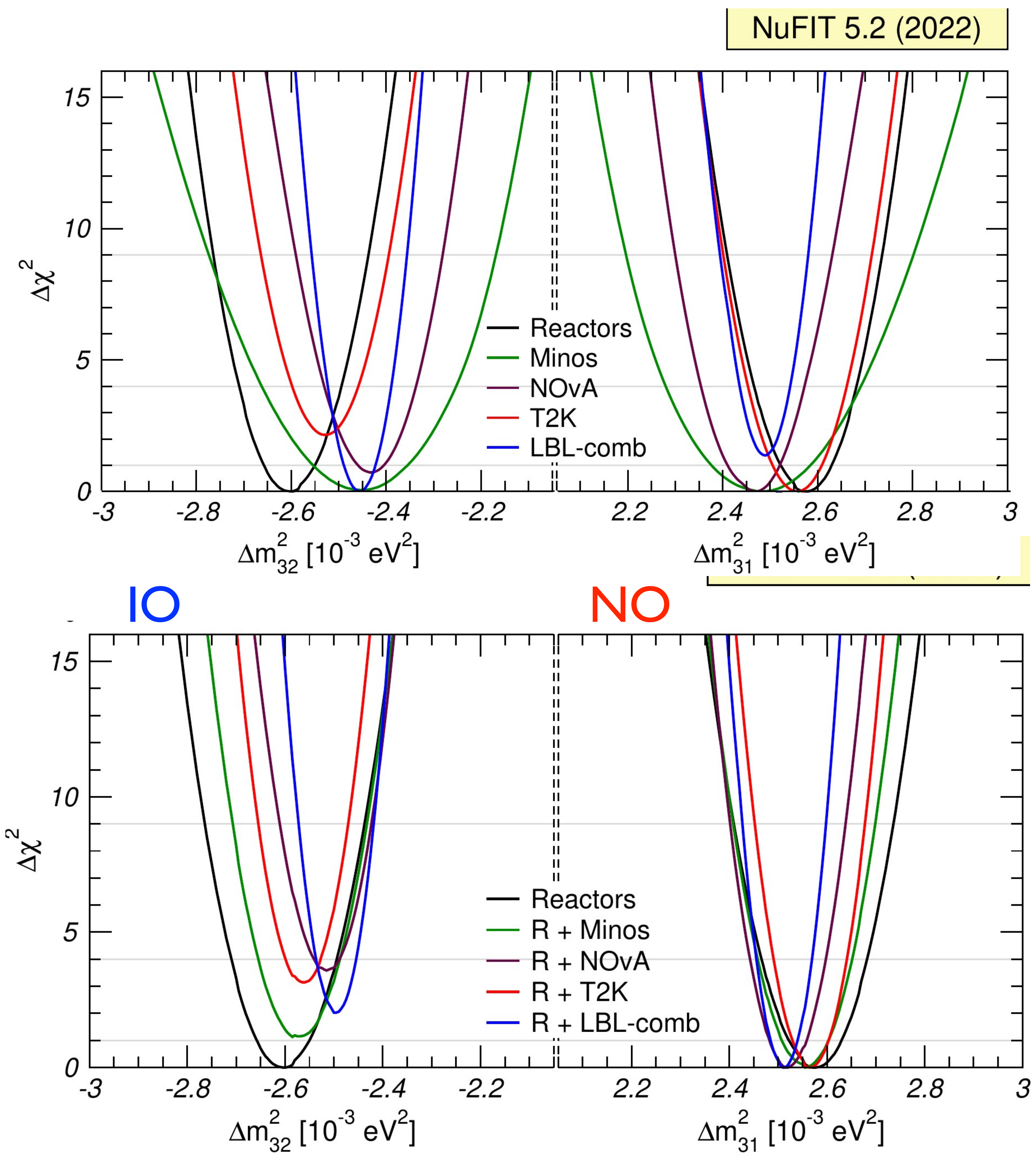
For JUNO: $|\Delta m^2_{ee}|^{IO} = 1.007\ |\Delta m^2_{ee}|^{NO}$

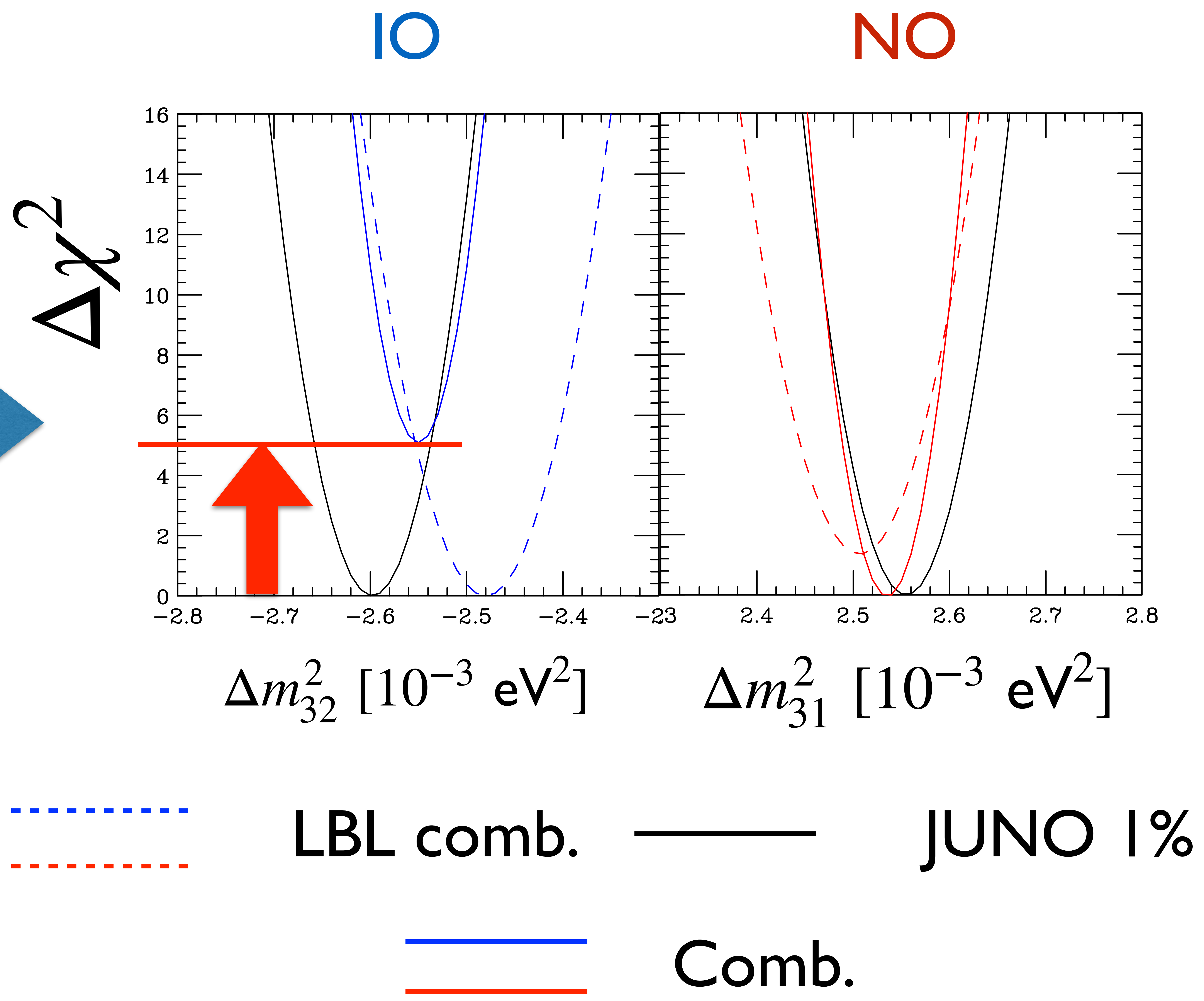
then $(2.4 - 0.9\cos\delta)\% \rightarrow (3.1 - 0.9\cos\delta)\%$

and experimental uncertainty on $|\Delta m^2_{ee}|$ drops to <1%. (Daya Bay 2.4%).

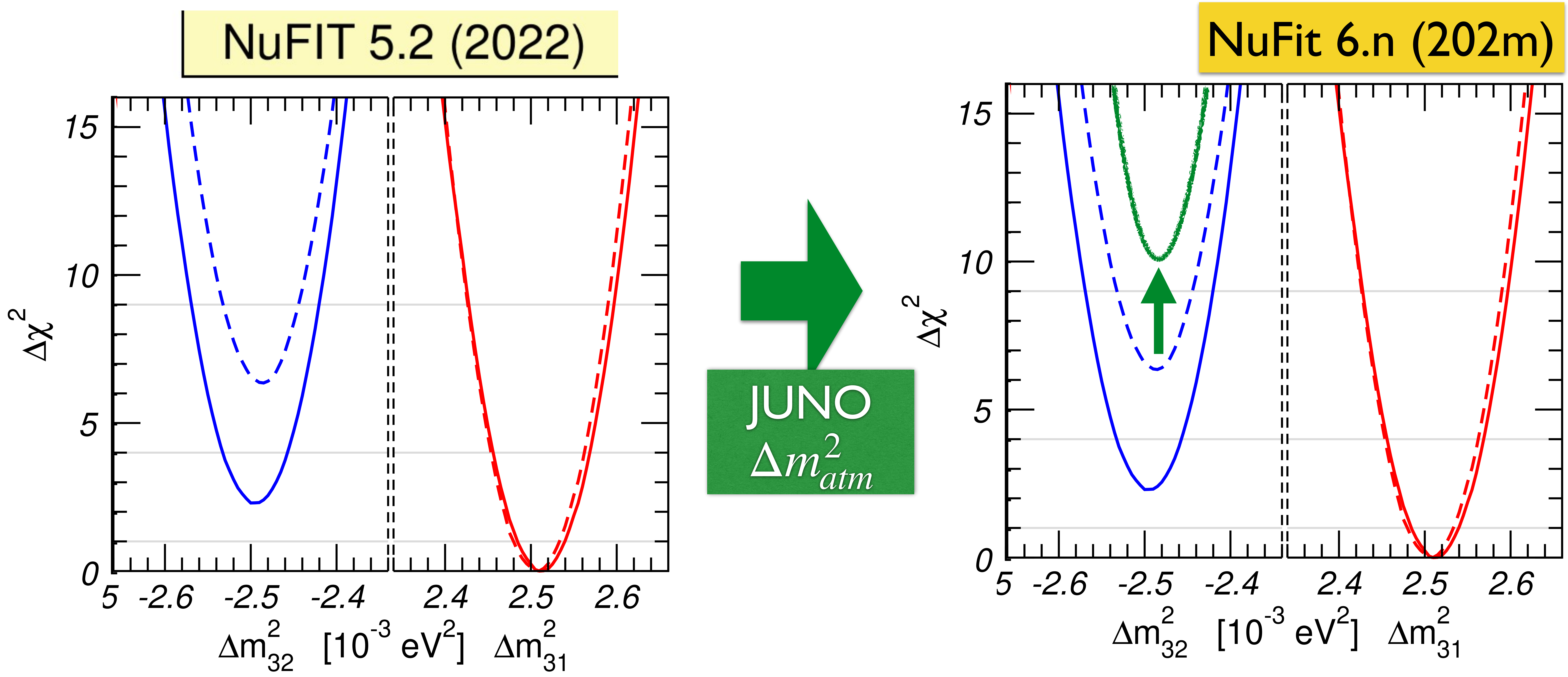


Preliminary NPZ++





Effect of JUNO's precision measurement on Δm^2_{atm}



my guess: Global Fits > 3σ at Nu 2026

- Nuclear Theory for Neutrino Physics
  - Matrix elements for $0\nu\beta\beta$
  - Nuclear Reactor $\bar{\nu}_e$ Spectra
  - Cross sections and Event Generators for Neutrino Interactions (esp. on Argon)

Summary:

- Flavor Models: Mass and Mixings and connection to Leptogenesis and other BSM physics are of paramount importance
- Understanding Neutrino Oscillation Physics, 3 or more flavors in matter, to match the precision of current and future experiments is crucial
- Nuclear Theory is important for extracting the most information out of the experiments

Extras

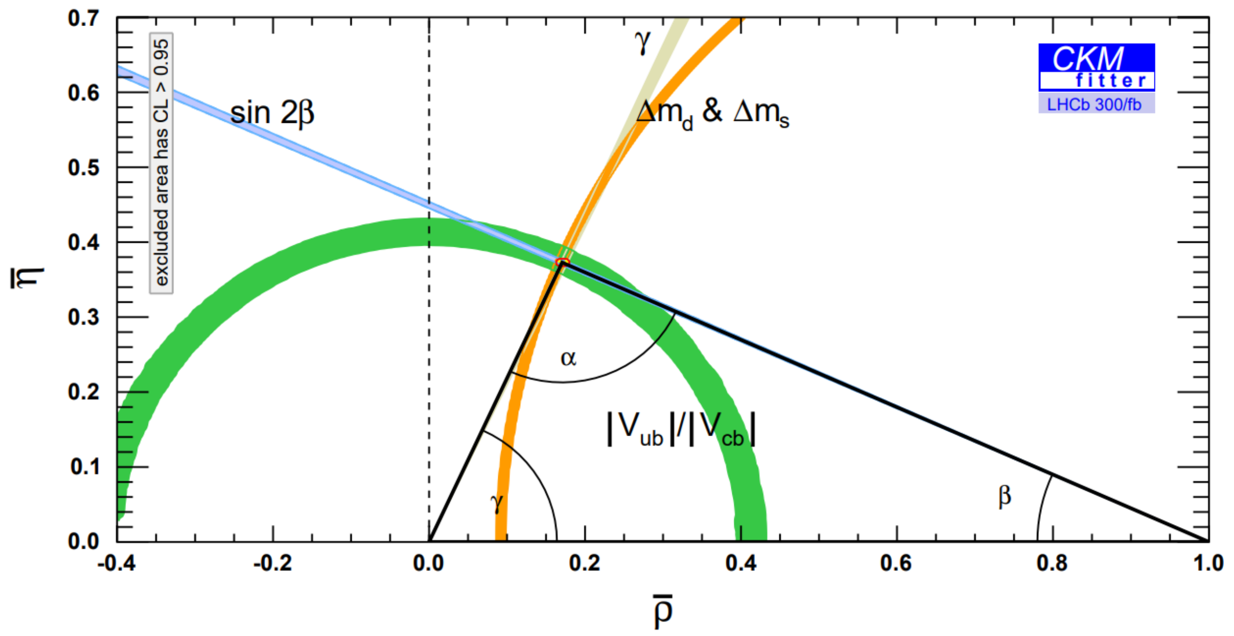
Jarlskog in Quark Sector: (see Yuehong Xie talk)

$$J_q = 2 \text{ Area} \left\{ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \right\}$$

Using Wolfenstein parameterization:

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$J_q = A^2\lambda^6\eta = (A^2\lambda^6) \times (2 \text{ Area of}$$



$$) = (3.08 \pm 0.14) \times 10^{-5}$$

where $(A^2\lambda^6) \approx 9 \times 10^{-5}$ is the scale factor for the area of Unitarity Triangle.

In the Lepton sector the Jarlskog Invariant (and hence the area of Unitarity Triangles) is potentially 1000 times larger !

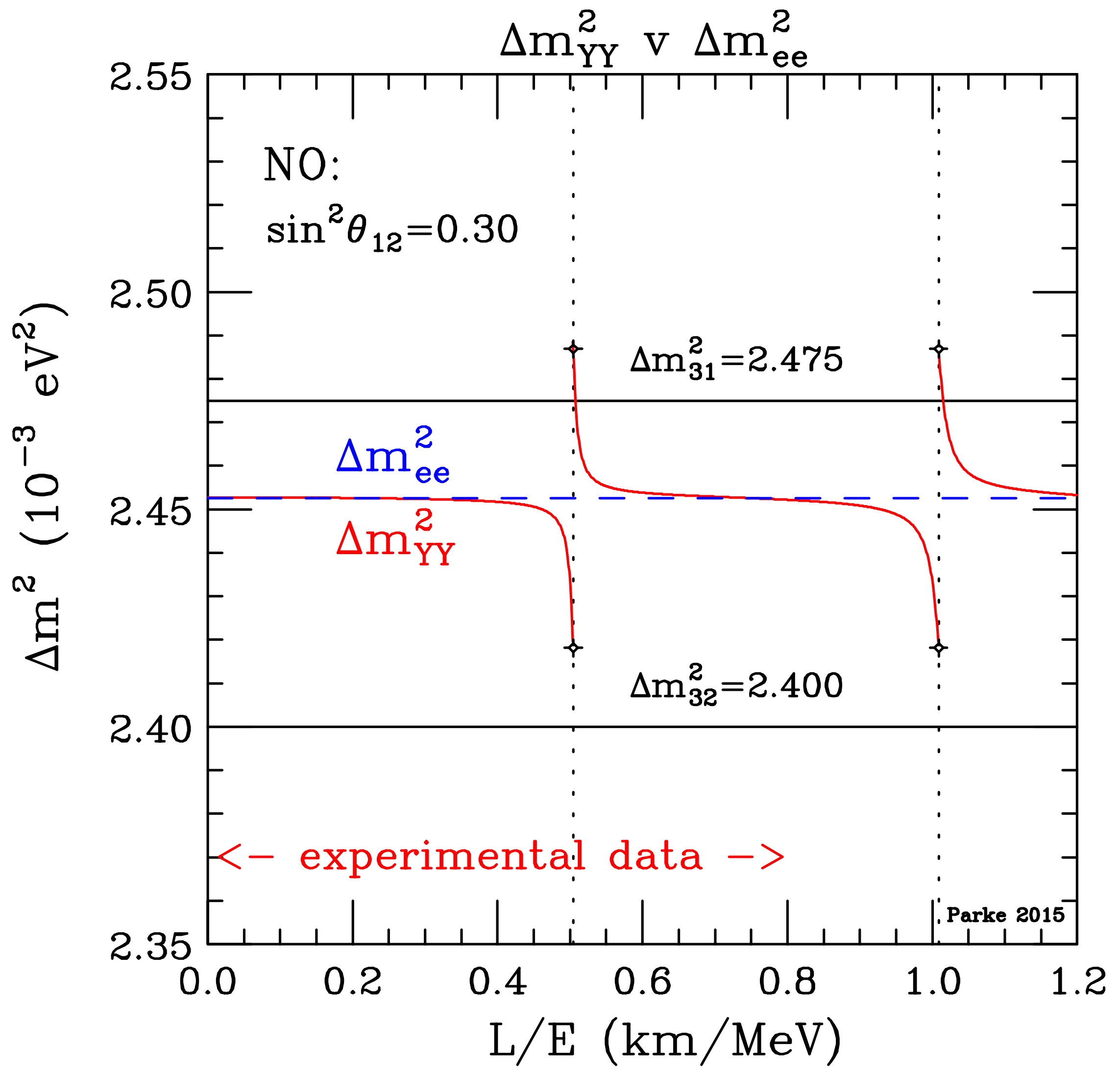
Daya Bay:

1.

$$\sin^2\Delta_{YY} \equiv \cos^2\theta_{12}\ \sin^2\Delta_{31} + \sin^2\theta_{12}\ \sin^2\Delta_{32}.$$

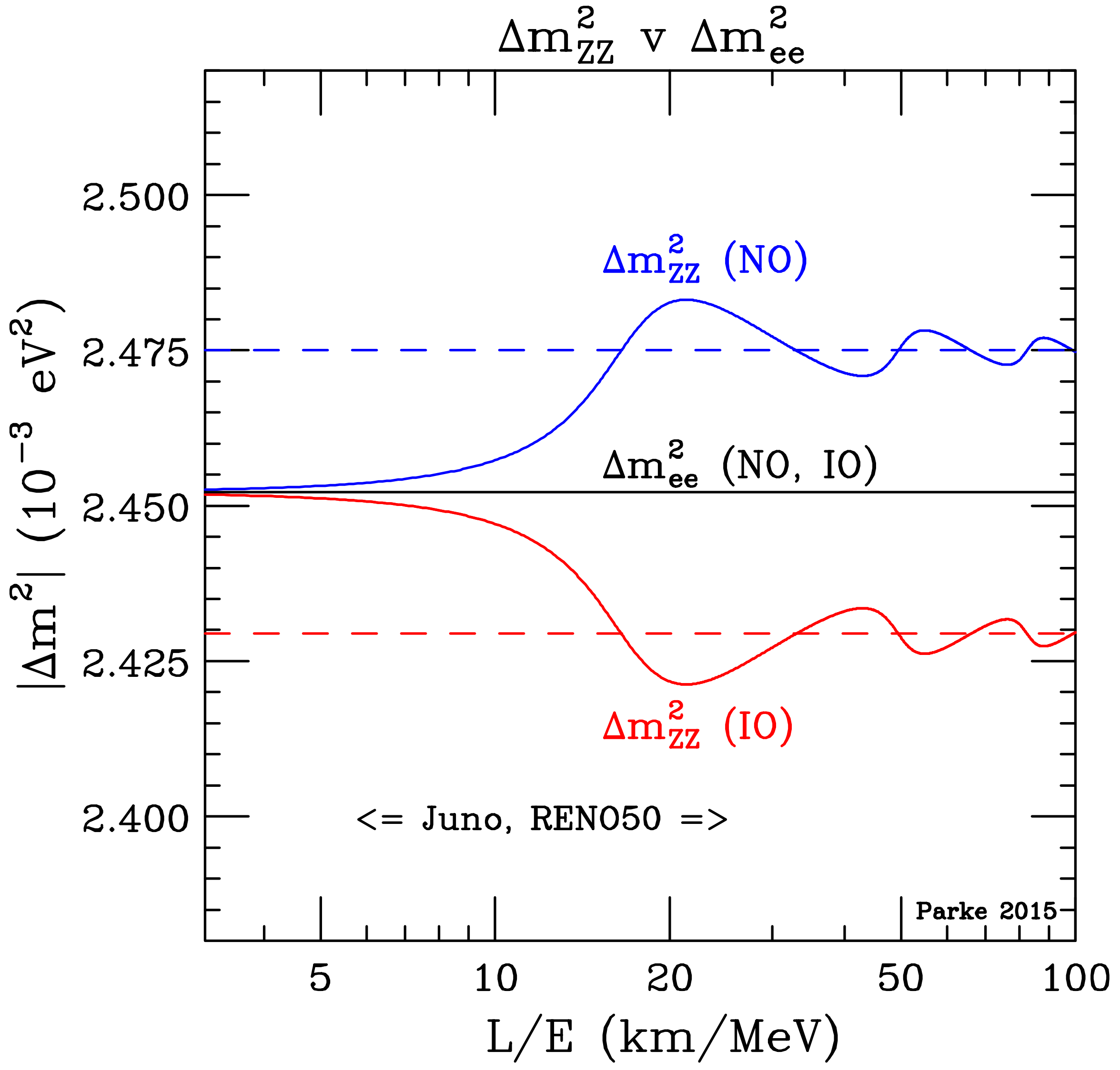
which implies that

$$\Delta m^2_{YY} \equiv \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})}\right].$$



2.

$$\Delta m^2_{ZZ} \equiv \frac{2E}{L}\left(\Delta_{31} + \Delta_{32} + \arctan[\cos 2\theta_{12}\tan\Delta_{21}]\right)$$

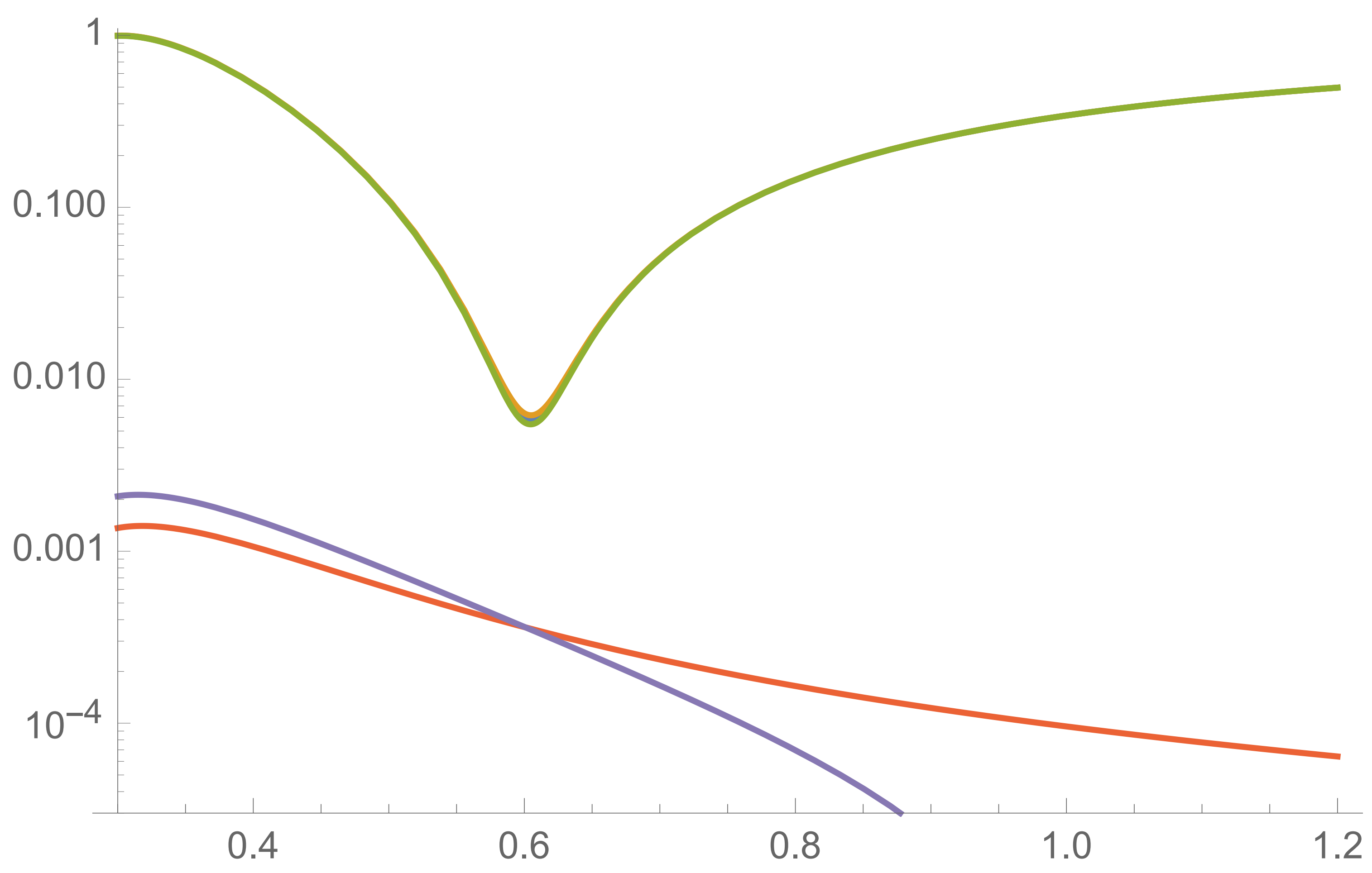


3.

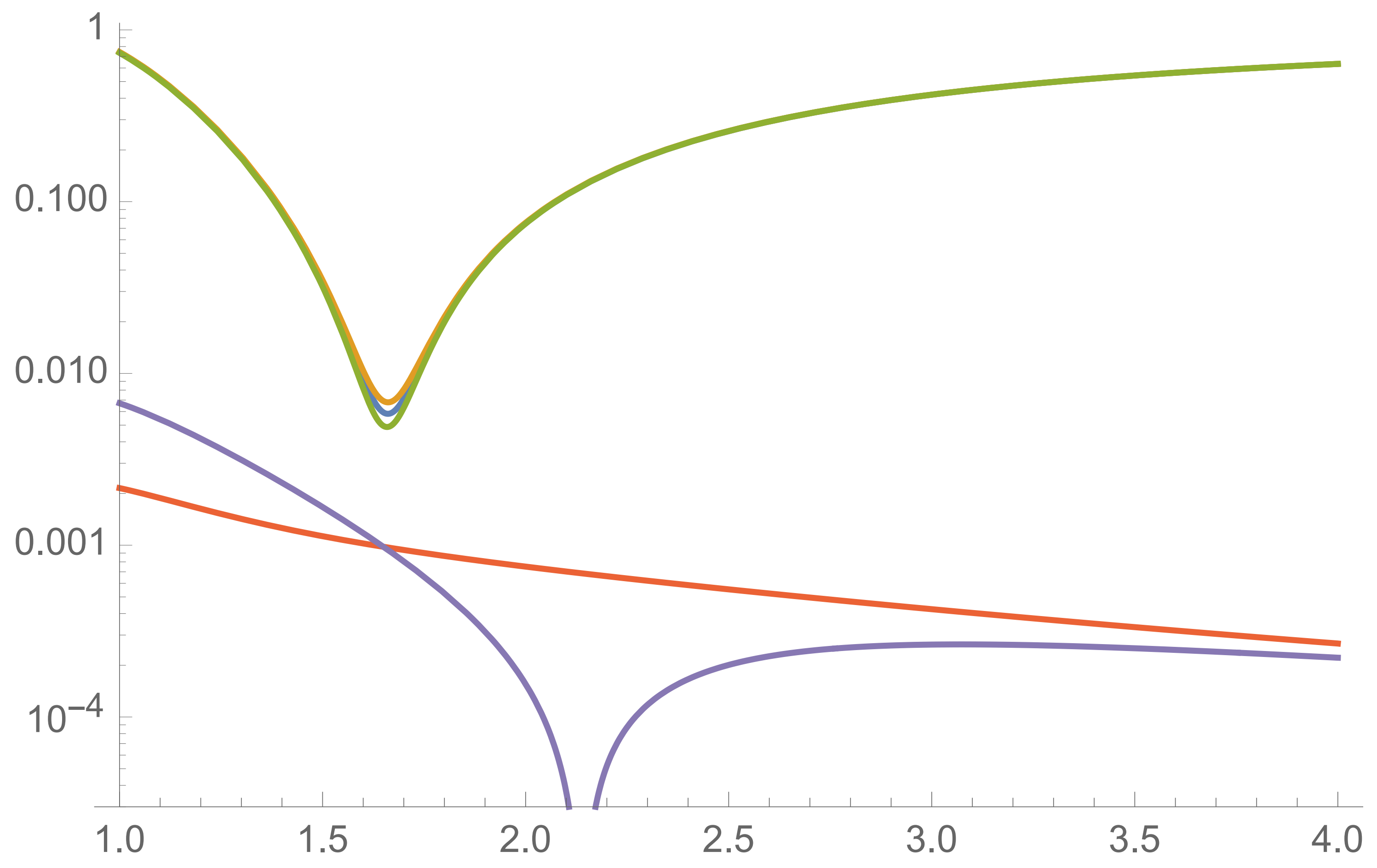
$$\Delta m^2_{ee} \equiv \frac{\partial}{\partial(L/2E)}\left(\Delta_{31} + \Delta_{32} + \arctan[\cos 2\theta_{12}\tan\Delta_{21}]\right)\Bigg|_{L/2E=0} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

NPZ'05

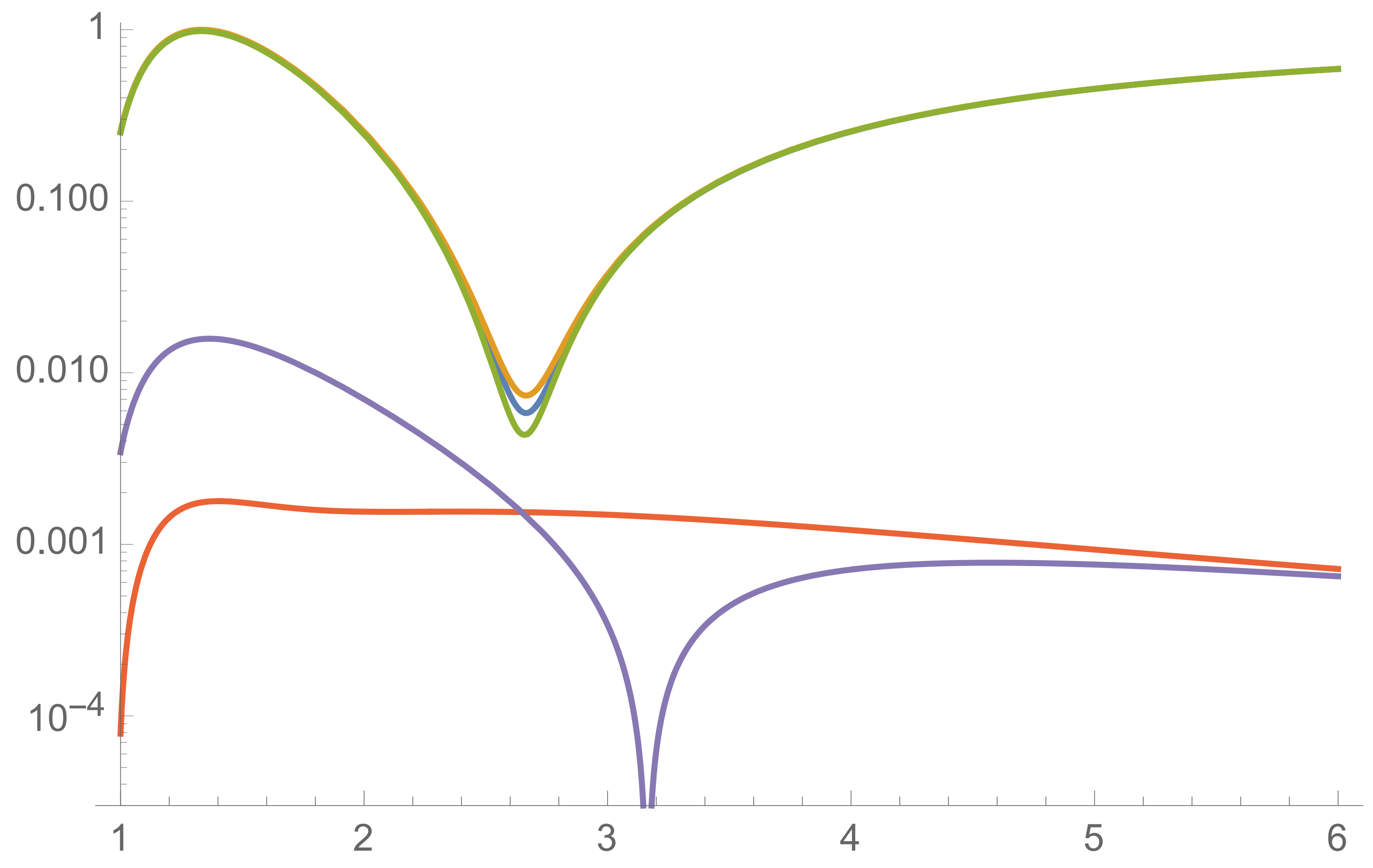
Vacuum v Matter:



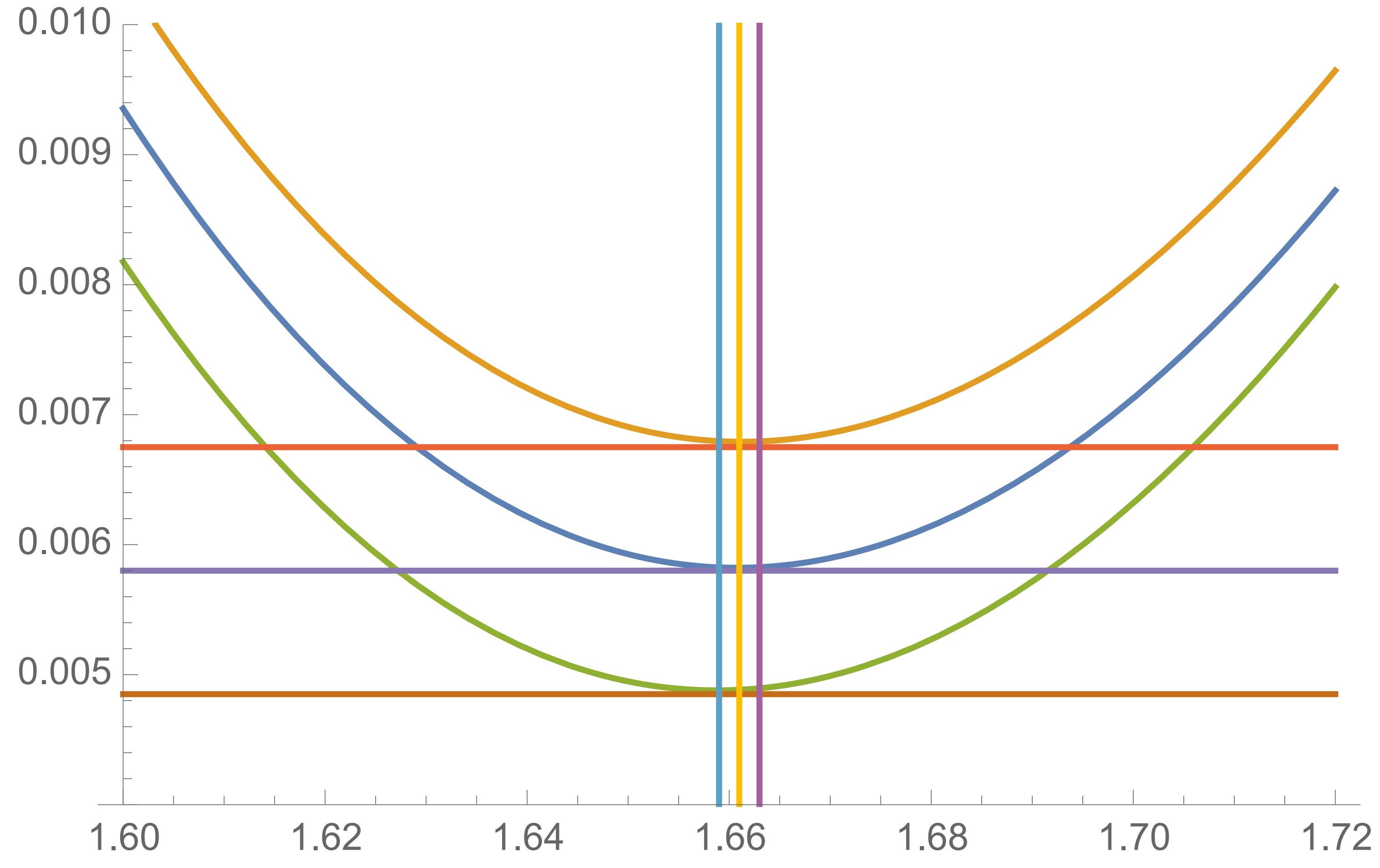
T2K



NOvA

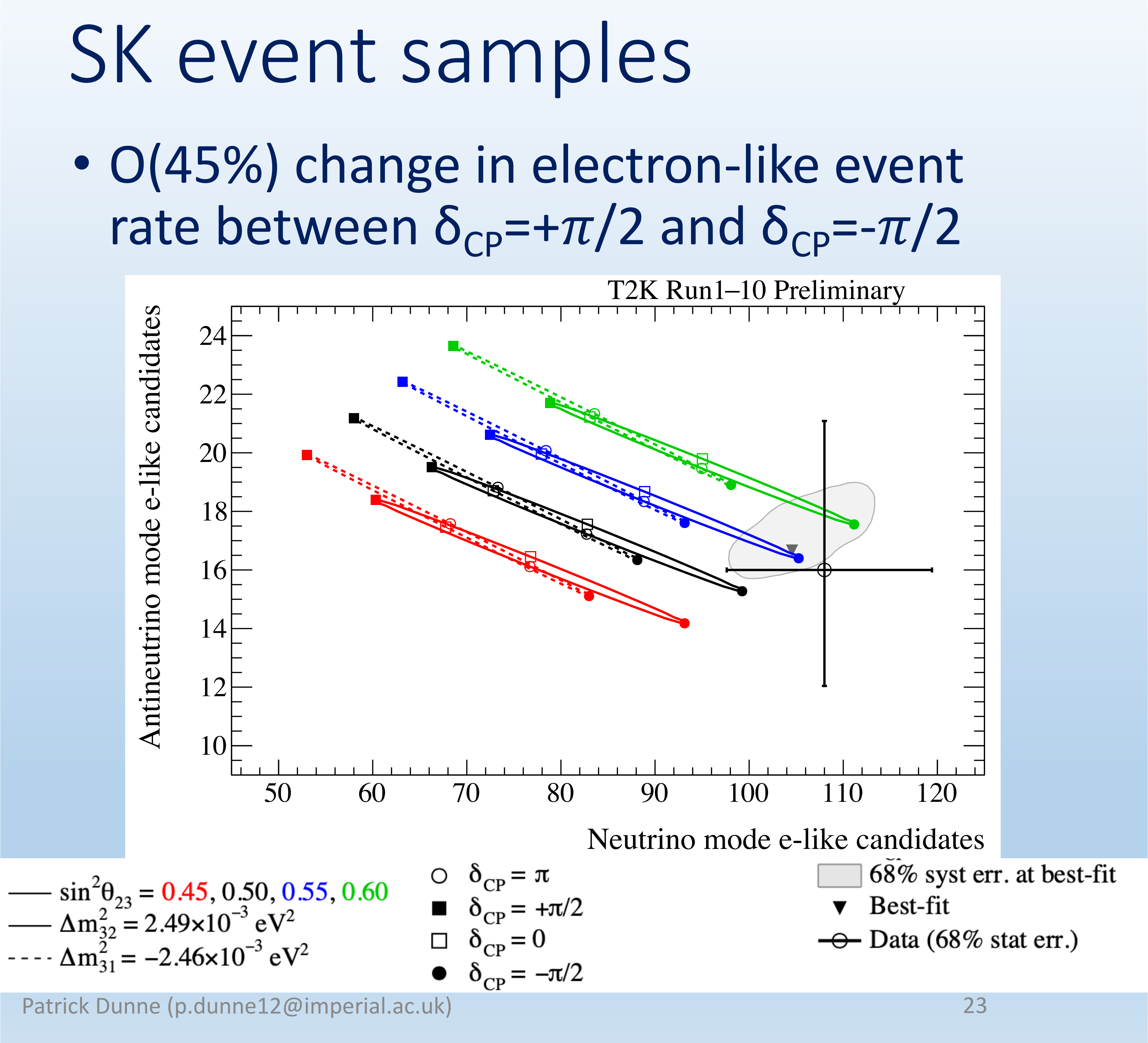


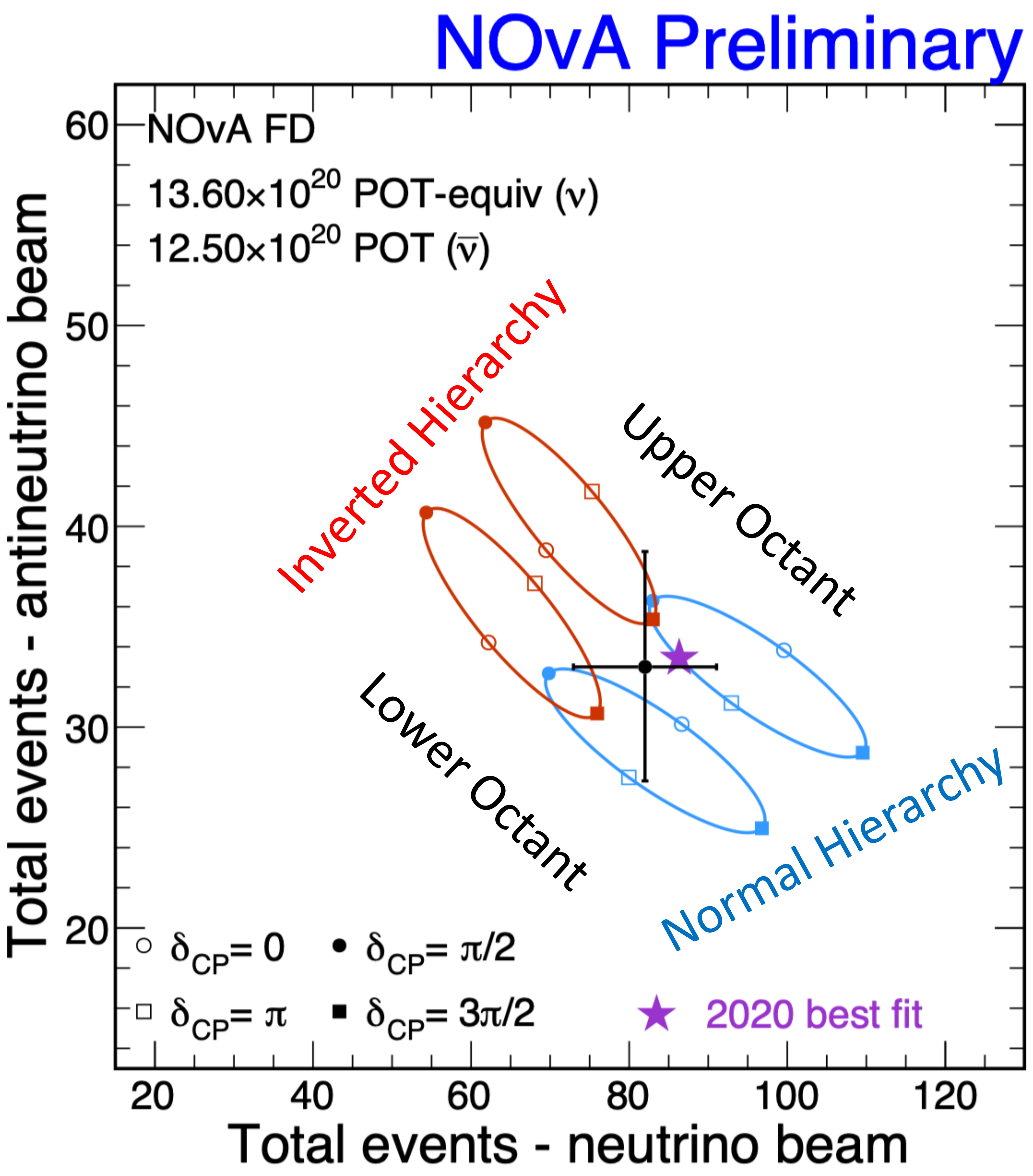
DUNE



T2K & NOvA

Number of Events proportional to Oscillation Probability





T2K NO prefer by ~2 units of χ^2

NOvA NO prefer by ~1 unit of χ^2

ν_e Disappearance:

$|\Delta m^2_{ee}|$ same for both orderings

Daya Bay:

ν_μ Disappearance:

$|\Delta m^2_{\mu\mu}|$ same for both orderings

NOvA, T2K:

$$-\Delta m^2_{32}|^{IO}_{DB} = \Delta m^2_{31}|^{NO}_{DB} + \cos 2\theta_{12}\Delta m^2_{21}$$

$$-\Delta m^2_{32}|^{IO}_{\mu dis} = \Delta m^2_{31}|^{NO}_{\mu dis} - \cos 2\theta'_{12}\Delta m^2_{21}$$

$$\cos 2\theta_{12} \approx 0.40$$

$$\cos 2\theta'_{12} = \cos 2\theta_{12} - 2 s_{13} \cos\delta \approx 0.40 - 0.30 \cos\delta$$

If IO then 0

If NO then 0

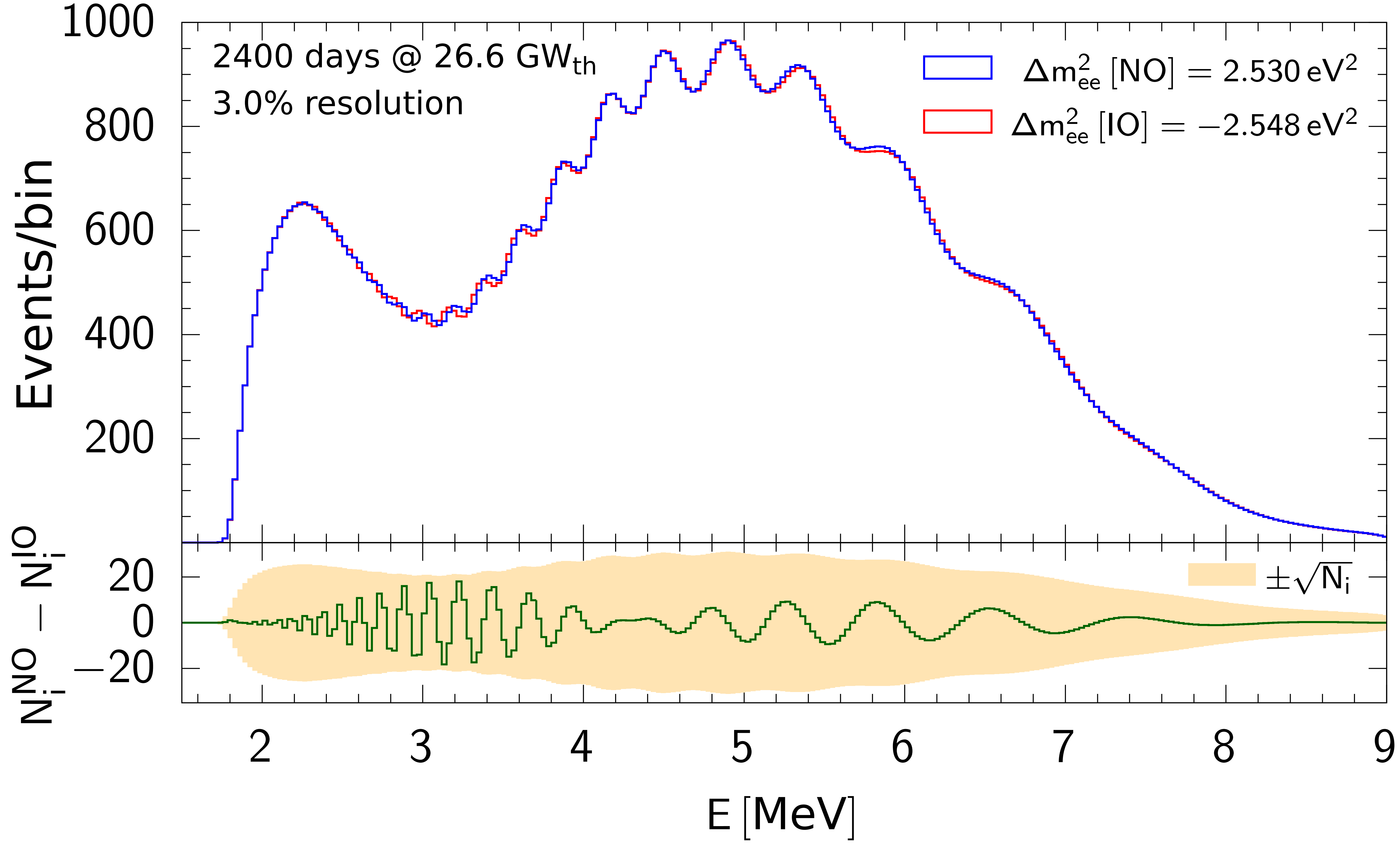
$$(\Delta m^2_{32}|^{IO}_{\mu dis} - \Delta m^2_{32}|^{IO}_{DB}) + (\Delta m^2_{31}|^{NO}_{\mu dis} - \Delta m^2_{31}|^{NO}_{DB}) = (2.4 - 0.9\cos\delta)\%\ \Delta m^2_{ee}$$

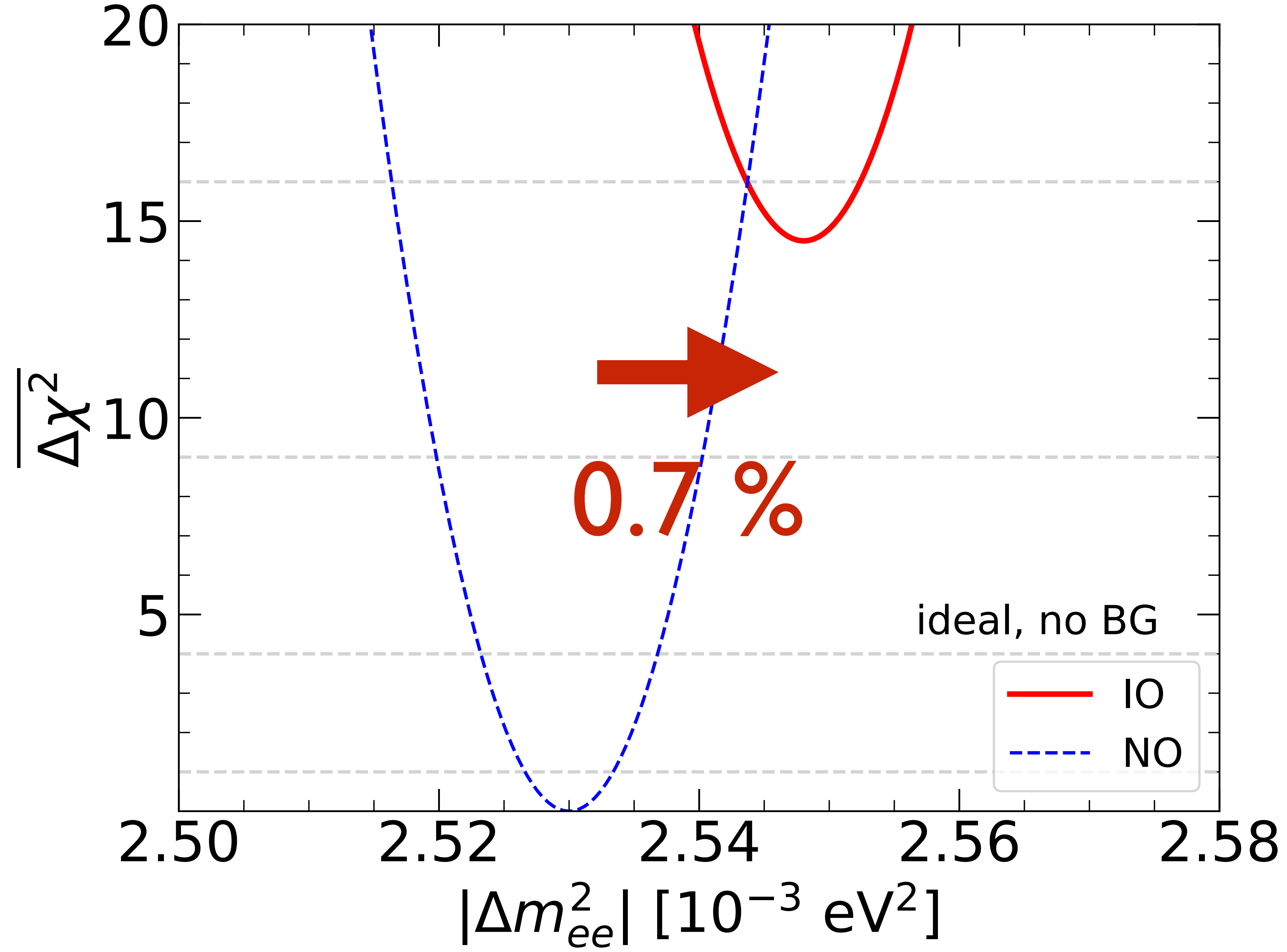
1.5 to 3.3 %

Unchanged if $31 \leftrightarrow 32$ in either or both MO's

JUNO Events Spectra

No backgrounds, No Systematics





8 years, 26.6 GW_th
baseline exactly 52.5 km
3.0 % resolution

If $|\Delta m^2_{32}|(IO) = |\Delta m^2_{32}|(NO)$, then $|\Delta m^2_{ee}|(IO) = 2.428$

If $|\Delta m^2_{31}|(IO) = |\Delta m^2_{31}|(NO)$, then $|\Delta m^2_{ee}|(IO) = 2.578$

If $|\Delta m^2_{32}|(IO) = |\Delta m^2_{31}|(NO)$, then $|\Delta m^2_{ee}|(IO) = 2.503$

Forero, SP, Ternes, Zukanovich 2107.12410