SCETlib Updated Level 3 Results.

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EWWG, January 27, 2022

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Recall: Small-p_T Power Expansion.

Expand in p_T (more precisely in powers of p_T/Q where here $Q \equiv \sqrt{q^2} = m_{\ell\ell}$)

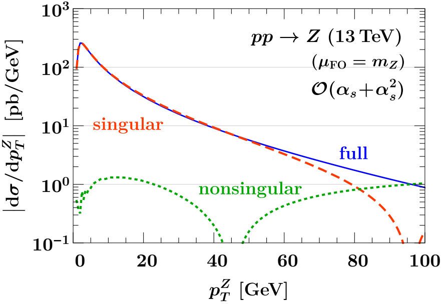
$$\frac{d\sigma}{dp_T} = \delta(p_T) + \alpha_s\Big[\frac{\ln p_T}{p_T} + \frac{1}{p_T} + \delta(p_T) + f_1^{\rm nons}(p_T)\Big]$$

$$+ \alpha_s^2\Big[\frac{\ln^3 p_T}{p_T} + \frac{\ln^2 p_T}{p_T} + \frac{\ln p_T}{p_T} + \frac{1}{p_T} + \delta(p_T) + f_2^{\rm nons}(p_T)\Big]$$

$$+ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \ddots \quad + \dots \Big]$$

$$\sim (1/p_T)\Big[\qquad \mathcal{O}(1) \qquad + \mathcal{O}(p_T)\Big]$$

- "singular" or "leading power"
  - To be resummed
- "nonsingular" or "subleading power"
  - Suppressed by relative p_T^2/m_V^2
  - To be supplied by matching to full FO

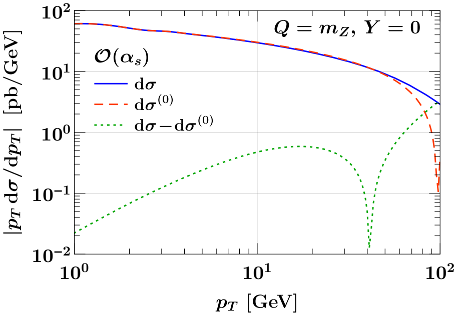


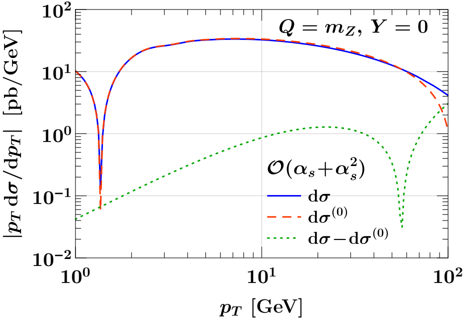
Matching to Fixed Order.

$$\mathrm{d}\sigma = \underbrace{\mathrm{d}\sigma^{(0)}(\mu_H, \mu_B, \nu_B, \mu_f, \mu_S, \nu_S)}_{\equiv\, \mathrm{d}\sigma^{\mathrm{resum}}} + \underbrace{\Big[\mathrm{d}\sigma^{\mathrm{FO}}(\mu_{\mathrm{FO}}) - \mathrm{d}\sigma^{(0)}(\mu_i, \nu_i \equiv \mu_{\mathrm{FO}})\Big]}_{\equiv\, \mathrm{d}\sigma^{\mathrm{nons}}}$$

- σ^{resum} and σ^{nons} are *separately* scale independent (args show residual dep.)
  - In particular, $\mathrm{d}\sigma^{\mathrm{nons}}$ *does not* depend on resummation scales and does not affect uncertainties of $\mathrm{d}\sigma^{\mathrm{resum}}$
- For $p_T \to Q$: $\mathrm{d}\sigma^{\mathrm{resum}} \to \mathrm{d}\sigma^{(0)}$
  - Reproduces $\mathrm{d}\sigma^{\mathrm{FO}}$ *exactly* at any given order (i.e. resummation should not induce any rogue higher-order corrections, because in general they would be unphysical and can be arbitrarily large)
  - Achieved by using profile scales: $\mu_i(p_T), \nu_i(p_T) \to \mu_{\mathrm{FO}}$ for $p_T \to Q$ which smoothly and intrinsically turn off resummation
- Relation to benchmark levels
  - Level 1: $\mathrm{d}\sigma^{\mathrm{resum}}$(canonical scales)
  - Level 2: $\mathrm{d}\sigma^{\mathrm{resum}}$(profile scales)
  - Level 3: $\mathrm{d}\sigma^{\mathrm{resum}}$(profile scales) $+\, \mathrm{d}\sigma^{\mathrm{nons}}(\mu_{\mathrm{FO}})$

Check against FO results from DYTurbo.





- Full $\mathrm{d}\sigma$ from DYTurbo
- Singular $\mathrm{d}\sigma^{(0)}$ from SCETlib
  - $p_T\,\mathrm{d}\sigma^{(0)}/\mathrm{d}p_T \sim \ln^n p_T + \mathrm{const}$ $\to$ grows/constant on log-log plot
- Nonsingular $\mathrm{d}\sigma - \mathrm{d}\sigma^{(0)}$ is indeed power-suppressed
  - $p_T\,\mathrm{d}\sigma^{\mathrm{nons}}/\mathrm{d}p_T \sim p_T^2$ $\to$ must vanish with negative slope on log-log plot
  - Strong check and best (only) way to identify (small) mismatches

Profile Scales and Perturbative Uncertainties.

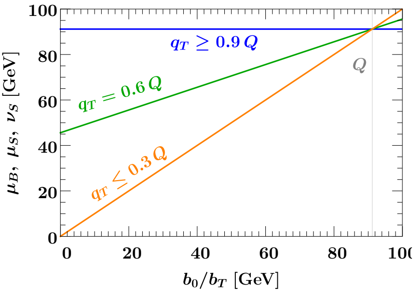
$$\mu_H = \nu_B = \mu_{\rm FO} = Q$$

$$\mu_B, \mu_S, \nu_S = \mu_{\rm FO} f_{\rm prof}\Big[\frac{p_T}{Q}, \frac{1}{Q}\mu_*\Big(\frac{b_0}{b_T}, \mu_i^{\rm min}\Big)\Big] \begin{cases} = b_0/b_T & p_T \ll Q \\ \to \mu_{\rm FO} \equiv \mu_R & p_T \to Q \end{cases}$$

$$\mu_f = \mu_F f_{\rm run}\Big[\frac{p_T}{Q}, \frac{1}{Q}\mu_*\Big(\frac{b_0}{b_T}, \mu_f^{\rm min}\Big)\Big] \begin{cases} = b_0/b_T & p_T \ll Q \\ \to \mu_F & p_T \to Q \end{cases}$$

$f_{\rm prof}$ steers transition from resummation on $\to$ resummation off

- Turn-off for $p_T \to Q$ *does not* alter canonical res. at $p_T \ll Q$
  - Transition driven by p_T/Q
    (b_T is just means to an end, we want to predict physical p_T spectrum not the b_T spectrum)
  - Transition points are based on relative size of leading-power vs. nonsingular (power) corrections
  - Their variation yields $\Delta_{\rm match}$



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Nonpert. cutoff prescription for $b_0/b_T \lesssim 1\,\mathrm{GeV}$ (freeze-out, local b^*, global b^*)

- New: Dedicated μ_* prescription
  - Similar to local b^* except each scale has its own cutoff

$$\mu_B^{\min} = \mu_S^{\min} = 1\,\mathrm{GeV} \qquad \text{(min scale appearing in } \alpha_s) \,,$$
$$\nu_S^{\min} = 0 \qquad \text{(no cutoff needed for rapidity scale)} \,,$$
$$\mu_f^{\min} = 1.65\,\mathrm{GeV} \qquad \text{(min scale of PDF, use } Q_0 \text{ of NNPDF3.1)}$$

  - And cutoffs are maintained (kept unchanged) under any scale variations

Profile Scales and Perturbative Uncertainties.

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$$\mu_B, \mu_S, \nu_S = \mu_{\mathrm{FO}} f_{\mathrm{prof}}\Big[\frac{p_T}{Q}, \frac{1}{Q}\mu_*\Big(\frac{b_0}{b_T}, \mu_i^{\min}\Big)\Big] \begin{cases} = b_0/b_T & p_T \ll Q \\ \to \mu_{\mathrm{FO}} \equiv \mu_R & p_T \to Q \end{cases}$$

$$\mu_f = \mu_F f_{\mathrm{run}}\Big[\frac{p_T}{Q}, \frac{1}{Q}\mu_*\Big(\frac{b_0}{b_T}, \mu_f^{\min}\Big)\Big] \begin{cases} = b_0/b_T & p_T \ll Q \\ \to \mu_F & p_T \to Q \end{cases}$$

Perturbative uncertainties via scale variations

- "Fixed-order" Δ_{FO}: Max envelope of varying μ_{FO} by factor of 2
  - Keeps all resummed scale ratios invariant, hence the name
  - Transitions into μ_R variation for $p_T \to Q$
- "Resummation" Δ_{resum}: Max envelope of varying μ_B, ν_B, μ_S, ν_S
  - 36 combinations that all possible scale ratios get probed and changed by factor 2 (but not 4) for $p_T \ll Q$ without changing $p_T \to Q$

Profile Scales and Perturbative Uncertainties.

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$$\mu_B, \mu_S, \nu_S = \mu_{\mathrm{FO}} f_{\mathrm{prof}}\Big[\frac{p_T}{Q}, \frac{1}{Q}\mu_*\Big(\frac{b_0}{b_T}, \mu_i^{\min}\Big)\Big] \begin{cases} = b_0/b_T & p_T \ll Q \\ \to \mu_{\mathrm{FO}} \equiv \mu_R & p_T \to Q \end{cases}$$

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Perturbative uncertainties via scale variations

- "PDF/DGLAP" Δ_f: Max envelope of varying μ_f by factor of 2
  - Probes unc. due to universal DGLAP evolution (at actual PDF scale)
  - Explicitly separated out now (was previously tied to μ_B)

    $\to$ Together with $\mu_f^{\min}$ largely avoids previously seen unphysical oscillations
  - Transitions into μ_F variation for $p_T \to Q$
  - Note: This leads to adding in quadrature separate max-envelopes of μ_F and μ_R variations also in FO limit

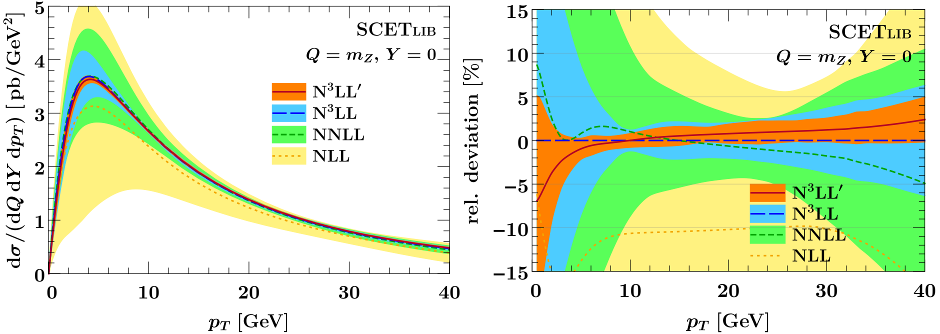
    $\to$ Actually the more sensible thing to do than usual envelope of 7-point variations

Nonperturbative Uncertainties (preliminary).

- Previously: Via simple cutoff variations, crude and not ideal
  - Relies on size of leftover, unresummed perturbative logs
  - Directly depends on perturbative order
- New: Use a basic nonperturbative model
  - Switch to quartic μ_* prescription $\mu_*(\mu, \mu_{\min}) = (\mu^4 + \mu_{\min}^4)^{1/4}$
    $\to$ Avoids inducing artificial quadratic OPE coefficient
    [see e.g. Scimemi, Vladimirov arXiv:1609.06047; Ebert, Michel, Stewart, Sun arXiv:2201.07237]
  - Otherwise current central-value predictions are not affected
    (i.e. model is turned off at central parameter values)
  - Use reasonably generous model parameter variations to estimate Δ_{np}

Results.

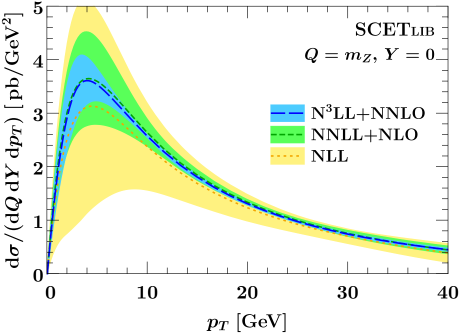
Level 2

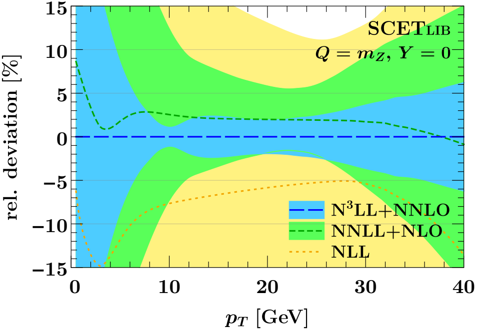


- Excellent coverage/convergence (except in pure FO region)
- Here: Total uncertainty = $\sqrt{\Delta^2_{\mathrm{FO}} + \Delta^2_f + \Delta^2_{\mathrm{resum}} + \Delta^2_{\mathrm{match}}}$

Results.

Level 3





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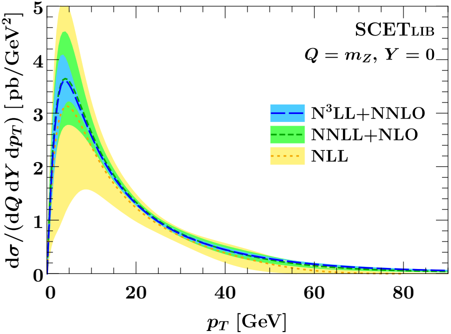
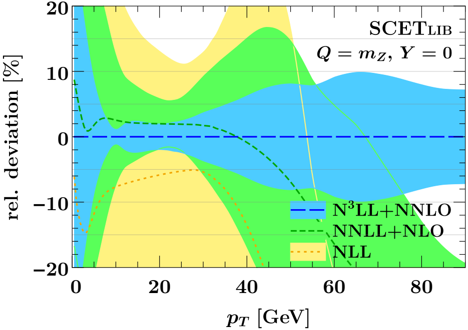
Results.

Level 3 (wide)





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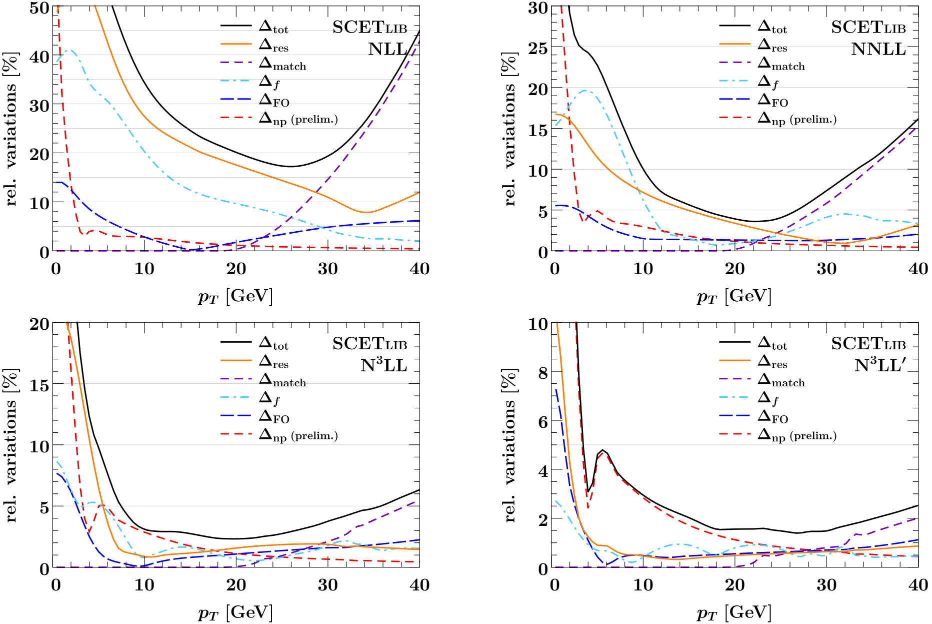
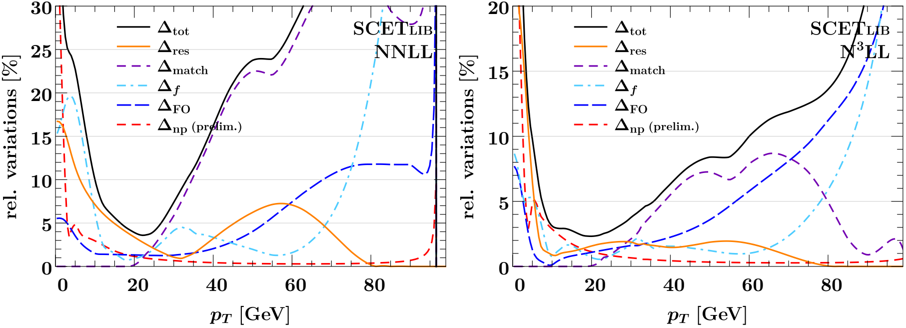
Breakdown of Uncertainties: Level 2.



Breakdown of Uncertainties: Level 2 → Level 3.

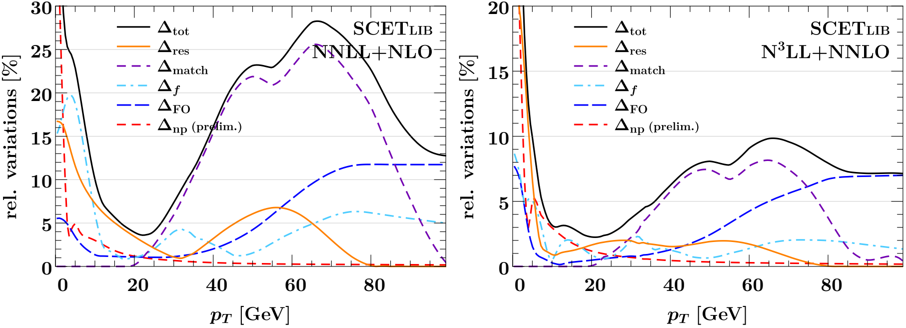
Level 2 (wide)



- Matching only affects uncertainties beyond $p_T \gtrsim 50\,\mathrm{GeV}$ (as it should)

Breakdown of Uncertainties: Level 2 → Level 3.

Level 3 (wide)



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Additional Slides

Factorization and Resummation at Leading Power.

Leading-power terms factorize into hard, collinear, and soft contributions

(with $Q \equiv \sqrt{q^2}$ and $x_{a,b} \equiv (Q/E_{\rm cm})e^{\pm Y}$)

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}p_T^2} = \sum_{a,b} H_{ab}(Q^2, \mu) \times [B_a B_b S](Q^2, x_a, x_b, \vec{p}_T, \mu)$$

$$[B_a B_b S] = \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta^{(2)}(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s)$$
$$\times\ B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$

- Most general forms with no hard-coded choices yet (and completely equivalent)

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$$\times\; B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$

$$\equiv \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)\, \tilde{S}(b_T, \mu, \nu)$$

$$\equiv \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2}\, e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{f}_a(x_a, b_T, \mu, \zeta_a)\, \tilde{f}_b(x_b, b_T, \mu, \zeta_b)$$

(where $\zeta_{a,b} \propto \omega_{a,b}^2$ with $\zeta_a \zeta_b = Q^4$ plays the role of ν)

- Most general forms with no hard-coded choices yet (and completely equivalent)

Schematic Resummation Structure.

$$\mathrm{d}\sigma^{(0)} = H(Q,\mu) \times B(p_T,\mu,\nu/Q)^2 \otimes S(p_T,\mu,\nu/p_T)$$

$$\ln^2\frac{p_T}{Q} = 2\ln^2\frac{Q}{\mu} + 2\ln\frac{p_T}{\mu}\ln\frac{\nu}{Q} + \ln\frac{p_T}{\mu}\ln\frac{\mu\, p_T}{\nu^2}$$

- For generic μ, ν, each function contains (potentially large) logs
- Resummation follows from solving RGEs, and evolving each function from some starting scales μ_i, ν_i to common arbitrary μ, ν

$$H(\mu) = H(\mu_H) \times U_H(\mu_H, \mu)$$
$$B(\mu,\nu) = B(\mu_B,\nu_B) \otimes U_B(\mu_B,\nu_B;\mu,\nu)$$
$$S(\mu,\nu) = S(\mu_S,\nu_S) \otimes U_B(\mu_S,\nu_S;\mu,\nu)$$

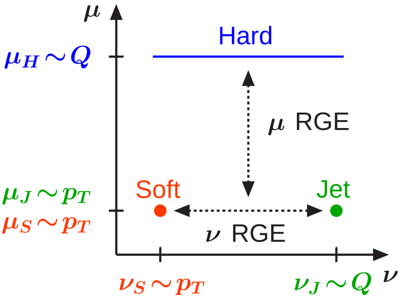
  - Dependence on overall arbitrary μ, ν cancels *exactly at each order* (it must do so = RGE consistency or path independence)
  - This is *not* SCET specific, it is exactly how CSS formula arises from solution of Collins-Soper equation

Schematic Resummation Structure.

$$\mathrm{d}\sigma^{(0)} = H(\mu_H) \times U_H(\mu_H, \mu) \times [B(\mu_B, \nu_B) \otimes U_B(\mu_B, \nu_B; \mu, \nu)]^2$$
$$\otimes S(\mu_S, \nu_S) \otimes U_B(\mu_S, \nu_S; \mu, \nu)$$

- *Boundary conditions* $H(\mu_H)$, $B(\mu_B, \nu_B)$, $S(\mu_S, \nu_S)$ can (must) be calcluated in (log-free) fixed order, so at

$$\mu_H \sim Q$$
$$\mu_B \sim p_T\,, \qquad \nu_B \sim Q$$
$$\mu_S \sim p_T\,, \qquad \nu_S \sim p_T$$



- RGE then really sums logs of ratios

$$\ln \frac{\mu_B}{\mu_H} \sim \ln \frac{\mu_S}{\mu_H} \sim \ln \frac{\nu_S}{\nu_B} \sim \ln \frac{p_T}{Q}$$

- Choice of boundary scales does matter
  - Determine precise form of resummed logarithms ("resummation" scales)
  - Their dependence *only cancels to the fixed order* of the boundary conditions

Resummation in Practice.

- Solving the complete RGE system for p_T distribution is (surprisingly) difficult
  - Exact distributional solution in $\vec{p}_T$ space is equivalent (up to different boundary terms) to solving RGE in b_T space with *canonical* b_T *scales* $(b_0 = 2e^{-\gamma_E})$

$$\mu_H = Q\,, \quad \mu_B = b_0/b_T\,, \quad \nu_B = Q\,, \quad \mu_S = \mu_\nu = \nu_S = b_0/b_T$$

  - Quite nontrivial statement, proven in [Ebert, FT; 1611.08610]
  - This corresponds to canonical logs (level 1)
- Once canonical scales are inserted, the dependence on μ_i, ν_i "disappears"
  - But important to remember that this was a choice
  - Typical CSS implementations are (roughly) equivalent to only retaining μ_H dependence
    (In addition, μ_R is reintroduced by reexpanding $\alpha_s(\mu_H)$ in terms of $\alpha_s(\mu_R)$, which typically leads to violating RGE consistency unless $\mu_H = \mu_R$)

Complete RGE System.

In virtuality scale μ

$$\mu \frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\mu} = \gamma_H(Q,\mu)\, H(Q,\mu)$$

$$\mu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_B(\mu,\nu)\, B(\vec{p}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_S(\mu,\nu)\, S(\vec{p}_T,\mu,\nu)$$

and rapidity scale ν

$$\nu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = -\frac{1}{2}\int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, B(\vec{p}_T-\vec{k}_T,\mu,\nu)$$

$$\nu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, S(\vec{p}_T-\vec{k}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\gamma_\nu(\vec{k}_T,\mu) = \nu \frac{\mathrm{d}}{\mathrm{d}\nu}\gamma_S(\mu,\nu)\delta(\vec{k}_T) = -4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)]\delta(\vec{k}_T)$$

- plus evolution equations for $\alpha_s(\mu)$ and PDFs(μ)
- plus consistency relations between different anomalous dimensions γ_i which encode RGE consistency