Longitudinal Hands-on Calculations

Longitudinal Tracking

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CERN

The CERN Accelerator School

Introduction to Accelerator Physics

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The CERN Accelerator School

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Agenda of the afternoon

15h00 – 15h30 **Introduction to exercises**	
15h00 – 16h00 **Longitudinal tracking**	
Legstretch	
16h30 – 17h45 **Longitudinal tracking**	
17h45 – 18h30 **Discussion on solutions of exercises**	

Outline

- **Introduction**
  - **Interaction between beam and RF system**
- **Longitudinal particle tracking**
  - **Basic tracking equations**
  - **Single and multiple particle tracking**
- **Summary**

Introduction

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical

- **Describe particle motion by differential equations**

→ **Continuous trajectories of particle motion**

→ **Deduce useful parameters for stable acceleration:**
  - → RF bucket
  - → Synchrotron frequency
  - → Stable phase
  - → …

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical	Numerical: tracking
• Describe particle motion by differential equations	• Track particle parameters from turn to turn
→ Continuous trajectories of particle motion	→ Profit from discretization of motion: turn-by-turn, RF station-by-RF station
→ Deduce useful parameters for stable acceleration: → RF bucket → Synchrotron frequency → Stable phase → …	→ No notion of RF bucket, synchrotron frequency, stable phase, etc.
	→ Follow ensemble of particles to study evolution of bunch

Study interaction between beam and RF

Complementary approaches for the same problem

(Semi-)Analytical	Numerical: tracking
• Describe particle motion by differential equations	• Track particle parameters from turn to turn
→ Continuous trajectories of particle motion → Deduce useful parameters for stable acceleration: → RF bucket → Synchrotron frequency → Stable phase → …	→ Profit from discretization of motion: turn-by-turn, RF station-by-RF station → No notion of RF bucket, synchrotron frequency, stable phase, etc. → Follow ensemble of particles to study evolution of bunch
→ Classical introduction of longitudinal beam dynamics	→ Flexible approach

Today

Objectives of longitudinal hands-on

1. **Design RF system (upgrade)**

   `LongitudinalHandsOnRFSystemCalculations_empty.ipynb`

   - Study boundary constraints
   - Derive requirements for RF system
   - Choose main components
   - Compare with existing facilities

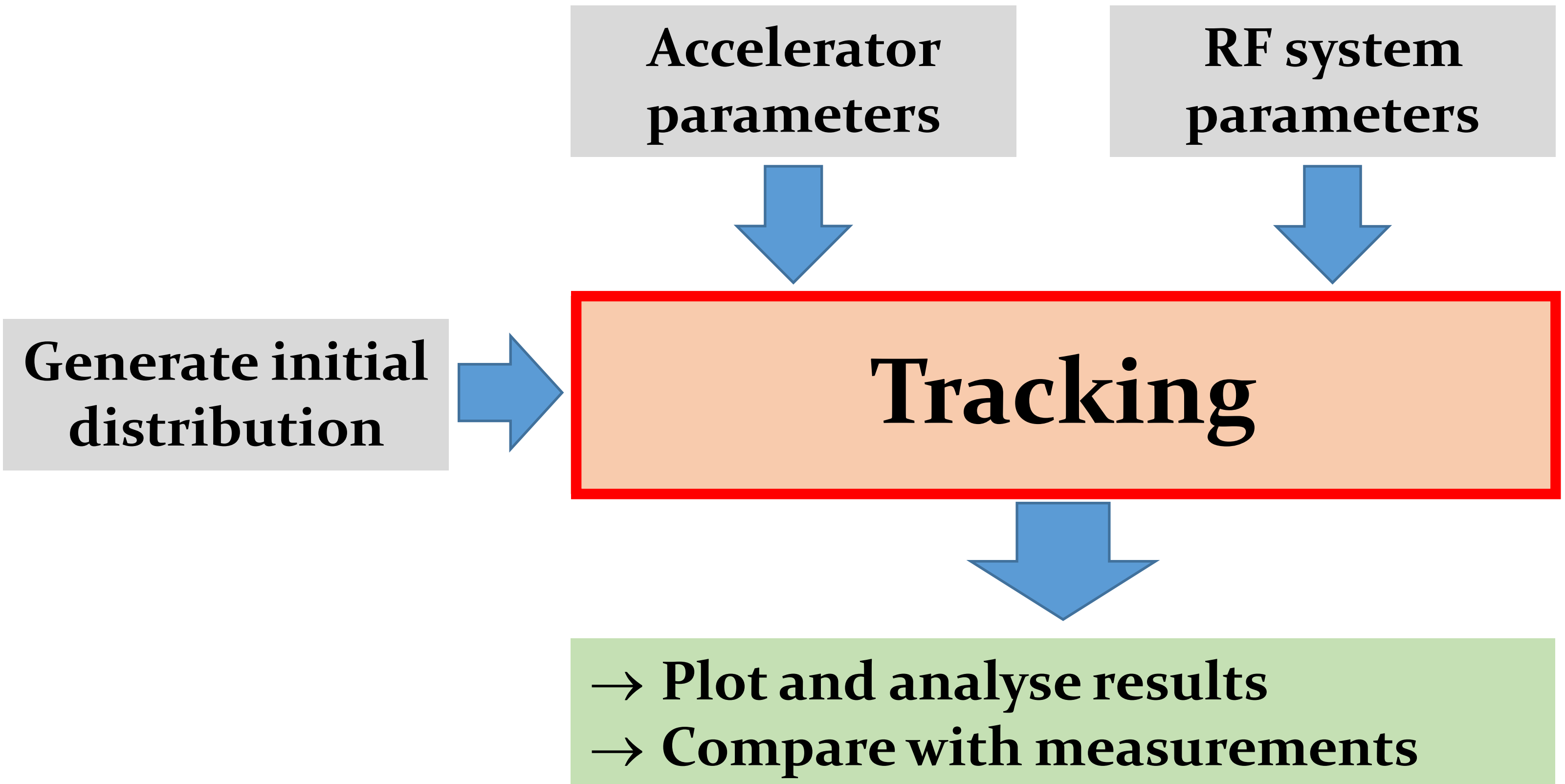
2. **Play with longitudinal beam dynamics**

   `LongitudinalHandsOnTracking_empty.ipynb`

   - **Build your own particle tracker**
   - **Understand motion of particles in longitudinal phase space**
   - **Transition from single particle motion to evolution of an entire bunch**

Longitudinal tracking

Tracking simulation flow

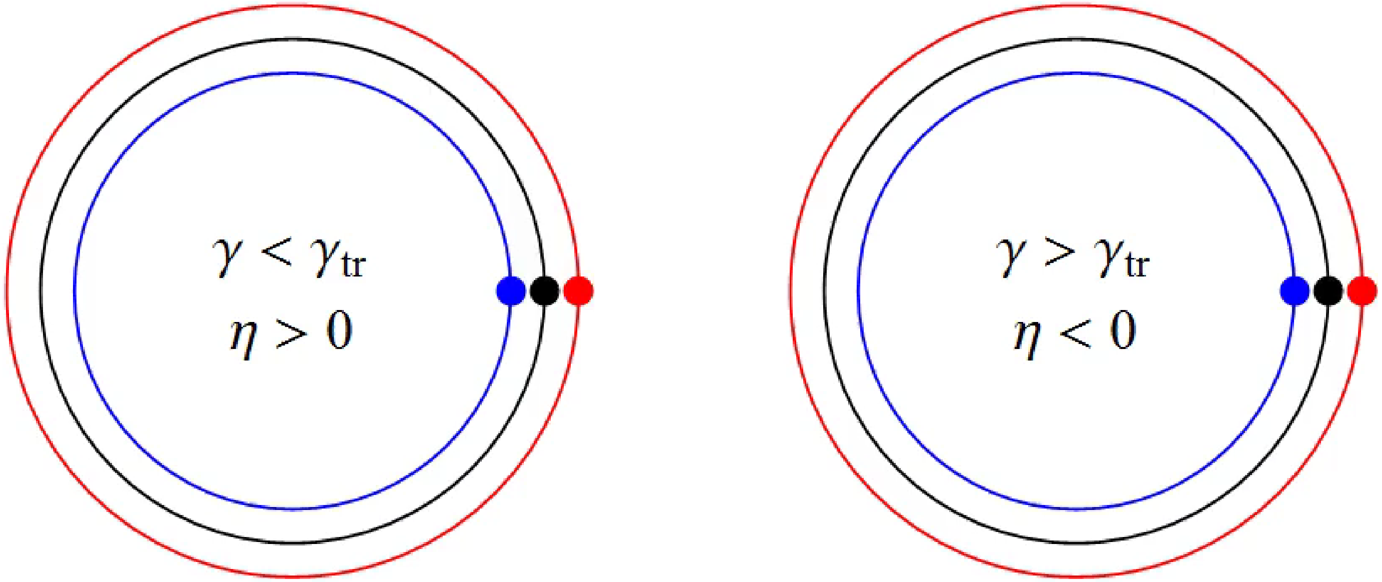


→ Follow the coordinates of one or more particles determine its behaviour

Circular ~~accelerator~~ without RF system
⮡ ring

- Particles with higher or lower momentum have a different orbit compared to a reference particle

→ Arrival time/phase depends on energy $$\frac{\Delta L}{L} = \frac{1}{\gamma_{tr}^2}\frac{\Delta p}{p}$$



$$\frac{\Delta f}{f} = -\eta\frac{\Delta p}{p}$$, phase slip factor: $$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$$

Arrival phase of a particle at next turn

$$\frac{\Delta f}{f} = -\eta \frac{\Delta p}{p}$$

$$\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta E}{E}$$

$$\frac{\Delta f}{f} = -\frac{\eta}{\beta^2}\frac{\Delta E}{E}$$

$$\frac{\Delta \theta}{\theta} = \frac{\eta}{\beta^2}\frac{\Delta E}{E}$$

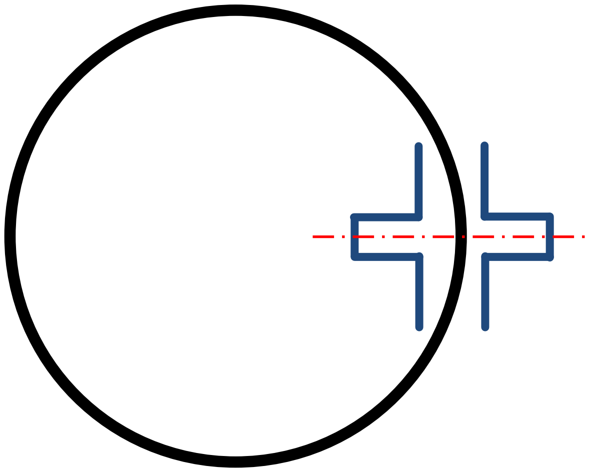
→ **Turn-by-turn drift equation**

$$\theta_{n+1} = \theta_n + 2\pi \frac{\eta}{\beta^2}\frac{\Delta E}{E}$$

$$\phi_{n+1} = \phi_n + 2\pi h \frac{\eta}{\beta^2}\frac{\Delta E}{E}$$

→ **Azimuth, θ or phase, ϕ a particles arrives next turn**

Circular accelerator with RF station



- Particle energy changes at passage through cavity

→ For **sinusoidal RF voltage**:

$$\Delta E_{n+1} = \Delta E_n + qV \sin \phi_{n+1}$$

→ With **acceleration**:

$$\Delta E_{n+1} = \Delta E_n + qV \left[\sin \phi_{n+1} - \sin \phi_\mathrm{S}\right]$$

Reference particle: $\phi = \phi_\mathrm{S}$

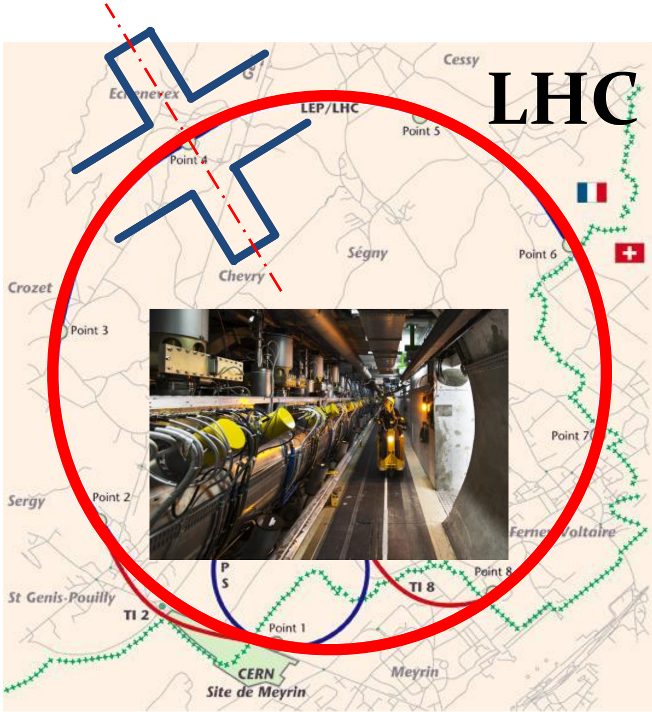
→ **General energy change:**

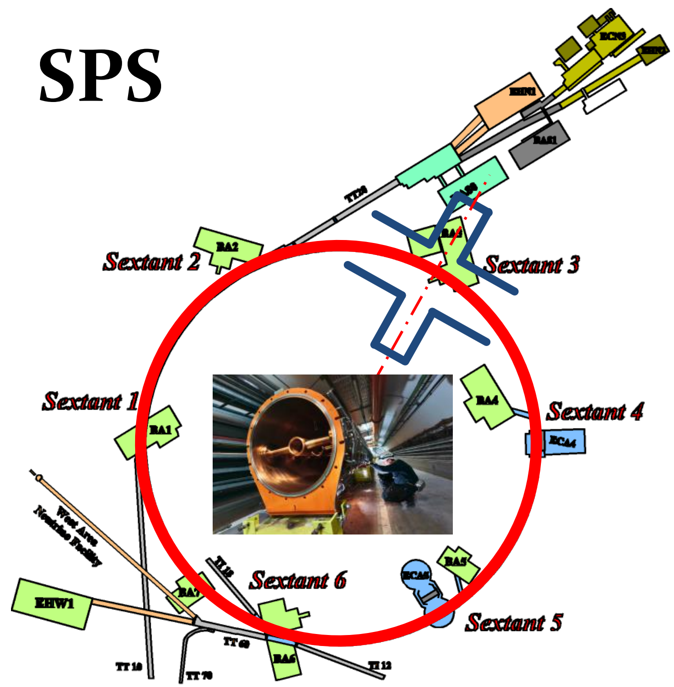
$$\Delta E_{n+1} = \Delta E_n + qV \left[g(\phi_{n+1}) - g(\phi_\mathrm{S})\right] + \Delta E_\mathrm{ext} + \Delta E_\mathrm{self}$$

Multiple RF stations

- RF systems modelled point-like mostly valid approximation

→ Valid in most cases





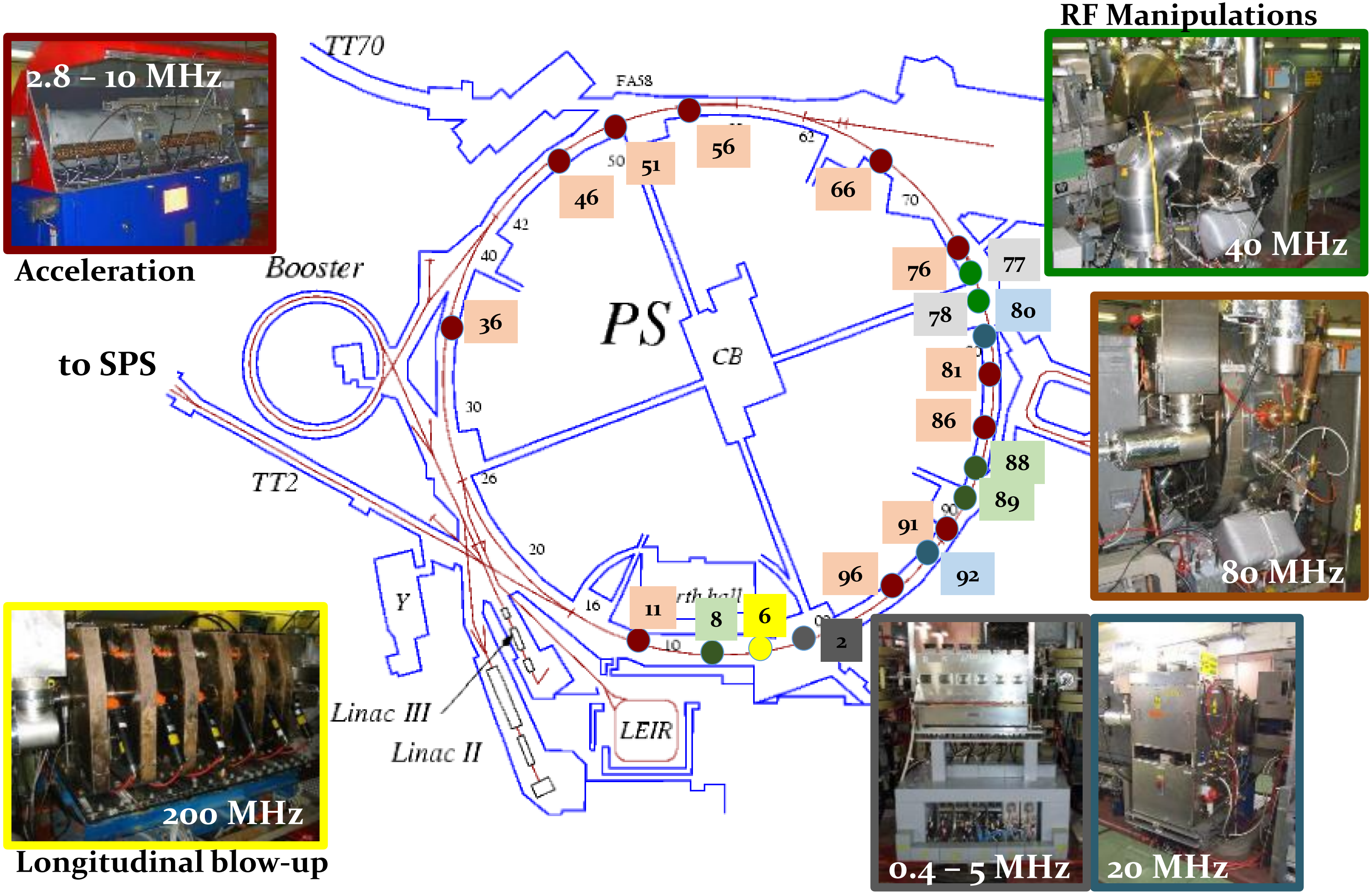
Exceptions:

→ Large synchrotron tune f_S/f_{rev}

→ Strong intensity effects: interaction within one turn

→ Beam energy changing during turn

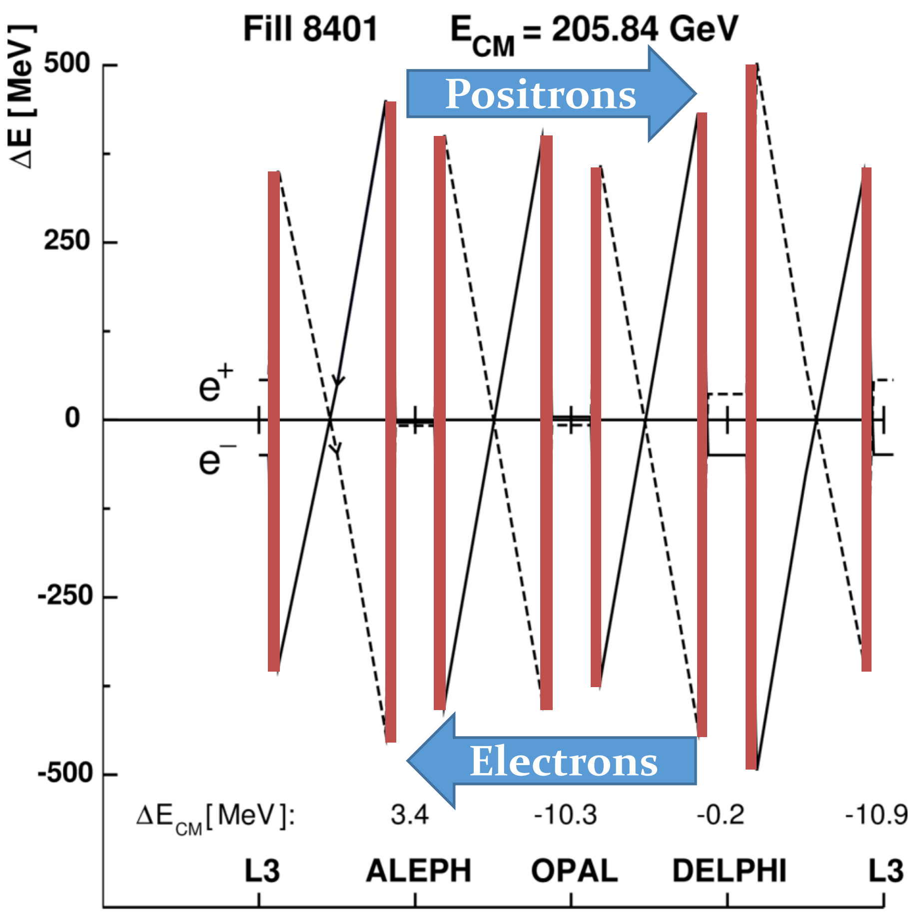
Multiple RF stations

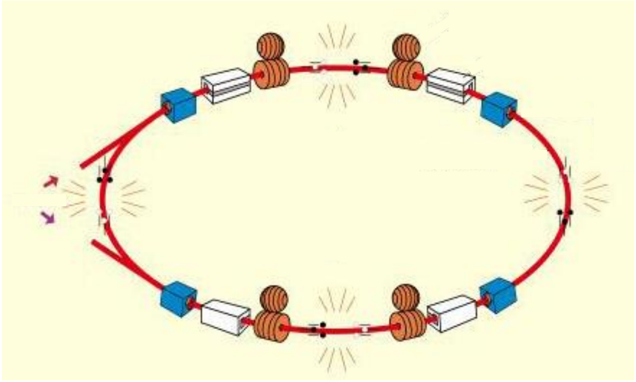


→ **Small $Q_S = f_S / f_{rev}$:** **Single kick per turn fully sufficient**

Example: Electrons and positrons in LEP

- Beam energy changed in LEP along turn due to strong synchrotron radiation





- 4×2 RF sections

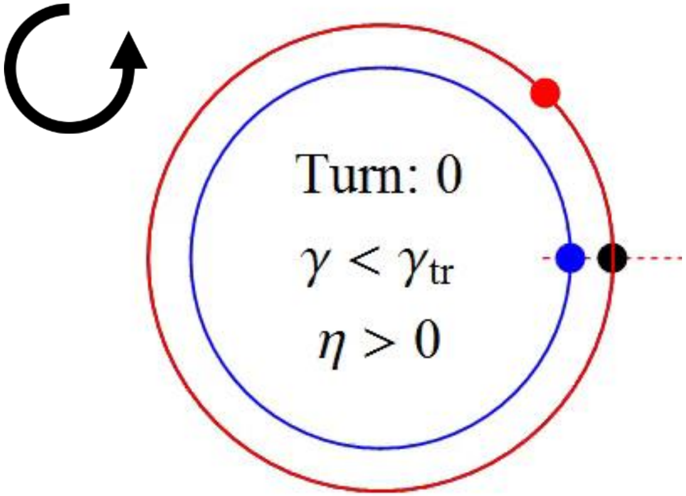
→ Energy loss in bending magnets

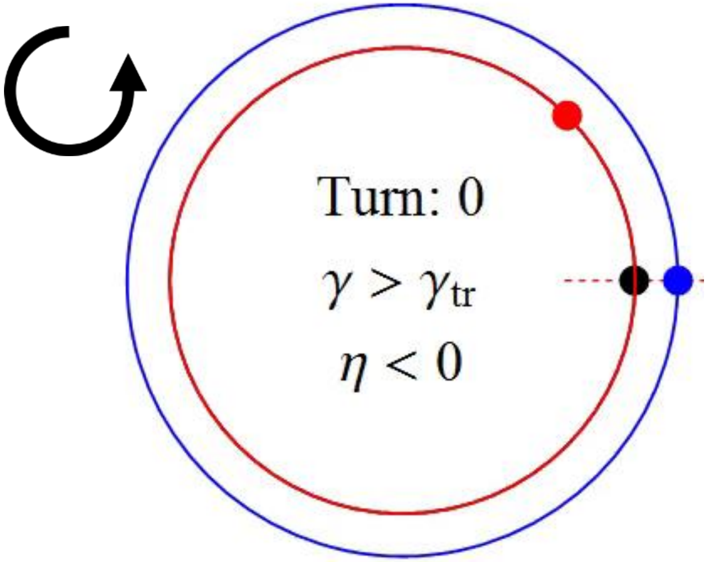
→ Track from RF section to RF section

Combining both tracking equations

- **Observe phase and energy error at each turn with respect to reference particle**
- **Test particles:**

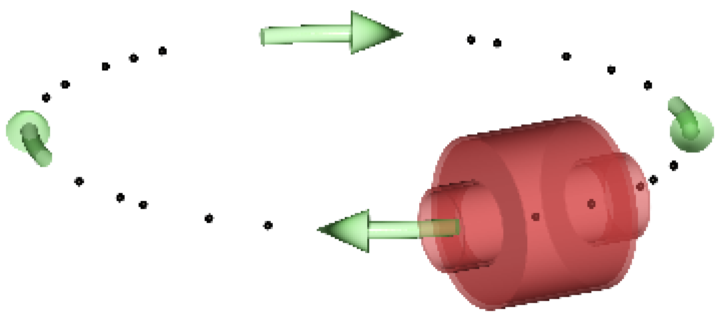
$\Delta\phi = \phi - \phi_S = 0$	$\Delta E = 0$
$\Delta\phi \neq 0$	$\Delta E = 0$
$\Delta\phi = 0$	$\Delta E \neq 0$





Longitudinal phase space

Simple accelerator model:

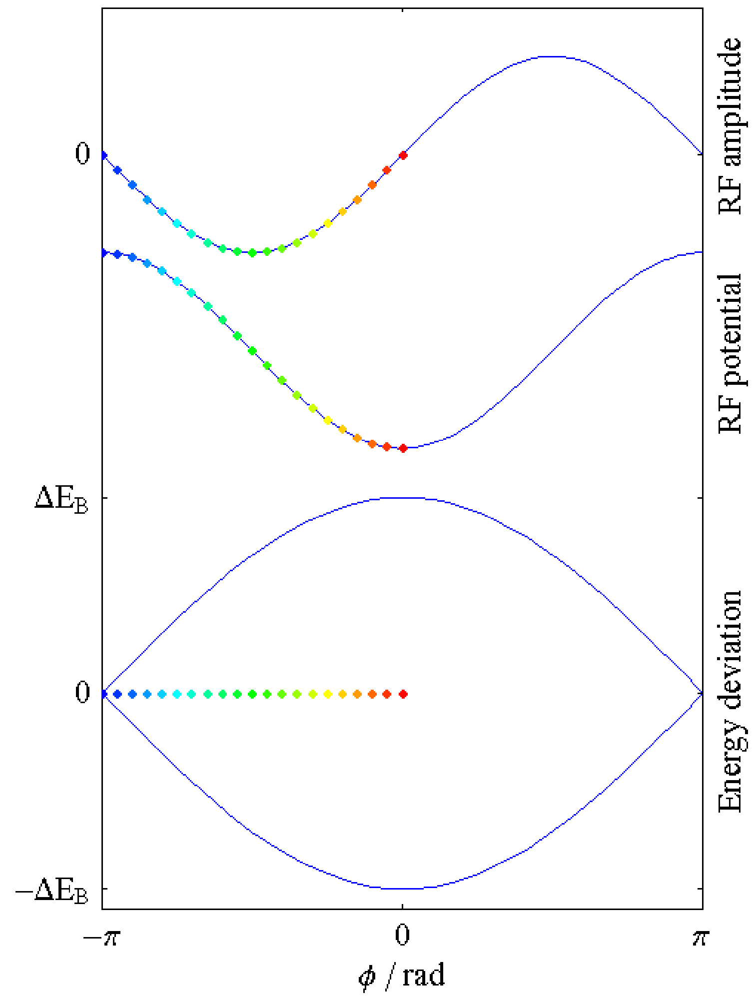


Energy dependent phase advance, ϕ:

$$\phi_{n+1} = \phi_n + 2\pi h \eta \frac{\Delta E_n}{\beta^2 E},\ \eta = \frac{1}{\gamma_{\mathrm{tr}}^2} - \frac{1}{\gamma^2}$$

Phase dependent energy gain, Δ*E*:

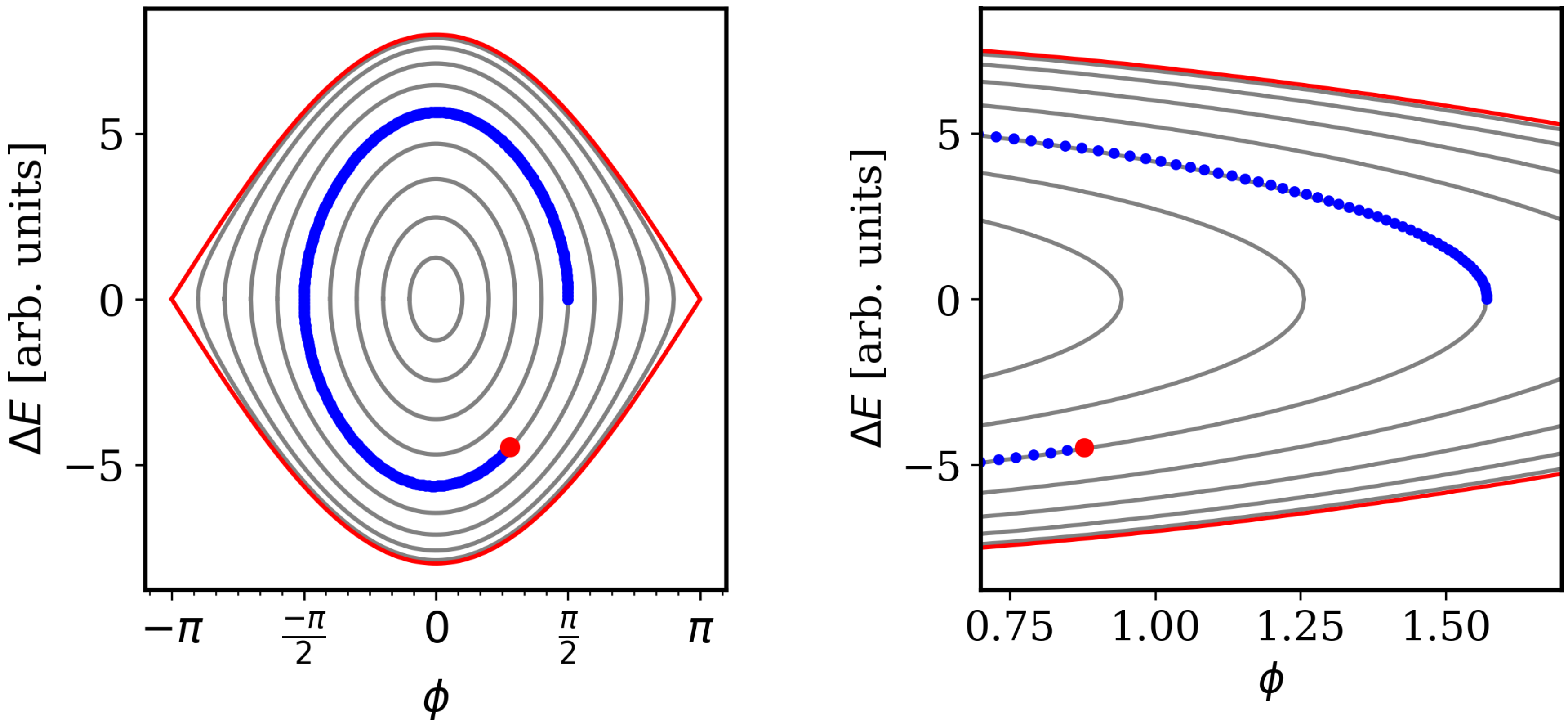
$$\Delta E_{n+1} = \Delta E_n + qVg(\phi_{n+1})$$



Works for arbitrary shape of acceleration amplitude *g*(ϕ)

Continuous versus discrete

- **Analytical solution describes static condition**
- **No notion of turn-by-turn evolution**



→ **Same result with both approaches for $Q_S = f_S/f_{rev} << 1$**

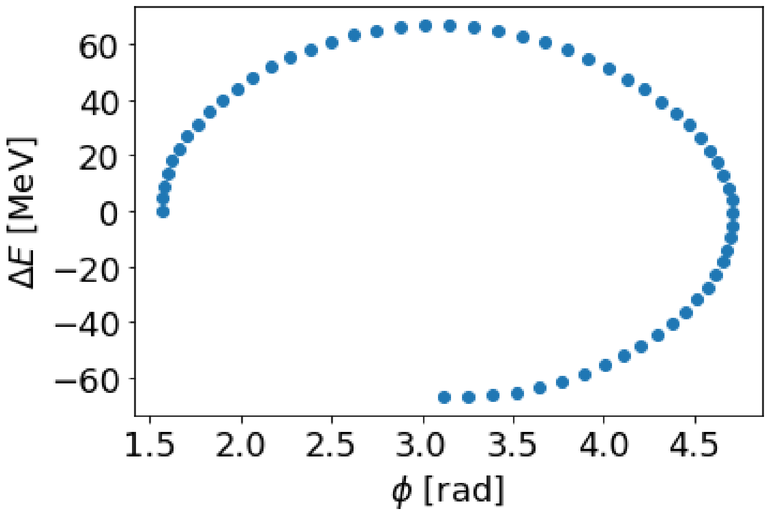
Example: simple tracking in Python

- **Follow the trajectory of a single particle**

```
# Tracking functions
def drift(phaseInitial, energyInitial, harmonic, eta, beta, energy):
    newPhase = phaseInitial + 2*np.pi*harmonic*eta*energyInitial/(beta**2*energy)
    return newPhase
def kick(energyInitial, phaseInitial, charge, voltage):
    newEnergy = energyInitial + charge*voltage*np.sin(phaseInitial)
    return newEnergy
```

Turn $n \rightarrow n$+1

```
n_turns = 60
particlePhase = 0.5*np.pi                                   # Initial particle phase
particleEnergy = 0                                          # Initial particle energy offset
particlePhaseArray = np.zeros(n_turns)                      # Initialize phase array
particleEnergyArray = np.zeros(n_turns)                     # Initialize energy offset array
for i in range(n_turns):
    particleEnergyArray[i] = np.array(particleEnergy)
    particlePhaseArray[i] = np.array(particlePhase)
    particlePhase = drift(particlePhase, particleEnergy, harmonic, eta, beta, energy)
                                                            # Track phase
    particleEnergy = kick(particleEnergy, particlePhase, charge, voltage)
                                                            # Track energy
```

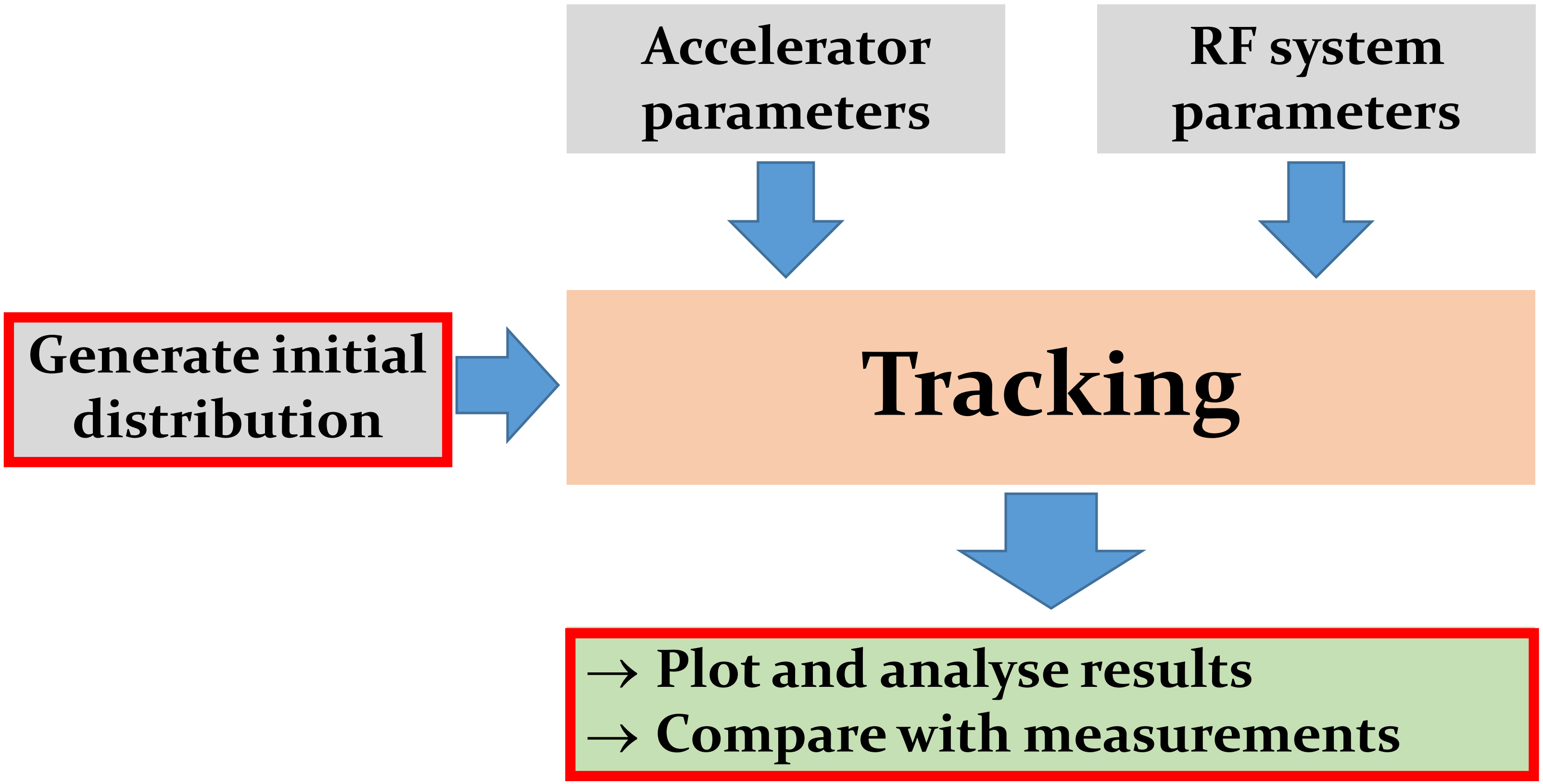


Choice of particle coordinates

- **Time or phase? Momentum or energy?**
- **Absolute or relative coordinates**

		Advantages	Disadvantages
t	E	• Most universal • Suitable for any tracking • Canonically conjugated	• Numerical precision: large absolute value • Relative bunch motion more difficult to follow
Δt	ΔE	• Relevant deviations only • Canonically conjugated • Most suited for multiple h	• Required synchronous particle as reference • Duration of turn may change
Φ	E, ΔE	• Turn length always 2π • Relevant deviations only	• Requires synchronous particle as reference • Not canonically conjugated
ϕ	E, ΔE	• RF bucket length always 2π • Relevant deviations only • **Most suited for single h**	• Requires synchronous particle as reference • Not canonically conjugated

Tracking simulation flow



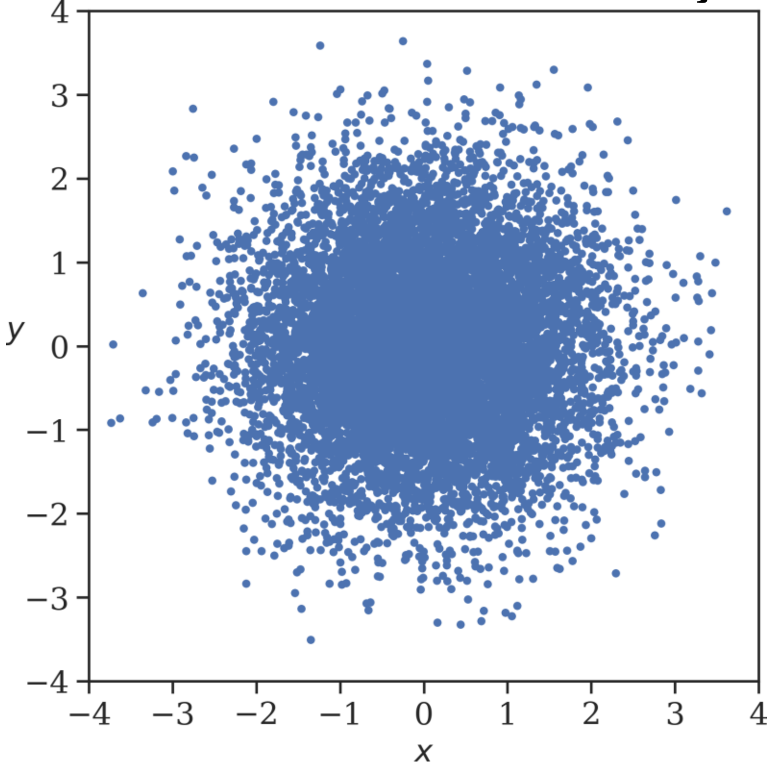
Distributions and projections

From single particle tracking to distribution

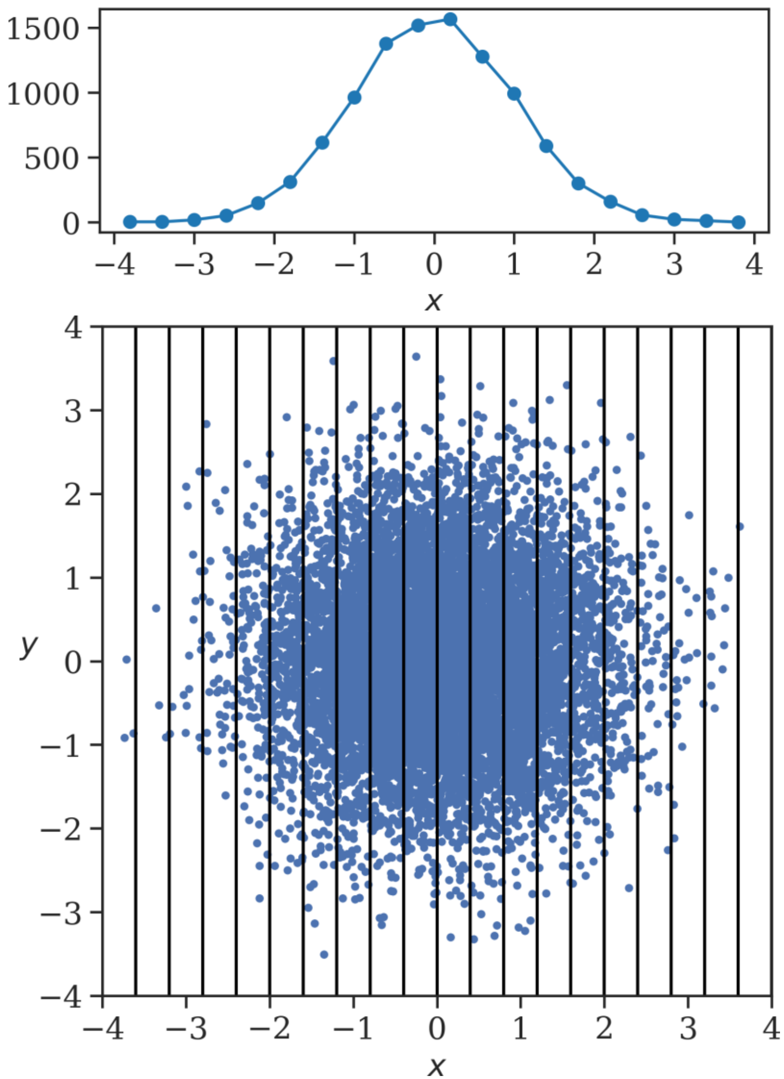
→ **10^{10}…10^{12} particles per bunch** → **too much computing power**

→ **Macro-particles to reduce** → **up to few 10^6 per bunch**

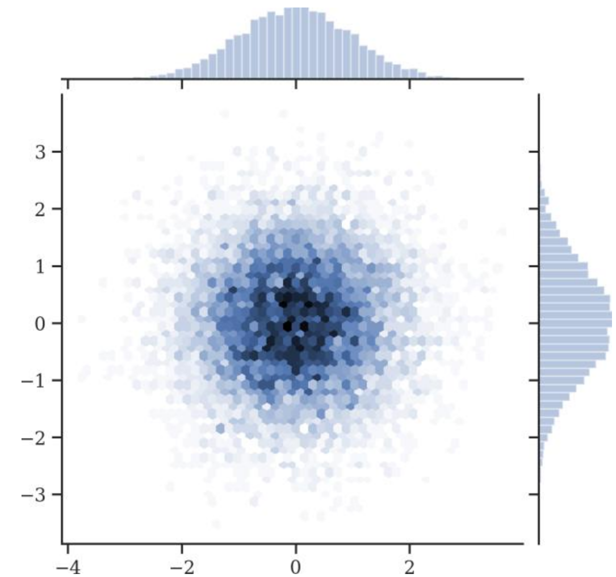
Normal distribution in *x*, *y*



Projections of distributions



- **Very common task:**
  - → **e.g. Python seaborn**
  - → `plot_phase_space_distribution`

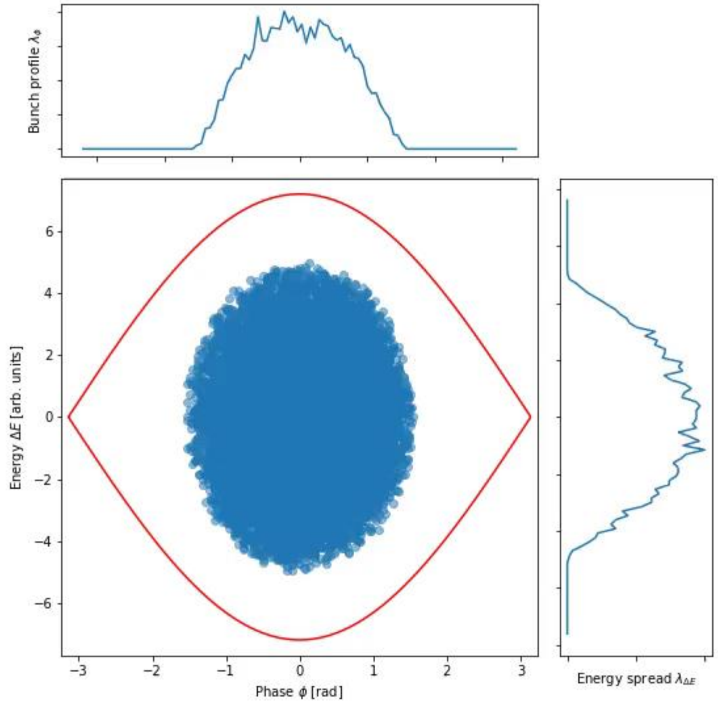


→ **Time projection directly observable: bunch profile**

Example: Tracking of a single bunch

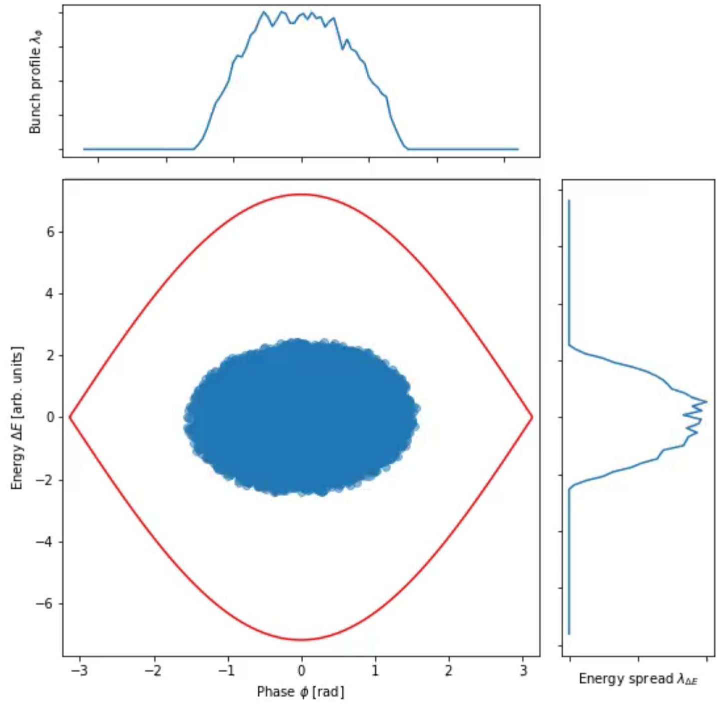
- **Set-up bunch with parabolic distribution: `generate_bunch`**
- **Most simple case: single harmonic RF without acceleration**

Correct voltage at correct phase



→ Matched bunch

Wrong voltage at correct phase

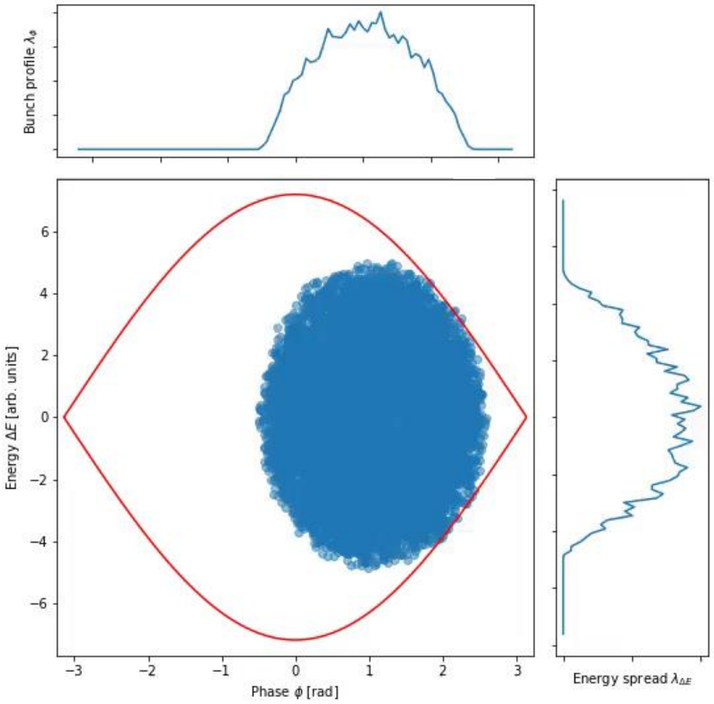


→ Breathing bunch (quadrupole)

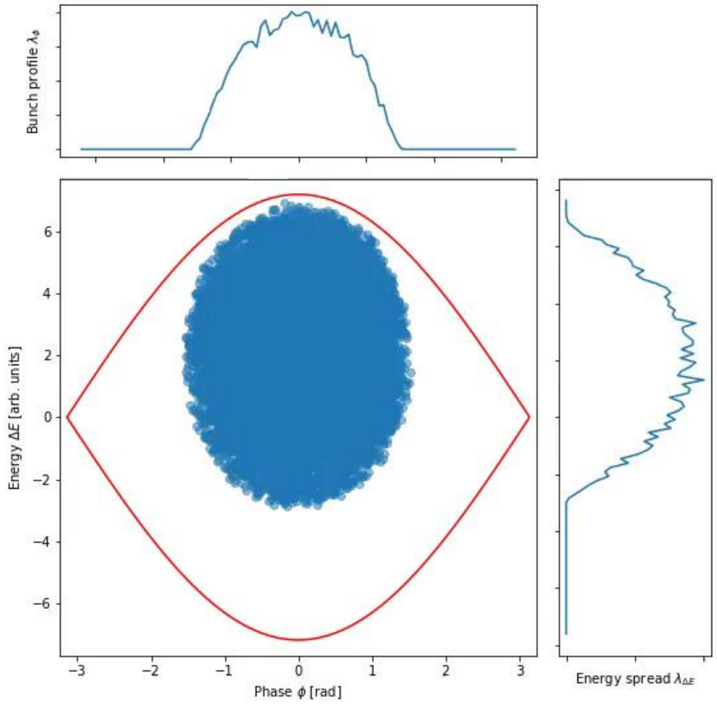
Example: Tracking of a single bunch

- **Set-up bunch with parabolic distribution:** `generate_bunch`
- **Most simple case: single harmonic RF without acceleration**

Correct voltage at wrong phase



All correct, but wrong energy



→ Dipole oscillations
→ Phase and energy offset for example at injection

Getting closer to reality

→ **State-of-the-art tracking may include much more**

Non-linear phase slip factor, e.g. transition

$$\phi_{n+1} = \phi_n + 2\pi h \eta \frac{\Delta E_n}{\beta^2 E}$$

Multiple RF systems with changing parameters

Energy loss: synchrotron radiation or impedances

Beam induced voltage

$$\Delta E_{n+1} = \Delta E_n + qV \left[g(\phi_{n+1}) - g(\phi_\mathrm{S}) \right] + \Delta E_\mathrm{ext} + \Delta E_\mathrm{self}$$

Global regulation loops for beam phase and radial position

Acceleration

Feedbacks around cavities

Longitudinal tracking codes

- **Dedicated to longitudinal dynamics: fast and focussed on RF aspects**
- **Combined transverse and longitudinal tracking**

Name	Remarks	
BLonD	• Widely used at CERN • Complex RF manipulations and feedbacks • Longitudinal intensity effects http://blond.web.cern.ch/	Longitudinal, 1D
ESME	• Longitudinal work horse code for many years • RF manipulations with multiple RF systems • Intensity effects esme.fnal.gov	Longitudinal, 1D
PyHeadTail	• Longitudinal and transverse combined simulation https://twiki.cern.ch/twiki/bin/view/ABPComputing/PyHEADTAIL	Combined, 3D
PyOrbit	• Longitudinal and transverse combined simulation https://twiki.cern.ch/twiki/bin/view/ABPComputing/PyORBIT	Combined, 3D
elegant	• Longitudinal and transverse combined simulation • Mainly used for electron accelerators https://ops.aps.anl.gov/elegant.html	Combined, 3D
…	…	

Summary

- **Longitudinal simulations using particle tracking**
  - → **Complementary approach to longitudinal beam dynamics**
  - → **Flexibility to change parameters during tracking**
  - → **Powerful technique to study**
    - **Multi-harmonic RF systems**
    - **Complicated intensity effects**
    - **Longitudinal dynamics with feedbacks and RF loops**

Google

...is your friend!

You will build a (small) longitudinal tracking code

A big Thank You

to all colleagues providing support, material and feedback

**Simon Albright, Maria-Elena Angoletta,
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Manfred Wendt, Jörg Wenninger, J. Wulff
and many more…**

Thank you very much for your attention!

References

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