## On using SRF cavities to detect GWs

## Diego Blas

based on A. Berlin, DB, R. T. D'Agnolo, S. Ellis, B.Harnik, X Kahn, U. Schütte-Engel 2112.11465

## The Gravitational Soundscape

By Christopher Moore, Robert Cole and Christopher Berry, formerly of the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge


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## The Gravitational Soundscape



## The Gravitational Soundscape at high frequencies

Crucial question: what sources above kHz ?


Ringwald et al, 2011.0473


## What are we looking for?

Interaction GWs with light

$$
\begin{gathered}
\mathcal{L}=\sqrt{-g}\left(R+F_{\mu \nu} F^{\mu \nu}\right) \supset \frac{1}{2} A_{\mu} j_{\mathrm{eff}}^{\mu}(h)+\eta^{\mu \alpha} \eta^{\nu \beta} F_{\mu \nu} F_{\alpha \beta}+O\left(h^{2}\right) \\
j_{\mathrm{eff}}^{\mu}=-\partial_{\beta}\left(\frac{1}{2} h F^{\mu \beta}+h_{\alpha}^{\beta} F^{\alpha \mu}-h_{\alpha}^{\mu} F^{\alpha \beta}\right)
\end{gathered}
$$


$h_{O\left(F^{\mu \nu}\right)}^{\mu \nu} \approx \sim A^{\mu}$

## What are we looking for?

Interaction GWs with light

analogy with axions
$a$ か~~~mun $A^{\mu}$


## Where are we looking for it?

## Cavities

'Empty'
(large static mode, ADMX-like)


## Where are we looking for it?

## Cavities

'Empty'
(large static mode, ADMX-like)

‘Loaded'

|17

Better suited for 'broad band exploration'

How does this happen?

## Cavities

EM-coupling
Mechanical-coupling (shaking the walls)


ลn

## How does this happen?

## Cavities

EM-coupling
Mes -coupling


other designs for
$\xi$


## How does this happen?

## Cavities

EM-coupling
Mecr -coupling

axion-mechanical?

## How does this happen?

## Cavities

EM-coupling
Mecr -coupling


## $\xi$

 are aso ossille

## Some (VERY IMPORTANT) details



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## Some (VERY IMPORTANT) details

Local inertial i) choice of frame

Laboratory frame

$$
\ddot{z}^{\mu}+\Gamma_{\nu \lambda}^{\mu} \dot{z}^{\nu} \dot{z}^{\lambda}=a^{\mu}
$$

laboratory coordinates accelerated wrt LIF

$$
\begin{aligned}
g_{00} & =-\left(1+a_{i} x^{i}\right)^{2}+(\vec{\omega} \times \vec{x})^{2}-h_{00}^{L I F}-2(\vec{\omega} \times \vec{x})_{\underline{i}} h_{0 i}^{L I F} \\
& -(\vec{\omega} \times \vec{x})_{\underline{i}}(\vec{\omega} \times \vec{x})_{j} h_{i j}^{L I F} \\
g_{0 i} & =(\vec{\omega} \times \vec{x})_{i}-\gamma_{0 i}-(\vec{\omega} \times \vec{x})_{j} h_{i j}^{L I F} \\
g_{i j} & =\delta_{i j}-h_{i j}^{L I F}
\end{aligned}
$$

## Some (VERY IMPORTANT) details

$$
R \sim \omega^{2} h
$$

i) choice of frame $R_{\mu \nu \rho \sigma}(h)=R_{\mu \nu \rho \sigma}\left(h^{T T}\right)+O\left(h^{2}\right)$

LIF at order $\quad O\left((\omega L)^{3}\right)$

$$
h_{00} \simeq-R_{0 i 0 j} x^{i} x^{j} \quad, \quad h_{i j} \simeq-\frac{1}{3} R_{i k j l} x^{k} x^{l}, \quad h_{0 i} \simeq-\frac{2}{3} R_{0 j i k} x^{j} x^{k}
$$

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$$

$$
\begin{aligned}
\left(h_{\alpha \beta}^{\mathrm{TT}}\right)=\left(\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & h_{x x} & h_{x y} & 0 \\
0 & h x y & -h x x & 0 \\
0 & 0 & 0 & 0
\end{array}\right) h_{00} & \simeq \frac{1}{2} \partial_{t}^{2} h_{a b}^{\mathrm{TT}} x^{a} x^{b}+\mathcal{O}\left(x^{3}\right) \\
h_{i j} & \simeq-\frac{1}{6} \partial_{t}^{2}\left[\left(\delta_{i z} h_{j a}^{\mathrm{TT}}+\delta_{j z} h_{i a}^{\mathrm{TT}}\right) z x^{a}-h_{i j}^{\mathrm{TT}} z^{2}-\delta_{i z} \delta_{j z} h_{a b}^{\mathrm{TT}} x^{a} x^{b}\right]+\mathcal{O}\left(x^{3}\right) \\
h_{0 i} & \simeq \frac{1}{3} \partial_{t}^{2}\left(h_{i a}^{\mathrm{TT}} z x^{a}-\delta_{i z} h_{a b}^{\mathrm{TT}} x^{a} x^{b}\right)+\mathcal{O}\left(x^{3}\right) \quad \text { (proper detector frame) }
\end{aligned}
$$

## Some (VERY IMPORTANT) details

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R \sim \omega^{2} h
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i) choice of frame $\quad R_{\mu \nu \rho \sigma}(h)=R_{\mu \nu \rho \sigma}\left(h^{T T}\right)+O\left(h^{2}\right)$

$$
\begin{aligned}
& \text { LIF at order } O\left((\omega L)^{3}\right) \\
& h_{00} \simeq-R_{0 i 0 j} x^{i} x^{j} \quad, \quad h_{i j} \simeq-\frac{1}{3} R_{i k j l} x^{k} x^{l}, \quad h_{0 i} \simeq-\frac{2}{3} R_{0 j i k} x^{j} x^{k} \\
&
\end{aligned}
$$

## Some (VERY IMPORTANT) details

$$
R \sim \omega^{2} h
$$

i) choice of frame $R_{\mu \nu \rho \sigma}(h)=R_{\mu \nu \rho \sigma}\left(h^{T T}\right)+O\left(h^{2}\right)$

$$
\begin{aligned}
& h_{00} \simeq-R_{0 i 0 j} x^{i} x^{j} \text { LIF at order } Q\left((\omega L)^{3}\right) \\
& h_{i j} \simeq-\frac{1}{3} R_{i k j l} x^{k} x^{l}, \quad h_{0 i} \simeq-\frac{2}{3} R_{0 j i k} x^{j} x^{k} \\
&(R x) \sim O((\omega L))
\end{aligned}
$$

## Some (VERY IMPORTANT) details

i) choice of frame $R_{\mu \nu \rho \sigma}(h)=R_{\mu \nu \rho \sigma}\left(h^{T T}\right)+O\left(h^{2}\right)$

LIF at all order in $h^{\mu \nu}$

$$
\begin{aligned}
& h_{00}=-2 x^{k} x^{\ell} \sum_{n=0}^{\infty} \frac{n+3}{(n+3)!} x^{m_{1}} \cdots x^{m_{n}} \partial_{m_{1}} \cdots \partial_{m_{n}} R_{0 k 0 \ell} \\
& h_{0 j}=-2 x^{k} x^{\ell} \sum_{n=0}^{\infty} \frac{n+2}{(n+3)!} x^{m_{1}} \cdots x^{m_{n}} \partial_{m_{1}} \cdots \partial_{m_{n}} R_{0 k j \ell} \\
& h_{i j}=-2 x^{k} x^{\ell} \sum_{n=0}^{\infty} \frac{n+1}{(n+3)!} x^{m_{1}} \cdots x^{m_{n}} \partial_{m_{1}} \cdots \partial_{m_{n}} R_{i k j \ell}
\end{aligned}
$$

## Some (VERY IMPORTANT) details

ii) the signal in terms of power


$$
Q=\omega \frac{\text { Stored energy }}{\text { Power loss }}
$$

Power extracted in a resonant cavity

## Some (VERY IMPORTANT) details

ii) the signal in terms of power

waveguide

$$
Q=\omega \frac{\text { Stored energy }}{\text { Power loss }}
$$

Power extracted in a resonant cavity


## Some (VERY IMPORTANT) details

ii) the signal in terms of power

$$
P=\beta \frac{\omega}{Q}\langle U\rangle
$$

$\vec{B}_{0}$

$$
B^{2} \sim\left(B^{0}\right)^{2}+2 B^{0} B^{1}+\left(B^{1}\right)^{2}
$$

$O(h) e^{i \omega t}$ unless $B^{0}$ resonates (in time) $\left\langle B^{0} B^{1}\right\rangle=0$ and $\langle U\rangle \sim O\left(h^{2}\right)$

## Some (VERY IMPORTANT) details

ii) the signal in terms of power

$$
P=\beta \frac{\omega}{Q}\langle U\rangle
$$

(aN)

$$
U=\int \mathrm{d}^{3} x \frac{1}{2}\left(\operatorname{Re}[\boldsymbol{E}(\boldsymbol{x}, t)]^{2}+\operatorname{Re}[\boldsymbol{B}(\boldsymbol{x}, t)]^{2}\right)
$$

$$
B(x, t)=B^{(0)}+B^{(1)}(x, t)
$$

$$
B(x, t)=E^{(1)}(x, t)
$$

$$
B^{2} \sim\left(B^{0}\right)^{2}+2 B^{0} B^{1}+\left(B^{1}\right)^{2}
$$

$$
\mathcal{O}(h)
$$

$$
\begin{aligned}
& O(h) e^{i \omega t} \text { unless } B^{0} \text { resonates (in time) }\left\langle B^{0} B^{1}\right\rangle=0 \\
& \text { and }\langle U\rangle \sim O\left(h^{2}\right)
\end{aligned}
$$

## Some (VERY IMPORTANT) details

ii) the signal in terms of power

$$
P=\beta \frac{\omega}{Q}\langle U\rangle
$$



## Some (VERY IMPORTANT) details

ii) the sianal in terms of power


## Back to our calculation: mode excitation

$$
\boldsymbol{E}(\boldsymbol{x}, t)=\sum \boldsymbol{E}_{s n}(\boldsymbol{x}, t)+\boldsymbol{E}_{i n}(\boldsymbol{x}, t)
$$


solenoidal irrotational

$$
\begin{aligned}
\boldsymbol{E}_{s n}(\boldsymbol{x}, t) & =e_{s n}(t) \boldsymbol{E}_{s n}(\boldsymbol{x}) \\
\boldsymbol{E}_{i n}(\boldsymbol{x}, t) & =e_{i n}(t) \boldsymbol{E}_{i n}(\boldsymbol{x})
\end{aligned}
$$

$$
\left(\omega_{s m}^{2}+\partial_{t}^{2}+\sigma_{s m} \partial_{t}\right) e_{s m}(t)=e^{-i \omega_{G} t} \eta_{s m}
$$

$$
\left(\partial_{t}^{2}+\sigma_{i m} \partial_{t}\right) e_{i m}(t)=e^{-i \omega_{G} t} \eta_{i m}
$$

$$
\eta \sim \int_{V} \mathrm{~d}^{3} x E J_{e f f}
$$

'source' (here we want to maximise)

## Back to our calculation: mode excitation

$$
\boldsymbol{E}_{s n}(\boldsymbol{x}, t)=e_{s n}(t) \boldsymbol{E}_{s n}(\boldsymbol{x})
$$

(only solenoidal modes are excited)

$$
\begin{gathered}
U=\int \mathrm{d}^{3} x \frac{1}{2}\left(\operatorname{Re}[\boldsymbol{E}(\boldsymbol{x}, t)]^{2}+\operatorname{Re}[\boldsymbol{B}(\boldsymbol{x}, t)]^{2}\right) \\
P=\beta \frac{\omega}{Q}\langle U\rangle
\end{gathered}
$$

when considering only thermal noise one gets

$$
h_{0} \gtrsim 3 \times 10^{-22} \times\left(\frac{1 \mathrm{GHz}}{\omega_{g} / 2 \pi}\right)^{3 / 2}\left(\frac{0.1}{\eta_{n}}\right)\left(\frac{8 \mathrm{~T}}{B_{0}}\right)\left(\frac{0.1 \mathrm{~m}^{3}}{V_{\text {cav }}}\right)^{5 / 6}\left(\frac{10^{5}}{Q}\right)^{1 / 2}\left(\frac{T_{\mathrm{sys}}}{1 \mathrm{~K}}\right)^{1 / 2}\left(\frac{\Delta \nu}{10 \mathrm{kHz}}\right)^{1 / 4}\left(\frac{1 \mathrm{~min}}{t_{\text {int }}}\right)^{1 / 4},
$$

which modes get excited?

## Back to our calculation: mode excitation

$$
\boldsymbol{E}_{s n}(\boldsymbol{x}, t)=e_{s n}(t) \boldsymbol{E}_{s n}(\boldsymbol{x})>\eta_{s m}=\frac{\int_{V} d V \boldsymbol{E}_{s m}^{*}(\boldsymbol{x})\left(i \omega_{G} \boldsymbol{J}_{\mathrm{eff}}(\boldsymbol{x})\right)}{\int_{V} d V\left|\boldsymbol{E}_{s m}(\boldsymbol{x})\right|^{2}}
$$




TM (121)


TE (212)
there is ALWAYS a response (even for longitudinal waves!)!

## Recall the(VERY IMPORTANT) details



$$
h_{i j}^{\mathrm{TT}}=\left(\begin{array}{ccc}
h_{+} & h_{\times} & 0 \\
h_{\times} & -h_{+} & 0 \\
0 & 0 & 0
\end{array}\right)_{i j} e^{i \omega(t-z)}
$$


in the LAB
$\quad$ e.g.at
$\left.\boldsymbol{O}\left((\omega L)^{3}\right)\right)$$\left\{\begin{array}{l}h_{00} \simeq \frac{1}{2} \partial_{t}^{2} h_{a b}^{\mathrm{TT}} x^{a} x^{b}+\mathcal{O}\left(x^{3}\right) \\ h_{i j} \simeq-\frac{1}{6} \partial_{t}^{2}\left[\left(\delta_{i z} h_{j a}^{\mathrm{TT}}+\delta_{j z} h_{i a}^{\mathrm{TT}}\right) z x^{a}-h_{i j}^{\mathrm{TT}} z^{2}-\delta_{i z} \delta_{j z} h_{a b}^{\mathrm{TT}} x^{a} x^{b}\right]+\mathcal{O}\left(x^{3}\right) \\ h_{0 i} \simeq \frac{1}{3} \partial_{t}^{2}\left(h_{i a}^{\mathrm{TT}} z x^{a}-\delta_{i z} h_{a b}^{\mathrm{TT}} x^{a} x^{b}\right)+\mathcal{O}\left(x^{3}\right) \quad \text { (proper detector frame) }\end{array}\right.$

Mode excitation at $\alpha=0$
Berlin et al $21|2 .| | 465$


Projected Sensitivities of Axion Experiments


## The Gravitational Soundscape at high frequencies

## Crucial question: detectability above kHz ?

$\left.\begin{array}{|c||c|c|c|}\hline \text { Technical concept } & \text { Frequency } & \begin{array}{c}\text { Proposed sensitivity } \\ \text { (dimensionless) }\end{array} & \begin{array}{c}\text { Proposed sensitivity } \\ \sqrt{S_{n}(f)}\end{array} \\ \hline \hline \text { Spherical resonant mass, Sec. 4.1.3 [282] } & & & \\ \hline \text { Mini-GRAIL (built) [289] } & 2942.9 \mathrm{~Hz} & \begin{array}{c}10^{-20} \\ 2.3 \cdot 10^{-23}(*)\end{array} & \begin{array}{c}5 \cdot 10^{-20} \mathrm{~Hz}^{-\frac{1}{2}} \\ 10^{-22} \mathrm{~Hz}^{-\frac{1}{2}}(*)\end{array} \\ \hline \text { Schenberg antenna (built) [286] } & 3.2 \mathrm{kHz} & \begin{array}{c}2.6 \cdot 10^{-20} \\ 2.4 \cdot 10^{-23}(*)\end{array} & \begin{array}{c}1.1 \cdot 10^{-19} \mathrm{~Hz}^{-\frac{1}{2}} \\ 10^{-22} \mathrm{~Hz}^{-\frac{1}{2}}(*)\end{array} \\ \hline \text { Laser interferometers } & & & \\ \hline \text { NEMO (devised), Sec. 4.1.1 [25, 272] } & {[1-2.5] \mathrm{kHz}} & 9.4 \cdot 10^{-26} & 100 \mathrm{MHz} \\ \hline \text { Akutsu's proposal (built), Sec. 4.1.2 [277,328] } & {[1-13] \mathrm{MHz}} & 8 \cdot 10^{-14}(*) & \begin{array}{c}10^{-24} \mathrm{~Hz}^{-\frac{1}{2}} \\ 10^{-20} \mathrm{~Hz}^{-\frac{1}{2}}(*)\end{array} \\ \hline \text { Holometer (built), Sec. 4.1.2 [279] } & & 10^{-22} & 10^{-21} \mathrm{~Hz}^{-\frac{1}{2}}\end{array}\right]$

## FoM:

$h_{c} \sim 10^{-23}(\mathrm{GHz} / f)$
$h_{s} \sim 10^{-30}(\mathrm{GHz} / f)$

## The Gravitational Soundscape at high frequencies

Crucial question: detectability above kHz ?

| Resonant polarization rotation, Sec. 4.2.4 [307] |  |  |  |
| :---: | :---: | :---: | :---: |
| Cruise's detector (devised) [308] | $\left(0.1-10^{5}\right) \mathrm{GHz}$ | $h \simeq 10^{-17}$ | $\times$ |
| Cruise \& Ingley's detector (prototype) [309, 310] | 100 MHz | $8.9 \cdot 10^{-14}$ | $10^{-14} \mathrm{~Hz}^{-\frac{1}{2}}$ |
| Enhanced magnetic conversion (theory), Sec. 4.2.5 [311] | 5 GHz | $h \simeq 10^{-30}-10^{-26}$ | $\times$ |
| Bulk acoustic wave resonators (built), Sec. 4.2.6 [316, 317] | $(\mathrm{MHz}-\mathrm{GHz})$ | $4.2 \cdot 10^{-21}-2.4 \cdot 10^{-20}$ | $10^{-22} \mathrm{~Hz}^{-\frac{1}{2}}$ |
| Superconducting rings, (theory), Sec. 4.2 .7 [318] | 10 GHz | $h_{0, n, \text { mono }} \simeq 10^{-31}$ | $\times$ |
| Microwave cavities, Sec. 4.2.8 |  |  |  |
| Caves' detector (devised) [320] | 500 Hz | $h \simeq 2 \cdot 10^{-21}$ | $\times$ |
| Reece's 1st detector (built) [321] | 1 MHz | $h \simeq 4 \cdot 10^{-17}$ | $\times$ |
| Reece's 2nd detector (built) [322] | 10 GHz | $h \simeq 6 \cdot 10^{-14}$ | $\times$ |
| Pegoraro's detector (devised) [323] | $(1-10) \mathrm{GHz}$ | $h \simeq 10^{-25}$ | $\times$ |
| Graviton-magnon resonance (theory), Sec. 4.2.9 [324] | $(8-14) \mathrm{GHz}$ | $9.1 \cdot 10^{-17}-1.1 \cdot 10^{-15}$ | $\left(10^{-22}-10^{-20}\right) \mathrm{Hz}^{-\frac{1}{2}}$ |

## FoM:

$$
\begin{aligned}
h_{c} & \sim 10^{-23}(\mathrm{GHz} / f) \\
h_{s} & \sim 10^{-30}(\mathrm{GHz} / f)
\end{aligned}
$$

Table 1: Summary of existing and proposed detectors with their respective sensitivities. See Sec. 4.3 for details.

## Two words on mechanical coupling



$$
P_{\mathrm{sig}} \simeq \frac{h_{+, \times}^{2}}{16}\left|\eta_{u}^{\text {(mech. })}\right|^{2}\left|\tilde{\eta}_{e}^{\text {(mech. })}\right|^{2} Q_{1} \omega_{1}\left\langle E_{\text {pump }}^{2}\right\rangle V_{0} \times \begin{cases}\frac{\omega_{h}^{4}}{\left(\omega_{h}^{2}-\omega_{m}^{2}\right)^{2}} & \left(\left|\omega_{h}-\omega_{m}\right| \gg \omega_{m} / Q_{m}\right) \\ Q_{m}^{2} & \left(\left|\omega_{h}-\omega_{m}\right| \ll \omega_{m} / Q_{m}\right)\end{cases}
$$

## Conclusions

- SRF cavities are a mature technology to look for GWs at GHz either
- 'ADMX' like
- Heterodyne

- As in any GR calculation: subtleties in working with a consistent gauge
- T gauge needs to be converted to laboratory frame
- The laboratory frame may need all orders in $(R x) \sim O((\omega L))$
- The way one reads out cavities is sensitive to time averaged $\langle U\rangle \sim O\left(h^{2}\right)$
- In the laboratory frame, there is sensitivity to ALL directions! (also longitudinal)
- Stay tuned for the connection to real world... (noise estimates + prospects)

