Probing GHz gravitational waves with graviton-magnon resonance

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Refs:
- AI, T. Ikeda, K. Miuchi, J. Soda, Eur. Phys. J. C 80 (2020) 3, 179
- AI, J.Soda, Eur. Phys. J. C 80 (2020) 6, 545

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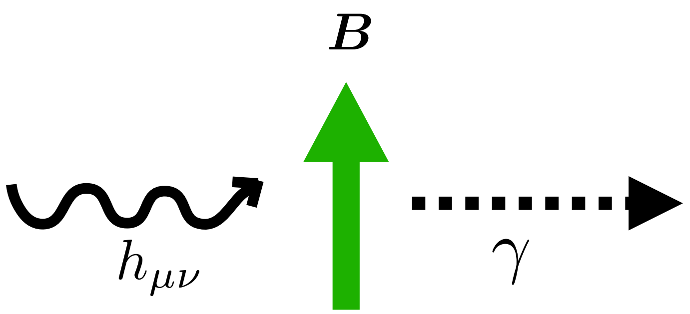
High-frequency GW detection

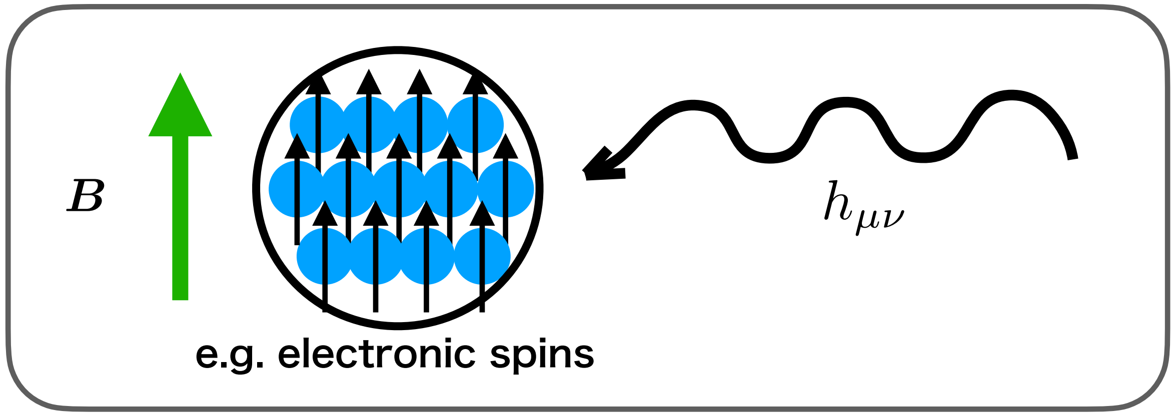
GWs around MHz-GHz range is theoretically interesting, but experiments are under development.
How to detect it?

- graviton-photon coupling (Ref: talks in this workshop)

$$\boxed{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \to -\frac{1}{4}F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}h^{\nu\beta}}$$



- graviton-spin coupling (Ref: AI, T.Ikeda, K.Miuchi, J.Soda (2020), AI, J.Soda (2020))



Fortunately, we have a “graviton-axion correspondence” in the both cases.

→ We can utilize well developed detectors for axion search to observe gravitational waves.

Talk plan

1. Interaction between gravitational waves and spins

2. Collective spin system: Magnon

3. Upper bounds on GHz gravitational waves from experiments of resonance fluorescence of magnons

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Dirac equation in curved spacetime

We study effects of gravitational waves on a spin of fermions.
It is described by the Dirac equation in curved spacetime,

$$\boxed{i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu\right)\psi = m\psi}$$

with a metric:

$$\boxed{\begin{aligned} g_{00} &= -1 - R_{0i0j}x^i x^j \ , \\ g_{0i} &= -\frac{2}{3}R_{0jik}x^j x^k \ , \\ g_{ij} &= \delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l \ , \end{aligned}}$$

Taking a non-relativistic limit and get a Hamiltonian
for the Schrodinger equation (Ref: AI, J.Soda (2020))

$$\boxed{\begin{aligned} \mathcal{H} &\ni -\frac{e}{m}S^i B^j \left[\delta_{ij}\left(1 + \frac{1}{2}R_{0k0l}x^k x^l + \frac{1}{6}R_{mkml}x^k x^l\right) - \frac{1}{6}R_{ikjl}x^k x^l\right] \\ &= -\frac{e}{m}S^i B^j \left[\delta_{ij}\left(1 - \frac{1}{3}\ddot{h}_{kl}x^k x^l\right) - \frac{1}{12}\left(h_{il,kj} + h_{kj,il} - h_{kl,ij} - h_{ij,kl}\right)x^k x^l\right] \end{aligned}}$$

Pauli term

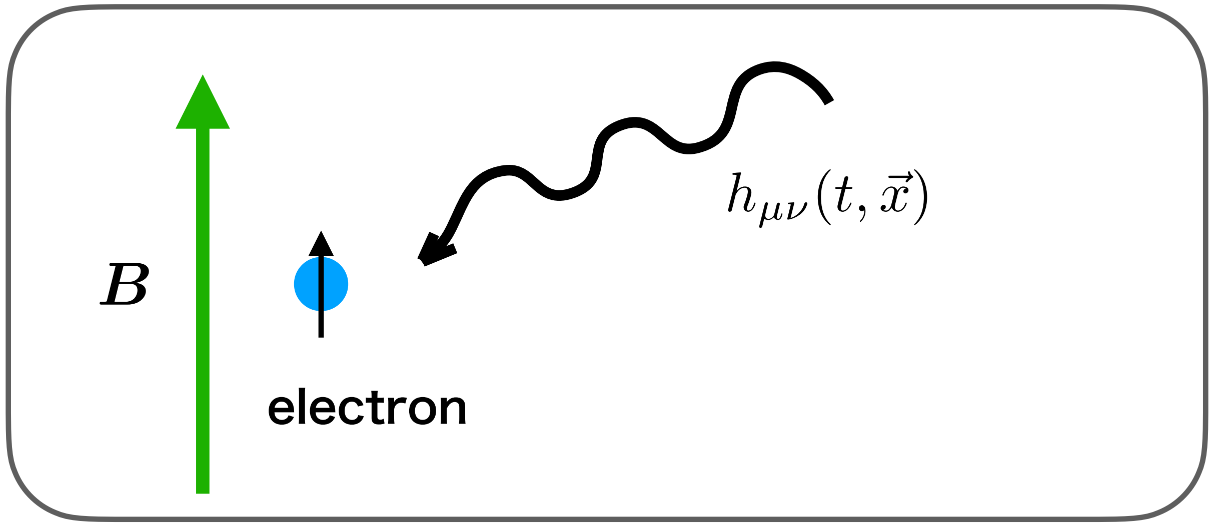
Interaction between a spin and GWs
in the presence of an external magnetic field

($\hat{S}^i$: spin, B^j : external magnetic field.)

Graviton-electron resonance

$$\boxed{\mathcal{H}_{spin} = -\frac{e}{m} S^i B^j \left[\delta_{ij} \left(1 - \frac{1}{3} \ddot{h}_{kl} x^k x^l \right) - \frac{1}{12} \left(h_{il,kj} + h_{kj,il} - h_{kl,ij} - h_{ij,kl} \right) x^k x^l \right]}$$

We can see that GWs can cause spin resonance in the presence of an external magnetic field



We now further consider a collective spin system (magnon)

→ We will see the resonance strength is enhanced by $\sqrt{N}$

(N is the number of spins)

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1. Interaction between gravitational waves and spins

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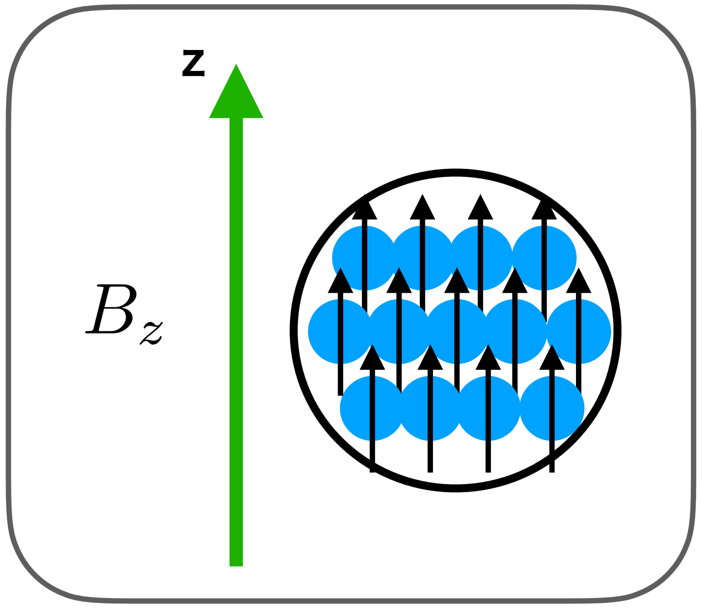
Collective spin system

We consider N electronic spins (e.g. a ferromagnetic sample) in an external magnetic field B_z .



It is well described by the Heisenberg model:

$$\boxed{\mathcal{H}_{mag} = -2\mu_B B_z \sum_i \hat{S}^z_{(i)} - \sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}}$$

$i = 1 \ldots N$ specify the sites of electrons.

J_{ij} : coupling constants between spins.

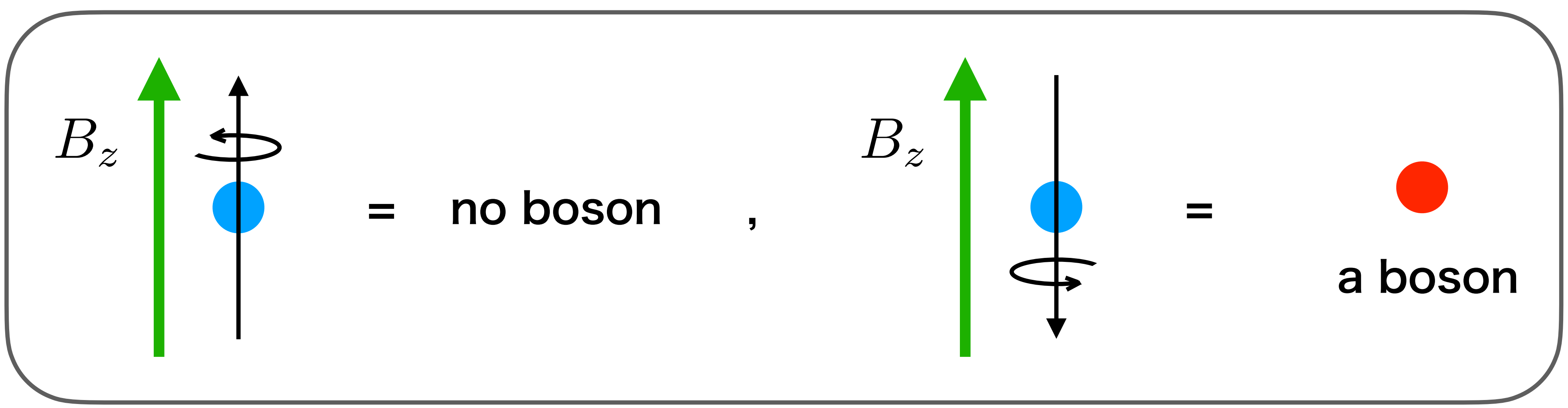
Holstein-Primakoff transformation

Spin operators can be rewritten in terms of bosonic operators
by using the Holstein-Primakoff transformation:

$$\begin{cases} \hat{S}^z_{(i)} = \frac{1}{2} - \hat{C}^\dagger_i \hat{C}_i \ , \\ \hat{S}^+_{(i)} = \sqrt{1 - \hat{C}^\dagger_i \hat{C}_i}\ \hat{C}_i \ , \\ \hat{S}^-_{(i)} = \hat{C}^\dagger_i \sqrt{1 - \hat{C}^\dagger_i \hat{C}_i} \ , \end{cases} \qquad \text{where} \quad [\hat{C}_i, \hat{C}^\dagger_j] = \delta_{ij}$$

Actually the SU(2) algebra is satisfied, $[\hat{S}^a_{(i)} \, , \hat{S}^b_{(i)}] = i\epsilon_{abc}\hat{S}^c_{(i)}$.

In the case of an electron,



Next, let us study the case of N electrons system.

Collective spin system

$$\mathcal{H}_{mag} = -2\mu_B B_z \sum_i \hat{S}^z_{(i)} - \sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}$$

Using the Holstein-Primakoff transformation

$$\begin{cases} \hat{S}^z_{(i)} = \frac{1}{2} - \hat{C}^\dagger_i \hat{C}_i \ , \\ \hat{S}^+_{(i)} = \sqrt{1 - \hat{C}^\dagger_i \hat{C}_i}\ \hat{C}_i \ , \\ \hat{S}^-_{(i)} = \hat{C}^\dagger_i \sqrt{1 - \hat{C}^\dagger_i \hat{C}_i} \ , \end{cases}$$

$$-2\mu_B B_z \sum_i \hat{S}^z_{(i)} = -2\mu_B B_z \sum_i (\frac{1}{2} - \hat{C}^\dagger_i \hat{C}_i)$$

$$\to 2\mu_B B_z \sum_i \hat{C}^\dagger_i \hat{C}_i$$

Translating to the Fourier space as

$$\boxed{\hat{C}_i = \sum_{\boldsymbol{k}} \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_i}}{\sqrt{N}} \hat{c}_k}$$

$$= 2\mu_B B_z \sum_i \sum_{\boldsymbol{k}} \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i}}{\sqrt{N}} \sum_{\boldsymbol{k}'} \frac{e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}_i}}{\sqrt{N}} \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}'}$$

$$= 2\mu_B B_z \sum_{\boldsymbol{k}} \sum_{\boldsymbol{k}'} \delta_{\boldsymbol{k}-\boldsymbol{k}'} \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}'} \qquad (\because \sum_i e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}_i} = N\delta_{\boldsymbol{k}-\boldsymbol{k}'})$$

$$= 2\mu_B B_z \sum_{\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}}$$

Collective spin system

$$\boxed{\mathcal{H}_{mag} = \underline{-2\mu_B B_z \sum_i \hat{S}^z_{(i)}} - \underline{\sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}}}$$

Now we have $\underline{-2\mu_B B_z \sum_i \hat{S}^z_{(i)}} = 2\mu_B B_z \sum_{\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}}$

Doing similar calculation yields $\underline{-\sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}} = \sqrt{N} \sum_{\boldsymbol{k}} \left(\tilde{J}(0) - \tilde{J}(\boldsymbol{k}) \right) \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}} \quad \left(\hat{C}^\dagger_j \hat{C}_j \ll 1 \right)$

(where $J(\boldsymbol{r}_j) = \sum_{\boldsymbol{k}} \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j}}{\sqrt{N}} \tilde{J}(\boldsymbol{k})$)

Therefore $\boxed{\mathcal{H}_{mag} = \sum_{\boldsymbol{k}} \hbar\omega_k \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}}}$

quantized "spin wave"

=

magnon

where the dispersion relation is given by $\boxed{\hbar\omega_k = 2\mu_B B_z + \sqrt{N} \left(\tilde{J}(0) - \tilde{J}(\boldsymbol{k}) \right)}$

Magnons in GWs

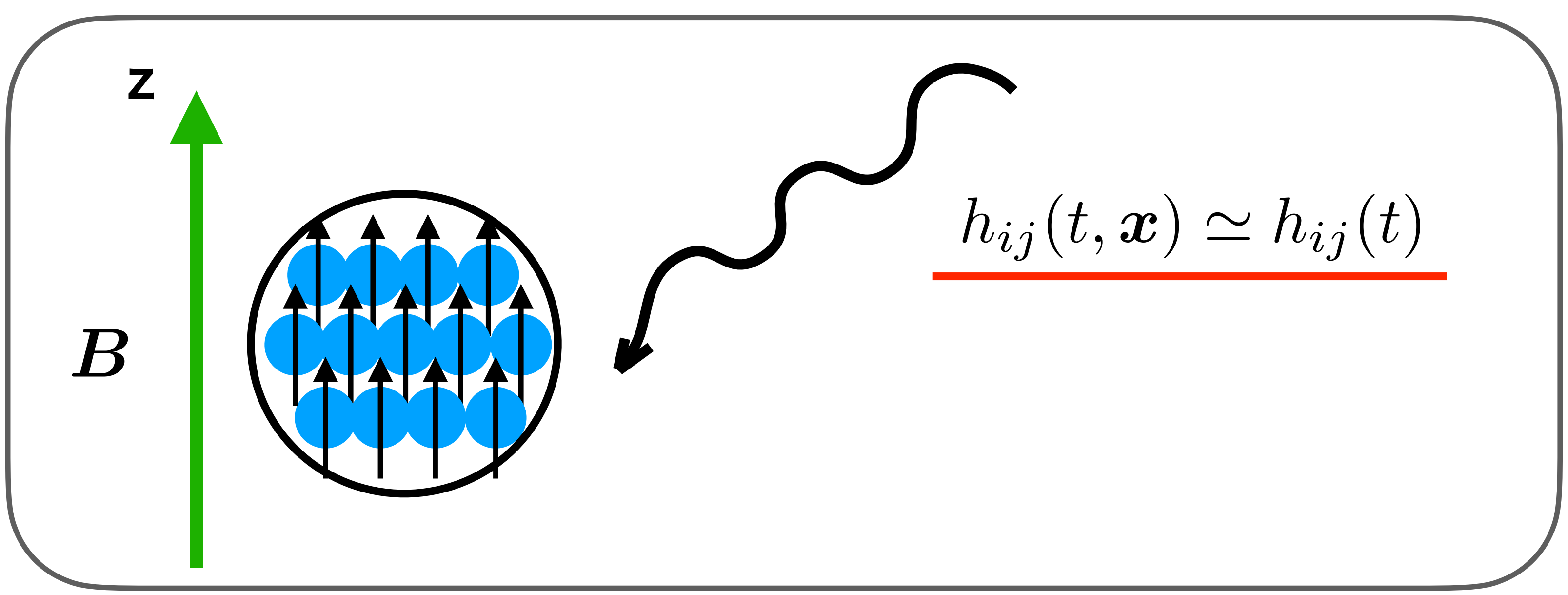
In terms of magnons, the hamiltonian of N spin system can be written as

$$\boxed{\mathcal{H}_{mag} = \sum_{\boldsymbol{k}} \hbar\omega_k \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}}, \qquad \hbar\omega_k = 2\mu_B B_z + \sqrt{N}\left(\tilde{J}(0) - \tilde{J}(\boldsymbol{k})\right)}$$

Let us consider effects of GWs on the magnons.

We assume that GWs we observe are homogeneous over the sample,
such situation is satisfied in experiments which we utilize for giving constraints.

Typically the size of a sample is ∼ 0.5 mm
and the weave length of GHz GWs which we observe is ∼ 5 cm



GW and magnon

Including the effect of GWs on the spin system, the total Hamiltonian is

$$H_{\mathrm{tot}} = -\mu_B \left(2\delta_{za} + \underline{Q_{za}}\right) B_z \sum_i \hat{S}^a_{(i)} - \sum_{i,j} J_{ij} \hat{\boldsymbol{S}}_{(i)} \cdot \hat{\boldsymbol{S}}_{(j)}$$

where $\quad Q_{ij} = -\frac{2}{3}\delta_{ij}\ddot{h}_{kl}|_{\boldsymbol{x}=0}\, x^k x^l - \frac{1}{6}\left(h_{il,kj} + h_{kj,il} - h_{kl,ij} - h_{ij,kl}\right)|_{\boldsymbol{x}=0}\, x^k x^l$

Transforming from spins to magnons
and considering a planar GW

$$H_{\mathrm{tot}} \simeq 2\mu_B B_z \hat{c}^\dagger \hat{c} + \underline{g_{eff}\left(\hat{c}^\dagger \underbrace{e^{-i\omega_h t}}_{\approx\, e^{i(2\mu_B B_z)t}} + \hat{c}\underbrace{e^{i\omega_h t}}_{\approx\, e^{-i(2\mu_B B_z)t}}\right)}$$

$$g_{eff} = \frac{\sqrt{2}\pi^2}{60}\left(\frac{l}{\lambda}\right)^2 \mu_B B_z \sin\theta \sqrt{N}\left[\cos^2\theta\,(h^{(+)})^2 + (h^{(\times)})^2 + 2\cos\theta\sin\alpha\, h^{(+)}h^{(\times)}\right]^{1/2}.$$

($h^{(+)}$, $h^{(\times)}$: amplitude, α : polarization angle, ω_h, λ : angular frequency and wavelength of a GW,
θ : angle between the GW and an external magnetic field B_z , l : radius of the sample.)

- When $\omega_h = 2\mu_B B_z$, resonance occurs
- the strength of interaction is amplified by $\sqrt{N}$ (typically $\sqrt{N} \sim \sqrt{10^{20}} \sim 10^{10}$)

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Magnons in GWs

GWs excite magnons!

Excited magnons radiate electromagnetic fields when they return to the ground state. (resonance fluorescence)

↓

Observing the resonance fluorescence of magnons, one can observe GWs.

↓

Utilizing the results of the resonance fluorescence of magnons for axion DM search

(QUAX (2018), G. Flower et al. (2018))

we gave upper limits on GWs

(AI, T. Ikeda, K. Miuchi, J. Soda, Eur. Phys. J. C 80 (2020) 3, 179
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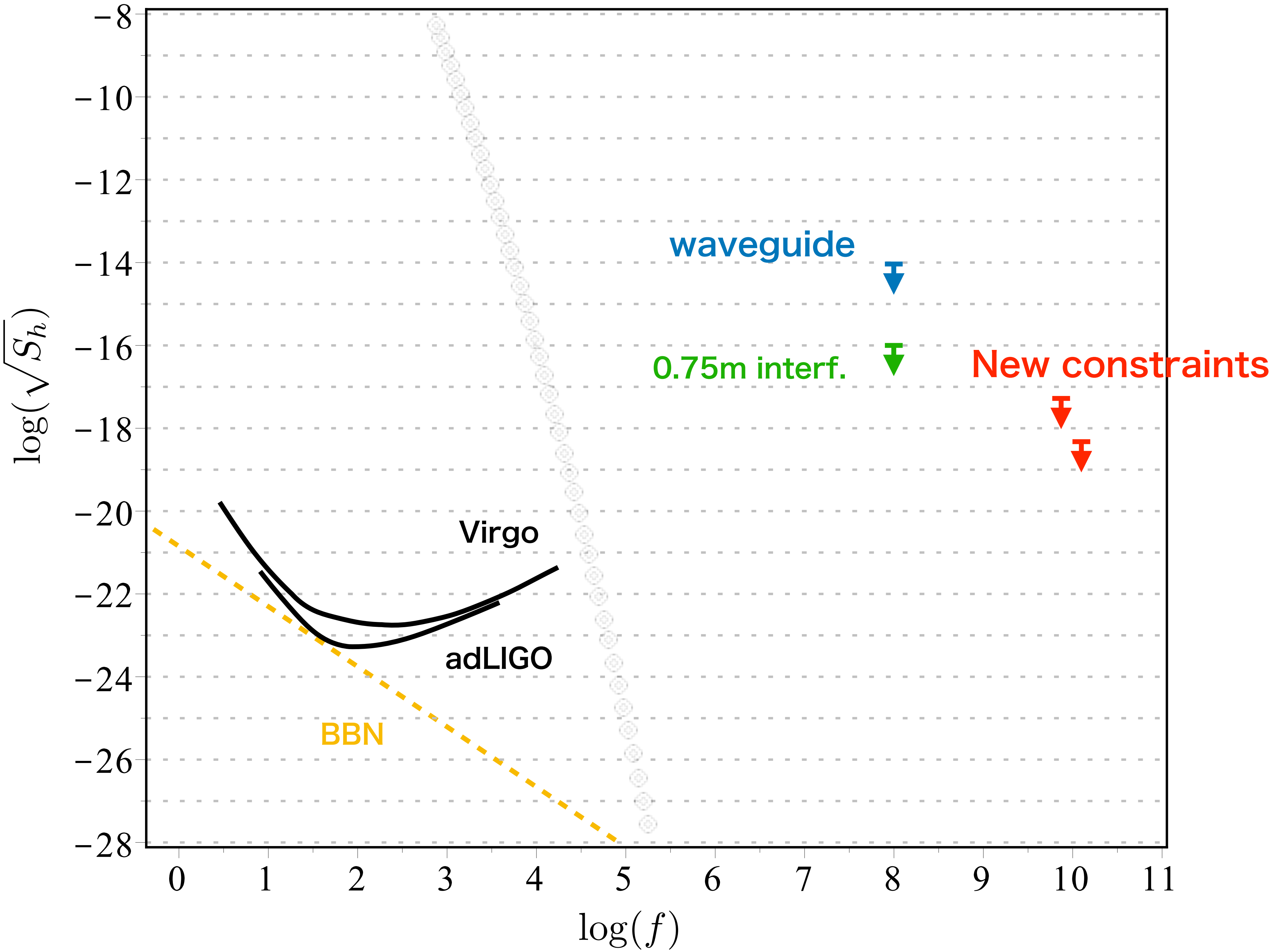
Upper limits on GHz GWs



Upper limits on GHz GWs

In terms of the spectral density, the upper limits are

$$\sqrt{S_h} \sim \begin{cases} 7.5 \times 10^{-19}\ [\mathrm{Hz}^{-1/2}] & \text{at } 14\ \mathrm{GHz}\ , \\ 8.7 \times 10^{-18}\ [\mathrm{Hz}^{-1/2}] & \text{at } 8.2\ \mathrm{GHz}\ . \end{cases}$$

In terms of the dimensionless amplitude, the upper limits are

$$h_c \sim \begin{cases} 1.3 \times 10^{-13} & \text{at } 14\ \mathrm{GHz}\ , \\ 1.1 \times 10^{-12} & \text{at } 8.2\ \mathrm{GHz}\ . \end{cases}$$

We have implicitly assumed that the coherence time of the GW is as same as that of axion DM $\Delta t_a \sim 1\mathrm{ms}$

In case of GWs, the sensitivity could be improved depending on the coherence time of GWs. From the Dicke radiometer equation, we see the following scaling

$$\boxed{h_c \propto \Delta t_h^{-1/4}}$$

Summary

- We formulated the graviton-spin interaction and showed that GWs can excite magnons (graviton-magnon resonance)

- The graviton-magnon resonance enables us to probe GWs at the frequency of magnon modes

- We gave upper limits on the spectral density $\sqrt{S_h}$ of continuous GWs,

$$\sqrt{S_h} \sim \begin{cases} 7.5 \times 10^{-19} \ [\mathrm{Hz}^{-1/2}] & \text{at } 14 \ \mathrm{GHz} \ , \\ 8.7 \times 10^{-18} \ [\mathrm{Hz}^{-1/2}] & \text{at } 8.2 \ \mathrm{GHz} \ . \end{cases}$$

at 95% C.L.

- One can perform all sky search of continuous GWs at the above sensitivity.

- We can also search for stochastic GWs and burst GWs with almost the same sensitivity.

- How to improve the sensitivity? e.g. increasing N