CAS – Introduction to Accelerator Physics

# Collective effects

**Part II: Space charge *cont.***

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In the last lecture, we have learned about the **dynamics and the representation of multiparticle systems**. We have seen how we differentiate between **incoherent and coherent motion**. Linked to this, we looked at the phenomenon of **filamentation with decoherence and emittance blow-up**.

We also discussed a first collective effect – **direct space charge**. We saw that direct space charge, for the case of a uniform coating beam, leads to an **tune shift of all witness particles**.

We will now look at the **space charge induced tune footprint** and into some of the **mitigation methods** for direct space charge and then discuss some of the effects of **indirect space charge**. We will then move to a phenomenon known as **wake fields**.

- Part 2: Direct- and indirect space charge
    - Direct space charge – impact on machine performance
    - Direct space charge – mitigation techniques
    - Indirect space charge
    - From indirect space charge to (resistive) wall wakes

We have seen the impact of direct space charge for the case of a uniform coasting beam. We also got a first idea of the scaling laws for direct space charge in general.

Let’s have a look now at the more realistic case of non-uniformly charged, bunched beams and the **impact of direct space charge** on the beam and on the **machine performance**.

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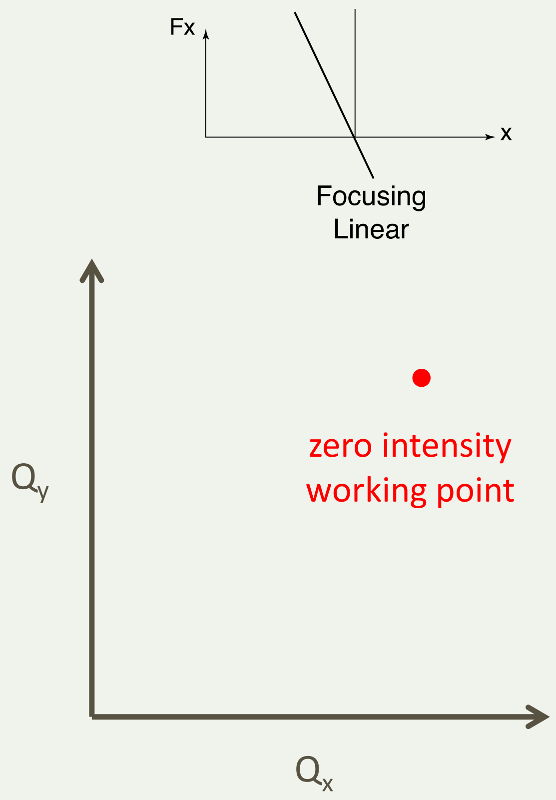
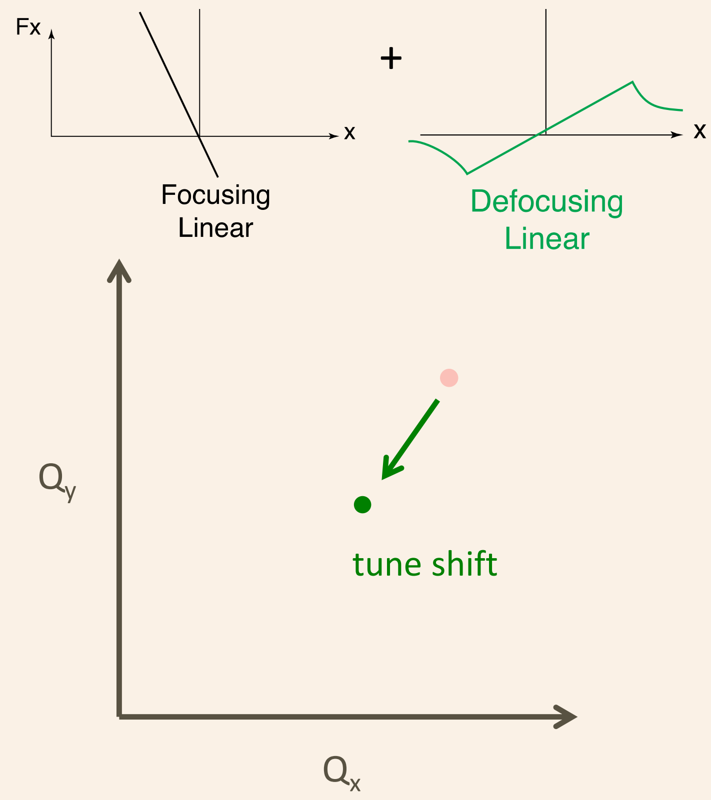
# Coasting beam tune shifts

without space charge



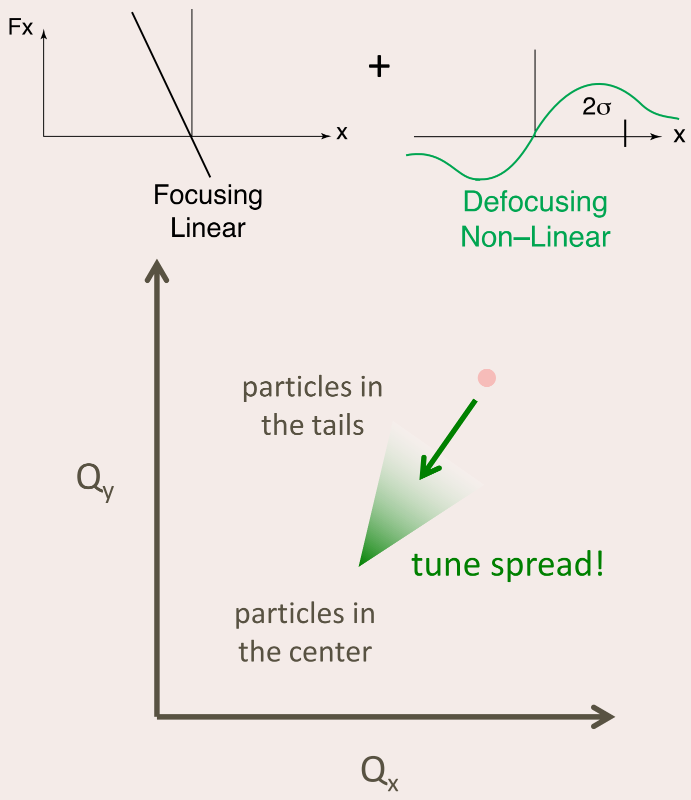

- All particles have the tunes $Q_x$ and $Q_y$ determined by the machine quadrupoles

with space charge
**uniform distribution**




- All particles have the tunes $Q_x$ and $Q_y$ determined by the machine quadrupoles and the **linear defocusing** from space charge

with space charge
**Gaussian distribution**




- Particles have a different tunes, since the space charge **defocusing depends on the particles' amplitude**
- The tune shift is largest for particles in the beam center

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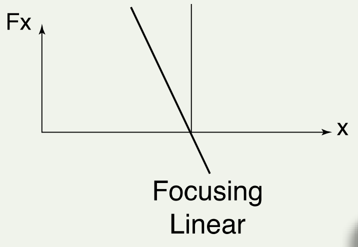
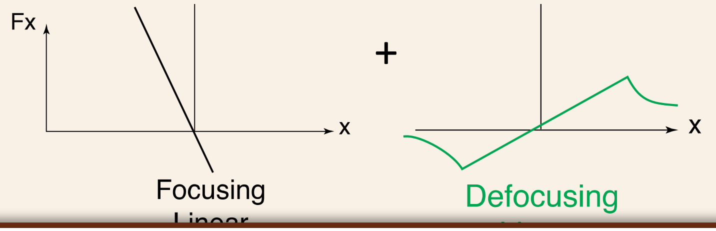
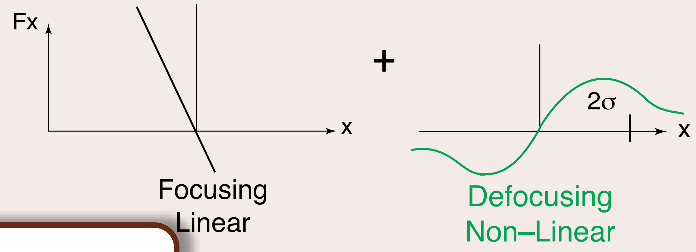
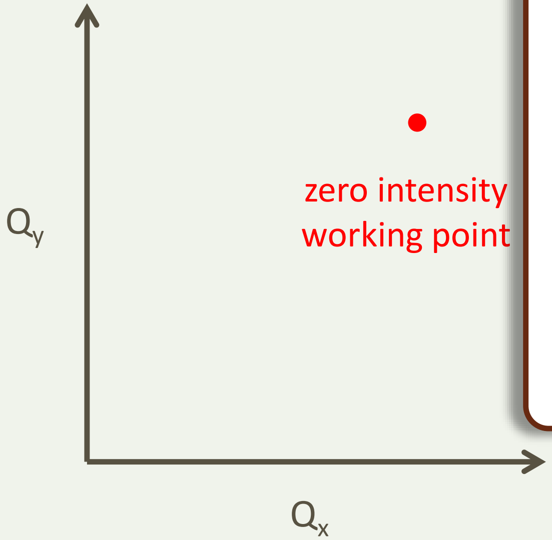



with space charge
**uniform distribution**

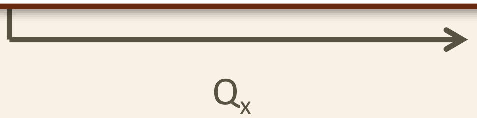


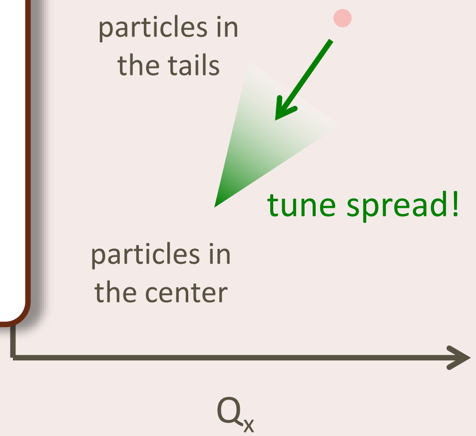

with space charge
**Gaussian distribution**




Note: the maximum space charge tune shift in a **Gaussian beam distribution is stronger compared to the detuning in a uniform beam distribution** when considering similar beam sizes due to the higher particle density in the beam core!

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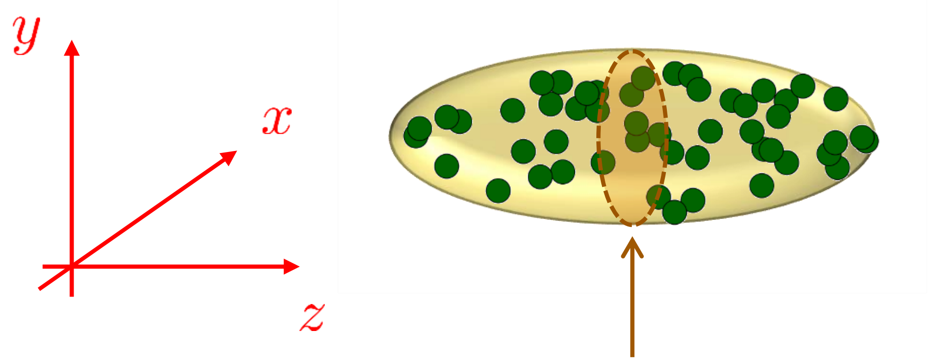
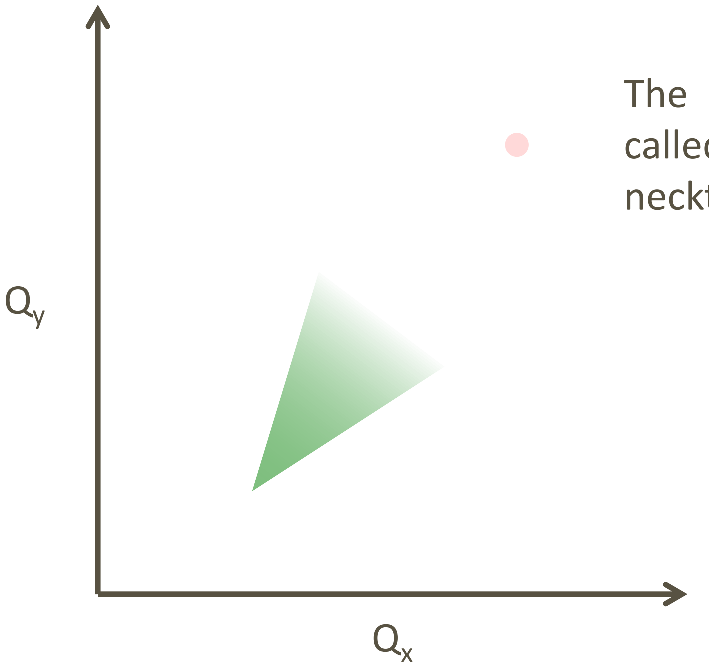
# Coasting vs. bunched – Gaussian

- In beams with **Gaussian transverse distribution** we observed already for coasting beams with constant line density a tune spread due to the nonlinear force and the resulting dependence on the transvers particle amplitude
- In case a Gaussian beam **is also bunched, an additional tune spread** is induced by the variation of the line density




Particles close to the peak line density (often in the bunch center) will have the largest tune spread




The tune footprint is also called the space charge necktie due to its shape

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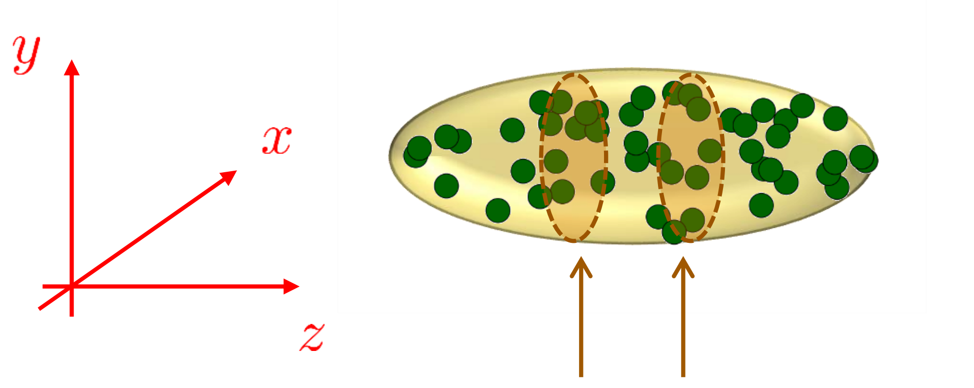
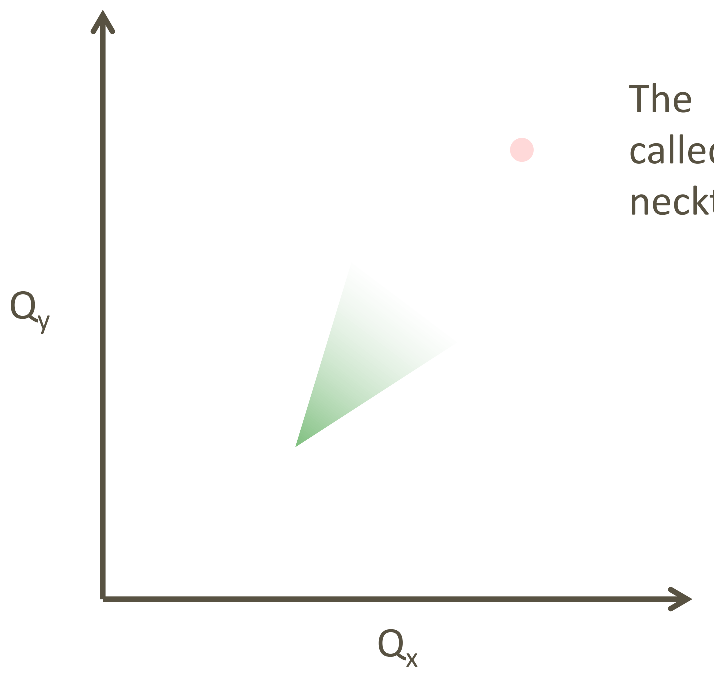



The tune spread is reduced for particles further away from the peak line density

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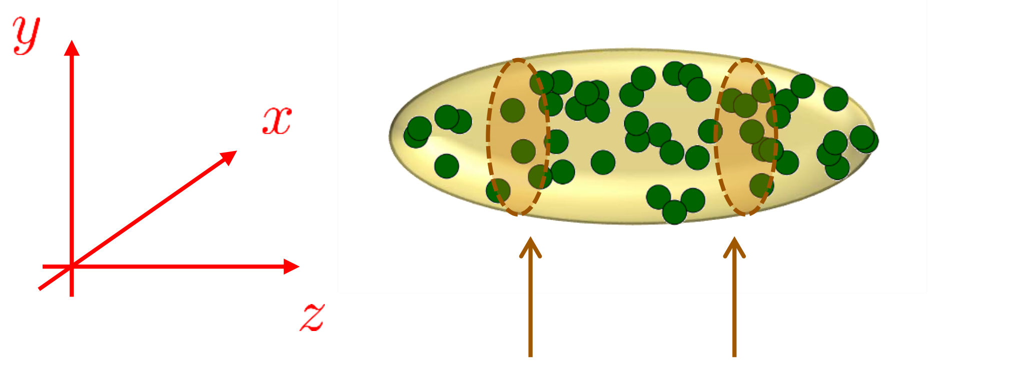


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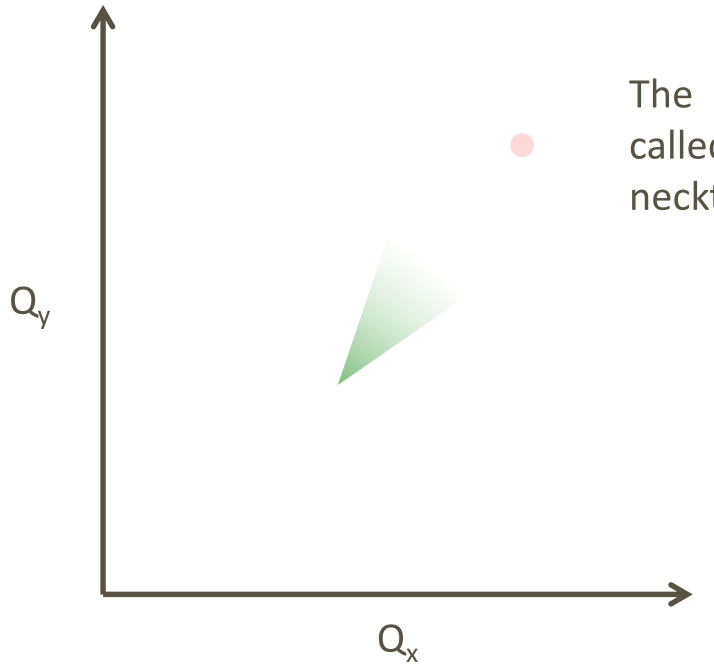


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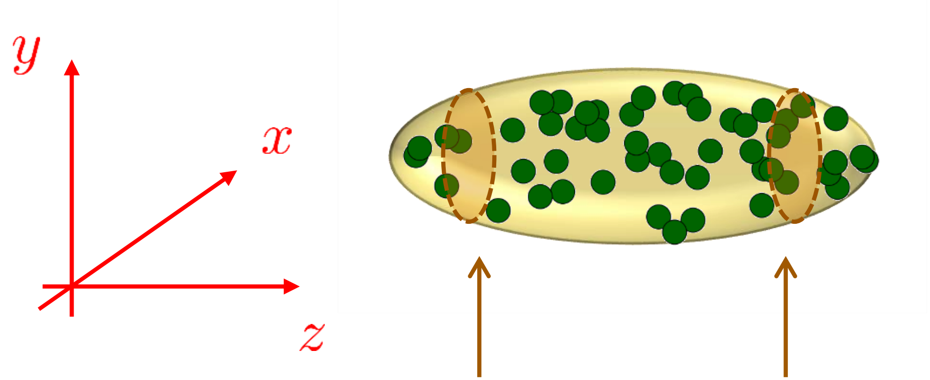


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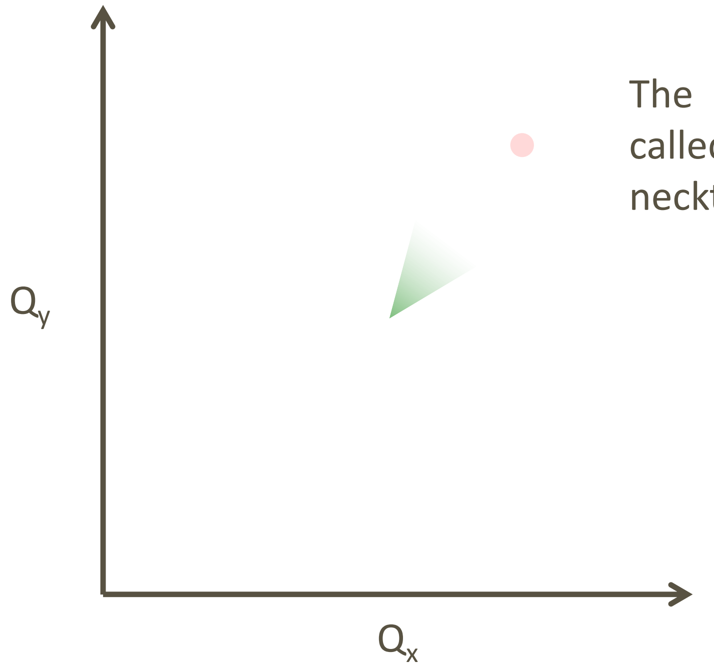


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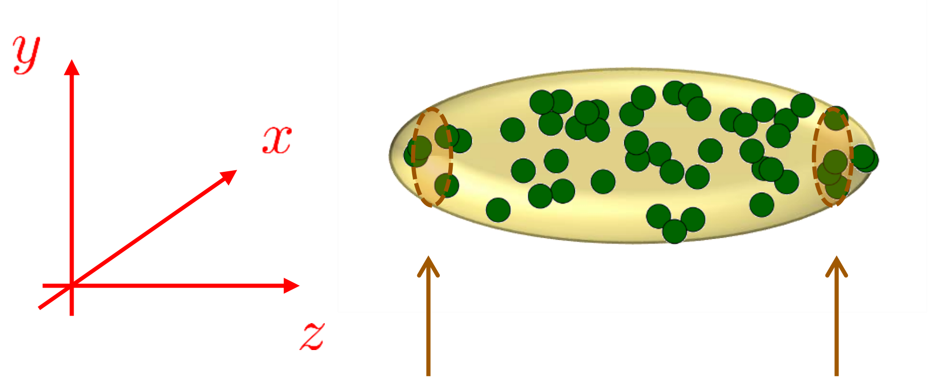


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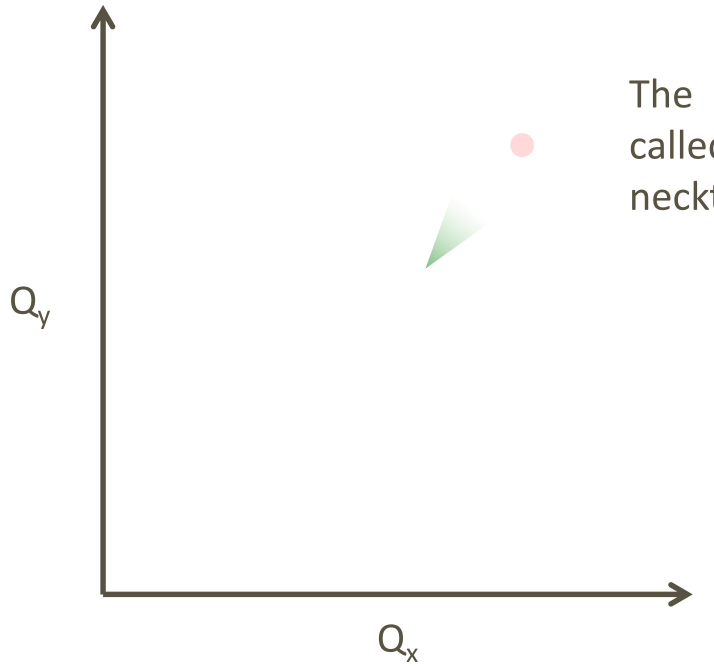


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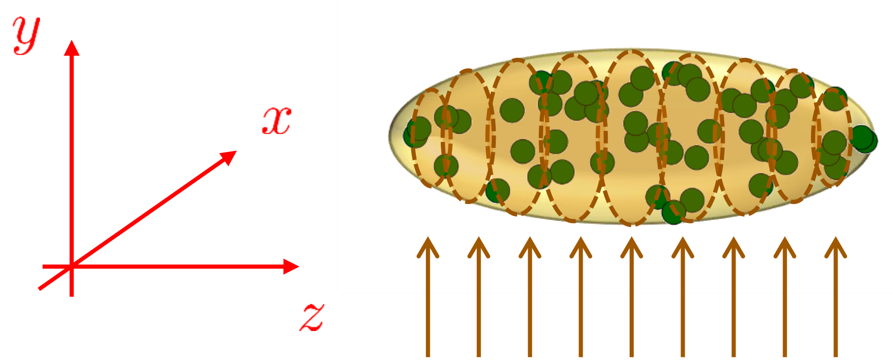
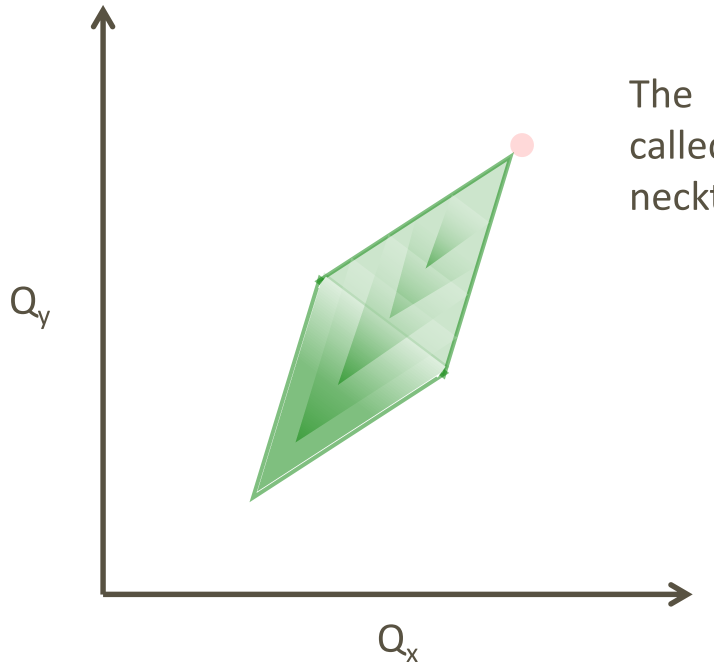
> - The longitudinal variation of the transverse space-charge force due to the line density fills the gap until the zero-intensity working point
> - The tune of individual particles is modulated by twice the synchrotron period




For bunched beams the tune spread is as big as the maximum tune shift!

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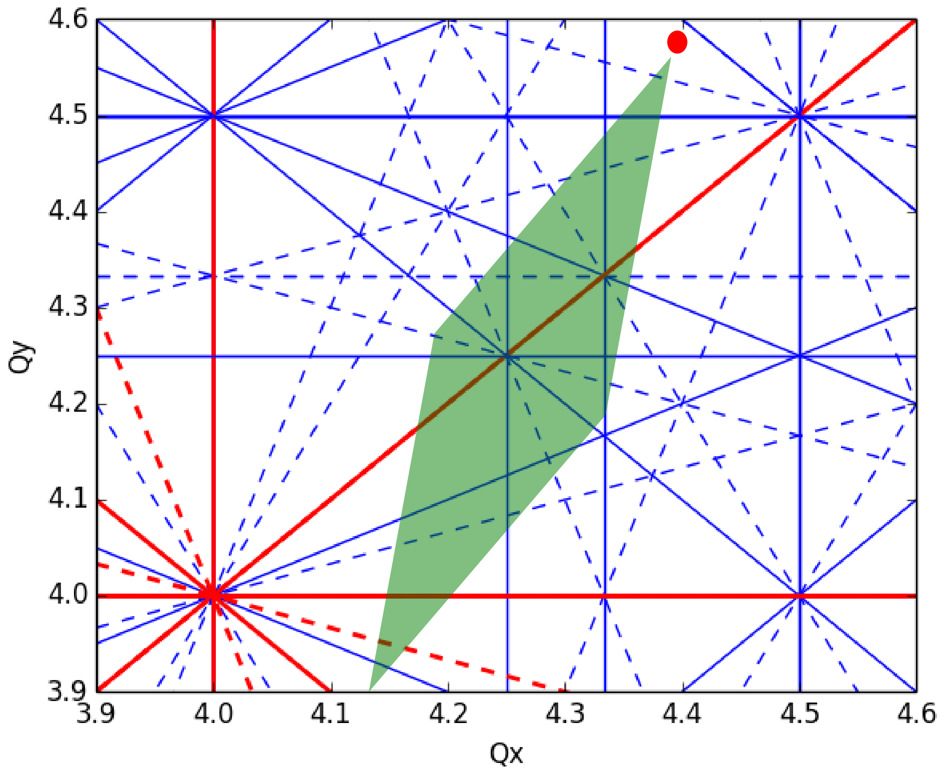
# Brightness limitation due to space charge

- A space charge tune spread beyond 0.5 can barely be tolerated without excessive **emittance blow-up and/or particle loss due to resonances**
  - Dipole errors in the machine excite the integer resonances (Q=n)
  - Quadrupole errors excite the half integer resonances (Q=n+1/2)
  - Higher order resonances can be excited due to sextupoles and multipole errors

- Imagine that a beam with a tune spread of beyond 0.5 is injected



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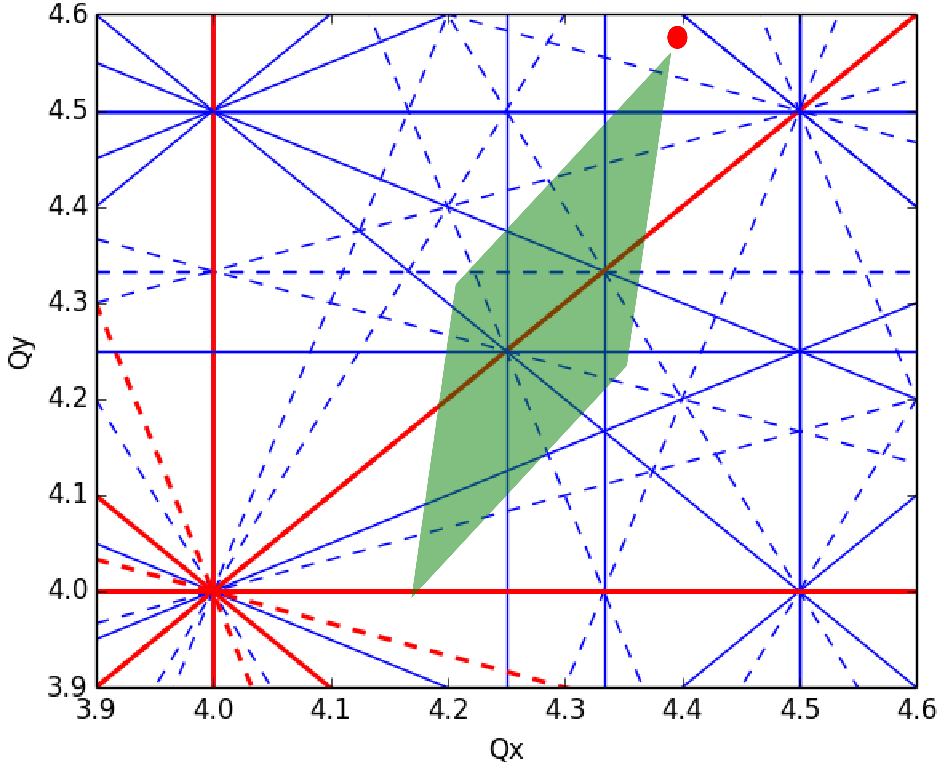
- Particles in the beam core will cross the integer resonance resulting in **emittance blow-up** and a reduction of the tune spread



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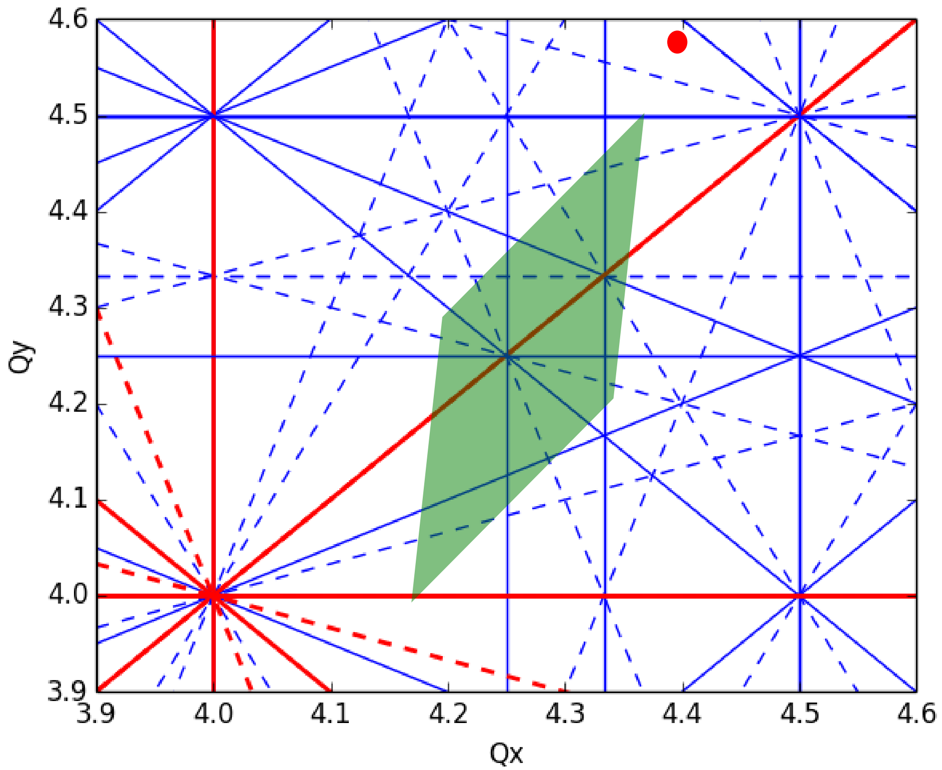
- Imagine that a beam with a tune spread of beyond 0.5 is injected
- Particles in the beam core will cross the integer resonance resulting in **emittance blow-up** and a reduction of the tune spread
- Particles in the beam tails can be pushed onto the half integer resonance resulting in **losses due to aperture restrictions**



Direct space charge effects have the undesired property of **generating incoherent tune spreads** which can be a problem for dynamic aperture and emittance preservation.

We will look at a few of the different techniques used for **the mitigation of direct space charge** effects.

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# Direct space charge tune shift

After some reshuffling we notice some of the **fundamental properties of the direct space charge tune shift**:

$$\left.\begin{aligned} \Delta Q_{x,y} &= -\frac{r_0 R \lambda}{e\beta^2\gamma^3}\left\langle \frac{\beta_{x,y}(s)}{a^2(s)} \right\rangle \\ a(s) &= \sqrt{\frac{\beta_{x,y}(s)\hat{\varepsilon}^n_{x,y}}{\beta\gamma}} \end{aligned}\right\} \implies \boxed{\Delta Q_{x,y} = -\frac{r_0 R \lambda}{e\beta\gamma^2\hat{\varepsilon}^n_{x,y}}}$$

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} = \begin{cases} 1.54 \cdot 10^{-18} \text{ m (proton)} \\ 2.82 \cdot 10^{-15} \text{ m (electron)} \end{cases}$$

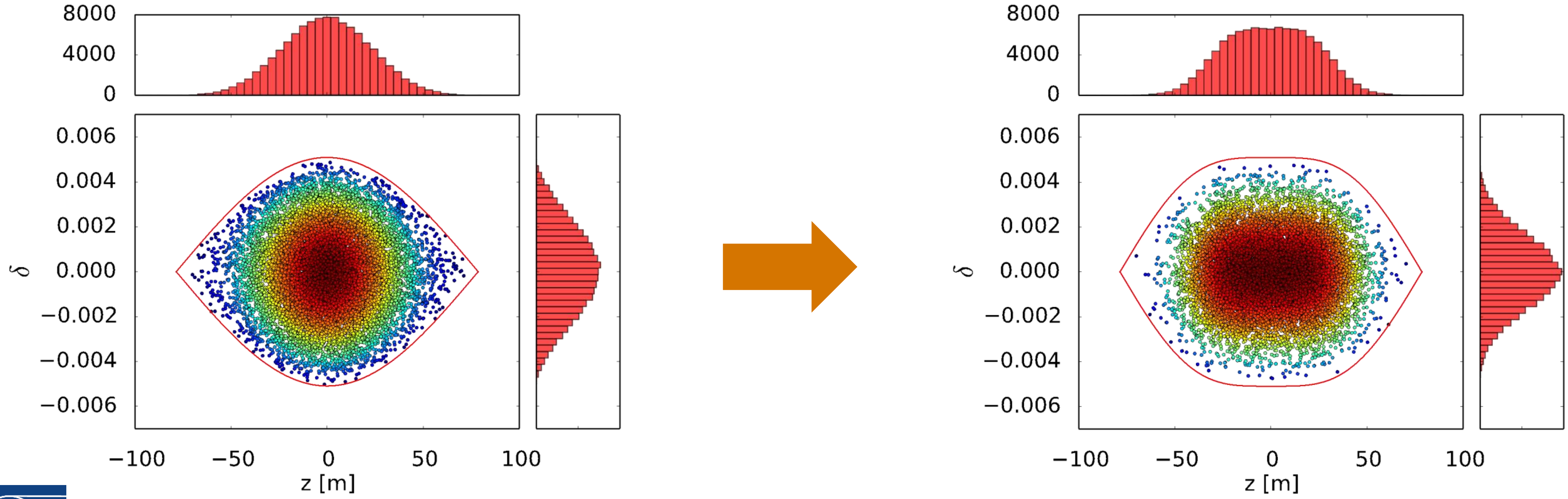
- is negative, because space charge transversely always defocuses
- is proportional **to the line density** and thus to the number of particles in the beam
- decreases **with energy like $1/(\beta\gamma^2)$** (when expressed in terms of normalized emittance) and therefore vanishes in the ultra-relativistic limit
- does not depend on the local beta functions or beam sizes but is inversely proportional to the normalized emittance (here the emittance includes all particles!)

# Mitigation techniques

Maximum tune shift (circular Gaussian)

$$\Delta\hat{Q}_{x,y} = -\frac{r_0 C\hat{\lambda}}{2\pi e\beta\gamma^2}\frac{1}{2\,\varepsilon^n_{x,y}}$$

- Decrease the peak line density by
  - maximizing the bunch length
  - flattening the bunch profile with a specially configured (double harmonic) RF system



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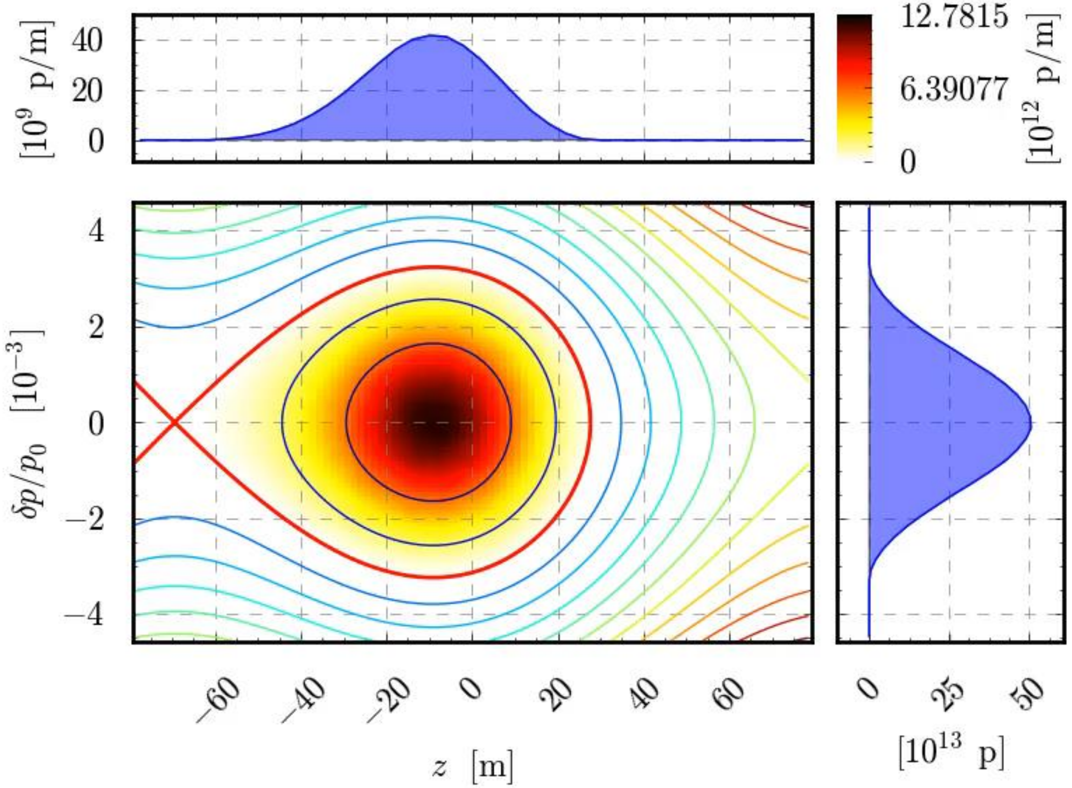
  - using **bunch distributions with small peak density** (e.g. parabolic instead of Gaussian)
  - reducing the central density of the particle distribution (e.g. "hollow bunches")



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- Increase the beam energy by
  - accelerating the beam as quickly as possible
  - increasing the injection energy (usually requires an upgrade of the pre-injector)

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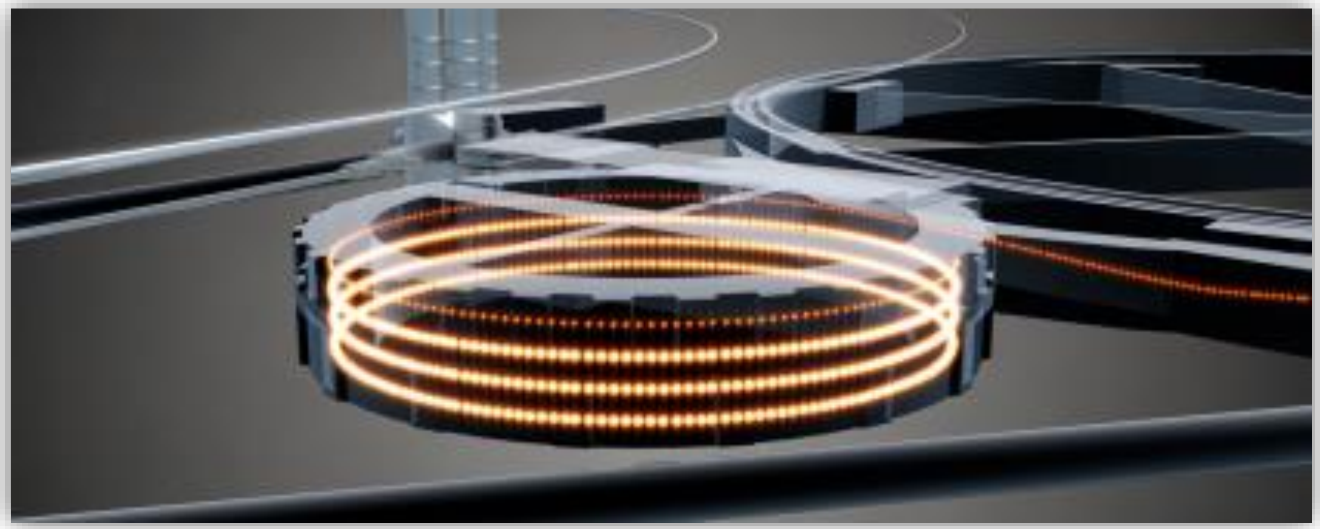
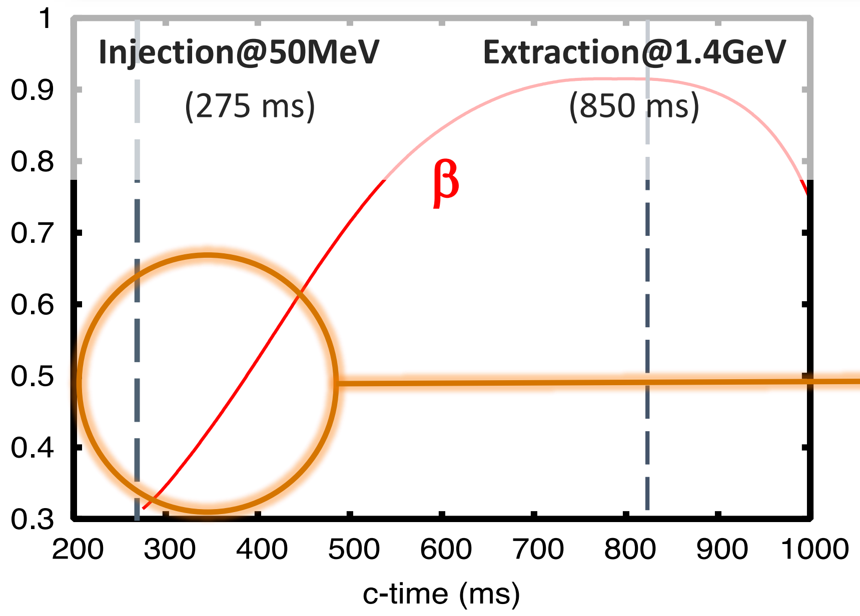
- Minimize the machine circumference
  - when designing/building a new accelerator since the **space charge detuning is an integrated effect**

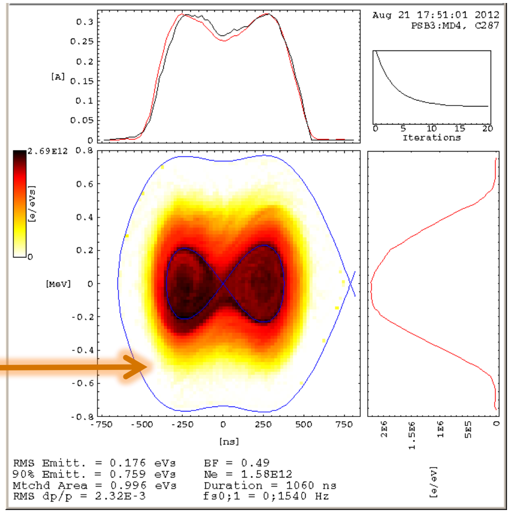
# Direct space charge in the CERN PSB

An accelerator that is particularly subject to space charge effects and makes use of **several mitigation schemes at the same time** is the CERN PS Booster:

- The PSB used to accelerate high brightness beams from 50 MeV to 1.4 GeV over 530 ms
- Instead of one large ring is it made up **of 4 smaller rings** to reduce the integrated effect
- Space charge is important, especially in first part of the cycle – **bunch is flattened** through a second harmonic RF system
- With the LIU **the injection energy** has been increased from 50MeV to 160MeV





Direct space charge leads to a **purely incoherent tune shift** and a tune spread leading to the characteristic tune footprint with its **typical necktie shape** for bunched beams. Direct space charge does not lead to any coherent tune shifts (on the centroid motion).

We will now look at the effect of **indirect space charge**. We will briefly look at the **different sources** of indirect space charge, that it can lead to both **incoherent as well as coherent tuneshifts** and how these tune shifts are usually parameterized using the **Laslett coefficients**.

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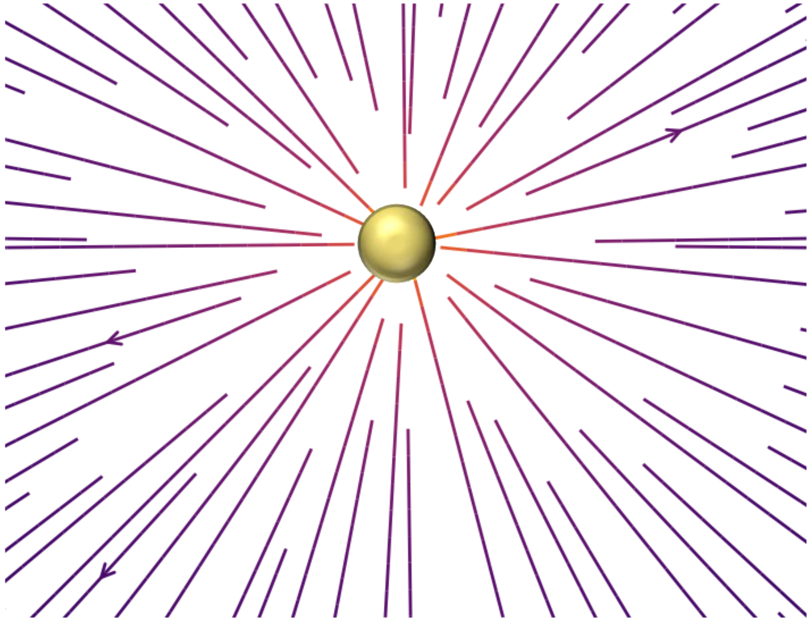
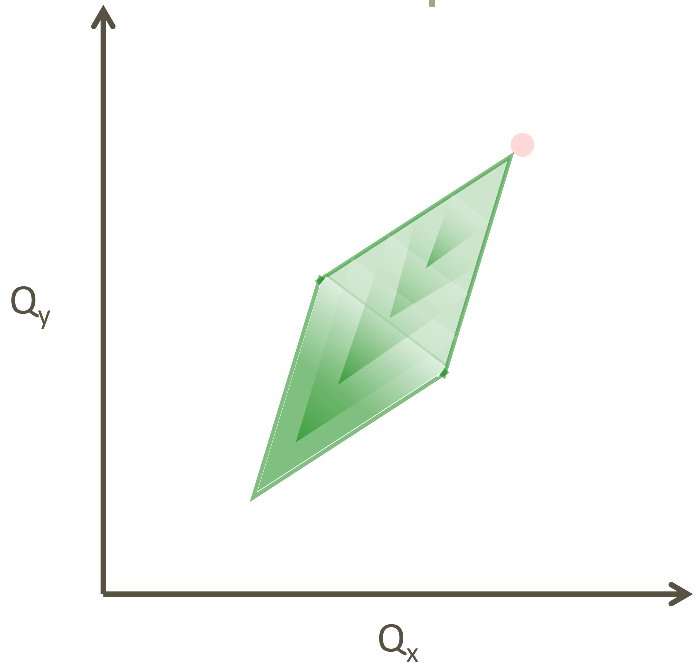
# Direct vs. indirect space charge

- Direct space charge – central force
- The space symmetry endowed by free space **eliminates any impact on the centroid** motion

**→ Exclusively incoherent tune shifts**



**Tune footprint - necktie**



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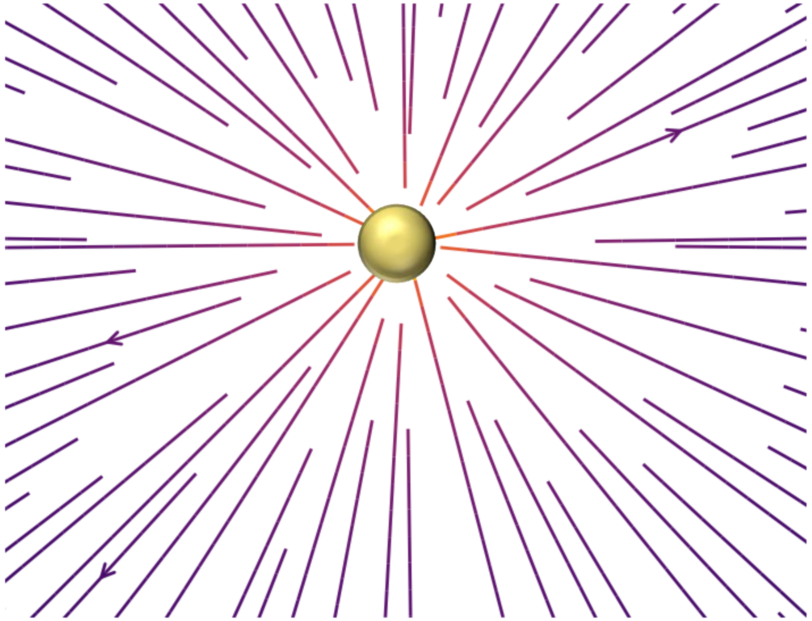
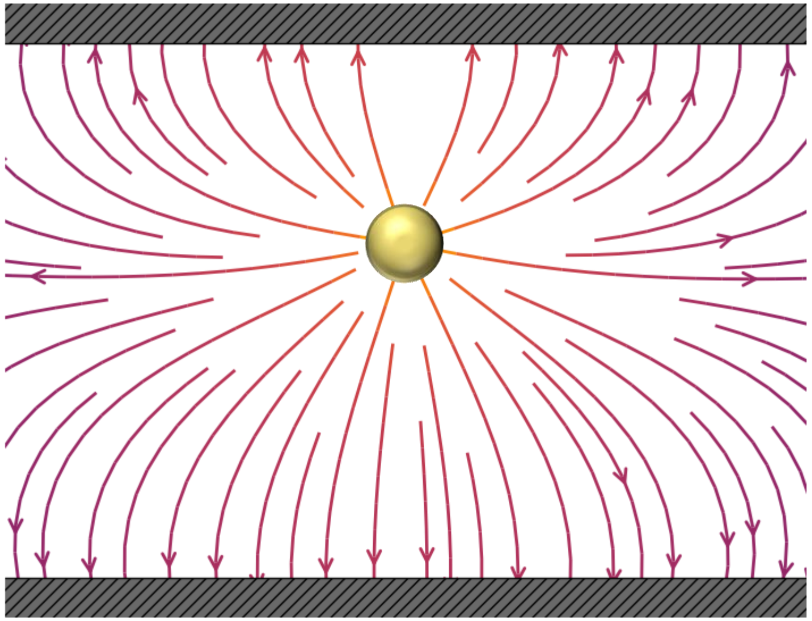


- Indirect space charge – image charge forces
- The free space symmetry is broken by the geometrical arrangement of the conducting parallel plates; this gives a **net impact on the centroid** motion

→ **Both incoherent and coherent tune shifts**



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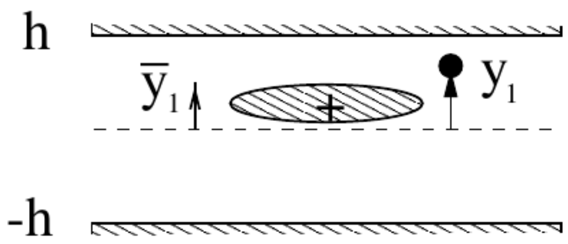
Electro- and magneto-static forces can be computed with the **method of image charges:**

- **We consider the beam as line charge density λ with infinite length**

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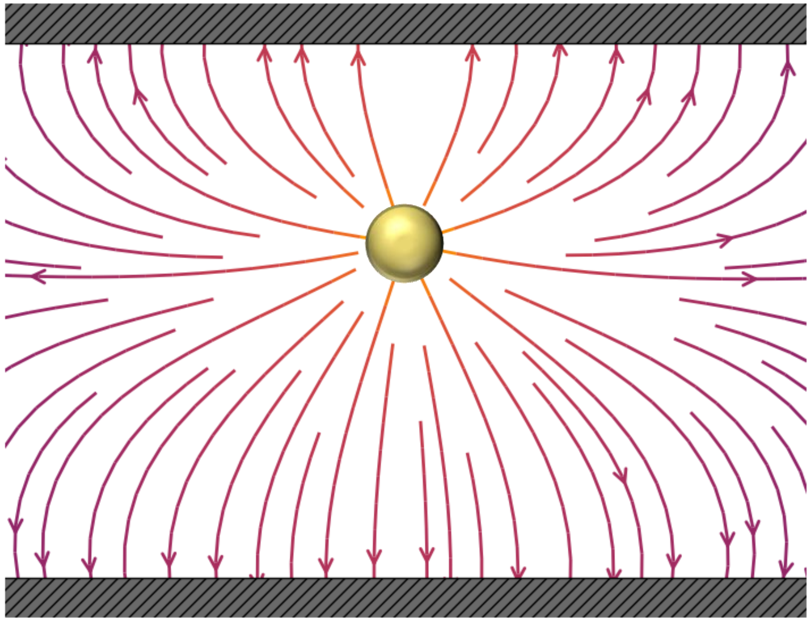

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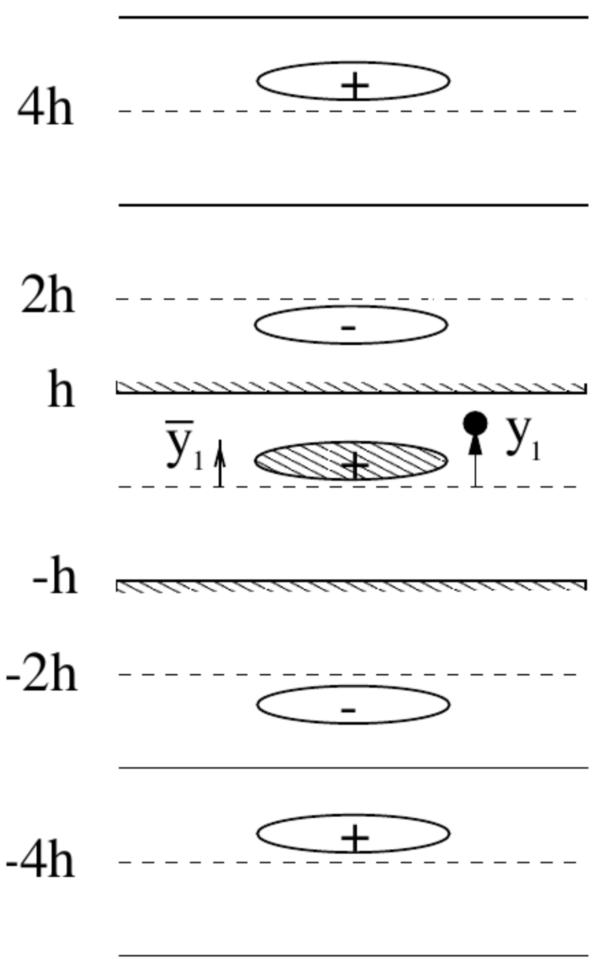


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Electro- and magneto-static forces can be computed with the **method of image charges:**

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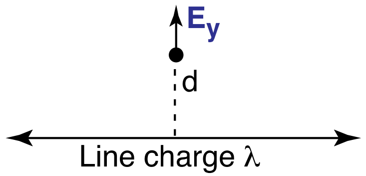
- **We iteratively add charges to satisfy the boundary conditions. The resulting sum of fields converges to the final net field.**
- **The electric field at distance d is given by Gauss' law**




$$E_y = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{d}$$

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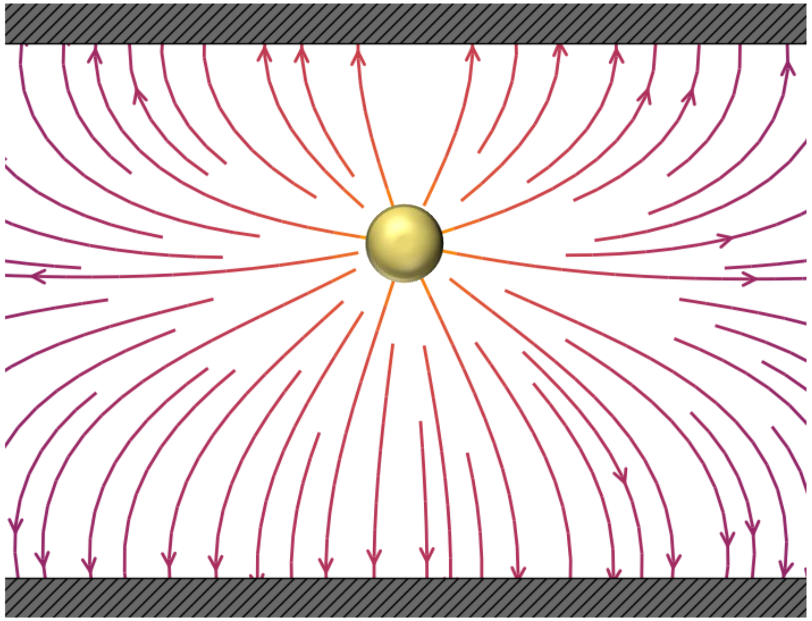

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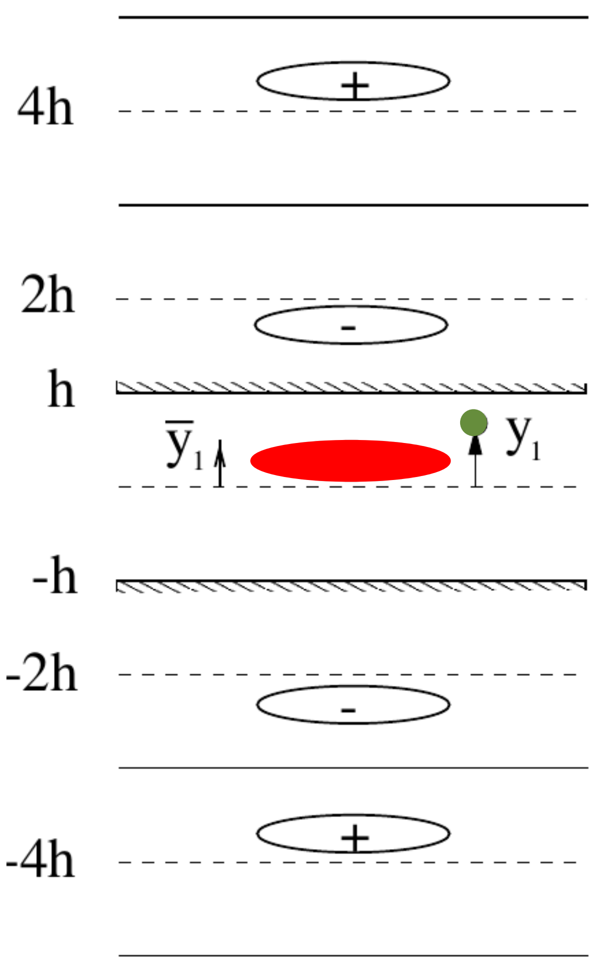


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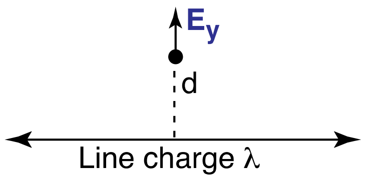
- **The electric field at distance d is given by Gauss' law**




$$E_y = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{d}$$

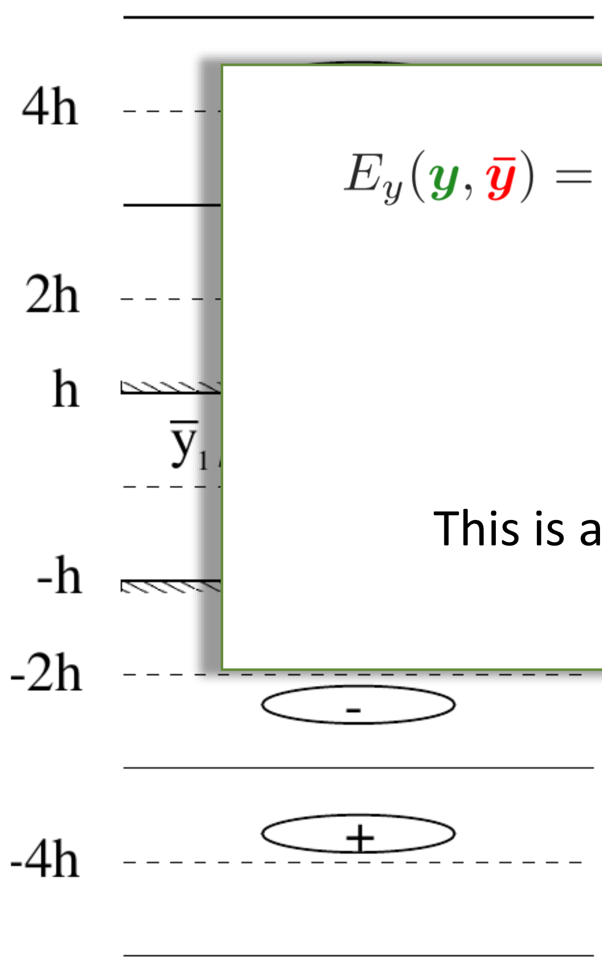
- A flat vacuum chamber can be approximated by two perfect conducting parallel plates placed at vertical positions +h and –h
- Let the **beam centroid be displaced vertically by $\bar{y}$**
- The **witness particle is at y**
- The **boundary condition** at the two perfect conducting plates is satisfied by superposing **an infinite number of image line charges** with alternating signs as shown in the sketch
- The resulting electric field as a **function of beam and witness particle offsets** can be computed as

# Direct vs. indirect space charge



Electro- and magneto-static forces can be computed with the **method of image charges:**

- A flat vacuum chamber can be approximated by two perfect conducting parallel plates placed at vertical positions +h and –h




$$E_y(\boldsymbol{y}, \boldsymbol{\bar{y}}) = \frac{\lambda}{2\pi\epsilon_0}\left[ + \frac{1}{2h - \bar{y} - y} - \frac{1}{2h + \bar{y} + y} + \frac{1}{6h - \bar{y} - y} - \frac{1}{6h + \bar{y} + y} + \dots \right.$$

$$\left. - \frac{1}{4h + \bar{y} - y} + \frac{1}{4h - \bar{y} + y} - \frac{1}{8h - \bar{y} - y} + \frac{1}{8h + \bar{y} + y} + \dots \right]$$

This is an infinite sum which, however, actually converges…

- **We iteratively add charges to satisfy the boundary conditions. The resulting sum of fields converges to the final net field.**

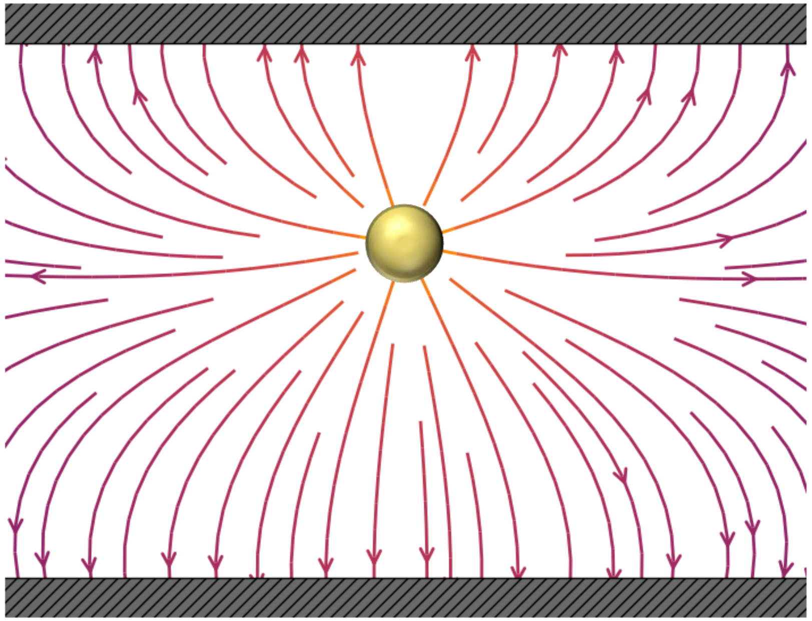
$$\boxed{E_y(\boldsymbol{y}, \boldsymbol{\bar{y}}) = \frac{\lambda}{\pi\epsilon_0 h^2}\left[(\bar{y} + y)\frac{\pi^2}{32} + (\bar{y} - y)\frac{\pi^2}{96}\right] = \frac{F_y}{e}}$$

$$\bar{y}_1, y_1 \ll h\,, \quad E_{\parallel} = 0$$

# Image charge forces

- Static **electric fields vanish** inside a conductor for any finite conductivity, while **static magnetic fields pass through** (unless in case of very high permeability)
- This is **no longer true for time varying fields** – electromagnetic fields will **penetrate the material according to the skin depth $\delta_w$** which depends on the frequency and the material properties
- We distinguish between **magnetic dc fields** (low frequency where $\delta_w >> \Delta_w$) and **magnetic ac fields** (high frequency where $\delta_w << \Delta_w$)

- We have seen how **electric image charge force** are induced by electrostatic fields in the presence of (perfectly) conducting boundaries
- Similarly, there are also **magnetics image current forces** – here we need to **differentiate between AC and DC forces**



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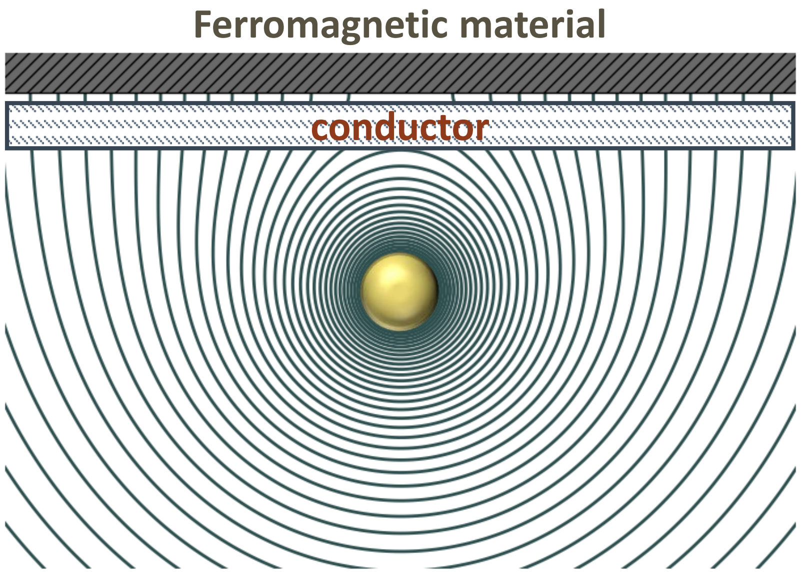
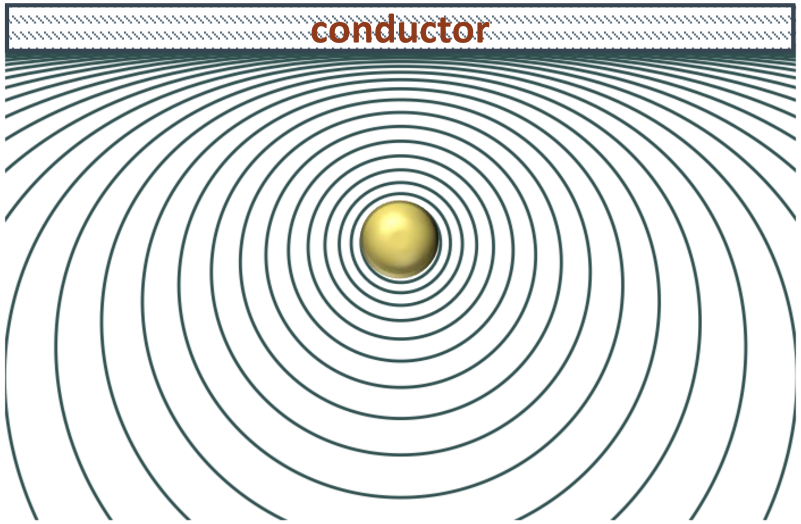



At **low frequencies magnetic fields penetrate** and pass through the vacuum chamber, they can interact with bodies behind the chamber




If the skin depth is very small (**rapidly varying fields**), **magnetic fields do not penetrate** and the field lines are tangent to the surface.

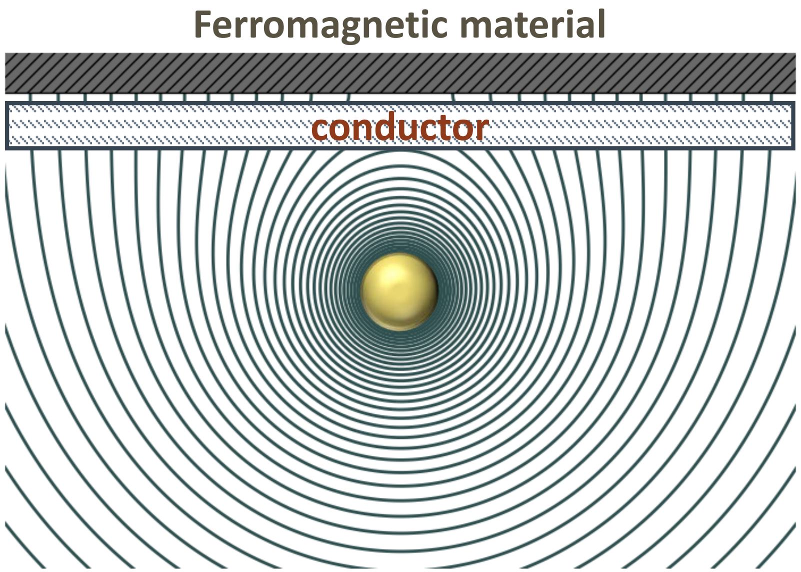
# Image current forces

- In a very similar fashion as done for the electric case we can also here deploy the **method of image currents** to solve for the magnetic fields and forces

- We obtain forces of the form

$$\frac{F_y}{e} = -\frac{\lambda\beta^2}{\pi\epsilon_0 h^2}\left[(\bar{y}+y)\,\frac{\pi^2}{32} + (\bar{y}-y)\,\frac{\pi^2}{96}\right]$$

$$\frac{F_y}{e} = +\frac{\lambda\beta^2}{\pi\epsilon_0 h^2}\left[(\bar{y}+y)\,\frac{\pi^2}{32} - (\bar{y}-y)\,\frac{\pi^2}{96}\right]$$

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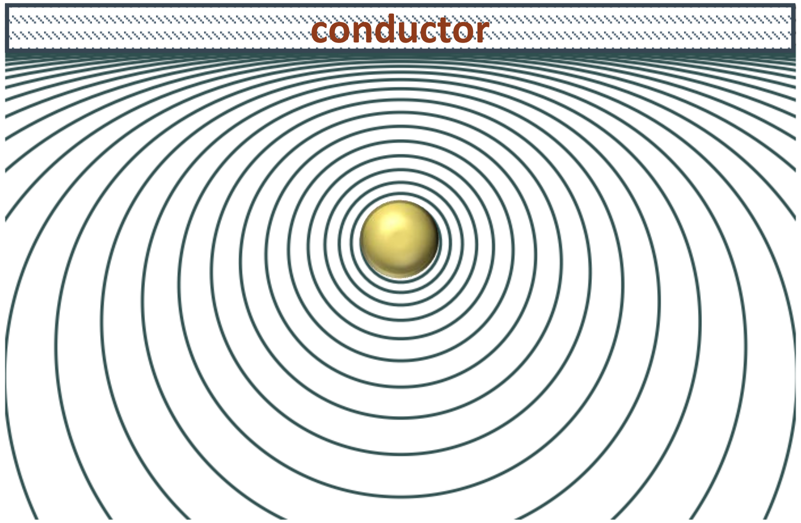


If the skin depth is very small (**rapidly varying fields**), **magnetic fields do not penetrate** and the field lines are tangent to the surface.

# Tune shifts and Laslett coefficients

- These electric image charge and magnetic ac and dc image current forces have the general similar form

$$F_x \propto \frac{e\lambda}{\pi\epsilon_0 h^2} f(\bar{x}, x) \qquad F_y \propto \frac{e\lambda}{\pi\epsilon_0 h^2} f(\bar{y}, y)$$

- These forces can lead to **both incoherent as well as coherent tune shifts** – from the forces, the tune shifts can be computed. In fact, it turns out that the **tune shifts can be parameterized** via the **Laslett coefficients**. For example, for electric image charge forces between two perfectly conducting parallel plates, the incoherent and coherent tune shifts can be expressed as:

$$\Delta Q_{x,y}^{\mathrm{inc}} = -\frac{2\,\langle\beta_{x,y}\rangle\, r_0 R}{e\beta^2\gamma}\,\frac{\varepsilon_1^{x,y}}{h^2} \qquad \Delta Q_{x,y}^{\mathrm{coh}} = -\frac{2\,\langle\beta_{x,y}\rangle\, r_0 R}{e\beta^2\gamma}\,\frac{\xi_1^{x,y}}{h^2}$$

The Laslett coefficients can be evaluated **for different geometries** and are classified in **incoherent and coherent tune shifts for electric, magnetic ac and magnetic dc** image charges and currents.

# Tune shifts and Laslett coefficients

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| Laslett coefficients | Circular $(w = h)$ | Elliptical (e.g. $w = 2h$) | Parallel plates $(h/w = 0)$ |
|---|---|---|---|
| $\varepsilon_1^x$ | $0$ | $-0.172$ | $-\pi^2/48$ |
| $\varepsilon_1^y$ | $0$ | $+0.172$ | $+\pi^2/48$ |
| $\xi_1^x$ | $+1/2$ | $0.083$ | $0$ |
| $\xi_1^y$ | $+1/2$ | $0.55$ | $+\pi^2/16$ |
| $\varepsilon_2^x$ | $-\pi^2/24$ | $-\pi^2/24$ | $-\pi^2/24$ |
| $\varepsilon_2^y$ | $+\pi^2/24$ | $+\pi^2/24$ | $+\pi^2/24$ |
| $\xi_2^x$ | $0$ | $0$ | $0$ |
| $\xi_2^y$ | $+\pi^2/16$ | $+\pi^2/16$ | $+\pi^2/16$ |

Assuming parallel plates for the ferro-magnetic boundary for all geometries …

* L. J. Laslett, LBL Document PUB-616, 1987, vol III

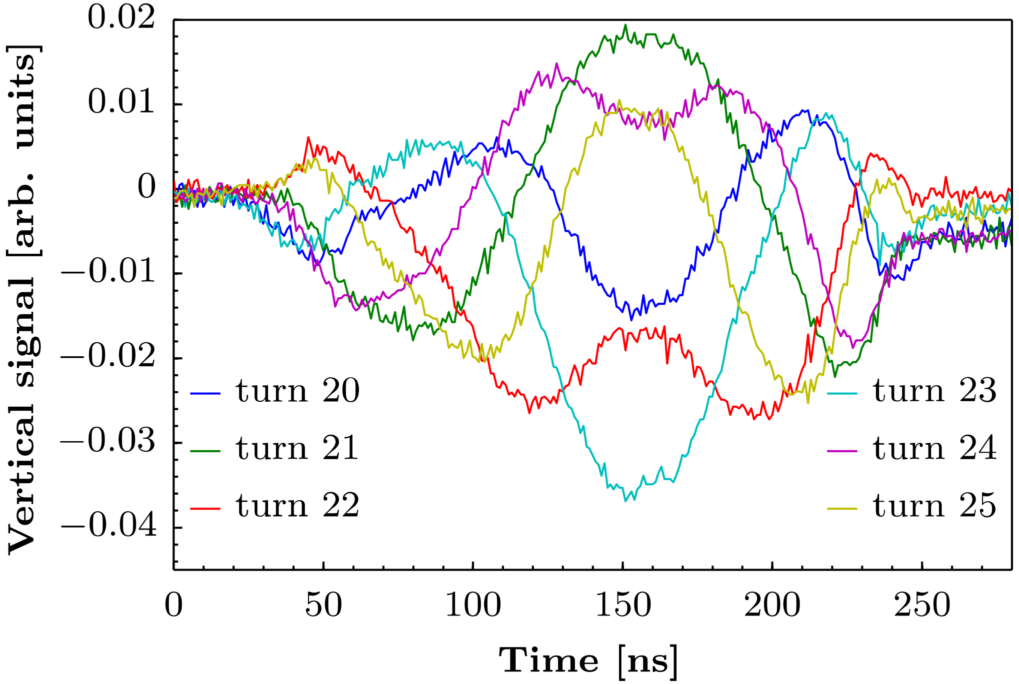
Note: they are always defined with respect to the *vertical half gap h or g*

The Laslett coefficients can be evaluated **for different geometries** and are classified in **incoherent and coherent tune shifts for electric, magnetic ac and magnetic dc** image charges and currents.

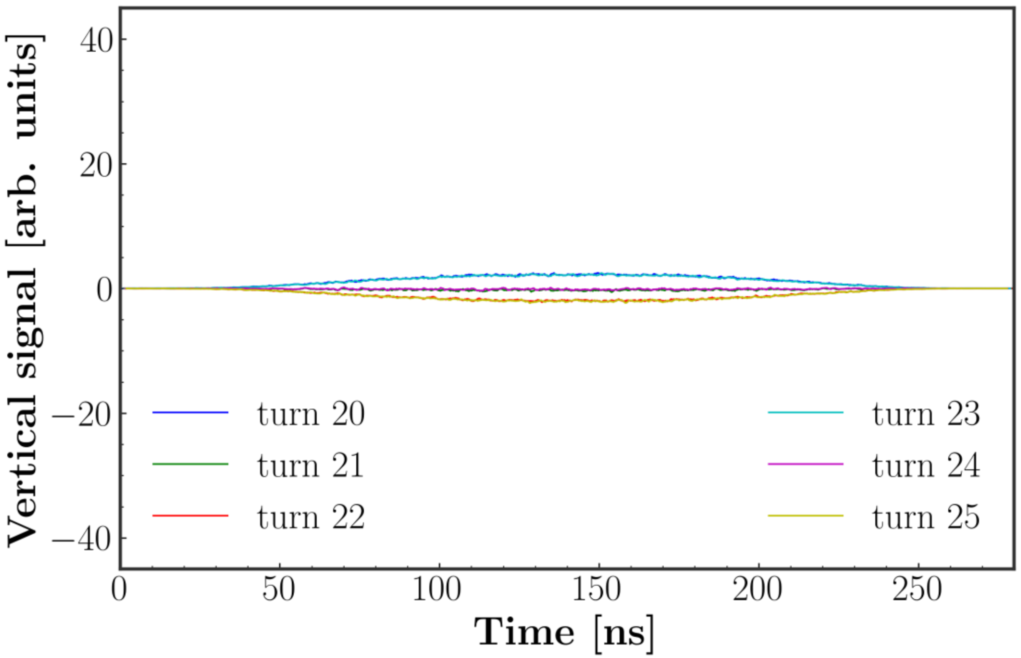
# Example: PS injection oscillations

- The observed intra-bunch motion was reproduced with an amazing precision with multi-particle simulations (HEADTAIL code) **including the indirect space charge effect** taking

**measurement**




**simulation**

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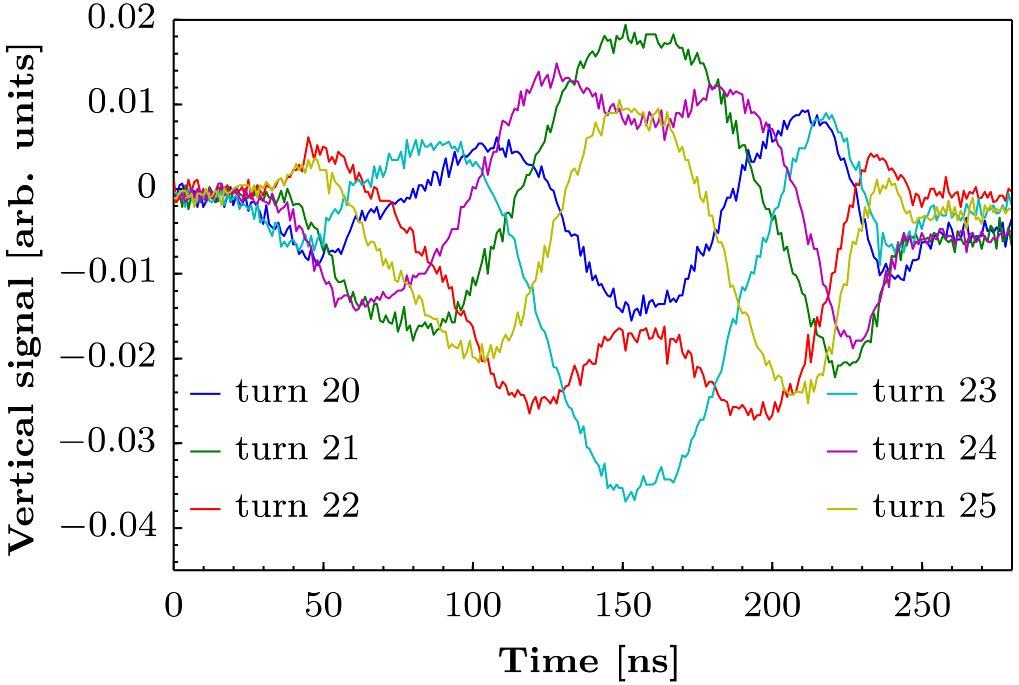
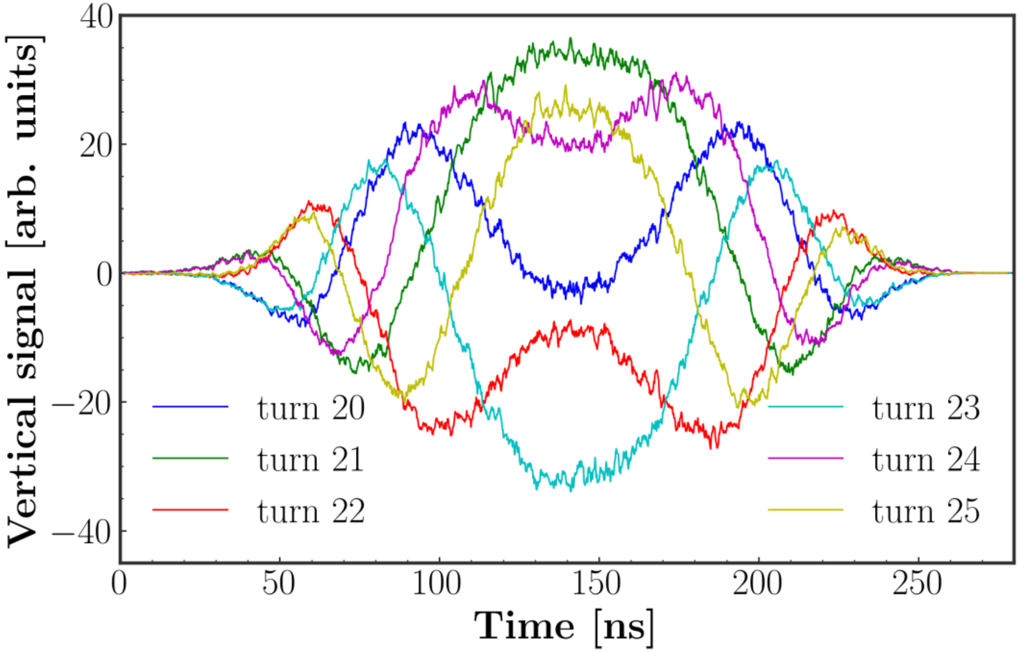



**simulation**

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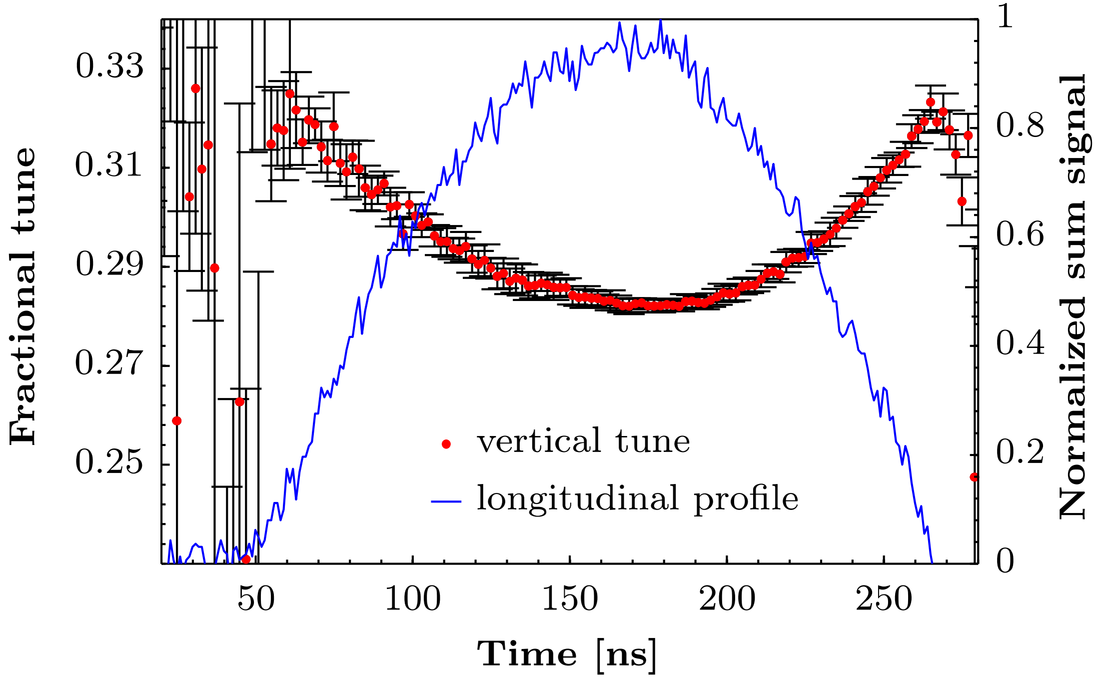
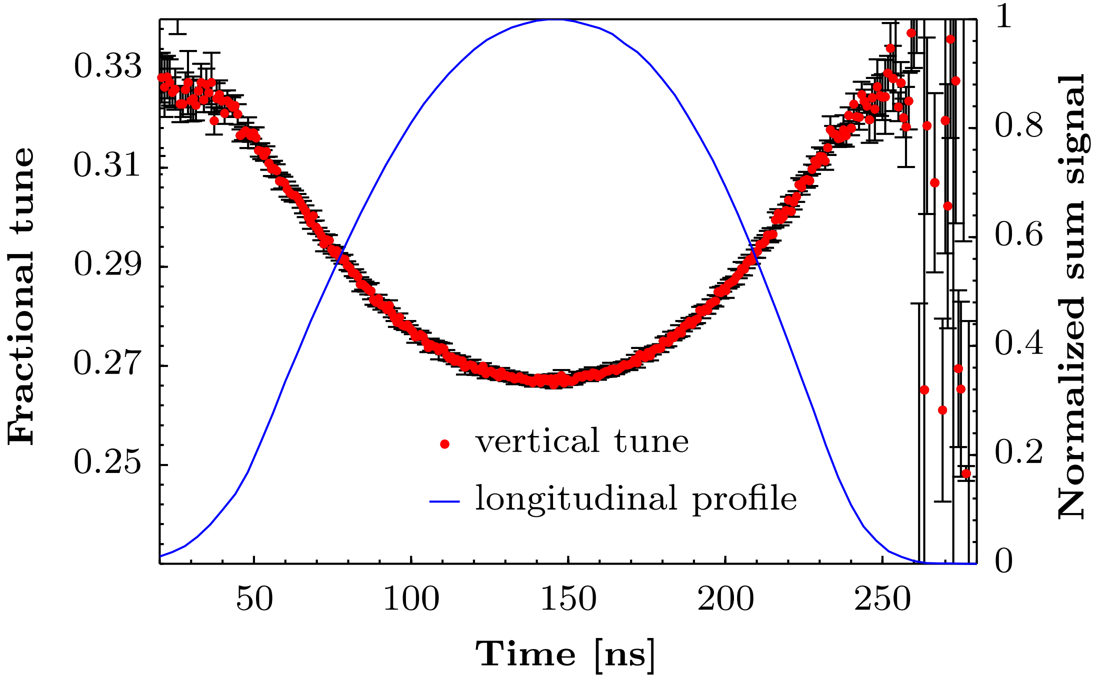
# Example: PS injection oscillations

- The observed intra-bunch motion was reproduced with an amazing precision with multi-particle simulations (HEADTAIL code) **including the indirect space charge effect** taking
- It was understood from simulations that the observed intra-bunch motion was induced by the beam injected **off-center in combination with the indirect space charge effect**, which causes a tune shift along the bunch proportional to the local charge density

**measurement**




**simulation**

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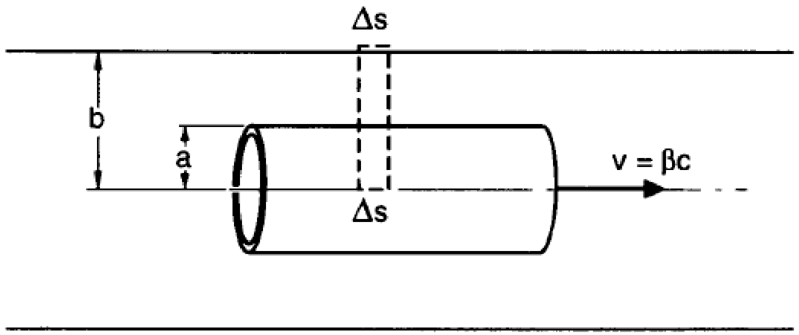
# Longitudinal space charge

- Longitudinal (direct) space charge can be also computed analytically (via Maxwell's equations) for cylindrically symmetric bunches of a given transverse density profile n(r):




$$E_s(r, s-\beta ct) = -\frac{1}{2\pi\epsilon_0\gamma^2}\,\lambda'(s-\beta ct)\left[\log\left(\frac{b}{r}\right) + \int_r^b 2\pi r' n(r') \log\left(\frac{r}{r'}\right)\,dr'\right]$$

- Note about this formula
  - For r=b the longitudinal electrical field vanishes as it should (perfect conducting pipe)
  - It diverges for b → ∞ because the analysis breaks down if the bunch length ≤ b/γ

# Longitudinal space charge

It has the following interesting dependencies:

- It decreases with energy like 1/γ2 and **vanishes in the ultra-relativistic limit**
- It is proportional to the **opposite of the derivative of the line density –λ'**. This can be understood intuitively because it must be directed from a region with higher charge density to a region with lower charge density (i.e. it pushes with the opposite of the gradient of the line charge)

$$E_s(r, s-\beta ct) = -\frac{1}{2\pi\epsilon_0\gamma^2}\lambda'(s-\beta ct)\left[\log\left(\frac{b}{r}\right) + \int_r^b 2\pi r' n(r')\log\left(\frac{r}{r'}\right)\,dr'\right]$$

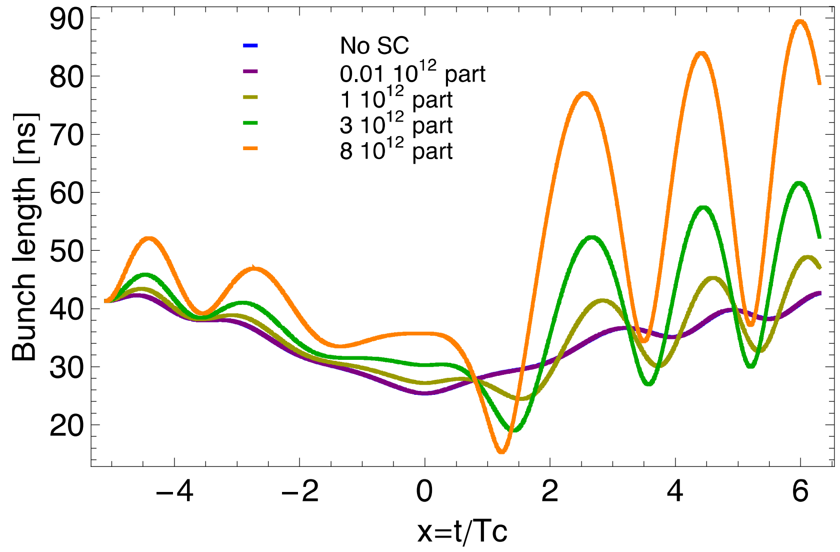
- Space charge would then spread out charge bumps. However, remember that only below transition energy, accelerated particles go faster and space charge has this smoothing action. Above transition, accelerated particles take a longer time to go around the accelerator and density peaks can be enhanced. This is the origin of the so-called **negative mass instability**. Momentum spread (unbunched beams) or synchrotron motion (bunched beams) can usually stabilize this effect.

# Longitudinal space charge: synchrotron tune shift

- Similarly to the transverse plane, longitudinal space charge also leads to **a synchrotron tune shift** which is proportional to number of particles in the bunch $N_b$ (linear)

$$\Delta Q_s \approx \frac{3\, e^2 g N_b \eta R^2}{8\pi\epsilon_0 \beta^2 \gamma^2 E_0 Q_{s_0} \hat{z}^3}$$

… example of synchrotron tune shift due to space charge in the radial center of a transverse parabolic bunch




The longitudinal space charge force **changes sign when crossing transition**:

- Quadrupolar oscillations are excited due to the sudden mismatch of the beam distribution at transition
- This effect gets stronger with increasing space charge force
- To avoid the mismatch, a jump of the RF voltage needs to be programmed similar to the phase jump

# Remarks

- Direct space charge
  - interaction of the bunch particles with the self induced electro-magnetic fields in free space
  - results in an incoherent tune shift (or spread)
  - the space charge force along a bunch is modulated with the local line density along the bunch and this results in an additional tune spread
  - decreases with energy like $\beta^{-1}\gamma^{-2}$
  - is a typical performance limitation for low energy machines
- Indirect space charge
  - interaction with image charges and currents induced in perfect conducting walls and ferromagnetic materials close to the beam pipe
  - results in incoherent and coherent tune shifts (or spreads), some of which are proportional to the average line density and others to the local line density
  - the contributions to the coherent and incoherent tune shifts for different standard geometries are expressed in terms of Laslett coefficients
  - decreases with energy like $\beta^{-2}\gamma^{-1}$

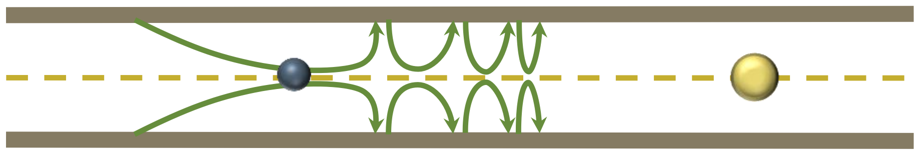
So far, we have introduced **direct and indirect space charge as collective effects**. The corresponding forces were not externally given but **dependent on the actual particle distribution** within the beam (remember, we looked at single particles as well as uniform and Gaussian distributions). The forces led to **incoherent and coherent tune shifts**.

We will now go a step further and investigate more complicated structures. We will try to find a smart way to deal with these structures. In the course of this, we will generalize and extend the direct and indirect space charge effects towards the **concept of wake fields and impedances**.

- Part 2: Direct- and indirect space charge
  - Direct space charge – impact on machine performance
  - Direct space charge – mitigation techniques
  - Indirect space charge
  - From indirect space charge to (resistive) wall wakes

# Electromagnetic fields of different sources


- Direct space charge
  - **Free space:** probe particles are effected directly by the source particles via the Lorentz force.



- Indirect space charge/**resistive wall wake**
  - **Smooth boundaries:** probe particles are effected by the source particles' induced image charges and currents.

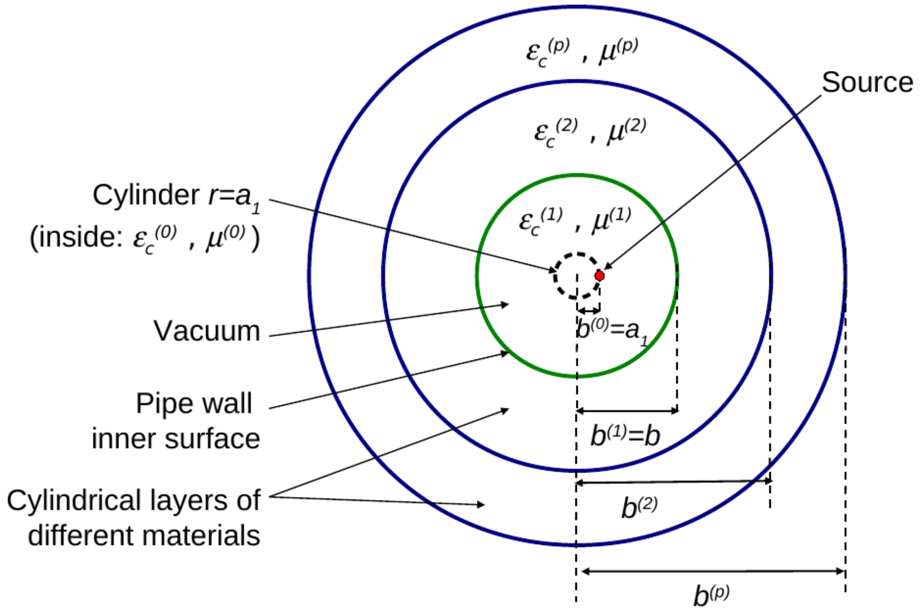
- Resonator wake fields
  - Discontinuities in boundaries: fields are excited by the source particles' distribution and can keep ringing, thus effecting trailing probe particles

**From (in-)direct space charge to resistive walls…**

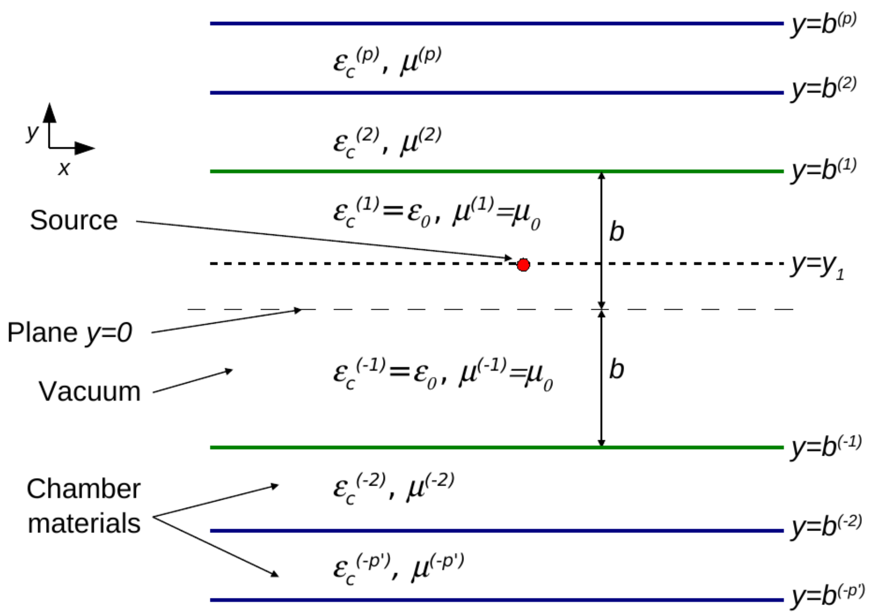
# From (in-)direct space charge to resistive wall

- We consider a smooth multilayer structure, **longitudinally translation invariant and transversely bounded**.
- We consider a charged point particle traveling through the smooth multilayered structure.
- The **induced electromagnetic fields can be computed** for certain geometries by means of Maxwell's equations (longitudinal and transverse electromagnetic fields) with **field matching at the boundaries**. Two examples of such geometries are shown below.

**Cylindrical**




**Flat**



# From (in-)direct space charge to resistive wall

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- The **induced electromagnetic fields can be computed** for certain geometries by means of Maxwell's equations (longitudinal and transverse electromagnetic fields) with **field matching at the boundaries**. Two examples of such geometries are shown below.
- It turns out, that the **resulting electromagnetic fields** can be decomposed into 3 components:

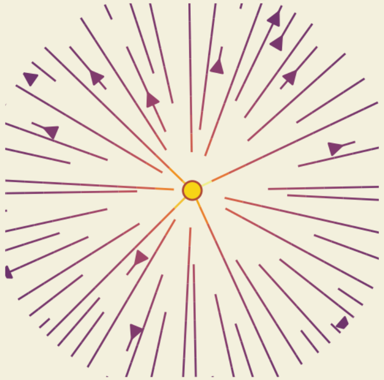
$$\vec{K}_{\text{Total}} = \vec{K}_{\text{direct}} + \vec{K}_{\text{boundaries}}$$

$$\sigma \to \infty \qquad \beta = 1$$

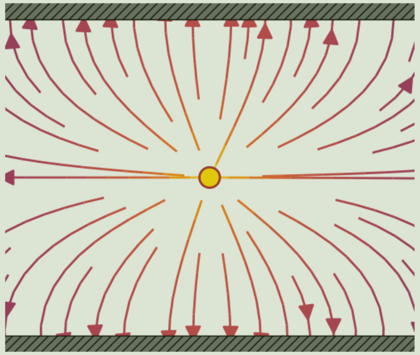
$$= \vec{K}_{\text{direct}} + \vec{K}_{\text{indirect}} + \vec{K}_{\text{resistive wall}}$$

# From (in-)direct space charge to resistive wall

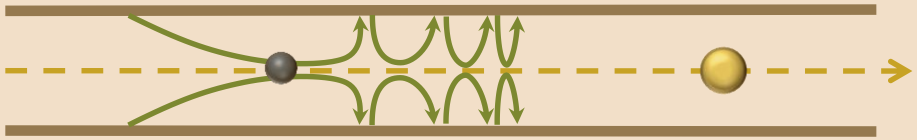
- A component which is **independent of the surrounding boundaries → direct space charge**



- A component which is independent of the surrounding material properties and purely **depended on the surrounding geometry → indirect space charge**



- A component which is **dependent on the surrounding material's electromagnetic properties → resistive wall**



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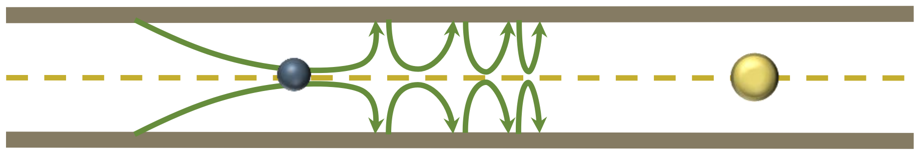
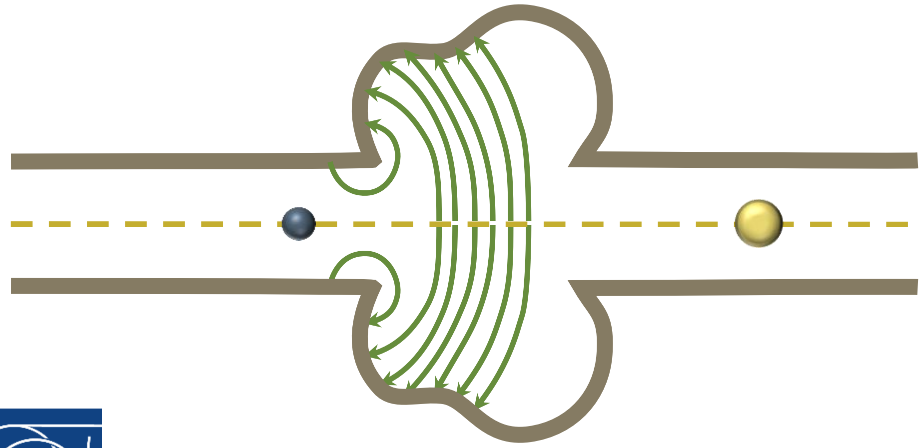
# Electromagnetic fields in different types of structures


- Direct space charge
  - **Free space:** probe particles are effected directly by the source particles via the Lorentz force.

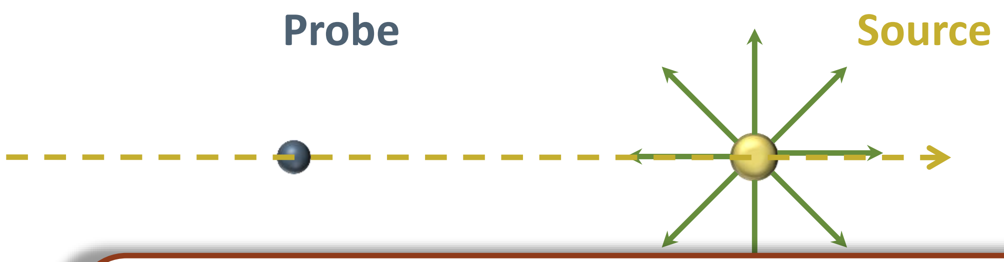


- Indirect space charge/**resistive wall wake**
  - **Smooth boundaries:** probe particles are effected by the source particles’ induced image charges and currents.



- Resonator wake fields
  - **Discontinuities in boundaries:** fields are excited by the source particles’ distribution and can keep ringing, thus effecting trailing probe particles

# Electromagnetic fields in different types of structures

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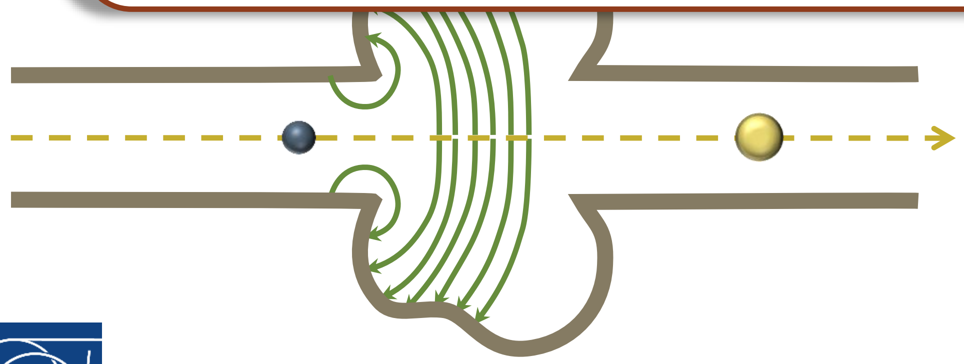
  - **Free space:** probe particles are effected directly by the source particles via the Lorentz force.

For more complicated geometries the (semi-) analytic methods no longer work. One must rely on **computational methods** to evaluate the induced electromagnetic fields (FDTD, FEM etc.).

*Some examples:*

…



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# Electromagnetic fields in complex structures

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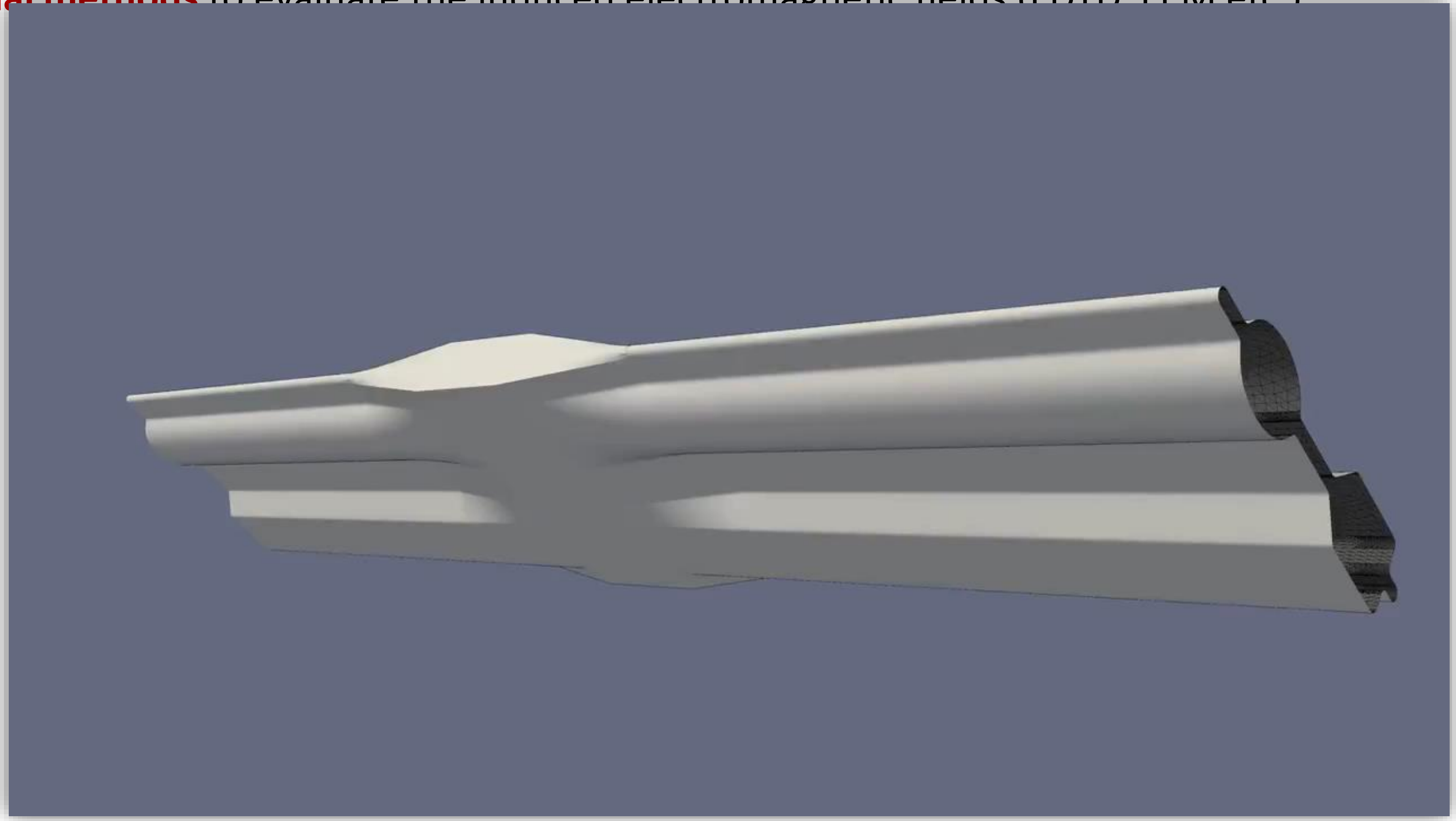
ERL vacuum chamber



# Electromagnetic fields in complex structures

For more complic

**computation**

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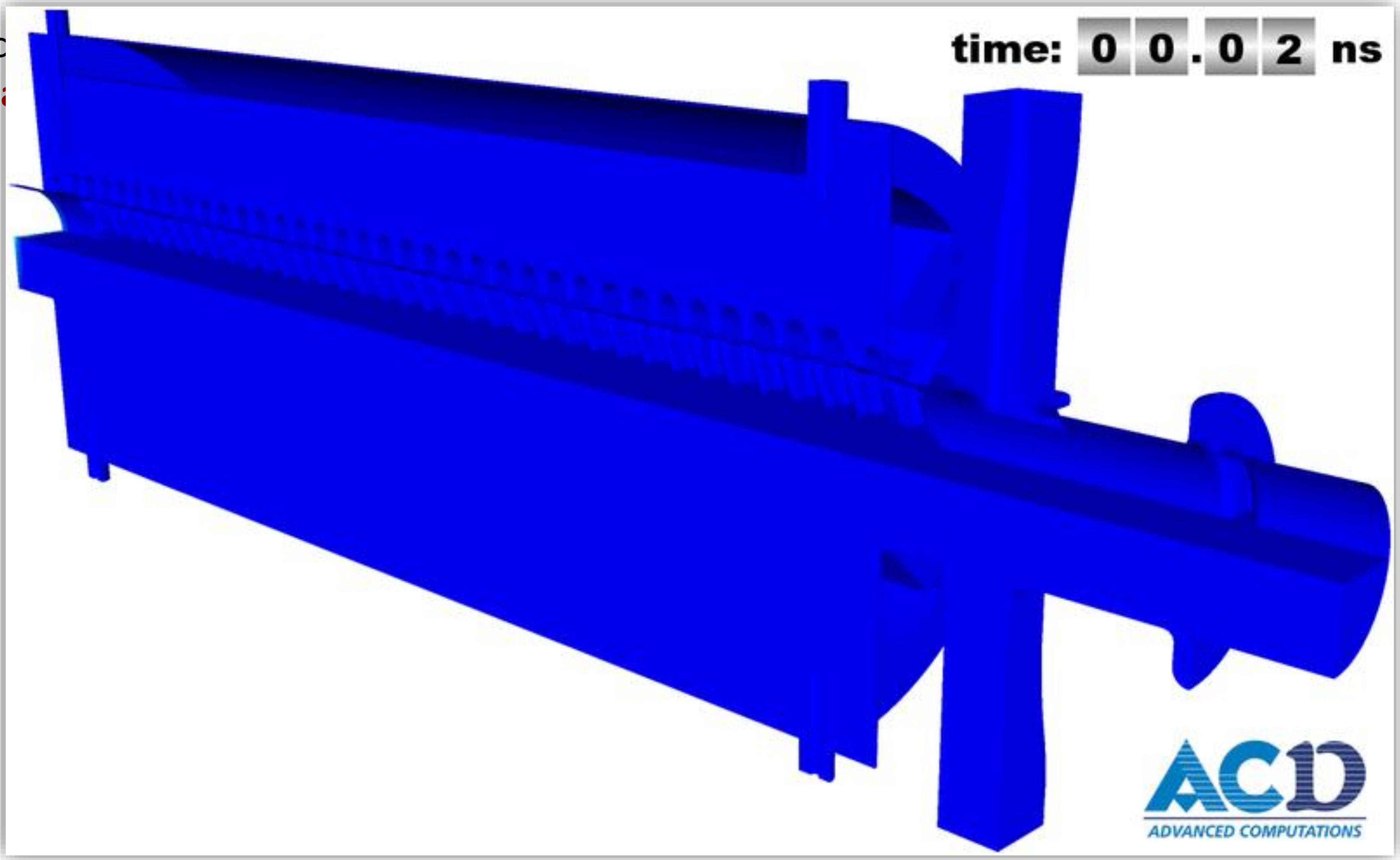

# Electromagnetic fields in complex structures

The CERN Accelerator School

For more complicated geometries the (semi-) analytic methods no longer work. One must rely on **computational methods** to evaluate the induced electromagnetic fields (FDTD, FEM etc.).

In principle, we need to solve the **full set of Maxwell's equations at every time step** to obtain the electromagnetic fields at every location within the structure in order to evaluate to forces at the probe particle's locations. This becomes very tedious and **virtually impossible for a 27km ring such as the LHC**.

**How can we treat these phenomena more effectively in our models?**

We normally **use a set of assumptions** to simplify the problem:

- **Rigid beam approximation:** the beam traverses the discontinuity of the vacuum chamber rigidly
- **Impulse approximation:** what the beam really cares about is the integrated impulse as it completes the traversal of the discontinuity

We have shown on the example of **multilayer structures** that one can in principle solve Maxwell’s equations to obtain the induced electromagnetic fields by a given charge distribution. We saw that it is **possible to decompose these fields**. We were able to identify the part that is dependent on the electromagnetic properties of the surrounding material as the **wall wake** which already led to the idea of wake fields.

We have seen examples of induced electromagnetic fields within complex structures. We will now look at how we can deal with these types of fields more practically by introducing the **concept of wake fields**.

- Part 2: Direct- and indirect space charge
    - Direct space charge – impact on machine performance
    - Direct space charge – mitigation techniques
    - Indirect space charge
    - From indirect space charge to (resistive) wall wakes

In this lecture we discussed **indirect space charge** and showed that this can lead **to both incoherent as well as coherent tune shifts**. We then moved on the a more general treatment of electromagnetic fields in simple structures where we were able to identify **yet another type of induced fields** originating from the **electromagnetic properties** of the surrounding material – **the wall wake**.

We also already looked at more general examples of induced fields in complex structures.

Next we will introduce the **concept of wake fields and impedances** and study the effect of these on the machine and on the beam.

- Part 2: Direct- and indirect space charge
    - Direct space charge – impact on machine performance
    - Direct space charge – mitigation techniques
    - Indirect space charge
    - From indirect space charge to (resistive) wall wakes

# End part 2

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# Backup

# Direct space charge tune shift

- The direct space charge force for a beam with uniform charge distribution is linear in x and y
  → results in a **direct space charge tune shift**
- We derive the general expression for the space charge induced tune shift in the vertical plane

  - Express $y''$ in terms of the space charge forces

$$y'' = \frac{1}{\beta^2 c^2} \frac{d^2}{dt^2} y = \frac{1}{\beta^2 c^2} \frac{F_y^{SC}}{m\gamma}$$

  - Linearize the space charge forces for small offsets

$$F_y^{SC}(y) \approx F_y^{SC}(0) + \frac{\partial}{\partial y} F_y^{SC}(0)\, y$$

  - Generalize Hill's equation to include the defocusing space charge term

$$\left.\begin{aligned} y'' + K_y(s)\, y + K_y^{SC}(s)\, y = 0 \\ K_y^{SC} = -\frac{1}{\beta^2 c^2} \frac{1}{m\gamma} \frac{\partial}{\partial y} F_y^{SC}(0)\, y \end{aligned}\right\} \implies y'' + K_y(s)\, y - \frac{1}{\beta^2 c^2} \frac{1}{m\gamma} \frac{\partial}{\partial y} F_y^{SC}(0)\, y = 0$$

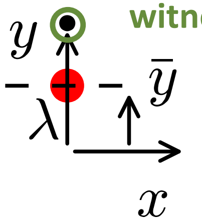
  - Calculate the tune shift treating space charge like a focusing error

$$\Delta Q_y = \int \beta_y(s) K_y^{SC}(s)\, ds$$

# Incoherent vs. coherent tune shift

- Indirect space charge generates forces via image charges – these forces are a function of the beam (centroid) offset and the witness particle offsets
- We can write down the equation of motion for the (indirect space charge perturbed) betatron motion in smooth approximation as

$$y'' + \left(\frac{Q_{y0}}{R}\right)^2 y = \frac{\langle F_y(y,\bar{y})\rangle}{m\gamma\beta^2 c^2}$$




- We linearize the forces for small offsets in y and $\bar{y}$

$$y'' + \left(\frac{Q_{y0}}{R}\right)^2 y = \frac{1}{m\gamma\beta^2 c^2}\left(\left.\frac{\partial \langle F_y\rangle}{\partial y}\right|_{\bar{y}=0} y + \left.\frac{\partial \langle F_y\rangle}{\partial \bar{y}}\right|_{y=0} \bar{y}\right)$$

- In case the induced tune shift is small, we can write

$$Q_y^2 = Q_{y0}^2 + 2\,Q_{y0}\Delta Q_y + (\Delta Q_y)^2 \approx Q_{y0}^2 + 2\,Q_{y0}\Delta Q_y$$

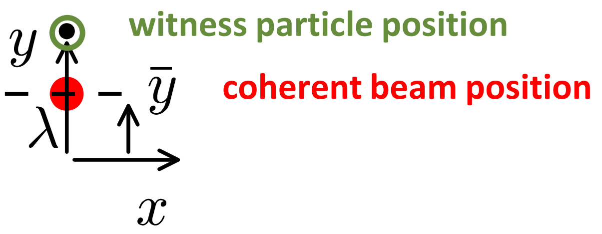
# Incoherent vs. coherent tune shift

- We linearize the forces for small offsets in y and ȳ

$$y'' + \left(\frac{Q_{y0}}{R}\right)^2 y = \frac{1}{m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0} y + \left.\frac{\partial\langle F_y\rangle}{\partial \bar{y}}\right|_{y=0} \bar{y}\right)$$

- The incoherent tune shift of an individual witness particle is obtained when considering the beam as a whole being centered; the second term in the general equation of motion becomes zero and we obtain:

$$y'' + \left(\frac{Q_{y0}}{R}\right)^2 y - \underbrace{\frac{1}{m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0}\right)}_{2\,Q_{y0}\Delta Q_y/R^2} y = 0$$




- Hence, the **incoherent tune shift becomes**

$$\Delta Q_y^{\text{incoh}} = \frac{R^2}{2Q_{y0}m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0}\right) = \frac{R\langle\beta_y\rangle}{2m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0}\right)$$

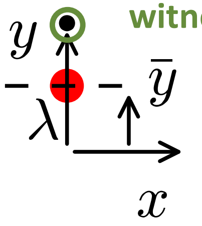
# Incoherent vs. coherent tune shift

- We linearize the forces for small offsets in y and $\bar{y}$

$$y'' + \left(\frac{Q_{y0}}{R}\right)^2 y = \frac{1}{m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0} y + \left.\frac{\partial\langle F_y\rangle}{\partial \bar{y}}\right|_{y=0}\bar{y}\right)$$

- The coherent equation of motion evaluates on the beam centroid motion as:

$$\bar{y}'' + \left(\frac{Q_{y0}}{R}\right)^2 \bar{y} - \underbrace{\frac{1}{m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0} + \left.\frac{\partial\langle F_y\rangle}{\partial \bar{y}}\right|_{y=0}\right)}_{2\,Q_{y0}\Delta Q_y/R^2}\bar{y} = 0$$




- Hence, the **coherent tune shift becomes**

$$\Delta Q_y^{\mathrm{coh}} = \frac{R^2}{2Q_{y0}m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0} + \left.\frac{\partial\langle F_y\rangle}{\partial \bar{y}}\right|_{y=0}\right)$$
$$= \frac{R\langle\beta_y\rangle}{2m\gamma\beta^2c^2}\left(\left.\frac{\partial\langle F_y\rangle}{\partial y}\right|_{\bar{y}=0} + \left.\frac{\partial\langle F_y\rangle}{\partial \bar{y}}\right|_{y=0}\right)$$

# Laslett tune shift from indirect space charge

- The coherent and incoherent tune shifts from indirect space charge are given by the **derivatives of the image charge forces**

$$\Delta Q_y^{\text{incoh}} = \frac{R\langle\beta_y\rangle}{2m\gamma\beta^2c^2}\left(\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0}\right), \quad \Delta Q_y^{\text{coh}} = \frac{R\langle\beta_y\rangle}{2m\gamma\beta^2c^2}\left(\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0} + \frac{\partial\langle F_{\text{beam}}\rangle}{\partial\bar{y}}\bigg|_{y=0}\right)$$

- In general the contribution from electric images to the **inoherent beam force** for a variety of geometries can be expressed in terms of **Laslett coefficients**

$$\frac{\partial\langle F_{\text{beam}}\rangle}{\partial x}\bigg|_{\bar{x}=0} = \frac{e\lambda}{\pi\epsilon_0}\frac{\varepsilon_1^x}{h^2}$$
$$\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0} = \frac{e\lambda}{\pi\epsilon_0}\frac{\varepsilon_1^y}{h^2}$$

$$\varepsilon_1^x,\, \varepsilon_1^y \ldots \begin{cases} \textit{incoherent electric} \\ \text{image coefficients} \end{cases}$$

- Similarly, contribution from electric images to the **coherent beam force** for a variety of geometries can be expressed in terms of **Laslett coefficients**

$$\frac{\partial\langle F_{\text{beam}}\rangle}{\partial x}\bigg|_{\bar{x}=0} + \frac{\partial\langle F_{\text{beam}}\rangle}{\partial\bar{x}}\bigg|_{x=0} = \frac{e\lambda}{\pi\epsilon_0}\frac{\xi_1^x}{h^2}$$
$$\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0} + \frac{\partial\langle F_{\text{beam}}\rangle}{\partial\bar{y}}\bigg|_{y=0} = \frac{e\lambda}{\pi\epsilon_0}\frac{\xi_1^y}{h^2}$$

$$\xi_1^x,\, \xi_1^y \ldots \begin{cases} \textit{coherent electric} \\ \text{image coefficients} \end{cases}$$

# Laslett tune shift from indirect space charge

- Incoherent electric Laslett tune shift - special case of two parallel perfect conducting plates

  - In the vertical plane we get the Laslett coefficient from the force

- Coherent electric Laslett tune shift - Special case of two parallel perfect conducting plates

  - In the vertical plane we get the Laslett coefficient from the force

$$F_y(y, \bar{y}) = \frac{e\lambda}{\pi\epsilon_0 h^2}\left[(\bar{y}+y)\,\frac{\pi^2}{32}\,(\bar{y}-y)\,\frac{\pi^2}{96}\right]$$

$$\Longrightarrow \varepsilon_1^y = \frac{\pi^2}{48}$$

$$\Longrightarrow \xi_1^y = \frac{\pi^2}{16}$$

- The horizontal force on the witness particle comes only from image charges and therefore it follows from the source free Gauss' law

$$\vec{\nabla}\cdot\vec{E} = 0 \;\Rightarrow\; \varepsilon_1^x = -\varepsilon_1^y$$

valid for all geometries

- Due to the translational invariance in the horizontal plane in the case of the two parallel plates it follows that

$$\xi_1^x \equiv 0$$

not valid in general

# Overview of force contributions

- The indirect space charge contributions from the different image charges and currents to the coherent and incoherent beam force are summarized below
  - As mentioned before, the magnetic force due to image currents in the vacuum chamber (ac components) are described by the electric image coefficients
  - The ac coherent component corresponds to betatron beam oscillations

| Beam force components | Images in vacuum chamber | | Images in pole faces | Comments |
|---|---|---|---|---|
| | electric | magnetic | magnetic | |
| $\left.\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\right\vert_{\bar{y}=0}$ | $\frac{\varepsilon_1^y}{h^2}$ | $-\beta^2\frac{\varepsilon_1^y}{h^2}$ | $\beta^2\frac{\varepsilon_2^y}{g^2}$ | incoherent, dc coherent |
| $\left.\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\right\vert_{\bar{y}=0}+\left.\frac{\partial\langle F_{\text{beam}}\rangle}{\partial \bar{y}}\right\vert_{y=0}$ | $\frac{\xi_1^y}{h^2}$ | $-\beta^2\frac{\xi_1^y}{h^2}$ | $\beta^2\frac{\xi_2^y}{g^2}$ | coherent |
| $\left.\frac{\partial\langle F_{\text{beam}}\rangle}{\partial \bar{y}}\right\vert_{y=0}$ | | $-\beta^2\frac{\xi_1^y-\varepsilon_1^y}{h^2}$ | $\beta^2\frac{\xi_2^y-\varepsilon_2^y}{g^2}$ | ac coherent |

# Indirect space charge: incoherent tune shift

- We found the expression for the coherent tune shift as function of the beam force as

$$\Delta Q_y^{\text{inc}} = \frac{R^2}{2Q_{y0}\beta^2 c^2 m\gamma}\left(\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0}\right) = \frac{R\langle\beta_y\rangle}{2\beta^2 c^2 m\gamma}\left(\frac{\partial\langle F_{\text{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0}\right)$$

  - In the second step the tune shift is written in terms of the average beta function instead of the betatron tune (as given by the smooth approximation) – like this we can refine our model and account for different geometries around the machine

- Collecting all contributions to the incoherent incoherent tune shift we find for the general case of bunched beams
  - For coasting beams the peak line density is equal to the average line density
  - The F factor corresponds to the ratio of the circumference surrounded by ferromagnetic material

$$\Delta Q_{x,y}^{\text{inc}} = -\frac{2\langle\beta_{x,y}\rangle r_0 R}{e\beta^2\gamma}\left[\underbrace{\frac{\varepsilon_1^{x,y}}{h^2}\hat{\lambda}}_{\text{electric image}} - \underbrace{\beta^2\frac{\varepsilon_1^{x,y}}{h^2}\left(\hat{\lambda}-\bar{\lambda}\right)}_{\text{ac magnetic image from bunching}} + \underbrace{\mathcal{F}\beta^2\frac{\varepsilon_2^{x,y}}{g^2}\bar{\lambda}}_{\text{magnetic image in magnet poles}}\right]$$

# Indirect space charge: coherent tune shift

- The general expression of the coherent tune shift is obtained as

$$\Delta Q_y^{\mathrm{coh}} = \frac{R\langle\beta_y\rangle}{2\beta^2 c^2 m\gamma}\left(\frac{\partial\langle F_{\mathrm{beam}}\rangle}{\partial y}\bigg|_{\bar{y}=0} + \frac{\partial\langle F_{\mathrm{beam}}\rangle}{\partial \bar{y}}\bigg|_{y=0}\right)$$

- Here we need to consider two cases
  - Betatron oscillations are of such low frequency that the induced magnetic field can penetrate through the vacuum chamber

$$\Delta Q_{x,y}^{\mathrm{coh}} = -\frac{2\langle\beta_{x,y}\rangle r_0 R}{e\beta^2\gamma}\left[\underbrace{\frac{\xi_1^{x,y}}{h^2}\hat{\lambda}}_{\substack{\text{electric}\\ \text{image}}} \underbrace{- \beta^2\frac{\xi_1^{x,y}}{h^2}\left(\hat{\lambda}-\bar{\lambda}\right)}_{\substack{\text{ac magnetic image}\\ \text{from bunching}}} + \underbrace{\mathcal{F}\beta^2\frac{\xi_2^{x,y}}{g^2}\bar{\lambda}}_{\substack{\text{magnetic image}\\ \text{in magnet poles}}}\right]$$

  - Magnetic fields from both betatron oscillations and longitudinal bunching cannot penetrate the vacuum chamber

$$\Delta Q_{x,y}^{\mathrm{coh}} = -\frac{2\langle\beta_{x,y}\rangle r_0 R}{e\beta^2\gamma}\left[\underbrace{\frac{\xi_1^{x,y}}{h^2}\hat{\lambda}}_{\substack{\text{electric}\\ \text{image}}} \underbrace{- \beta^2\frac{\xi_1^{x,y}}{h^2}\left(\hat{\lambda}-\bar{\lambda}\right)}_{\substack{\text{ac magnetic image}\\ \text{from bunching}}} \underbrace{- \beta^2\frac{\xi_1^{x,y}-\varepsilon_1^{x,y}}{h^2}\bar{\lambda}}_{\substack{\text{ac magnetic image}\\ \text{from transverse motion}}} + \underbrace{\mathcal{F}\beta^2\frac{\varepsilon_2^{x,y}}{g^2}\bar{\lambda}}_{\substack{\text{magnetic image}\\ \text{in magnet poles}}}\right]$$