NUCLEON STRUCTURE FUNCTIONS FROM A LATTICE COMPTON AMPLITUDE CALCULATION

K. U. Can (Adelaide), M. Batelaan (Adelaide), A. Hannaford-Gunn (Adelaide), R. Horsley (Edinburgh), Y. Nakamura (RIKEN|Kobe), H. Perlt (Leipzig), P. E .L. Rakow (Liverpool), G. Schierholz (DESY), H. Stüben (Hamburg), R. D. Young (Adelaide), J. M. Zanotti (Adelaide)

(CSSM & QCDSF/UKQCD Collaborations)

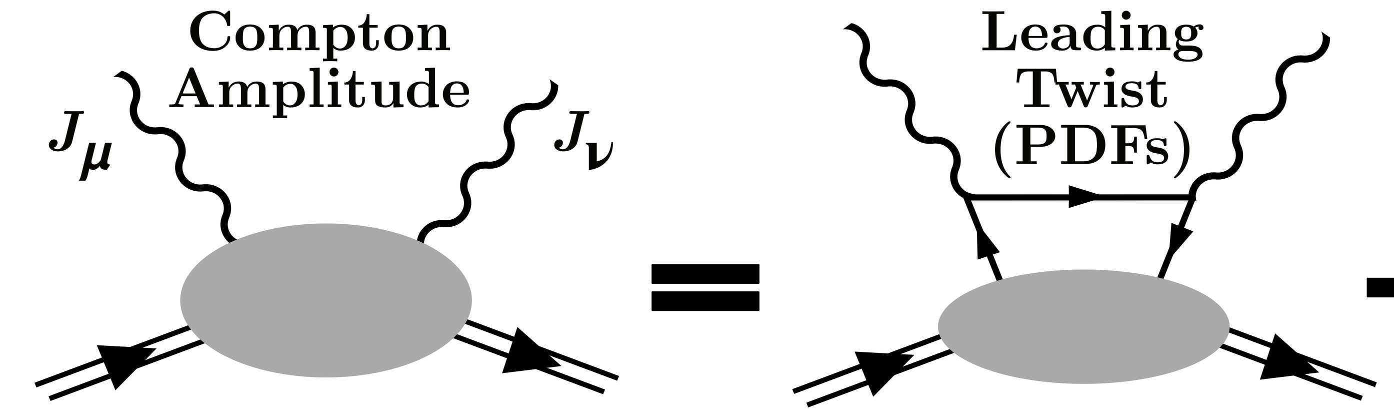
Based on:

- M. Batelaan et al. Phys.Rev.D 107 (2023) 5, 054503 [2209.04141]
- K. U. Can et al., Phys.Rev.D 102 (2020) 114505 [2007.01523]

Motivation

- Nucleon is a composite object of (anti-)quarks and gluons,
- Longitudinal distribution of its constituents (partons) are encoded in structure functions (SFs).
- At energies $Q^2 \gg M_N^2$, and excluding the resonances, structure functions reduce to parton distribution functions (PDFs).
- We aim to: Determine the SFs, from a first-principles approach,
- Constraint the low- and high-x regions of PDFs better,
- Identify the Q^2 region where power corrections become relevant.

Forward Compton Amplitude



$$= \ldots + \mathcal{O}\left(\frac{M_N}{Q^2}, \frac{1}{Q^2}\right)$$

- The forward Compton amplitude is described by the time-ordered product of electromagnetic currents sandwiched btw. nucleon states,

$$T_{\mu\nu}(p,q) = \int d^4z e^{iq\cdot z} \rho_{ss'} \langle p, s' | \mathcal{T}\{J_\mu(z) J_\nu(0)\} | p, s\rangle$$
$$= \left(g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \mathcal{F}_1(\omega, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{p\cdot q} \mathcal{F}_2(\omega, Q^2)$$

- $T_{\mu\nu}$ is parametrised by two Compton structure functions (SFs), $\mathcal{F}_i$, and we have defined, $\hat{P}_\mu \equiv p_\mu - q_\mu (p\cdot q/q^2)$, with $Q^2 = -q^2$ and $\omega = 2(p.q)/Q^2$.
- Compton SFs can be expanded in terms of the moments of DIS structure functions, e.g.,

$$\mathcal{F}_2(\omega, Q^2) = \sum_{n=1}^{N} 4\omega^{2n-1} \underbrace{\int_0^1 dx x^{2n-2} F_2(x, Q^2)}_{M_{2n}^{(2)}(Q^2)} \qquad \textbf{(Eq. 1)}$$

Method: Feynman-Hellmann Theorem (FHT)

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

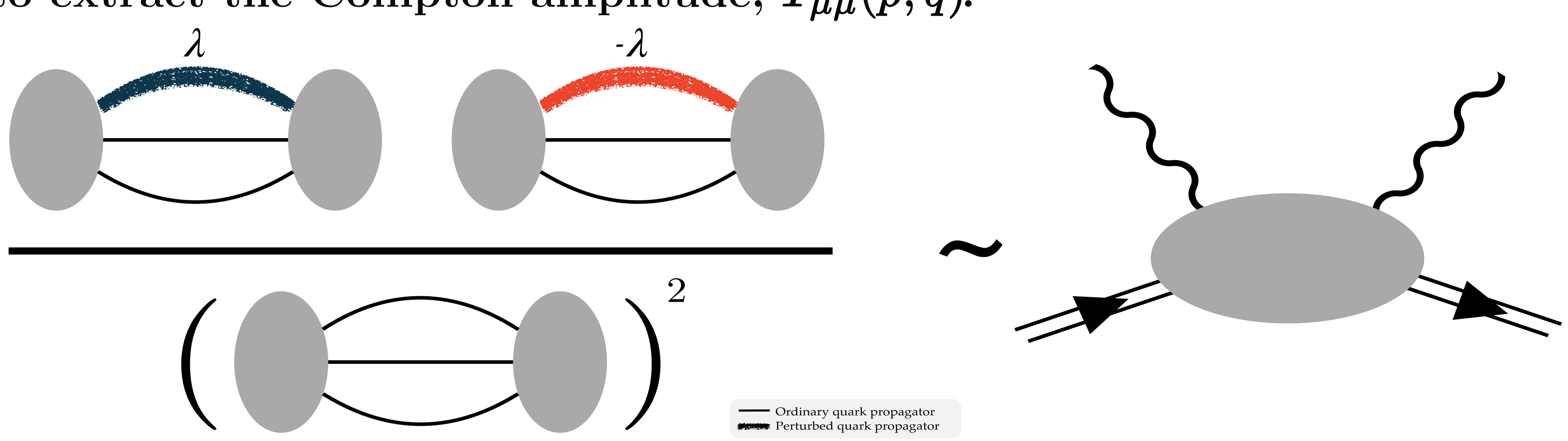
- FHT: In quantum mechanics, at 1st order, expectation value of a perturbed system is related to the energy shift.
- In QCD, we add an oscillating EM background field to the action,

$$S_{QCD} \to S_{QCD}(\lambda) = S_{QCD} + \lambda \int d^4z \cos(\mathbf{q}\cdot\mathbf{z})\, J_\mu(z)$$

- and extend the FHT to 2nd order to access the Compton amplitude,

$$\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial \lambda^2}\right|_{\lambda=0} = -\frac{1}{2E_N(\mathbf{p})} \overbrace{\int d^4z e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p) | \mathcal{T}\{J_\mu(z) J_\mu(0)\} | N(p)\rangle + (q \to -q)}^{T_{\mu\mu}(p,q)}$$

- Finally, we calculate a ratio of 2-point Green's functions as sketched, to extract the Compton amplitude, $T_{\mu\mu}(p,q)$.



Results

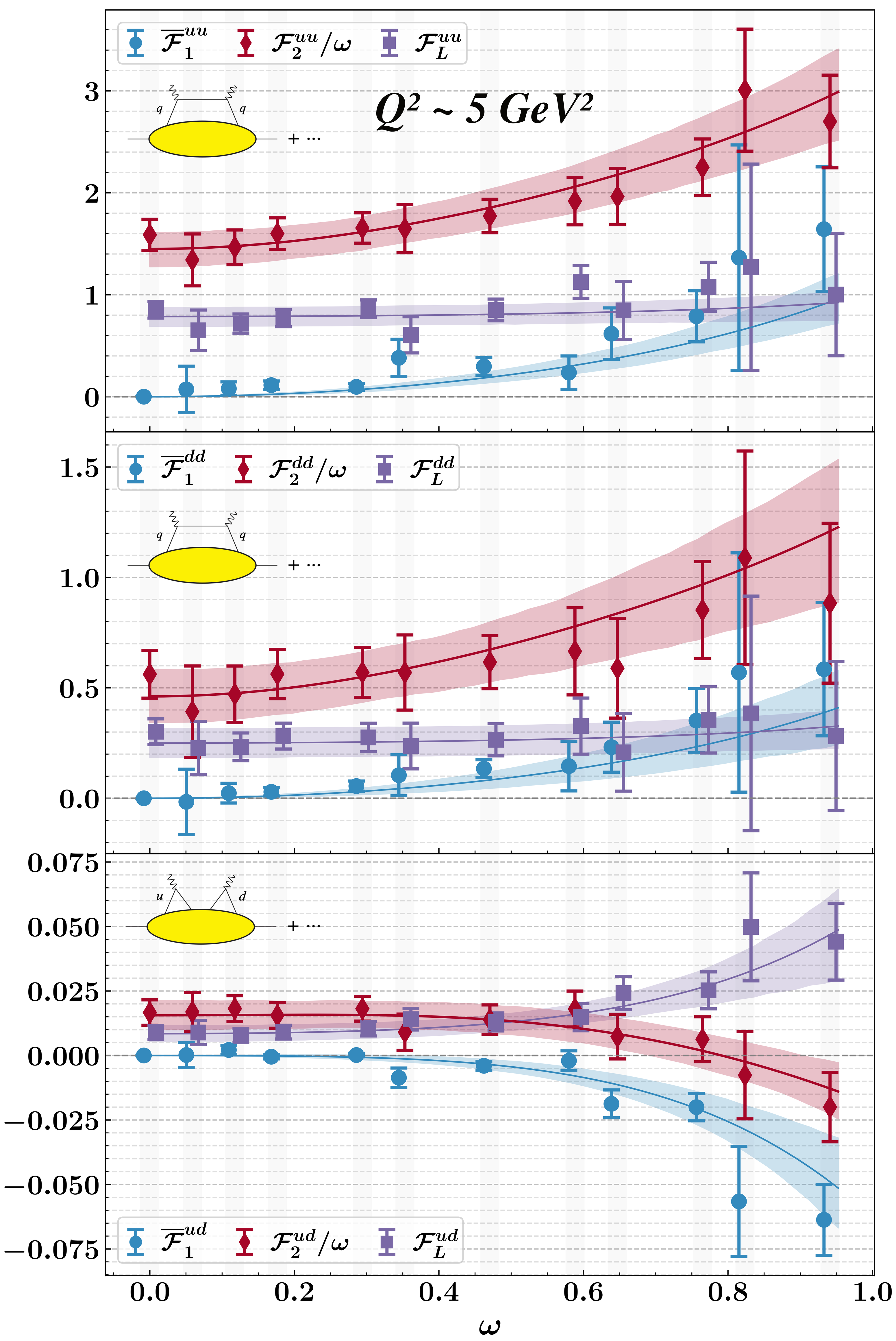


Fig. 1: ω dependence of the Compton SFs. Shaded bands are fits in the form of Eq. 1.

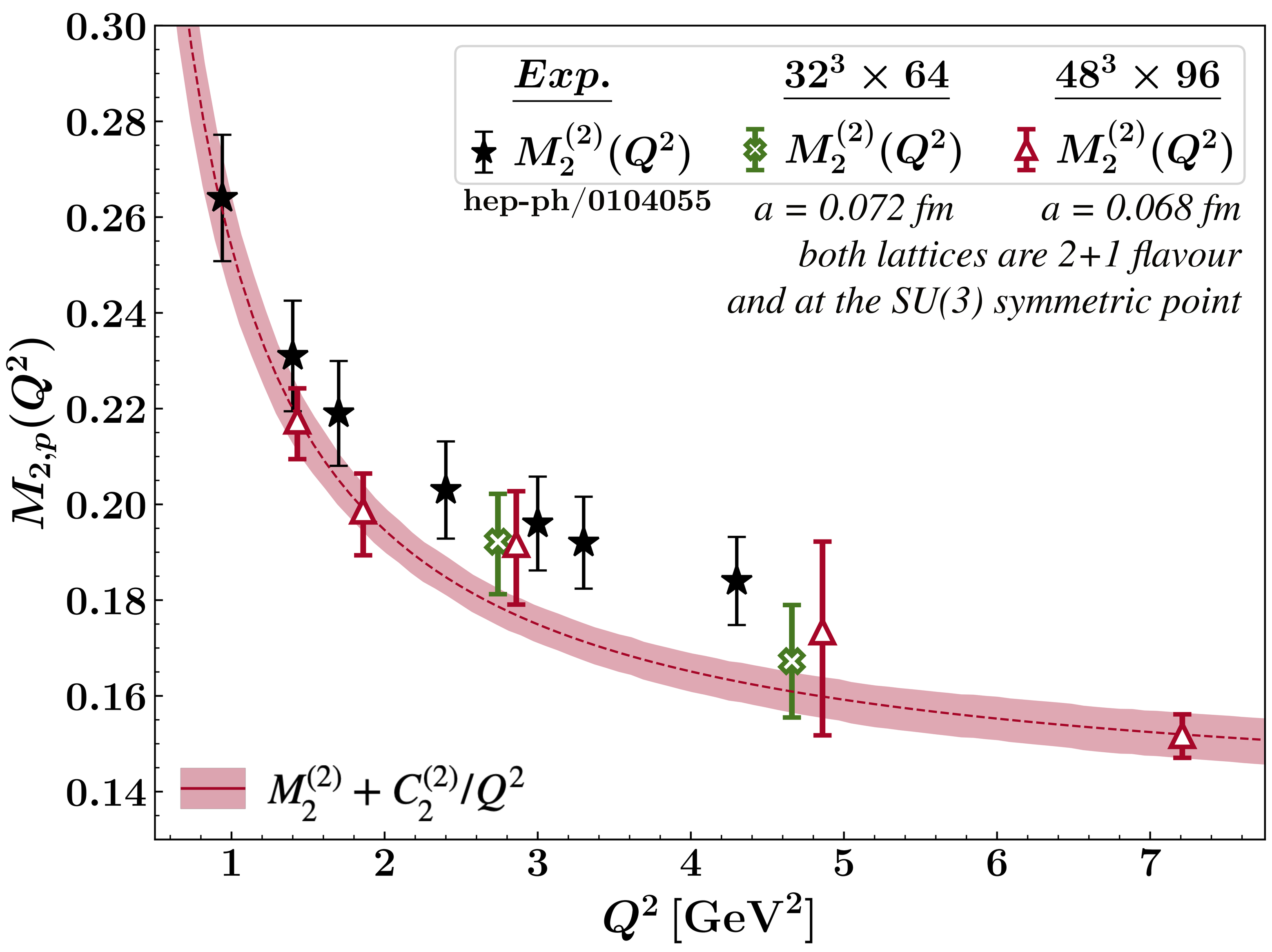


Fig. 2: Q^2 dependence of the lowest moments of $F_2(x,Q^2)$.

Outlook

- Determining the x-dependence of PDFs is a work in progress.
- We are extending our calculations to extract the moments of polarised, g_1, g_2, and the unpolarised parity-violating, F_3, structure functions.