# The Microstrip SQUID Amplifier

Michael Mück,ez SQUID Mess- und Analysegeraete

1. Introduction

2. The dc SQUID

3. Using a dc SQUID as a high-frequency amplifier

4. A SQUID amplifier with microstrip input coupling

5. Gain, Noise Temperature and Input and Output Impedance of Microstrip SQUID Amplifiers

6. Summary

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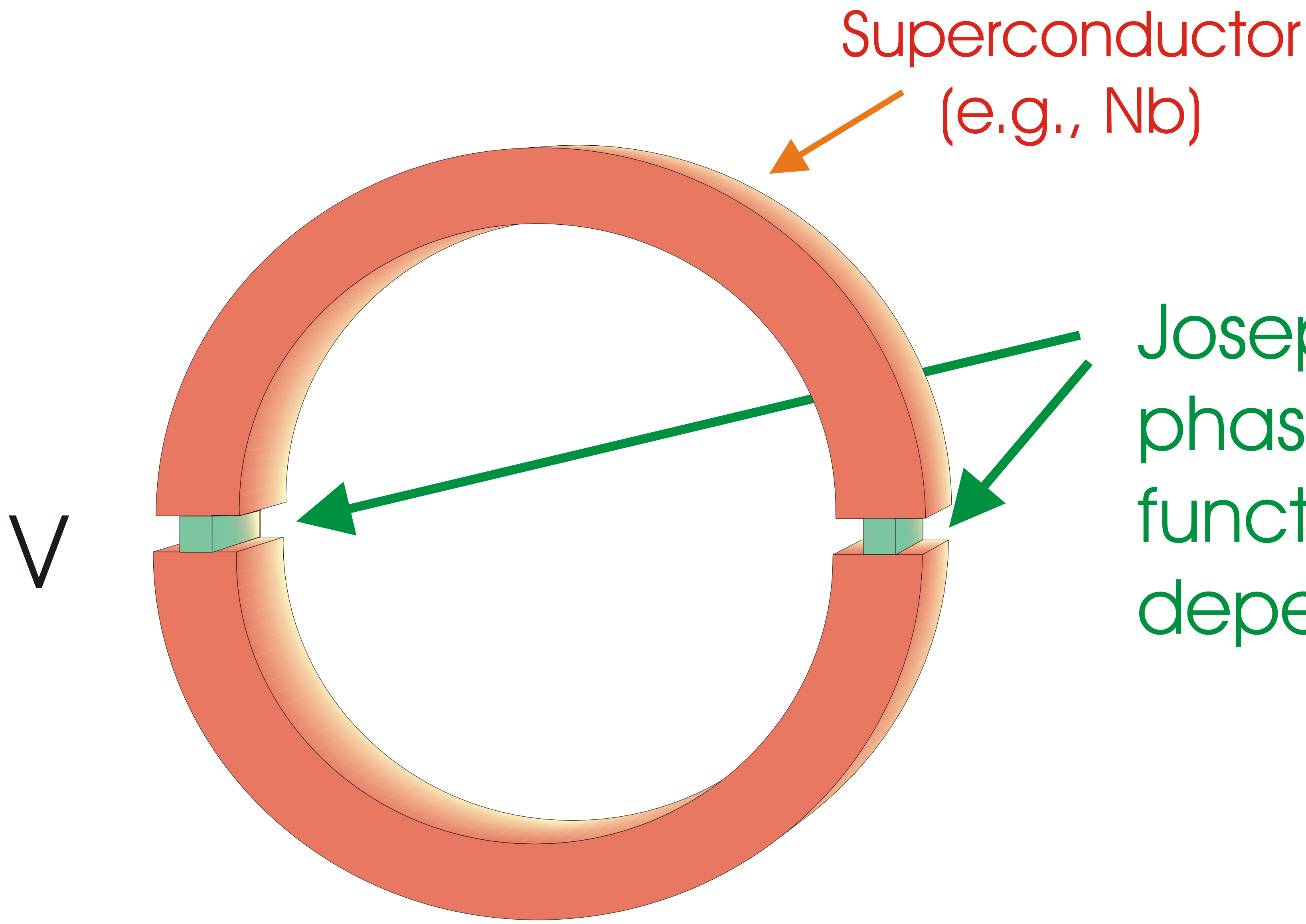
**This talk is dedicated to**
**John Clarke**
**on the occasion of his**
**80th birthday.**

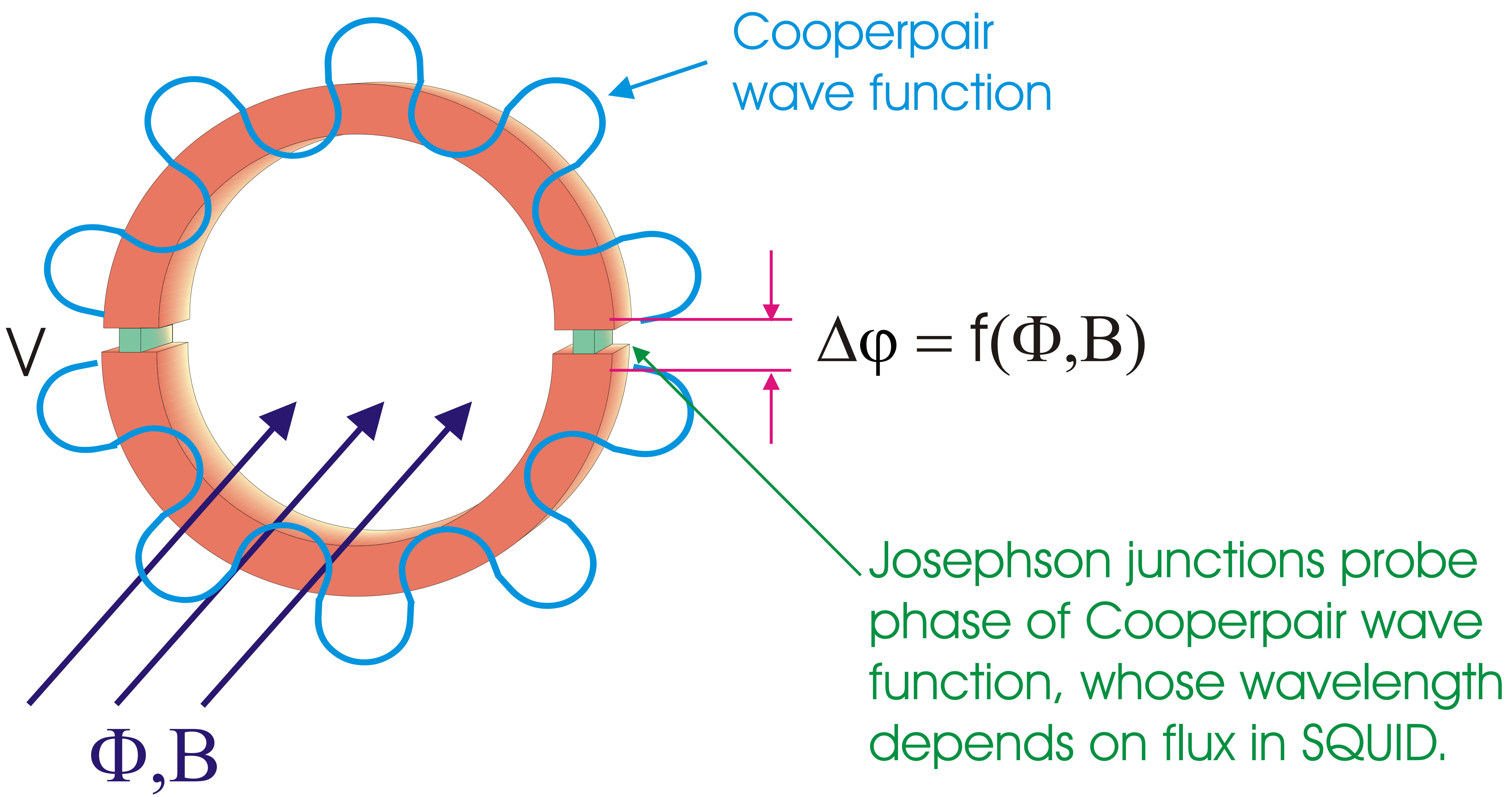
- LLNL needed a very low noise amplifier (~ 500 MHz) for their axion project.
- The noise temperature of a transistor amplifier scales as the physical temperature of the transistor, so cooling the transistor increases sensitivity.
- As a transistor requires current of ~ mA for low noise, there is a lower limit for the temperature of the transistor chip.
- Idea: Use a superconducting quantum interference device (SQUID) as rf amplifier. Noise temperature scales as SQUID temperature, but dissipation is much lower than in transistor. Physical temperature of SQUID can be 100 mK or lower.
  $\Rightarrow$ Noise could be ten times (or more) lower than noise of transistor.

# *The dc SQUID*

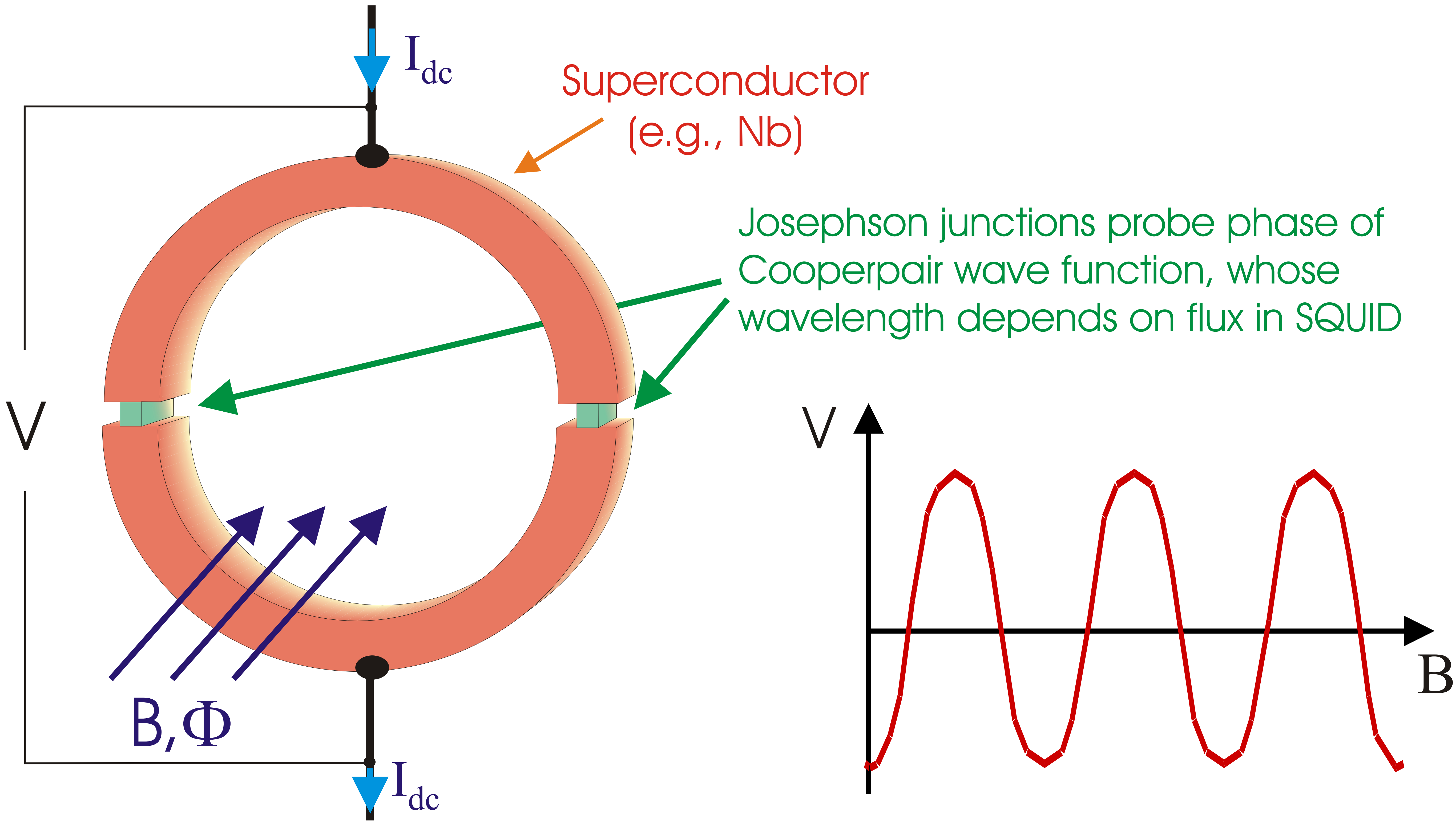



Josephson junctions probe phase of Cooperpair wave function, whose wavelength depends on flux in SQUID.

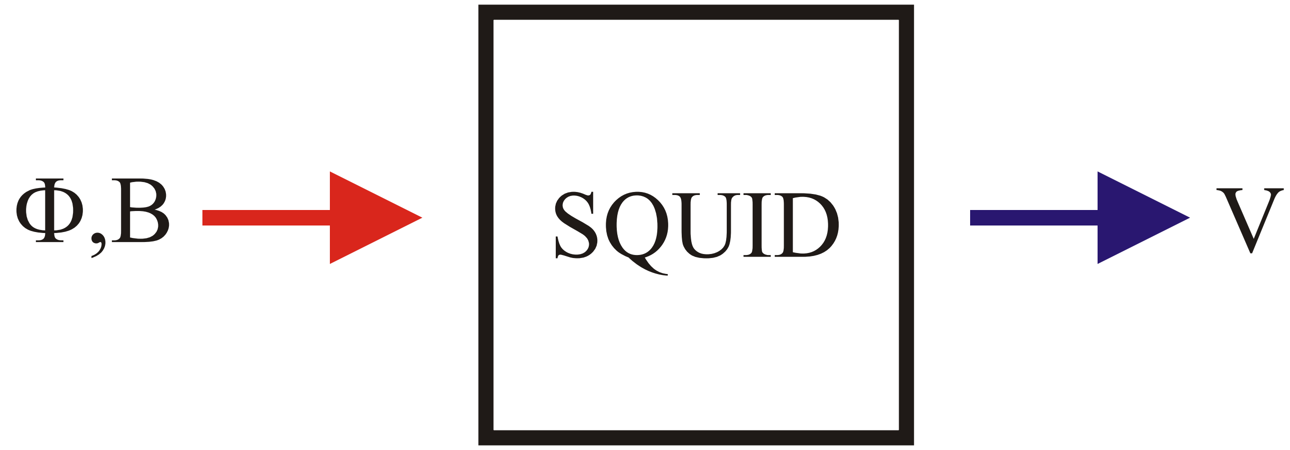
# The dc SQUID


Cooperpair
wave function
V
$\Delta\varphi = f(\Phi, B)$
Josephson junctions probe
phase of Cooperpair wave
function, whose wavelength
depends on flux in SQUID.
$\Phi$,B

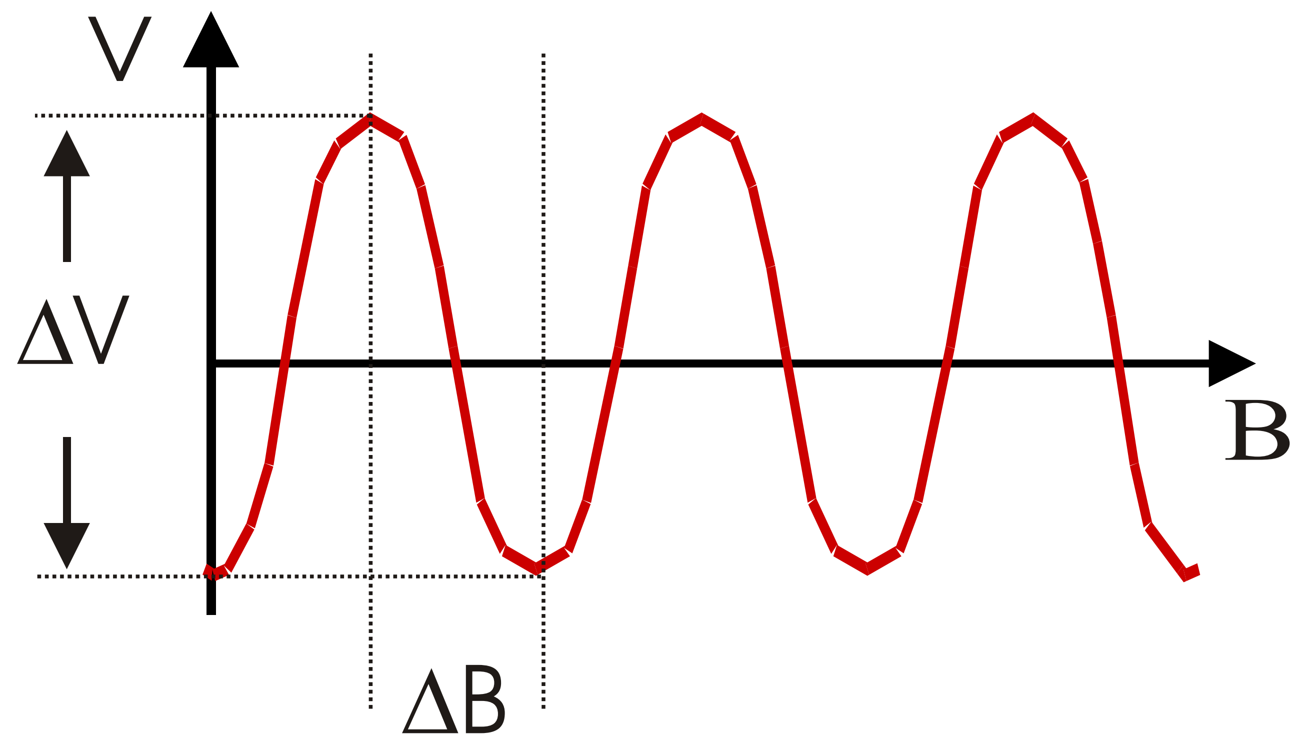
# *The dc SQUID*




Typical values: $I_c \sim 10\ \mu A$, $\Delta I_c \sim 5\ \mu A$, $\Delta V \sim 50\ \mu V_{pp}$,
$A \sim 100 \times 100\ \mu m^2$, $L \sim 150$ pH




Due to periodic nature of Cooperpair wave function, dependence of voltage on magnetic flux (field) is periodic, too.



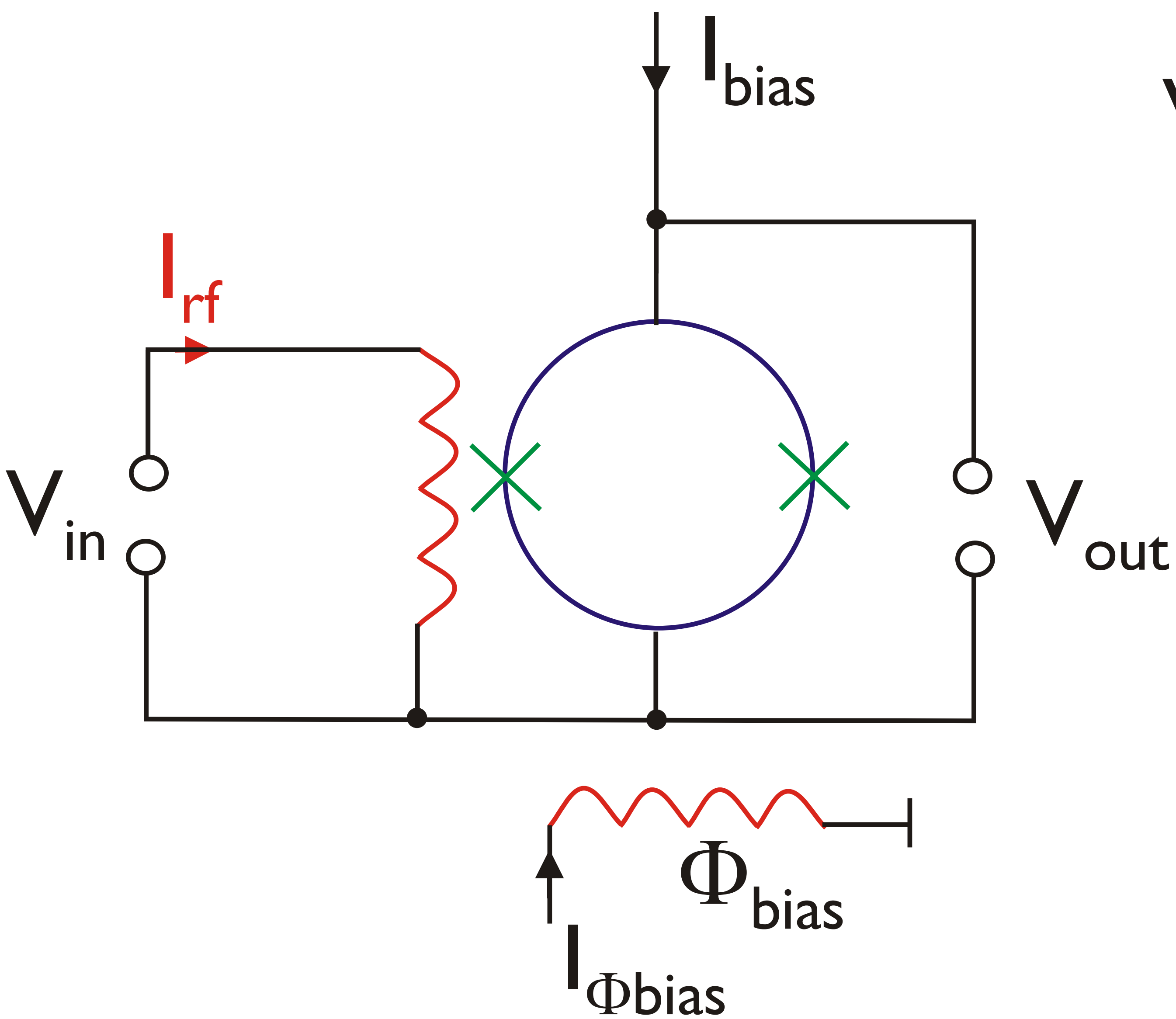

Typical values: $\Delta V \sim 50\ \mu V_{pp}$, for $\Delta B \sim 100$ nT

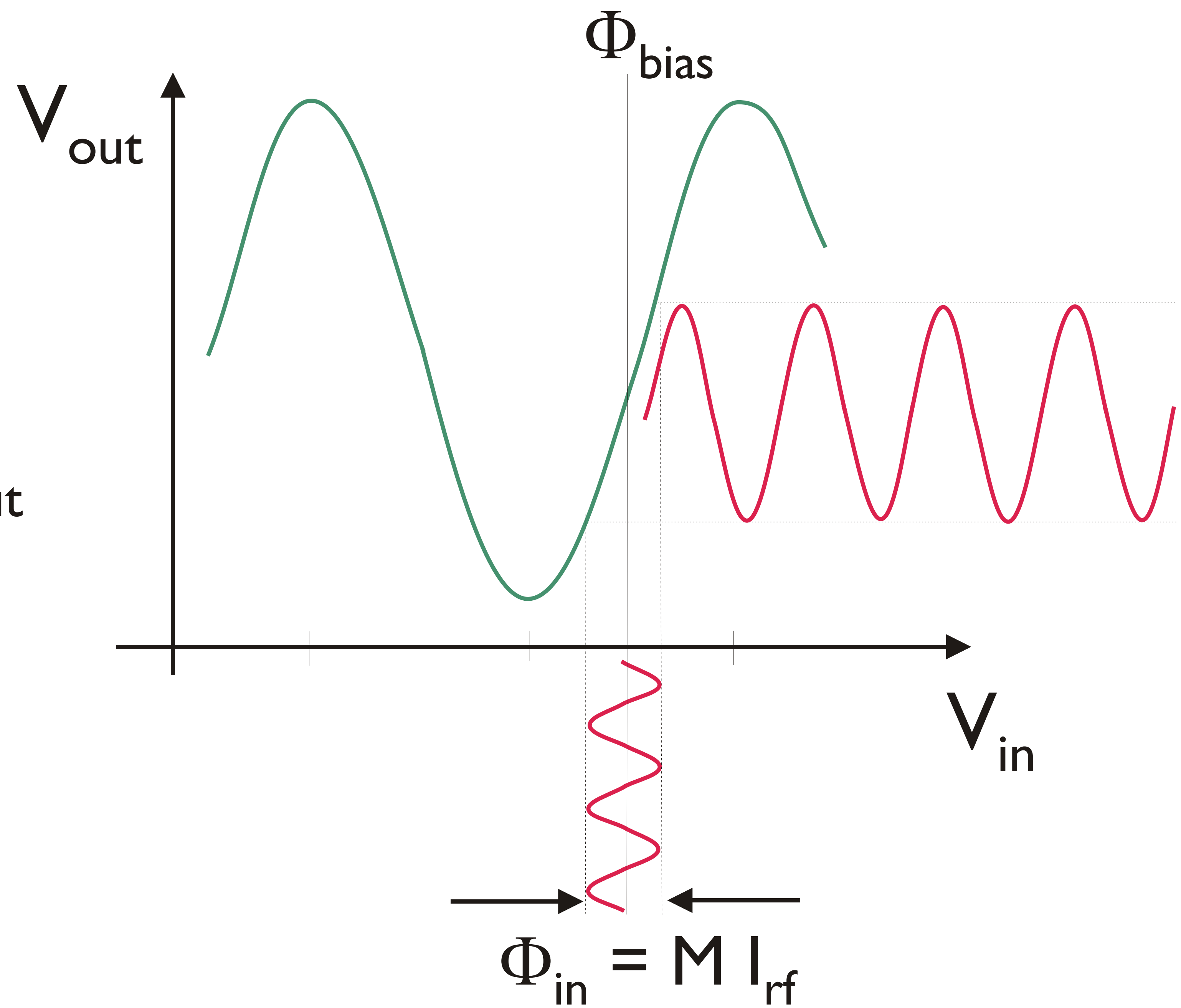
$A \sim 100 \times 100\ \mu m^2$, $L \sim 150$ pH

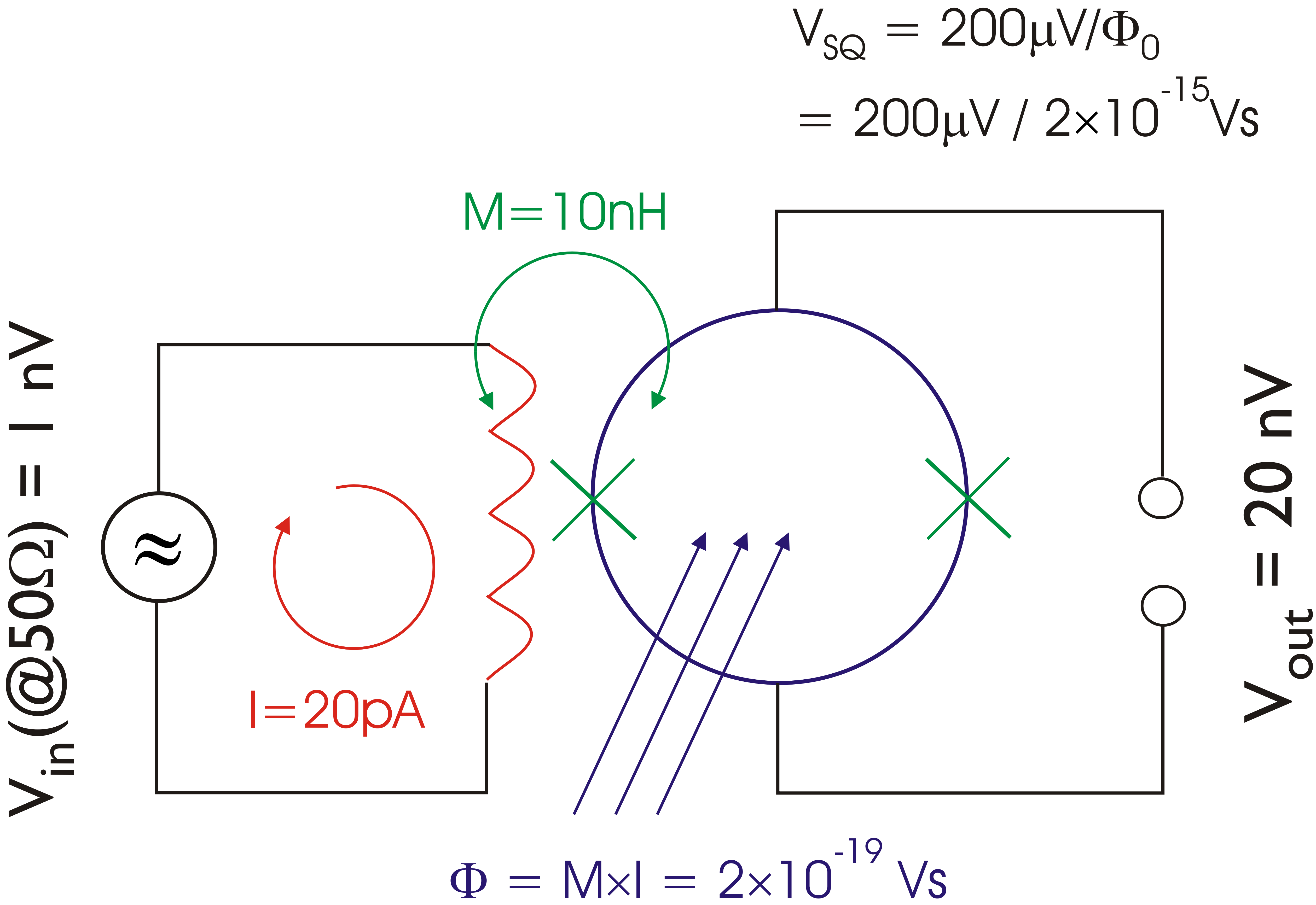
# Using a dc SQUID as a radio frequency amplifier

A SQUID measures magnetic flux: use an (integrated) input coil to convert rf current to magnetic flux.

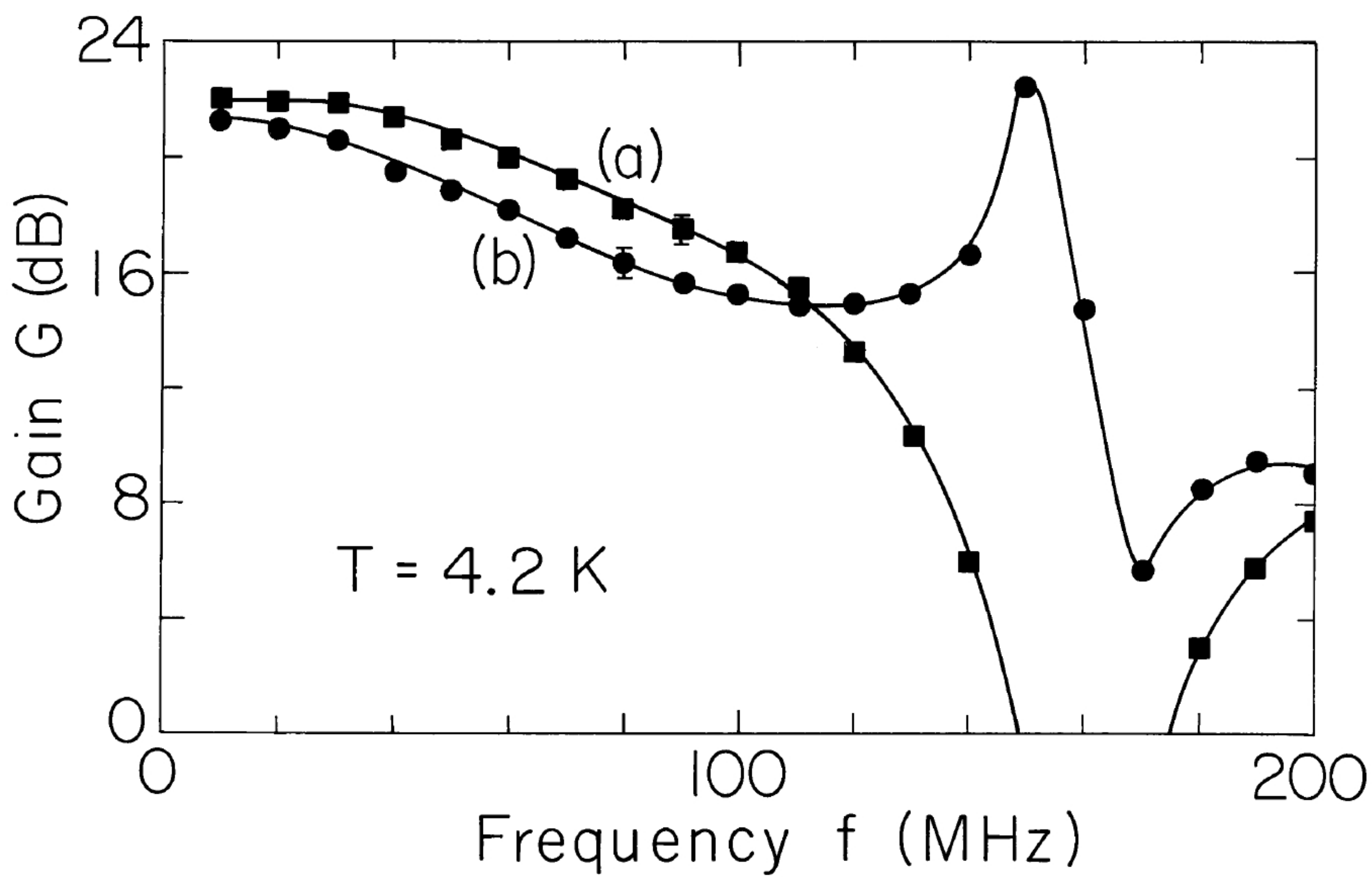
For small input power levels, the amplifier is linear.

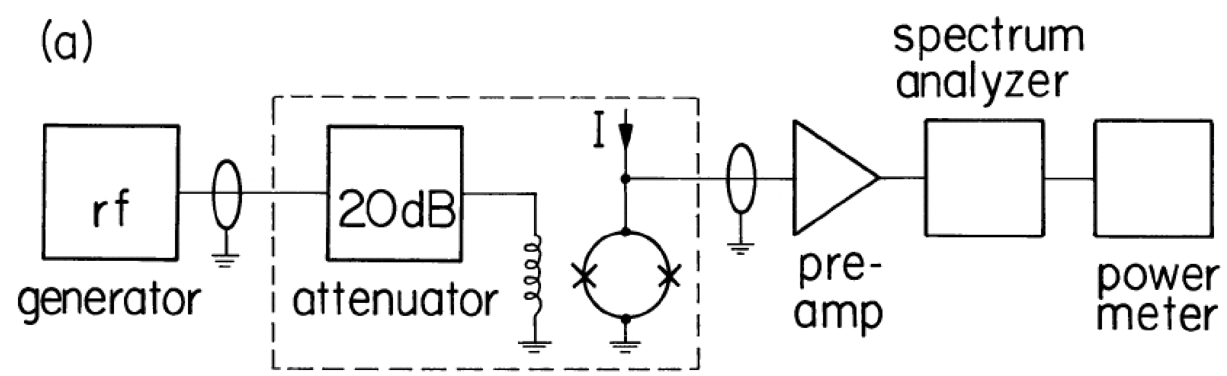




$V_{SQ} = 200\mu V/\Phi_0$
$= 200\mu V / 2\times10^{-15} Vs$
M=10nH
I=20pA
$V_{in}(@50\Omega) = 1\ nV$
$V_{out} = 20\ nV$
$\Phi = M\times I = 2\times10^{-19}\ Vs$

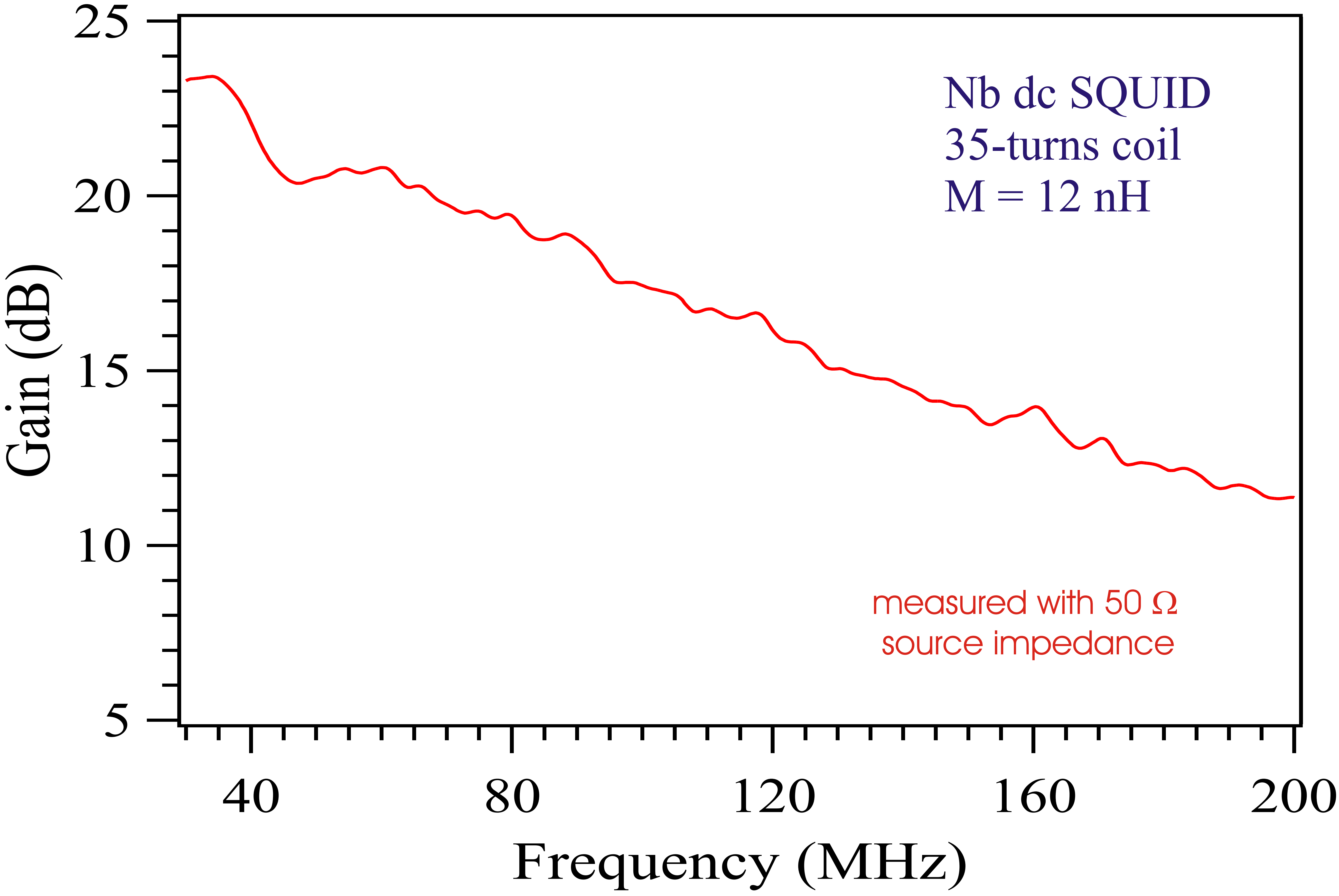


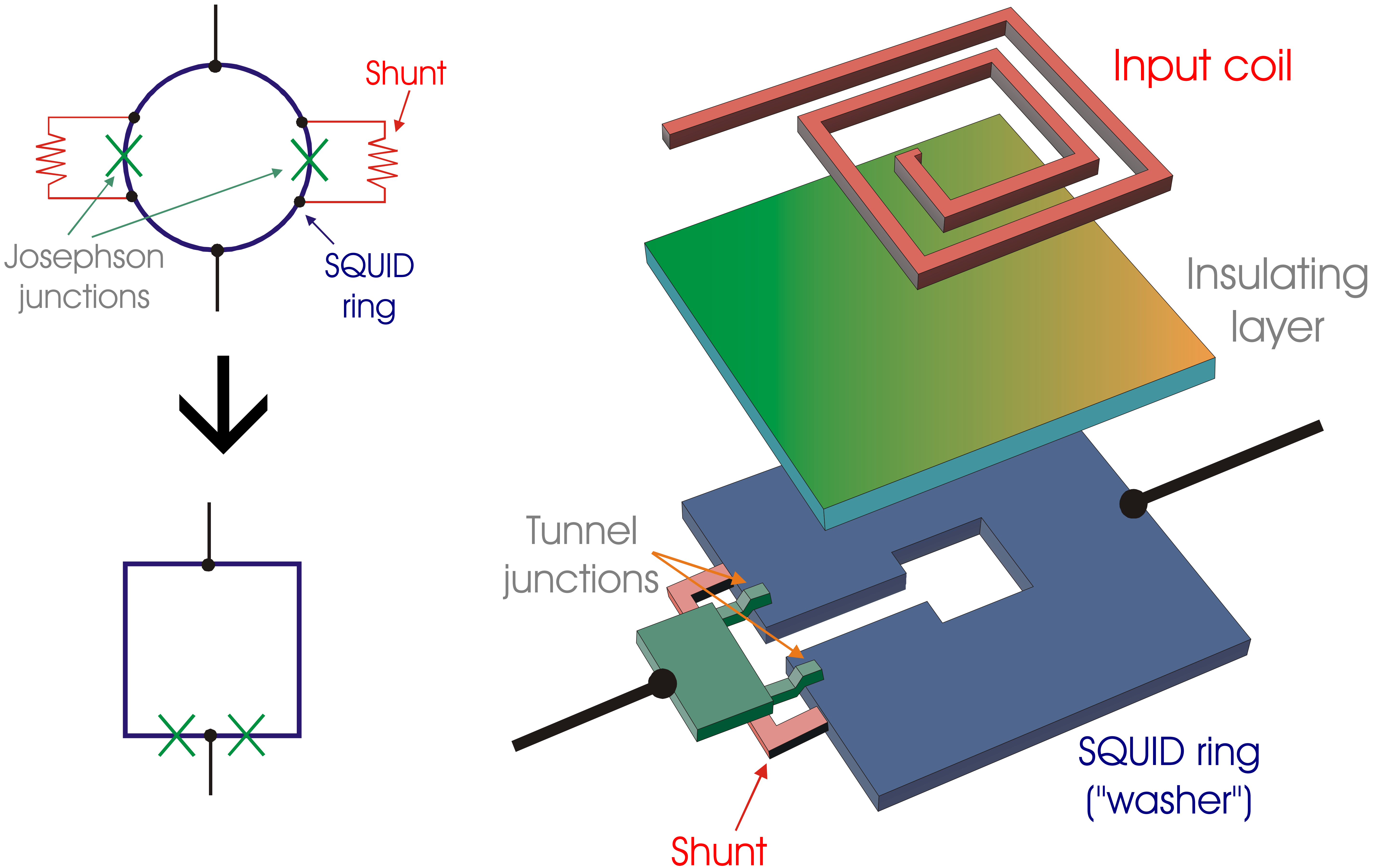




| | Frequency (MHz) | $G_p$ (dB) | | $T_N$ (K) | |
|---|---|---|---|---|---|
| | | Measured | Predicted | Measured | Predicted |
| $T = 4.2$ K (tuned) | 93 | 18.6 ± 0.5 | 17 | 1.7 ± 0.5 | 1.1 |
| $T = 1.5$ K (untuned) | 60 | 24.0 ± 0.5 | – | 1.2 ± 0.3 | – |
| | 80 | 21.5 ± 0.5 | – | 0.9 ± 0.3 | – |
| | 100 | 19.5 ± 0.5 | 18.5 | 1.0 ± 0.4 | 0.9 |
| $T = 4.2$ K (untuned) | 60 | 20.5 ± 0.5 | – | 4.5 ± 0.6 | – |
| | 80 | 18.0 ± 0.5 | – | 4.1 ± 0.7 | – |
| | 100 | 16.5 ± 0.5 | 16.5 | 3.8 ± 0.9 | 2.5 |

*C.Hilbert and J.Clarke, DC SQUIDs as radiofrequency amplifiers, J. Low Temp. Phys. 61, 263–280 (1985).*

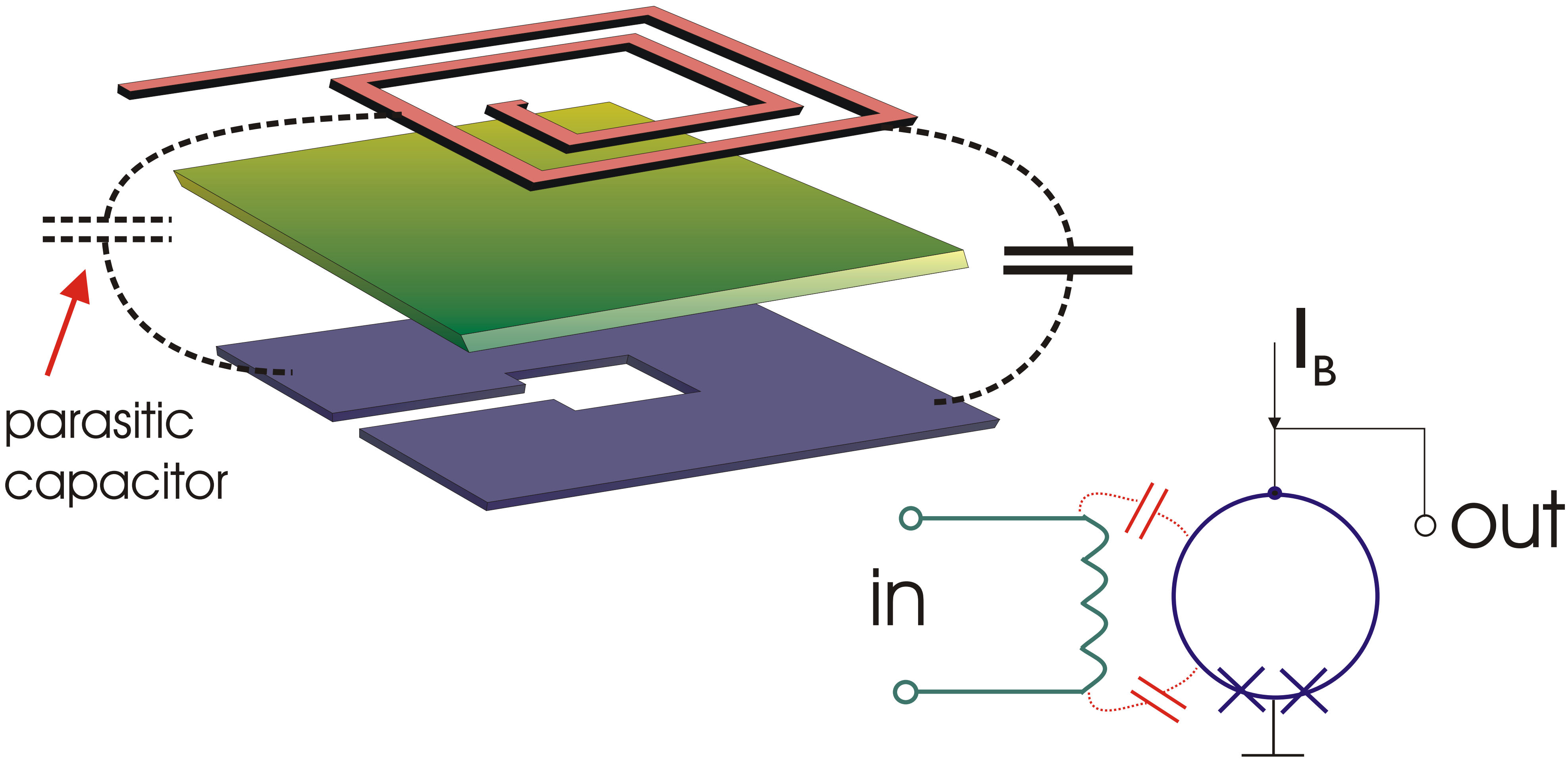
# Gain vs frequency of a ‘conventional’ dc SQUID amplifier




Shunt
Josephson
junctions
SQUID
ring
Input coil
Insulating
layer
Tunnel
junctions
SQUID ring
("washer")
Shunt

# The SQUID rf amplifier

At higher frequencies, parasitic capacitance between the integrated input coil and the SQUID washer reduces the gain substantially.



# Ways to reduce the effects of parasitic capacitance

- operate the amplifier at the resonance frequency of the input coil, which can be modified by external components (Takami et al.).

  *T.Takami, T.Noguchi, and K.Hamanaka, IEEE Trans.Mag. MAG-25, 1030 (1989)*

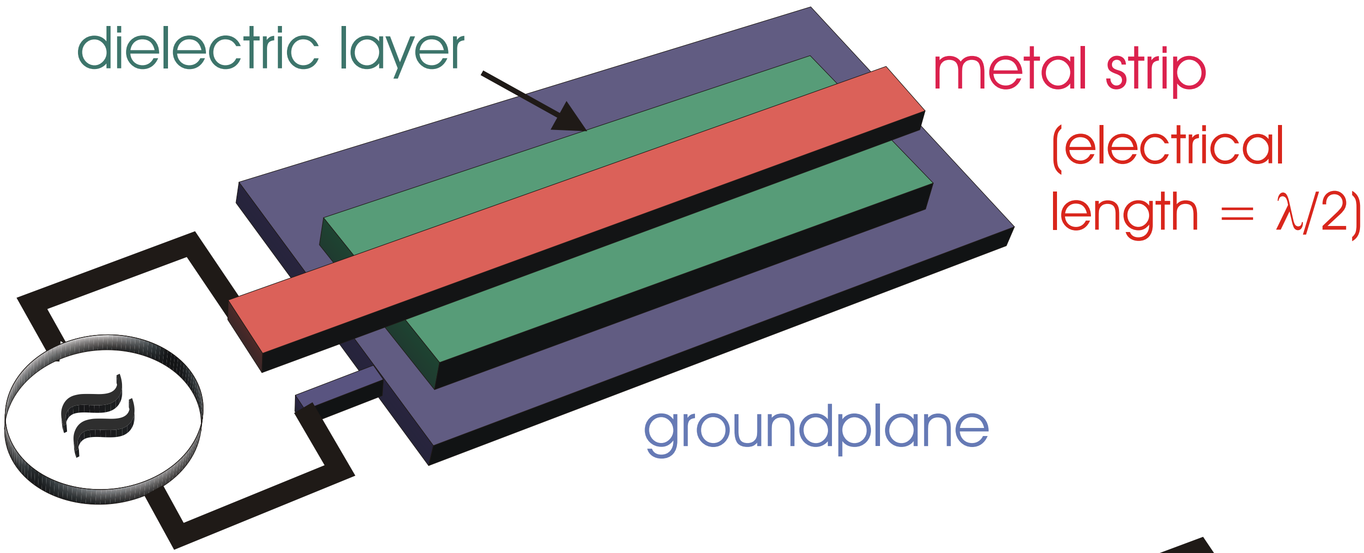
- reduce the parasitic capacitance by placing the input coil into the SQUID hole (Prokopenko et al.).

  *G.V.Prokopenko et al, IEEE Trans.Appl.Supercond. AS-13, 1042-1045 (2003)*

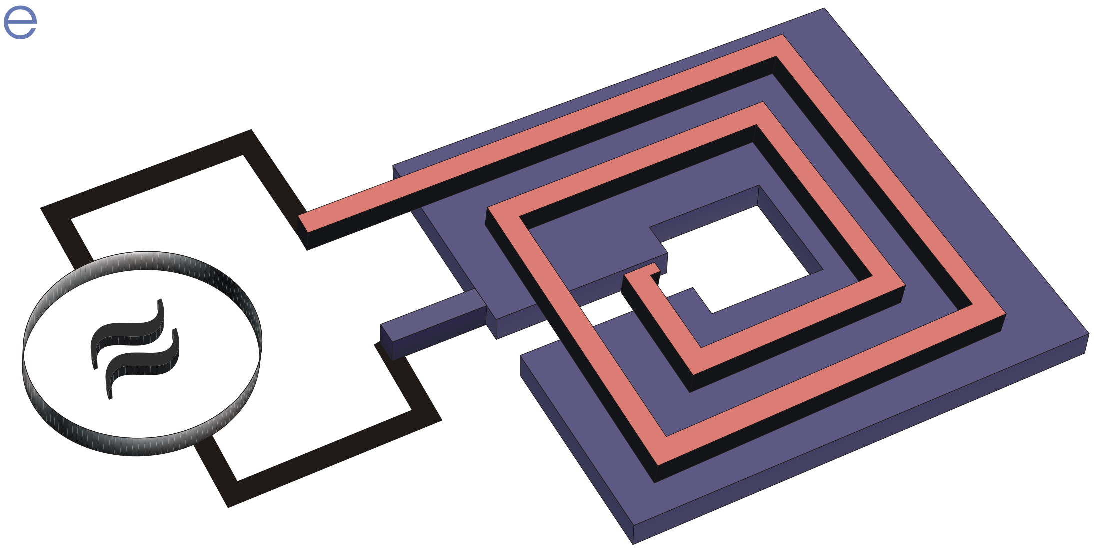
- make the input-coil inductance small enough to shift the resonance to above the operating frequency (Spietz et al.).

  *Lafe Spietz, Kent Irwin, and José Aumentado APPLIED PHYSICS LETTERS 93, 082506 (2008)*

- incorporate the parasitic capacitance into a microstrip transmission line.

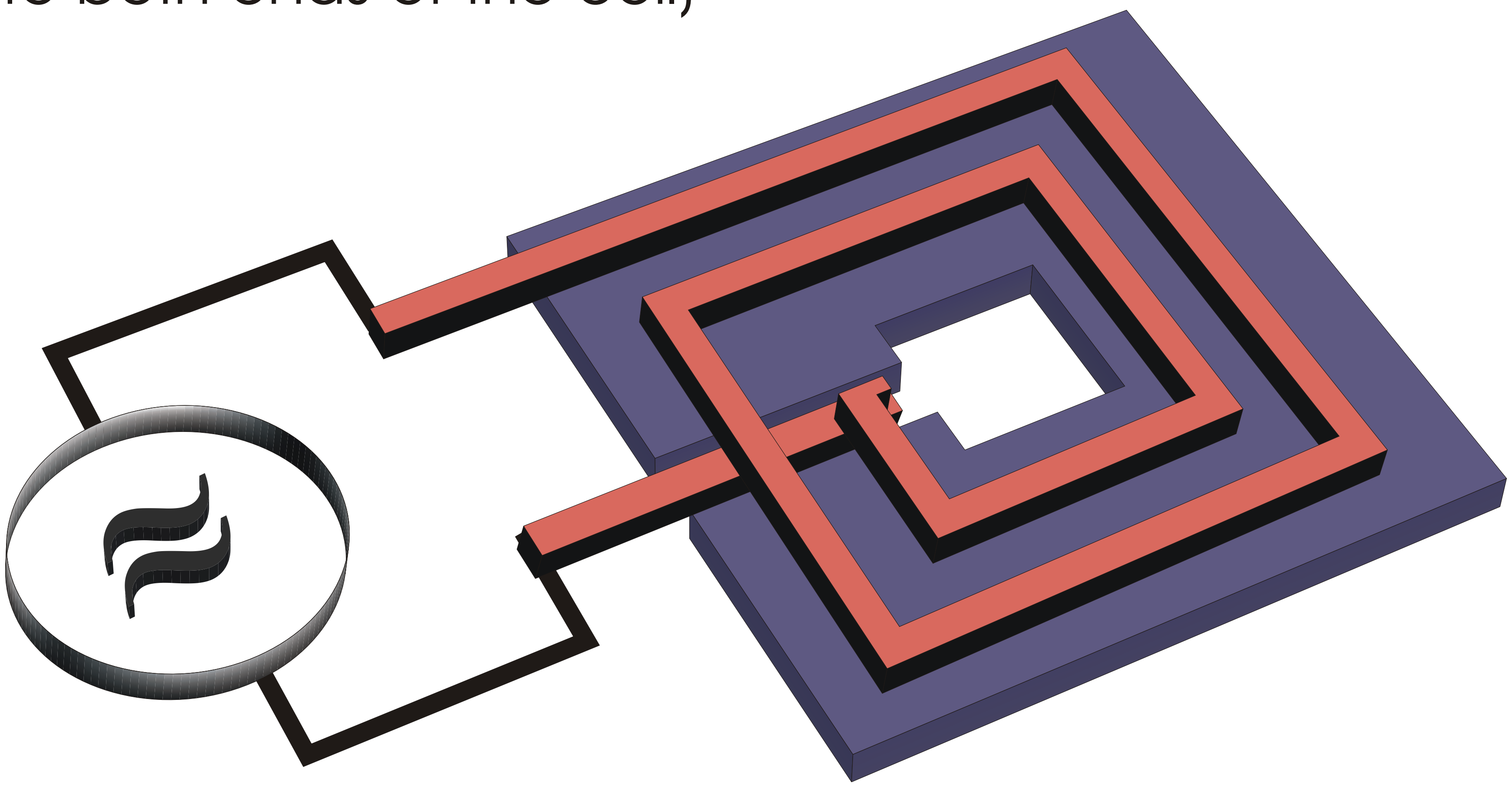



The λ/2 microstrip resonator

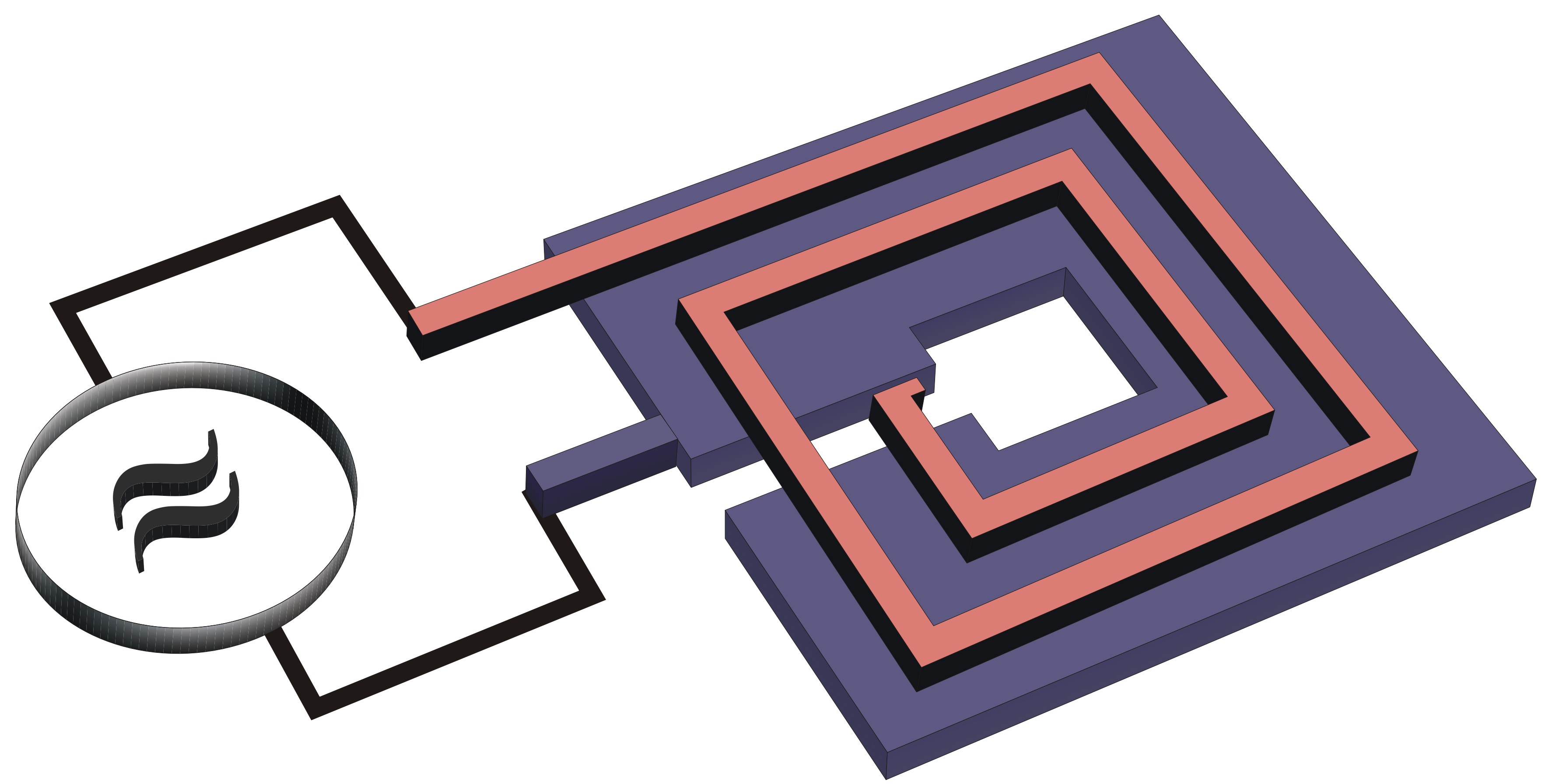


We have taken advantage of the parasitic capacitance by operating the input coil as a microstrip resonator.  In a microstrip SQUID amplifier, the input signal is applied between one end of the coil and the SQUID washer; the other end of the coil is left open. The SQUID washer acts as a groundplane for the microstrip resonator.

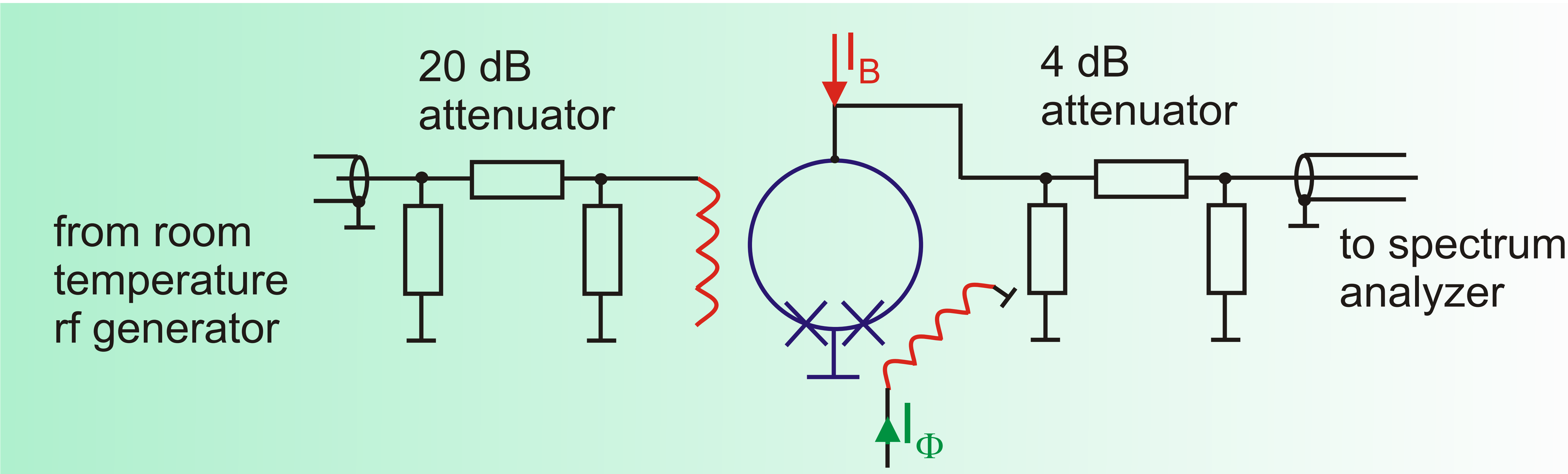
Conventional SQUID Amplifier (source connected to both ends of the coil)



Microstrip SQUID Amplifier (source connected to one end of the coil and SQUID washer; the other end of the coil is left open)

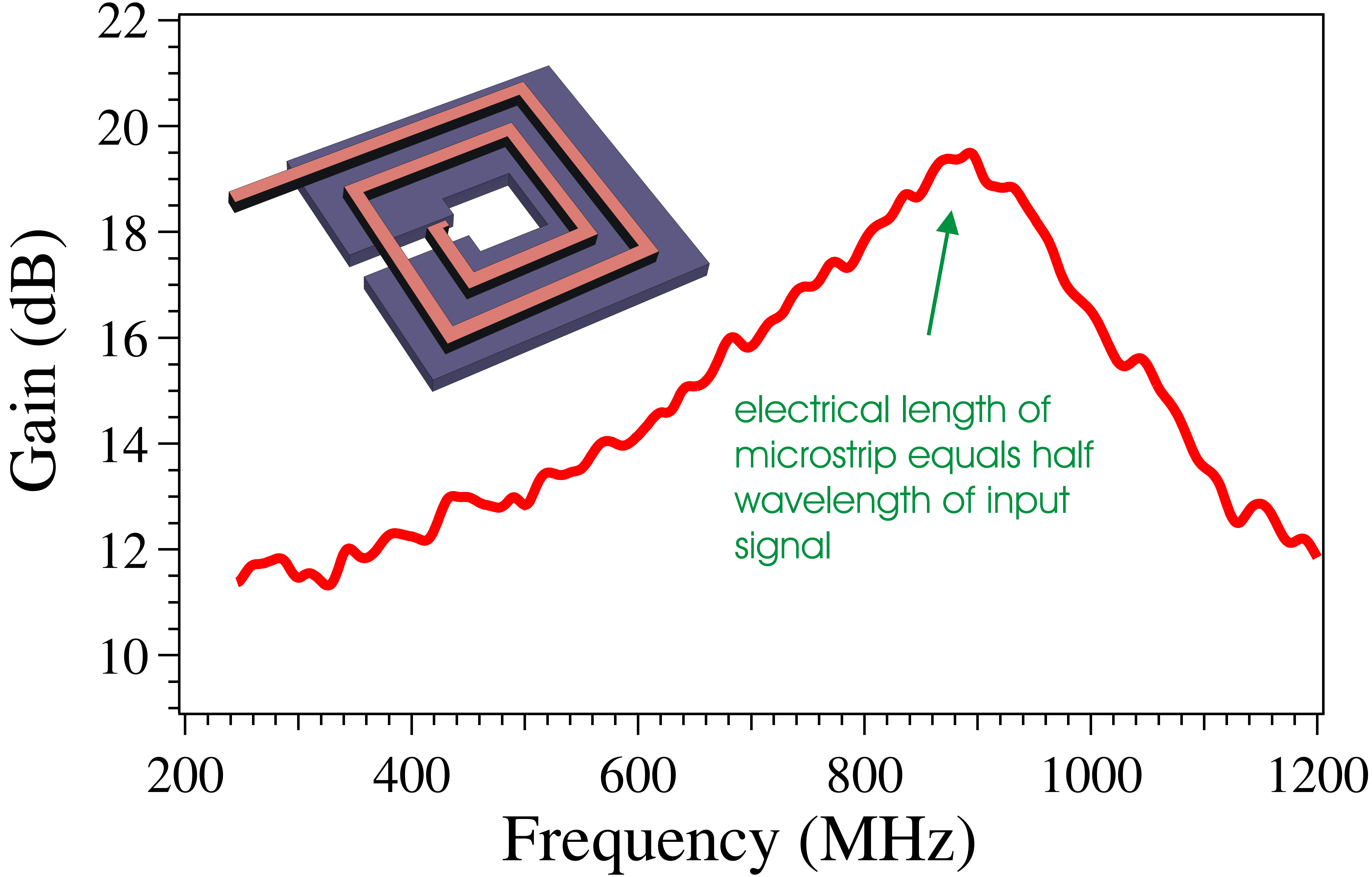


# *Gain Measurement Configuration*

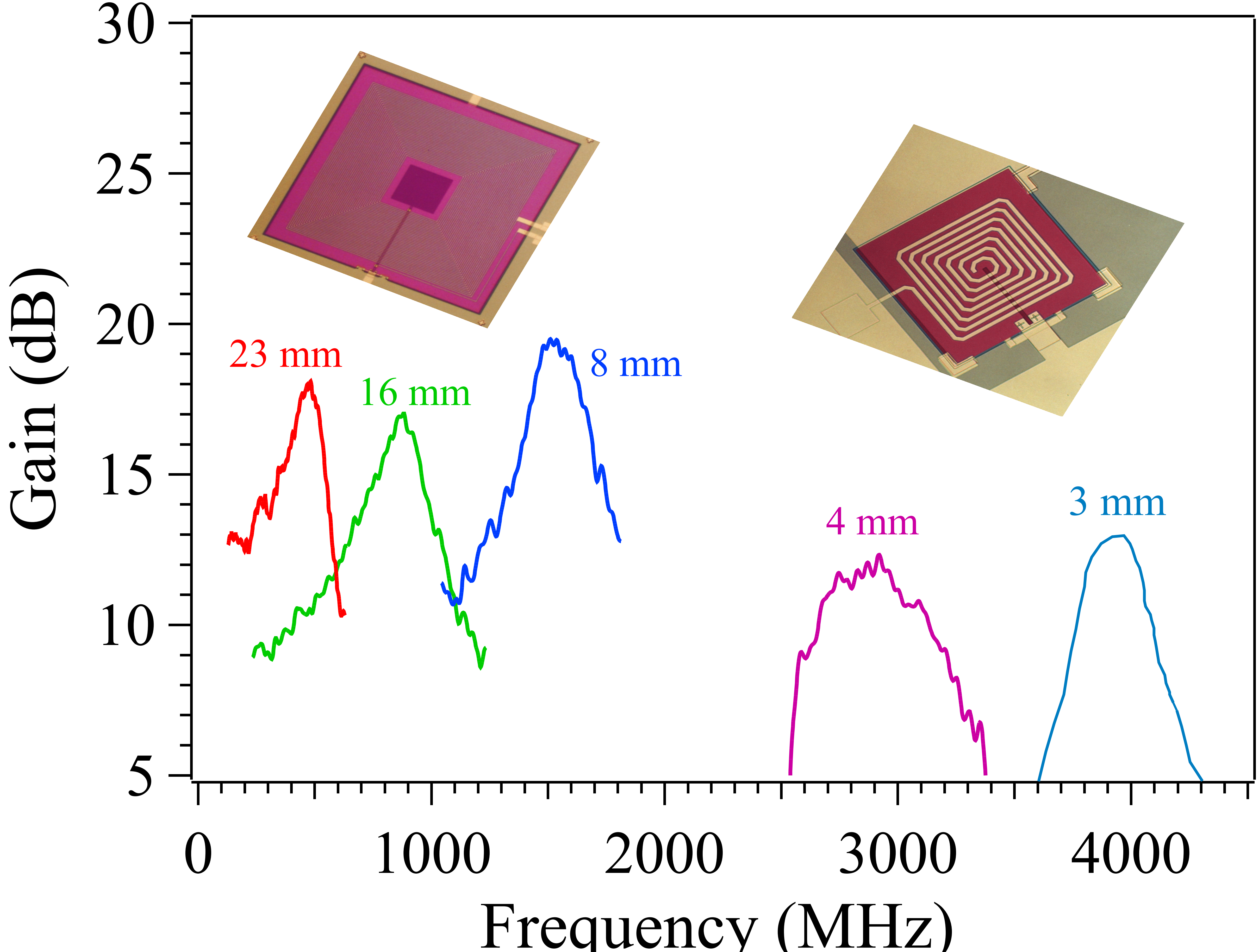


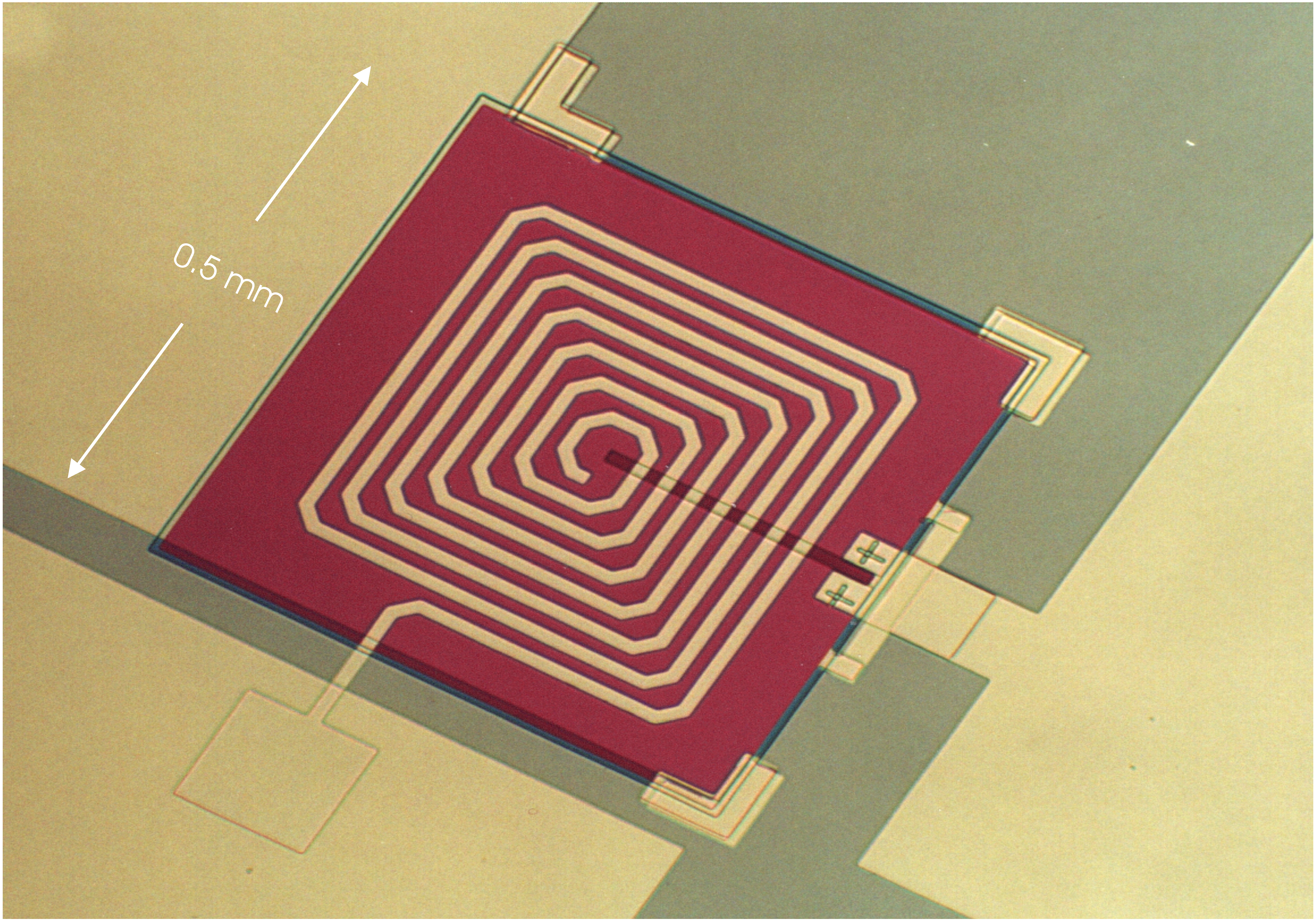

*Configuration used to measure gain. The two attenuators prevent noise from the room temperature equipment to saturate the SQUID.*

# Gain of a microstrip SQUID amplifier



# Gain of microstrip SQUID amplifier vs length of input resonator (coil)




0.5 mm

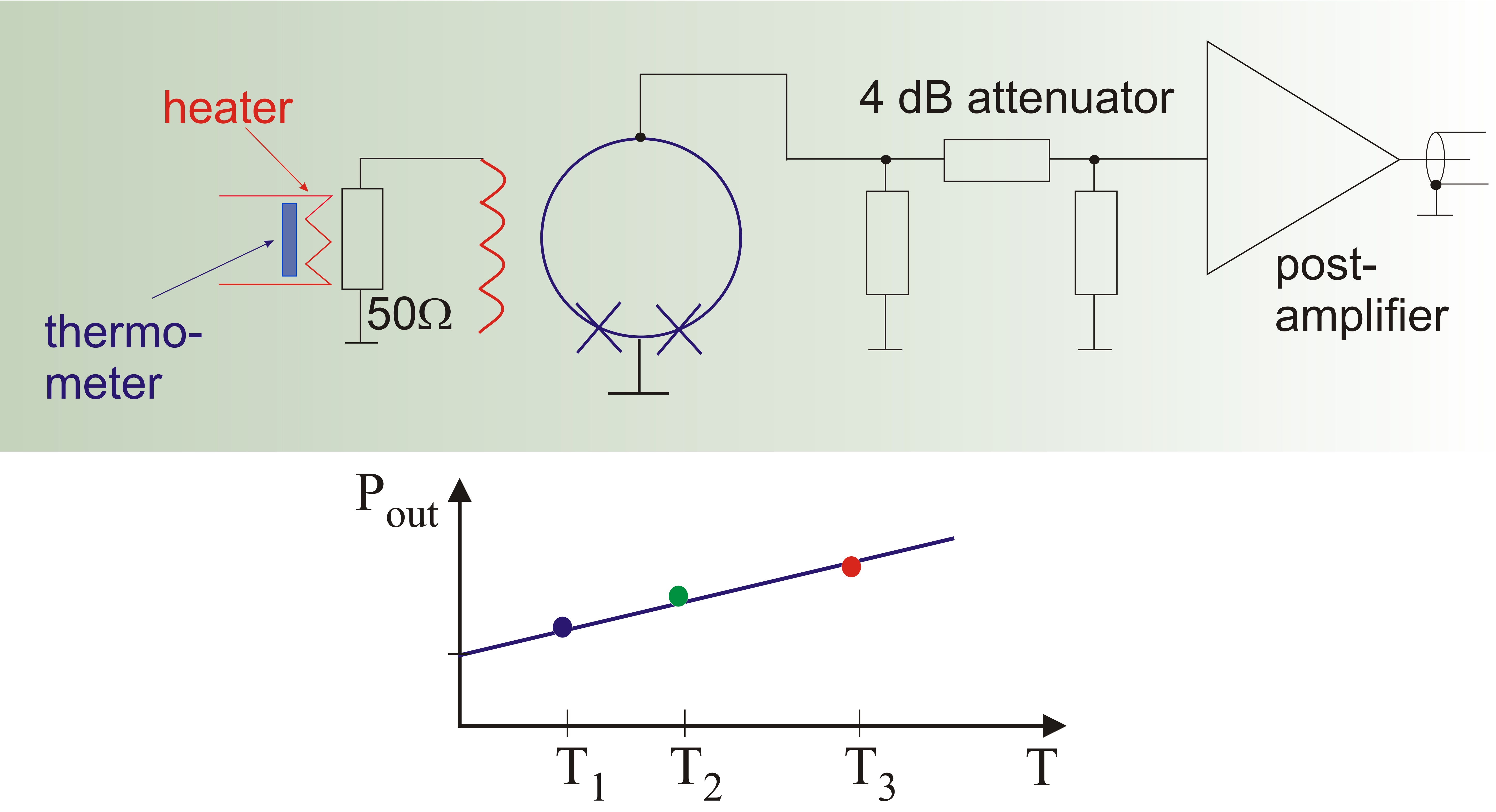
# *The Sensitivity and Noise Temperature of SQUID Amplifiers*

To characterize the sensitivity of an amplifier at high frequencies, its noise temperature is specified. One assumes the noise of the amplifier is produced by a virtual resistor at the input of the amplifier.

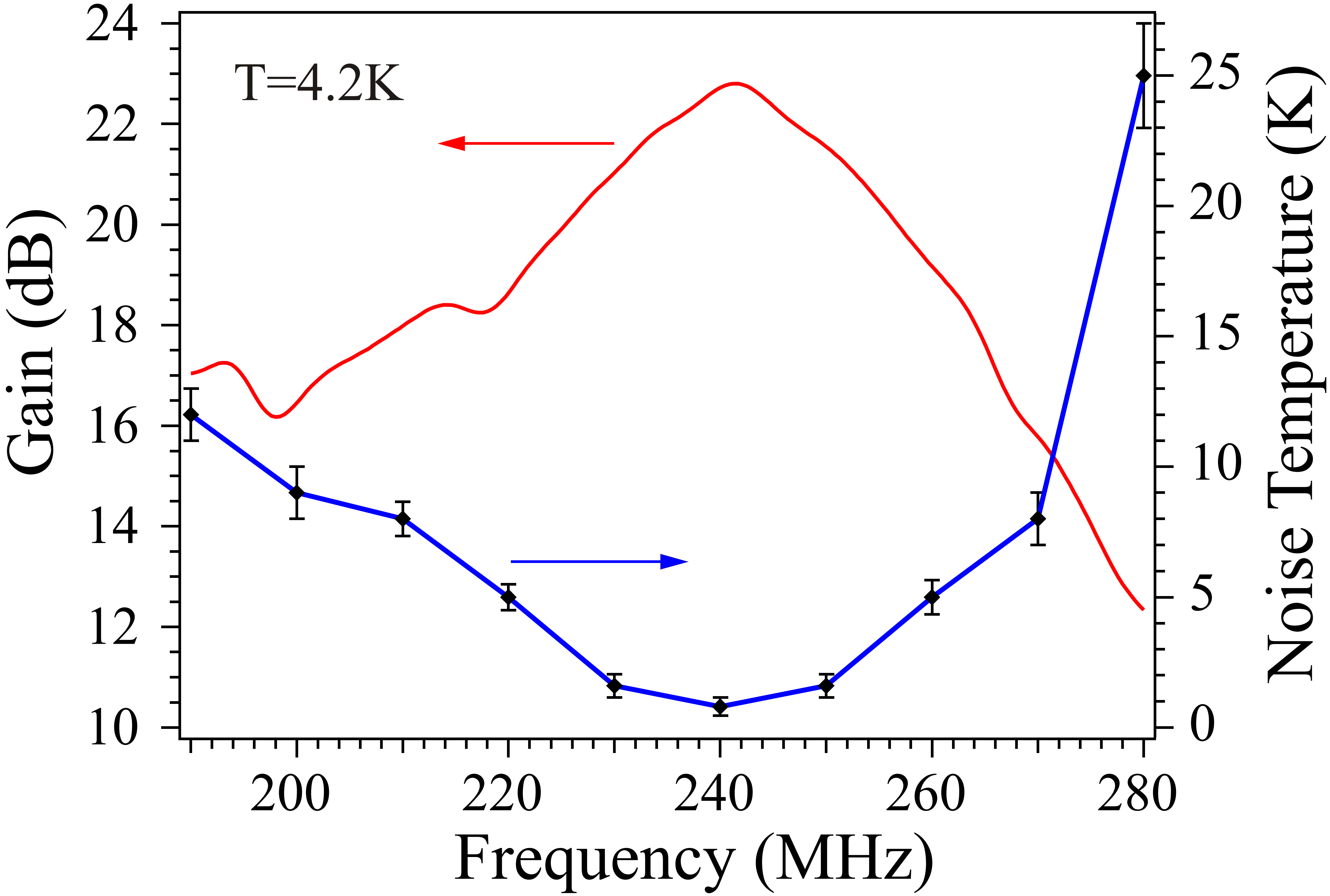
The temperature at which that resistor had to be in order to produce the same noise as the amplifier is the noise temperature of the amplifier.

An amplifier producing a voltage noise referred to its input of 0.1 $nV/Hz^{1/2}$ and having an input impedance of 50 Ω has a noise temperature of 4 K.

*Configuration used to measure the noise temperature $T_N$. A 50 Ω resistor acts as noise source; by comparing the noise powers at the output of the SQUID for different resistor temperatures, one can calculate $T_N$.*

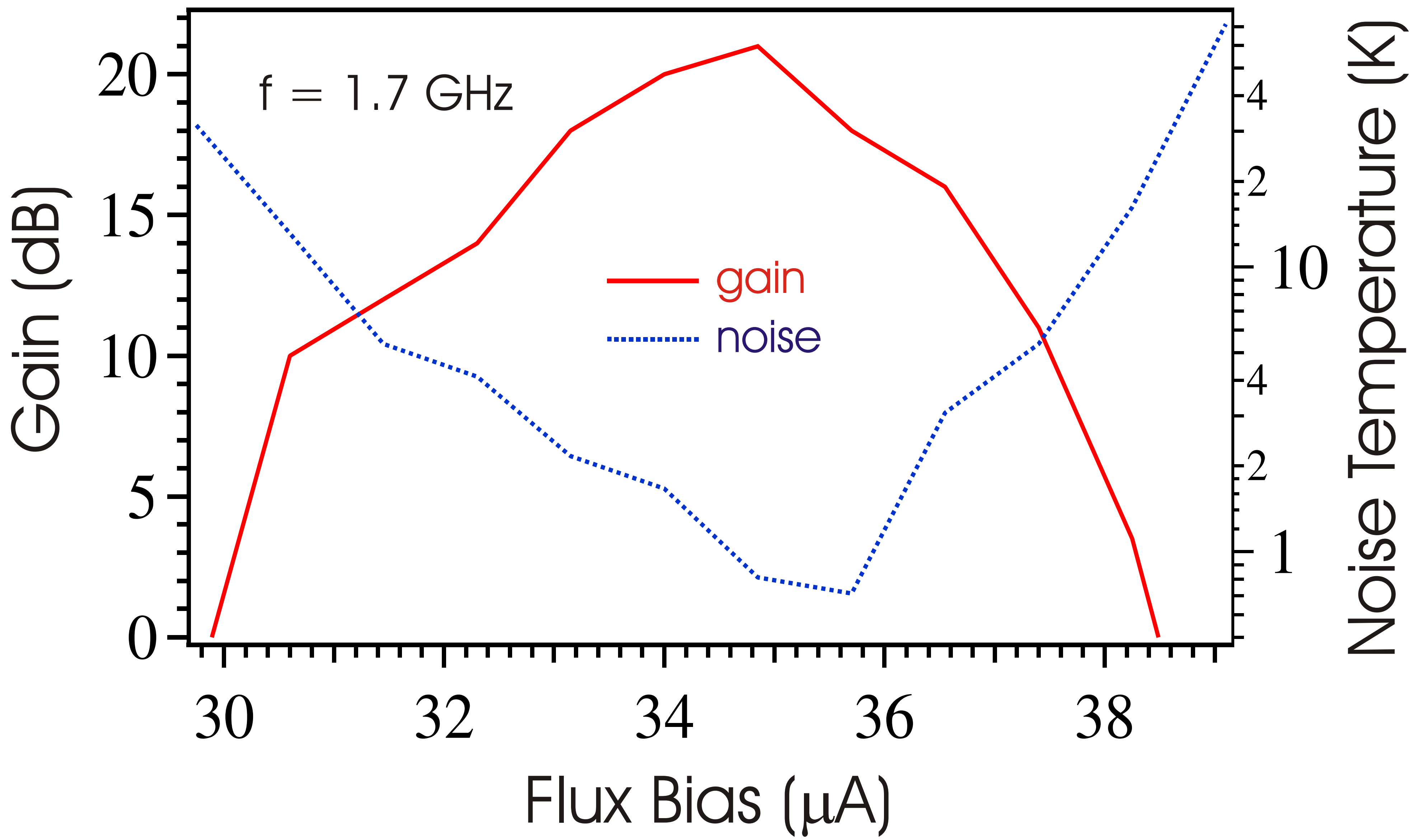


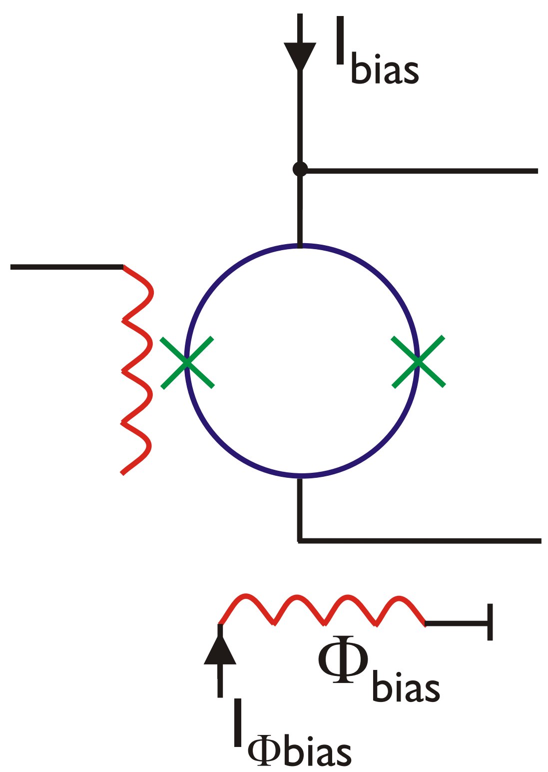
# *Noise Temperature Measurements:  4.2 K*

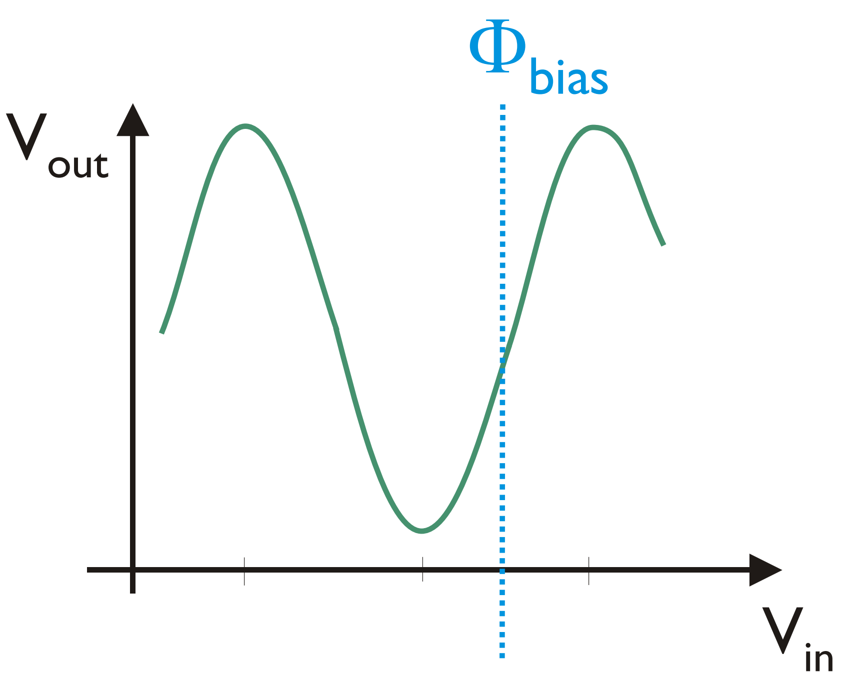



Gain and noise temperature vs frequency of
a microstrip SQUID with 71 mm long microstrip resonator.

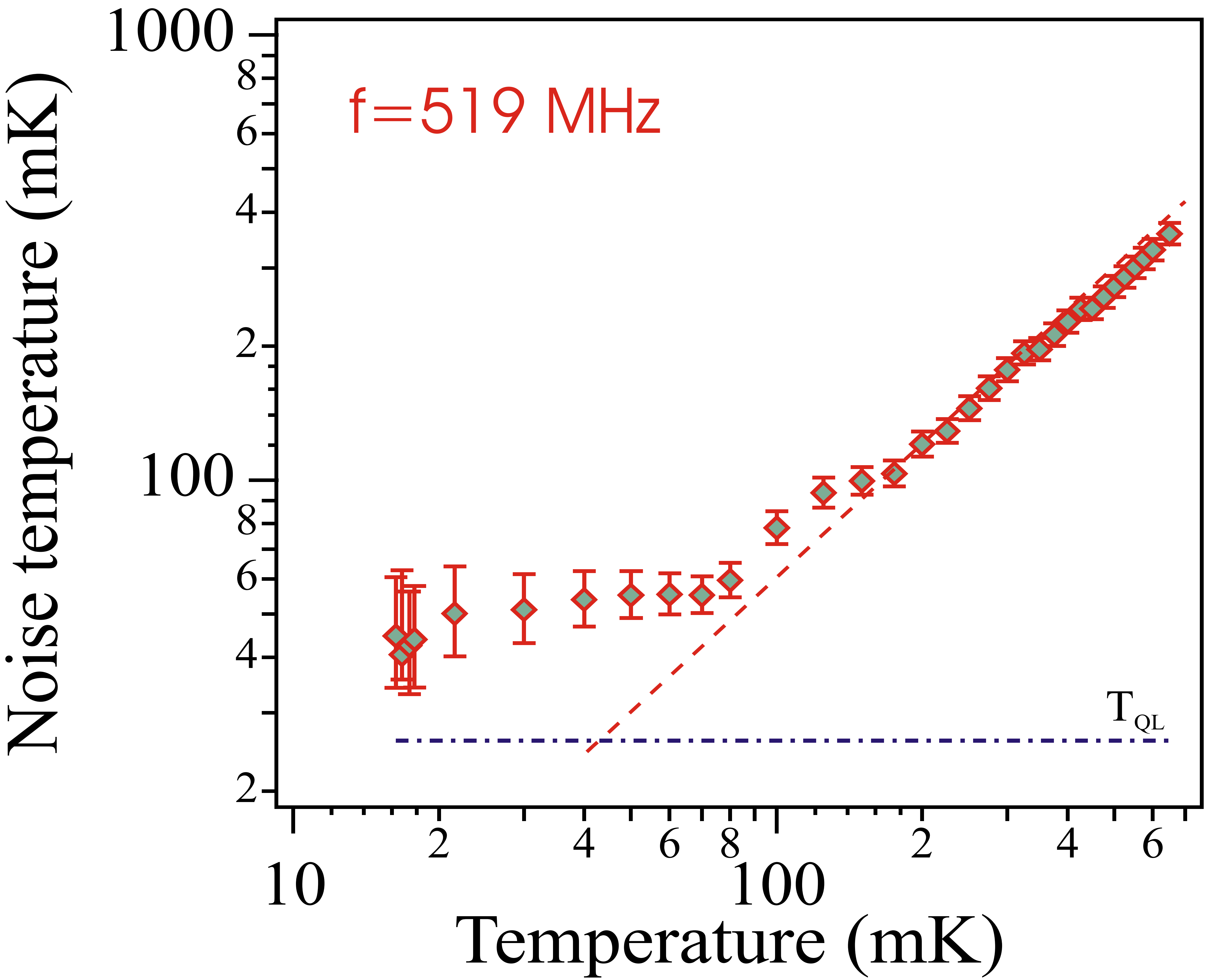
# *Noise Temperature Measurements: 4.2 K*



# *Noise Temperature Measurements: 20 mK to 1 K*



# *Noise Temperature Measurements*

Theoretical estimates for lowest noise temperature (classical):

$$T_n^{opt} \approx 7T\,\omega_0 / V_\Phi$$

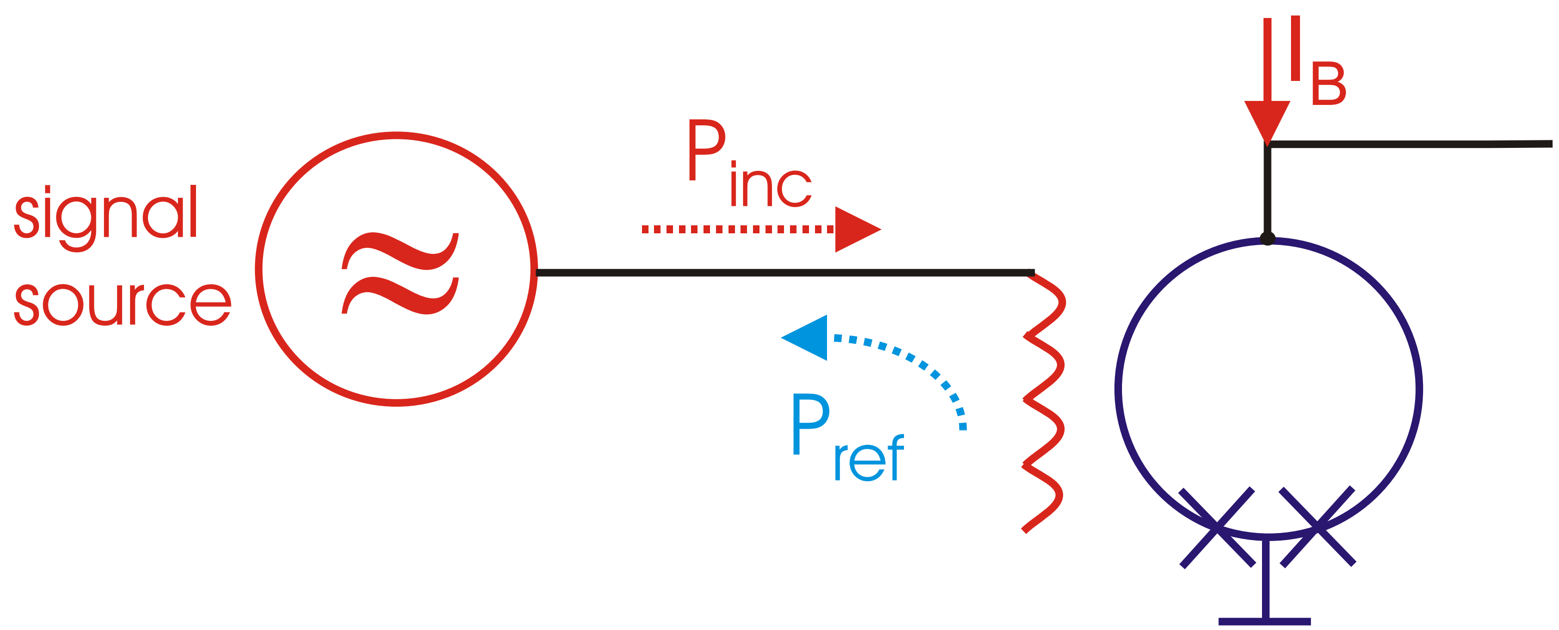
$$G_p^{opt} \times T_n^{opt} \approx 7T$$

$$\Rightarrow T_n^{opt} \approx 7T / G_p^{opt}$$

C.D. Tesche and J. Clarke, *J. Low Temp. Phys.* *27* 301 (1977).

# Input and Output Impedance: Why Care ?

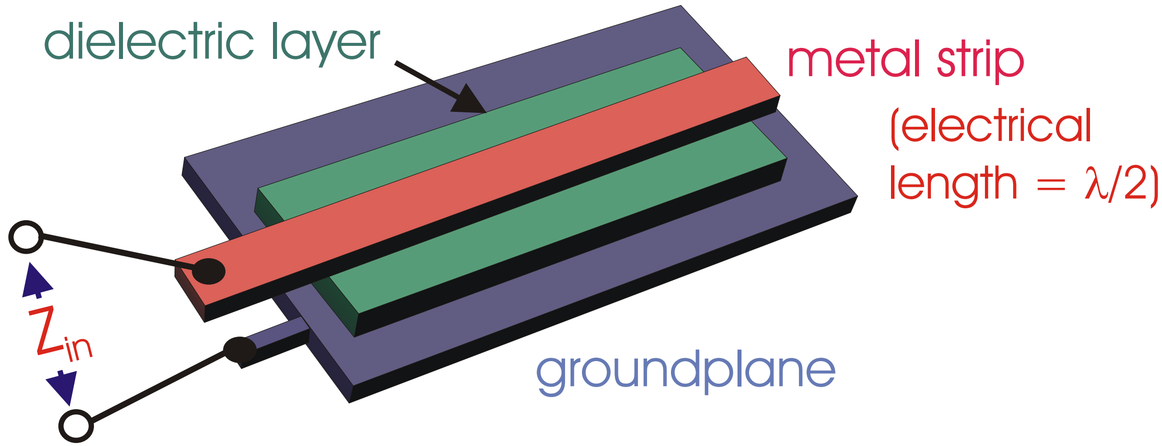
At 'high frequencies', if the input impedance of an amplifier is different from that of the signal source (or the coaxial line connecting the source to the amplifier), part of the source power is reflected from the input of the amplifier back to the source.



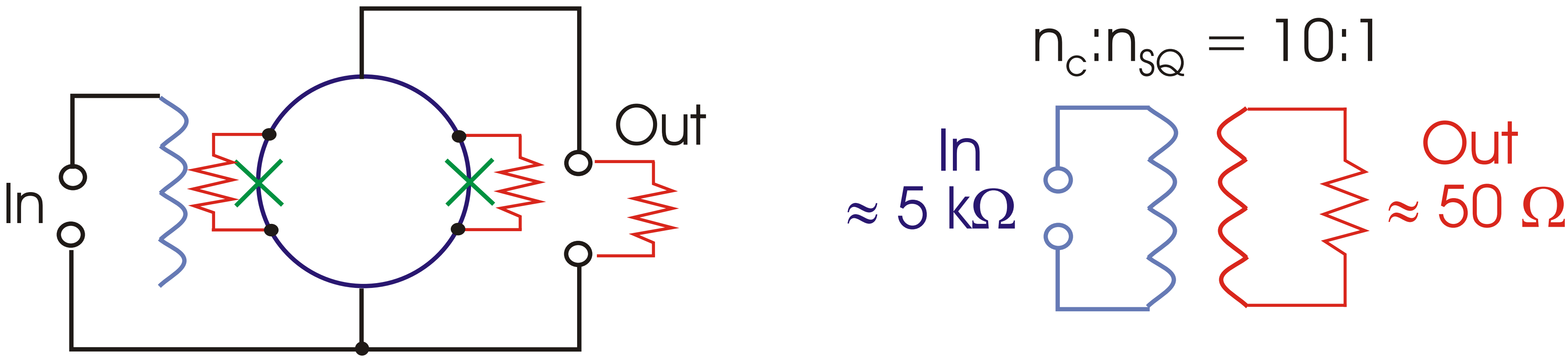

This results in a loss of available power (usually no problem), but can also result in an increased noise and in a deterioration of the properties of the source (if the source is sensitive to the load impedance, or to power reflected back from the load to the source, as is the case in, e.g., a qubit).

→ *Try to make input impedance of amplifier similar to the source impedance.*

# The input impedance of a microstrip SQUID amplifier

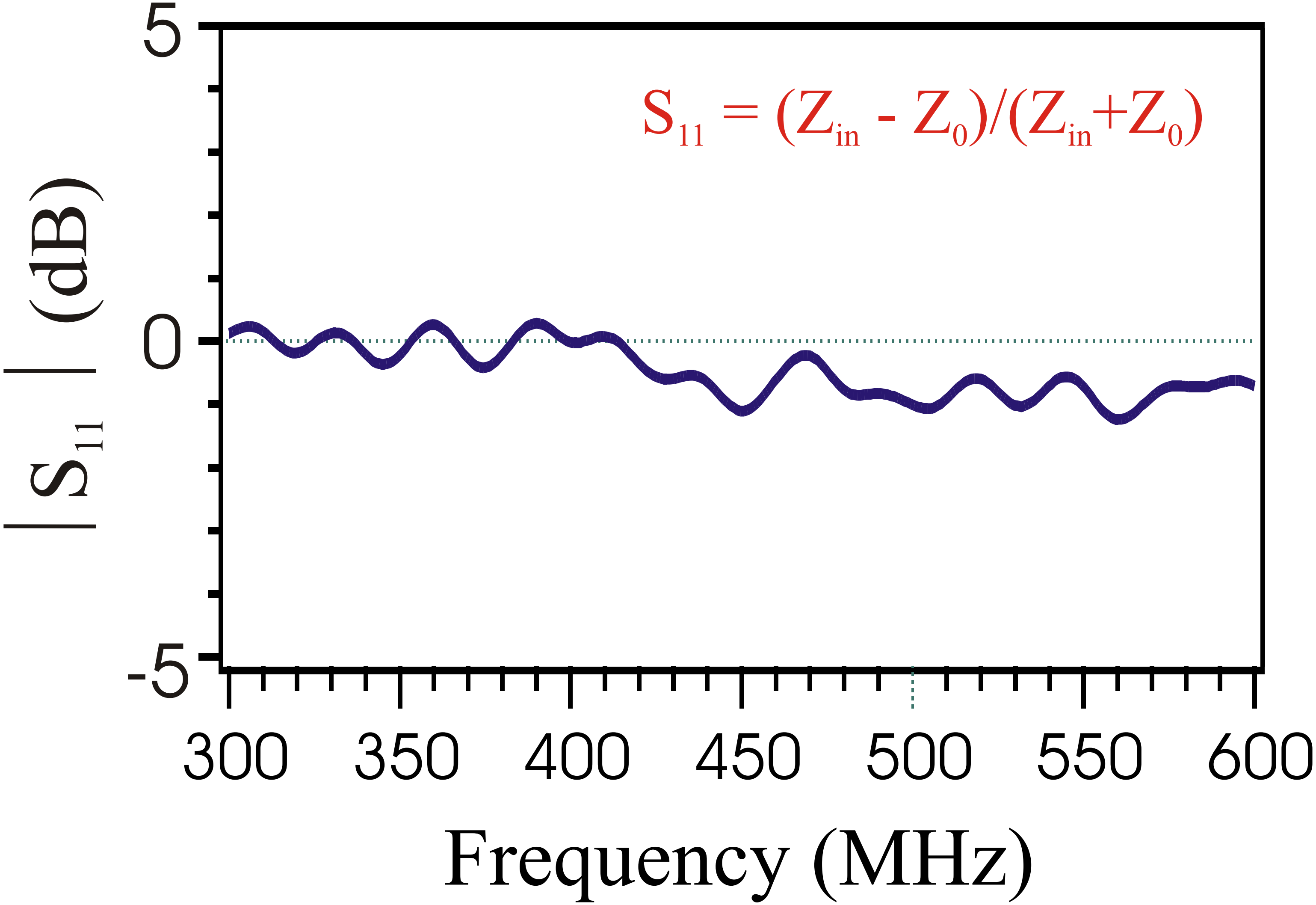



The input impedance $Z_{in}$ of a $\lambda/2$ microstrip resonator with open end is infinite.

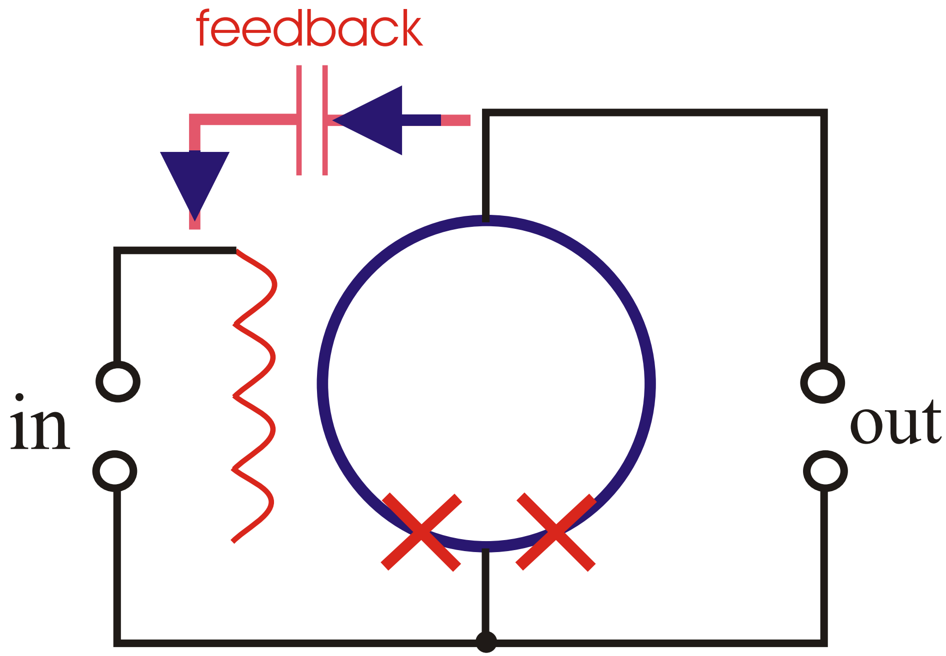


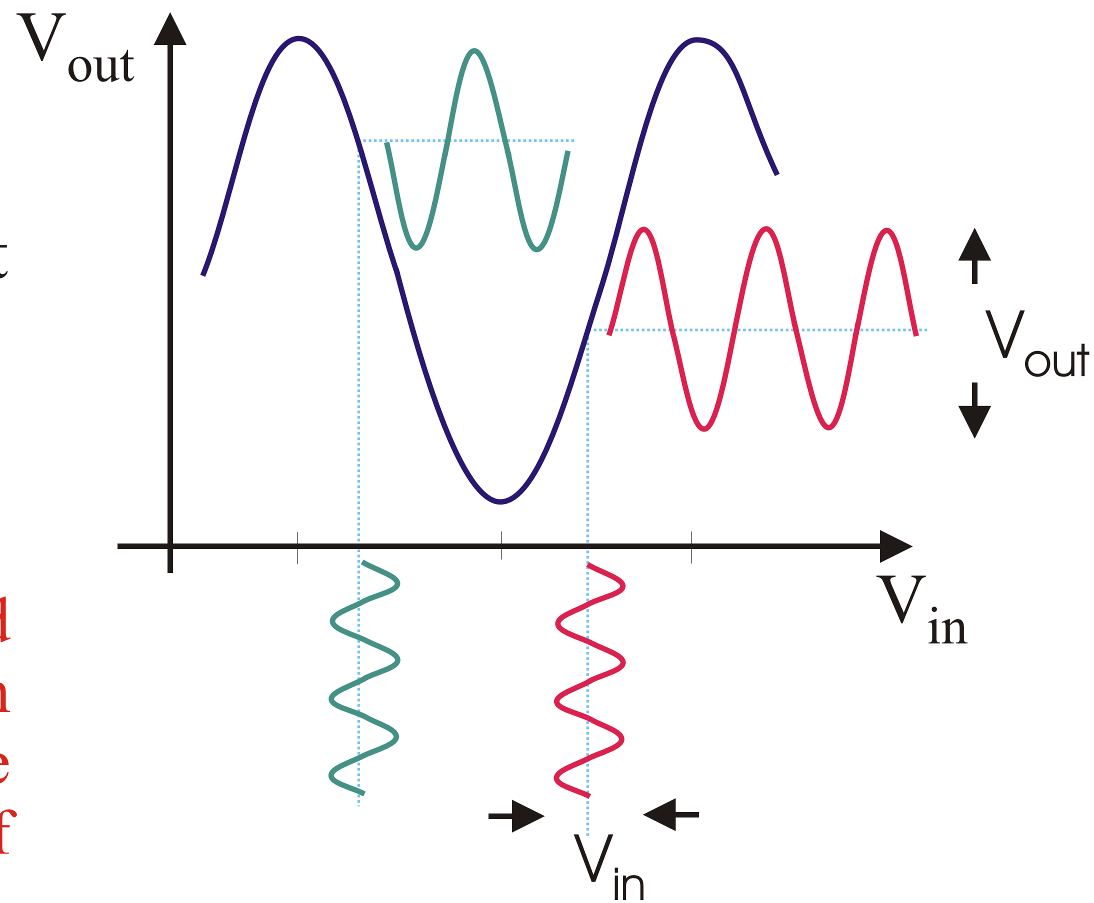

$$Z_{in} \approx 5\ \mathrm{k\Omega} \Rightarrow S_{11} \approx -0.2\ \mathrm{dB}$$

# Input return loss $|S_{11}|$ of a microstrip SQUID amplifier



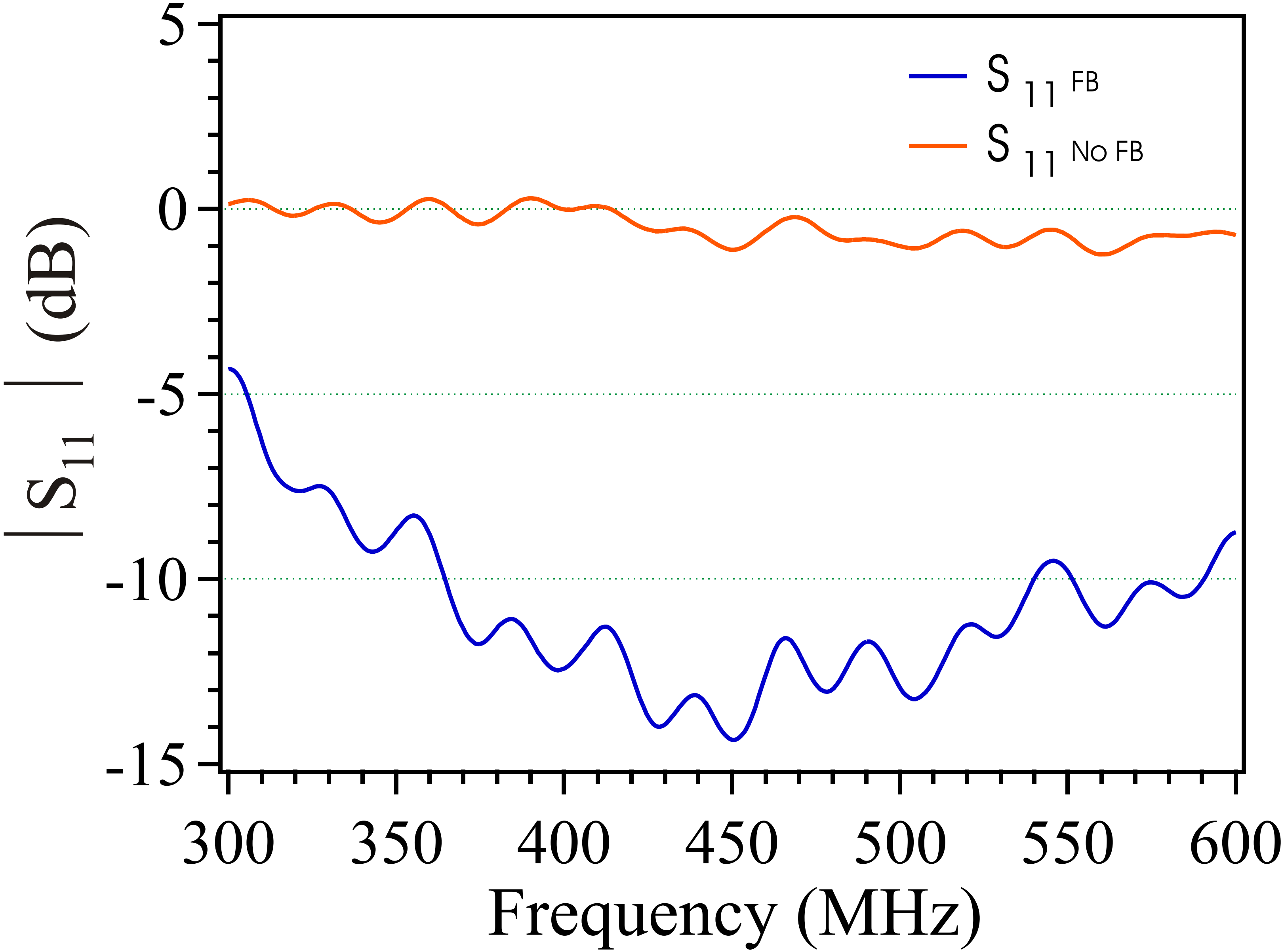
# Applying positive or negative feedback by feeding part of the output signal back to the input via a capacitor



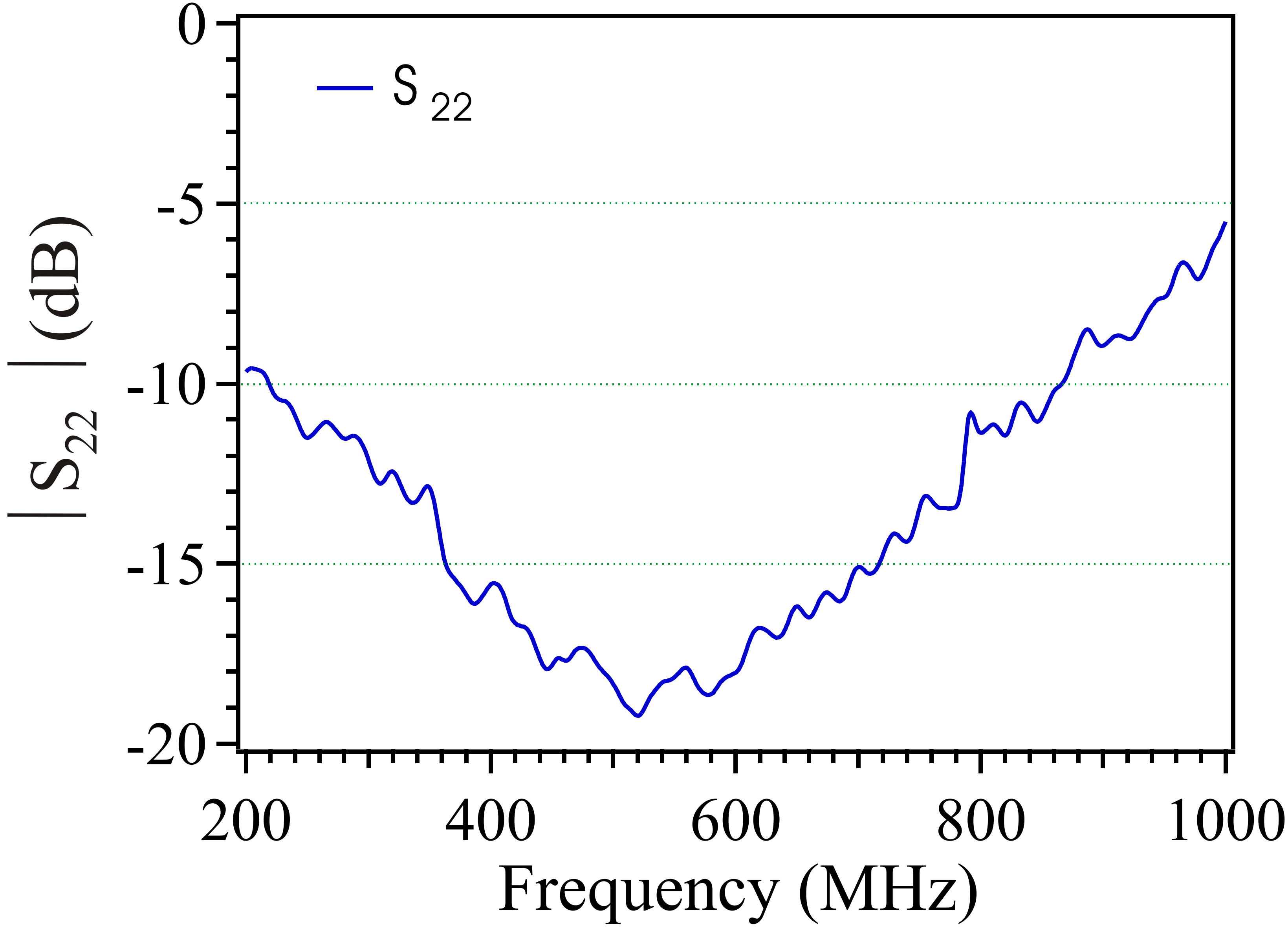




If part of the output signal is fed back to the input, one can obtain either negative or positive feedback, depending on the sign of $dV_{out}/dV_{in}$.

# $|S_{11}|$ of a microstrip SQUID amplifier for 500 MHz with external feedback



The output impedance of a microstrip SQUID amplifier for 500 MHz with external feedback, measured as $|S_{22}|$



# Nonlinearities in the SQUID amplifier: harmonics and gain compression

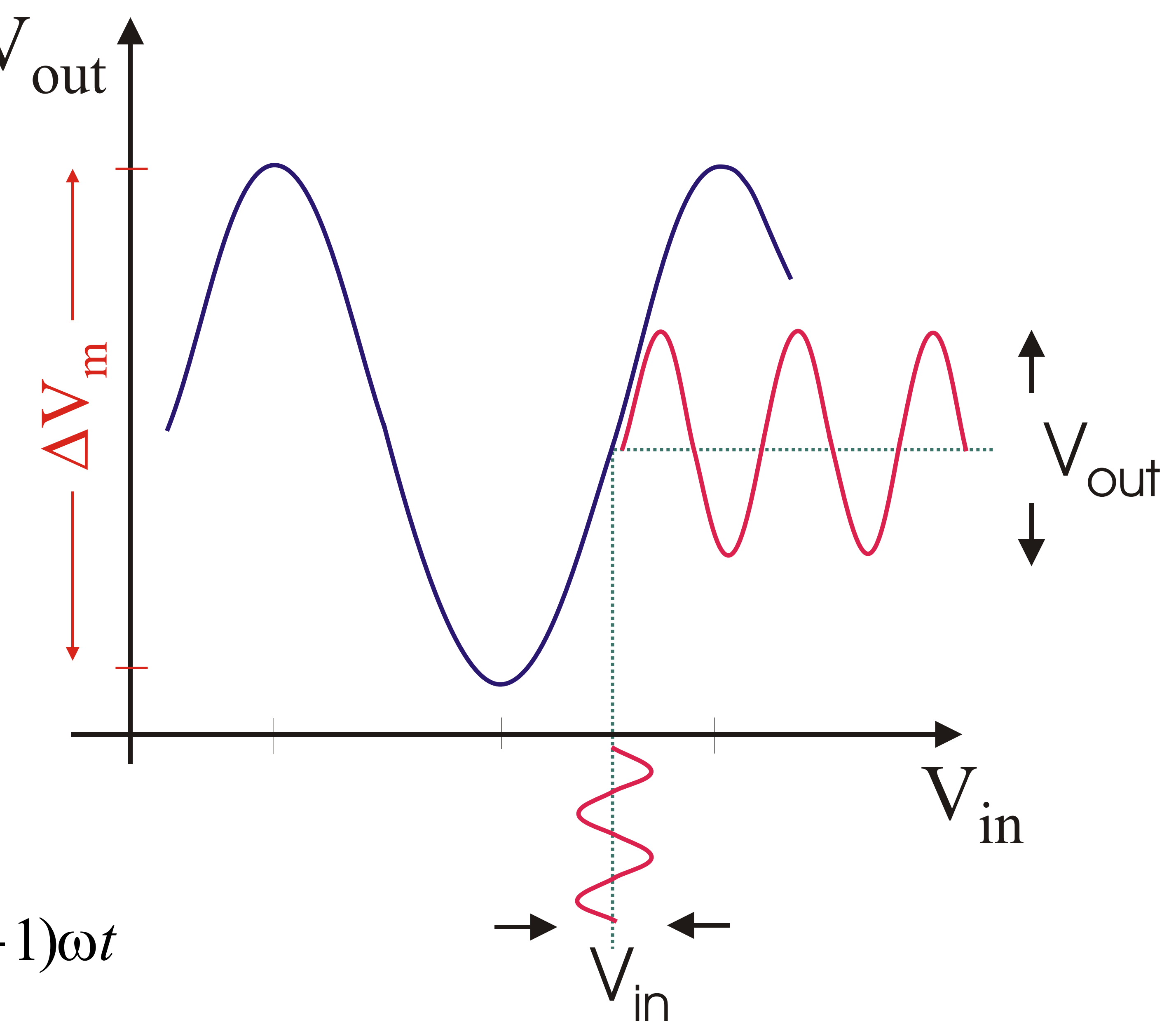
For large signals, the transfer function becomes a function of the input flux $\Phi_i$. The output voltage is

$$V_0(\Phi) = (\Delta V_m/2)\cos(2\pi\Phi_i/\Phi_0).$$

An input flux $\Phi_i = \Phi_m \cos\omega t$ produces a time-varying output voltage

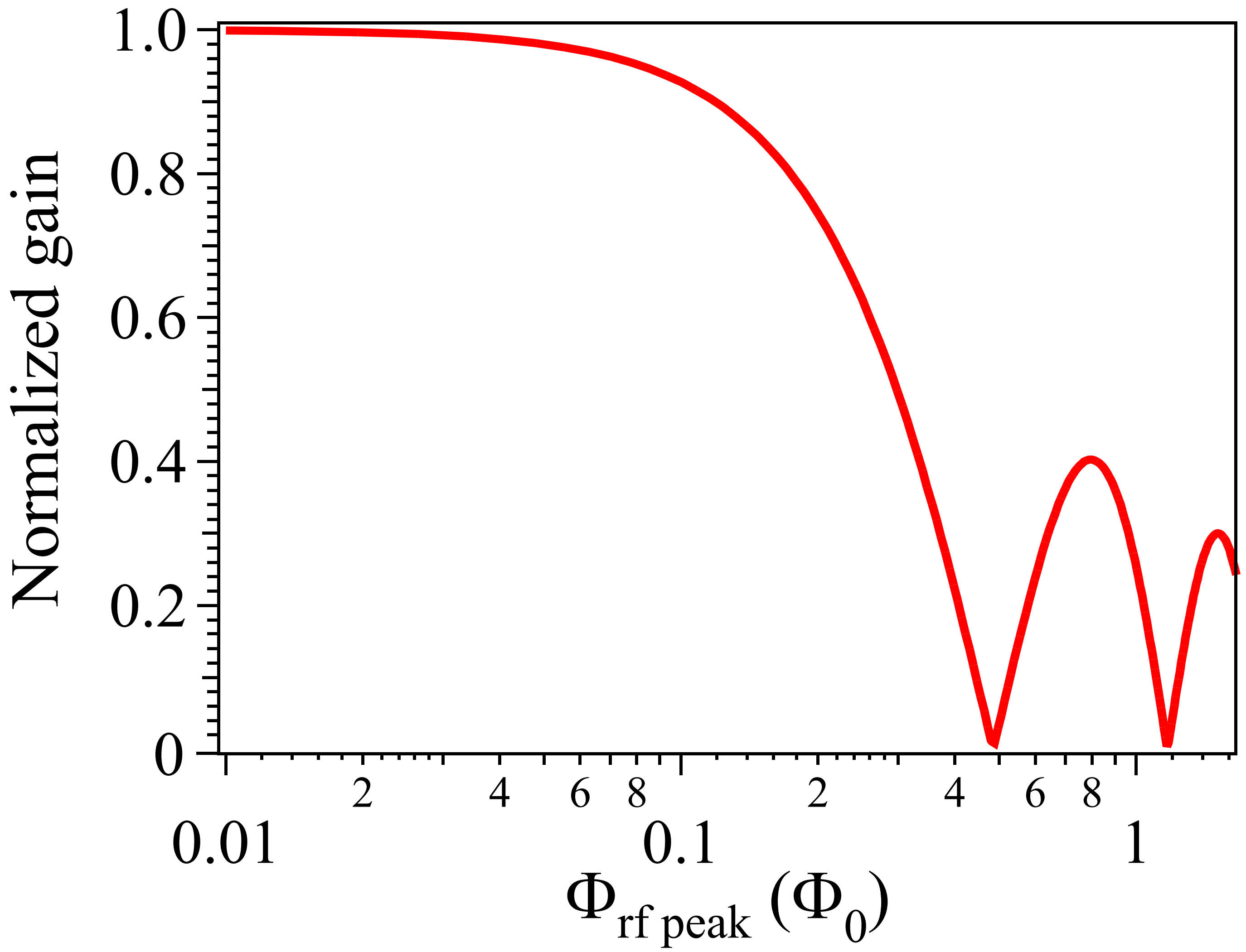
$$V_0(t) = (\Delta V_m/2)\sin[(2\pi\Phi_m/\Phi_0)\cos\omega t] =$$

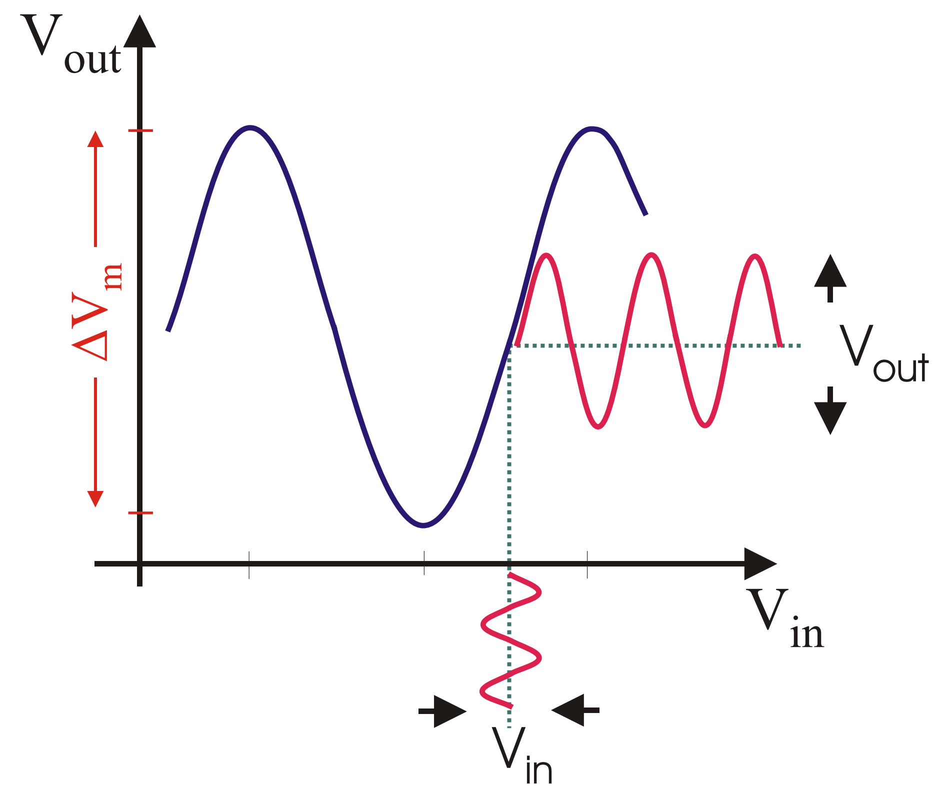
$$V_0(t) = \Delta V_m \sum_{n=0}^{\infty} (-1)^{n+1} J_{2n-1}(2\pi\,\Phi_m/\Phi_0)\cos(2n-1)\omega t$$

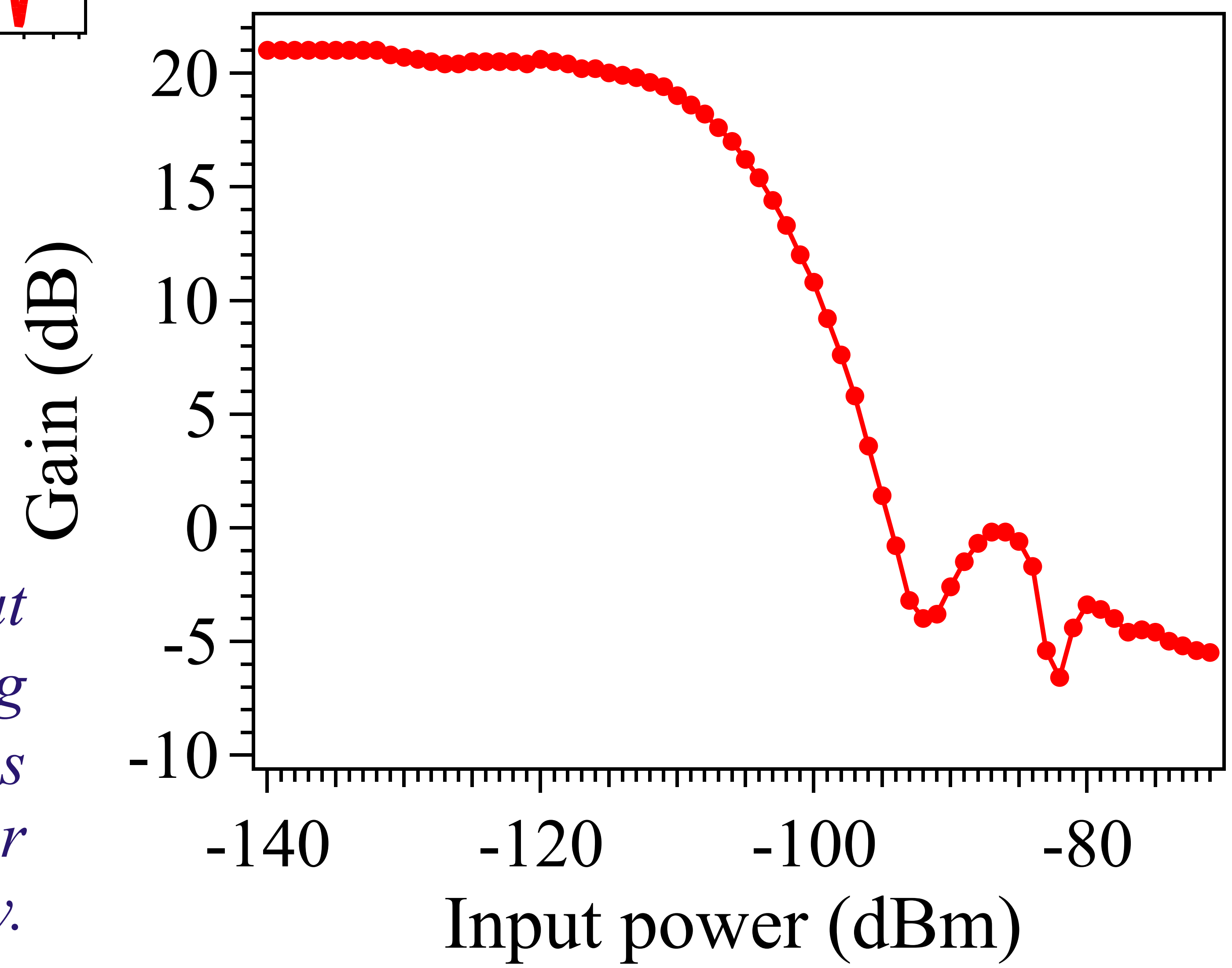



$V_0(t)$ contains odd harmonics $3\omega$, $5\omega$, $7\omega$ ... in addition to the fundamental frequency $\omega$. The gain is constant up to $\Phi_m < 0.1\Phi_0$ and decreases to zero for $\Phi_m \approx 0.6\,\Phi_0$.

# *Nonlinearities in the SQUID amplifier: gain compression*

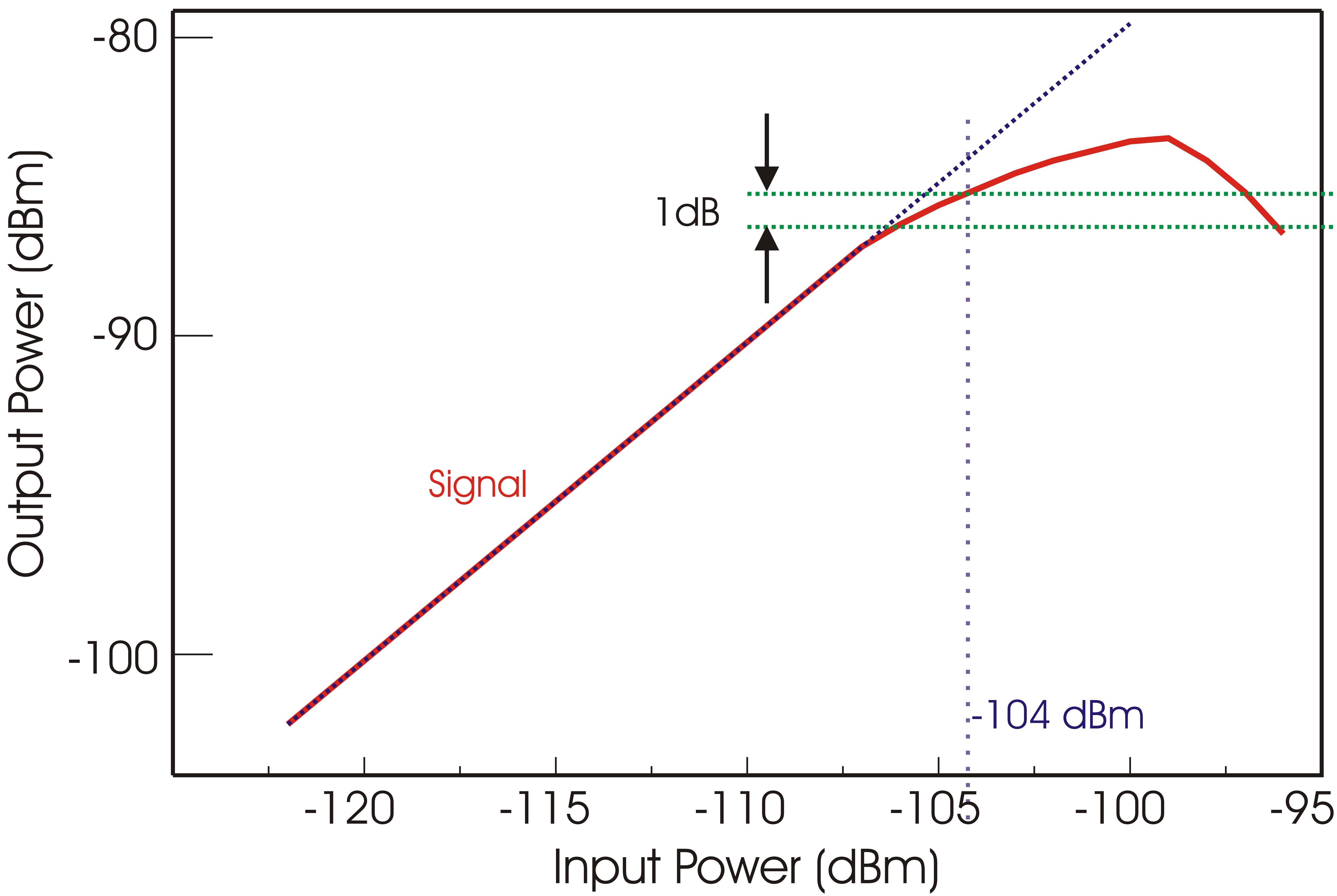


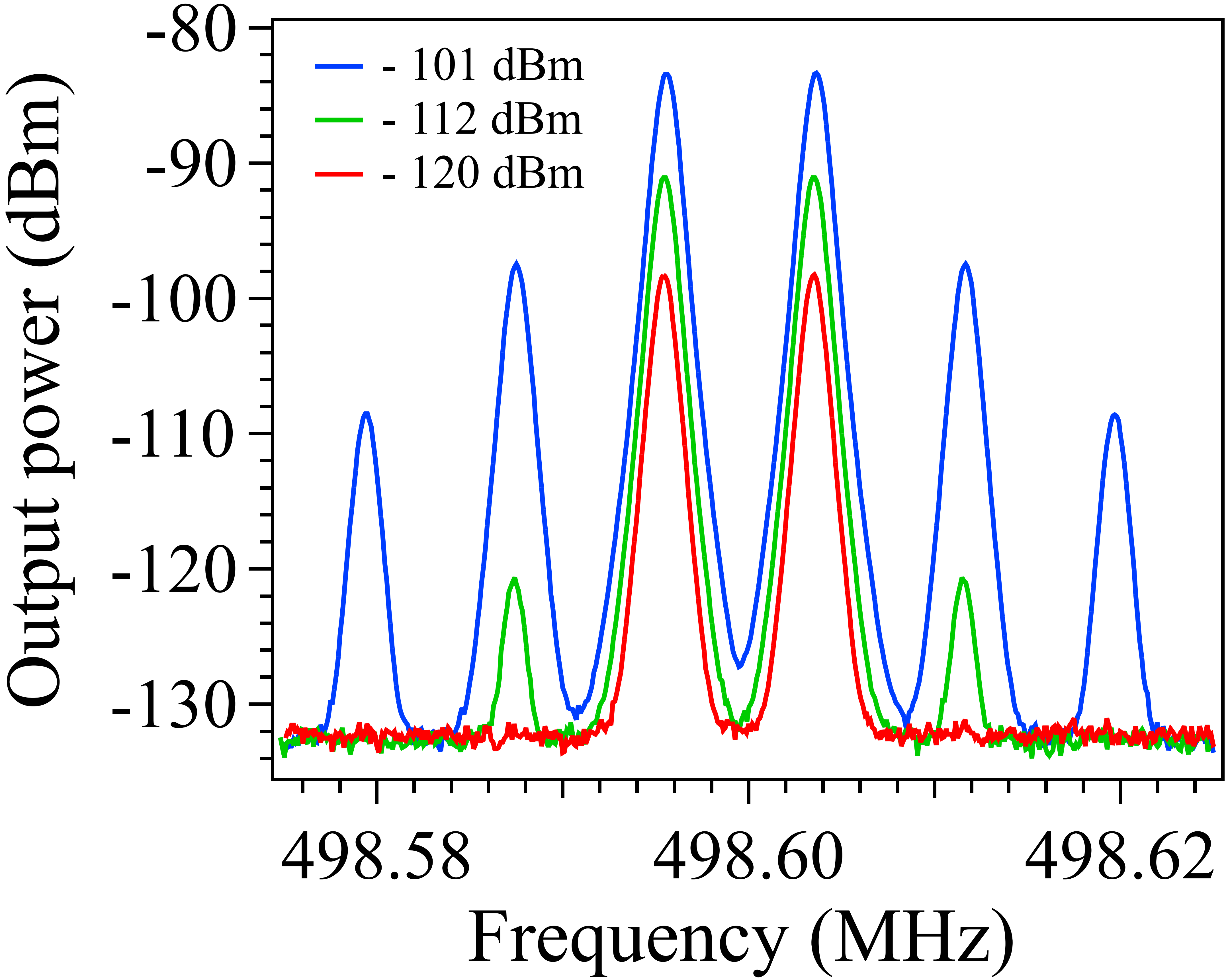




*Gain vs input power. As long as the input flux is below about 0.1 $\Phi_0$ (corresponding to an input power of -110 dBm in this case), the gain is constant. For higher input power, the gain drops considerably.*

# Determining the 1dB compression point of a SQUID amplifier



# Nonlinearities in the SQUID amplifier: Intermodulation




*Output spectrum for three different, equal-amplitude input signals at 498.5955 and 498.6035 MHz, showing the fundamental frequencies and IP3 and IP5 intermodulation products.*

# Conclusion

A dc SQUID can be used as an amplifier in the frequency range from dc at least up to several GHz.

Power gains of ~ 100 (20 dB) can be achieved with noise temperatures (in the optimum case at mK temperatures) of slightly above the quantum limit.

The gain of a SQUID amplifier is usually high enough to render the noise of a semiconductor post amplifier negligible.

A disadvantage is the small dynamic range of the SQUID. The input signal to a SQUID amplifier should not exceed ~ -100 dBm (0.1 pW).

The SQUID has to be sufficiently shielded from external magnetic fields to avoid changes in gain with that interference.