

# Nucleon axial formfactors from lattice QCD

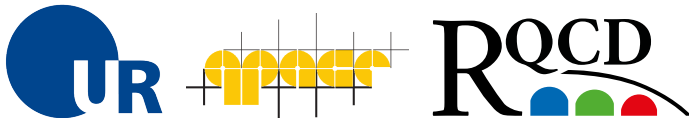
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# Motivation

In NOvA, T2K, DUNE, Hyper-Kamiokande (HK) muon (anti-)neutrinos are/will be scattered off  $\text{H}_2\text{O}$ ,  $^{12}\text{C}$  or  $^{40}\text{Ar}$  targets.

Formfactor predictions from QCD are needed. In addition (except for H) nuclear effects play a role (quasi-elastic scattering).

Here we address elastic scattering  $\bar{\nu}_\ell p \rightarrow \ell^+ n$ ,  $\nu_\ell n \rightarrow \ell^- p$  via charged current interactions.

Next step: pion production  $\nu n \rightarrow \ell^- p \pi^0$ ,  $\ell^- n \pi^+$  etc.

Flavour separation for  $\nu N \rightarrow \nu N$ ,  $N \in \{n, p\}$  would be interesting too.

Present constraints suggest that there is maximum oscillation  $\nu_\mu \rightarrow \nu_e, \nu_\tau$  for  $L/E \approx 500 \text{ km/GeV}$ .

NOvA: 810 km/2 GeV

T2K & HK: 295 km/0.6 GeV

DUNE: 1300 km/2.5 GeV

Aims: resolving the neutrino mass hierarchy, constraining elements of the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix, BSM physics.

# Neutrinos in the Standard Model

All fermions are massless at high energy scales.

They acquire masses via their Yukawa couplings to the Higgs.

Left-handed fermions (spin antiparallel to momentum) form doublets:  
 $(\nu_e, e^-)$ ,  $(u, d)$ ,  $(\nu_\mu, \mu^-)$ ,  $(c, d)$ ,  $(\nu_\tau, \tau^-)$ ,  $(t, b)$ . These interact **weakly**.

Right-handed fermions are singlets under the **weak** interaction.

Right-handed neutrinos can only interact **gravitationally**.

⇒ If they exist, right-(left-)handed (anti)neutrinos are “**sterile**”.

If neutrinos have no mass (i.e. they do not couple to the Higgs), right-handed neutrinos cannot be created.

We now know that left-handed neutrinos have (very small) masses.

⇒ Chirality is not conserved.

Obviously, detecting right handed neutrinos would be difficult.

∃ extra mass terms for right-handed neutrinos (seesaw mechanism)??

Are neutrinos always left-handed and antineutrinos right-handed? Is the neutrino its own antiparticle?

2-component **Majorana** spinor instead of 4-component **Dirac**-spinor?

⇒ **neutrino-less double  $\beta$ -decay** is possible in this case.

# Formfactors important to measure neutrino oscillations

Neutrinos  $|\nu_\alpha\rangle$  are created by **weak** interaction processes in a weak interaction eigenstate of definite **flavour**  $\alpha \in \{e, \mu, \tau\}$ .

The weak interaction violates time reversal  $T = CP$ . Therefore, weak flavour eigenstates are no energy eigenstates  $|\nu_j\rangle$  of the time evolution operator.

Unitary PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix **U**:

$$|\nu_j\rangle = \sum_{\alpha} \mathbf{U}_{\alpha j} |\nu_\alpha\rangle$$

$\Rightarrow$  Neutrino mixing in the time evolution.

Dirac neutrinos: the PMNS matrix has 4 independent entries (incl. one  $CP$  violating phase), in analogy to the CKM (Cabibbo-Kobayashi-Maskawa) matrix in the quark sector.

Majorana neutrinos: the PMNS matrix has 6 parameters (3 violate  $CP$ ).

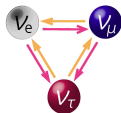
$\vec{p}_j = p_j \vec{e}_z$  momentum of neutrino of mass  $m_j \ll p_j$

$$\Rightarrow E - p_j = p_j \left[ \sqrt{1 + m_j^2/p_j^2} - 1 \right] \approx p_j [1 + m_j^2/(2p_j^2) - 1] \approx m_j^2/(2E).$$

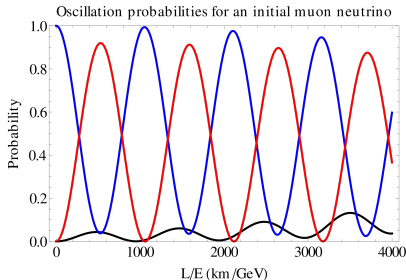
Time evolution ( $L = z \approx t$ )

$$|\nu_j(t)\rangle = \exp[-i(Et - p_j z)] |\nu_j(0)\rangle = \exp[-i m_j^2 L/(2E)] |\nu_j(0)\rangle$$

The oscillation probability between different neutrino flavours  $\alpha$  depends both on the entries of  $\mathbf{U}$  and on  $\Delta m_{jk}^2 = m_j^2 - m_k^2$ :



$$P_{\alpha \rightarrow \beta} = |\langle \nu_\beta(t) | \nu_\alpha \rangle|^2 = \left| \sum_j \mathbf{U}_{\alpha j}^* \mathbf{U}_{\beta j} e^{-i m_j^2 L/(2E)} \right|^2$$



cosmological constraints:

$$m_1 + m_2 + m_3 < 0.3 \text{ eV.}$$

“normal” hierarchy:  $m_1 < m_2 < m_3$

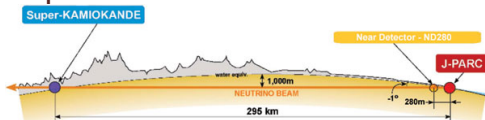
$$m_2 \approx m_1 + 0.0086 \text{ eV,}$$

$$m_3 \approx m_1 + 0.0504 \text{ eV}$$

“inverted” hierarchy:  $m_3 < m_1 < m_2$

$$m_1 \approx m_2 \approx m_3 + 0.0500 \text{ eV}$$

# Experiments



T2K: Tokai to Super-Kamiokande,  
 $E = 0.6 \text{ GeV}$ ,  $L/E \approx 500 \text{ km/GeV}$ .

Also NOvA,  $L/E \approx 400 \text{ km/GeV}$ , DUNE  $L/E \approx 520 \text{ km/GeV}$ , HK(=T2K).

Muon neutrino beam: proton on nucleus  $\rightarrow$  pions and kaons  $\rightarrow \mu^+ \nu_\mu$  or  $\mu^- \bar{\nu}_\mu$ .

Near and far detectors.

$$N_{\text{far}}^\mu(E_\nu) = N_{\text{near}}^\mu(E_\nu) \times [\text{flux}(L)] \times [\text{detector}] \times [1 - \sum_{\beta} P_{\mu \rightarrow \beta}(E_\nu)]$$

$E_\nu$  has to be reconstructed from the momentum of the detected charged lepton.

Trivial for  $\nu_\mu + n \rightarrow \mu^- + p$  if the initial momenta of  $n$  and of  $\nu_\mu$  are known.

But...

The neutrino beam is not monochromatic but has a momentum distribution.

The nucleon is bound in a nucleus and has  $|\mathbf{p}_{\text{Fermi}}| \sim 200 \text{ MeV}$ .

The lepton momentum reconstruction is often incomplete.

Misidentification of inelastic scattering as elastic scattering.

Monte-Carlo simulation needs input regarding the differential cross section.

## Formfactors (FFs)

$\nu_\ell n \rightarrow \ell^- p$  goes via the  $V - A$  current. The non-perturbative matrix elements that enter  $d\sigma/d\Omega$  can be decomposed in terms of four FFs.

Kinematics:  $q_\mu = p'_\mu - p_\mu$ ,  $Q^2 = -q_\mu q^\mu \geq 0$ ,  $p'^2 = p^2 = m_N^2 \approx m_n^2 \approx m_p^2$ .

$$\begin{aligned}\langle p(p') | \bar{u} \gamma_\mu d(0) | n(p) \rangle &= \bar{u}_p(p') \left[ F_1(Q^2) \gamma_\mu + \frac{F_2(Q^2)}{2m_N} \sigma_{\mu\nu} q^\mu \right] u_n(p), \\ \langle p(p') | \bar{u} \gamma_\mu \gamma_5 d(0) | n(p) \rangle &= \bar{u}_p(p') \left[ G_A(Q^2) \gamma_\mu + \frac{G_{\tilde{P}}(Q^2)}{2m_N} q_\mu \right] \gamma_5 u_n(p).\end{aligned}$$

Note that  $\langle p | \bar{u} \Gamma d | n \rangle = \langle p | (\bar{u} \Gamma u - \bar{d} \Gamma d) | p \rangle$  if  $m_u = m_d$ ,  $e_u = e_d$  (isospin limit).

Dirac (vector) FF  $F_1$ , Pauli FF  $F_2$ , axial FF  $G_A$ , induced pseudoscalar FF  $G_{\tilde{P}}$ .

$F_1$  and  $F_2$  are relatively well-known (using isospin symmetry) from lepton-proton and lepton-neutron/deuteron scattering (but not their slope at  $Q^2 = 0$ !).

$g_A = G_A(0)$  is well determined from  $\beta$ -decay.  $G_A(Q^2)$  information from neutrino scattering and (indirectly) through exclusive pion electroproduction  $e^- p \rightarrow \pi^- \nu p$ .

Using  $G_A(0.88 m_\mu^2) \approx g_A$ ,  $F_1$  and  $F_2$  as input,  $G_{\tilde{P}}(0.88 m_\mu^2)$  can be determined from muon capture  $\mu^- p \rightarrow \nu_\mu n$  in muonic hydrogen.

## PCAC and PPD relations

The impact of  $G_{\tilde{P}}(Q^2)$  on the cross section is suppressed by a factor  $m_\ell^2/m_N^2 \approx 0.01$  for  $\ell = \mu$ . Therefore, it is only relevant for very small  $Q^2$ , where this formfactor is large (e.g., at the muon capture point).

We define the pseudoscalar FF:

$$\langle p(p') | \bar{u} i \gamma_5 d(0) | n(p) \rangle = \bar{u}_p(p') G_P(Q^2) i \gamma_5 u_n(p).$$

Abbreviations to be used:  $A_\mu = \bar{u} \gamma_\mu \gamma_5 d$ ,  $P = \bar{u} i \gamma_5 d$ .

Consequence of the PCAC relation, i.e. the axial Ward-Takahashi identity ( $\partial_\mu A_\mu = 2m_{ud}P$  and  $\bar{u}_N \gamma_\mu \gamma_5 u_N = 2m_N \bar{u}_N i \gamma_5 u_N$ ):

$$2m_N G_A(Q^2) = 2m_{ud} G_P(Q^2) - \frac{Q^2}{2m_N} G_{\tilde{P}}(Q^2).$$

With complete non-perturbative order- $a$  improvement, this relation will receive  $\mathcal{O}(a^2 \Lambda^2, a^2 Q^2, a^2 m_{ud} \Lambda, \dots)$  lattice spacing corrections.

Current algebra gives the Pion pole dominance (PPD) relation

$$G_{\tilde{P}}(Q^2) \approx \frac{4m_N^2}{M_\pi^2 + Q^2} G_A(Q^2).$$

Unless  $M_\pi^2 = 0$ , also in the continuum this relation is only approximate.

# Lattice calculation

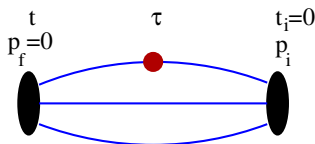
Ideally, one would compute the FFs directly from QCD via lattice simulation, without additional assumptions.

Apart from attaining meaningful statistical errors, this requires taking the following limits:

- ▶ Continuum limit:  $a^2 \rightarrow 0$ .
- ▶ Infinite volume limit:  $L = N_s a \rightarrow \infty$ . Due to the mass gap, these effects are exponential  $\sim \exp(-LM_\pi)$ , however, large  $N_s$  become necessary at small  $M_\pi$  and at small  $a$ . ChPT  $\rightarrow$  FVE are most relevant at small  $Q^2$  (better not to normalize wrt  $g_A = G_A(0)$ ).
- ▶ Physical point: Results must be extra-/interpolated to physical quark masses or, equivalently, physical pion and kaon masses.
- ▶ Extrapolation to infinite Euclidean time separations, where the ground state dominates.

# Spectral decomposition

In our case Wick contractions give only **connected diagrams** (isospin limit).



$$C_{2pt}(\mathbf{p}, t) = (Z_1^{\mathbf{p}})^2 e^{-E_N(\mathbf{p})t} \left[ 1 + \sum_{k>1} \left( \frac{Z_k^{\mathbf{p}}}{Z_1^{\mathbf{p}}} \right)^2 e^{-\Delta E_k(\mathbf{p})t} \right],$$

$$C_{3pt}(\mathbf{p}' = \mathbf{p}, \tau, t) = (Z_1^{\mathbf{p}})^2 \langle 1|A|1 \rangle e^{-E_N(\mathbf{p})t} \left[ 1 + \sum_{k>1} \frac{Z_k^{\mathbf{p}}}{Z_1^{\mathbf{p}}} \frac{\langle k|A|N \rangle}{\langle 1|A|1 \rangle} (e^{-\Delta E_k(\mathbf{p})(t-\tau)} + e^{-\Delta E_k(\mathbf{p})\tau}) \right. \\ \left. + \left( \frac{Z_k^{\mathbf{p}}}{Z_1^{\mathbf{p}}} \right)^2 \frac{\langle k|A|k \rangle}{\langle 1|A|1 \rangle} e^{-\Delta E_k(\mathbf{p})t} \right].$$

For simplicity, above  $q = 0$ .

$\langle 1|A|1 \rangle$  is the (purely real or imaginary) Euclidean matrix element of interest for the nucleon ground state  $|1 \rangle = |N \rangle$ ,  $Z_k \propto \langle \Omega | \mathcal{N} | k \rangle = Z_k^*$  are the so-called overlap factors and  $\Delta E_k = E_k - E_1$  with  $E_1 = E_N$ .

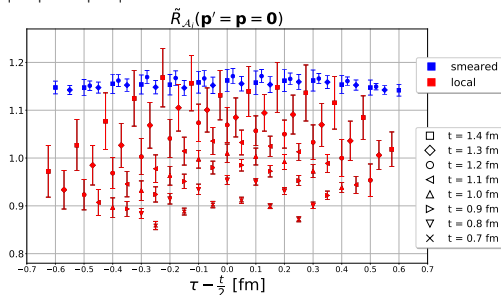
2pt-function: excited state suppression with  $\delta_k^2 = Z_k^2 / Z_1^2$ .

3pt-function: suppression only with  $\delta_k \langle k|A|1 \rangle / \langle 1|A|1 \rangle$ . What if one  $\langle k|A|1 \rangle$  is large?

# Excited state pollution

The signal decreases exponentially, **noise/signal increases exponentially** with the source-sink separation time. So one cannot achieve arbitrarily large separations between source, current and sink.

“Smearing” enables the construction of an “interpolator”  $\bar{O}_N$  that creates a combination of energy eigenstates,  $\bar{O}_N|\Omega\rangle = c_1|N\rangle + c_2|N'\rangle + \dots$ , with  $|c_1| \gg |c_k| \quad \forall \quad k > 1$ . Then all  $\delta_k$  are small.



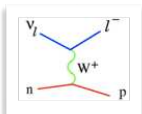
Example of a ratio of 3pt-over 2pt-functions for  $g_A$ :  
A fit gives  $g_A = 1.166(13)$   
for this particular ensemble  
( $a \approx 0.098$  fm,  $M_\pi \approx 429$  MeV).

Problem with the axial current: it can couple well to pions since  $\langle \Omega | A_\mu | \pi^+(q) \rangle = i\sqrt{2}F_\pi q_\mu$ . Matrix elements “ $\langle N\pi | A_\mu | N \rangle$ ” may be enhanced!

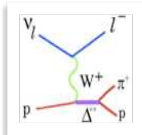
# Neutrino-nucleus scattering

This also happens in experiment. Our interpolators do not couple to  $\Delta$ , i.e.  $N\pi$  with spin 3/2, but they couple to  $P$ -wave  $N\pi$  with spin 1/2.

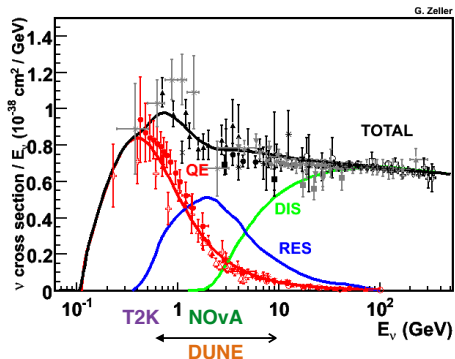
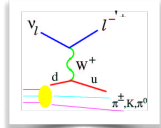
Quasi-elastic scattering (QE)



Resonance production (RES)



Deep Inelastic scattering (DIS)



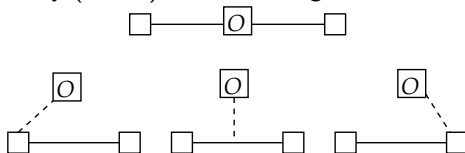
J.A. Formaggio, G. Zeller, Reviews of Modern Physics, 84 (2012)

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After understanding excited states in  $N \rightarrow N$ , one should also compute axial and vector  $N \rightarrow N\pi$  transition formfactors.

# Origin of $N \rightarrow N\pi$ pollution in the axial formfactors

Chiral perturbation theory (ChPT) tree-level diagrams:



Top diagram:

$$\sim G_A$$

$$= 0$$

$$\text{for } \mathcal{O} = A_\mu$$

$$\text{for } \mathcal{O} = P$$

Bottom centre diagram:

$$\sim G_{\bar{P}} + \text{excited states}$$

$$\sim G_P + \text{excited states}$$

$$\text{for } \mathcal{O} = A_4$$

$$\text{for } \mathcal{O} = P$$

Other diagrams: only contribute to the excited states.

In ChPT these contributions are enhanced by a factor  $m_N/M_\pi$  for  $A_4$  relative to the ground state but also present in  $A_j$ .

ChPT predicts the dominant  $N\pi$  energy level (all momentum transferred to  $\pi$  at tree-level) and the coupling.

## $N\pi$ excited state contributions

[Bär,1906.03652,1812.09191]:  $N\pi$  contributions to a combination  $R_4$  of  $C_{3pt}^O$  and  $C_{2pt}$  for  $O = A_4$  in leading one-loop order of SU(2) covariant ChPT.

[Bär,1907.03284]

$R_4$  vs.  $\tau/a - t_f/(2a)$

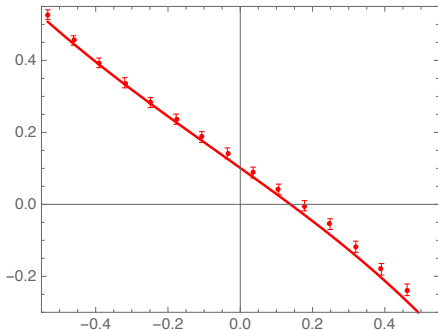
$t_f \rightarrow \infty$ :  $R_4 \rightarrow \text{const.}$

Data: [RQCD,1810.05569]:

$M_\pi \sim 150$  MeV,  $a = 0.07$  fm,

$t_f = 1.06$  fm,  $\mathbf{p}' = \mathbf{0}$ ,

$|\mathbf{q}| = 2\pi/(64a)$



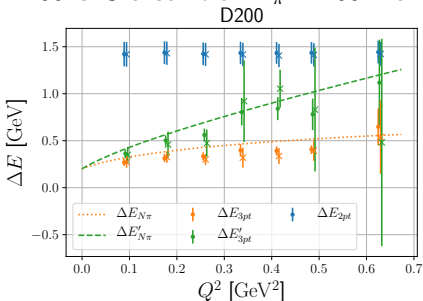
$$C_{3pt}^{A_4}(\mathbf{p}'=\mathbf{0}, \mathbf{p}=-\mathbf{q}, t_f, \tau) = C_{3pt,N}^{A_4}(\mathbf{q}, t_f, \tau) + C_{3pt,N\pi}^{A_4}(\mathbf{q}, t_f, \tau) = \mathcal{O}\left(\frac{M_\pi}{m_N}\right) + \mathcal{O}(1)$$

In this channel  $N(\mathbf{0})\pi(-\mathbf{q}) \rightarrow N(\mathbf{0})$  is enhanced, relative to  $N(-\mathbf{q}) \rightarrow N(\mathbf{0})$ .

Maybe correct lattice data by subtracting the expectation? Problem: systematics.

# Instead: simultaneous fit to different channels

D200 CLS ensemble:  $M_\pi \sim 200$  MeV,  $a = 0.064$  fm.

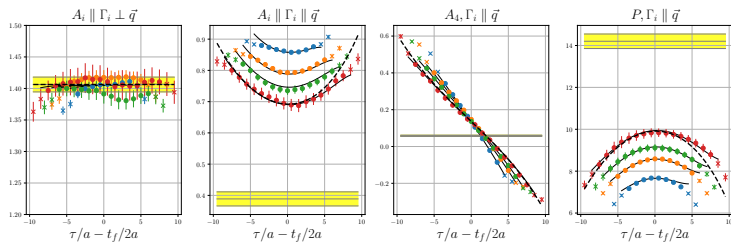


$$\Delta E_{N\pi} \approx E_\pi(-\mathbf{q}) + E_N(\mathbf{0}) - E_N(-\mathbf{q})$$

$$\Delta E'_{N\pi} \approx E_\pi(\mathbf{q}) + E_N(-\mathbf{q}) - E_N(\mathbf{0})$$

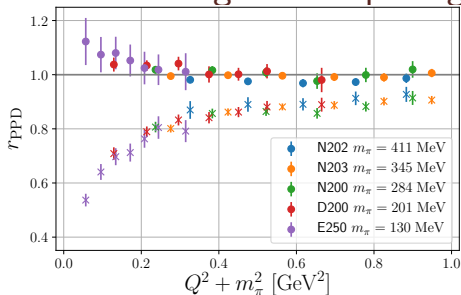
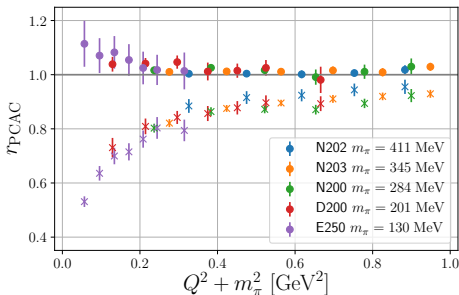
since  $\mathbf{p}' = \mathbf{0}$  and  $\mathbf{p} = -\mathbf{q}$ .

Include  $N\pi$  levels of tree-level ChPT leads to reasonable  $\chi^2/\text{d.o.f}$  ( $|\mathbf{q}| = 2\pi/(64a)$ .)



The yellow result respects the known PCAC relation (up to lattice artefacts).

# PCAC and PPD relations at a non-vanishing lattice spacing



Crosses: using the excited state gap from the two-point function.

Circles: simultaneous fit to all 3pt-functions, including  $A_4$ .

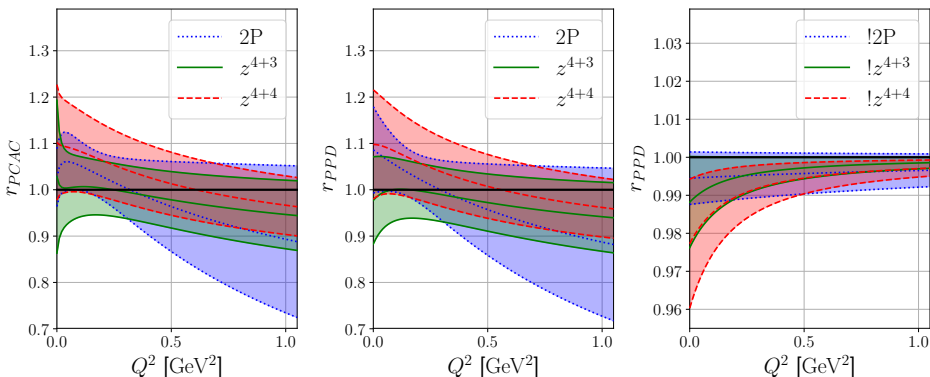
The simultaneous fit was carried out with a free second excited state gap on either side and

- ▶ fixing  $\Delta E_{N\pi}$  and  $\Delta E'_{N\pi}$  from the non-interacting levels,
- ▶ fitting these energies from the 3pt-function data.

The difference is included in the systematic error of the final result.

Note that  $G_A(Q^2)$ , extracted from  $A_i$  alone via a naive fit, is only marginally different. However, the effect on  $G_{\tilde{P}}$  and  $G_P$  is huge at small  $Q^2$  and  $M_\pi^2$ .

# Results: PCAC and PPD relations in the continuum limit

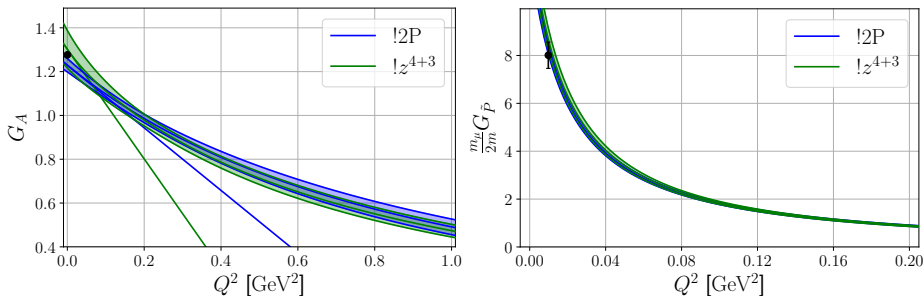


Right: PCAC is imposed.

$$r_{PCAC} = \frac{2m_{ud} G_P(Q^2) + \frac{Q^2}{2m_N} G_{\tilde{P}}(Q^2)}{2m_N G_A(Q^2)} = 1, \quad r_{PPD} = \frac{M_\pi^2 + Q^2}{4m_N^2} \frac{G_{\tilde{P}}(Q^2)}{G_A(Q^2)} \approx 1.$$

Violations of the pion pole dominance relation are very small.

## Results: physical point, continuum limit



Black points are experimental values for  $g_A = G_A(0)$  and  $g_P = m_\mu/(2m_N)G_{\bar{P}}(0.88m_\mu^2)$  at the muon capture point.

Straight lines are the slopes at  $Q^2 = 0$ : the axial radius is parametrization dependent and smaller for dipole fits.

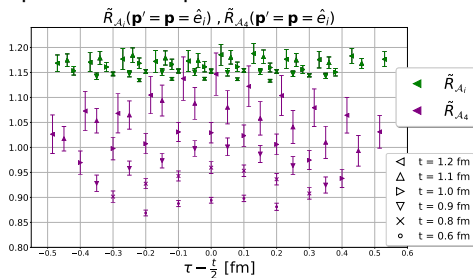
The axial radius is uninteresting for neutrino scattering! Even the electric charge radius of the proton is not very well known from charged lepton scattering.

Results look fine but we would like to confirm this picture, improve on the method and extend the work to pion production.

# Problem with $A_4$ and $P$

Forward limit  $\mathbf{q} = 0$  but moving frame  $\mathbf{p}' = \mathbf{p} = \hat{e}_i$ ,  $|\hat{e}_i| = 2\pi/L \approx 525$  MeV.

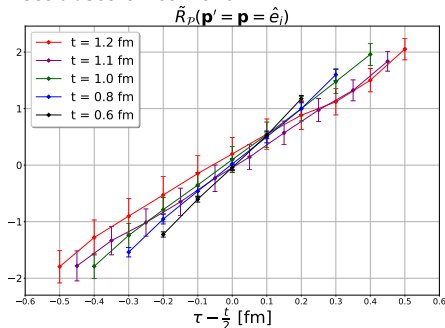
Spatial and temporal axial currents.



Ratio should give  $g_A$  at  $t \gg \tau \gg 0$ .

As expected by ChPT, there are large excited state contributions in  $A_4$  and  $P$ !

Pseudoscalar current.



Ratio has to vanish for the ground state.

# Inclusion of 5-quark interpolators to resolve $N\pi$ states

- ▶ Compute 2- and 3-point functions including 5-quark interpolators  $O_{N\pi}$ .
- ▶  $O_{N\pi}$  must be projected onto  $O_h$  or little group irrep  $G_1$  to give spin 1/2.
- ▶ Project  $O_{N\pi}$  on the neutron ( $n$ ) isospin ( $I = -I_z = 1/2$ ):  
$$O_{N\pi}^{(n)} = \frac{1}{\sqrt{3}} O_{n\pi^0} - \sqrt{\frac{2}{3}} O_{p\pi^-}.$$
- ▶ Solve GEVP to obtain eigenvectors corresponding to the physical  $N$  and  $N\pi$  states.
- ▶ Use these to construct  $p \xrightarrow{\mathcal{J}^-} n$  3-point functions.
- ▶ In progress: also construct 3-point functions for pion production.

$$C_{3pt}^{\mathcal{J}, p\pi^-}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{p\pi^-}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle$$

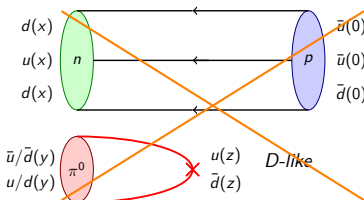
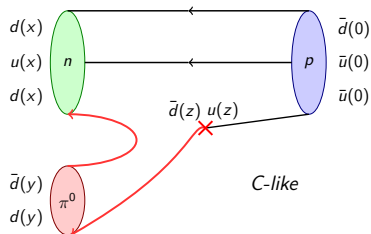
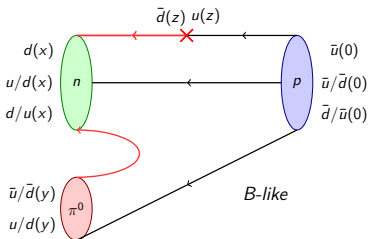
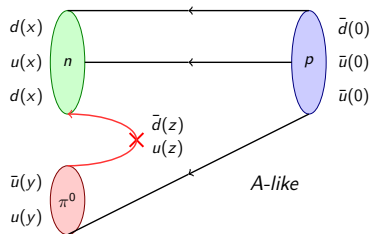
→ 12 Wick contractions

$$C_{3pt}^{\mathcal{J}, n\pi^0}(\mathbf{p}', t; \mathbf{q}, \tau) = P_i^+ \langle O_{n\pi^0}(\mathbf{p}', t) \mathcal{J}^-(\mathbf{q}, \tau) \bar{O}_p(\mathbf{p}, 0) \rangle$$

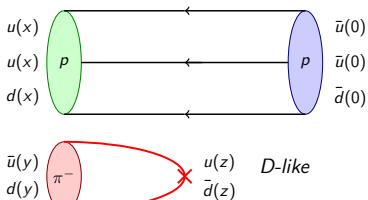
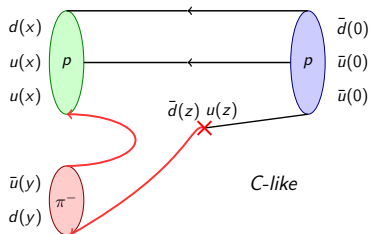
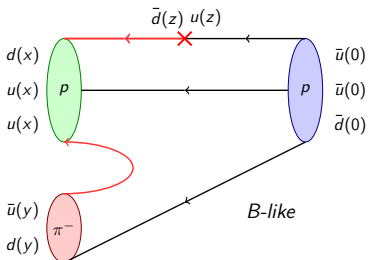
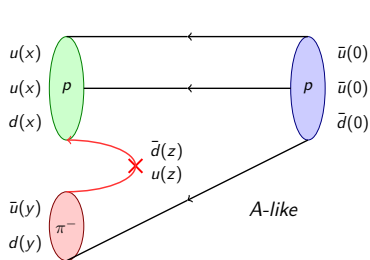
→ 16 Wick contractions

We neglect 3-point functions of the type  $\langle O_{N\pi} \mathcal{J} \bar{O}_{N\pi} \rangle$  as their contribution is strongly suppressed (see below).

# Topologies for $p \xrightarrow{\mathcal{T}^-} n + \pi^0$



# Topologies for $p \xrightarrow{\mathcal{T}^-} p + \pi^-$



$$\sim e^{-E_N t} e^{-E_\pi(t-\tau)}$$

corresponds to tree-level ChPT prediction!

[arXiv:1911.13150 (RQCD)]

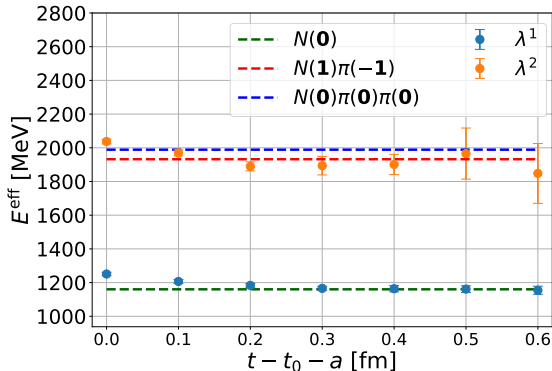
## GEVP for the 2-point functions

2-point function matrix  $C_{ij}(\mathbf{p}, t) = \langle O_i(\mathbf{p}, t) \bar{O}_j(\mathbf{p}, 0) \rangle$ ,  $O_i \in \{O_N, \Phi O_N, O_{N\pi}, \Phi O_{N\pi}\}$ .  $\Phi$  indicates quark smearing.

Here we only consider the  $2 \times 2$  sub-matrix constructed from  $\Phi O_N$  and  $\Phi O_{N\pi}$ .

GEVP:  $C(t)V(t, t_0) = C(t_0)V(t, t_0)\Lambda(t, t_0)$ ,  $V(t, t_0) = (v^1(t, t_0), v^2(t, t_0))$ .

$\Lambda(t, t_0) = \text{diag}(\lambda^1(t, t_0), \lambda^2(t, t_0))$ ,  $\lambda^\alpha(t, t_0) \approx d_\alpha(t_0)e^{-E_\alpha t}$ ,



$$E_\alpha^{\text{eff}} = \ln\left(\frac{\lambda^\alpha(t)}{\lambda^\alpha(t+a)}\right)$$

$$t_0 = 2a, \mathbf{p} = \mathbf{0}.$$

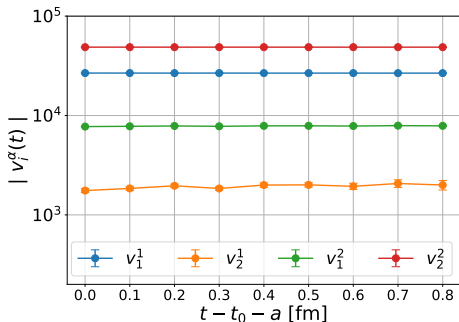
Similarly for  $\mathbf{p} \neq \mathbf{0}$ .

$E_2^{\text{eff}}$  is very close to non-interacting  $N\pi$  P-wave energy.

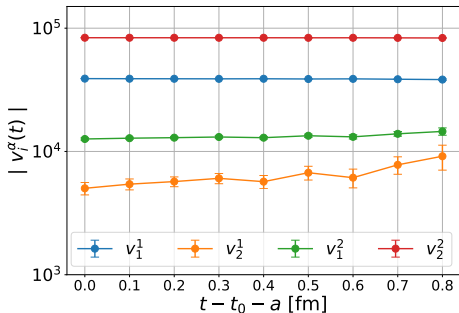
Possible interpretation:  $v^1 \sim N$ ,  $v^2 \sim N\pi$ .

# GEVP eigenvectors

Restframe



Moving frame



Little  $|N\pi\rangle$  in  $\bar{O}_N|\Omega\rangle$  and little  $|N\rangle$  in  $\bar{O}_{N\pi}|\Omega\rangle$ !

(Quantitative statements depend on interpolator basis used. Here: both smeared.)

Therefore,  $N\pi$  is not visible in  $\langle\Omega|O_N(t)\bar{O}_N(0)|\Omega\rangle$  2-point function.

All topologies (A, B, C, D) are important for the determination of the eigenvectors.

In the  $\langle\Omega|O_{N\pi}(t)\mathcal{J}(t)\bar{O}_N(0)|\Omega\rangle$  3-point functions,  $D$  can be the dominant contribution (momentum of current directly transferred to the  $\pi$ ).

# GEVP-projected correlation functions

We had

$$C(\mathbf{p}, t) v^\alpha(\mathbf{p}, t; t_0) = C(\mathbf{p}, t_0) v^\alpha(\mathbf{p}, t; t_0) \lambda^\alpha(\mathbf{p}, t; t_0).$$

GEVP-improved correlation functions for  $\alpha, \beta \in \{N, N\pi\}$ :

$$\begin{aligned} C_{2pt}^\alpha(\mathbf{p}, t) &= v_i^\alpha(\mathbf{p}, t; t_0) C_{ij}(\mathbf{p}, t) v_j^\alpha(\mathbf{p}, t; t_0) \\ C_{3pt}^{\alpha\beta}(\mathbf{p}, t, \mathbf{q}, \tau; P_k; \mathcal{J}) &= v_i^\alpha(\mathbf{p}', t; t_0) C_{ij}^{3pt}(\mathbf{p}', t, \mathbf{q}, \tau; P_k; \mathcal{J}) v_j^\beta(\mathbf{p}, t; t_0) \end{aligned}$$

$|v_2^N|$  and  $|v_1^{N\pi}|$  are small, however, ChPT predicts an enhancement of  $N \xrightarrow{\mathcal{J}} N\pi$  for some polarizations and currents  $\mathcal{J}$ .

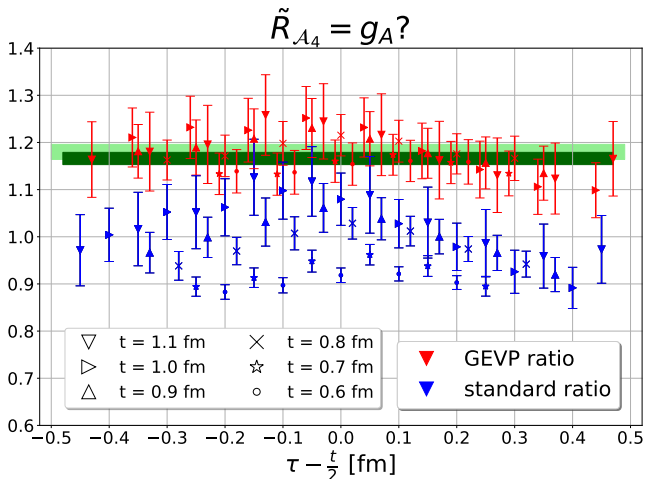
We neglect terms  $\sim |v_2^N|^2$  (small<sup>2</sup>). Also no enhancement expected from ChPT!  
 $\rightarrow$  no need to compute  $\langle \Omega | O_{N\pi}(t) \mathcal{J}(0) \bar{O}_{N\pi} | \Omega \rangle$ .

Improved nucleon matrix elements can be obtained as

$$\langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle \leftarrow \frac{C_{3pt}^{NN}(\mathbf{p}', t, \mathbf{q}, \tau; P_k; \mathcal{J})}{C_{2pt}^N(\mathbf{p}', t)} \sqrt{\frac{C_{2pt}^N(\mathbf{p}', \tau) C_{2pt}^N(\mathbf{p}', t) C_{2pt}^N(\mathbf{p}, t - \tau)}{C_{2pt}^N(\mathbf{p}, \tau) C_{2pt}^N(\mathbf{p}, t) C_{2pt}^N(\mathbf{p}', t - \tau)}}.$$

# Results in the forward limit (moving frame)

$$\mathcal{J} = \mathcal{A}_4, \mathbf{p}' = \mathbf{p} = \hat{e}_z \ (\mathbf{q} = \mathbf{0}).$$



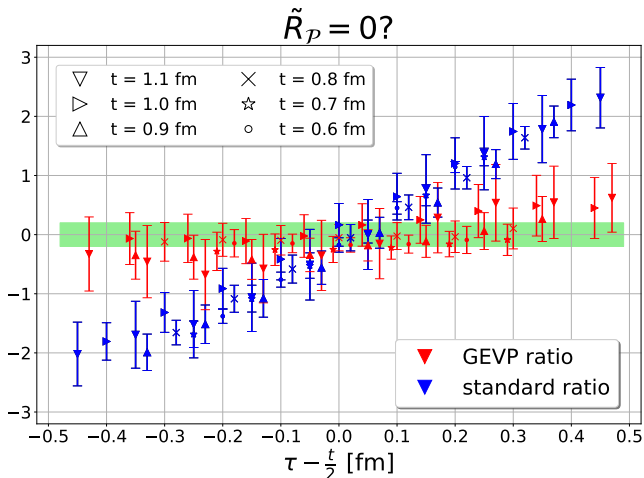
Light green band is  $g_A$  from  $\mathcal{A}_z$  with  $\mathbf{p}' = \hat{e}_z$ , dark green band from  $\mathbf{p}' = \mathbf{p} = \mathbf{0}$ .

Large contribution from D:  $\langle N(\hat{e}_z)\pi(\mathbf{0})|\mathcal{A}_4(\mathbf{0})|N(\hat{e}_z)\rangle$ ,  $\langle N(\hat{e}_z)|\mathcal{A}_4(\mathbf{0})|N(\hat{e}_z)\pi(\mathbf{0})\rangle$ .

Note that  $\langle N|\mathcal{A}_4|N\pi\rangle \sim \langle N|N\rangle\langle\Omega|A_4|\pi\rangle = \langle N|N\rangle\sqrt{2}E_\pi F_\pi$ .

## Results in the forward limit 2 (moving frame)

$$\mathcal{J} = \mathcal{P}, \mathbf{p}' = \mathbf{p} = \hat{\mathbf{e}}_z \ (\mathbf{q} = \mathbf{0}).$$



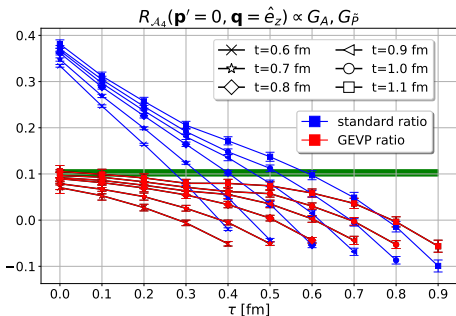
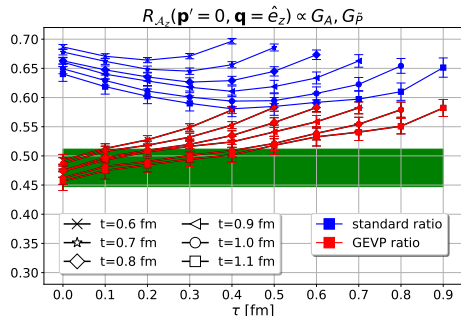
Light green band illustrates the expectation for the ground state (zero!).

Again large contribution from D (for the same reason as for  $A_4$ ):

$$\langle N(\hat{\mathbf{e}}_z)\pi(\mathbf{0})|\mathcal{P}(\mathbf{0})|N(\hat{\mathbf{e}}_z)\rangle, \langle N(\hat{\mathbf{e}}_z)|\mathcal{P}(\mathbf{0})|N(\hat{\mathbf{e}}_z)\pi(\mathbf{0})\rangle \text{ are big.}$$

... and with momentum transfer

$$\mathcal{J} = \mathcal{A}_z, \mathcal{A}_4, \mathbf{p}' = \mathbf{0} \ (\mathbf{q} = -\mathbf{p} = \hat{e}_z)$$



Prediction of  $R_{\mathcal{A}_z}/R_{\mathcal{A}_4} = \frac{E_N + m_N}{p_z} \approx 4.7$  consistent with the data!

$G_A$  from  $R_{\mathcal{A}_i}^{P_i}(q_{j \neq i})$  and  $G_P, G_{\tilde{P}}$  extracted from GEVP-improved ratio satisfy PCAC and PPD relation up to  $\mathcal{O}(a^2)$  effects.

This can also be achieved without the GEVP, using a ChPT inspired fit ansatz.

In the moving frame no excitations can be detected within our uncertainty.

In the rest frame (large  $\tau$ ), contaminations from higher excitations are still present.

# Summary

- ▶ We have determined the axial formfactor in the range of momentum transfers that is relevant for long baseline neutrino experiments.
- ▶ Using this result can improve our understanding of nuclear effects and help to entangle different processes, ultimately leading to higher precision.
- ▶ A problem in the determination of the induced pseudoscalar and pseudoscalar formfactors is the enhancement of  $N\pi$  states. This cannot be eliminated by “smearing”: also the most ideal nucleon interpolator will couple to  $N\pi$ ! Once this was understood, all formfactors could be determined reliably.
- ▶ The PCAC relation between the formfactors is satisfied in the continuum limit. It is still slightly violated at  $a > 0$ . Violations of pion pole dominance are found to be below a few per cent at physical quark masses.
- ▶ We further confirmed this picture by explicitly computing axial  $N \rightarrow N\pi$  matrix elements, also using five-quark interpolators. This (and the determination of transition formfactors) is in progress.
- ▶ The chiral effective theory provides good guidance for understanding the couplings of interpolators and currents to excited states.
- ▶ It would be great to reduce noise over signal for baryonic  $n$ -point functions!