



RF Systems

Christine.Vollinger@cern.ch

CAS - Introduction to Accelerator Physics

26 September 2022



Copyright statement and speaker's release for video publishing

The author consents to the photographic, audio and video recording of this lecture at the CERN Accelerator School. The term “lecture” includes any material incorporated therein including but not limited to text, images and references.

The author hereby grants CERN a royalty-free license to use his image and name as well as the recordings mentioned above, in order to post them on the CAS website.

The author hereby confirms that to his best knowledge the content of the lecture does not infringe the copyright, intellectual property or privacy rights of any third party. The author has cited and credited any third-party contribution in accordance with applicable professional standards and legislation in matters of attribution. Nevertheless the material represent entirely standard teaching material known for more than ten years. Naturally some figures will look alike those produced by other teachers.

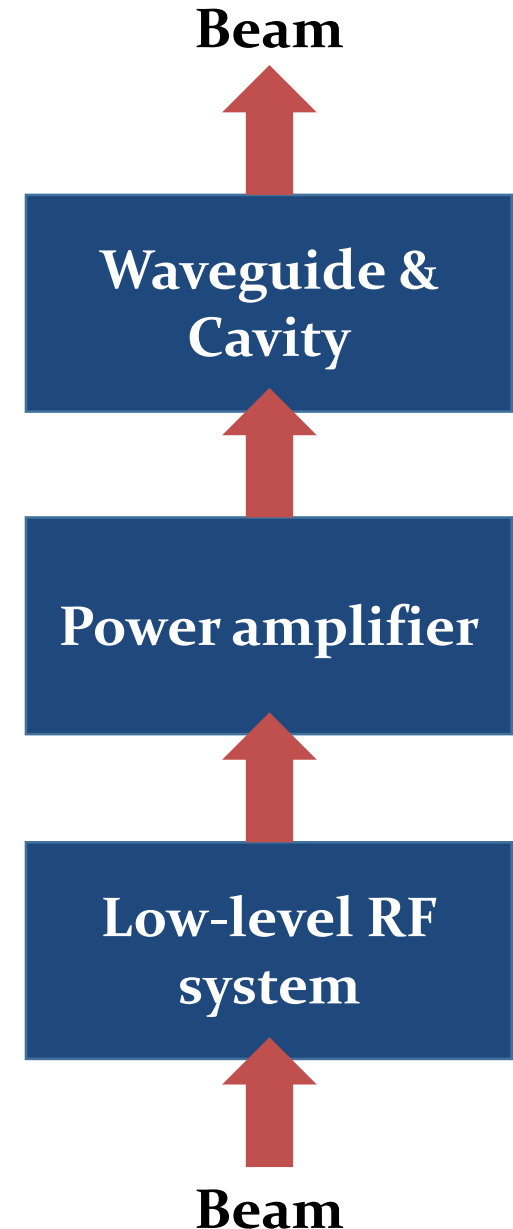
Outline

- **Introduction**
- **Choice of parameters for an RF system**
 - Frequency and voltage
- **RF cavity parameters**
 - Shunt impedance, beam loading, power coupling
- **Power amplifiers**
 - Tube or solid state
- **Summary**

Introduction

Introduction

- The **radiofrequency (RF)** system does the actual **acceleration** - it transforms a string of magnets into an **accelerator**
 - Cavity is the most visible part of an RF system
 - On top of the RF system **food chain**
 - Interacts directly with beam
 - Also RF gymnastics!
- What is below?
- How are RF signals generated which make the beam feel comfortable?



Frequency and wavelength ranges



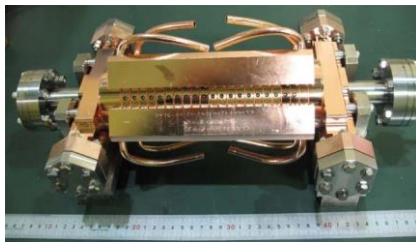
PS longitudinal damper



PS main RF system



SPS 200 MHz



CLIC 12 GHz

100 kHz
3 km

1 MHz
300 m

10 MHz
30 m

100 MHz
3 m

1 GHz
30 cm

10 GHz
3 cm

100 GHz
3 mm

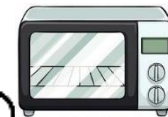


Long wave

**Medium/
short wave**



VHF

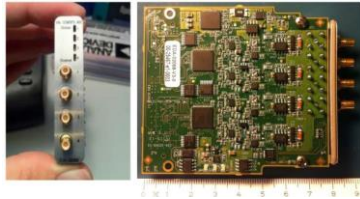


**Microwave
links**



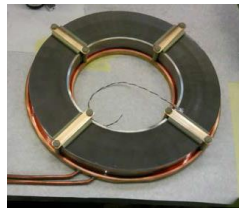
Amplitude ranges

Signals from beam
pick-ups

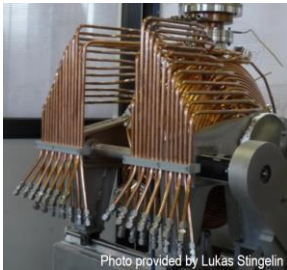


LLRF systems

Low/Medium
energy hadron RF



SLS



LHC: 16 MV



LEP: 3.6 GV total

1 μ V

1 mV

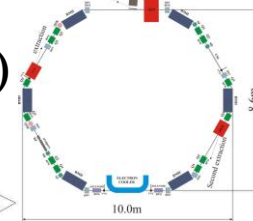
1 V

1 kV

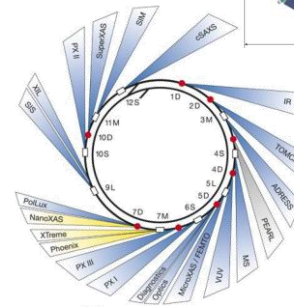
1 MV

1 GV

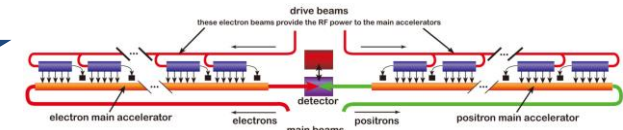
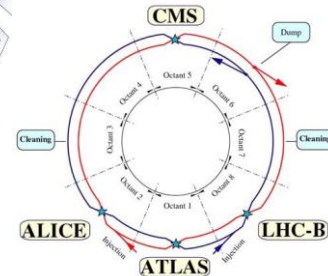
Cooled hadron
beams
(ELENA)



Electron light
sources



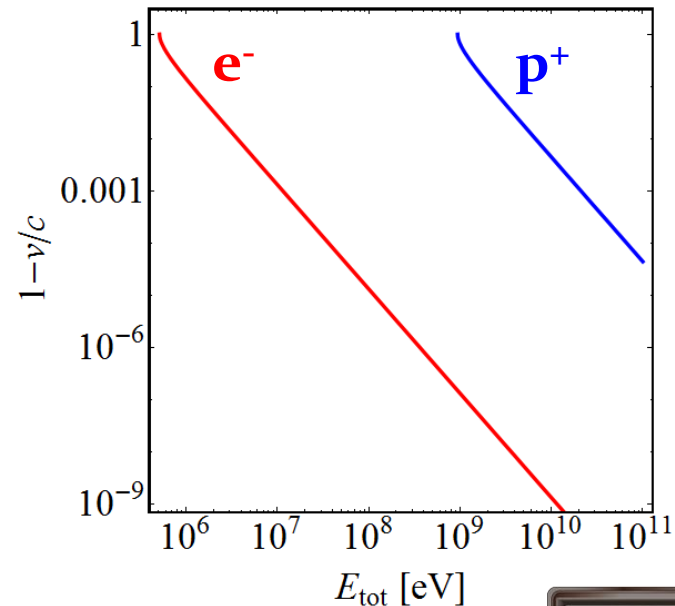
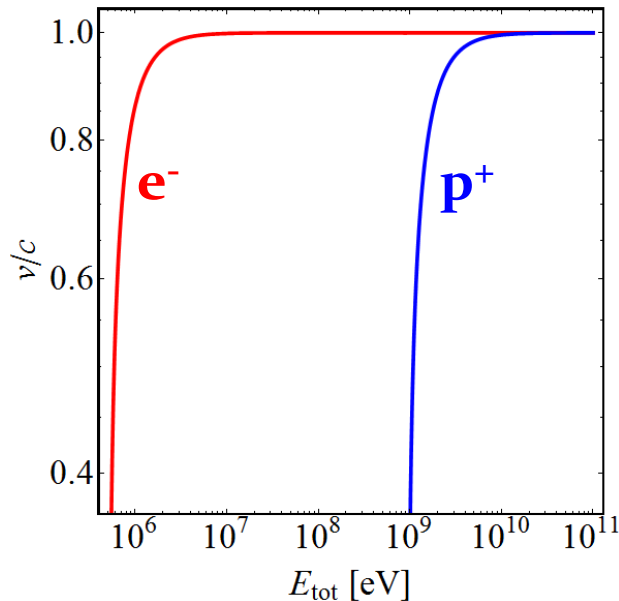
LHC



ILC and CLIC: several TV

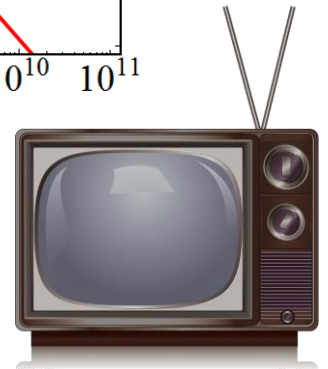
Particle velocity

- Particle velocity depends on its type: $\beta = v/c = \sqrt{1 - (E_0/E)^2}$



- Old television set (30 kV): **Electrons** at 30% of c_0
Protons just at 0.7%
- Small synchrotron (500 MeV): **Electrons** at 99.99995%
Protons at 75.8%

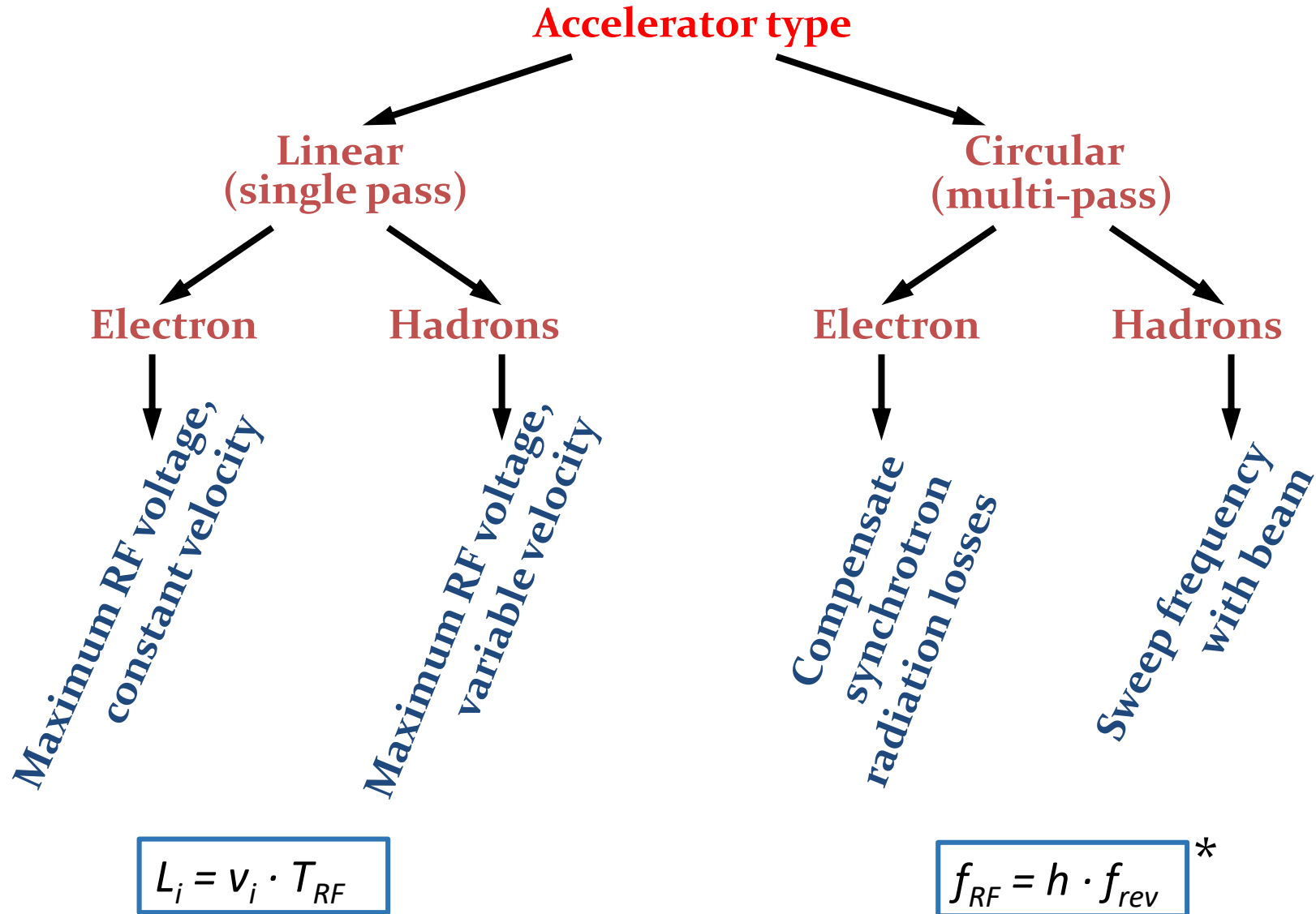
→ Most *electron* accelerators at 'fixed' frequency
(example here: accelerator in Bonn, 1996) →



Parameter choices

RF system for high-energy accelerators

9



L_i = Distance between two adjacent gaps

*Exceptions (rare) exist,
e.g. “fixed frequency” acceleration in SPS

**Choice of frequency (range),
or: shall we try high or low RF
frequency?**

Why choose a **low** RF frequency?


| Advantages | Disadvantages |
|--|---|
| <ul style="list-style-type: none"> • Large beam aperture • Long RF buckets, large acceptance • Wide-band or wide range tunable cavities possible • Power amplification and transmission straightforward | <ul style="list-style-type: none"> • Bulky cavities, size scales $\propto 1/f$, volume $\propto 1/f^3$ • Lossy material to downsize cavities • Moderate or low acceleration gradient • Short particle bunches difficult to generate |

RF frequencies
below ~200 MHz for



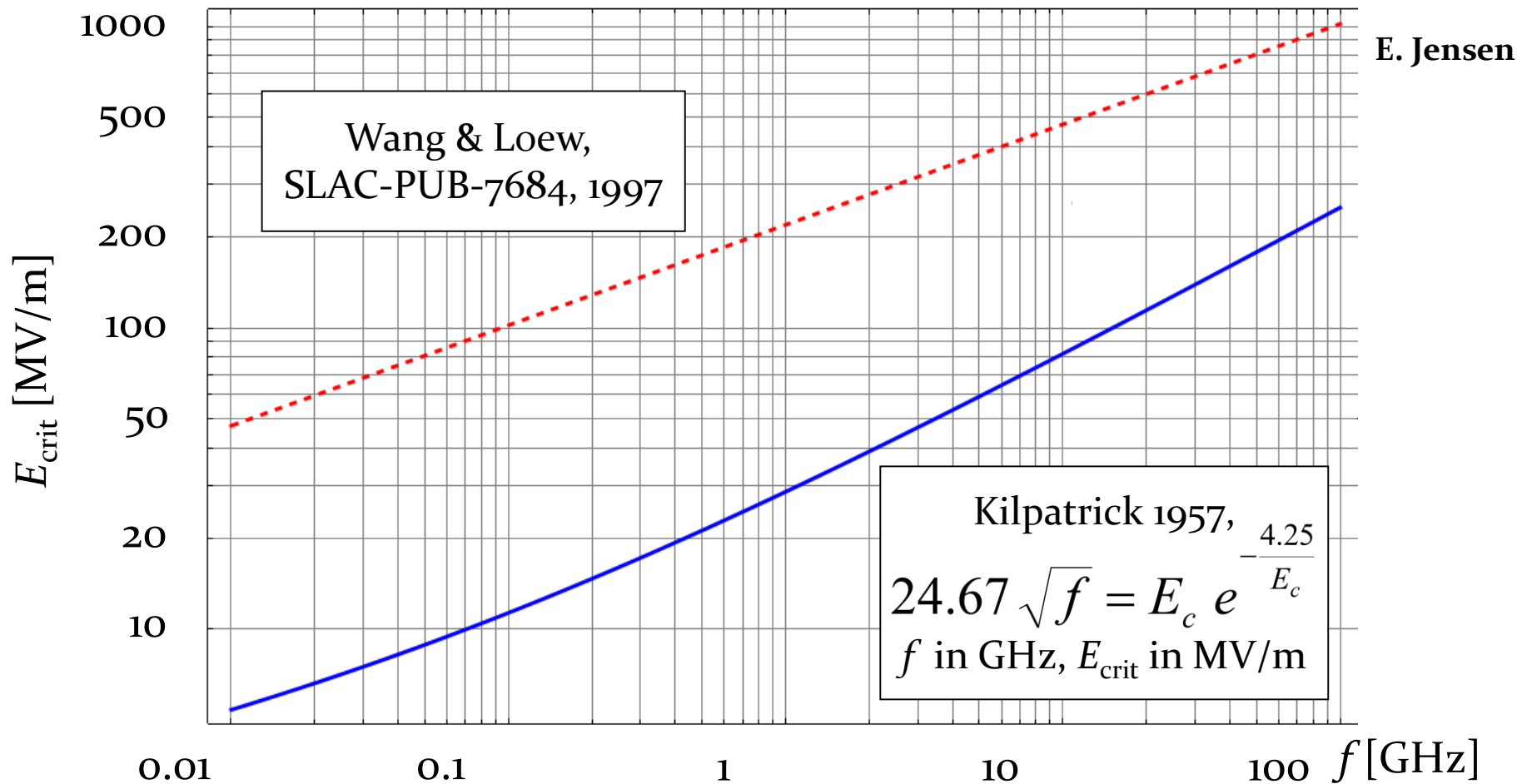
- **Mainly electrons, some hadron linear accelerators**
- **Cyclotrons**
- **Low- and medium energy hadron synchrotrons**

Why choose a **high** RF frequency?

| Advantages | Disadvantages |
|--|--|
| <ul style="list-style-type: none">• Reasonable cavity size scales $\propto 1/f$, volume $\propto 1/f^3$• Break down voltage increases• High gradient per length• Particle bunches are short | <ul style="list-style-type: none">• Maximum beam available aperture scales $\propto 1/f$• No technology for wide-band or tunable cavities• Power amplifiers more difficult• Power transmission losses |
| <p>RF frequencies above ~200 MHz used for</p>  | |
| <ul style="list-style-type: none">→ Linear accelerators→ Electron storage rings→ High energy hadron storage rings | |

Limits to maximum gradient

- Surface electric field in vacuum



→ High frequencies preferred for large gradient.

→ Today also other criteria are used, e.g. considering local field quantity, see: Grudiev et al., PRAB #102001, 2009, [1] & Wuensch [2]

Some standard frequencies = your choice

If exact RF frequency not critical, choose standard value

| Accelerator | Frequency |
|---|-------------------|
| Hadron synchrotrons (PSB, PS, JPARC RCS, MR) | <10 MHz |
| Hadron accelerators and storage rings (RHIC, SPS) | ~200 MHz |
| Electron storage rings (LEP, ESRF, Soleil) | 352 MHz |
| Electron storage rings (DORIS, BESSY, SLS,...) | 499.6...499.8 MHz |
| Superconducting electron linacs and FELs (X-FEL, ILC) | 1300 MHz |
| Normal conducting electron linacs (SLAC) | 2856 MHz |
| High-gradient electron linac (CLIC) | 11.99 GHz |

- Off-the-shelf **RF components easily available** in frequency ranges used by industry.
- **Exchange of developments and equipment** amongst research laboratories.
- Exchange expertise with colleagues from other labs!

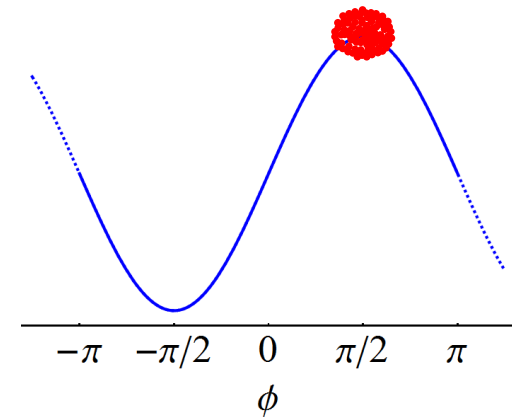
Required RF voltage

Minimum voltage requirement

- RF system expected to provide certain energy gain

$$qV = \Delta E$$

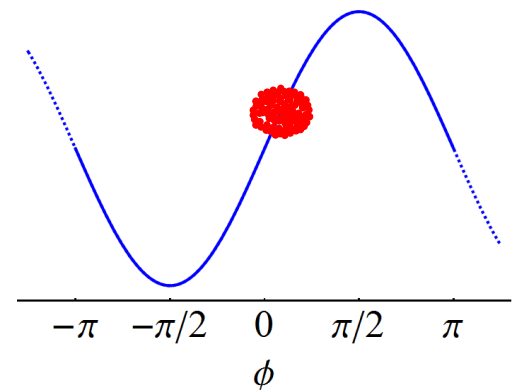
- On-crest acceleration
- Used mostly in linear accelerators
- Insufficient in a circular accelerator, (stable phase condition)



- More voltage provided to avoid on-crest acceleration

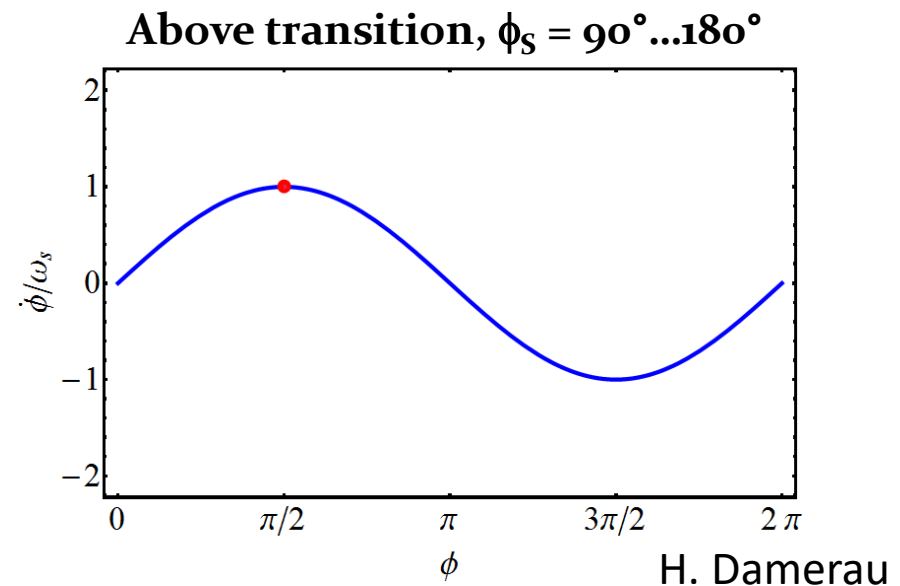
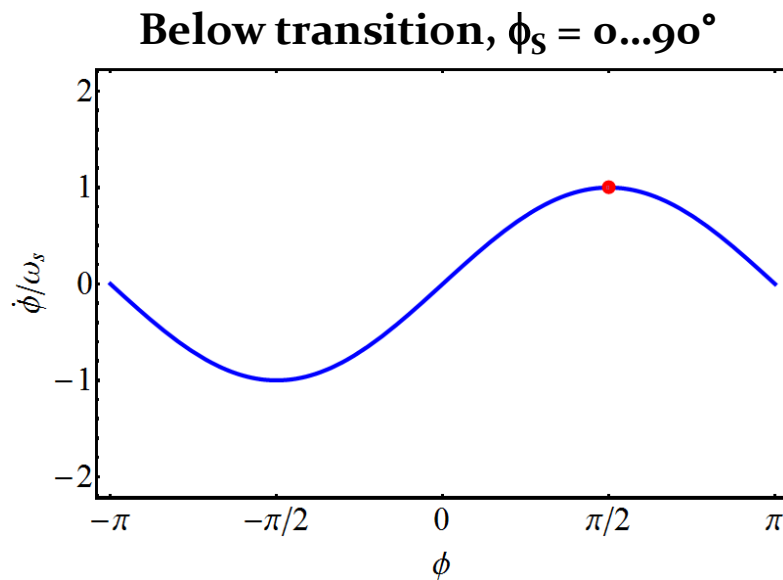
$$qV > \Delta E \rightarrow qV \sin(\phi_S) = \Delta E$$

- Off-crest acceleration
- Needed for circular accelerator
- Higher voltage for given energy gain



Bucket area dependence on stable phase

- In a circular accelerator the area in energy-time phase space (bucket area) depends on the stable phase



- Typical synchronous phase with respect to 0° or 180°
 - Hadron accelerators: $< 40^\circ$
 - Electron storage rings: $\sim 20^\circ$

Minimum voltage requirement (circular)

The RF system must compensate

1. Energy gain per turn due to changing magnetic field



$$F_Z = F_L \rightarrow \frac{p}{q} = \rho B \rightarrow \dot{p} = q\rho\dot{B}$$

$$\dot{p} = \frac{\Delta p}{\Delta t} = \frac{m_0 c^2 \beta}{2\pi R} (\beta \Delta \gamma + \gamma \Delta \beta) = \frac{\Delta E_{\text{turn}}}{2\pi R}$$

$$\Delta E_{\text{turn}} = 2\pi q \rho R \dot{B}$$

The Synchrotron - Frequency change

During the energy ramping, the RF frequency increases to follow the increase of the revolution frequency:

$$\omega = \frac{\omega_{RF}}{h} = \omega(B, R_s)$$

Hence: $\frac{f_{RF}(t)}{h} = \frac{v(t)}{2\pi R_s} = \frac{1}{2\pi R_s} \frac{c\beta}{E(t)} B(t)$ (using $p(t) = cB(t)qR_s$, $E = mc^2$)

Since $E^2 = (m_0 c^2)^2 + p^2 c^2$ the RF frequency must follow the variation of the B field with the law

$$\frac{f_{RF}(t)}{h} = \frac{c}{2\pi R_s} \frac{1}{\sqrt{1 + (m_0 c^2 / (ec\beta))^2}} \frac{B(t)^2}{B(t)^2} \frac{0^{1/2}}{b}$$

RF frequency program during acceleration determined by B-field

This asymptotically tends towards $f_r \rightarrow \frac{c}{2\pi R_s}$ when B becomes large compared to $m_0 c^2 / (ec\beta)$ which corresponds to $v \rightarrow c$

Introductory CAS, Kaunas, September 2022 22

Recall talk of F. Tecker

2. Energy loss, e.g., due to synchrotron radiation (electrons)

$$\Delta E_{\text{turn}} = \frac{e^2}{3\epsilon_0 (m_0 c^2)^4} \frac{E^4}{\rho}$$

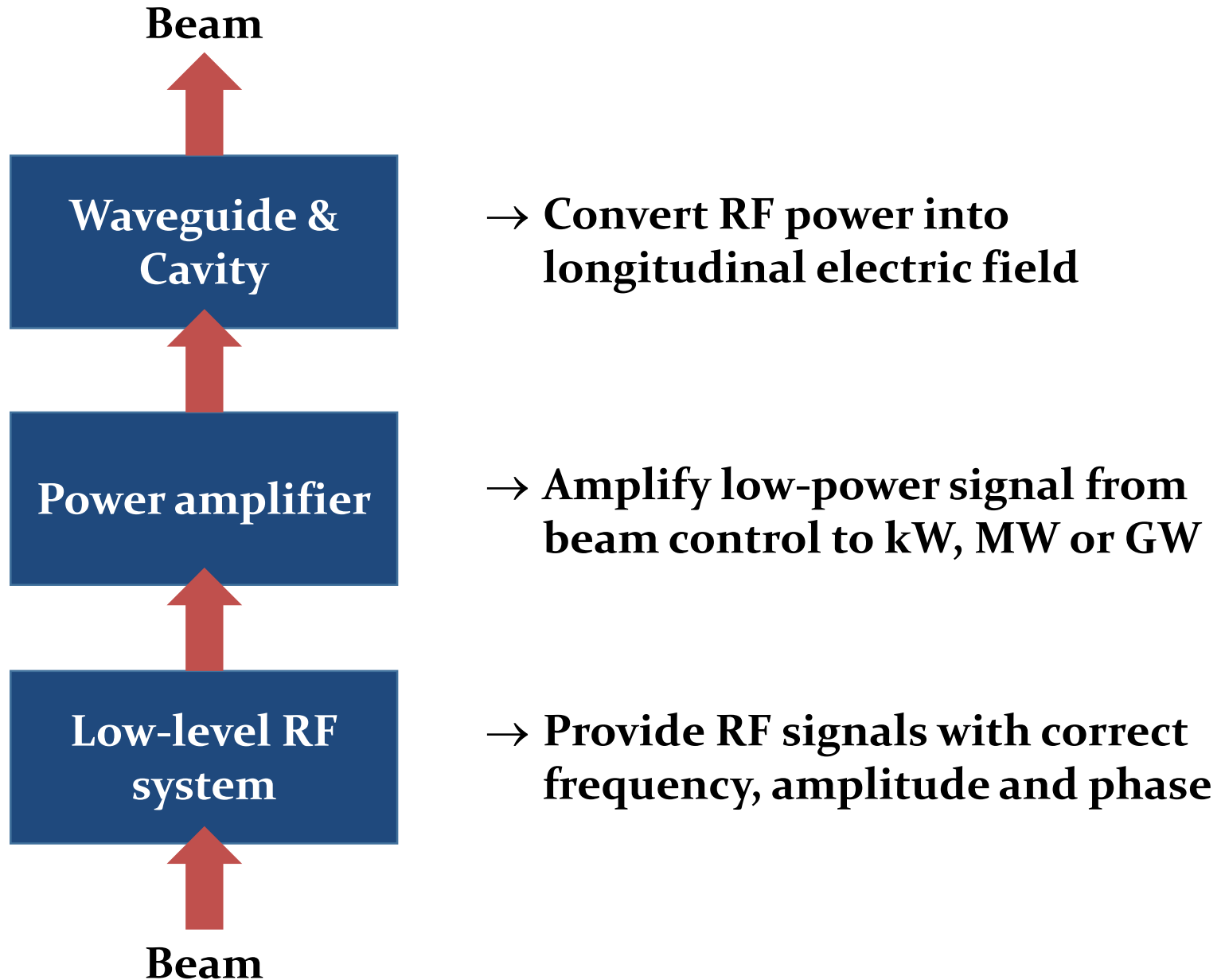
Particle energy

Recall talk of L. Rivkin

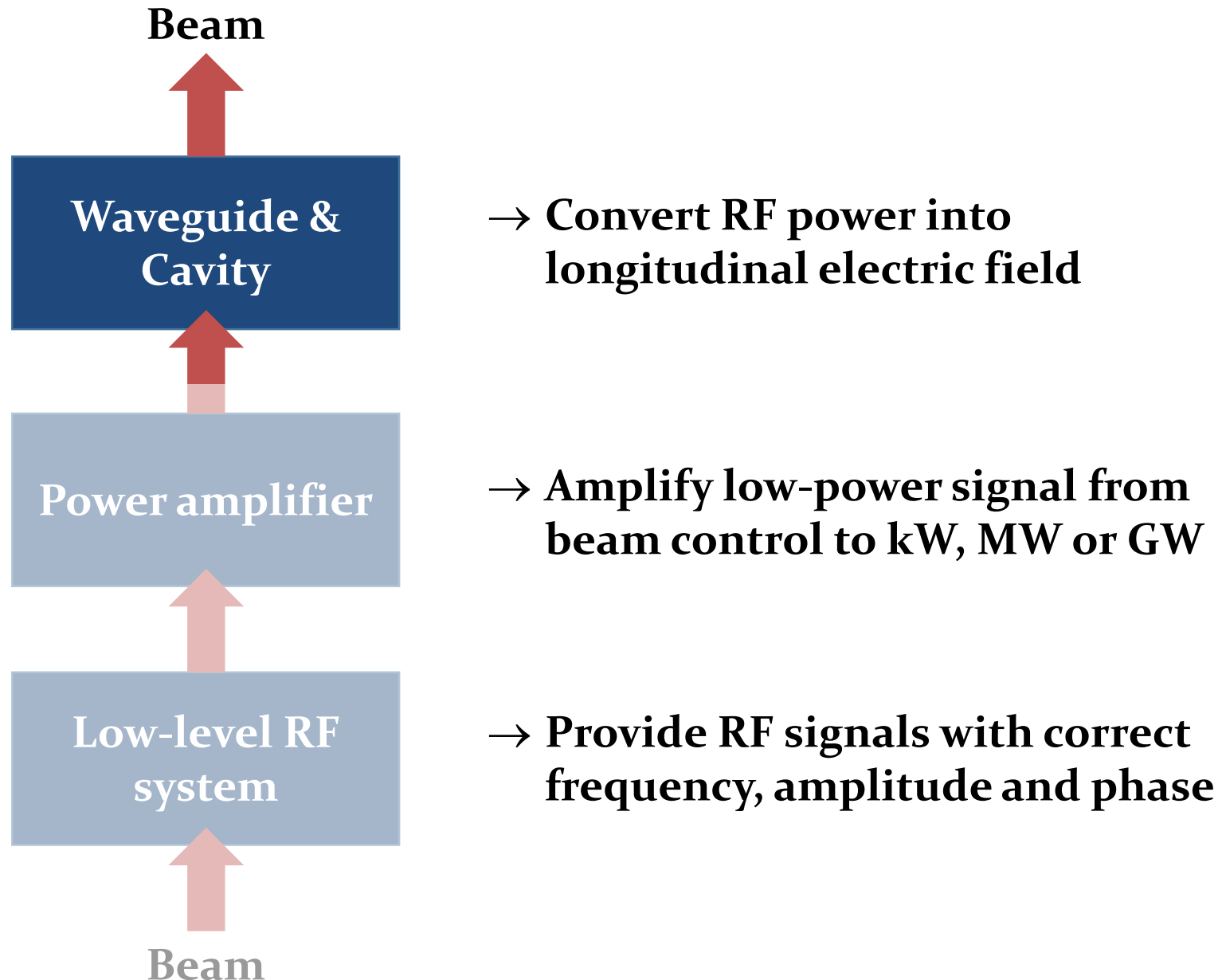
$$\Delta E_{\text{turn}} [\text{keV}] = 88.5 \cdot \frac{E^4 [\text{GeV}]^4}{\rho [\text{m}]} \quad \Delta P_{\text{loss}} [\text{kW}] = 88.5 \cdot \frac{E^4 [\text{GeV}]^4}{\rho [\text{m}]} \cdot I_B [\text{A}]$$

$\rightarrow (m_p/m_e)^4 = 1836^4 \sim 1.1 \cdot 10^{13}$ times less for protons

RF system overview



RF system overview



RF components: Waveguide and Cavity

Waveguides

Rectangular waveguide



Circular waveguide



Arbitrarily shaped cross-section waveguide



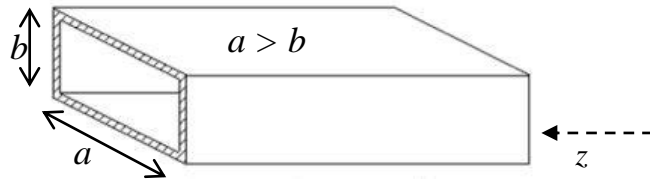
Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

- Waveguides are hollow metallic tubes with uniform cross-sections of different shapes.
- The metallic waveguide is a completely enclosed system without any radiation loss.
- Waveguides present a one-conductor system, so *no TEM-mode propagation* is possible. Propagation happens in TE-mode (*Transverse-Electric*) and TM-mode (*Transverse-Magnetic*). These modes have a lower frequency limit, the so-called cut-off frequency below which propagation is not possible. The cut-off frequency depends on the dimensions of the waveguide cross-section.



- Waves are following the waveguide shape, even if it is bend.
- Propagating waves need to fulfill boundary conditions on the waveguide walls.

Rectangular Waveguide



Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, 2nd ed., Springer

$|\vec{k}| = \frac{2\pi}{\lambda} = \frac{\omega}{c}$
 $\lambda = \frac{c}{f}$
 f
 $\omega = 2\pi f$

wave-number vector

wave length

frequency

angular frequency

$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$ and $\mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$

WAVE FUNCTION

\mathbf{k} – the wave-number vector with $|\mathbf{k}| = k$, which gives the direction of propagation of the wave.
 ω is more properly called the angular frequency (f – frequency)

$\omega^2 = c^2 k^2$
↔
 dispersion relation
 →
 $c = \frac{\omega}{|\mathbf{k}|} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$\mathbf{E}_0, \mathbf{B}_0$ – constant vectors, the amplitude of the wave

$\lambda = 2\pi/k$ – the wavelength of the wave

Short wave length → high frequency → high energy

-67-

- The rectangular waveguide is characterized by a propagation constant, attenuation constant and characteristic impedance.

TE-modes

- TE-modes are characterized by a zero *electric* field in propagation direction. The electric field is in the transverse plane, only.
- Modes have a cut-off frequency (no propagation below this frequency), identical for TE or TM:

TM-modes

- TM-modes are characterized by a zero *magnetic* field in propagation direction. The magnetic field is in the transverse plane, only.

$$\omega_{c,mn} = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

The mode with the *lowest cut-off frequency* is called *the dominant mode*.

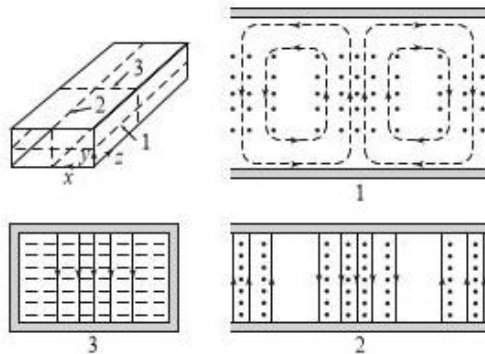
Hybrid mode happens when TE- and TM-modes start to couple.

A waveguide is called *overmoded*, if more than one mode is propagating in the waveguide at the same time.

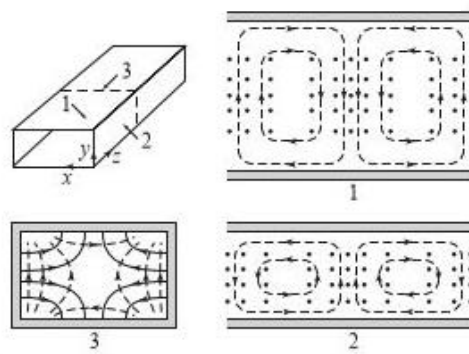
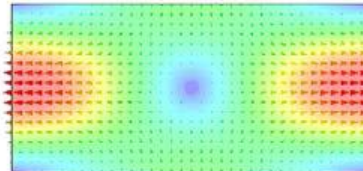
Rectangular Waveguide

Fundamental and higher waveguide modes and field patterns for TE-modes

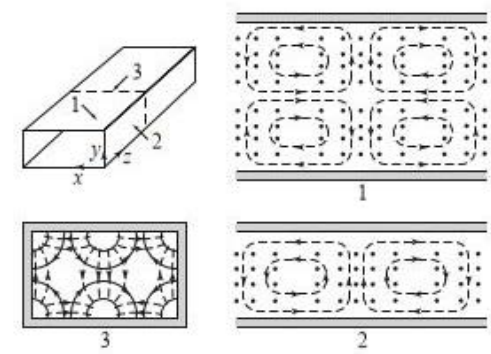
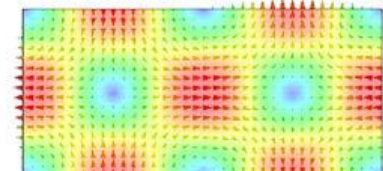
TE₁₀-mode



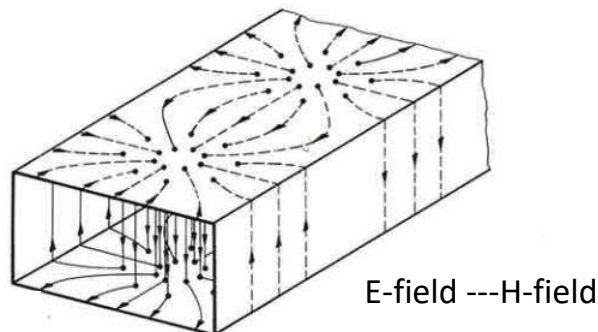
TE₁₁-mode



TE₂₁-mode



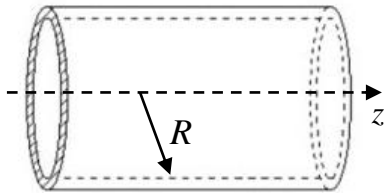
Simulation pictures: courtesy E. Jensen, field pattern source: Pozar, *Microwave engineering*, 4th ed., Wiley



E-field ---H-field

Circular Waveguide

- We know the cut-off frequencies for the different modes:

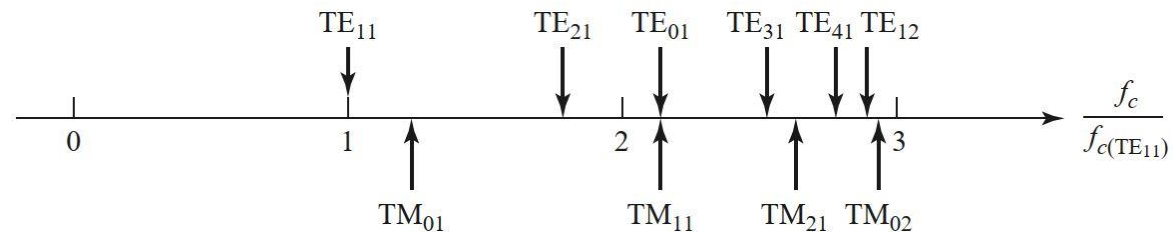


$$f_{c,nm} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left(\frac{\overset{\text{TE-mode}}{p'_{nm}} \text{ or } \overset{\text{TM-mode}}{p_{nm}}}{R} \right)$$

p_{nm} → Roots of the Bessel-function for TM-mode $J_n(p_{nm}) = 0$

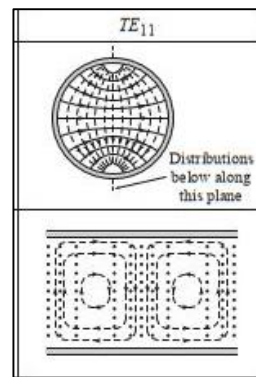
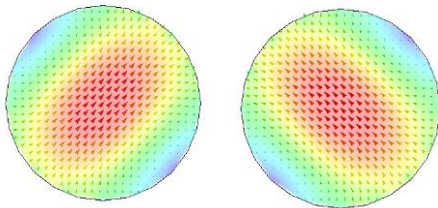
p'_{nm} → Roots of the derivative of the B.F. for TE-mode $J'_n(p'_{nm}) = 0$

Chart of cut-off frequencies relative to f_c of TE₁₁-mode:

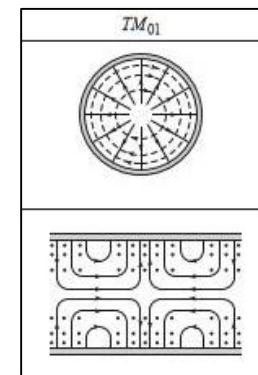
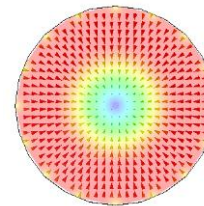


Source: Pozar, *Microwave engineering*, 4th ed., Wiley

1st mode is TE₁₁. Electric field is transverse. Mode has 2 polarisations (orientations of the electrical field).



- 2nd mode is TM₀₁. Magnetic field is transverse.



Pillbox Cavity Resonator

A cylindrical cavity resonator can be seen as a section of circular waveguide shorted at both ends.

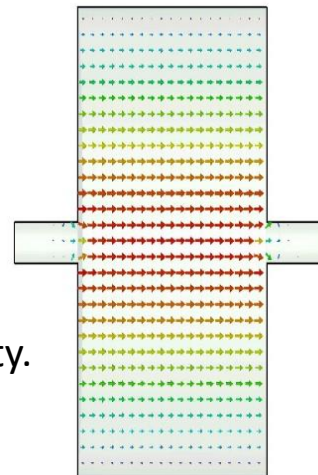
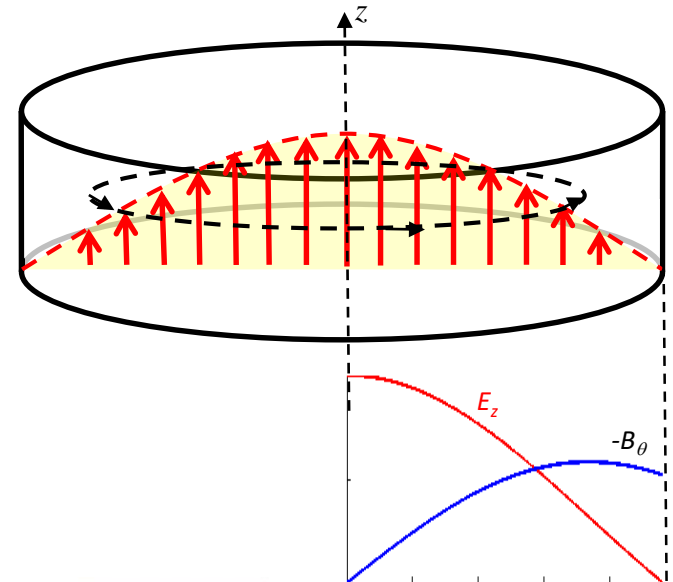
From the TE_{11} -mode (dominant mode for circular waveguide)

→ TE_{111} -mode is the dominant TE-mode for a cylindrical cavity.

→ TM_{010} -mode is the dominant TM-mode for a cylindrical cavity.

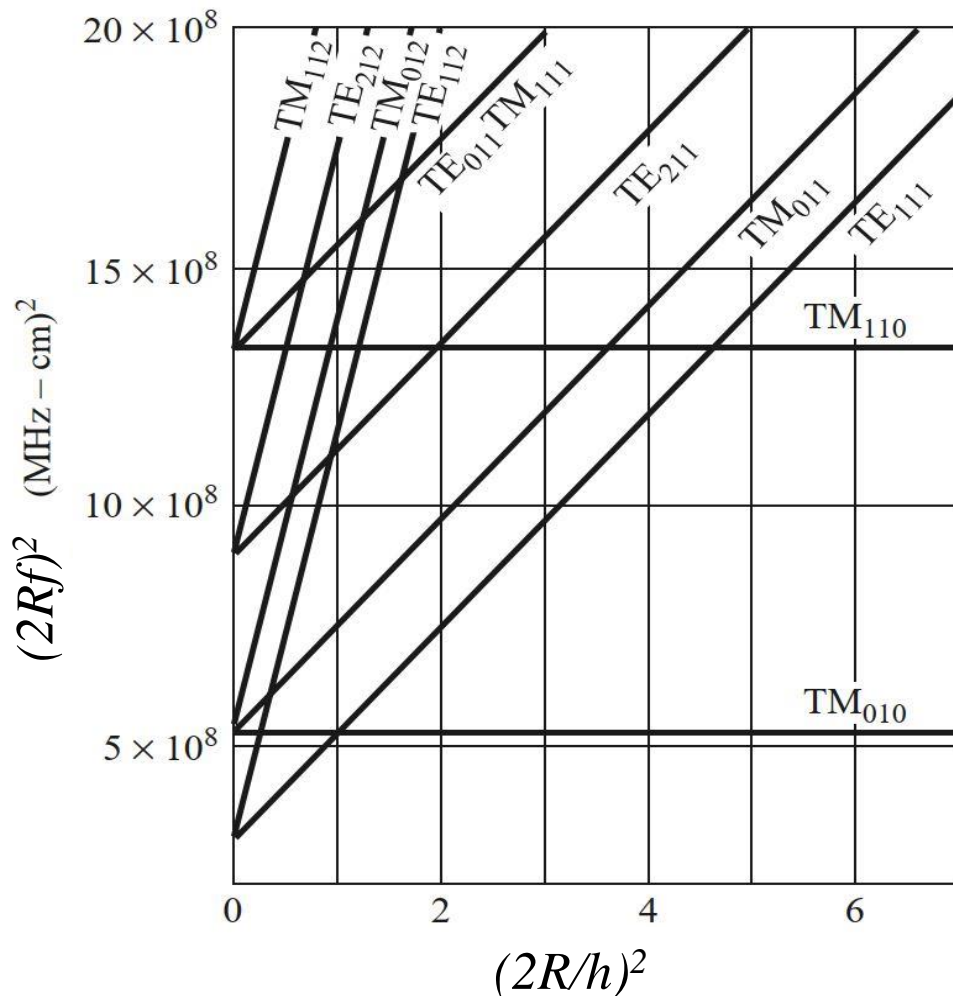
→ TM_{010} -mode mode is used for acceleration as it has a large electric field along the z-axis.

→ Electric field is described by Besselfunction J_0 .



Pillbox cavity is flat compared to a “long” cylindrical cavity. Also, beam-pipe connection is needed.

Pillbox Cavity Resonator



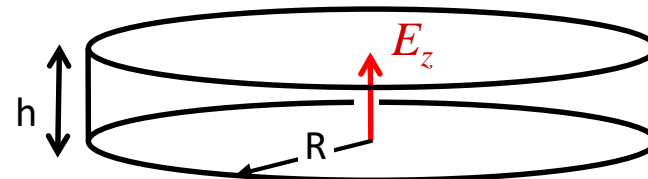
For the case of the TM_{010} -mode, we have no dependence on cavity height, so we get a very simple formulae:

$$0.383 \lambda_{TM,010} = R$$

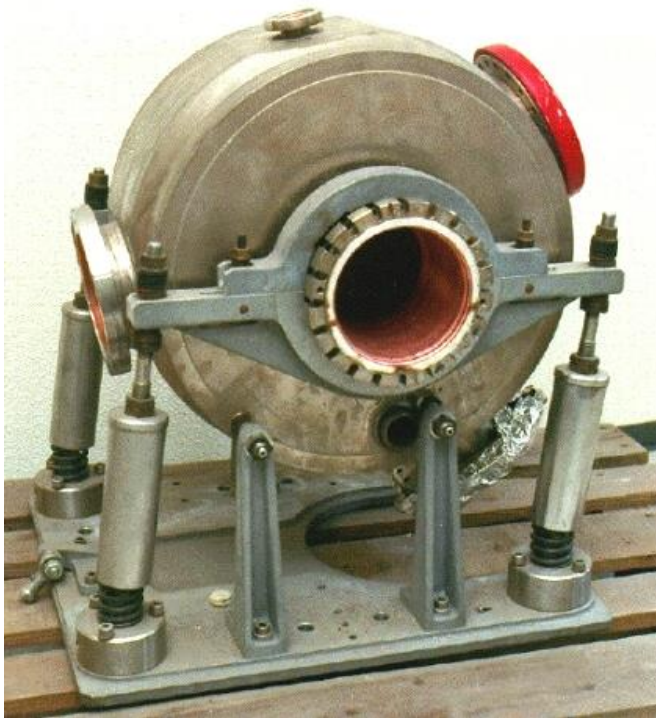
Resonant chart for a general cylindrical cavity shows the excited modes as a function of cavity dimensions.

→ 1st TE-mode is TE_{111}

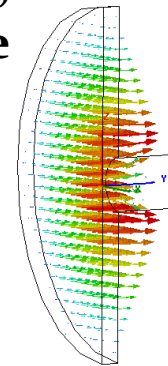
→ 1st TM-mode is TM_{010} , and shows up for ratios $(2R/h)^2 > 1$



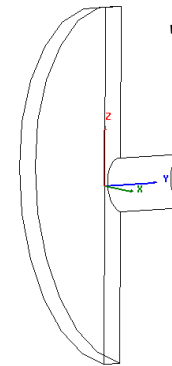
Example “true” Pillbox Cavity



Electric field,
 TM_{010} -mode



Magnetic field,
 TM_{010} -mode



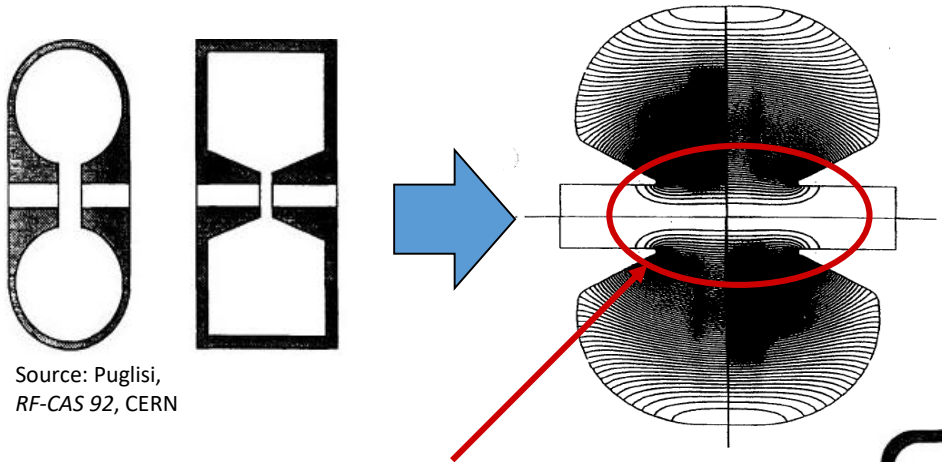
E. Jensen

Cavity from DORIS Storage ring
(1970-ish, very early
electron/positron collider)

Pillbox Cavity Design Feature

In practice, a “pure” pillbox cavity is not very efficient for acceleration. A simple shape modification can be done by using so-called “nose cones”.

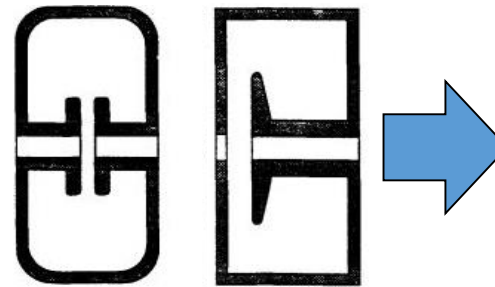
Nose cone is a protrusion on the cavity wall that causes a concentration of the electrical field in the gap.



Source: Puglisi,
RF-CAS 92, CERN

Enhancement of E-field

Nose cones also help to improve the Transit-time factor...

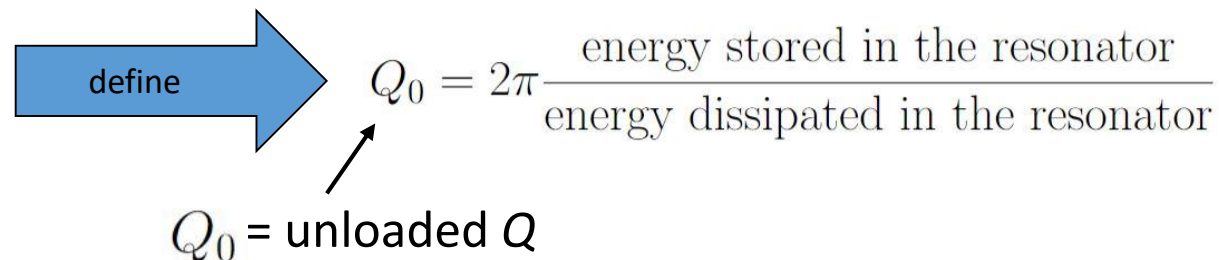


Source: Puglisi,
RF-CAS 92, CERN

PS 80 MHz cavity

Cavity Resonators and Q-value (1/2)

- Resonators are classified by their quality factor Q .
- The quality factor (or Q -value) can be used as a measure of “how well the cavity is resonating” (more details will come).


$$Q_0 = 2\pi \frac{\text{energy stored in the resonator}}{\text{energy dissipated in the resonator}}$$

$Q_0 = \text{unloaded } Q$

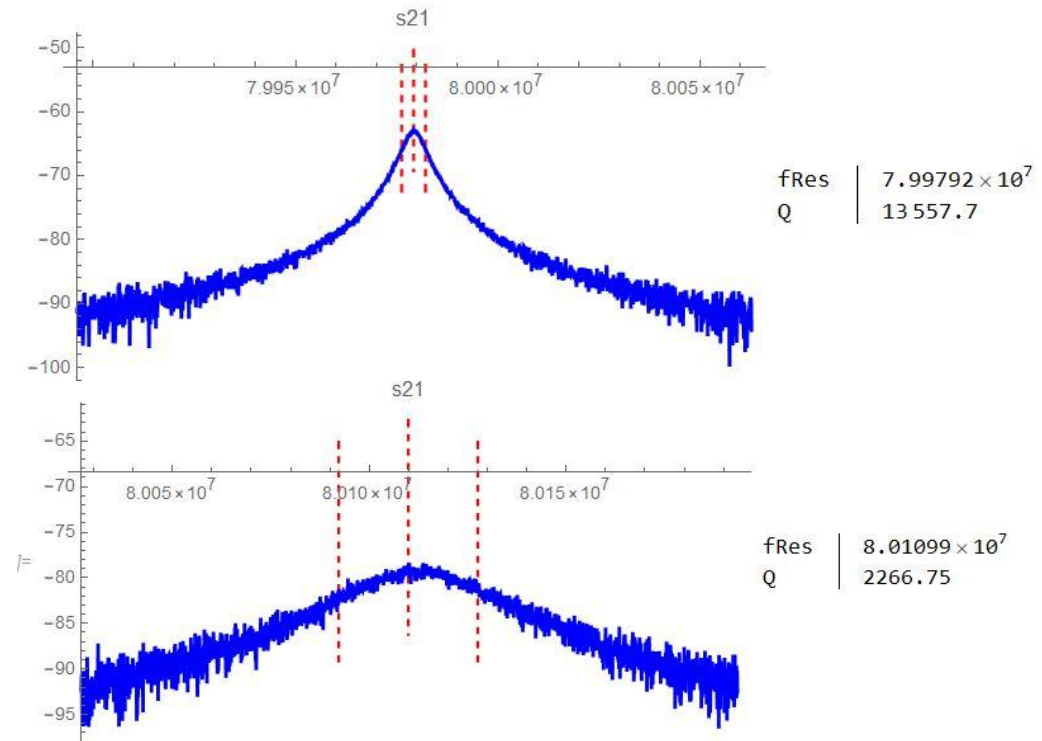
- High Q -value is desired in accelerating cavities; Q -value is one of the accelerator efficiency figures-of-merit.
- Q -value reduces e.g. due to the power dissipated in the metallic walls or other loss mechanisms.
- The connection of the cavity resonator to the outer world will reduce its Q -value as well (*we say “it loads” the cavity with an additional loss mechanism*).

Cavity Resonators and Q-value (2/2)

31

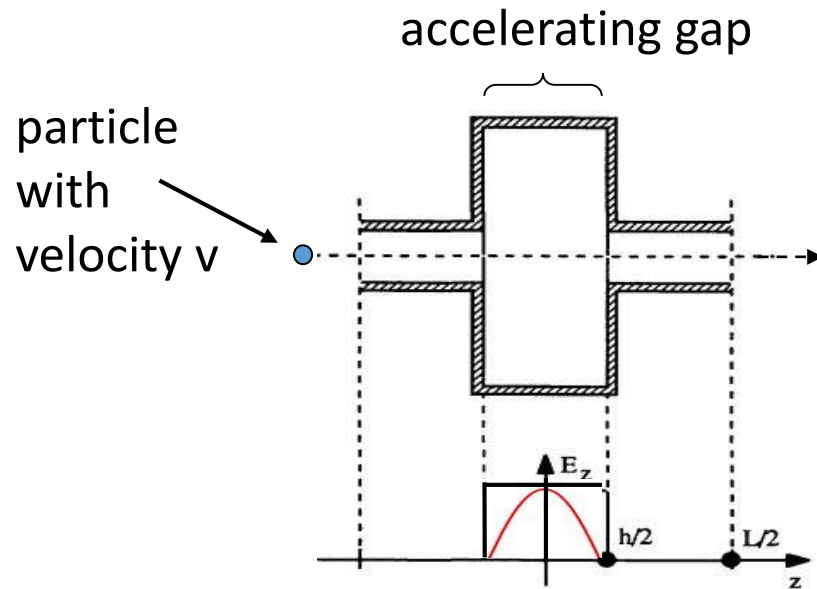


Example: measurements taken on the PS 80MHz pillbox cavity



- We tested this cavity for Q -deterioration and shifting of its fundamental mode (~ 80 MHz).
- Q -values obtained from 3-dB-measurement (see dashed red lines).
→ Plot on the top has a higher Q -value than the plot below.

Transit Time Factor (1/2)



transit-time factor:

$$T = \frac{\text{energy gained in time-varying RF-field}}{\text{energy gained in a DC field of voltage } V_0}$$

$$T = \frac{\int_{-L/2}^{L/2} E(0, z) \cos(2\pi z / \beta \lambda) dz}{\int_{-L/2}^{L/2} E(0, z) dz}$$

- Particle in the harmonic time-varying field will see less energy gain compared to a constant DC field.
- This is called transit-time effect and is described by a factor T (so-called transit-time factor).

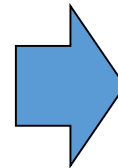
Distance the particle travelled in one RF-period

Transit Time Factor (2/2)

$$T = \frac{\int_{-L/2}^{L/2} E(0, z) \cos(2\pi z / \beta\lambda) dz}{\int_{-L/2}^{L/2} E(0, z) dz}$$

Transit time factor

$E(0, z) dz = \text{constant}$
(as it is for an ideal pillbox cavity!)



gap length

$$T = \frac{\sin \pi g / \beta\lambda}{\pi g / \beta\lambda}$$

- To achieve max. energy gain from this formulae, we want that $T = 1 \rightarrow g = 0$
- Leads to design request: *gap as small as possible*.
- But other considerations as: risk of RF electric breakdown also impact on optimum gap geometry.
- Note that it is assumed that the particle does not change velocity along the gap length.

Accelerator efficiency figures-of-merit (1/2)

Several figures-of-merit are commonly used to characterize accelerating cavities:

- *Q-value*

unloaded Q_0 - measure of the resonance quality

$$Q_0 = \frac{\omega U}{P}$$

energy stored in the resonator

energy dissipated in the resonator

- *Shunt impedance $[M\Omega]$*

measure of effectiveness to produce an axial voltage V_0

$$r_s = \frac{V_0^2}{P}$$

Design goal is a high shunt impedance

- *Effective Shunt impedance $[M\Omega/m]$*

Measure of effectiveness per unit power loss to deliver energy to a particle

$$r_{s,\text{eff}} = \frac{(V_0 T)^2}{P} = r_s T^2$$

- *"R-over-Q" $[\Omega]$*

Measure of cavity acceleration efficiency at a given frequency – geometry dependent only!

$$r/Q = \frac{(V_0 T)^2}{\omega U}$$

Accelerator efficiency figures-of-merit (2/2)

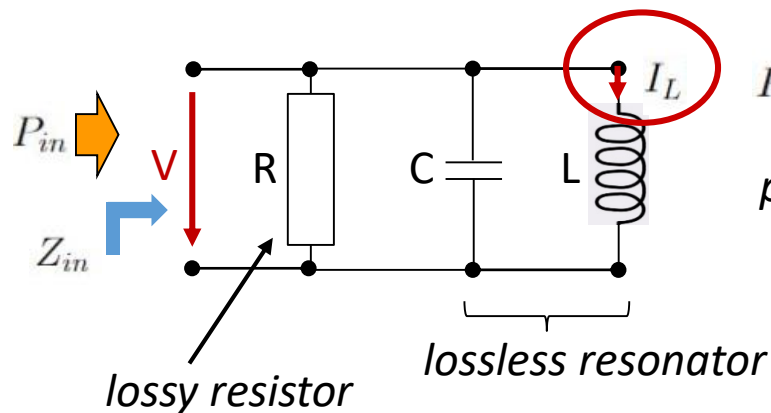
Typical values for different cavities:

| Cavity type | R/Q | Q_0 | R |
|---|--------------|---------------|-----------------|
| Ferrite loaded cavity (low frequency, rapid cycling) | 4 k Ω | 50 | 200 k Ω |
| Room temperature copper cavity (type 1 with nose cone) | 192 Ω | $30 * 10^3$ | 5.75 M Ω |
| Superconducting cavity (type 2 with large iris) | 50 Ω | $1 * 10^{10}$ | 500 G Ω |

Cavity equivalent circuit (1/4)

At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

For a cavity with the desired high shunt impedance, only a parallel resonant circuit is suited → require to model a large voltage.



$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} |V|^2 \frac{1}{Z_{in}^*}$$

power to the resonator
circuit

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1}$$

input impedance

$$P_{in} = |V|^2 \left(\frac{1}{R} + j\frac{1}{\omega L} - j\omega C \right)$$

Power dissipated in the resistor: $P_{loss} = \frac{1}{2} \frac{|V|^2}{R}$

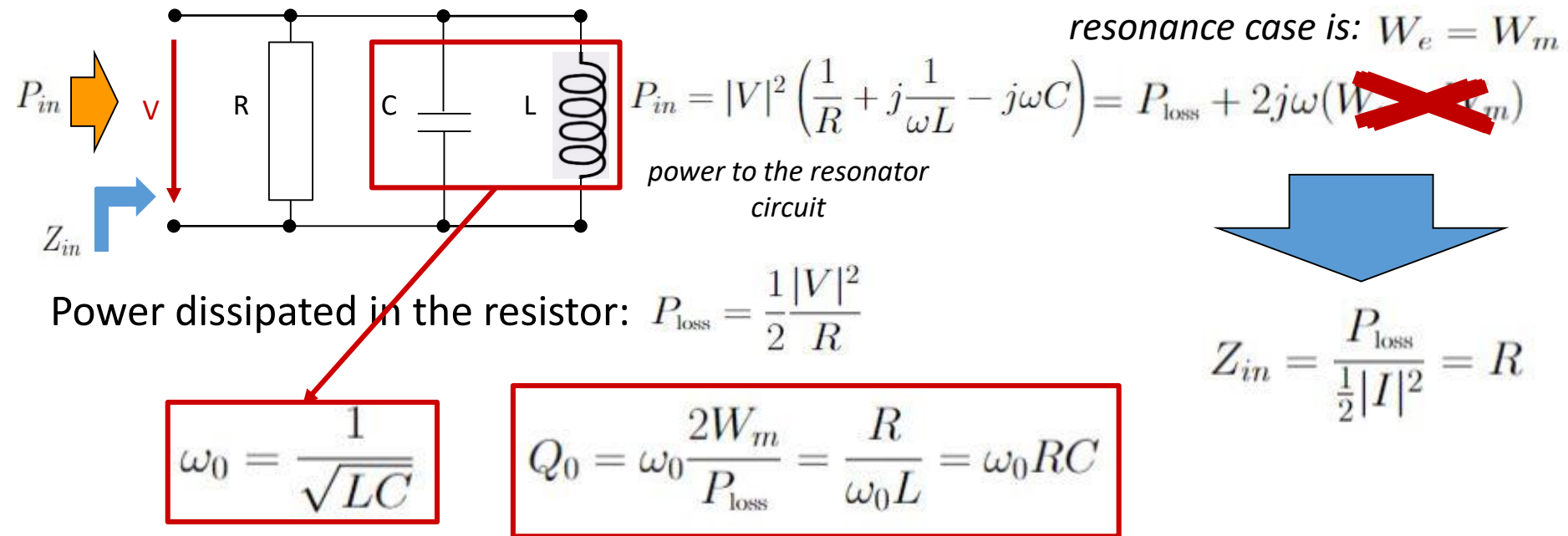
Energy stored in capacitor: $W_e = \frac{1}{4} |V|^2 C$

Energy stored in inductor: $W_m = \frac{1}{4} |I_L|^2 L = \frac{1}{4} |V|^2 \frac{1}{\omega^2 L}$

Cavity equivalent circuit (2/4)

At frequency near resonance, the cavity resonator can be modeled by a lumped-element circuit.

For a cavity with the desired high shunt impedance, only a parallel resonant circuit is suited → require to model a large voltage.

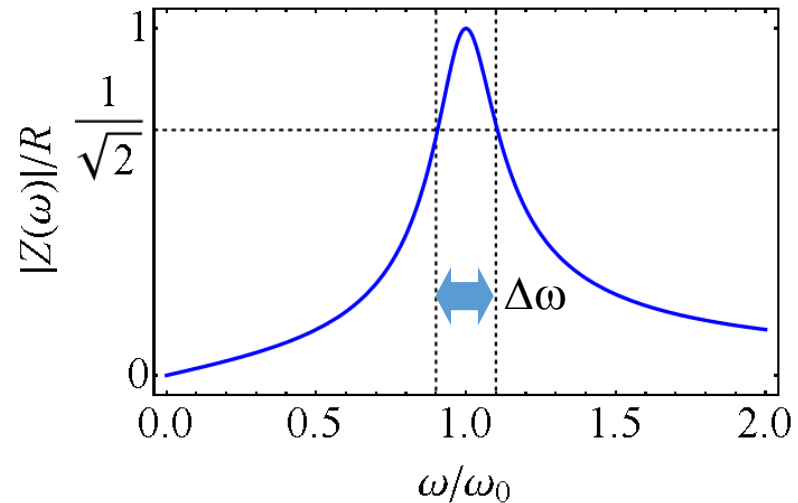
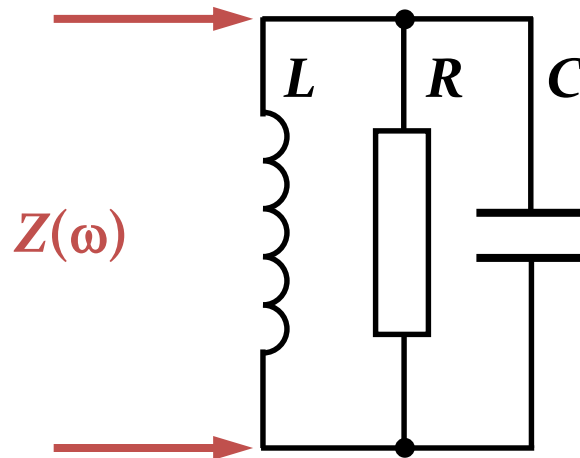


3 Parameters are sufficient to describe the cavity

$$Z(\omega) = \frac{R}{1 + iQ \left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0} \right)} \simeq \frac{R}{1 + 2iQ \frac{\Delta\omega}{\omega_0}}$$

Cavity equivalent circuit (3/4)

- The resonance of a cavity can be understood as simple parallel resonant circuit described by R, L, C

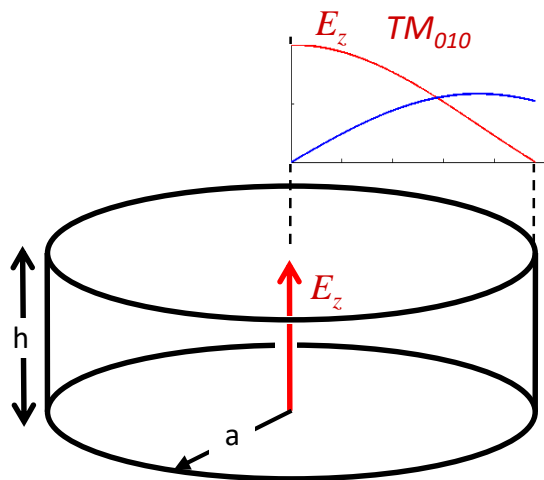


$$Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC$$

$$Z(\omega) = \frac{R}{1 + iQ \left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0} \right)} \simeq \frac{R}{1 + 2iQ \frac{\Delta\omega}{\omega_0}}$$

→ Resonant circuit can also be described by $R, R/Q, \omega_0$ or any other set of three parameters – and this is what cavity designers use.

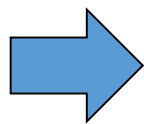
Pillbox Cavity Resonators



For the case of the TM_{010} -mode in a pillbox, where we have no dependence on cavity height, we get a very simple formulae:

$$0.383 \lambda_{TM,010} = a$$

$$Q = \left(0.383 \frac{\lambda_{TM,010}}{\delta} \right) \left[1 + \left(0.383 \frac{\lambda_{TM,010}}{h} \right) \right]^{-1}$$



$$Q = \frac{0.383 \lambda_{TM,010}}{\delta} \left[1 + \frac{a}{h} \right]^{-1} = \frac{q}{\delta} \left[1 + \frac{a}{h} \right]^{-1}$$

Gives an idea of expected Q for a normal-conducting, single-gap cavity.

Remember?
skindepth is:

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}}$$

frequency
conductivity
permeability

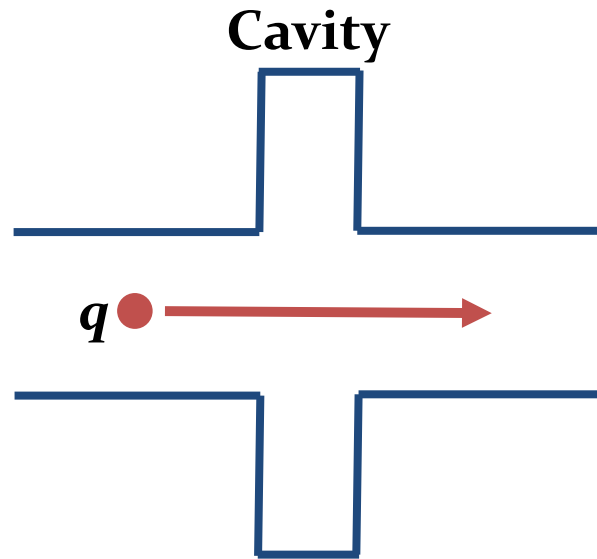
Cavity equivalent circuit (4/4) - Summary

- Most common choice by cavity designers ω_o , R , R/Q – why?
- **Resonance frequency, ω_o**
 - Exactly defined for given application, e.g.,: $h \cdot f_{\text{rev}}$
- **Shunt impedance, r_{sh} or R**
 - High shunt impedance required to produce a given voltage **without beam**.
- **“R-upon-Q”, R/Q**
 - Defined only by the cavity geometry
 - Criterion to optimize a geometry
 - Detuning with beam proportional to R/Q

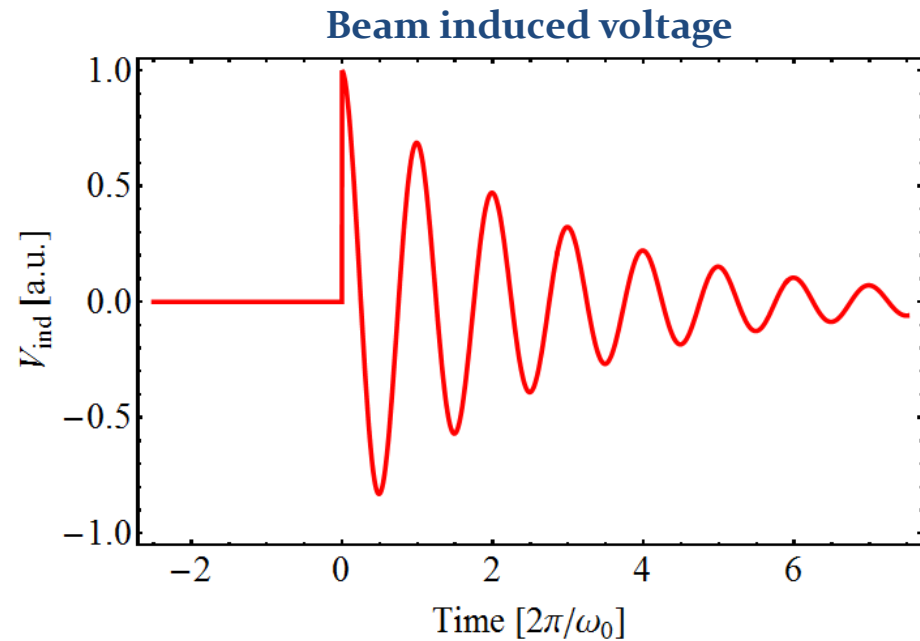


What about this R/Q-criteria?

→ Charged particle experiences cavity gap as capacitor



$$q = V_{\text{ind}} C$$



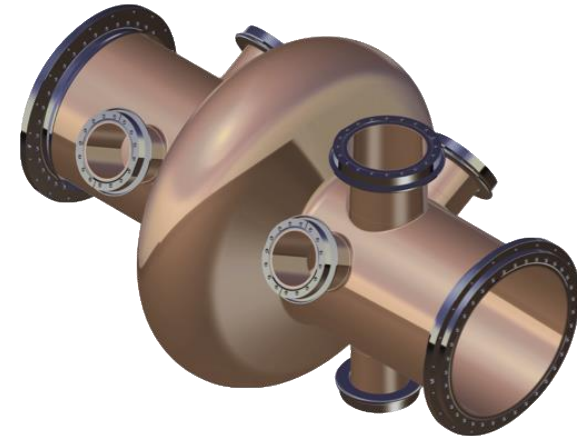
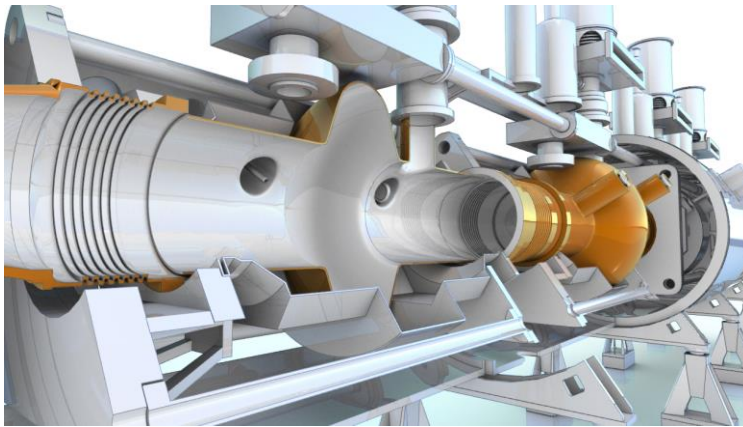
$$Q = \omega_0 RC \quad \rightarrow \quad \frac{1}{C} = \left(\frac{R}{Q} \right) \omega_0$$

$$V_{\text{ind}} = \frac{q}{C} \propto \frac{R}{Q}$$

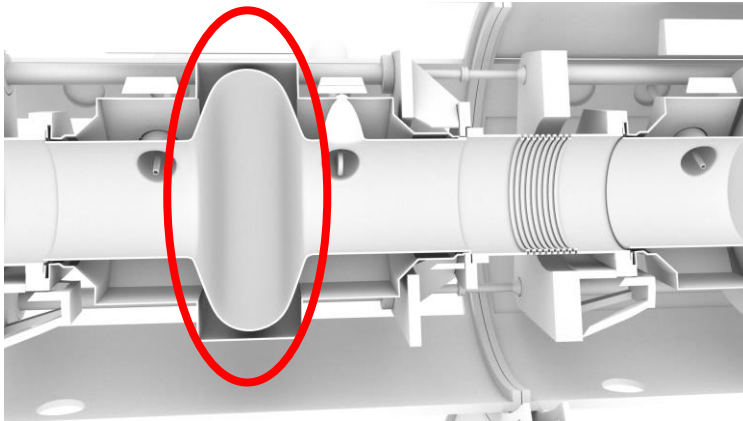
→ Goal: cavity geometry with small R/Q to reduce beam loading

Example: 400 MHz cavities in LHC

- Very high beam currents, need to reduce beam loading in RF cavities
- Shunt impedance, R is *high* but small R/Q is needed
- Possible with superconducting cavities in LHC



Bell shape: $R/Q \sim 44 \Omega$, 400 MHz



→ 2×8 cavities, 5.3 MV/m

$$\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}$$

~0

What is Cavity loading...

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

resonance frequency

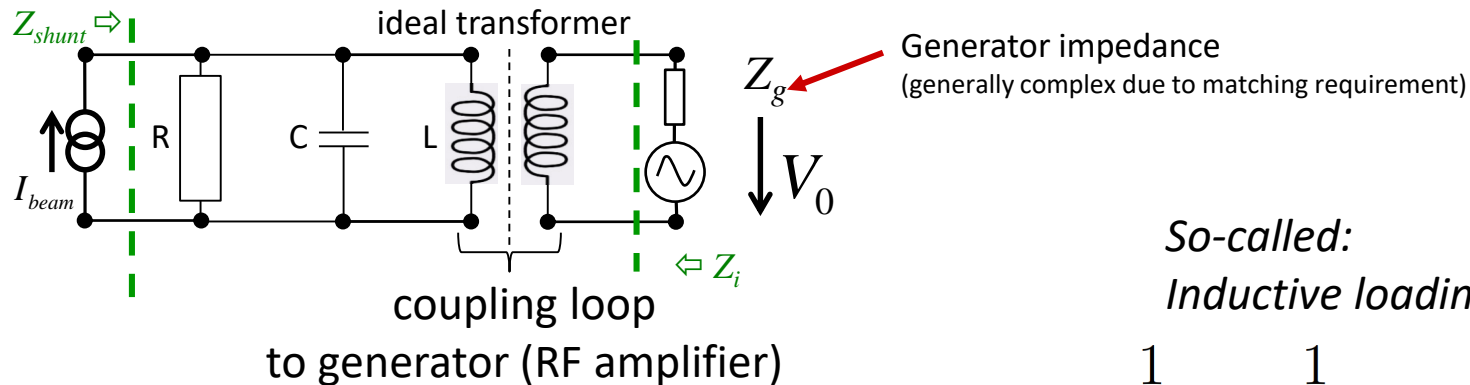
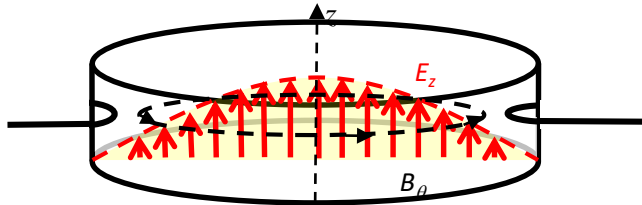
$$Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \frac{R}{\omega_0 L} = \omega_0 RC$$

unloaded Q

$$C_{\text{par}} = \frac{Q_0}{\omega_0 r_{\text{shunt}}}$$

$$L_{\text{par}} = \frac{r_{\text{shunt}}}{\omega_0 Q_0}$$

*parallel capacitance
and inductance*



*So-called:
Inductive loading*

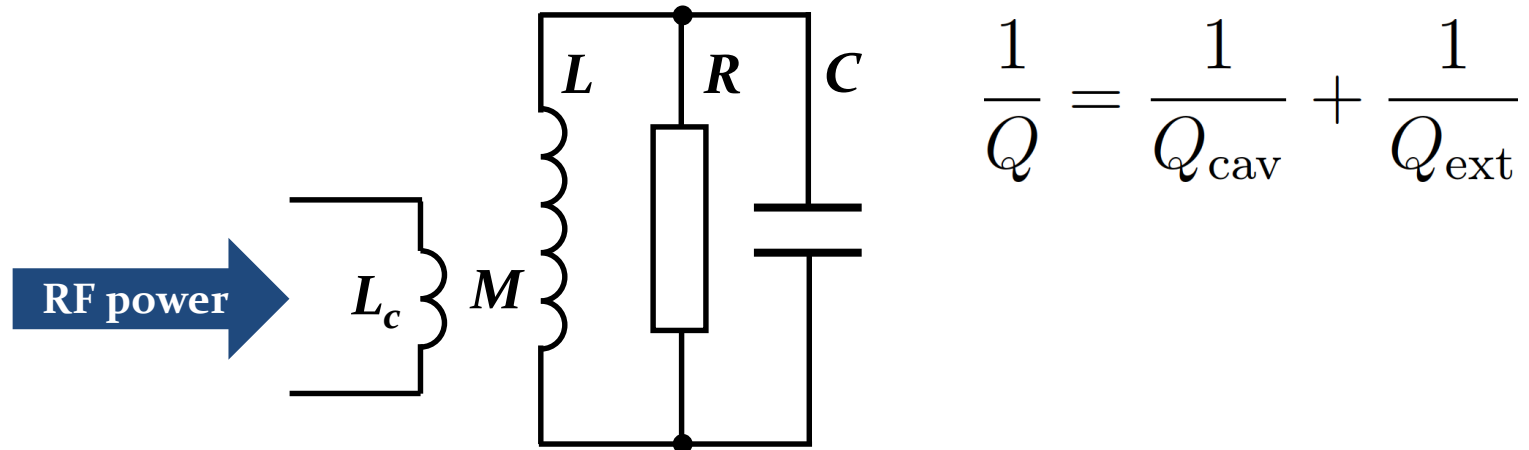
$$\frac{1}{Q} = \frac{1}{Q_{\text{cav}}} + \frac{1}{Q_{\text{ext}}}$$

- Beam is usually modelled as a current source and sees a (generally complex) Z_{shunt} .
- Via the transformer, the coupling to the cavity can be adjusted to “*matching*” / in practice, we would rotate our coupling loop to modify the coupling strength.

Principles of coupling power into a cavity

Coupling power into a cavity

- Attach **inductivity** or capacitance of resonator, or combined



→ Coupling **loop** forms **transformer** with **resonator** inductivity

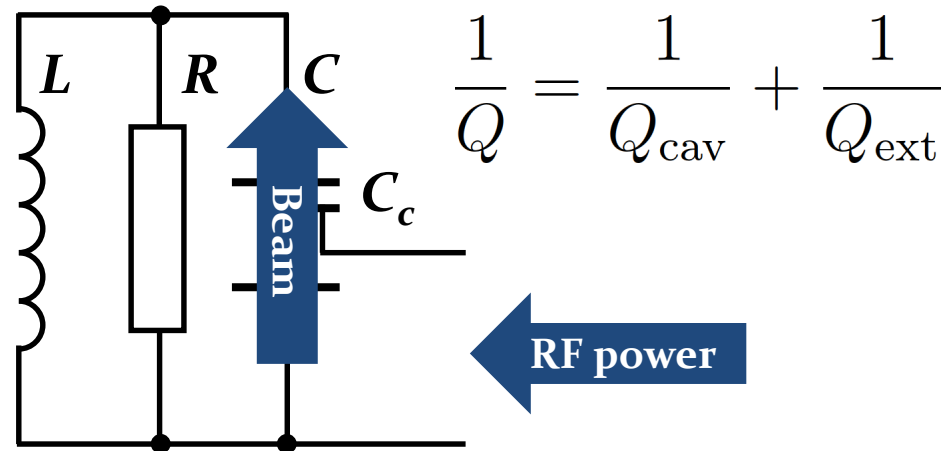


L. Stigelin

- Main coupler
PSI cyclotron
→ **~1 MW at 50 MHz**
- By far the most
common method,
allows also to adopt
the matching.

Coupling power into a cavity

- Attach inductivity or **capacitance** of resonator, or combined



- **Capacitive divider** to gap to transform generator impedance to cavity shunt impedance
- Advantage: allows to DC isolate the coupler (if required by amplifier).
- Disadvantage: coupling is fixed.
- Beam also **couple**s capacitively via the gap

Coupler of CERN PS 40 MHz



- Coupler forms one **half of** capacitor with the gap

Capacitive (electric) coupling

- Coupling through an electric antenna

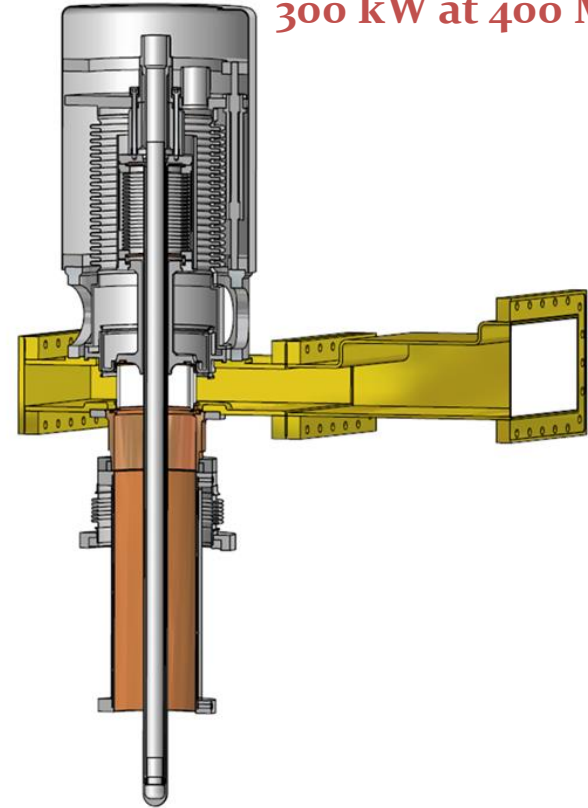
Electrical coupler to space



https://en.wikipedia.org/wiki/Transmitter_Solt

Power coupler of LHC cavities

300 kW at 400 MHz



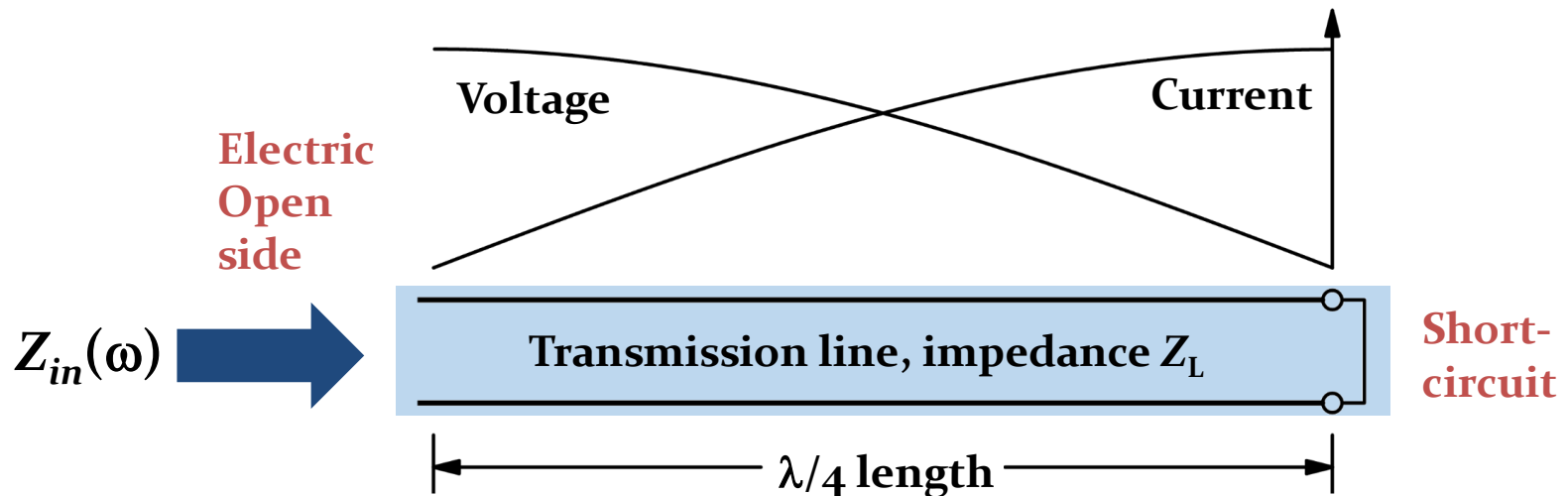
- 2 MW at 540 kHz
- Used to transmit radio broadcast in Hungarian language around the world.
- *claims to have reached Michigan...

- Coupler antenna transmits directly into the cavity

Cavity Design Issues

RF cavities in low frequency range

- **RF wavelength large below ~10 MHz: >30 m**
- With a pillbox design, we would need huge cavities → too large for accelerators
- Line resonators: **$\lambda/4$ resonator**

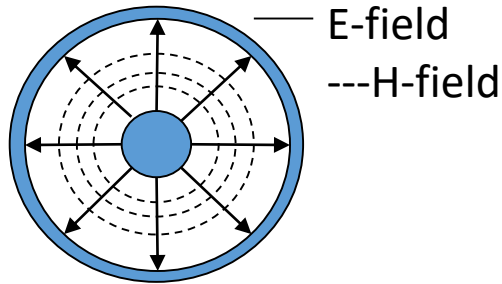


- Short circuit on one side
- Open end on other
- Voltage is zero
- No current but voltage

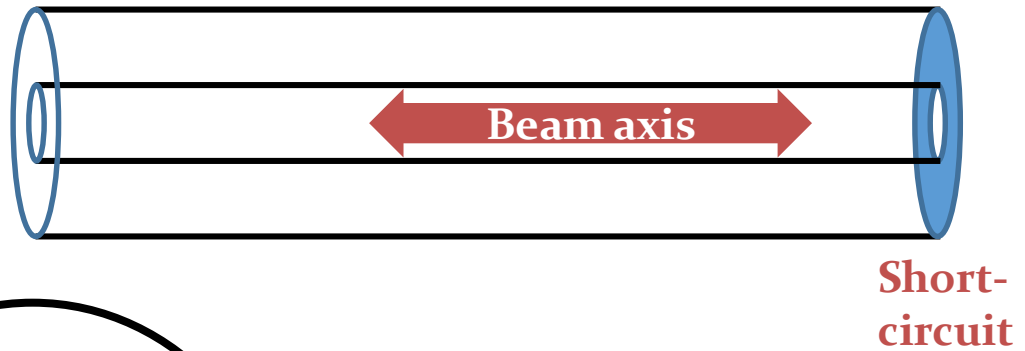
Why is this resonator so common in particle accelerators?

RF cavities in low frequency range

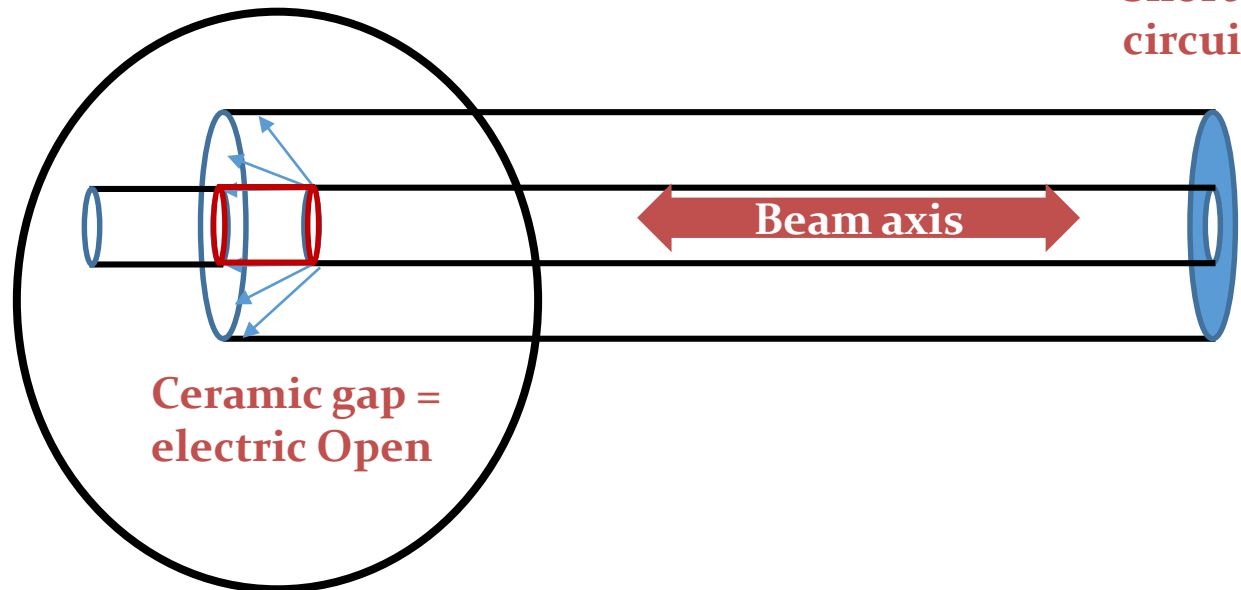
- Coaxial structure with inner conductor as beam pipe



$Z_{in}(\omega)$



This side acts like a capacitor, we speak of capacitive loading.



→ Still rather long geometry, 7.5 m at 10 MHz

RF cavities in low frequency range

- Coaxial structure with inner conductor as beam pipe

→ Add more capacitive

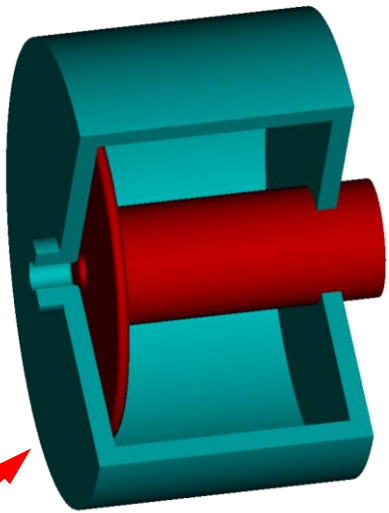
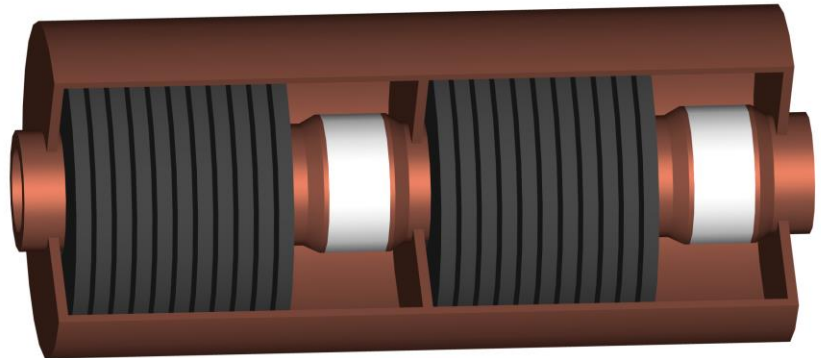


Plate
capacitor

or inductive loading



Ferrite
inductivity

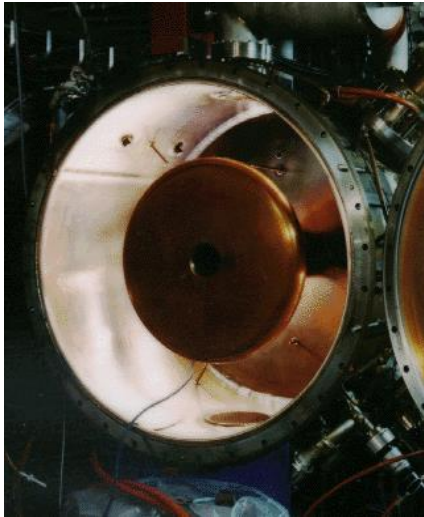
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

resonance frequency

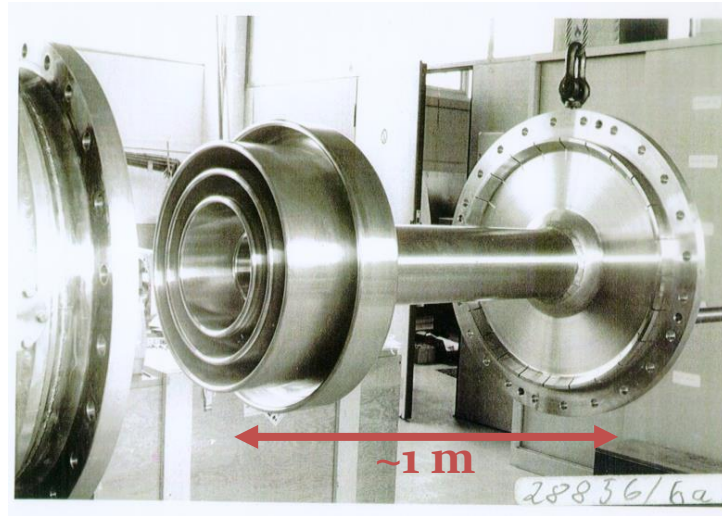
Capacitive loading in real world

→ Add capacitor at gap of cavity to shorten the resonator

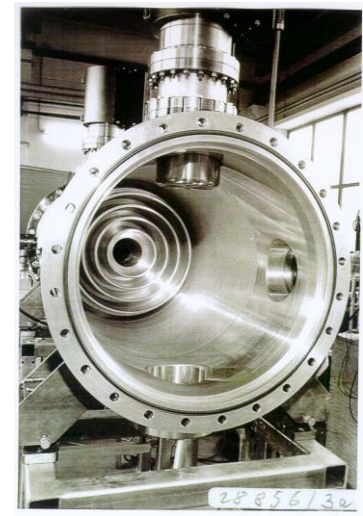
NSLS, 52.88 MHz



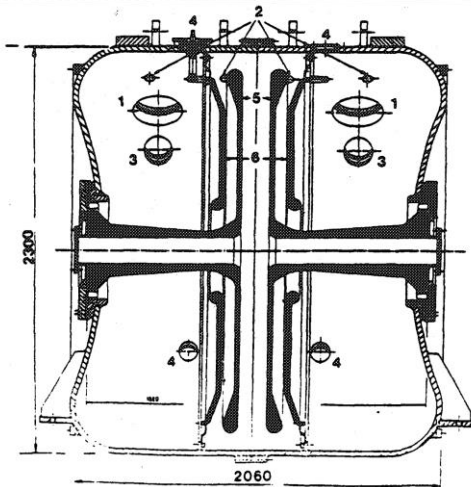
DESY PIA, 10.4 MHz, inner cond.



Outer cond.



ACOL, 9.53 MHz

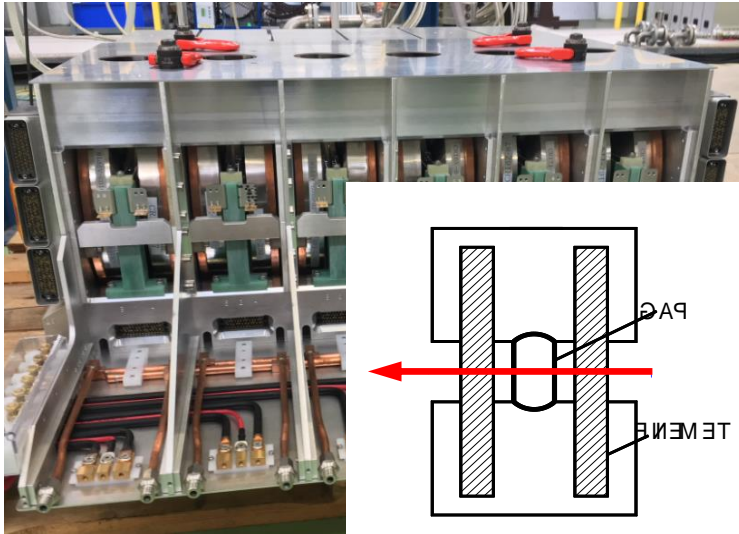


- Significantly reduces cavity size
- Fixed frequency only
- Adds small losses due to capacitor
- Advantage: entire cavity in vacuum

Inductive loading in real world

- Inductive loading with magnetic material shortens resonator from tens of meters to a device, **lossy though**

CERN PSB Finemet cav., 0.6-18 MHz



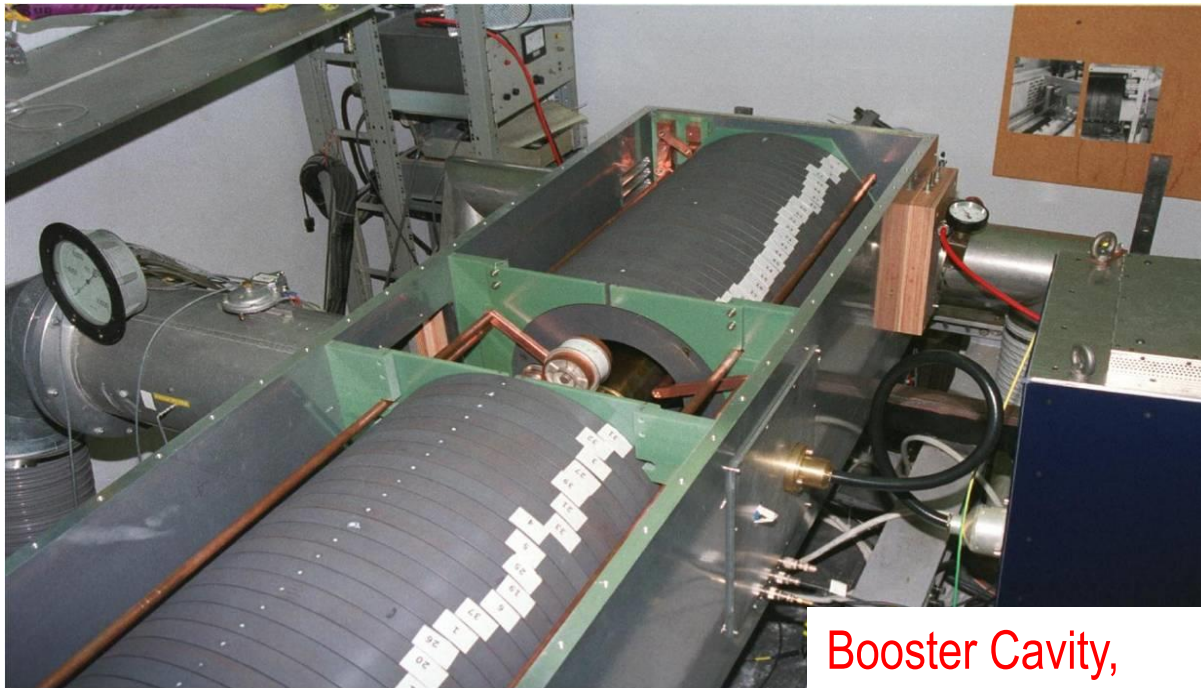
M. Paoluzzi

CERN PS, double gap, 2.8-10 MHz



- Additional advantage: permeability of ferrite can be controlled by DC bias current → **variable inductivity**
 - Cavity with programmable resonance frequency
 - Essential for hadron acceleration in low-energy accelerators

Tunable cavities – the PSB

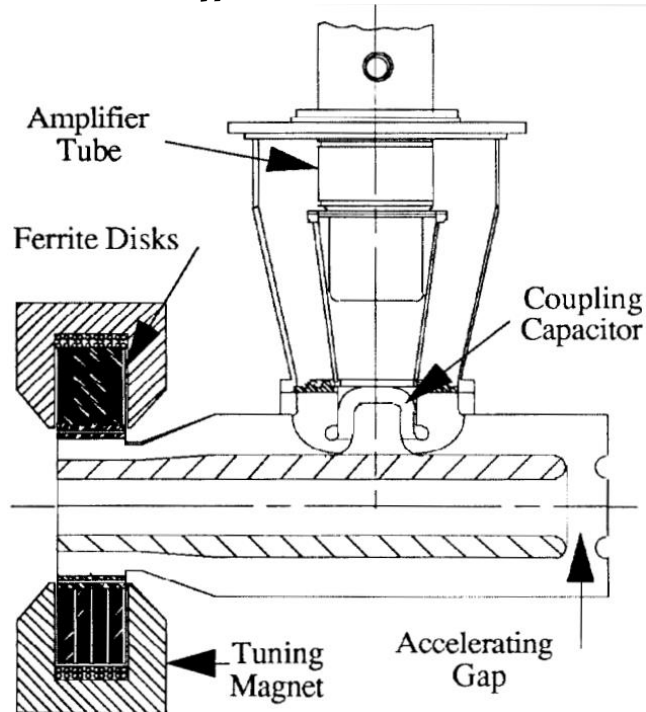


Booster Cavity,
below 20 MHz

Tunable cavities at higher frequencies

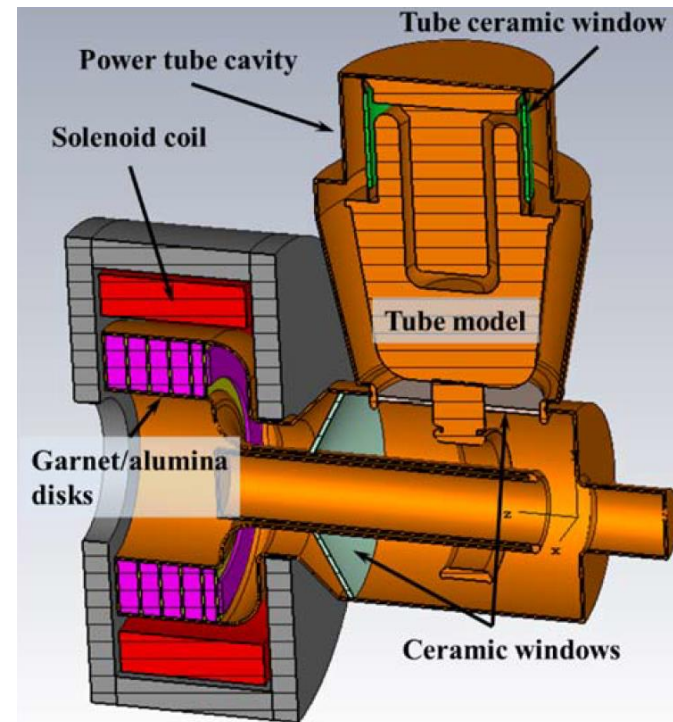
→ Remove inductive or capacitive loading

SSC Low Energy Booster,
~47 MHz to 60 MHz



C. C. Friedrichs et al., PAC91, p. 1020

FNAL Booster 2nd harmonic,
76 MHz – 106 MHz, 100 kV

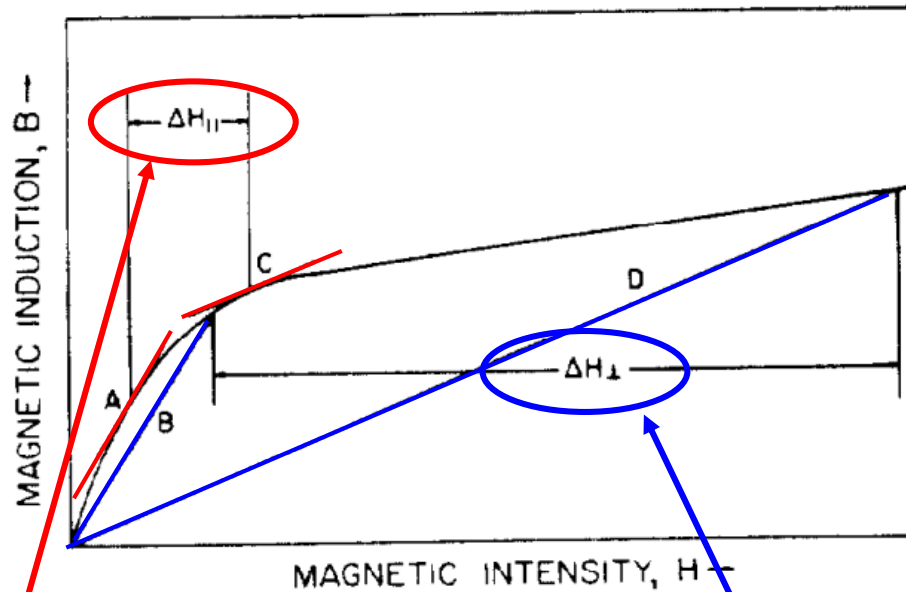


R. L. Madrak, IPAC16, p. 130

→ Upper frequency limit for cavities with large tuning range

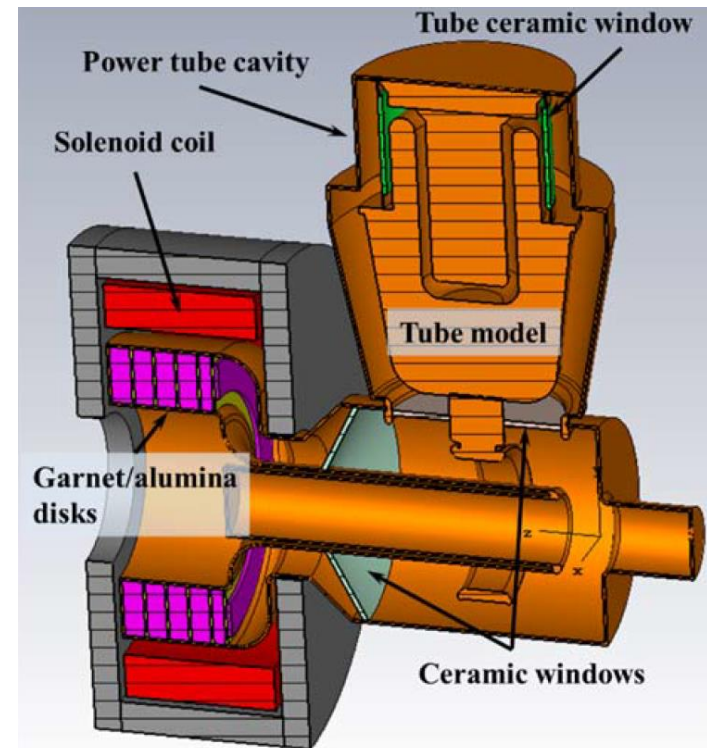
Tunable cavities at higher frequencies

From: Smythe, IEEE, TNS [3]



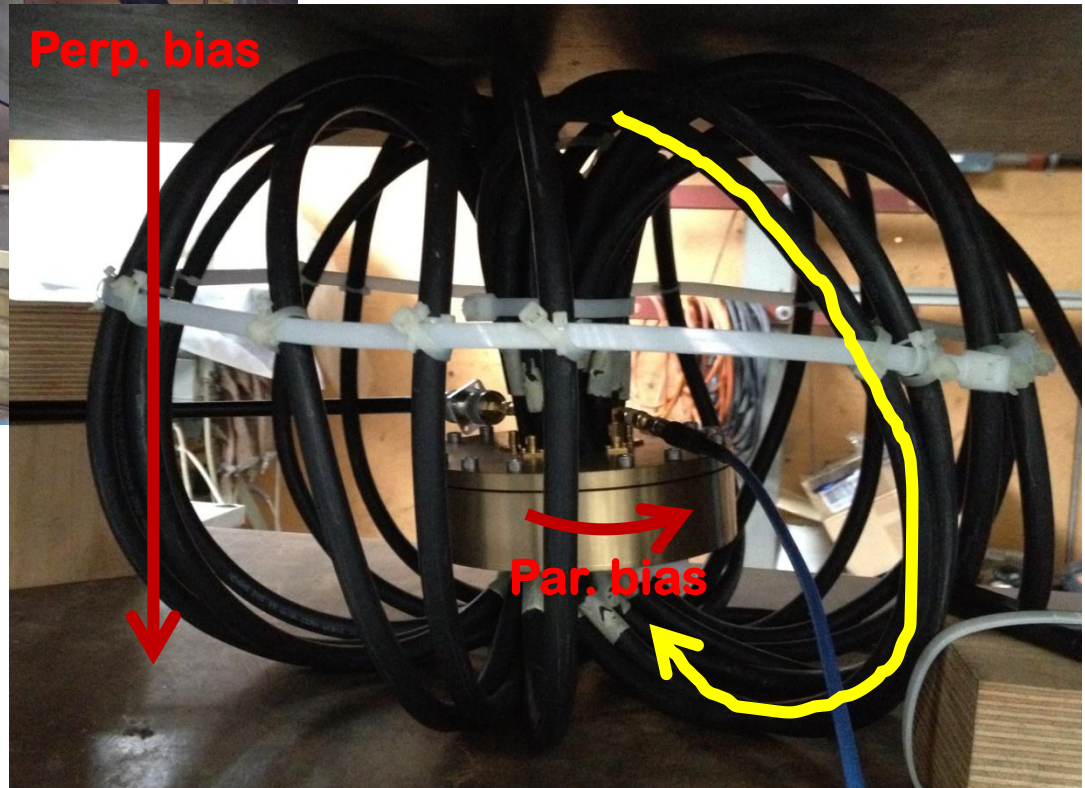
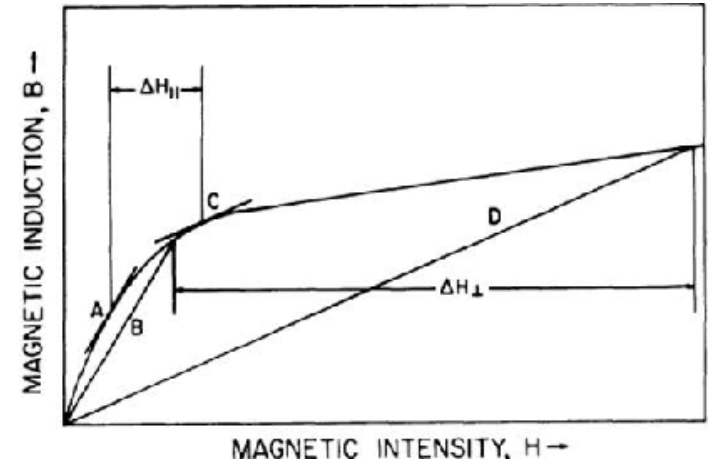
Parallel bias
(= $H_{\text{mag,bias}}$ is parallel H_{RF})

Perpendicular bias
(= $H_{\text{mag,bias}}$ is perpendicular H_{RF})

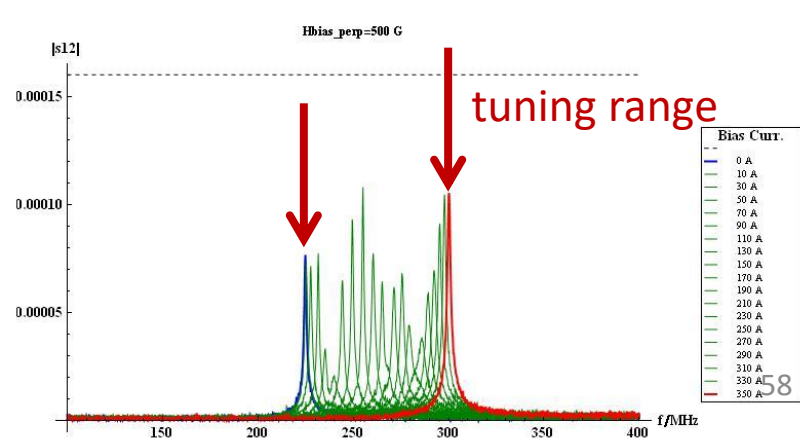
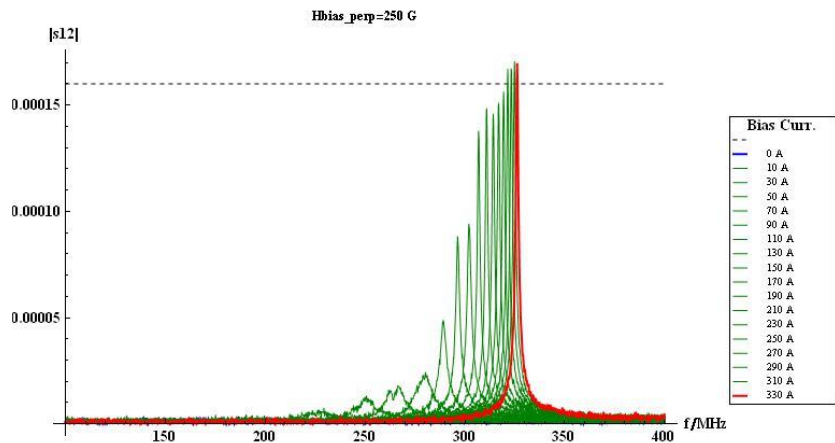
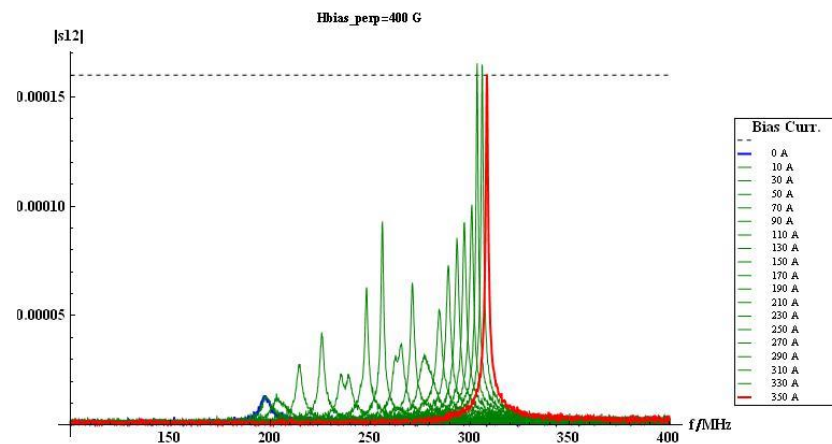
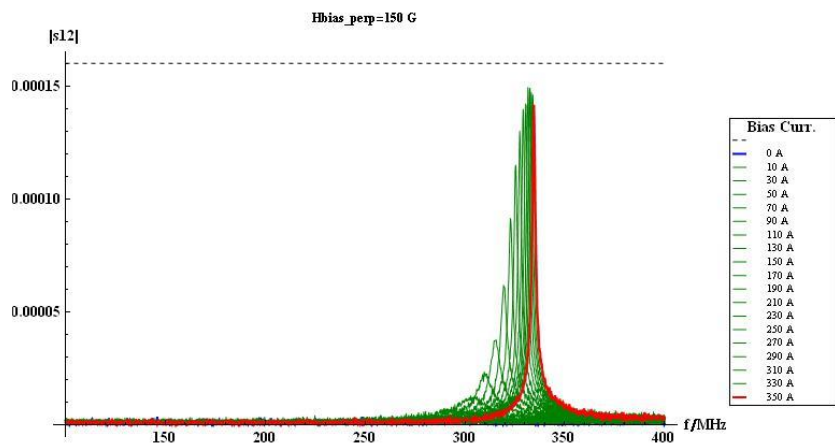
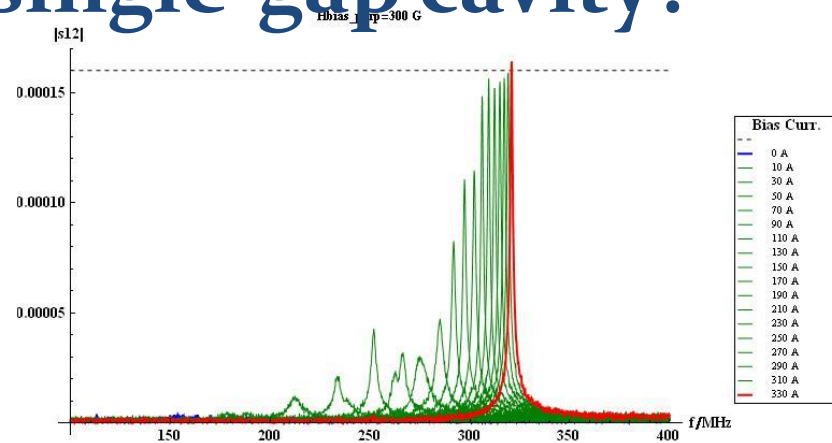
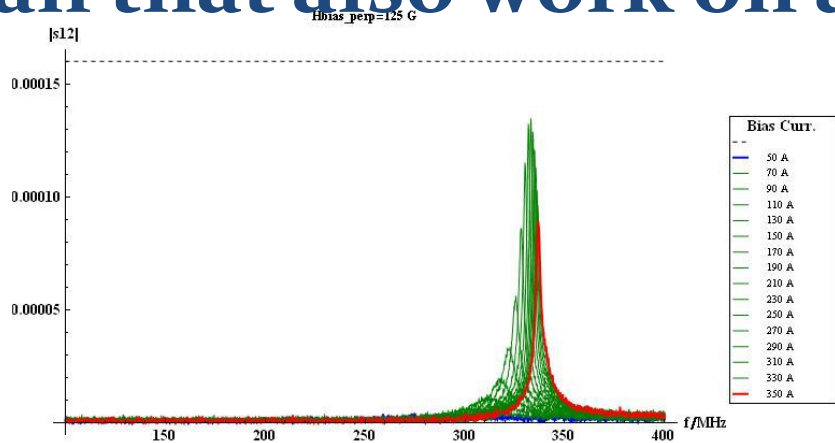


- Garnets are ferrites with a magnetic field-dependent relative permeability.
- Exposed to a (slowly varying) magnetic field, their μ -value covers a certain range until they saturate.
- Problems are the garnet losses (cannot be in vacuum) and the limited Q-value of the material (reduces the cavity Q).

Can that also work on a single-gap cavity?



Can that also work on a single-gap cavity?



Applying the tuning principle to a single-gap cavity



Ferrite-filled part
(makes use of area of
homogeneous magnetic field)



Coupler to cavity

Support structure

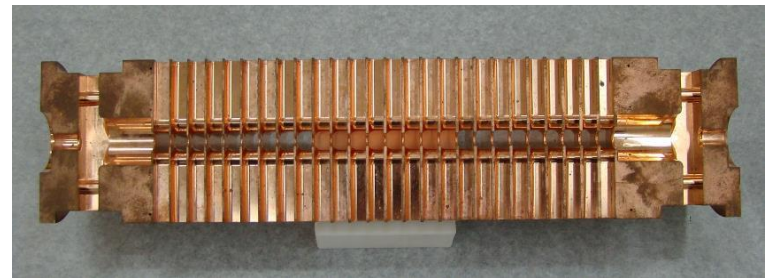
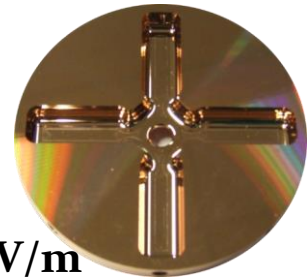
RF cavities in linear accelerators

- Beam only passes once → **Maximize gradient**
- Many accelerating cells to best reuse RF voltage

SuperHILAC, ~70 MHz, Berkley

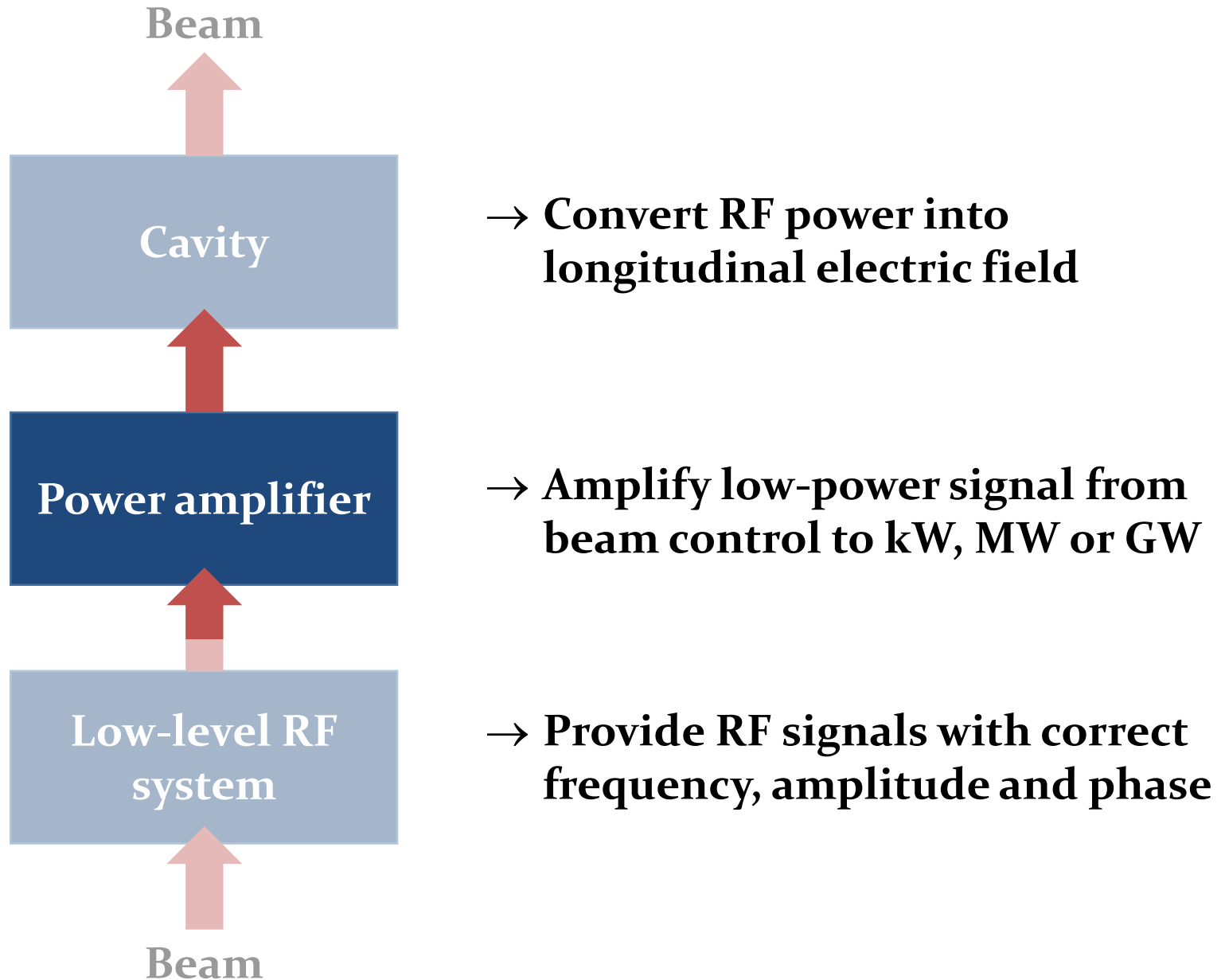


CLIC, 12 GHz, ~100 MV/m



- Cavity is the **contrary** to 'one size fits all'
- Many, many more variants

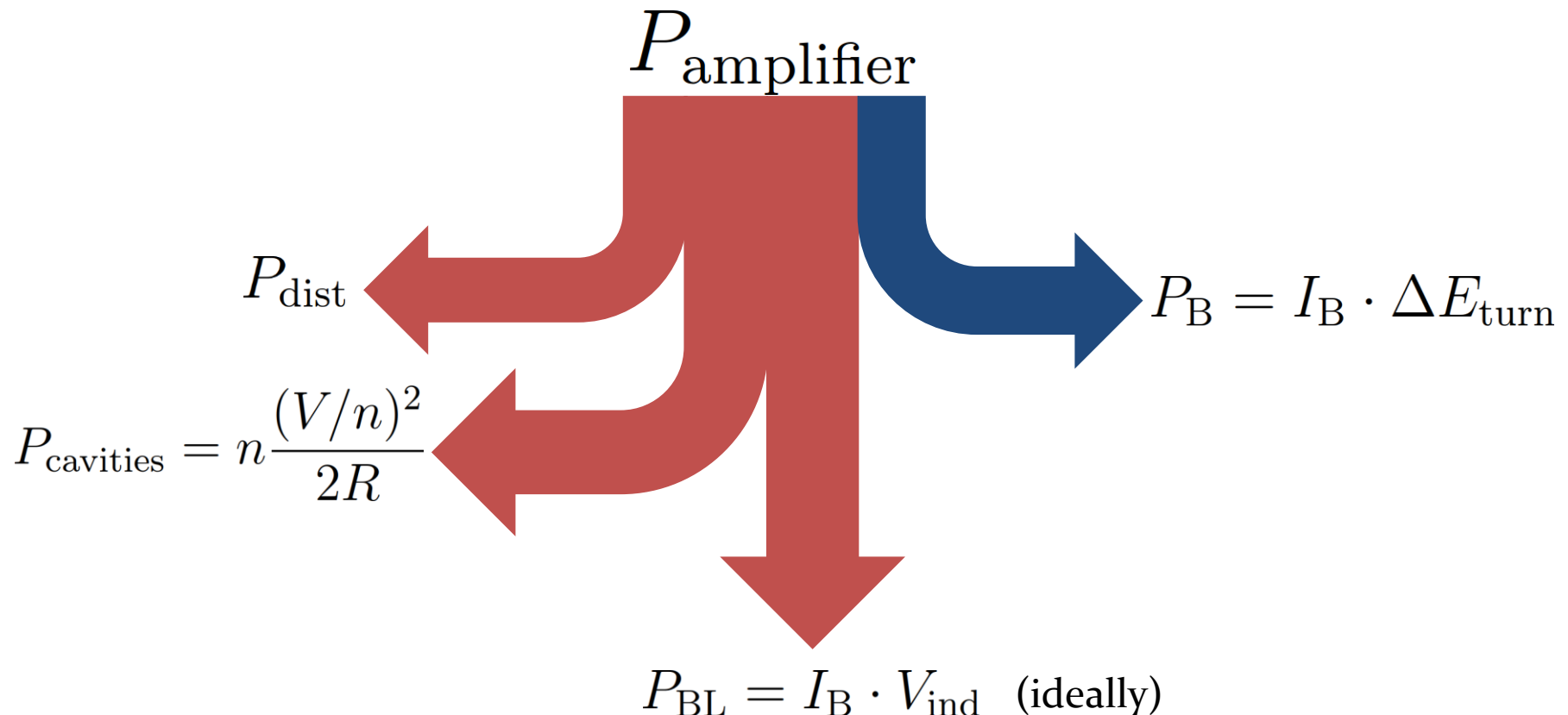
RF system overview



Power amplifiers

How much power is required?

1. Power to accelerate beam → **Wanted**
2. Compensate beam-induced voltage → **Refl. P**
3. Compensate electrical losses in cavity → **Heat**
4. Compensate electrical losses in distribution → **Heat**



Power amplifiers

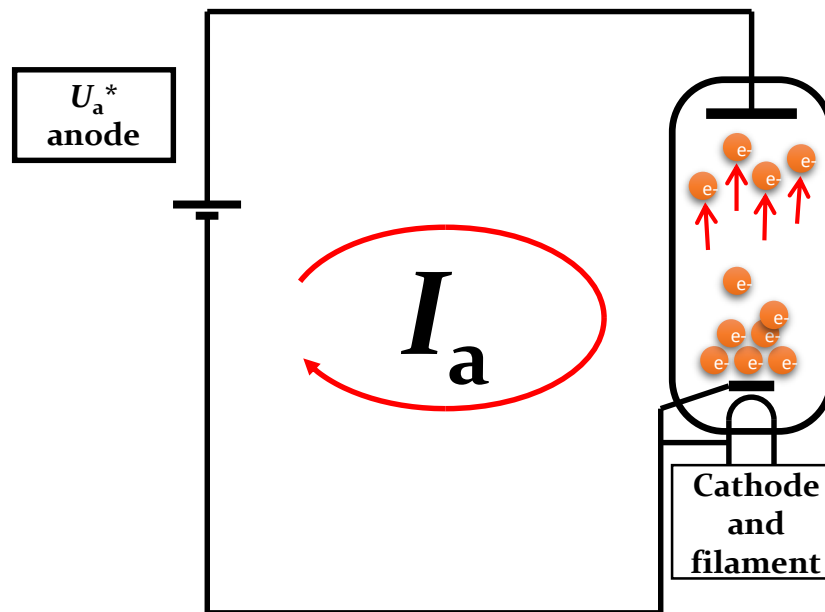
- Basically

$$P_{\text{out}} = g \cdot P_{\text{in}} \quad \text{or} \quad V_{\text{out}} = \sqrt{g} \cdot V_{\text{in}}$$

- The ideal power amplifier
 - Large bandwidth: amplifies all frequencies equally
 - No saturation, infinite power
 - Zero delay
 - No added noise
 - Unconditionally stable and resistant to reverse power
 - Radiation-hard
- Unfortunately such a device has not been invented yet
- Let us have a look at some real amplifiers

Basics of grid tube

- From diode to tetrode amplifier



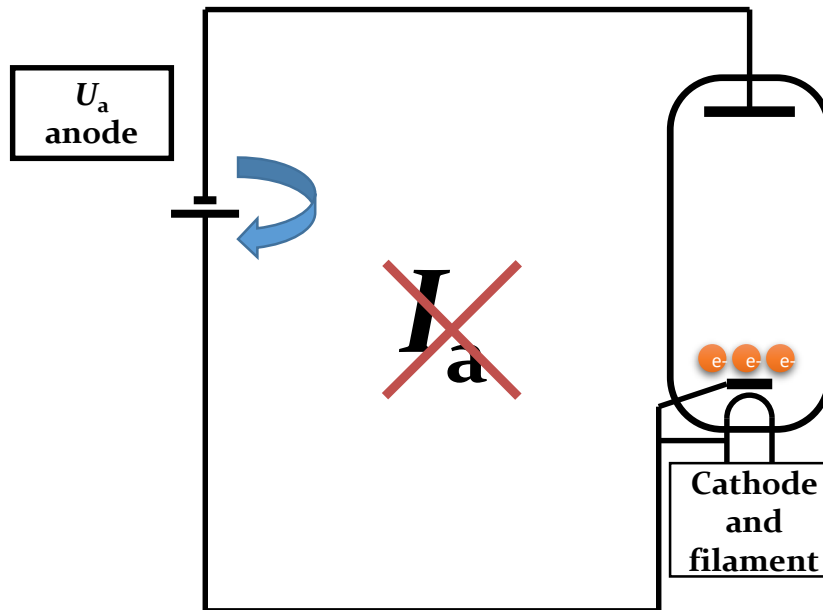
- Vacuum tube
- Heater + Cathode
- Heated cathode
 - Coated metal, carbides, borides,...
- thermionic emission
- Electron cloud
- Anode

→ Diode

*For tube amplifier designs
voltages are named U instead of V

Basics of grid tube

- From diode to tetrode amplifier

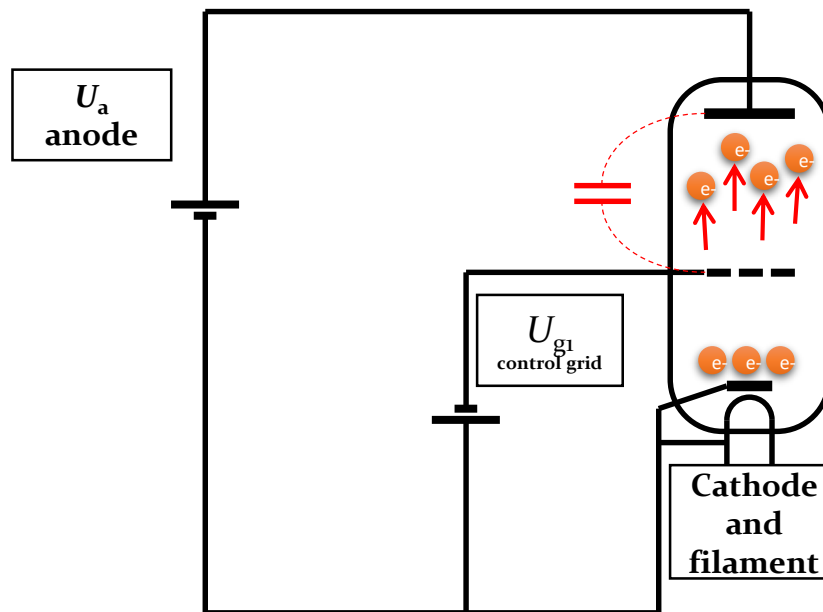


- Vacuum tube
- Heater + Cathode
 - Heated cathode
 - Coated metal, carbides, borides,...
 - thermionic emission
 - Electron cloud
- Anode

→ Diode! (but no amplification ...yet)

Basics of grid tube

- From diode to tetrode amplifier



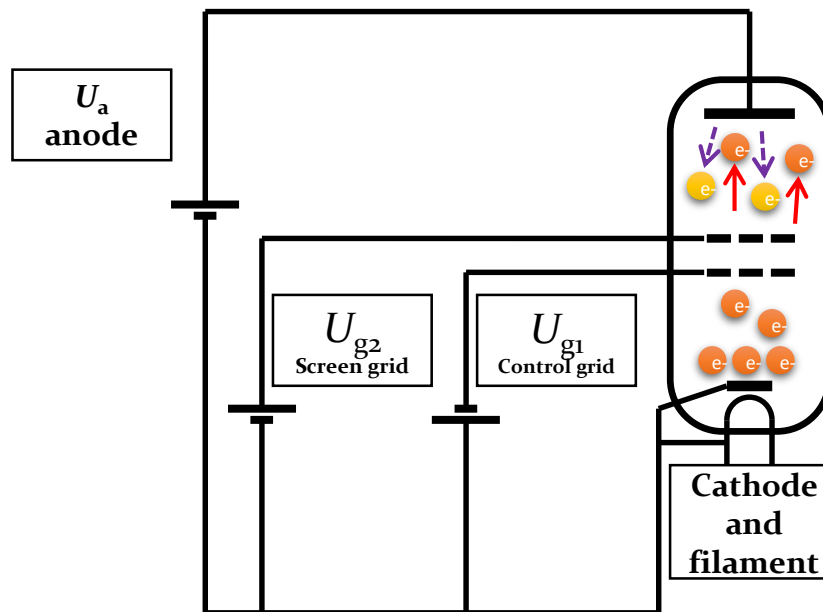
→ From Diode to Triode

- Adding a negatively charged control grid (e- won't catch)
- Modulating the grid voltage proportionally modulates the anode current
- Transconductance
 - By changing the voltage at grid → steering current at anode
- Limitations**
 - Parasitic capacitor from anode to control grid (g_1)
 - Tendency to oscillate

Basics of grid tube

- From diode to tetrode amplifier

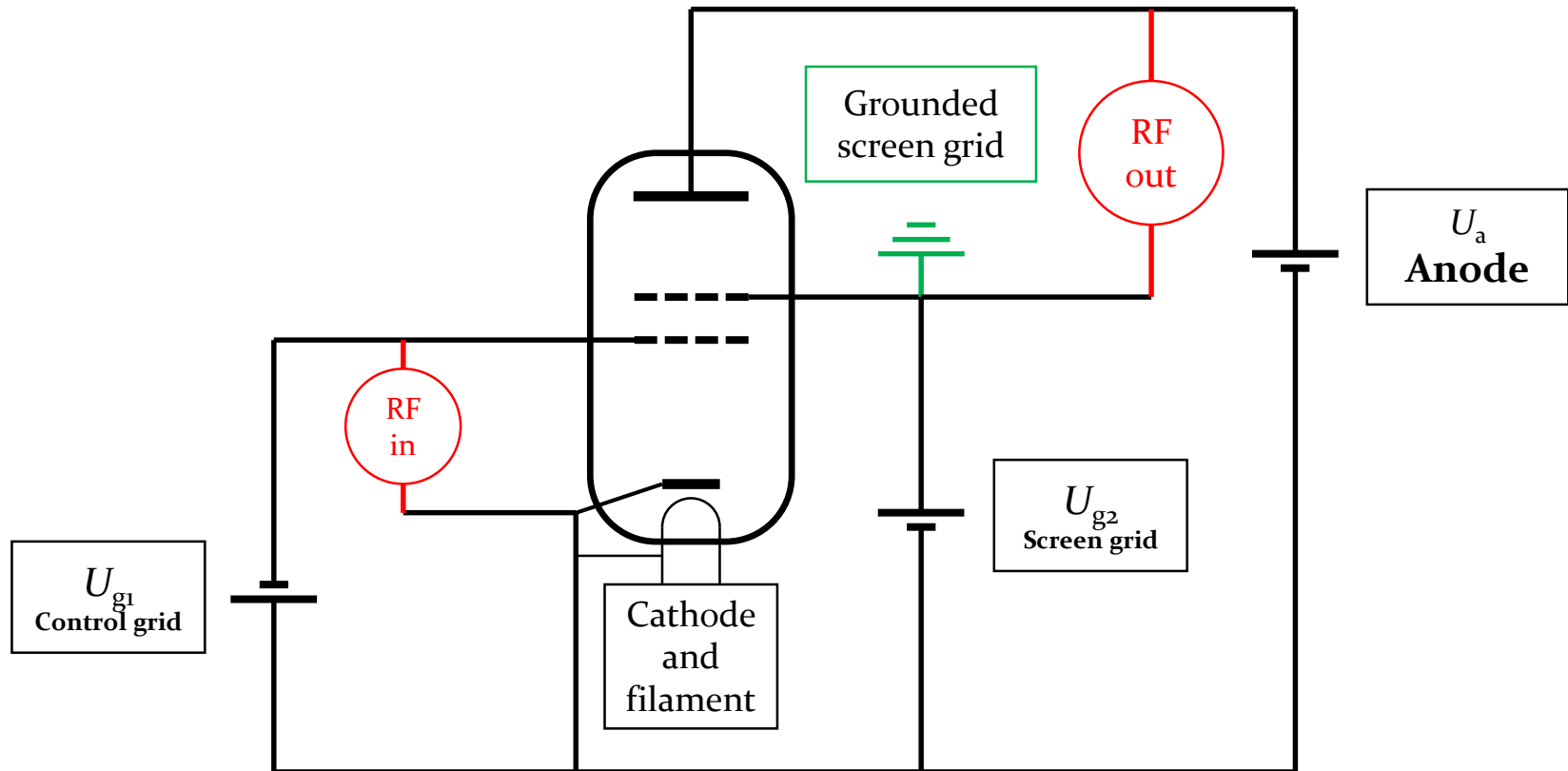
→Tetrode



- To avoid oscillation, add a screen grid (g2)
 - Positive (lower anode)
 - decouples anode from control grid g1
 - Higher gain
- Limitations**
 - Secondary electrons due to anode bombardment
 - To avoid this the anode is treated to reduce secondary emission

Tetrode based power amplifier

- Example of SPS 200 MHz amplifier, tetrode RS2004



→ Very simplified block diagram
small signal RF going in and amplified RF coming out

Example: Tetrode amplifier driving SPS RF ⁷⁰

- Two transmitters, 2×1 MW at 200 MHz (almost continuous)
- Eight tetrodes per amplifier

RS2004 tetrode



Amplifier trolley



Complete transmitter



- Consists of 8 individual amplifiers.

➤ In operation since 1976

E. Montesinos

Example: Tetrode amplifier driving SPS RF and transfer to the cavity

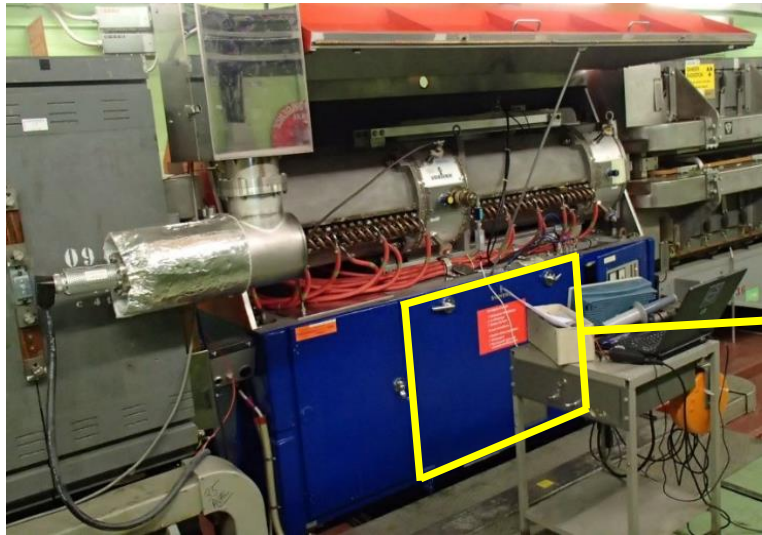
Amplifiers of the 200 MHz travelling wave system in the CERN SPS and their coaxial power lines leading to the underground installation (feeder lines)



Having the amplifiers on the surface and the cavity underground is only possible because no strict matching from amplifier to cavity is needed.

Tetrode amplifier driving PS RF

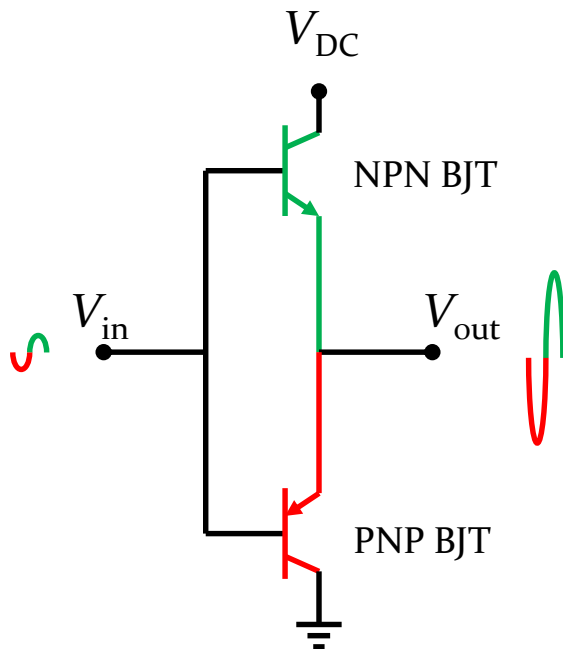
- Frequency range 2.8...10 MHz, ~60 kW per cavity, 11 units
- Space constraints to have amplifier installed below cavity



Amplifier trolley underneath cavity

- Tetrode is obvious choice
 - High power in small volume is needed
 - Operates in radioactive environment (consists of ceramics and metal parts)

Basics of RF solid state amplifiers

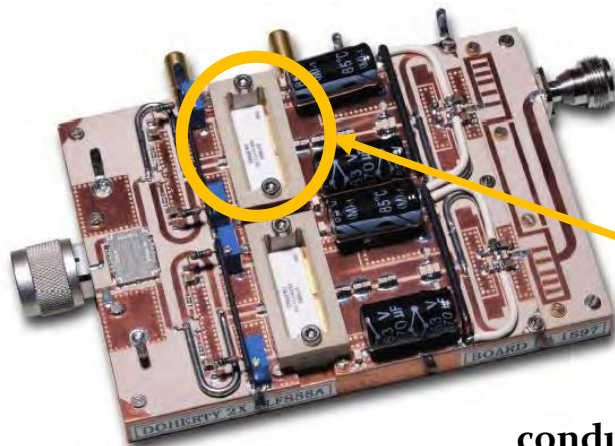
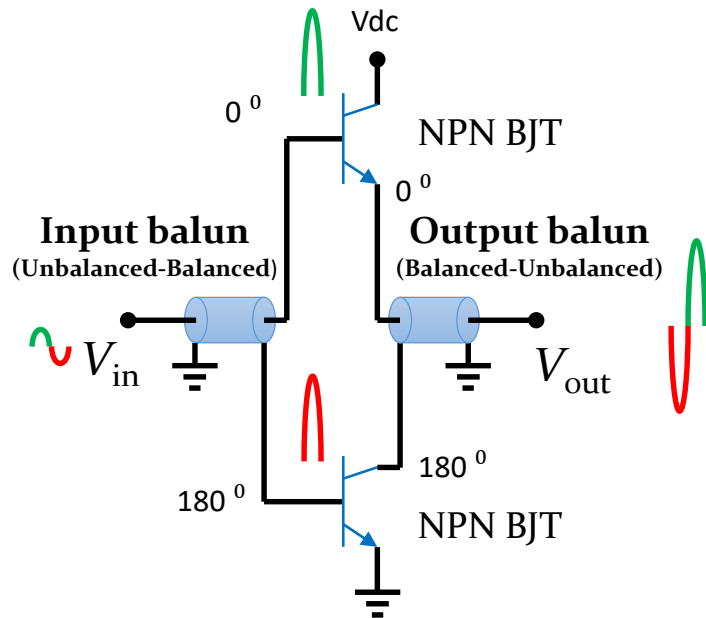


BJT: Bipolar Junction Transistor

- In a **push-pull** circuit the RF signal is applied to **two devices**
 - One of the devices is acting on the positive voltage swing and off during the negative voltage swing
 - The other device works in the opposite manner so that the two devices conduct half the time
- **The full RF signal is then amplified and need to be recombined.**
- **Needs two different type of devices that work as “ matched pairs” - and this cannot be obtained in RF.**

Basics of RF solid state amplifiers

BJT: Bipolar Junction Transistor



NXP Semiconductors AN11325
2-way Doherty amplifier with BLF888A

- Another **push-pull configuration** is to use two identical transistors and a balun (balanced-unbalanced “transformer”)
 - Power splitter, equally dividing the input power between the two transistors
 - Balun keeps one port in phase and inverts the second port in phase (sign flip!)
 - Since the signals are out of phase only one device is on at a time
- This configuration is easier to manufacture since only one type of BJT is required

Two semiconductor BJTs on one ceramic.

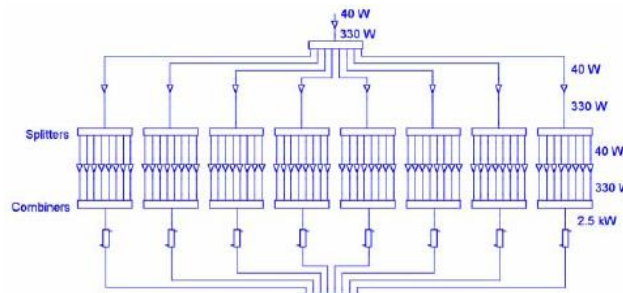
E. Montesinos

Example: Soleil 45 kW (1st Generation)

For the electron storage ring running at 352 MHz

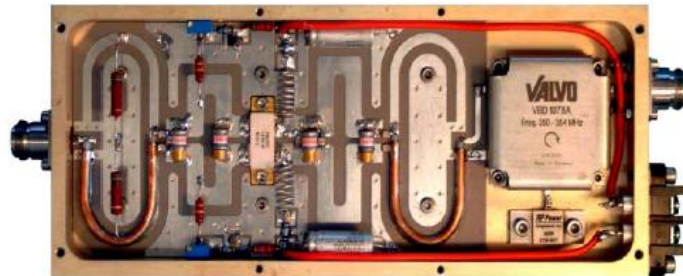


Set of splitters and combiners.



330 W for one amplifier module

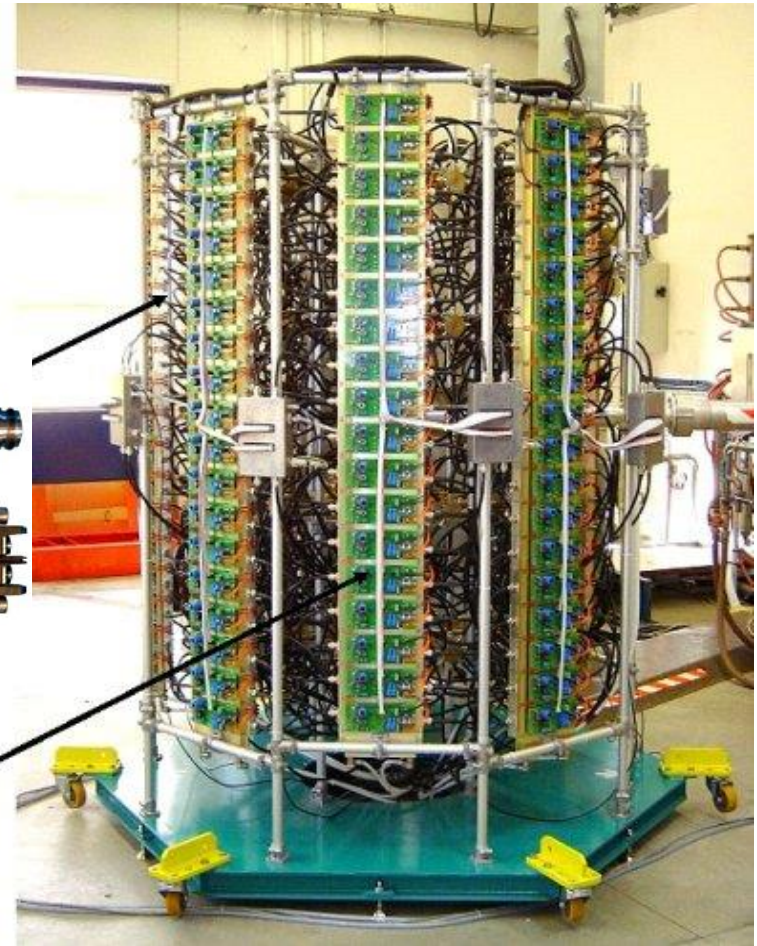
Modules are mounted into towers.



600 W, 300 V_{DC}/30 V_{DC} converter



Converters need to be near by to avoid transport of high currents over long distance.



Problem: Solid state amplifiers has high current requirement → heating and high losses.

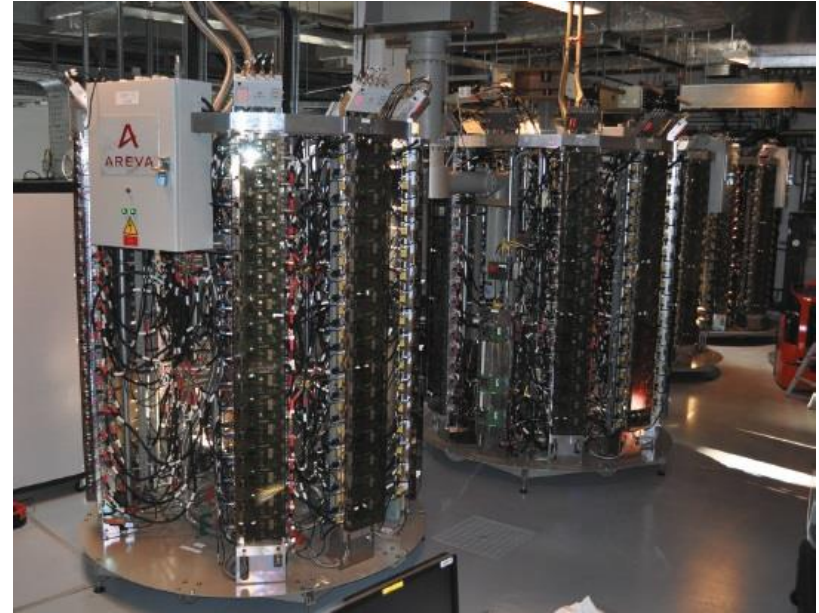
Example: Soleil for 352 MHz

Large scale solid state amplifier installations

45 kW per tower (2004 and 2007)



150 kW per tower (2012)

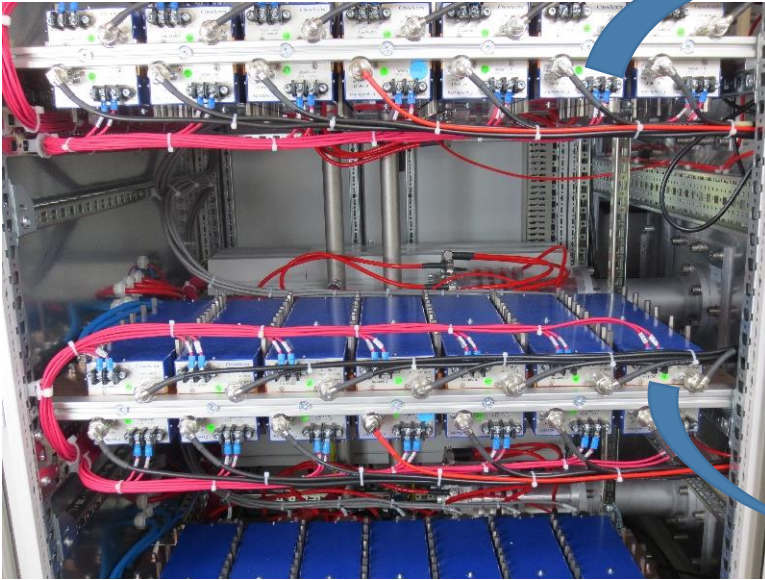


→ Requires a series of power combiners to moderate power per amplifier module to several tens of kilowatts

E. Montesinos

Example: BESSY II (more traditional looks...)

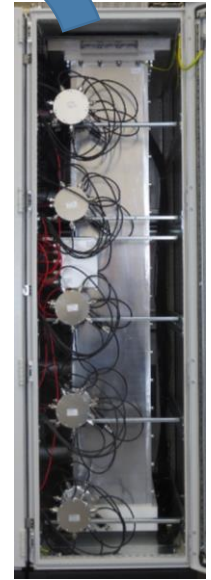
500 MHz solid state amplifiers: 4×80 kW for storage ring,
40 kW for booster synchrotron



Amplifier modules



80 kW unit

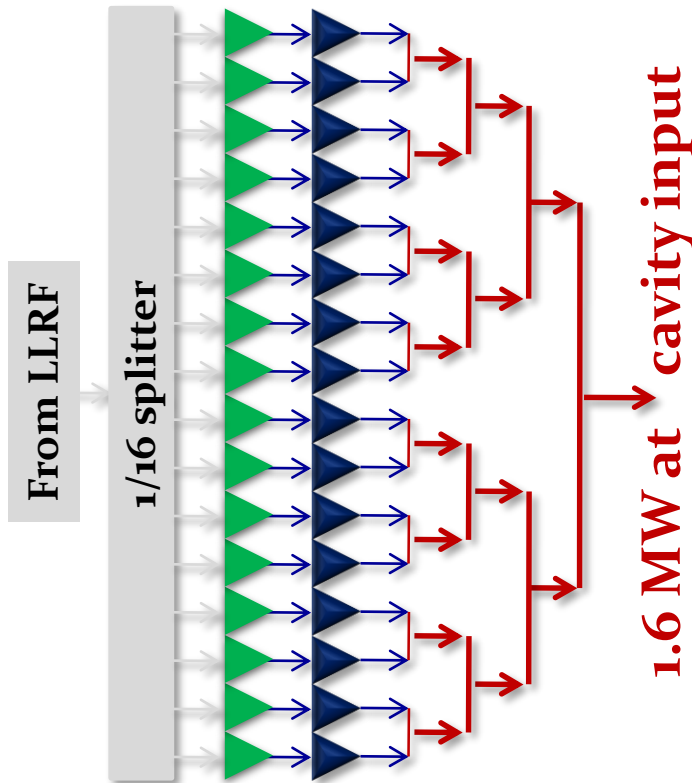


Combiner

- Power per module limited by RF transistors
- Increasing with modern semiconductor devices

Example: SPS

200 MHz solid state amplifiers: **2 amplifiers with 1.6 MW** peak power,
16 towers per amplifier



- 80 modules per tower, 1280 modules with **5120 transistors** per amplifier
- Presently the **largest RF installation** in a particle accelerator

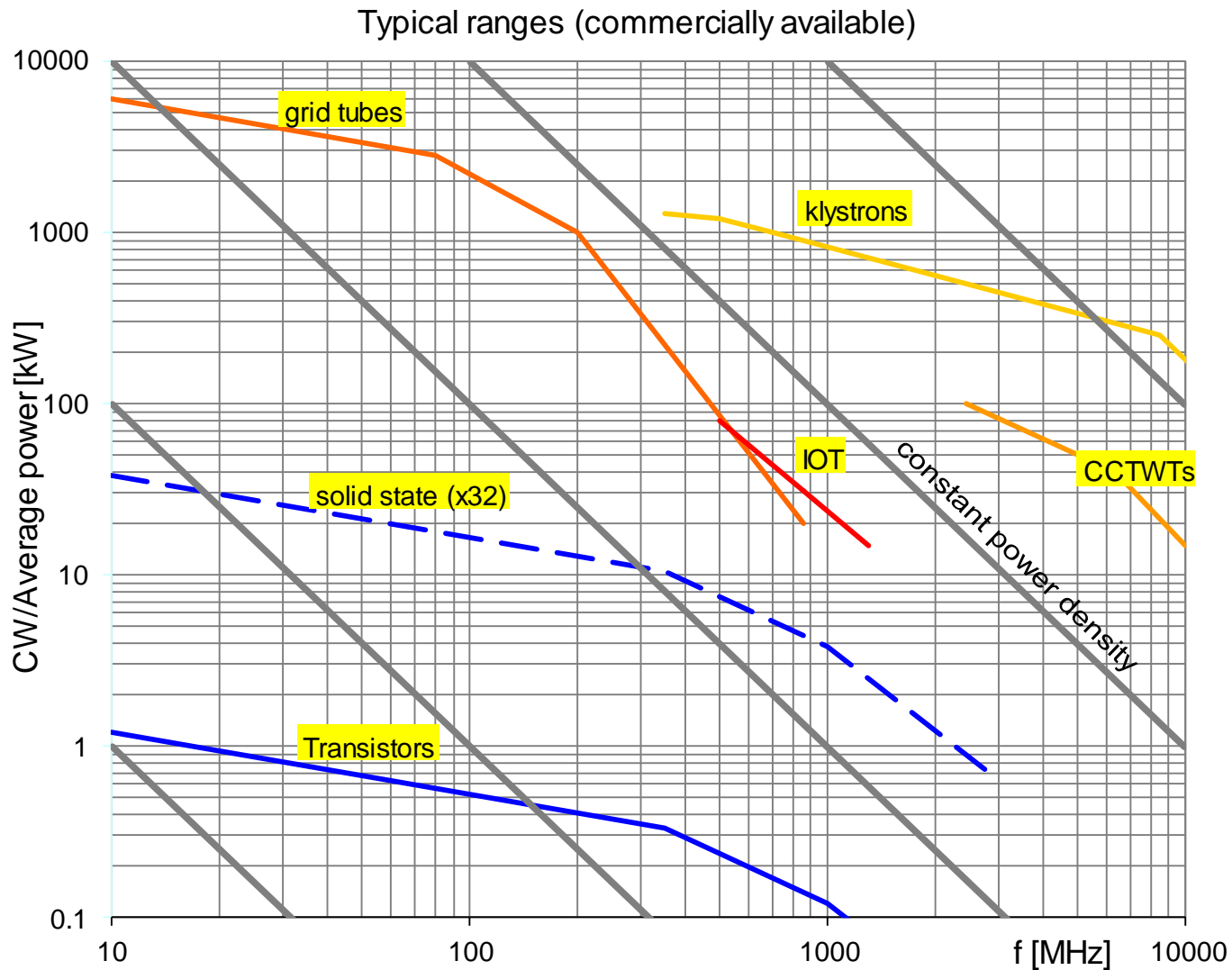


**All these slides were from Eric Montesinos
– BIG thanks!**

**AND to Heiko Damerau for giving
additional explanations.**

RF power amplifier

Power capability of commercially available amplifier types



How to choose the right RF amplifier?

| Prefer tube amplifier, when | Prefer solid-state amplifier, when |
|---|--|
| <ul style="list-style-type: none"> • Amplifier must be installed in the accelerator tunnel • Expecting important spikes from beam induced voltage • Large output power of a single device is required, without combiners • Not much space is available • High peak power in pulsed mode • Amplifier must be compact and/or close to cavity | <ul style="list-style-type: none"> • Amplifier can be located in non-radioactive environment • Circulator can be installed to protect the amplifier • Delay due to unavoidable combiner stages is little issue • Sufficient space can be made available • Continuous operation • Amplifier can be separate from the cavity |

→ Mostly no hard criteria → decide on case by case basis

Summary

- RF system parameters
 - Choose frequency and voltage wisely
- Parameters of RF cavities
 - R , R/Q
 - No 'one-size fits' all
- Power amplifier
 - Ideal amplifier does not (yet) exist
 - Tube or solid-state based

References and Recommended Literature

83

- [1] A. Grudiev, S. Calatroni, W. Wuensch: *New local field quantity describing the high gradient limit of accelerating cavities*,
Physical Review Special Topics – Accelerators and Beams, vol. 12, no. 102001, 2009.
- [2] W. Wuensch: *High Gradient Breakdown in Normal-Conducting RF Cavities*, CERN/PS 2002-028 (RF),
<http://cds.cern.ch/record/2833138/files/CERN-PS-2002-028-RF.pdf>
- [3] Smythe: *Reducing Ferrite Tuner Loss by Bias-Field Rotation*, IEEE, Trans Nuclear Science, Vol 30,
no. 4, 1983.
- J. W. Wang, G. A. Loew, *Field Emission and RF Breakdown in High-Gradient Room-Temperature Linac Structures*, SLAC-PUB-7684, 1997, <http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-7684.pdf>
- W. D. Kilpatrick, *Criterion for vacuum sparking designed to include both rf and dc*, Rev. Sci. Instrum. 28 (1957), 1957, <http://inspirehep.net/record/44645>

Books:

- POZAR, David M., *“Microwave Engineering”*, 4th edition, Wiley and sons.
- ZHANG, Keqian, *“Electromagnetic Theory for Microwaves and Optoelectronics”*, 2nd edition, Springer.

Walk through the CERN accelerators by using Google street view!

PS machine

<https://home.cern/science/accelerators/proton-synchrotron>

Use an electronic Smith-chart!

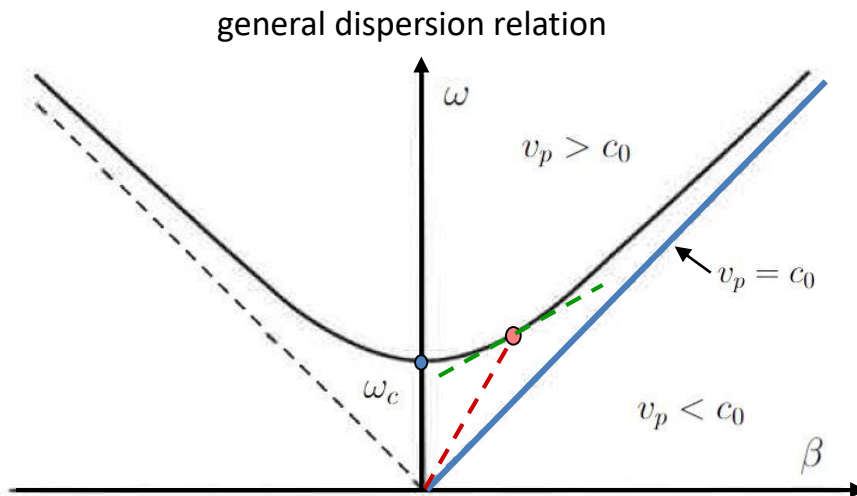
Smith-Chart Software “Dellsperger”

<https://www.fritz.dellsperger.net/smith.html>

**Thank you very much
for your attention!**

From waveguide to slow wave structure...

Dispersion Diagram



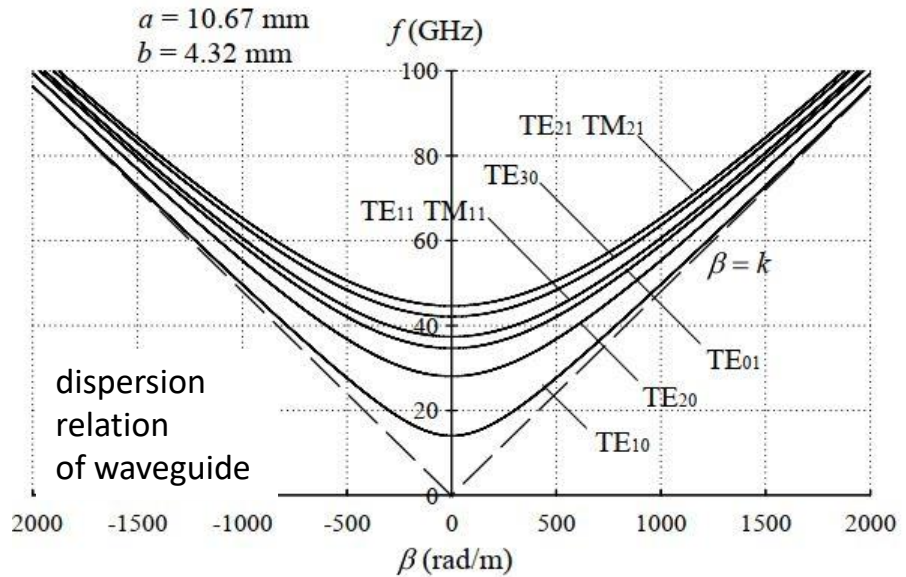
Tangent at point on the dispersion curve

$$v_p = \frac{\omega}{\beta}$$

From origin to point on the dispersion curve

$$v_g = \frac{\partial \omega}{\partial \beta}$$

- v_g is zero at cut-off frequency
- v_g is smaller than c_0
- v_p can be larger than c_0

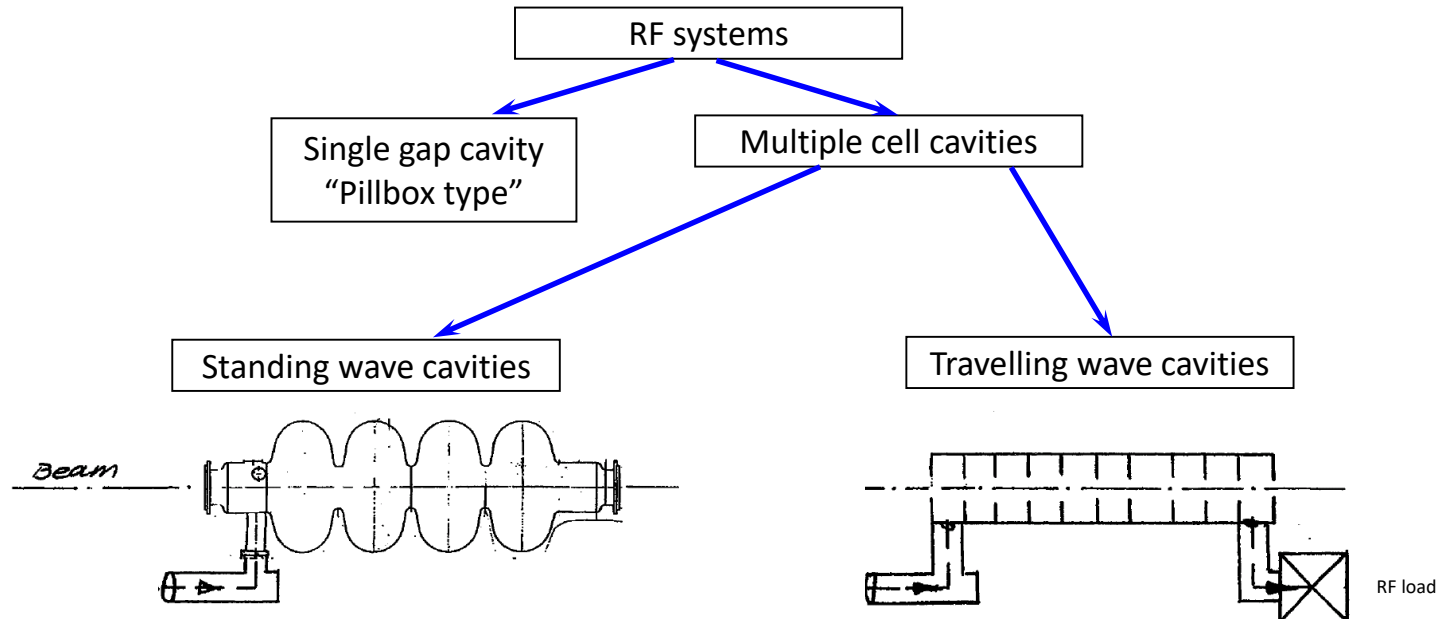


Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, Springer

NOTE that “we”, the RF engineers often use BETA for the imaginary part of the propagation constant (formerly “k”).

RF-systems

source: F. Caspers et al., *JUAS 2021*

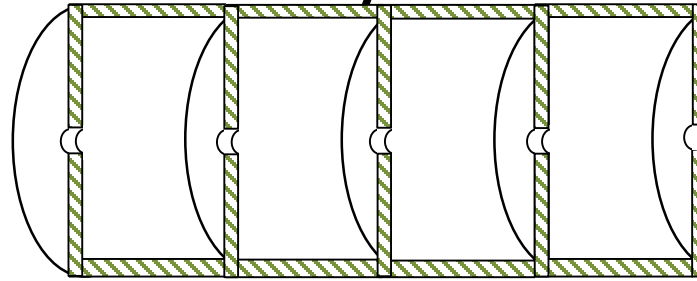


Typically used in applications with low beam loading,
e.g. ring accelerators (storage rings)

Typically used in applications with high beam loading, e.g.
linear accelerators (linacs)

Multiple cell cavities

Disc-loaded circular waveguide (1/2)

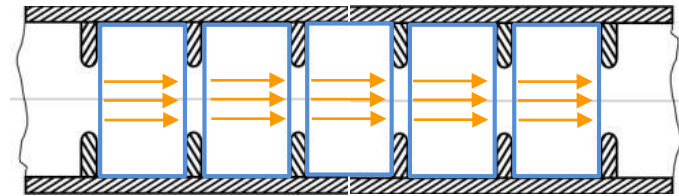
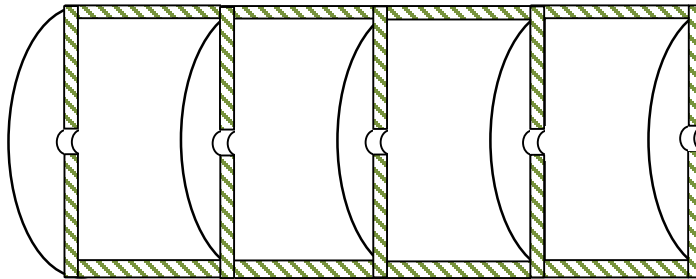


- Disc-loaded waveguide is a circular metallic waveguide with periodically added metallic discs and holes for particle passage and for coupling of the cells.
- We speak of a coupled-cavity chain structure.
- EM-fields need to fulfill boundary conditions on the metallic discs.
- Disc-loaded waveguide *can operate in travelling-wave as slow wave structure or in standing-wave mode.*
- In standing wave mode, the cells present a “concatenation” of TM_{010} -mode “pillbox”-type cavities.

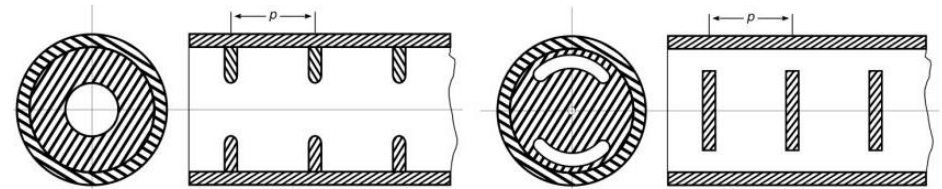
Disc-loaded circular waveguide

(2/2)

Source: Kramer, *Studies of HOM-couplers for the Upgrades Travelling Wave Acceleration System in the CERN SPS*, PhD, CERN-THESIS-2019-371



Example: Pi-mode structure in standing wave (SW) ...
can be derived from pillbox TM_{010} -mode.



(a) Iris coupled structure.

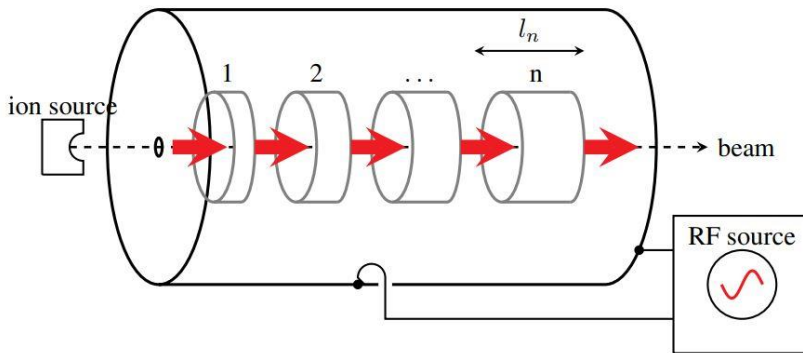
(b) Slot coupled structure.

- Different coupling mechanisms between the individual cells exits, either on the side or in the center.
- We speak of iris-coupled-structure and slot-coupled-structure.
- For slot-coupled-structures, the center opening for the beam can be very small and is often ignored in the EM-calculations.
- *The name of the resonant mode is given by the phase advance between two consecutive cells.*

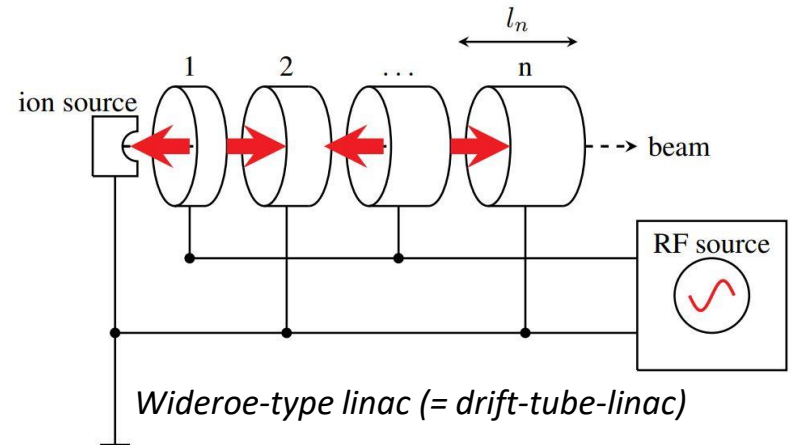
Example of Pi-mode Cavities (SW, 1/3)

How does this work?

source: F. Gerigk, CAS Ebeltoft, *Cavity Types*, 2010



Alvarez-type linac (= drift-tube-linac)

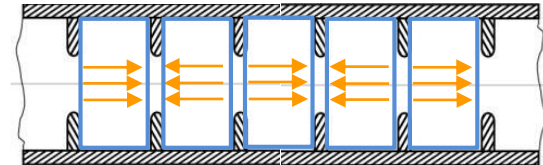
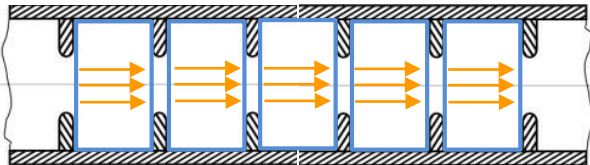


Wideroe-type linac (= drift-tube-linac)

0-mode structure

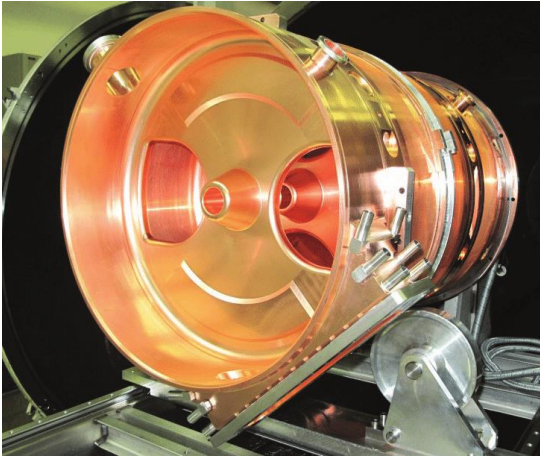
Pi-mode structure

Remember: The name of the resonant mode is given by the phase advance between two consecutive cells.



Example of Pi-mode Cavities (SW, 2/3)

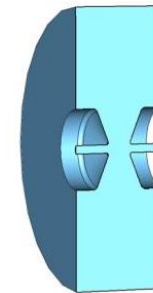
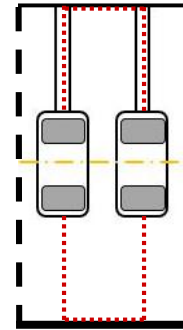
How does this look in reality?



*Slot-coupled-structure (PIMS)
at CERN*



Drift tube structure, CERN



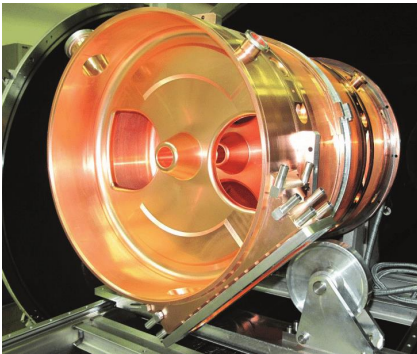
Picture of Linac1, CERN

Note that a 'cell' is not necessarily a pillbox-type shape. In drift tube structures, a cell is the area between two drift tubes and looks in principle like a pillbox with nose cones.

sources: P. Bourquin et al., *Development Status of the Pi-mode Accelerating Structure (PIMS) for LINAC4*, Proc. Of LINAC08, Canada.
C. Plostinar (ed.), *Comparative Assessment of HIPPI Normal Conducting Structures*, CARE-report-2002-0771

Example of Pi-mode Cavities (SW, 3/3)

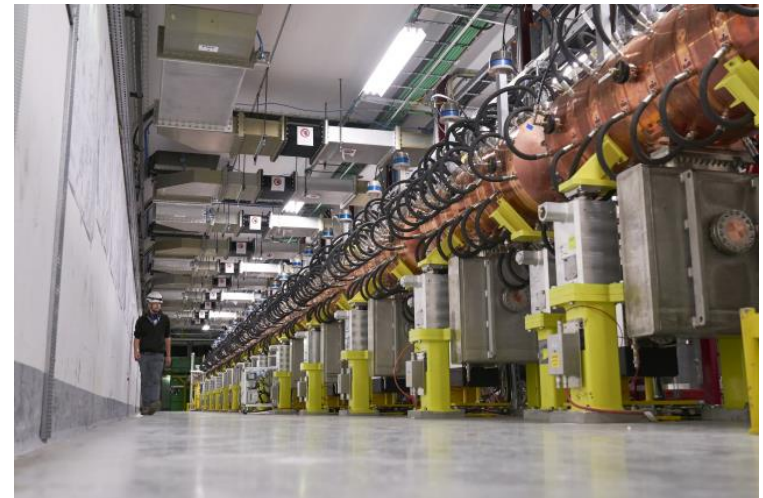
Slot-coupled-structure (PIMS) at CERN



PIMS cell



PIMS test set-up



Completed LINAC4, CERN

Picture sources: CERN cds

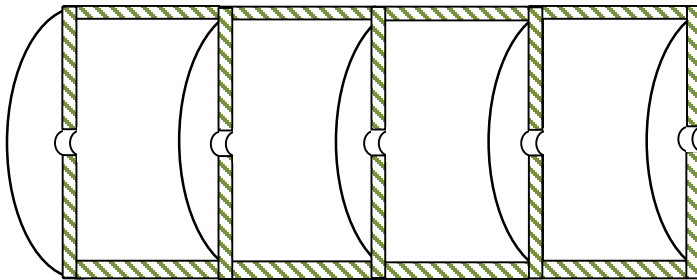
R. Wegner et al., *Linac4 PIMS Construction and First Operation*, IPAC2017, Copenhagen.

P. Bourquin et al., *Development Status of the Pi-mode Accelerating Structure (PIMS) for LINAC4*, Proc. Of LINAC08, Canada

Waveguide becomes Slow Wave Structure

- To get power transferred to the beam, *the accelerating field needs to be kept in phase* with the charged particle.
- For single gap cavities, this means that the resonance needs to be synchronous with the particle phase. We can then concatenate many “pillbox cavities” to one multi-cell pattern.
- Another way to reduce phase velocity is to build the disc-loaded waveguide where discs are added in a periodic pattern (schematic pictures look the same).
- As for the concatenated “pillbox cavities”, a multi-cell pattern builds up. The addition of discs inside the cylindrical waveguide induces multiple reflections between the discs and results in a change of the dispersion curve. The structure can then be used in travelling wave mode.
- If the wave is travelling with speed-of-light, and we slow down this wave (*=reduce phase velocity*), *we obtain* so-called slow wave structures (SWS).
- Slow wave structures are known from dielectrics (where the EM-wave slows down due to permittivity ϵ_r), but here we will discuss *slow-wave structures with metallic boundaries used for particle acceleration*.

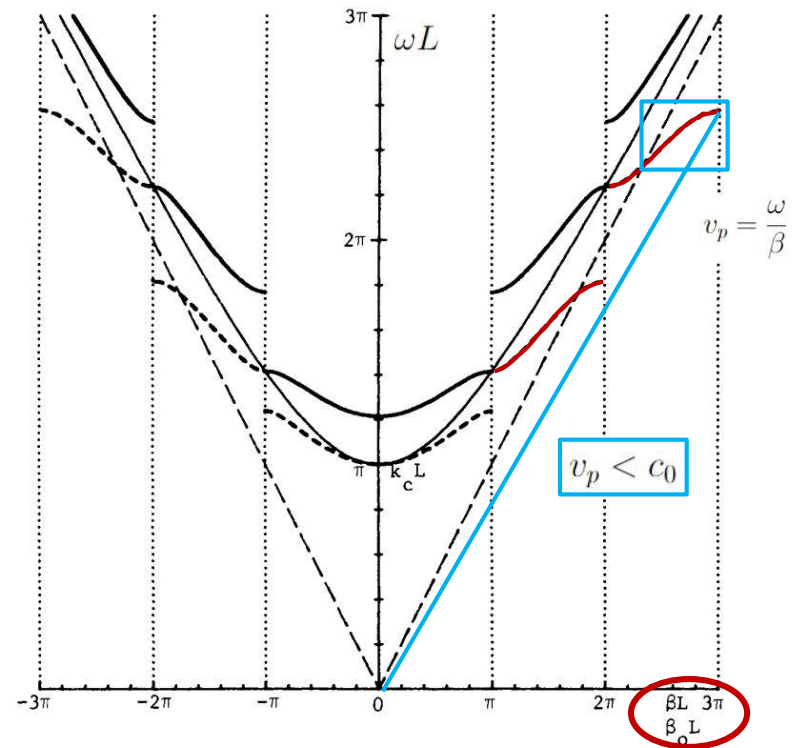
Slow wave structure (1/7)



- Slow wave structures work in the range of $v_p < c_0$.
- The addition of discs inside the cylindrical waveguide induces multiple reflections between the discs and results in a change of the dispersion curve.
- The disc-loaded structure can then be used in travelling wave mode.

The dispersion curve changes from continuous to splitting up into different modes which are slowed down. We speak of pass-bands. These modes are separated by stopbands.

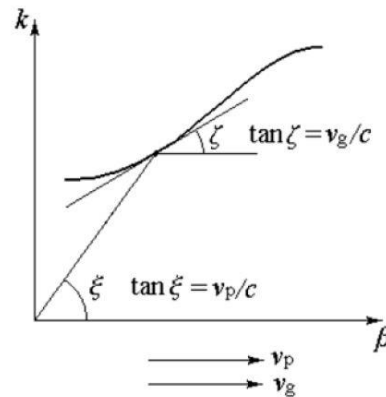
source: G. Dome, *RF Systems: Waveguides and Cavities*, aip-conf-proc.153-1296



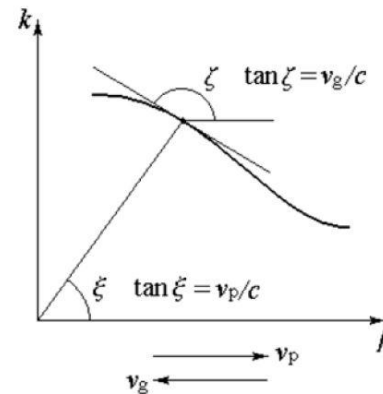
Slow wave structure (2/7)

Dispersion Curves, phase- and group velocity for FW and BW wave in periodic structure.

*Forward wave (FW)
 v_g and v_p point in the
 same direction.*



*Backward wave (BW)
 v_g and v_p point in the
 opposite direction.*



For the advanced RF-fans:

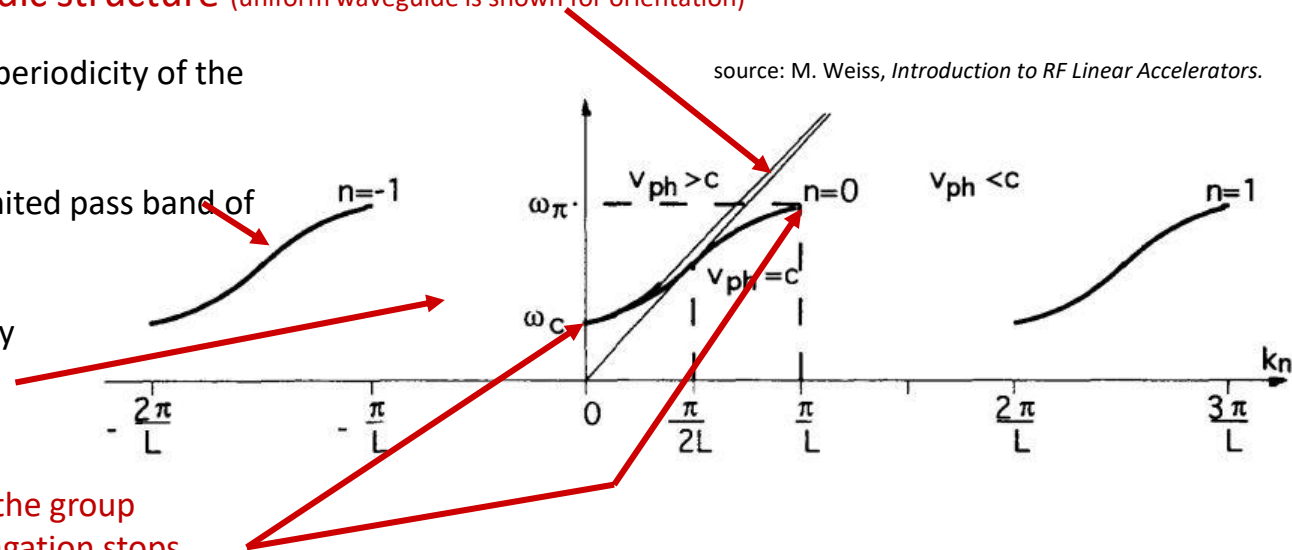
1. All guides modes in common transmission lines, metallic and dielectric waveguides are forward waves.
2. For transmission lines of “high-pass filter” type, the phase constant β decreases with increasing frequency.
3. “High-pass filter” type lines are modelled with a distributed series capacitance and a shunt inductance (i.e., opposite of what is usually done in the transmission line modelling!).
4. Forward and backward type of waves – this makes no difference for the direction of the beam.

Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, Springer

Slow wave structure (3/7)

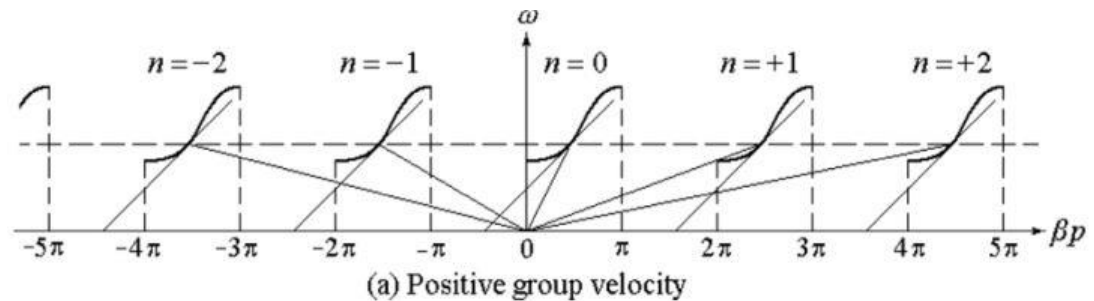
Dispersion diagram of periodic structure (uniform waveguide is shown for orientation)

- Dispersion diagram shows the periodicity of the structure.
- For a given mode, there is a limited pass band of possible frequencies.
- The passbands are separated by stopbands where the specific mode cannot propagate.
- At both ends of the passband, the group velocity is zero and wave propagation stops.
- When group velocity and phase velocity are in the same direction, we speak about forward waves, if they are in different direction, we speak about backward waves. This has nothing to do with the direction of the particle travelling.

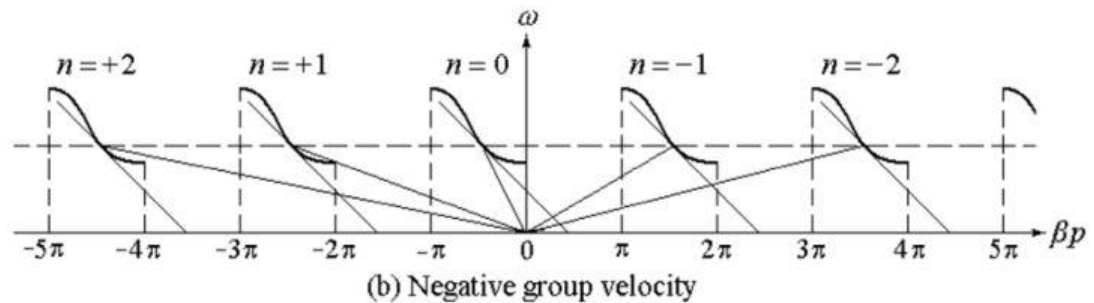


Slow wave structure (4/7)

- The request to fulfill the periodic boundary conditions leads to the concept of *space harmonics* (instead of wave modes).



- Space harmonics are closely connected the so-called *Floquet Theorem* (we will not cover this here).

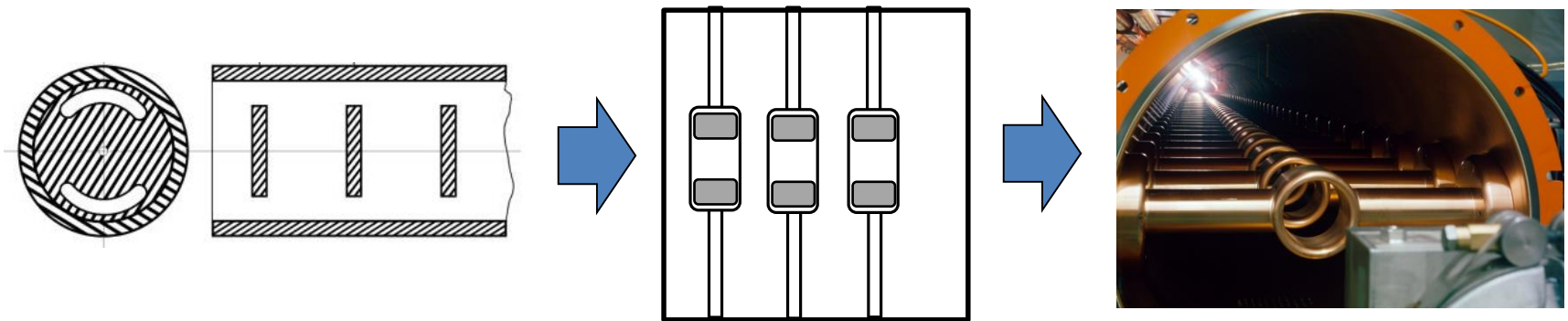


- The *Floquet Theorem* describes the relation between the phase constant of the n -th space harmonic and is a fundamental theorem of periodic structures.

Source: Zhang, *Electromagnetic Theory for Microwaves and Optoelectronics*, Springer

Slow wave structure (5/7)

- As was shown before, drift tubes can be used instead of separating discs to slow down the EM-field. The theory for drift tubes is very similar to calculating a slot-coupled structure. Just imagine that the slot is very large.



Inside view of the SPS travelling wave structure © CERN.

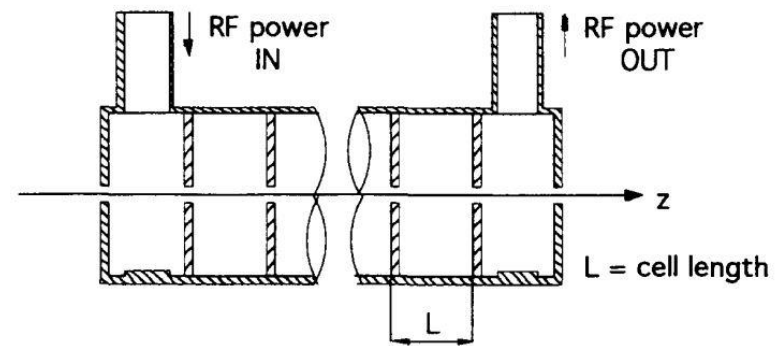
- In travelling wave (TW) mode, the field propagates through each cell.
- The phase advance per cell (distance between the discs, or periodic spacing of drift tubes) determines the length of the cell:

$$\Delta\varphi = \frac{2\pi}{\beta\lambda}l$$

← Remember? Is distance that a particle travels during one RF-period.

Slow wave structure (6/7)

- The only missing component for our travelling wave system are the input and output couplers.
- These have to be matching the structure to avoid that standing waves are building up inside the periodic structure.



- Often, higher order modes (HOMs) are building up in the structure due to the finite length (end covers are put there).

Note that a matching network is used to obtain the travelling state. This network is only matching the fundamental mode and not the HOMs!

I.e. the fundamental mode is travelling, others might be travelling, partially travelling or standing modes.

Slow wave structure (7/7)

SPS 200 MHz travelling wave cavity (16m long).

Backward wave structure

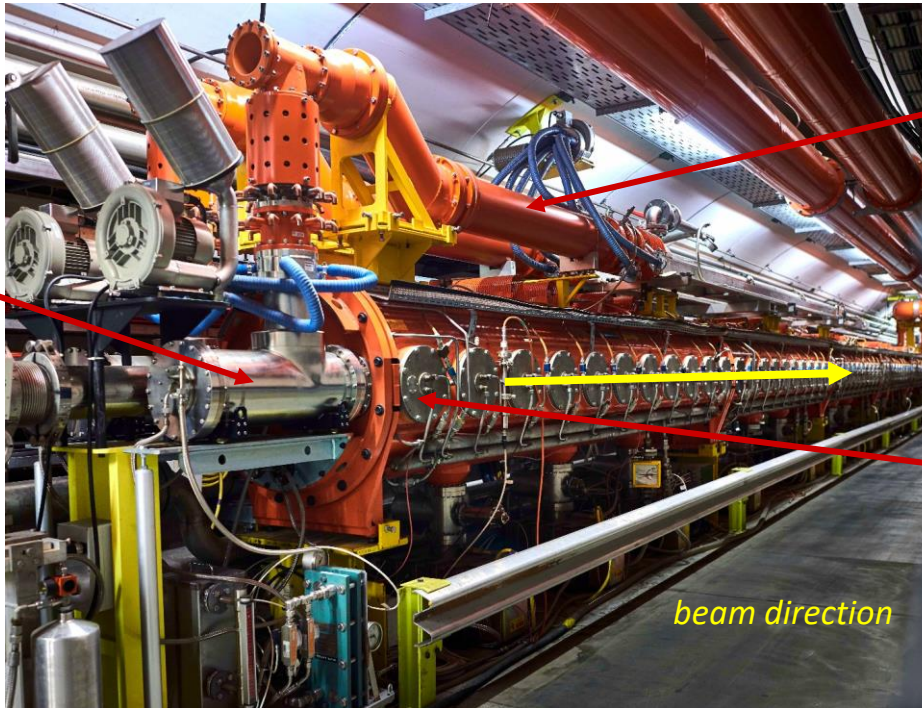
RF OUT side

Water cooled coaxial load

RF IN side

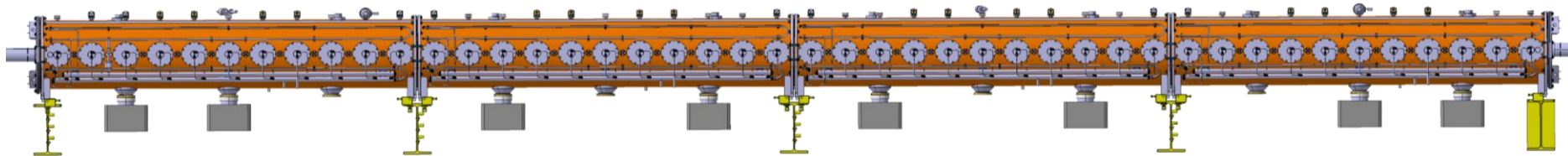
Drift tube with water cooling

beam direction

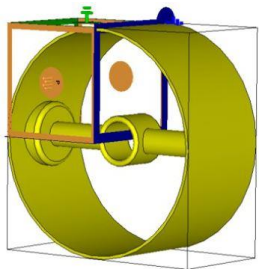


200 MHz TWC of SPS (1/2)

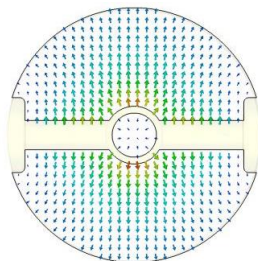
courtesy: E. Montesinos, CERN



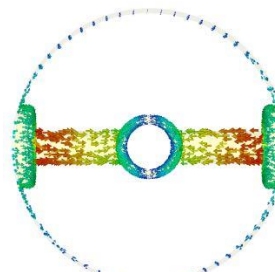
- Cavity is part of the LHC injector chain and was upgraded recently and equipped with new power amplifiers to reach a higher accelerating voltage.
- The cavity was entirely modelled in CST, so that we could well see the fields of the fundamental and the HOMs.



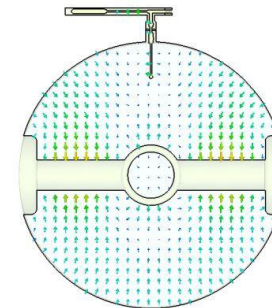
Single cell CST model with symmetries (no HOM coupler)



Electric field of fundamental mode in the Cross-section



Surface currents on the Drift tube

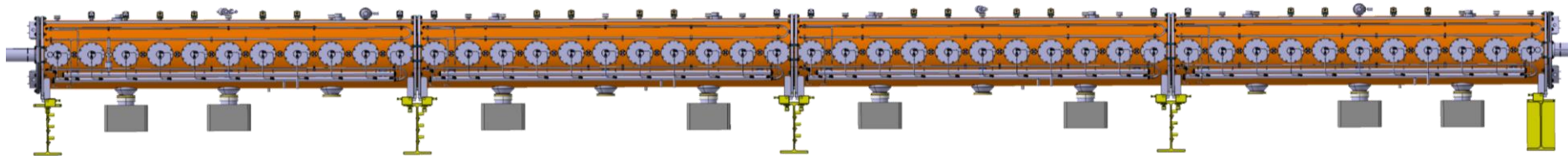


E-field in a cell with HOM-coupler ($22\pi/33$ mode)

Modelling in CST (P. Kramer, CERN)

200 MHz TWC of SPS (2/2)

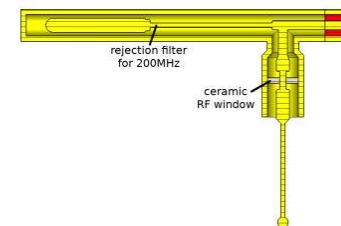
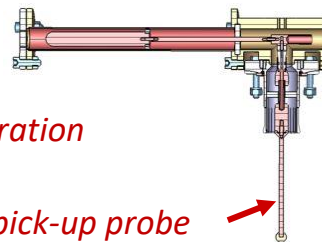
courtesy: E. Montesinos, CERN



- Cavity is part of the LHC injector chain and was upgraded recently and equipped with new power amplifiers to reach a higher accelerating voltage.
- A number of HOMs are developing, mostly as standing wave and these were taken out by HOM-couplers. Most harmful was the mode at 630 MHz, each section of the cavity has a number of these Hom couplers installed:

Schematic illustration

E-field pick-up probe



*Modelling in CST
(P. Kramer, CERN)*