Oblique Bernstein Wave Propagation in Electron-Ion Plasma with Electron Quantization Effects

Azhar Hussain

Department of Physics

Mirpur University of Science and Technology (MUST), Mirpur AJK

Co-workers: A. Majeed, G. Murtaza and M. Ayub

Layout of the Talk

- Introduction
- Motivations to study Oblique Electron Bernstein Waves
- Mathematical Model
- Landau-Kelly distribution function
- Oblique Bernstein Waves Dispersion Relation
- Analysis of Dispersion Relation of Oblique EBWs
- Damping of Oblique Bernstein Waves
- Conclusions

Introduction: Electron Bernstein Waves

The Electron Bernstein Waves (EBWs) are electrostatic waves that propagate at right angles to the ambient magnetic field and interacts strongly with electrons in the vicinity of electron cyclotron resonance or its harmonics.

These are high-frequency waves with the ions at neutralizing background. These waves damp for small deviations from perpendicular propagation, therefore we should analyze the $k_{||} \neq 0$ case.

EBWs have the wavelength of the order of electron Larmor radius so it is difficult to detect such waves.

Introduction: Electron Bernstein Waves

The dispersion relation for general Bernstein waves is

$$1+\sum_s \frac{k_D^2}{k^2}\exp[-b]\sum_{n=-\infty}^{\infty} I_n(b)[1+\xi_{0s}Z(\xi_{0s})]=0$$

Where $b=\frac{\kappa_\perp v_{th}}{2\omega_c^2}$ and $Z(\xi_{ns})$ is plasma dispersion function.

The dispersion relation for electron Bernstein waves is

$$1=\sum_{n=1}^{\infty}\frac{\omega_{pe}^2}{\Omega_e^2}\frac{1}{b}I_n(b)\exp[-b]\left(\frac{2n^2\Omega_e^2}{\omega^2-n^2\Omega_e^2}\right)$$

It can also be written as

$$\frac{k_\perp^2}{k_D^2}=\sum_{n=1}^{\infty} I_n(b)\exp[-b]\left(\frac{2n^2\Omega_e^2}{\omega^2-n^2\Omega_e^2}\right)$$

Introduction: Quantization Effects in Plasmas

- When we consider dynamic of a charged particle gas in the presence of a strong magnetic field, its momenta in parallel and perpendicular directions will be different, and consequently charged particles will occupy quantized cyclotron orbits, this is what we call Landau quantization effect.

- These discrete energy levels occupied by the charged particles are known as Landau levels.

- Landau quantization is a function of applied magnetic field and some changes that occur in the electronic properties of a material are caused by this effect.

- The effects of Landau quantization can only be observed if the separation between energy levels is greater than the average thermal energy of electrons $\hbar\Omega_c \gg k_B T$.

Motivations to Study Oblique EBWs

- N. J. M. Horing, "*Nondegenerate Plasma Modes in Quantizing Magnetic Field*", International Journal of Quantum Chemistry **5**, 763 (1971).

- B. Shokri and A. A. Rukhadze, "*Quantum Drift Waves*", Physics of Plasmas **6**, 4467 (1999).

- R. J. Armstrong, J. Juul Rasmussen, R.L. Stenzel and J. Trulsen, "*Observations of Obliquely Propagating Electron Bernstein Waves*", Physics Letters **85A**, 281 (1981).

- L. Tsintsadze, G. Peradze and N. Tsintsadze, "*Landau-Kelly Representation of Statistical Thermodynamics of Quantum Plasma and Magnetic String Waves*, Bulletin of the Georgian National Academy of Sciences **11**, 49 (2017).

- M. Adnan et al., *"On the Characteristics of Obliquely Propagating Electrostatic Structures in Non-Maxwellian Plasmas in the Presence of Ion Pressure Anisotropy"*, Physics of Plasmas **24**, 032114 (2017).

- M. F. Bashir and G. Murtaza, "*Effect of Temperature Anisotropy on Various Modes and Instabilities for a Magnetized Non-Relativistic Bi-Maxwellian Plasma*", Brazilian Journal of Physics **42**, 487 (2012).

Mathematical Model

We start with the Vlasov-Poisson system

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{x}} + \left[\frac{q_\alpha}{m}(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c})\right] \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = 0$$

$$\boldsymbol{\nabla}.\mathbf{E}_1 = 4\pi q n_{\alpha 1}$$

$$f_{1\alpha} = 2\pi \sum_{n=-\infty}^{\infty} \frac{-1}{s + ik_{\parallel} v_{\parallel} + in\Omega_\alpha} \times \left[\frac{q_\alpha}{m}\left\{\left(E_{1x}\frac{n}{z}[J_n(z)]^2 + E_{1y} i J_n(z) J'_n(z)\right)\frac{\partial f_{0\alpha}}{\partial v_\perp} + J_n(z)]^2 E_{1z}\frac{\partial f_{0\alpha}}{\partial v_{\parallel}}\right\}\right.$$

$$\vec{E} = -\vec{\nabla}\varphi$$

Landau-Kelly Distribution Function *

$$f_{0s} = C \exp\left[\frac{-p_{\parallel}^{2}}{2m_s T} - \frac{-p_{\perp}^{2}}{2m_s \varepsilon_{\perp}}\right]$$

Here $$C = \frac{1}{(2\pi m_s T)(2\pi m_s \varepsilon_{\perp})}$$

and

$$\varepsilon_{\perp} = \frac{\hbar\Omega_s}{2}\left|\coth\frac{\hbar\Omega_s}{2T}\right|$$

For $\hbar\Omega_s << 2T$ we have $\varepsilon_{\perp} = T.$

For $\hbar\Omega_s >> 2T$ we have $\varepsilon_{\perp} = \frac{\hbar\Omega_s}{2}.$

Coth[0.1] = 10.0333

Coth[6] = 1.00001

---

* D. C. Kelly, Dielectric Tensor for a Quantum Plasma, Phys. Rev. 134, A641 (1964).

Dispersion Relation of Oblique Electron Bernstein Waves

Without the loss of generality, we can take electric field only in the x- and z-direction we can derive the following general dispersion relation (sometimes also called Harris dispersion relation) of electrostatic waves in plasmas:

$$1 = -\sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{k^2} 2\pi m_s \int_0^{\infty} p_\perp J_n^2\left(\frac{k_\perp v_\perp}{\Omega_s}\right) dp_\perp \int_{-\infty}^{\infty} \frac{dp_\parallel}{\omega - k_\parallel v_\parallel - n\Omega_s}\left[\frac{\partial f_0}{\partial p_\parallel} k_\parallel + \frac{n\Omega_s}{v_\perp}\frac{\partial f_0}{\partial p_\perp}\right]$$

Substituting Landau-Kelly Distribution in the above equation, and after performing all integrations we obtain

$$1 = \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{k^2} m_e I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right]\left[\frac{1}{2T} Z'(\xi_{ne}) - \frac{2n}{\hbar\omega}\xi_{0e} Z(\xi_{ne})\right].$$

where

$$\mu_e = \frac{k_\perp^2 v_{the}^2}{\Omega_e^2}, \quad v_r = \frac{v_{the}^2}{v_{Qe}^2} \quad \text{and} \quad v_{Qe}^2 = \frac{\hbar\Omega_e}{2m_e}.$$

Approach-II: Dispersion Relation of Oblique EBWs

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + q\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J},$$

$$\omega^2 \mathbf{E} - c^2 k^2 \mathbf{E} + c^2 \mathbf{k}(\mathbf{k} \cdot \mathbf{E}) = -4\pi \mathrm{i} \omega \mathbf{J},$$

$$J_i = \sigma_{ij} E_j,$$

$$[(\omega^2 - c^2 k^2)\delta_{ij} + c^2 k_i k_j + 4\pi \mathrm{i} \omega \sigma_{ij}] E_j = 0,$$

$$R_{ij} = (\omega^2 - c^2 k^2)\delta_{ij} + c^2 k_i k_j + 4\pi \mathrm{i} \omega \sigma_{ij},$$

Approach-II: Dispersion Relation of Oblique EBWs

$$\epsilon_{xx} = 1 + \sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{\omega^2} \frac{n^2}{\mu_s} I_n(\mu_s) \exp[-\mu_s] \left[\xi_{0s} Z(\xi_{ns}) - \frac{1}{2} Z'(\xi_{ns}) \left(\frac{\epsilon_\perp}{T} - 1 \right) \right]$$

here $\mu_s = \frac{k_\perp^2 \epsilon_\perp}{m_s \Omega_s^2}$. Taking $\epsilon_\perp \rightarrow \frac{\hbar \Omega_s}{2}$ (i.e., $\frac{\hbar \Omega_s}{2T} \gg 1$)

$$\epsilon_{xx} = 1 + \sum_{n=-\infty}^{\infty} v_r \frac{\omega_{pe}^2}{\omega^2} \frac{n^2}{\mu_e} I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \left[\xi_{0e} Z(\xi_{ne}) - \frac{1}{2} Z'(\xi_{ne}) \left(\frac{\hbar \Omega_e}{2T} - 1 \right) \right]$$

where

$$\mu_e = \frac{k_\perp^2 v_{the}^2}{\Omega_e^2}, \quad v_r = \frac{v_{the}^2}{v_{Qe}^2} \quad \text{and} \quad v_{Qe}^2 = \frac{\hbar \Omega_e}{2 m_e}.$$

$$\epsilon_{xz} = -\sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{\omega^2} \frac{v_{the}}{v_{Qe}} \frac{n}{2\sqrt{\frac{\mu_e}{v_r}}} I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] Z'(\xi_{ns}) \left[\xi_{0s} + \xi_{ns} \left(\frac{\hbar \Omega_e}{2T} - 1 \right) \right].$$

$$= \epsilon_{zx}.$$

F. F. Chen, Introduction to Plasma Physics and Controlled Fusion, 3rd Ed., p#258 (2016).

Approach-II: Dispersion Relation of Oblique EBWs

$$\epsilon_{xx} = 1 + \sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{\omega^2} \frac{n^2}{\mu_s} I_n(\mu_s) \exp[-\mu_s] \left[\xi_{0s} Z(\xi_{ns}) - \frac{1}{2} Z'(\xi_{ns}) \left(\frac{\epsilon_\perp}{T} - 1 \right) \right]$$

here $\mu_s = \frac{k_\perp^2 \epsilon_\perp}{m_s \Omega_s^2}$. Taking $\epsilon_\perp \to \frac{\hbar \Omega_s}{2}$ (i.e., $\frac{\hbar \Omega_s}{2T} \gg 1$)

$$\epsilon_{xx} = 1 + \sum_{n=-\infty}^{\infty} v_r \frac{\omega_{pe}^2}{\omega^2} \frac{n^2}{\mu_e} I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \left[\xi_{0e} Z(\xi_{ne}) - \frac{1}{2} Z'(\xi_{ne}) \left(\frac{\hbar \Omega_e}{2T} - 1 \right) \right]$$

where

$\mu_e = \frac{k_\perp^2 v_{the}^2}{\Omega_e^2}$, $v_r = \frac{v_{the}^2}{v_{Qe}^2}$ and $v_{Qe}^2 = \frac{\hbar \Omega_e}{2 m_e}$.

$$\epsilon_{xz} = -\sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{\omega^2} \frac{v_{the}}{v_{Qe}} \frac{n}{2\sqrt{\frac{\mu_e}{v_r}}} I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] Z'(\xi_{ns}) \left[\xi_{0s} + \xi_{ns} \left(\frac{\hbar \Omega_e}{2T} - 1 \right) \right].$$

$$= \epsilon_{zx}.$$

F. F. Chen, Introduction to Plasma Physics and Controlled Fusion, 3rd Ed., p#258 (2016).

Approach-II: Dispersion Relation of Oblique EBWs

$$\epsilon_{zz} = 1 - \sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{\omega^2} I_n(\mu_s) \exp[-\mu_s] \xi_{0s} \xi_{ns} Z'(\xi_{ns}) \left\{ 1 - \frac{n\Omega_s}{\omega} \left(-\frac{T}{\epsilon_\perp} + 1 \right) \right\}.$$

here $\mu_s = \frac{k_\perp^2 \epsilon_\perp}{m_s \Omega_s^2}$. Taking $\epsilon_\perp \to \frac{\hbar\Omega_s}{2}$ (i.e., $\frac{\hbar\Omega_s}{2T} \gg 1$)

$$\epsilon_{zz} = 1 - \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{\omega^2} I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \xi_{0e} \xi_{ne} Z'(\xi_{ne}) \left\{ 1 - \frac{n\Omega_e}{\omega} \left(-\frac{2T}{\hbar\Omega_e} + 1 \right) \right\}.$$

$$k_x^2 \epsilon_{xx} + 2k_x k_z \epsilon_{xz} + k_z^2 \epsilon_{zz} = 0.$$

$$1 = \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{k^2} m_e I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \left[\frac{1}{2T} Z'(\xi_{ne}) - \frac{2n}{\hbar\omega} \xi_{0e} Z(\xi_{ne}) \right].$$

F. F. Chen, Introduction to Plasma Physics and Controlled Fusion, 3rd Ed., p#258 (2016).

Approach-II: Dispersion Relation of Oblique EBWs

$$\epsilon_{zz} = 1 - \sum_s \sum_{n=-\infty}^{\infty} \frac{\omega_{ps}^2}{\omega^2} I_n(\mu_s) \exp[-\mu_s] \xi_{0s}\xi_{ns} Z'(\xi_{ns}) \left\{1 - \frac{n\Omega_s}{\omega}\left(-\frac{T}{\epsilon_\perp} + 1\right)\right\}.$$

here $\mu_s = \frac{k_\perp^2 \epsilon_\perp}{m_s \Omega_s^2}$. Taking $\epsilon_\perp \rightarrow \frac{\hbar\Omega_s}{2}$ (i.e., $\frac{\hbar\Omega_s}{2T} \gg 1$)

$$\epsilon_{zz} = 1 - \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{\omega^2} I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \xi_{0e}\xi_{ne} Z'(\xi_{ne}) \left\{1 - \frac{n\Omega_e}{\omega}\left(-\frac{2T}{\hbar\Omega_e} + 1\right)\right\}.$$

$$k_x^2 \epsilon_{xx} + 2k_x k_z \epsilon_{xz} + k_z^2 \epsilon_{zz} = 0.$$

$$1 = \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{k^2} m_e I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \left[\frac{1}{2T} Z'(\xi_{ne}) - \frac{2n}{\hbar\omega} \xi_{0e} Z(\xi_{ne})\right].$$

F. F. Chen, Introduction to Plasma Physics and Controlled Fusion, 3rd Ed., p#258 (2016).

Analysis of Dispersion Relation of Oblique EBWs

Perpendicular Propagation: Set $k_{||} = 0$.

$$1 = \sum_{n=1}^{\infty} \frac{\omega_{pe}^2}{\Omega_e^2} \frac{1}{\mu_{Qe}} I_n\left(\mu_{Qe}\right) \exp\left[-\mu_{Qe}\right] \left(\frac{2n^2\Omega_e^2}{\omega^2 - n^2\Omega_e^2}\right).$$

where $\mu_{Qe} = \frac{k_\perp^2 \hbar}{2m_e\Omega_e}$.

$$1 = \sum_{n=1}^{\infty} \frac{\omega_{pe}^2}{\Omega_e^2} \frac{1}{b} I_n(b) \exp[-b] \left(\frac{2n^2\Omega_e^2}{\omega^2 - n^2\Omega_e^2}\right)$$

Here $b = \frac{k_\perp^2 v_{th}^2}{2\omega_c^2}$.

Analysis of Dispersion Relation of Oblique EBWs

Oblique Propagation: $k_{||} \neq 0.$

Taking into account the imaginary parts in the expansion of plasma dispersion function for large argument we have

$$D_r(\omega) = 1 - \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{k^2} m_e I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right]$$

$$\left[\frac{1}{2T}\left(\frac{1}{\xi_{ne}^2} + \frac{3}{2\xi_{ne}^4}\right) + \frac{2n}{\hbar\omega}\xi_{0e}\left(\frac{1}{\xi_{ne}} + \frac{1}{2\xi_{ne}^3}\right)\right]$$

$$D_i(\omega) = \sqrt{\pi} \sum_{n=-\infty}^{\infty} \frac{\omega_{pe}^2}{k^2} m_e I_n\left(\frac{\mu_e}{v_r}\right) \exp\left[-\frac{\mu_e}{v_r}\right] \exp\left[-\xi_{ne}^2\right] \left[\frac{1}{T}\xi_{ne} + \frac{2n}{\hbar\omega}\xi_{0e}\right]$$

where $\xi_{ne} = \frac{\omega_r - n\Omega_e}{k_{||} v_{the}}.$

Analysis of Dispersion Relation of Oblique EBWs

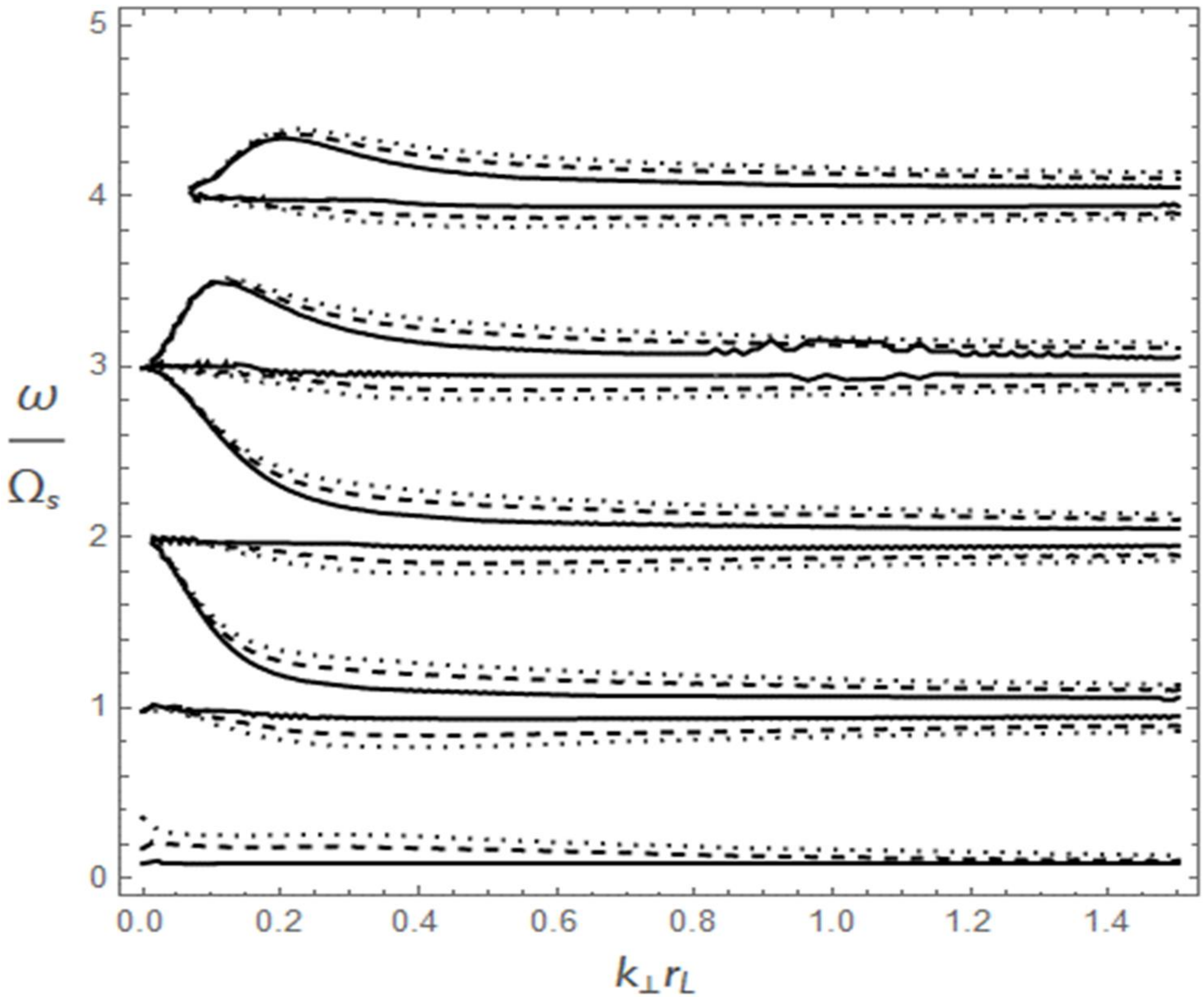
Normalized result for the real part of dispersion relation can be written as

$$\frac{k^2}{k_D^2} = \exp\left[-\frac{\mu_e}{v_r}\right]\left[I_0\left(\frac{\mu_e}{v_r}\right)\left\{\frac{k_r\mu_e}{2\left(\frac{\omega_r}{\Omega_e}\right)^2}+\frac{3k_r^2\mu_e^2}{4\left(\frac{\omega_r}{\Omega_e}\right)^4}\right\}+\sum_{n=1}^{\infty} I_n\left(\frac{\mu_e}{v_r}\right)\left\{\frac{v_r n^2}{\frac{\omega_r^2}{\Omega_e^2}-n^2}+\frac{k_r\mu_e\left(\frac{\omega_r^2}{\Omega_e^2}+n^2\right)}{\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^2}+\frac{n^2 v_r k_r\mu_e\left(3\frac{\omega_r^2}{\Omega_e^2}+n^2\right)}{2\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^3}+\frac{3k_r^2\mu_e^2\left(\frac{\omega_r^4}{\Omega_e^4}+n^4+6n^2\frac{\omega_r^2}{\Omega_e^2}\right)}{2\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^4}\right\}\right]$$

where $\mu_e = \frac{k_\perp^2 v_{the}^2}{\Omega_e^2}$, $k_r = \frac{k_\parallel^2}{k_\perp^2}$, $v_r = \frac{v_{the}^2}{v_{Qe}^2}$, $v_{the}^2 = \frac{2T}{m_e}$ and $v_{Qe}^2 = \frac{\hbar\Omega_e}{2m_e}$.

The last two terms appear here have been added to refine the results, however, these can be neglected.

Analysis of Dispersion Relation of Oblique EBWs



v_r=0.01, $\beta = 8$.

……. $k_r = 0.09$

------ $k_r = 0.05$

______ $k_r = 0.01$

Plot of $\frac{\omega}{\Omega_e}$ vs. $k_\perp r_L$, with variation in $k_r = \frac{k_\parallel^2}{k_\perp^2}$. Here β=$\frac{\omega_{pe}^2}{\Omega_{ce}^2}$

Damping of Oblique EBWs

Damping (γ) of oblique EBW can be found employing the following standard relation

$$\gamma = \frac{-D_i(\omega)}{\frac{\partial D_r(\omega)}{\partial \omega_r}},$$

Using D_r and D_i from the last slide we have

$$\frac{\gamma}{\Omega_e} = -\frac{\sqrt{\pi}\sum_{n=-\infty}^{\infty} I_n\left(\frac{\mu_e}{v_r}\right)\exp\left[-\frac{\left(\frac{\omega_r}{\Omega_e}-n\right)^2}{k_r\mu_e}\right]\left[\frac{\frac{\omega_r}{\Omega_e}-n}{\sqrt{k_r\mu_e}}+\frac{n v_r}{2\sqrt{k_r\mu_e}}\right]}{\sum_{n=-\infty}^{\infty} I_n\left(\frac{\mu_e}{v_r}\right)\left[\frac{n v_r}{2\left(\frac{\omega_r}{\Omega_e}-n\right)^2}+\frac{k_r\mu_e}{\left(\frac{\omega_r}{\Omega_e}-n\right)^3}+\frac{3 n v_r k_r \mu_e}{4\left(\frac{\omega_r}{\Omega_e}-n\right)^4}+\frac{3k_r^2\mu_e^2}{\left(\frac{\omega_r}{\Omega_e}-n\right)^5}\right]}.$$

where $k_r = \frac{k_\parallel^2}{k_\perp^2}$.

Damping of Oblique EBWs

Separating n , the normalized growth rate becomes

$$\frac{\gamma}{\Omega_e} = -\frac{\sqrt{\pi}\left[t_1 + \sum_{n=1}^{\infty} I_n\left(\frac{\mu_e}{v_r}\right)[t_2 + t_3]\right]}{t_4 + \sum_{n=1}^{\infty} I_n\left(\frac{\mu_e}{v_r}\right) t_5},$$

$$t_1 = I_0\left(\frac{\mu_e}{v_r}\right)\exp\left[-\frac{\left(\frac{\omega_r}{\Omega_e}\right)^2}{k_r\mu_e}\right]\frac{\frac{\omega_r}{\Omega_e}}{\sqrt{k_r\mu_e}},$$

Damping of Oblique EBWs

$$t_2 = \exp\left[-\frac{\left(\frac{\omega_r}{\Omega_e} - n\right)^2}{k_r\mu_e}\right]\left[\frac{\frac{\omega_r}{\Omega_e} - n}{\sqrt{k_r\mu_e}} + \frac{n v_r}{2\sqrt{k_r\mu_e}}\right]$$

$$t_3 = \exp\left[-\frac{\left(\frac{\omega_r}{\Omega_e} + n\right)^2}{k_r\mu_e}\right]\left[\frac{\frac{\omega_r}{\Omega_e} + n}{\sqrt{k_r\mu_e}} - \frac{n v_r}{2\sqrt{k_r\mu_e}}\right]$$

$$t_4 = I_0\left(\frac{\mu_e}{v_r}\right)\left[\frac{k_r\mu_e}{\left(\frac{\omega_r}{\Omega_e}\right)^3} + \frac{3k_r^2\mu_e^2}{\left(\frac{\omega_r}{\Omega_e}\right)^5}\right]$$

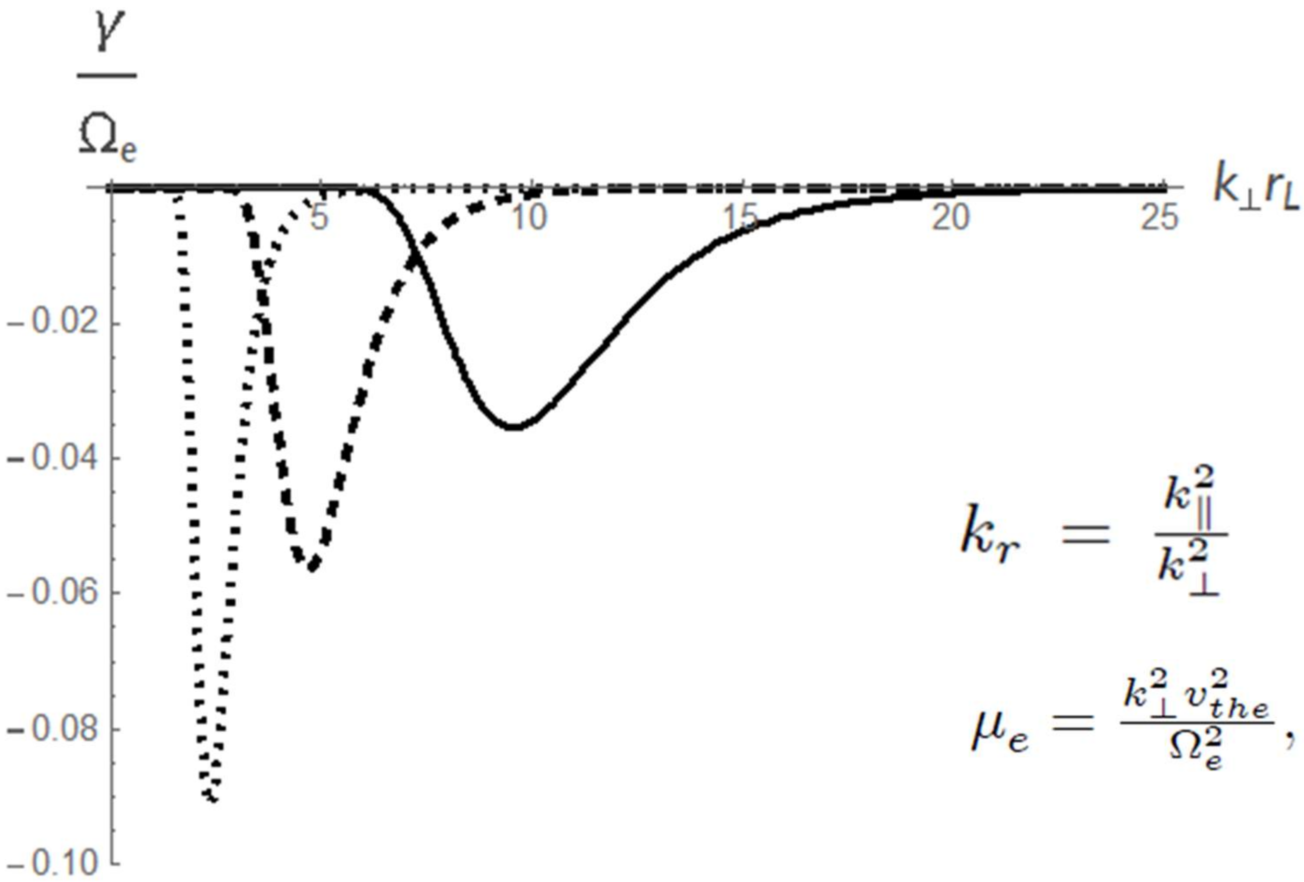
Damping of Oblique EBWs

$$t_5 = \left[\frac{nv_r\left(\frac{\omega_r^2}{\Omega_e^2}+n^2\right)}{\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^2}+\frac{2k_r\mu_e\left(\frac{\omega_r^3}{\Omega_e^3}+3n^2\frac{\omega_r}{\Omega_e}\right)}{\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^3}\right.$$

$$+\frac{12n^2v_rk_r\mu_e\left(\frac{\omega_r}{\Omega_e}\right)\left(\frac{\omega_r^2}{\Omega_e^2}+n^2\right)}{\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^4}$$

$$\left.+\frac{3k_r^2\mu_e^2\left\{\left(\frac{\omega_r}{\Omega_e}+n\right)^5+\left(\frac{\omega_r}{\Omega_e}-n\right)^5\right\}}{\left(\frac{\omega_r^2}{\Omega_e^2}-n^2\right)^5}\right].$$

Damping of Oblique EBWs



$$k_r = \frac{k_{\parallel}^2}{k_{\perp}^2}$$

$$\mu_e = \frac{k_{\perp}^2 v_{the}^2}{\Omega_e^2}, \; v_r = \frac{v_{the}^2}{v_{Qe}^2}$$

Plot of normalized growth rate for a fixed value of v=0.01 and with the variation of normalized k (=0.0001 for solid, =0.001 for dashed and =0.01 for dotted.)

Conclusions

- Propagation range of real frequencies can be enhanced by incorporating the obliqueness and large parallel wavenumbers have more strong influence on the wave propagation.

- Quantum signatures in the real part of oblique Bernstein waves exist and may be observed for the magnetic fields where $\hbar\Omega_c >> k_B T$ holds.

- Quantum signatures can take part in the damping of oblique Bernstein waves and are important for observation point of view.

- With the small deviation from the perpendicular propagation, we have strong electron cyclotron damping of the wave.

- For large ratio of normalized wavenumber, we observed that damping exists but it is limited to the small range of wavenumbers only.

Thanks!

Acknowledgements: Financial support form the Higher Education Commission (HEC) Pakistan to execute this work under the project No: 9184/AJK/ NRPU/R&D/HEC/2017 is gratefully acknowledged.