Regional E-conference on Physics

Effects of Exchange Interaction on Spin Acoustic Waves with Separated Spin Evolution in Spin Polarized Quantum Plasma

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Outline

- Introduction
- Motivation
- Mathematical Modelling
- Linear Dispersion Relation
- Spin Magneto-acoustic Waves
- Conclusions

Quantum Plasma

Quantum effects appear when

$$R_{in} \approx \lambda_B = h/\sqrt{2\pi k T e}$$

Degeneracy parameter:

$$\chi \cong \left(\frac{T_F}{T}\right)^{\frac{3}{2}} = n\lambda_B^3$$

$$n_e \lambda_B^3 \geq 1$$

This condition corresponds to **$T<T_F$** T_F Fermi temperature.

Screening length:

$$\lambda_F = \frac{v_F}{\omega_p}$$

Quantum coupling parameter: (E_{pot}/E_F)

$$\Gamma^Q \propto n^{-\frac{1}{3}}$$

Introduction

- Quantum Fermi Pressure

$$P_F = \frac{(3\pi^2)^{2/3}\hbar^2 n_s^{5/3}}{5m_s}$$

$$P_{spin} = \frac{\upsilon_{3D}(3\pi^2)^{2/3}\hbar^2 n_s^{5/3}}{5m_s}$$

- Particle dispersion/spreading

$$V_B = \frac{\Gamma_D \hbar^2}{2m_s}\left(\frac{\nabla^2\sqrt{n_s}}{\sqrt{n_s}}\right)$$

$$\Gamma_D = \frac{D-2}{3D}$$

- Spin Force

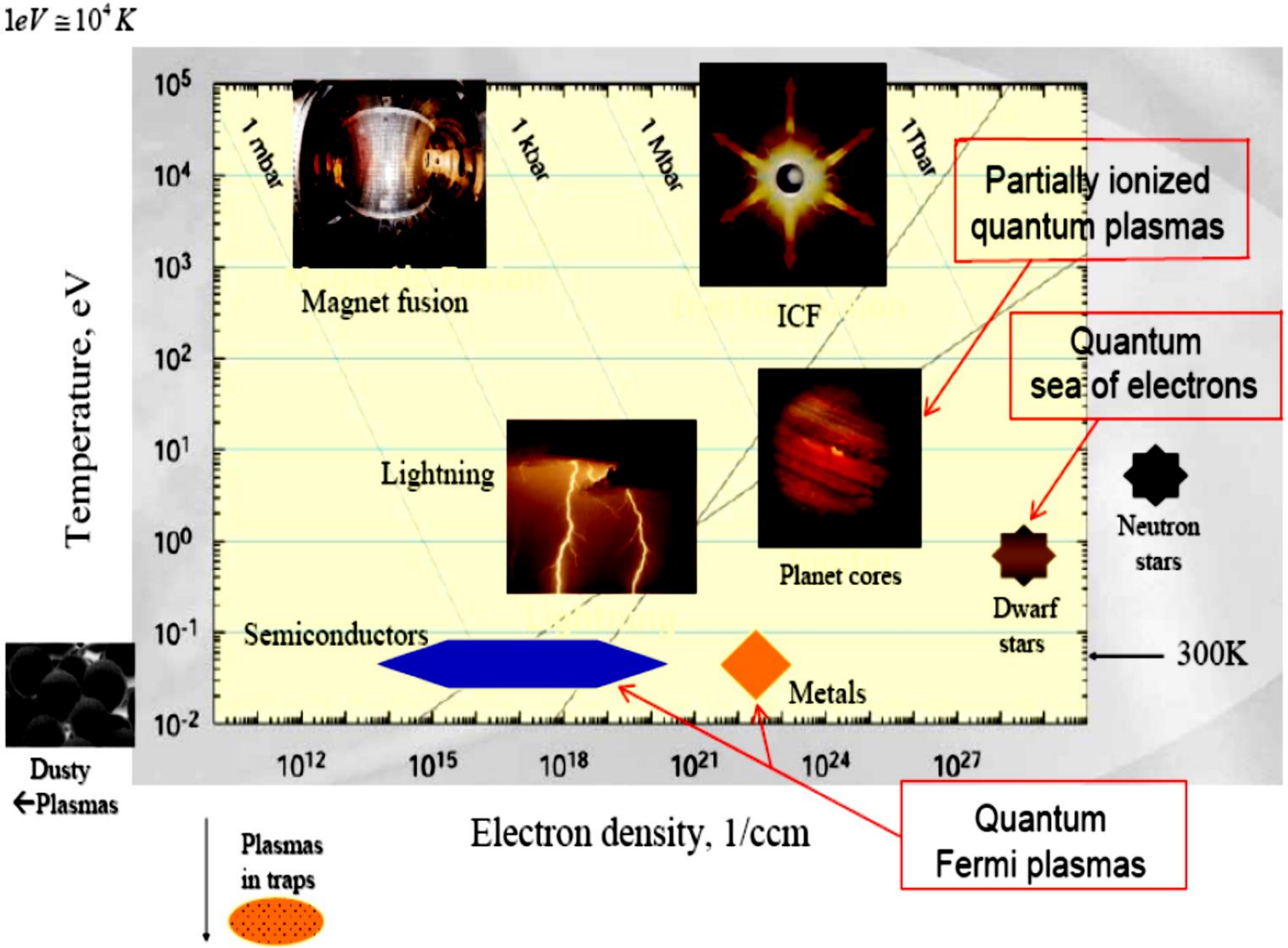
$$F_{spin} = \frac{2\gamma_e}{\hbar}\nabla\left(\vec{S}\,.\,\vec{B}\right)$$

- Exchange Potential:

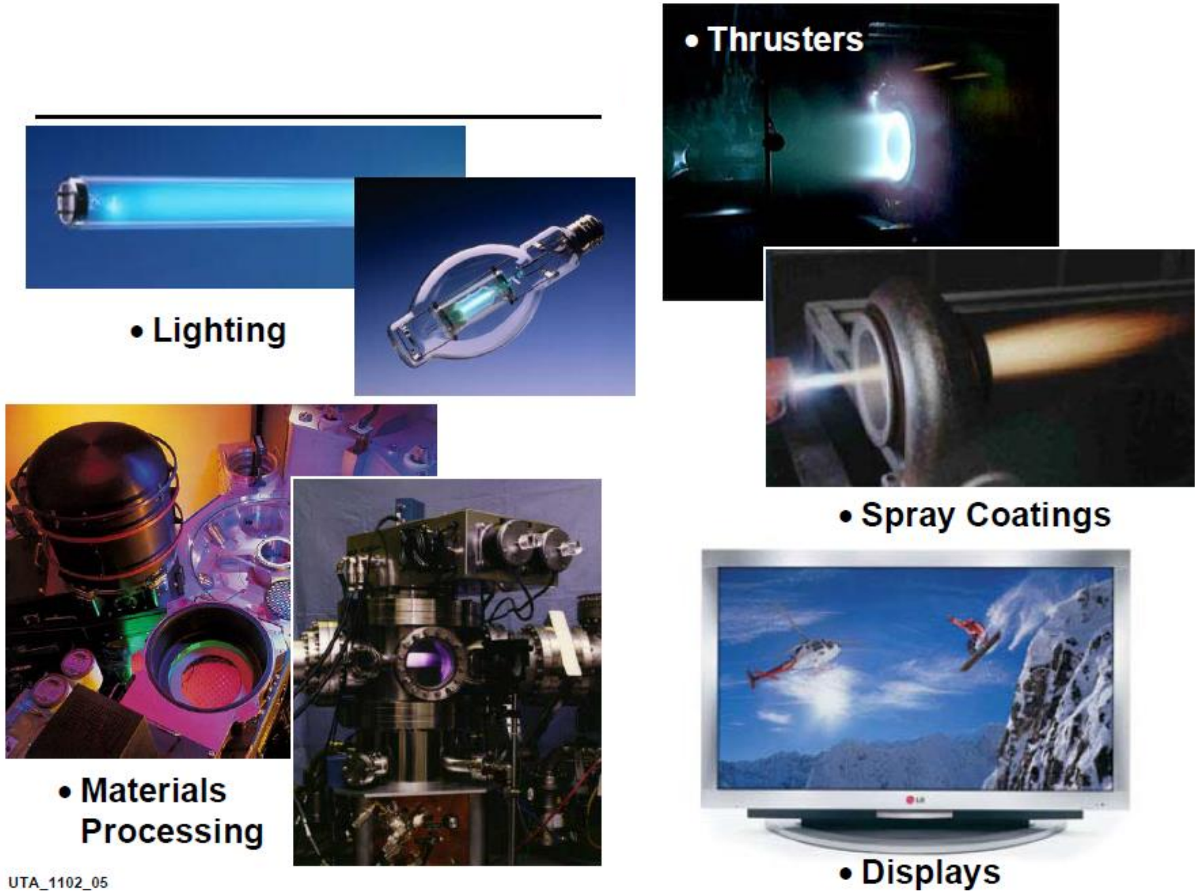
$$V_{Xe\downarrow} = 0.985\zeta_{3D}e^2 n_{e\downarrow}^{1/3}$$

$$\zeta_{3D} = (1+\kappa)^{4/3} - (1-\kappa)^{4/3}$$

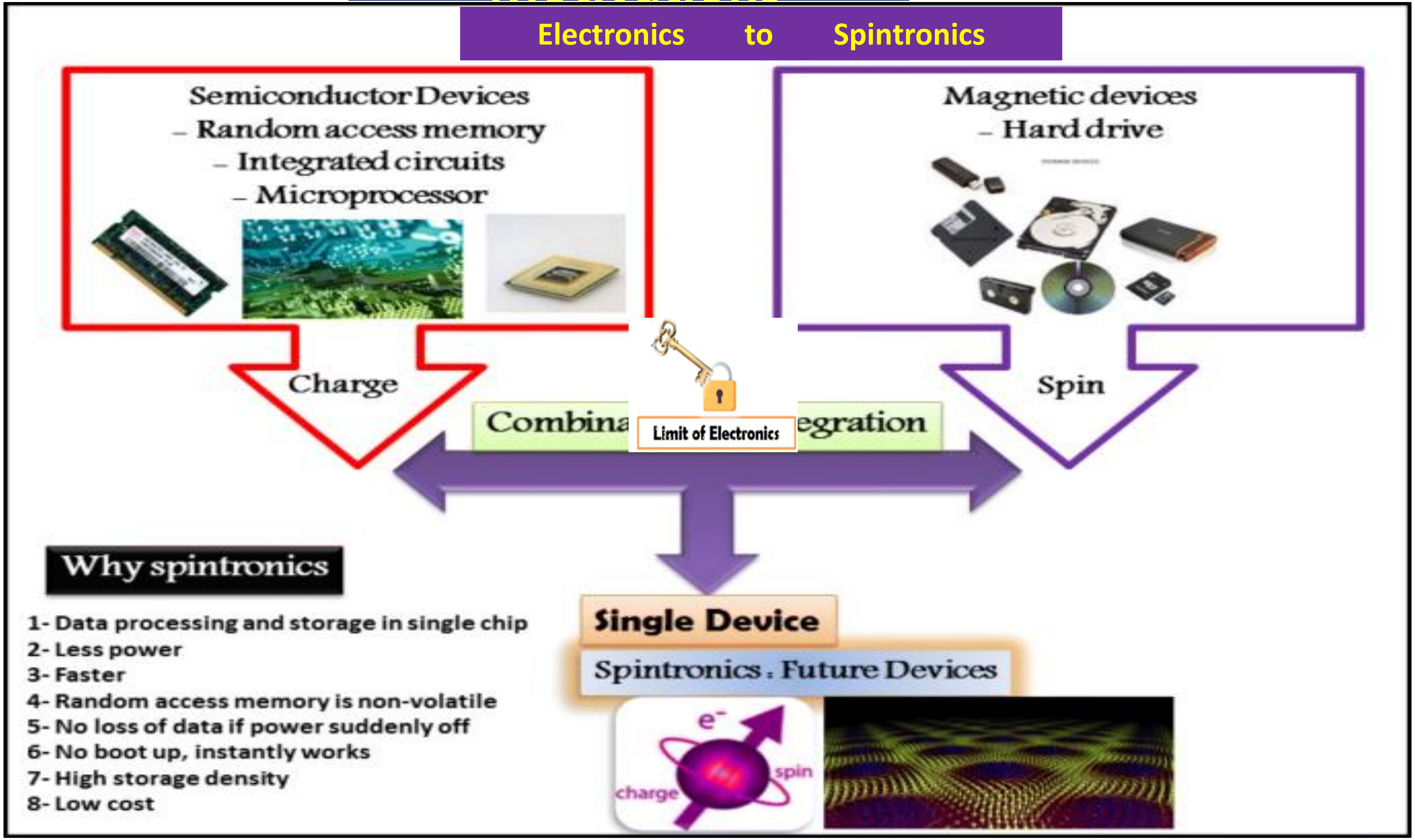
Degenerate Quantum Plasmas in Space and Laboratory



Collisional Low Temperature Plasmas Technology



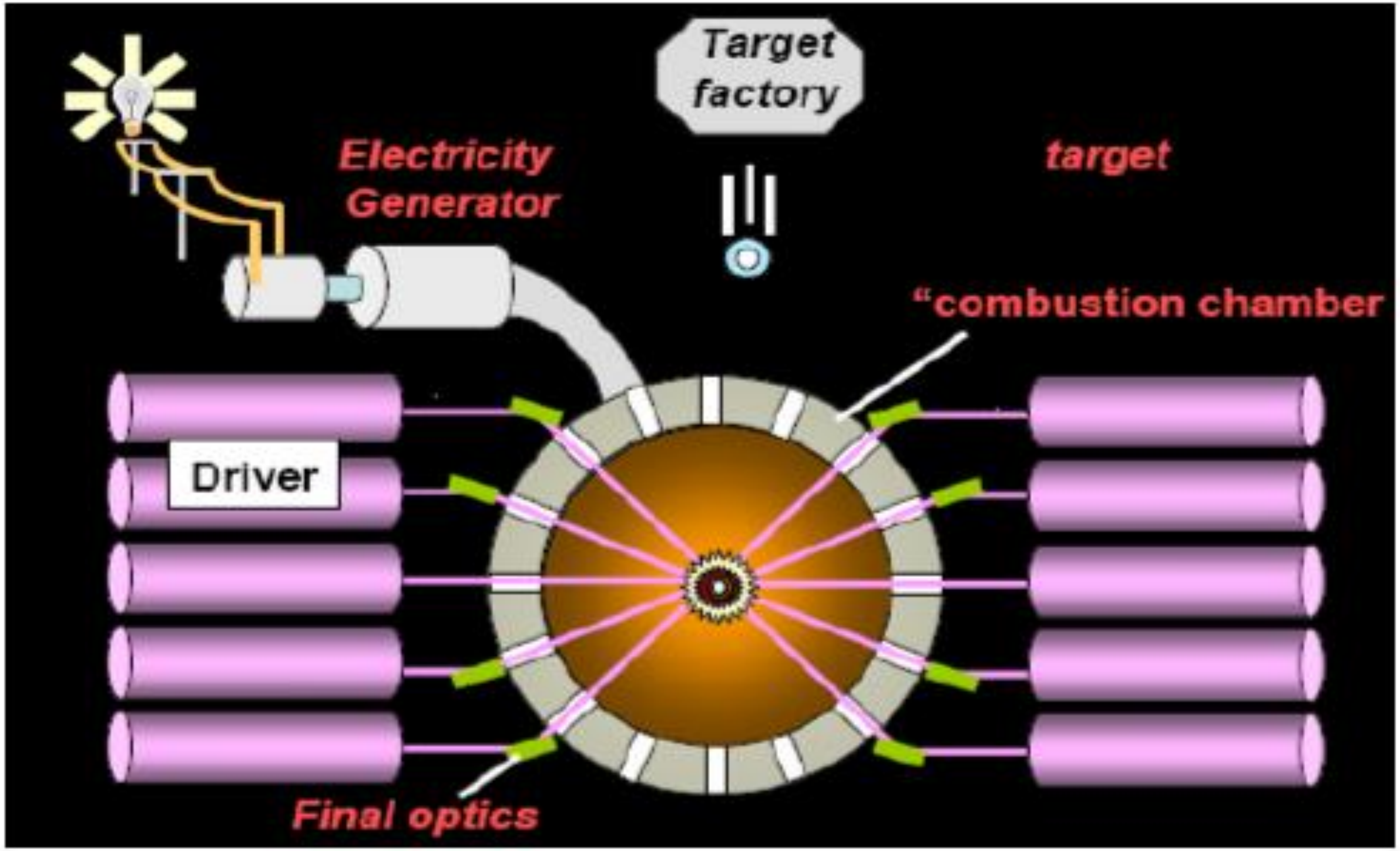
Motivations



Ref: https://doi.org/10.1007/s10948-020-05545-8

Motivations

Plasma Inertial Confinement Fusion and Spin Effects

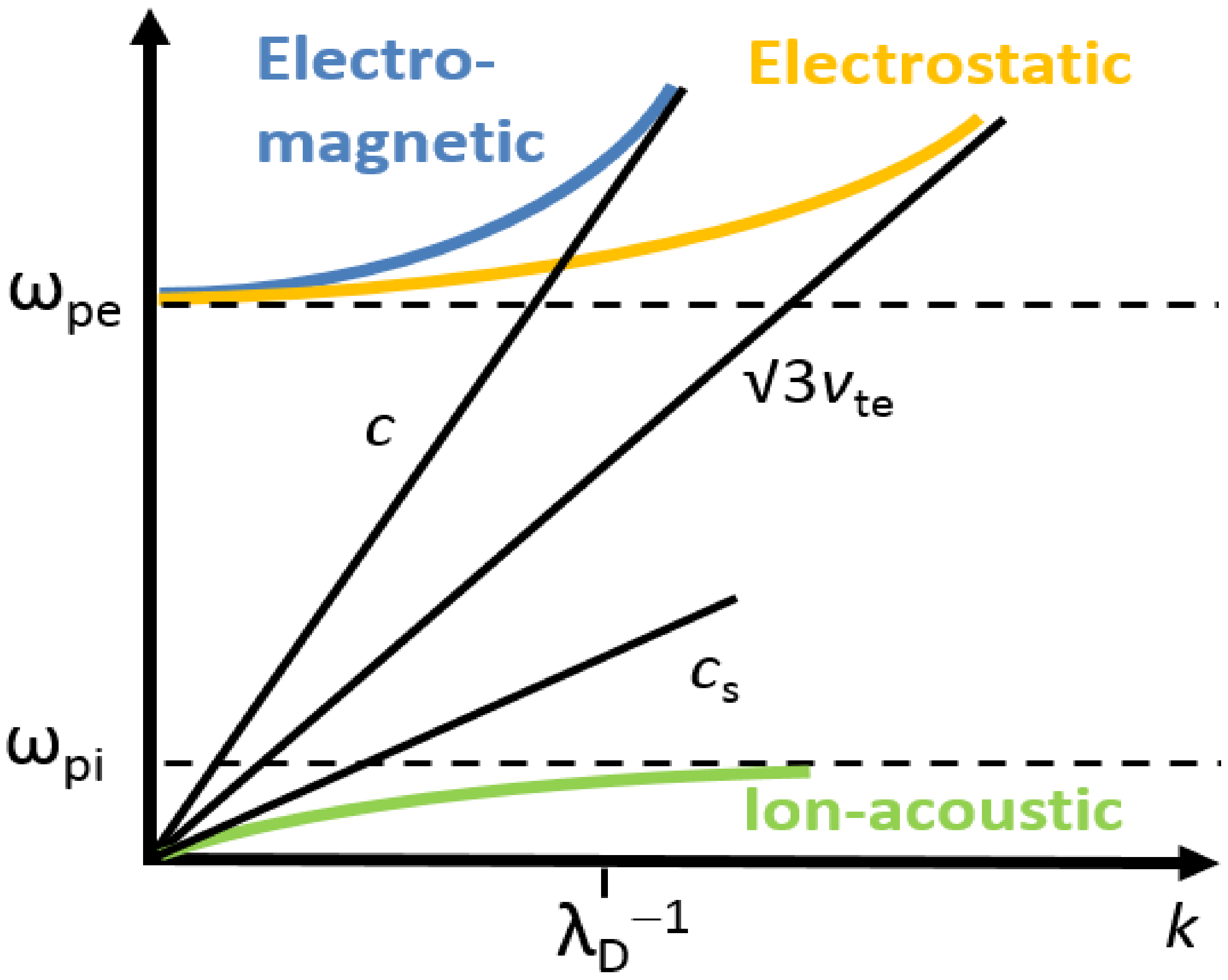


Ref: http://dx.doi.org/10.1088/0029-5515/52/10/103011; G. Bruhaug and A. Kish 2021

Low Frequency Modes

Frequency Range: $|\partial_t| \ll \omega_{pe}$, $\omega \ll \Omega_{cj}$, $\Omega_{cj} = \frac{eB_0}{m_j c}$; $T_i, T_e, T_{Fi} \ll T_{Fe}$.

- For low frequency the **ion motion** should be taken into account.



Linear Structures

- Ion acoustic, Electrostatic,
- Electromagnetic, **Magnetosonic, Alfven waves**

Ref: Francis F. Chen

Mathematical Modelling

- Quantum Magneto-Hydrodynamic Separated Spin Evolution (**QMHD-SSE**) model for macroscopic variable n (number density), **v** (velocity) and spin vector **S** of the l^{th} specie:

$$m_l n_l \frac{\mathrm{d}\vec{\mathrm{v}}_\mathrm{l}}{\mathrm{dt}} = n_l q_l \left(\vec{E} + \frac{1}{c} \vec{v}_l \times \vec{B} \right) - \nabla P_l + n_l \nabla V_B + n_l \nabla V_x + F_{spin}$$

$$\frac{\partial\, \mathrm{n_l}}{\partial \mathrm{t}} + \nabla . (n_l \vec{v}_l) = 0$$

$$\frac{d\,\vec{\mathrm{S}}}{\mathrm{d}t} = \frac{2\gamma_e}{\hbar} (\vec{S} \times \vec{B})$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial \mathrm{t}} + \frac{4\pi}{c} (\vec{J}_p + c\vec{J}_m)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi \sum_l q_l n_l$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v}_l \cdot \vec{\nabla})$$

$$\vec{J}_p = \sum_l q_l n_l\, \vec{v}_l$$

$$\vec{J}_m = \vec{\nabla} \times \vec{M}, \qquad and \quad \vec{M} = \gamma_e \vec{S}$$

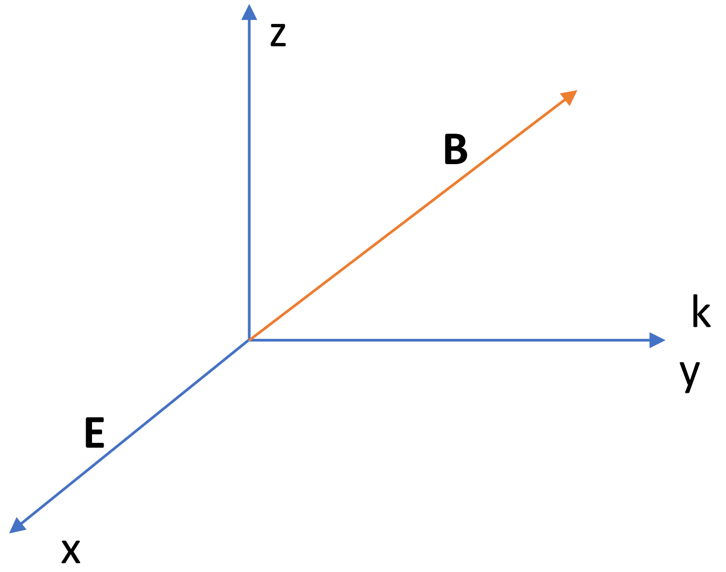
- For **high densities /low temperature**: only electrons above Fermi level contribute to magnetization; magnetization reduces through the Brillion function

$$\mathbf{M} = \mu_B n_e Tanh(\mu_B B / T_e) \hat{B}$$

- For **high temperature** M-> 0

General dispersion relation in spin polarized plasma

- The external magnetic field $\boldsymbol{B}_0 = B_0[\cos\theta\hat{y} + \sin\theta\hat{z}]$and electric field $\boldsymbol{E}_1 = E_1\hat{x}$ wave vector y-axis, the linear excitations δf are proportional to $F_A e^{i(ky-\omega t)}$;



- General **Dispersion Relation**

$$\omega^2 - c^2k^2 = \frac{\omega_{pi}^2\omega^2\left(\omega^2 - k^2 v_{Ti}^2\right)}{\omega^2\left(\omega^2 - k^2 v_{Ti}^2\right) - \Omega_i^2\left(\omega^2 - k^2 v_{Ti}^2\cos^2\theta\right)} + \frac{\omega_{pe}^2\delta_\uparrow\omega^2\left(\omega^2 - Q_{e\uparrow}k^2\right)}{\omega^2\left(\omega^2 - k^2 Q_{e\uparrow}\right) - \Omega_e^2\left(\omega^2 - Q_{e\uparrow}k^2\cos^2\theta\right)}\left(1 + \frac{\Omega_e\mu_e c k^2\sin\theta^2}{e\left(\omega^2 - Q_{e\uparrow}k^2\right)}\right)$$

$$+ \frac{\omega_{pe}^2\delta_\downarrow\omega^2\left(\omega^2 - Q_{e\downarrow}k^2\right)}{\omega^2\left(\omega^2 - k^2 Q_{e\downarrow}\right) - \Omega_e^2\left(\omega^2 - Q_{e\downarrow}k^2\cos^2\theta\right)}\left(1 - \frac{\Omega_e\mu_e c k^2\sin\theta^2}{e\left(\omega^2 - Q_{e\downarrow}k^2\right)}\right) - \frac{\omega_{pe}^2\hbar^2k^2\sin\theta}{4m_e\varepsilon_{Fe}}.$$

$$Q_{e\uparrow} = \frac{(2\delta_\uparrow)^{2/3}v_{Fe}^2}{3v_A^2} + \frac{H_e^2k^2}{36}, \qquad Q_{e\downarrow} = \frac{(2\delta_\downarrow)^{2/3}v_{Fe}^2}{3v_A^2} + \frac{H_e^2k^2}{36} - \frac{0.985e^2\zeta_{3D}\delta_\downarrow^{1/3}n_0^{1/3}}{3m_e v_A^2}.$$

$H_e = \hbar\Omega_i/m_e v_A^2$

$$\delta_\uparrow = n_{0e\uparrow}/n_{0i} = (1+\kappa)/2, \delta_\downarrow = n_{0e\downarrow}/n_{0i} = (1-\kappa)/2$$

$$\kappa = \frac{n_{e\uparrow} - n_{e\downarrow}}{n_{e\uparrow} + n_{e\downarrow}}, \qquad \kappa \in (0 \to 1)$$

Parallel Propagation

For $\theta = 0$, the mode is Alfvén wave

$$\omega^2 - c^2k^2 = \frac{\omega_{pi}^2\omega^2}{\omega^2 - \Omega_i^2} + \frac{\omega_{pe}^2\omega^2}{\omega^2 - \Omega_e^2}.$$

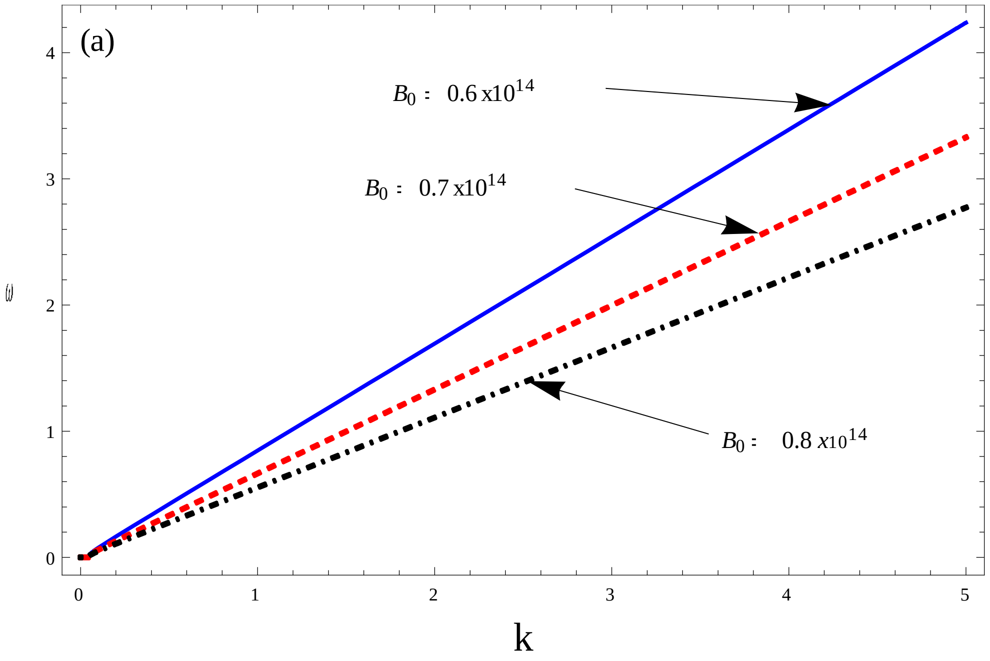
In low-frequency limit, the dispersion relation is $\omega = v_A k$, which is the same result as for a classical Alfvén propagation mode

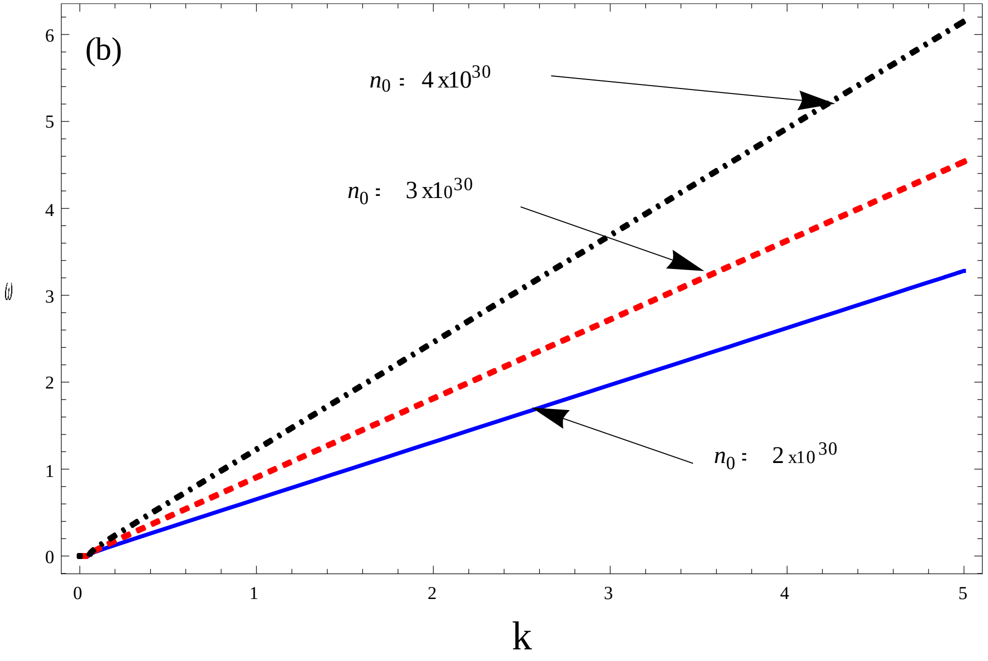
Perpendicular Propagation

- Dispersion relation reduces to **Magnetosonic mode** affected by quantum parameters; in low frequency limit such as;

$\omega^2 \ll \Omega_i^2, \Omega_e^2, \omega^2 \ll k^2 Q_s$ and $\omega^2 \gg k^2 v_{Ti}^2,$

$$\omega^2 - \frac{c^2}{v_A^2}k^2 = -\frac{\omega_{pe}^2 \hbar^2 k^2}{4 m_e^2 v_{Fe}^2} - \frac{c^2 \omega^2}{v_A^2} + \frac{\omega_{pe}^2 k^2 \delta_\uparrow}{\Omega_e^2}\left(Q_{e\uparrow} - \frac{\Omega_e \mu_e c}{e v_A^2}\right) + \frac{\omega_{pe}^2 k^2 \delta_\downarrow}{\Omega_e^2}\left(Q_{e\downarrow} + \frac{\Omega_e \mu_e c}{e v_A^2}\right).$$



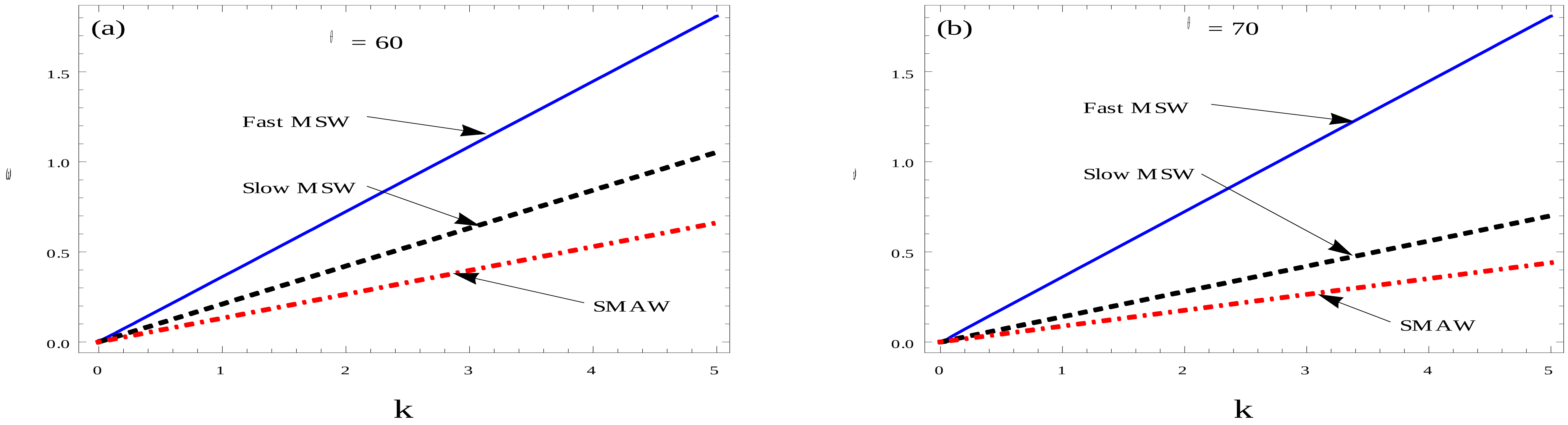


Obliquely propagating waves

In low frequency range dispersion relation reduces to 3[rd] order equation.

$$\omega^2 - \frac{c^2}{v_A^2}k^2 = -\frac{\omega_{pe}^2\hbar^2k^2\sin\theta}{4m_e^2v_{Fe}^2} - \frac{c^2\omega^2}{v_A^2} + \frac{\omega_{pe}^2k^2\delta_\uparrow\omega^2}{\Omega_e^2(\omega^2 - k^2Q_{e\uparrow}\cos^2\theta)}\left(Q_{e\uparrow} - \frac{\Omega_e\mu_e c\sin\theta^2}{ev_A^2}\right) + \frac{\omega_{pe}^2\omega^2k^2\delta_\downarrow}{\Omega_e^2(\omega^2 - k^2Q_{e\downarrow}\cos^2\theta)}\left(Q_{e\downarrow} + \frac{\Omega_e\mu_e c\sin\theta^2}{ev_A^2}\right)$$

Effect of obliqueness on frequency

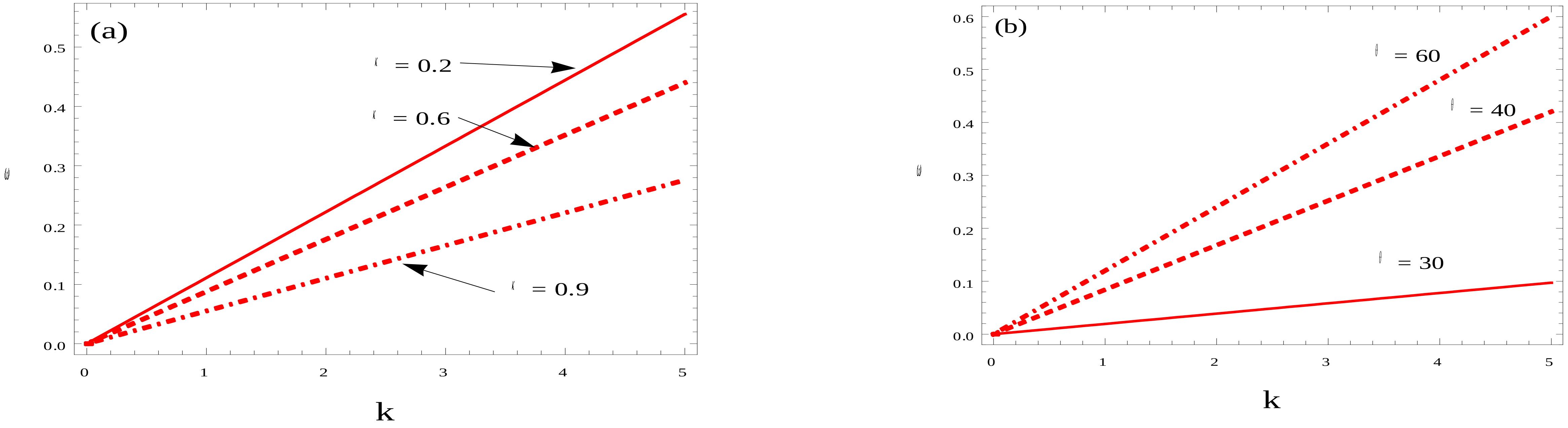


Obliquely propagating modes angular frequency ω against k at different angle (θ), (a) for θ=60°, (b) for θ=70° at $n_0=10^{30}\,cm^{-3}$, $B_0=10^{14}$G,κ=0.6.

Obliquely propagating Spin Magneto-acoustic waves

Effects of spin polarization density ratio on frequency

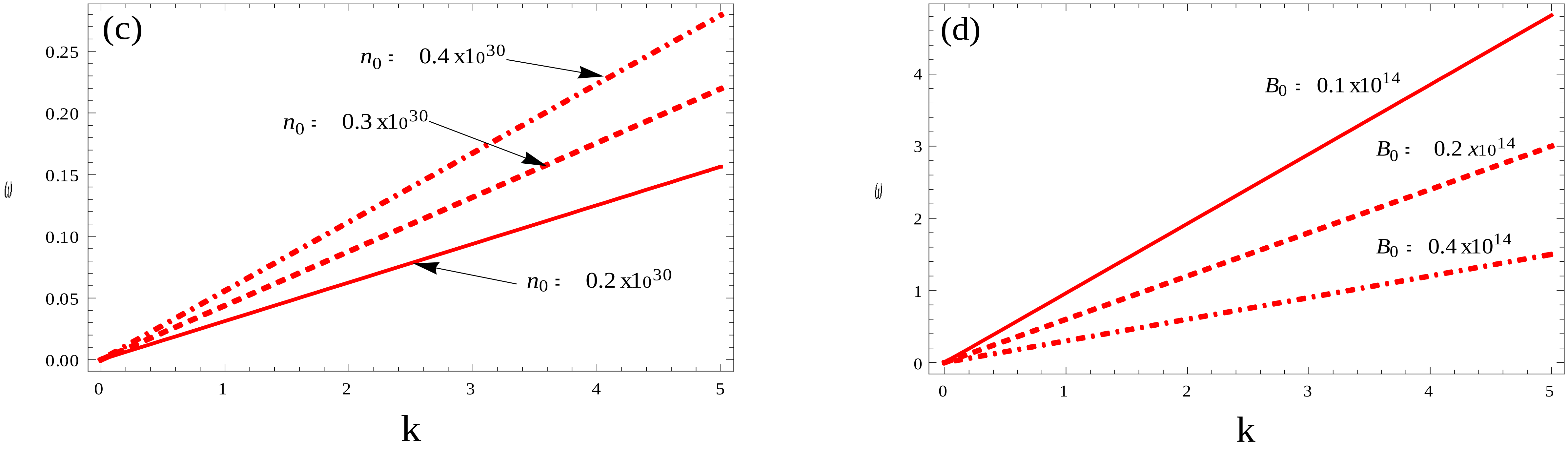
Effects of obliqueness on frequency



Angular frequency ω of SMAW against k for different (a) spin polarization (b) angle (in degrees), Chosen values in plots are $n_0 = 10^{30}$ cm^{-3}, $B_0 = 10^{14}$G, κ=0.6.

Obliquely propagating Spin Magneto-acoustic waves

Effects of number density on frequency | **Effects of magnetic field strength on frequency**



Angular frequency ω of SMAW against k for different (c) density (d) magnetic field, Chosen values in plots are $n_0 = 10^{30}$ cm^{-3}, $B_0 = 10^{14}$G, κ=0.7, θ=60.

Conclusions

- The contribution of spin and exchange quantum effects in our model change the frequency of waves.
- The spin term vanishes for parallel propagation, and we have only Alfven waves.
- The perpendicular propagation mode at large wavelength limit is strongly influenced by exchange, spin, and other quantum effects.
- We have observed that a large angle of propagation and density enhance the spin magnetoacoustic wave frequency.
- We have observed that in high spin-polarized plasma, the frequency of the obliquely propagating waves decreases and also low values are noticed for maximum magnetic field strength.
- High spin magneto-acoustic wave frequency is noticed in low spin polarized plasma.

Thank you for your time and attention