Injection and Extraction (+Kickers, Septa)

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based on lectures by Matthew Fraser and M.J. Barnes, W. Bartmann, J. Borburgh, V. Forte, B. Goddard, V. Kain and M. Meddahi

Many THANKS!!

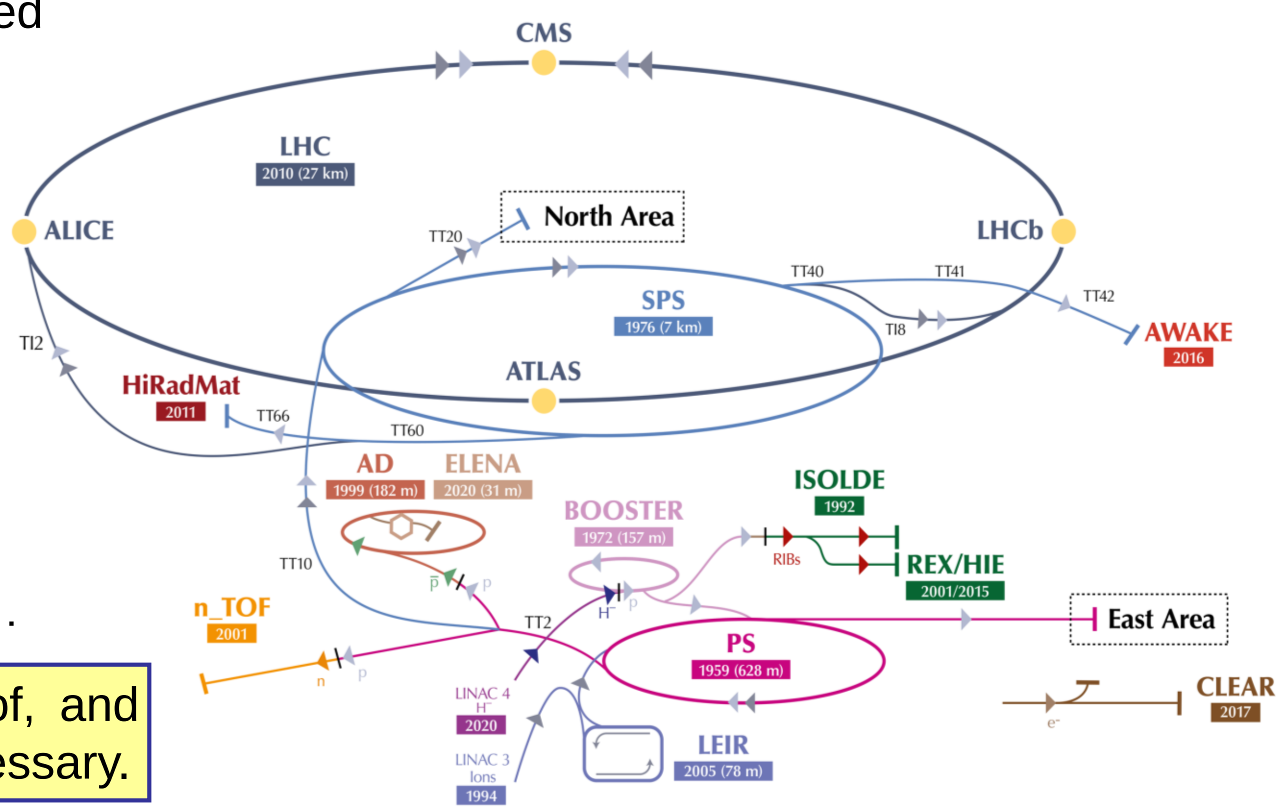
- Introduction: Kickers, septa and normalised phase-space
- Injection methods
  - Single-turn hadron injection
  - Injection errors, filamentation and blow-up
  - Multi-turn hadron injection
  - Charge-exchange H- injection
  - Lepton injection
- Extraction methods
  - Single-turn (fast) extraction
  - Non-resonant and resonant multi-turn (fast) extraction
  - Resonant multi-turn (slow) extraction
- Linking machines

Injection and extraction

- An accelerator has limited dynamic range
- Chain of stages needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External facilities and experiments:
  - e.g. ISOLDE, HIRADMAT, AWAKE…

Beam transfer (into, out of, and between machines) is necessary.

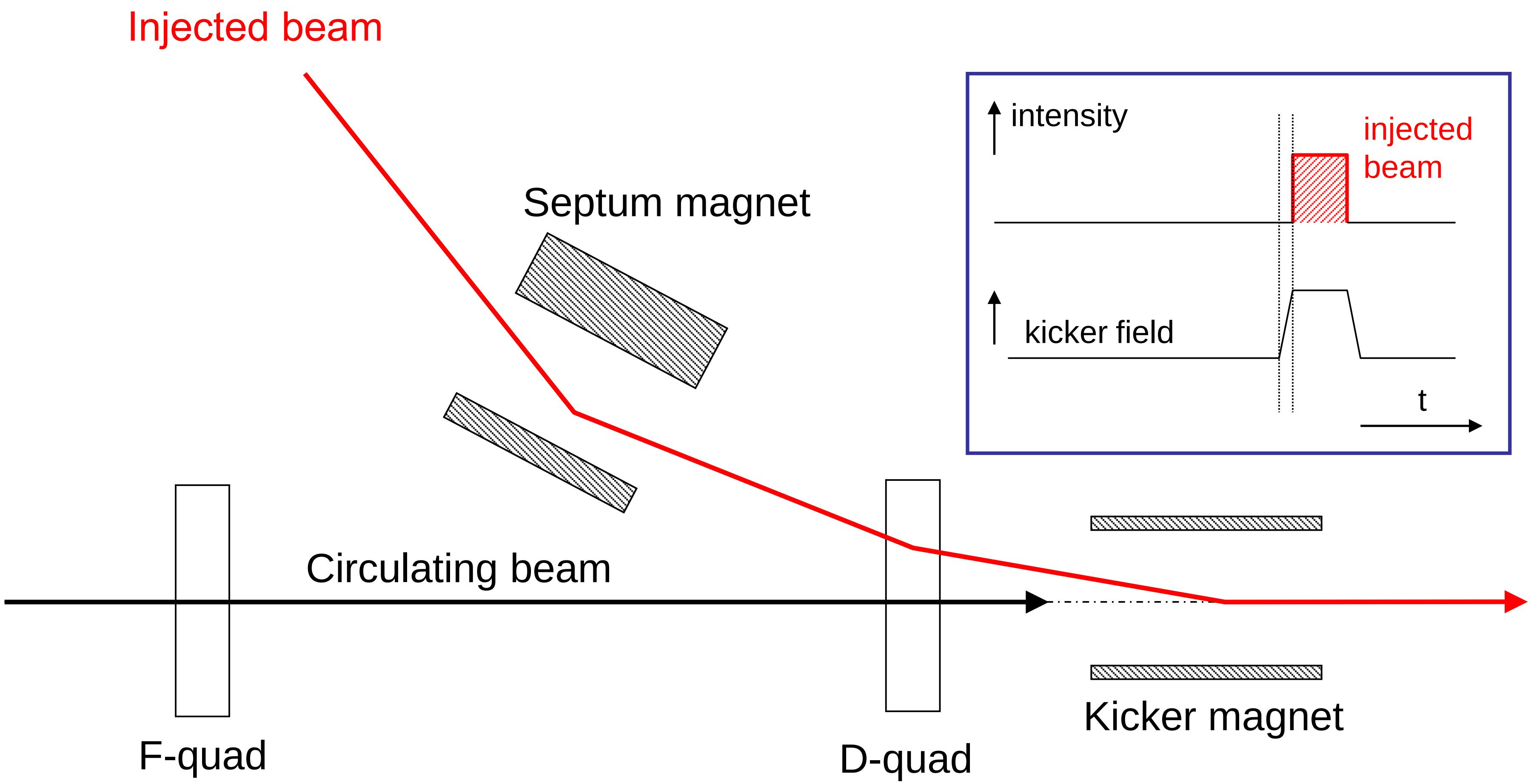
CERN Accelerator Complex



H⁻ (hydrogen anions) p (protons) ions RIBs (Radioactive Ion Beams) n (neutrons) p̄ (antiprotons) e⁻ (electrons)

LHC - Large Hadron Collider // SPS - Super Proton Synchrotron // PS - Proton Synchrotron // AD - Antiproton Decelerator // CLEAR - CERN Linear Electron Accelerator for Research // AWAKE - Advanced WAKefield Experiment // ISOLDE - Isotope Separator OnLine // REX/HIE - Radioactive EXperiment/High Intensity and Energy ISOLDE // LEIR - Low Energy Ion Ring // LINAC - LINear ACcelerator // n_TOF - Neutrons Time Of Flight // HiRadMat - High-Radiation to Materials

Basics: injection, septum and kicker



- **Kickers** produce fast pulses, rising their field within the particle-free gap in the circulating beam **(temporal separation)**
- **Septa** compensate for the relatively low kicker strength, and approach closely the circulating beam **(spatial separation)**

Google

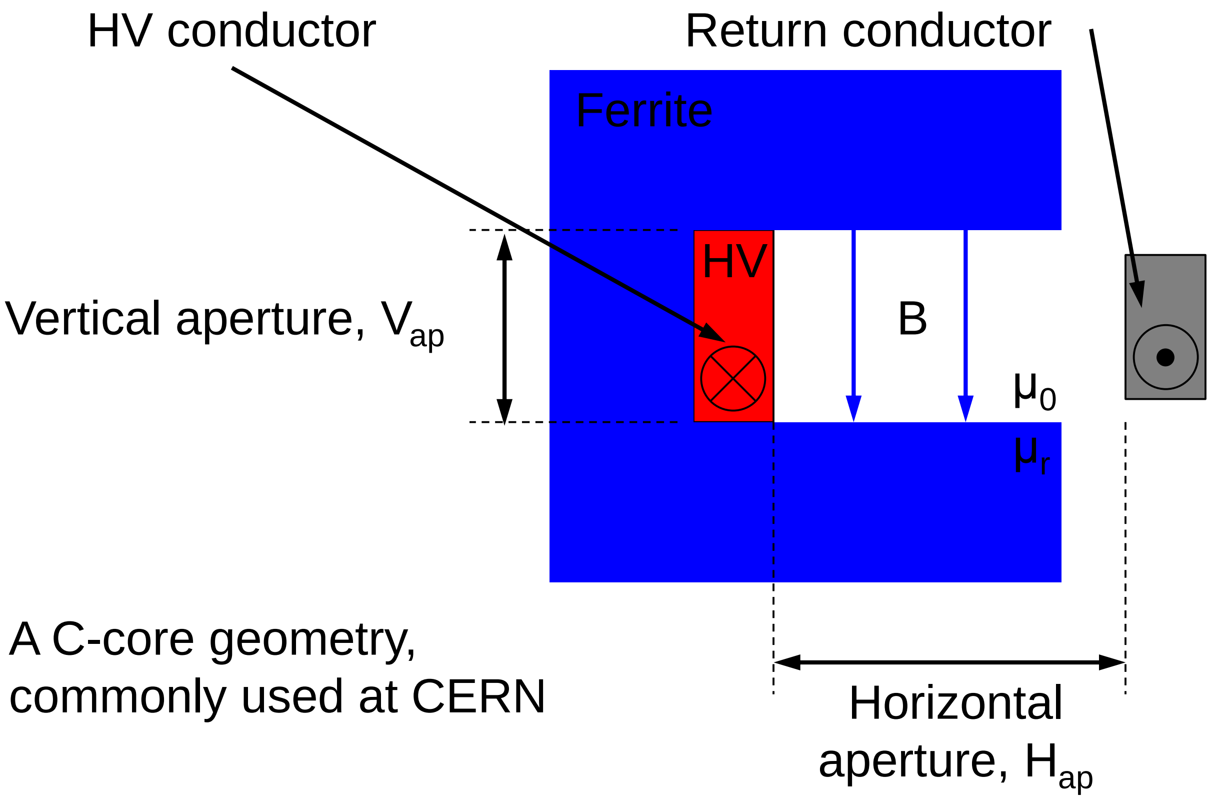
- Kicker

- Septum

…so we also call them
"**Fast Pulsed Magnets**"

Kickers - Magnetic parameters

Pulsed magnet with **very fast rise time** (<100 ns – few μs)



A C-core geometry, commonly used at CERN

Magnetic field

$$B_y \approx \mu_0 \left(\frac{N \cdot I}{V_{ap}} \right)$$

Derivation: remember Ampère's Law:

$$\oint_C \vec{B}.d\vec{l} = \mu_0 I_{enc}$$

Magnet inductance [per unit length]

$$L_{mag/m} \approx \mu_0 \left(\frac{N^2 \cdot H_{ap}}{V_{ap}} \right)$$

Derivation: remember Faraday's Law:

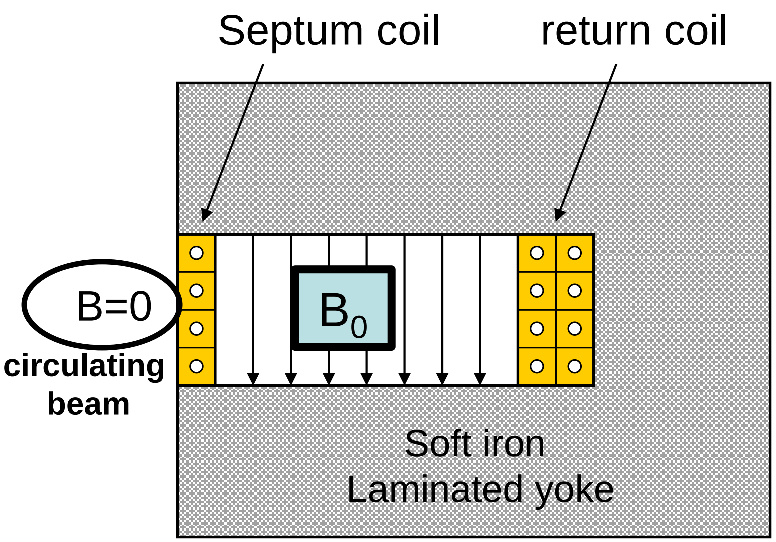
$\Phi_B = \int V\,dt$ *and* $V = L\,dI/dt$

- Dimensions H_{ap} and V_{ap} specified by beam parameters at kicker location
- Ferrite (permeability $\mu_r \approx 1000$) reinforces magnetic circuit and field uniformity in the gap
- For fast rise-times the inductance must be minimised: typically the number of turns, $N = 1$
- Kickers are often split into several magnet units, powered independently

Magnetic and electrostatic septum

Magnetic

Septum coil: **2 – 20 mm**

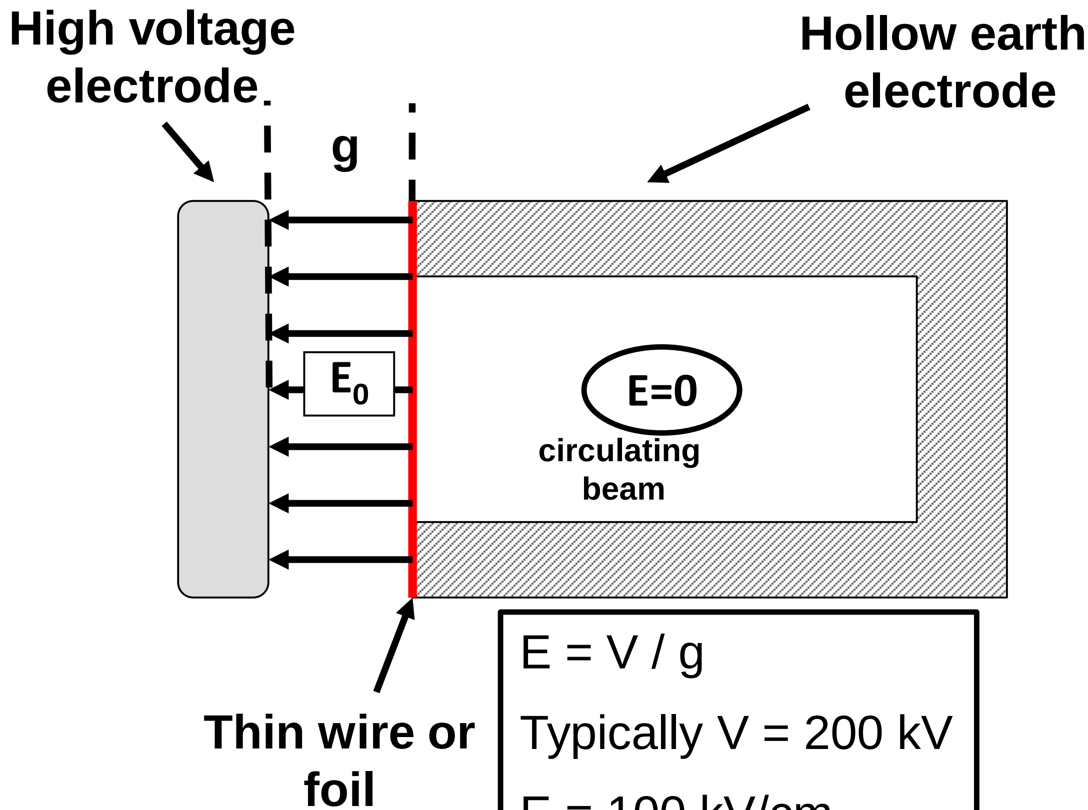


$B_o = \mu_0 I / g$

Typically I 5 - 25 kA

Electrostatic

Thin wire or coil: **~0.1 mm**

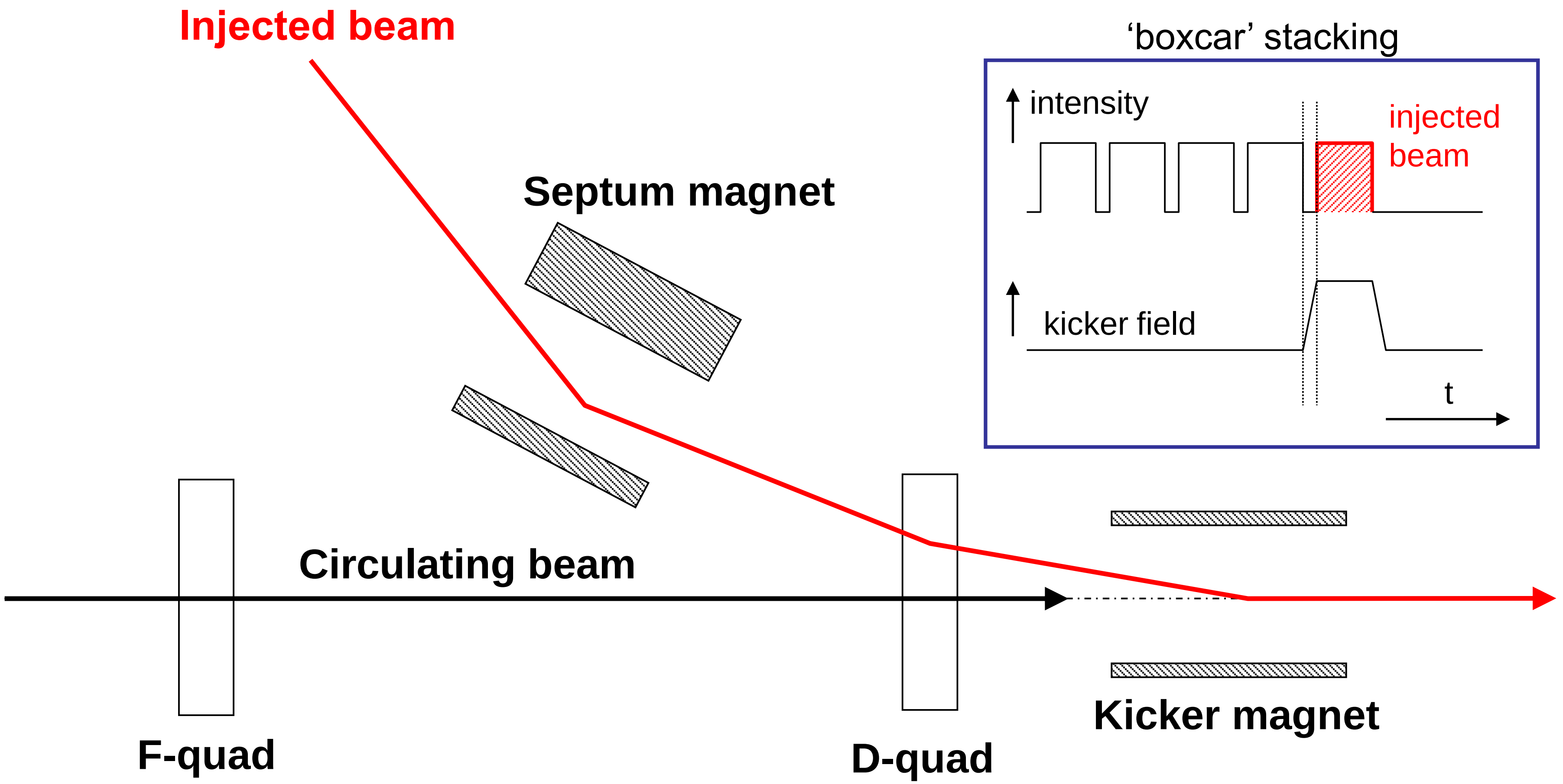


E = V / g

Typically V = 200 kV

E = 100 kV/cm

Single-turn injection – same plane



- Septum deflects the beam onto the closed orbit at the centre of the kicker
- Kicker compensates for the remaining angle
- Septum and kicker either side of D quad to minimise kicker strength

Normalised phase space

- Transform real transverse coordinates (*x*, *x'*, *s*) to normalised co-ordinates ($\bar{X}$, $\bar{X}'$, μ) where the independent variable becomes the phase advance *μ*:

$$\begin{bmatrix} \bar{X} \\ \bar{X}' \end{bmatrix} = \boldsymbol{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta(s)}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha(s) & \beta(s) \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos[\mu(s) + \mu_0]$$

$$\mu(s) = \int_0^s \frac{ds}{\beta(s)}$$

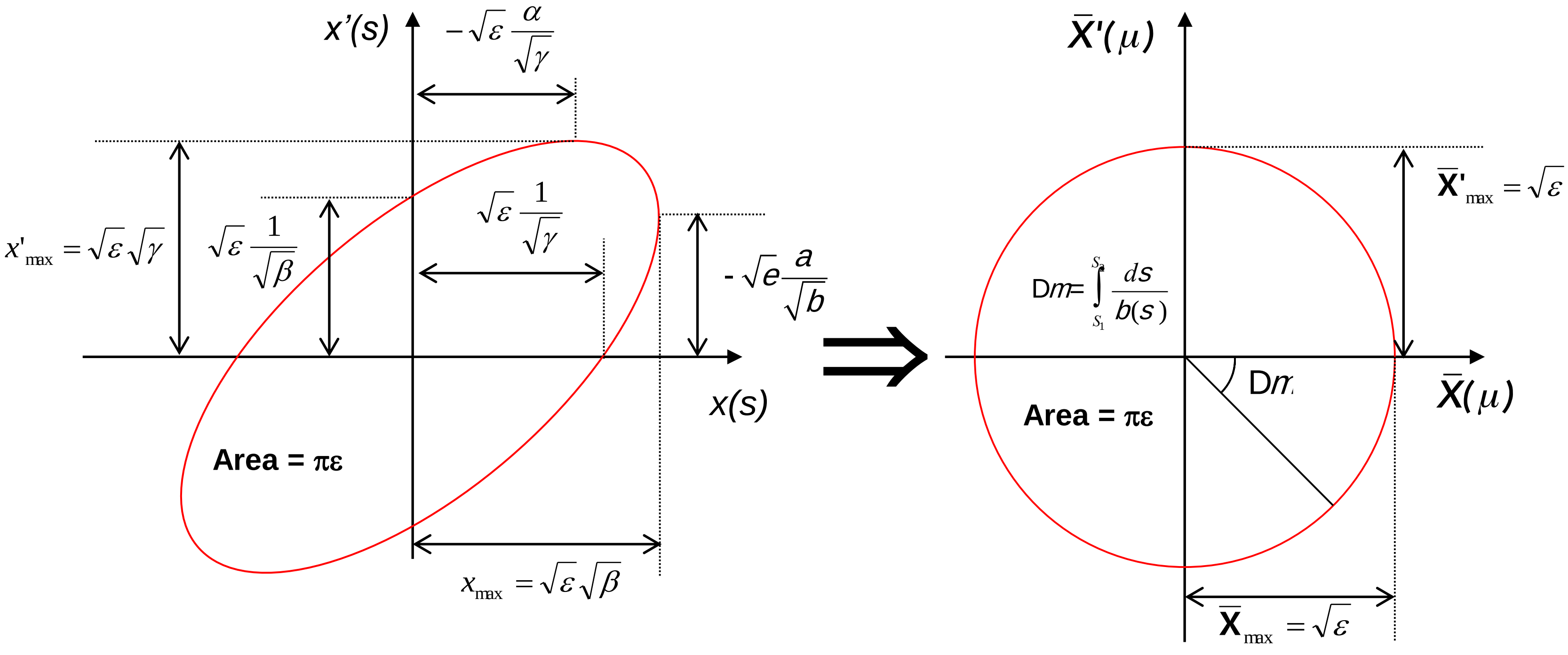
$$\bar{X}(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot x = \sqrt{\varepsilon}\cos[\mu + \mu_0]$$

$$\bar{X}'(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot \alpha(s)x + \sqrt{\beta(s)}x' = -\sqrt{\varepsilon}\sin[\mu + \mu_0] = \frac{d\bar{X}}{d\mu}$$

Normalised phase space

Real phase space

Normalised phase space

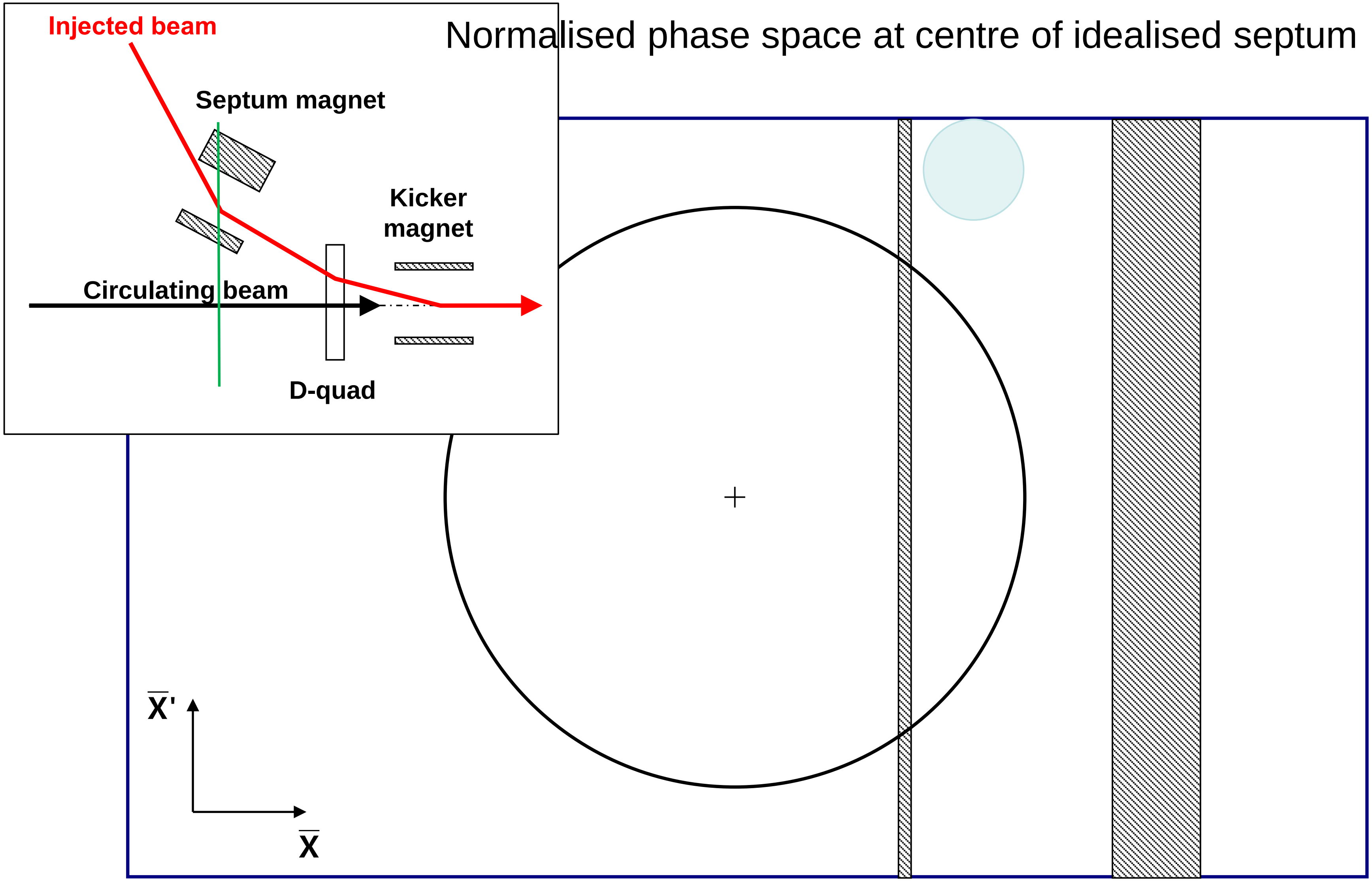


$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

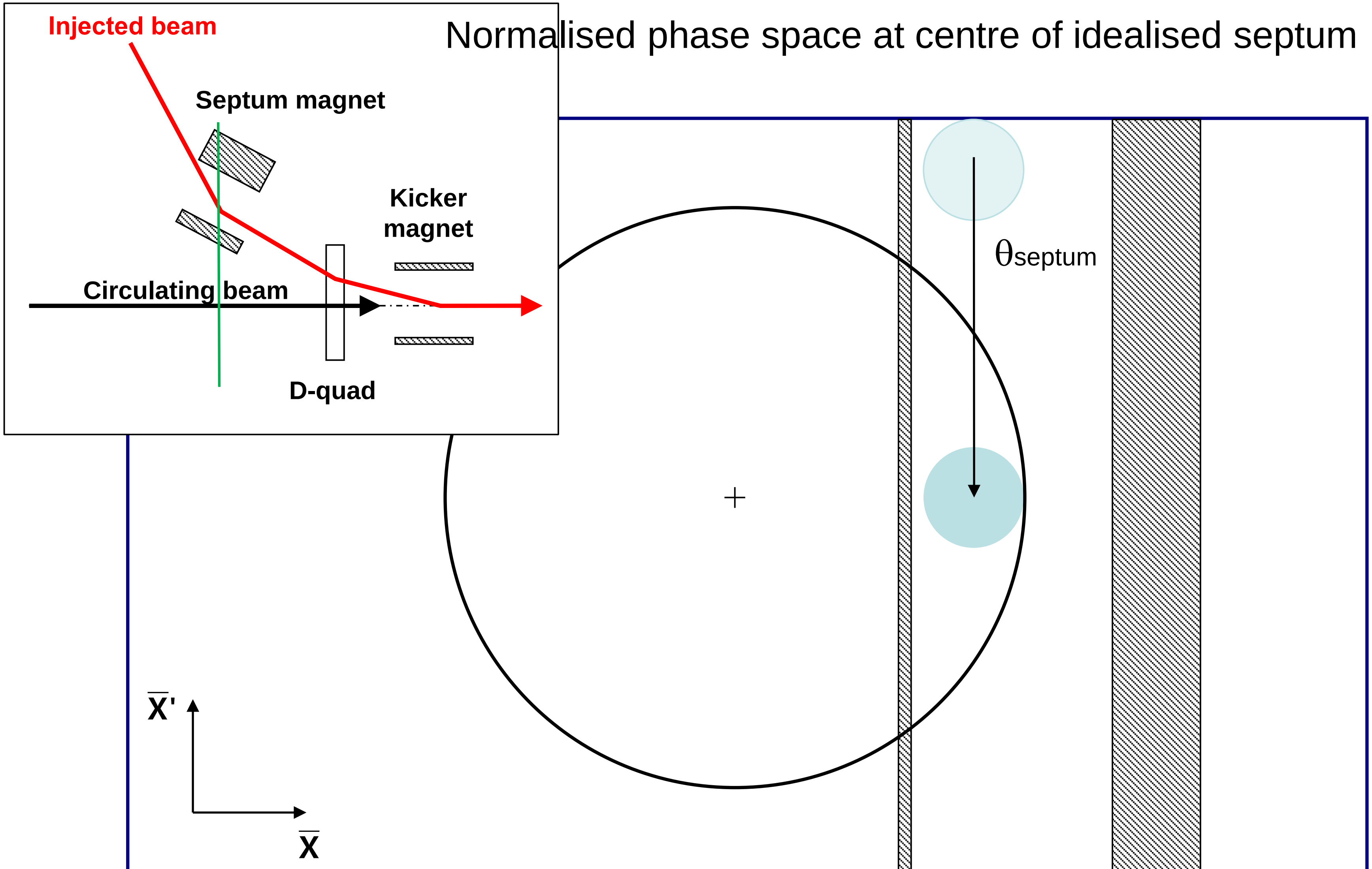
$$\varepsilon = \bar{\mathbf{X}}^2 + \bar{\mathbf{X}}'^2$$

Single-turn injection

Normalised phase space at centre of idealised septum

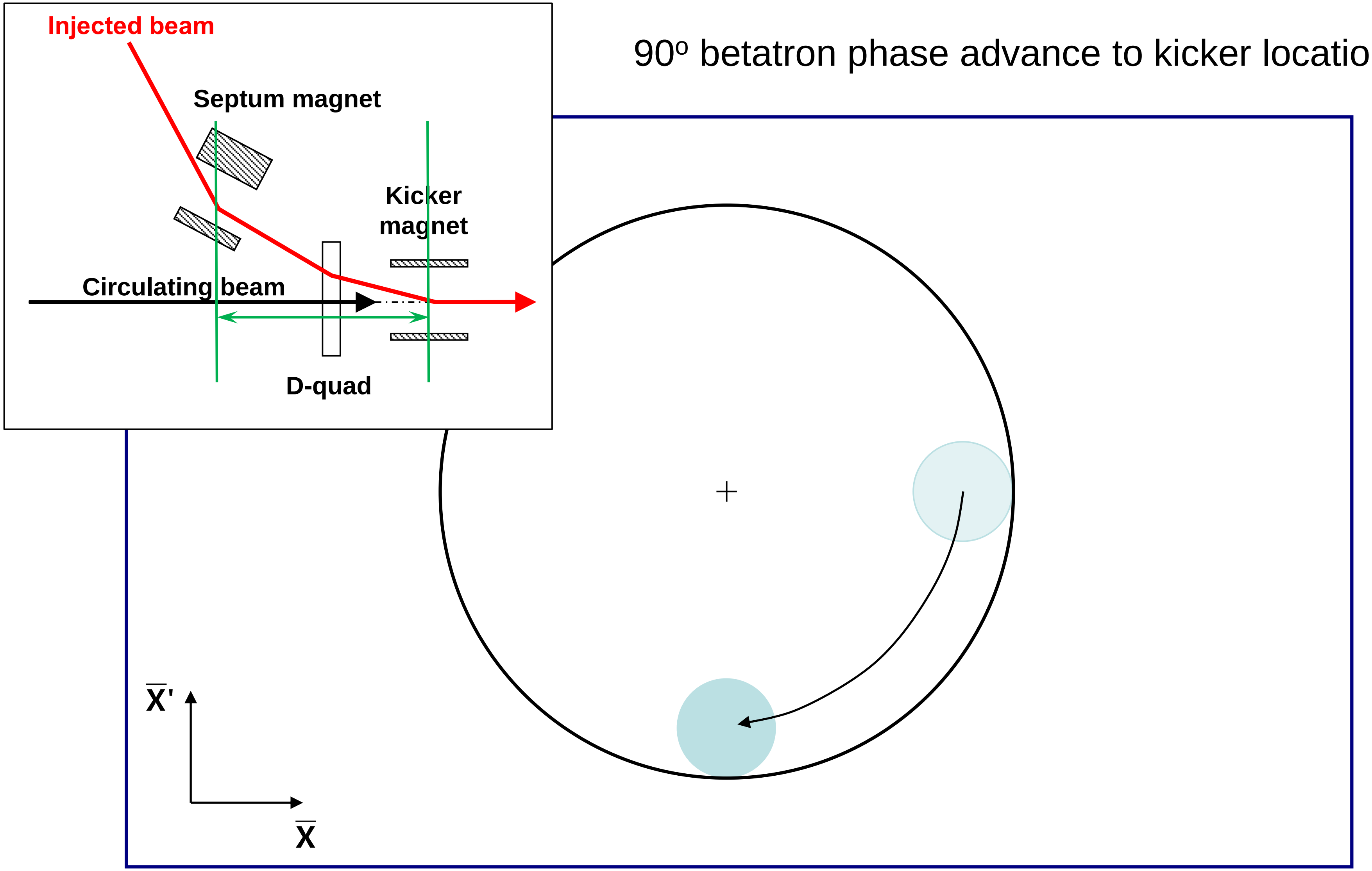


Single-turn injection



Single-turn injection

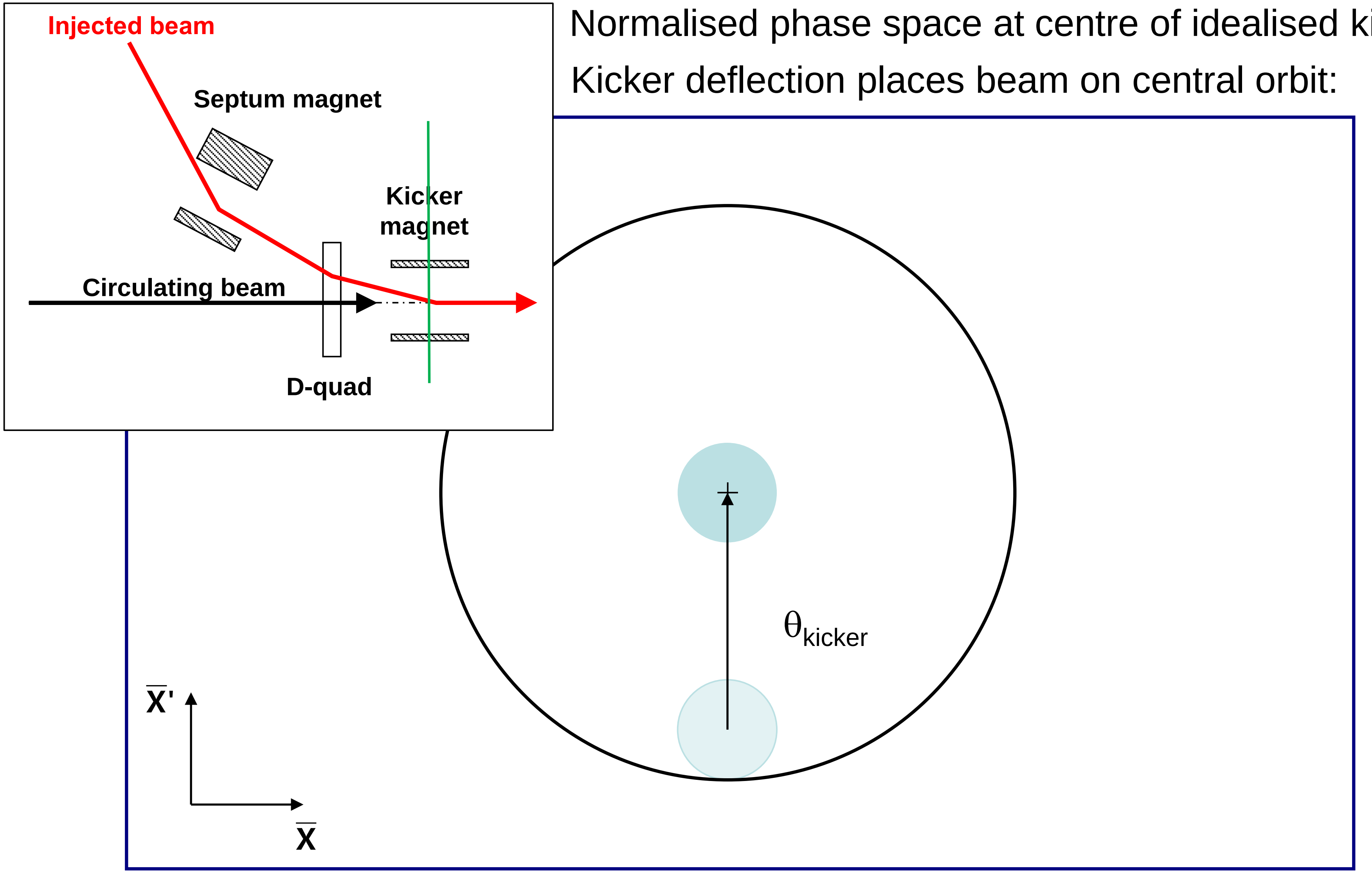
90° betatron phase advance to kicker location



Single-turn injection

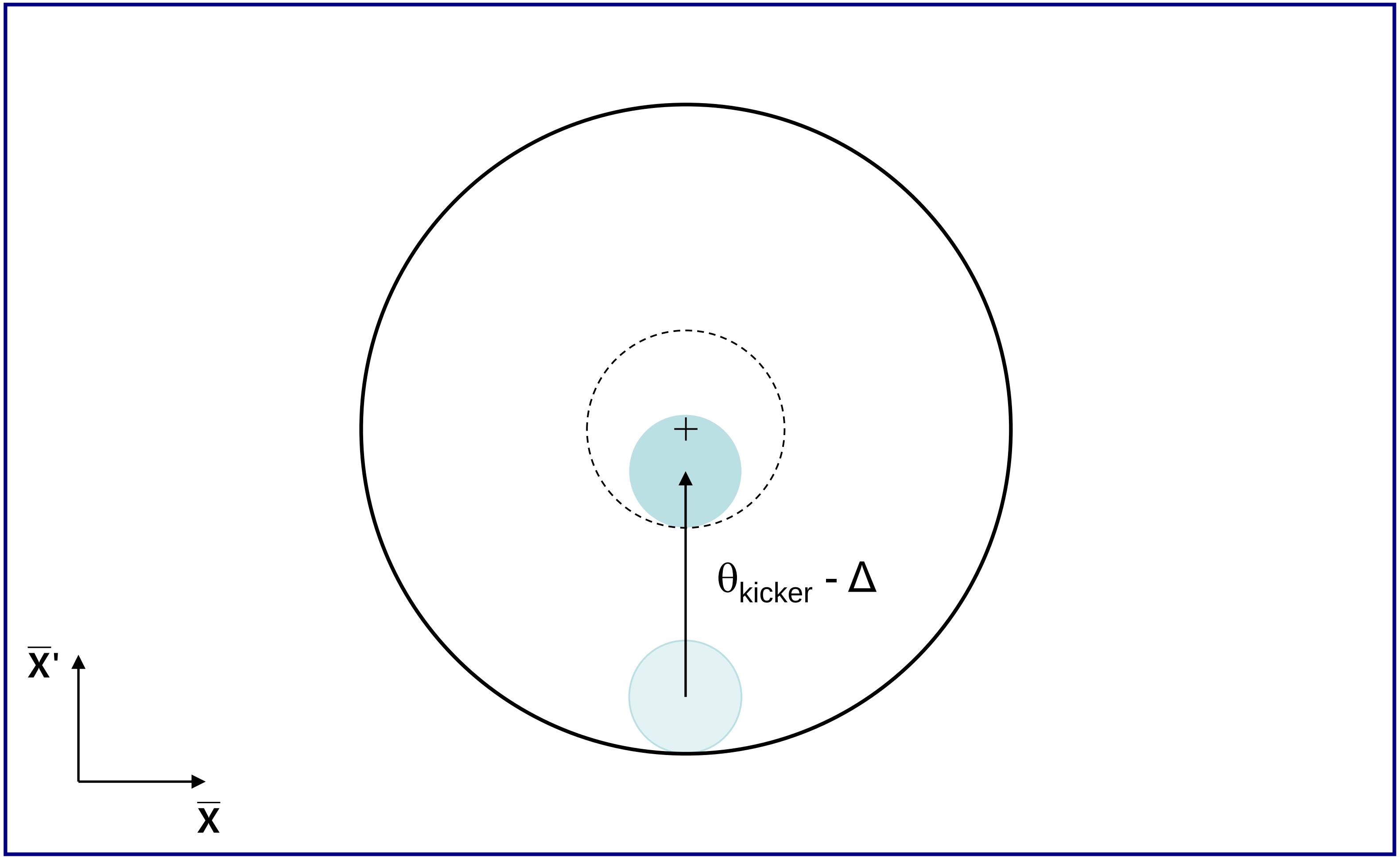
Normalised phase space at centre of idealised kicker

Kicker deflection places beam on central orbit:



Injection oscillations

For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ:



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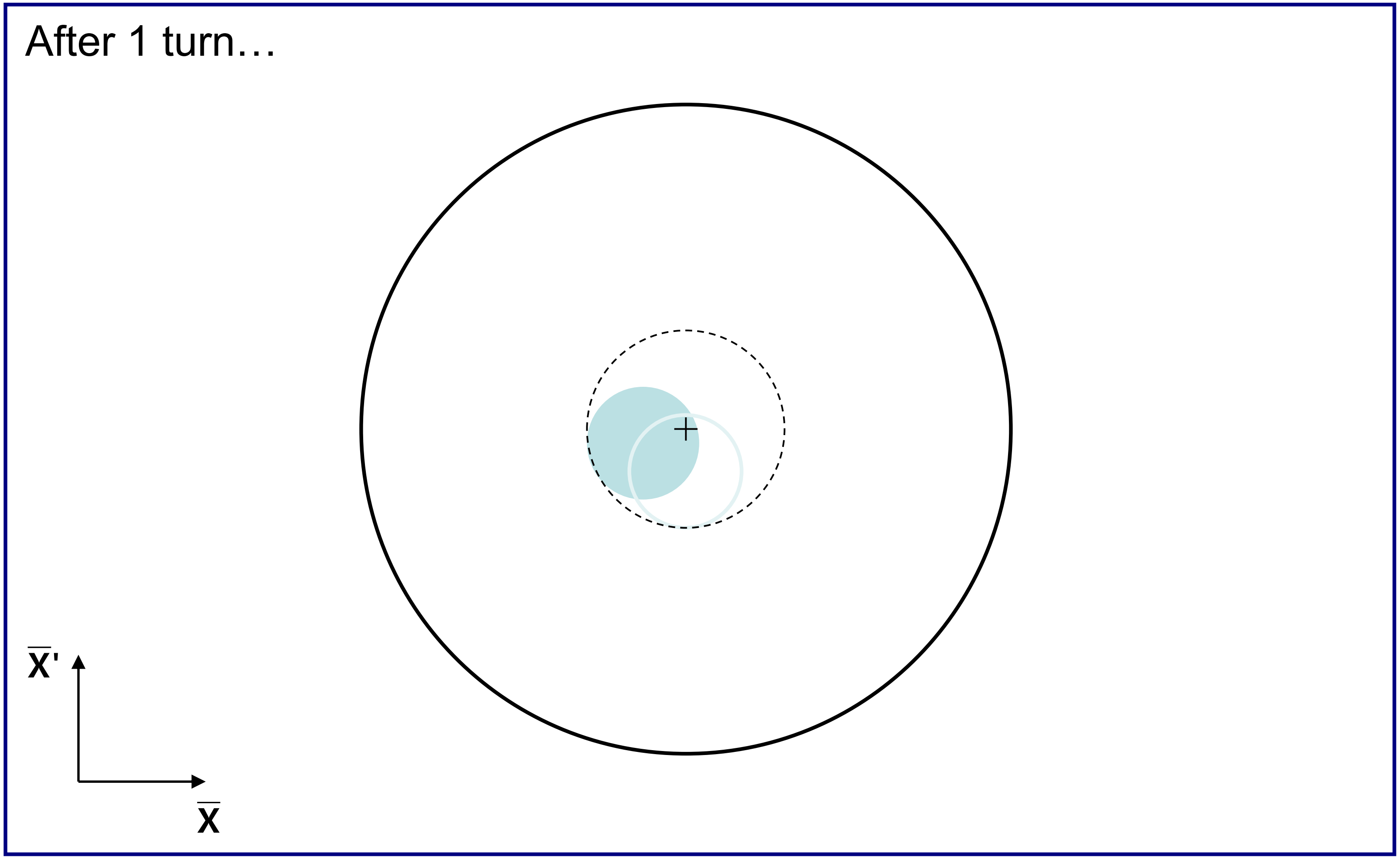
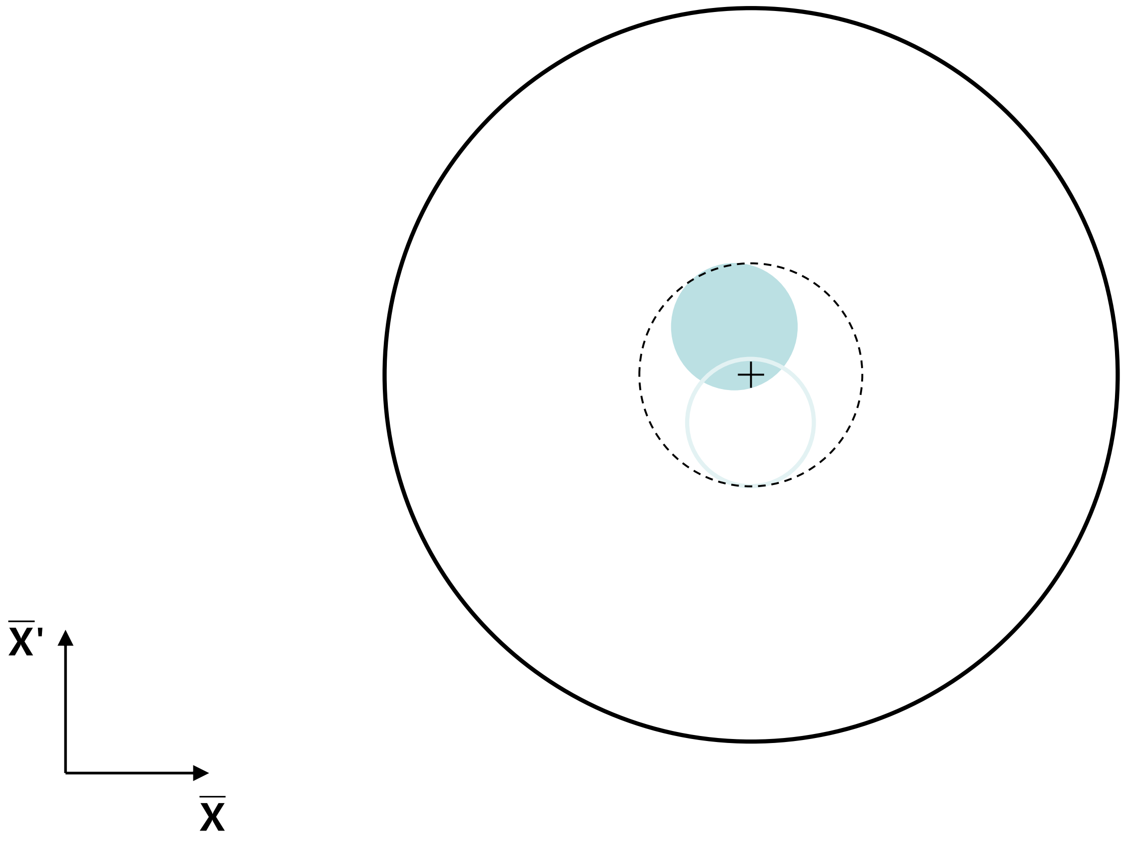


Injection oscillations

For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ:

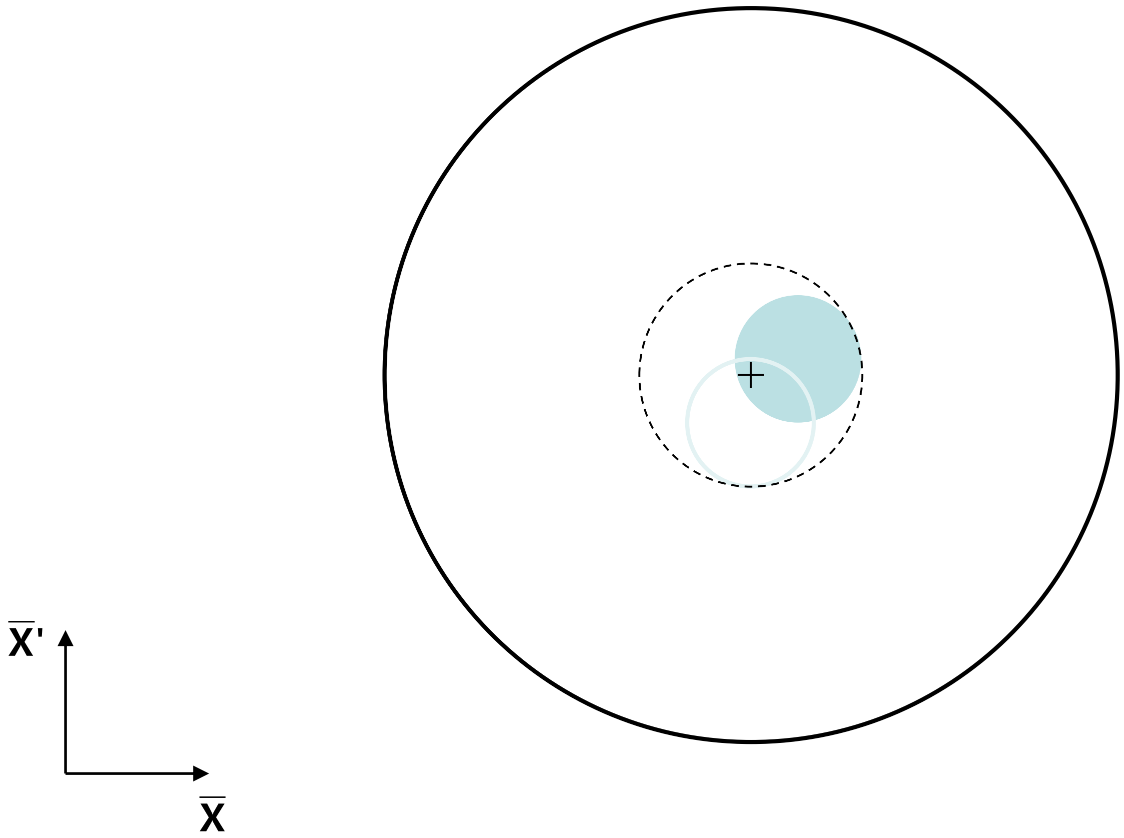
After 2 turns…



Injection oscillations

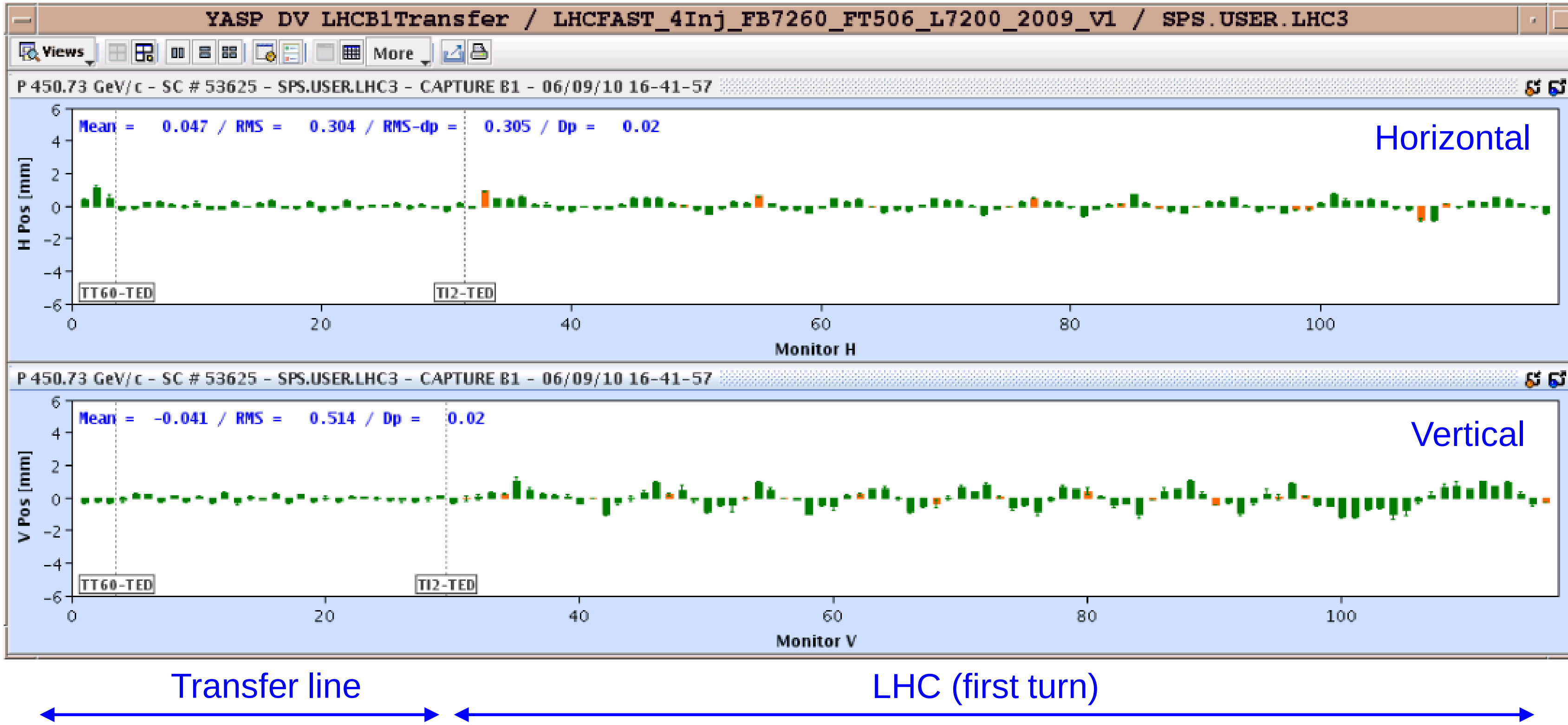
For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ:

After 3 turns etc…



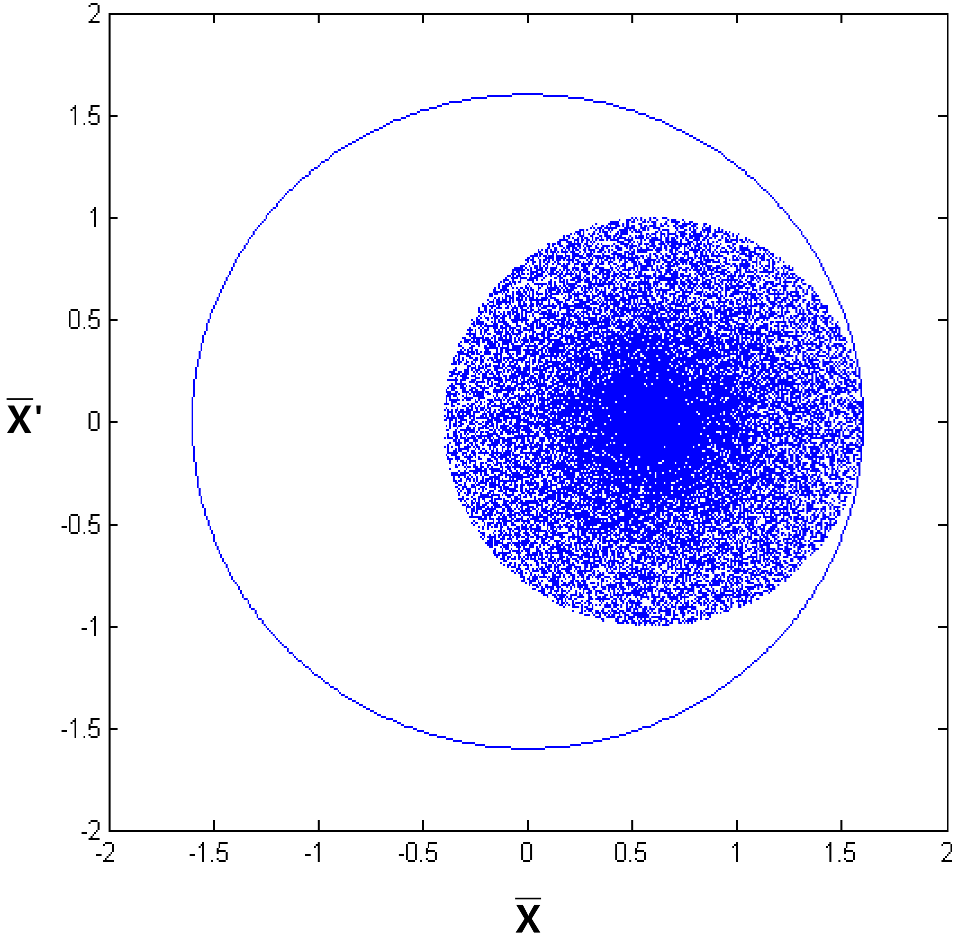
Injection oscillations

- Betatron oscillations with respect to the Closed Orbit:

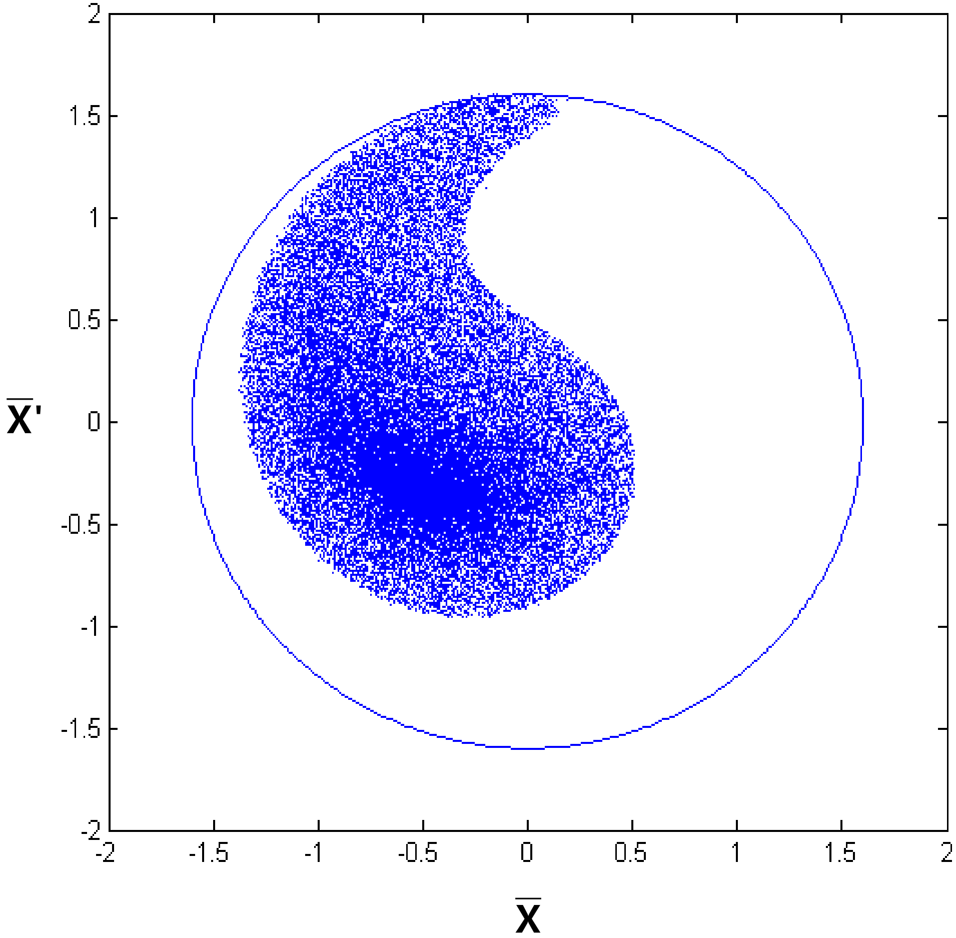


- Angular errors from septa and kicker have different orbit pattern
- Correct the difference between injected beam and closed orbit or 1st and 2nd turn

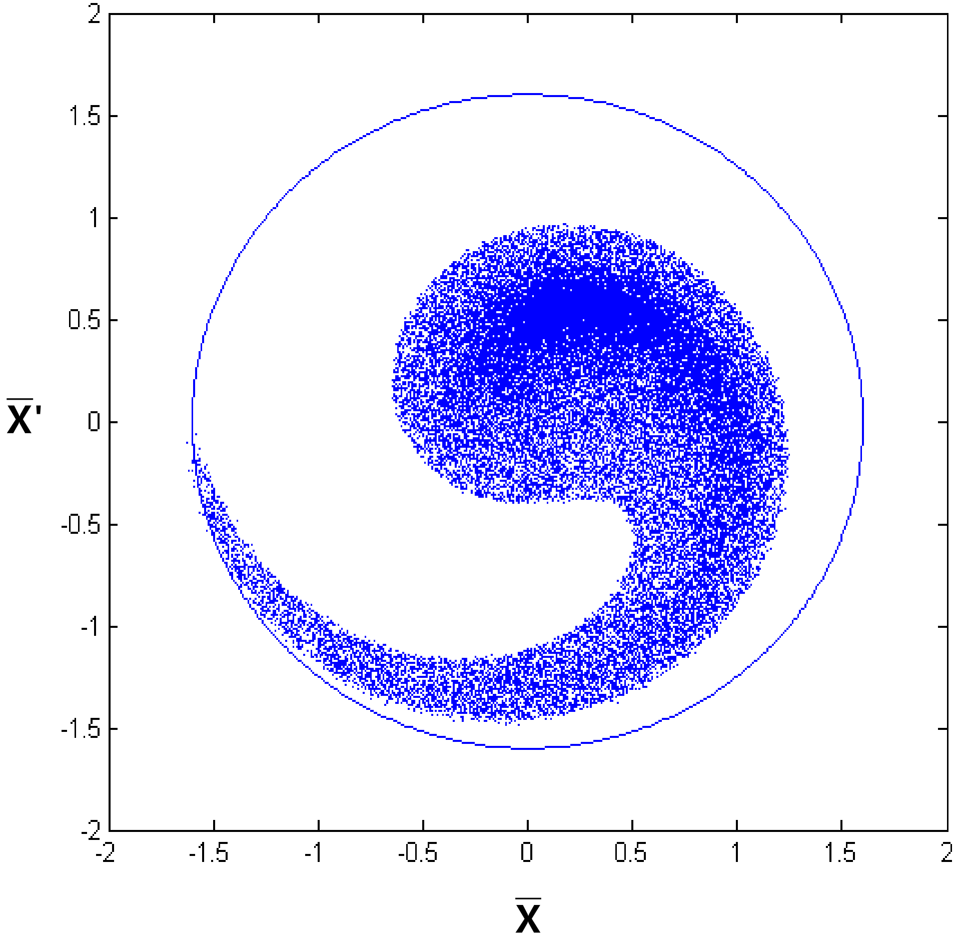
Filamentation



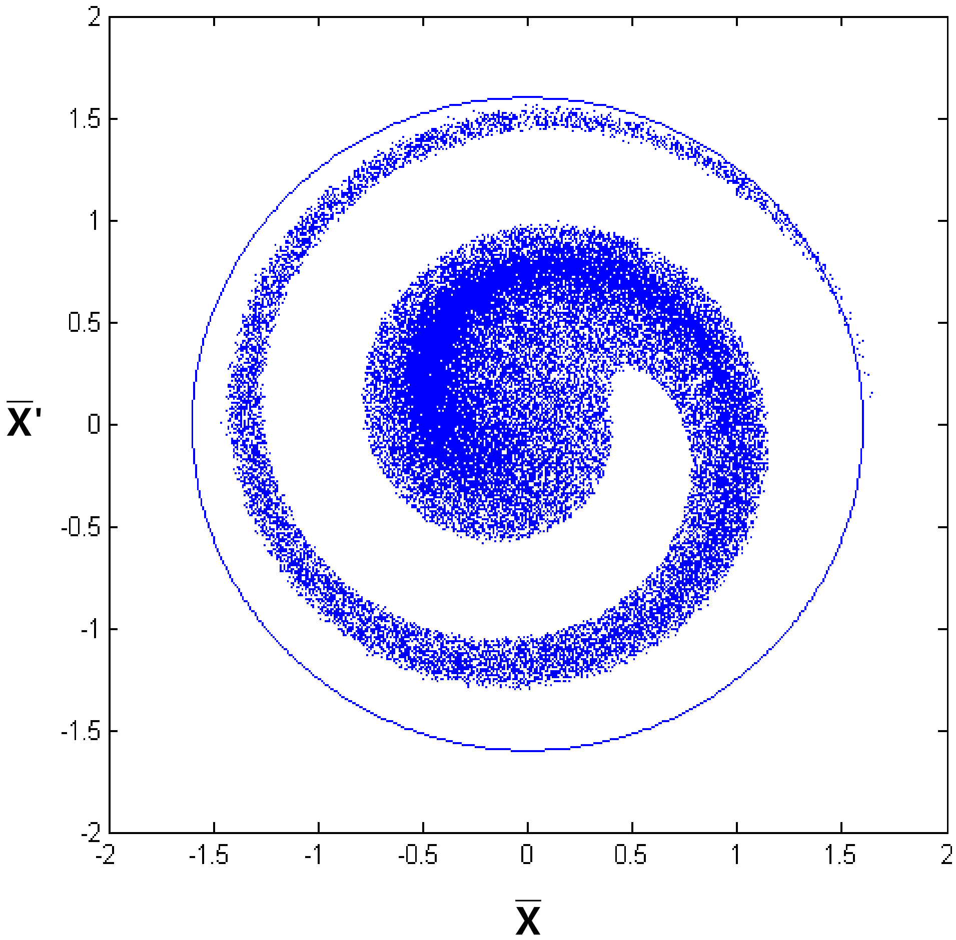
Filamentation



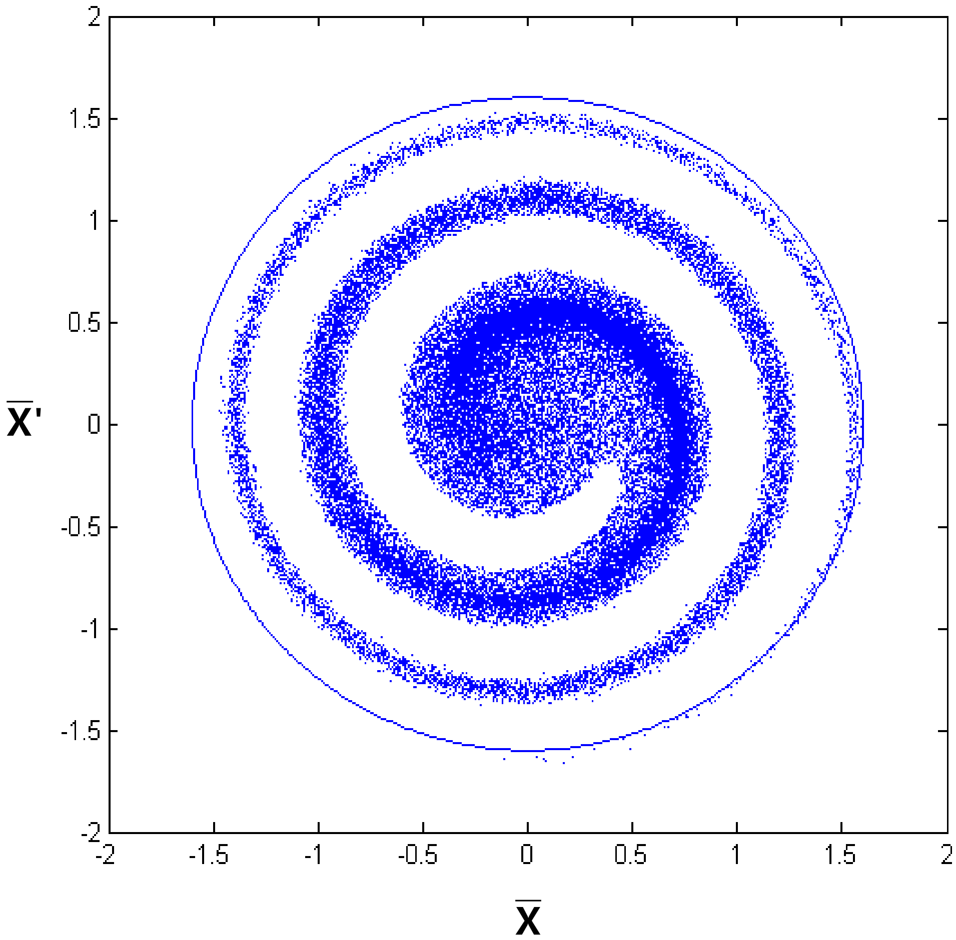
Filamentation



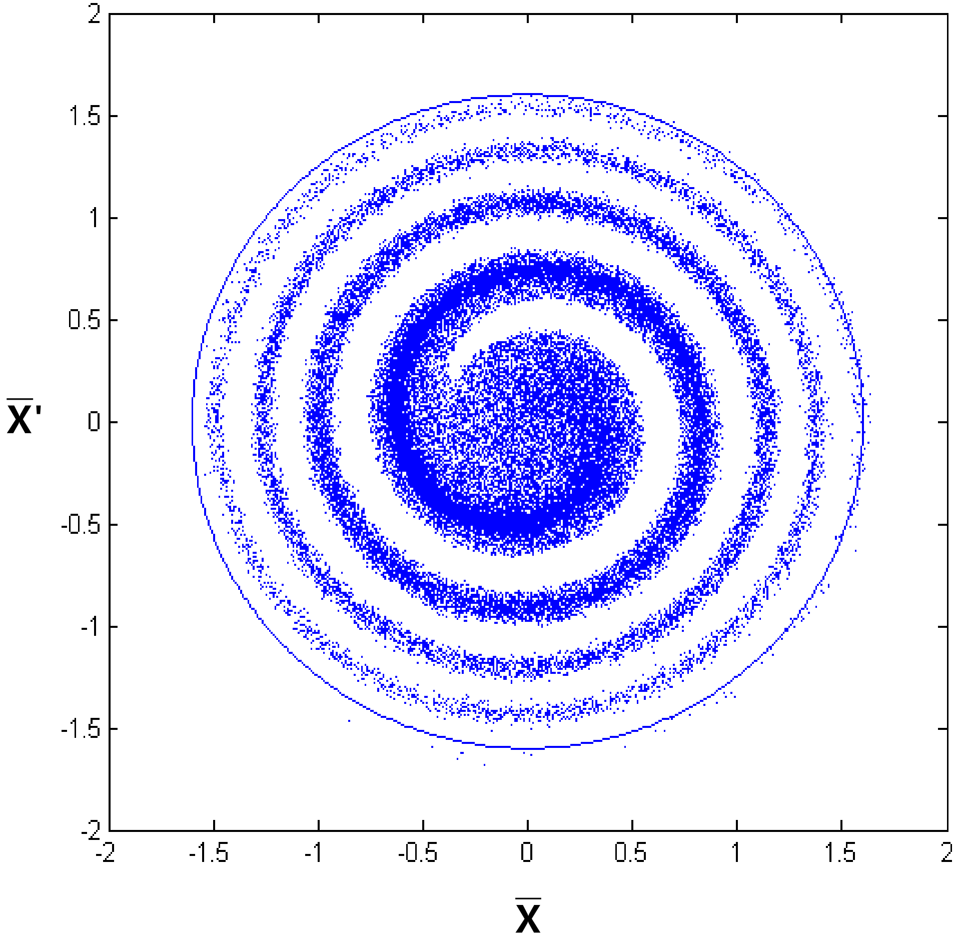
Filamentation



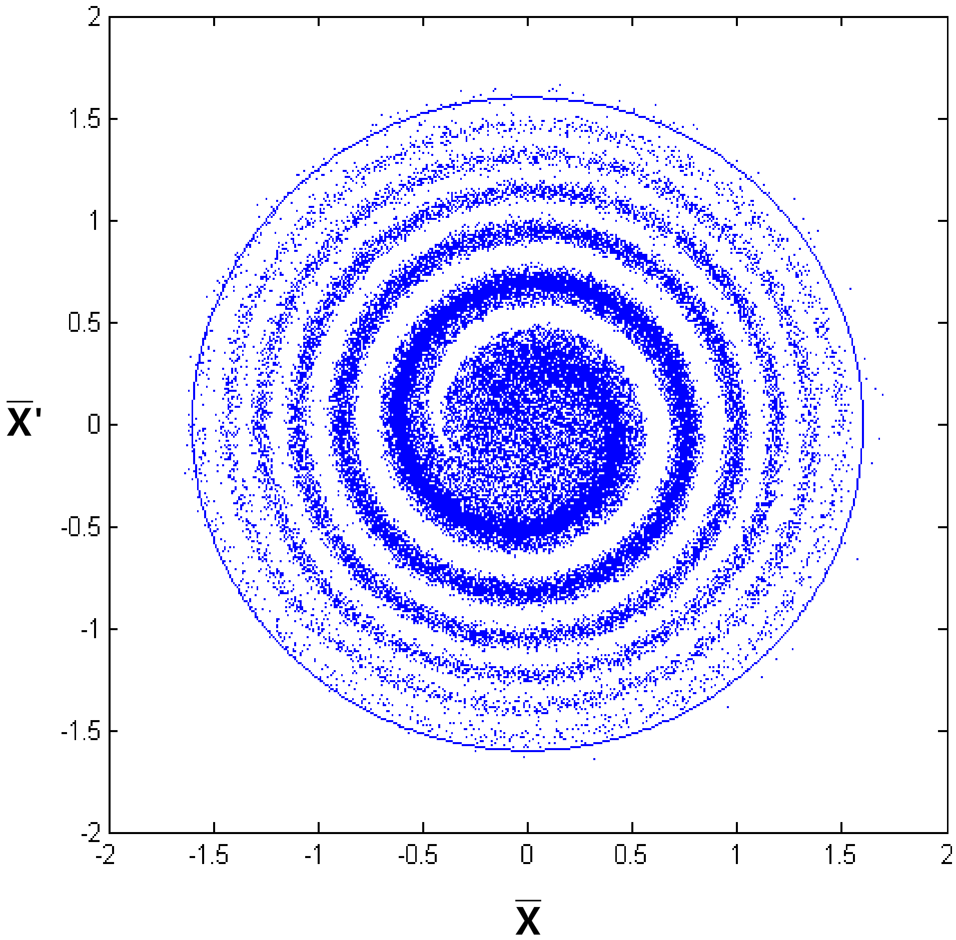
Filamentation



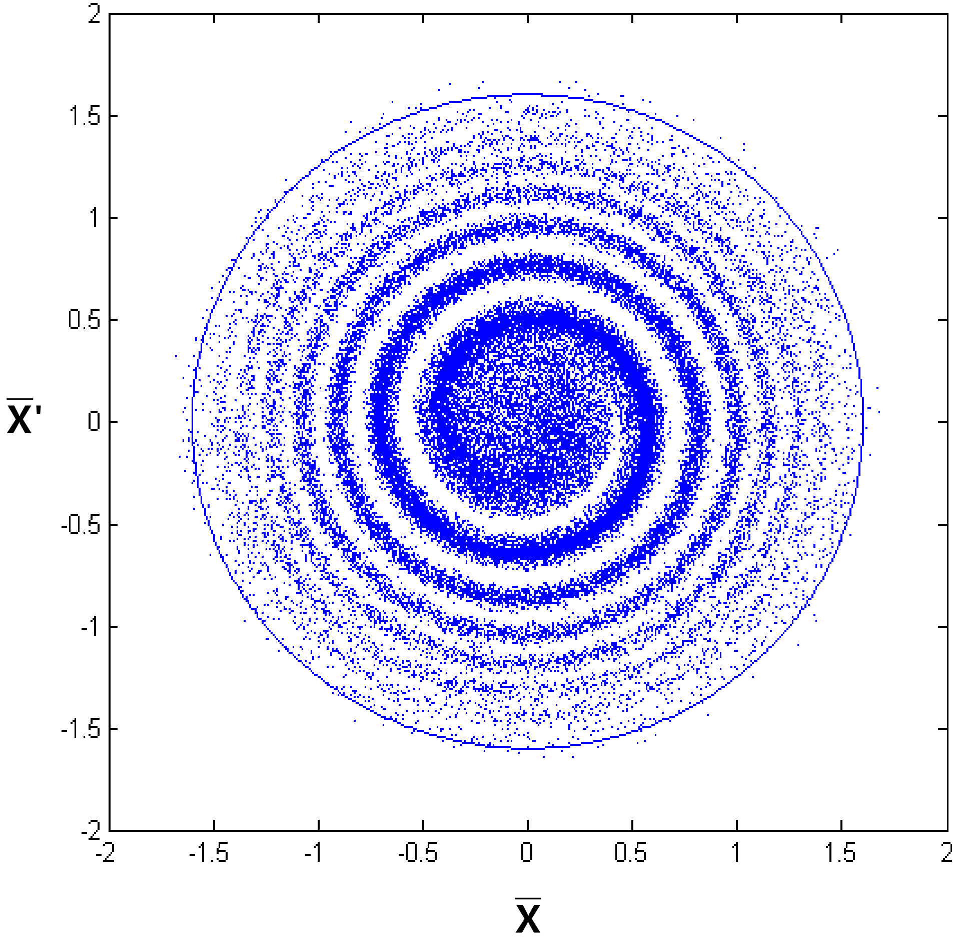
Filamentation



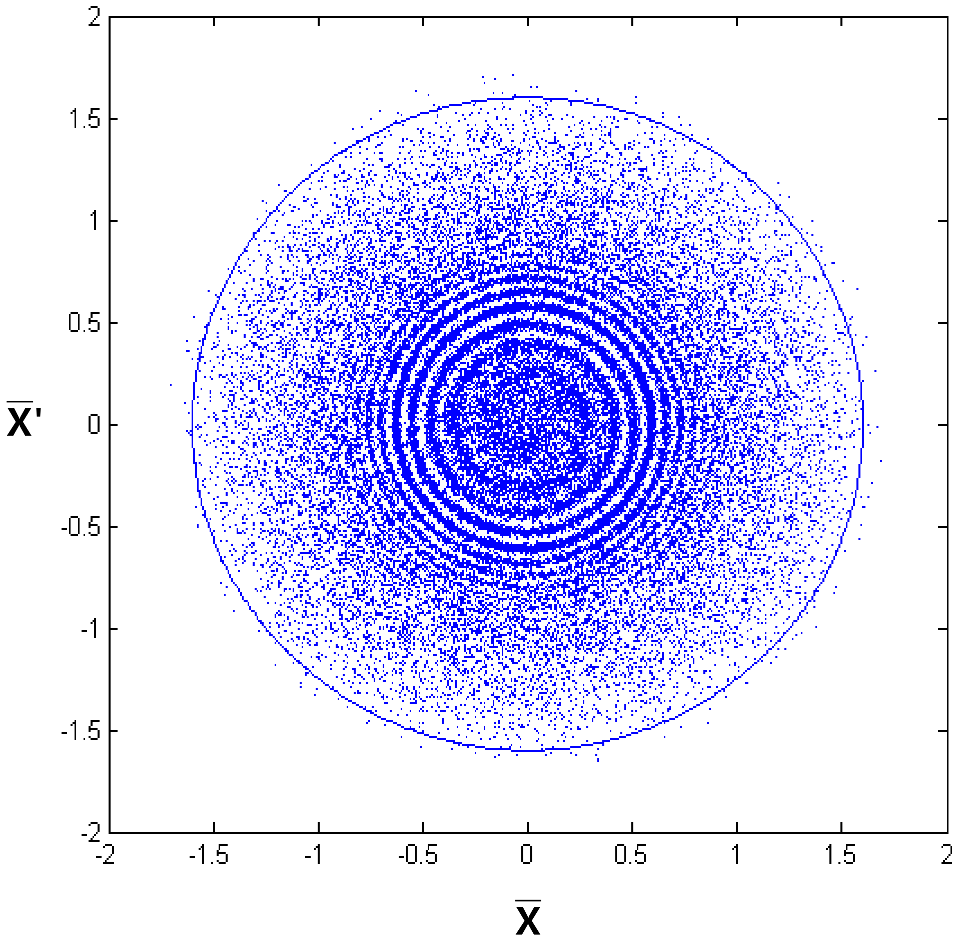
Filamentation



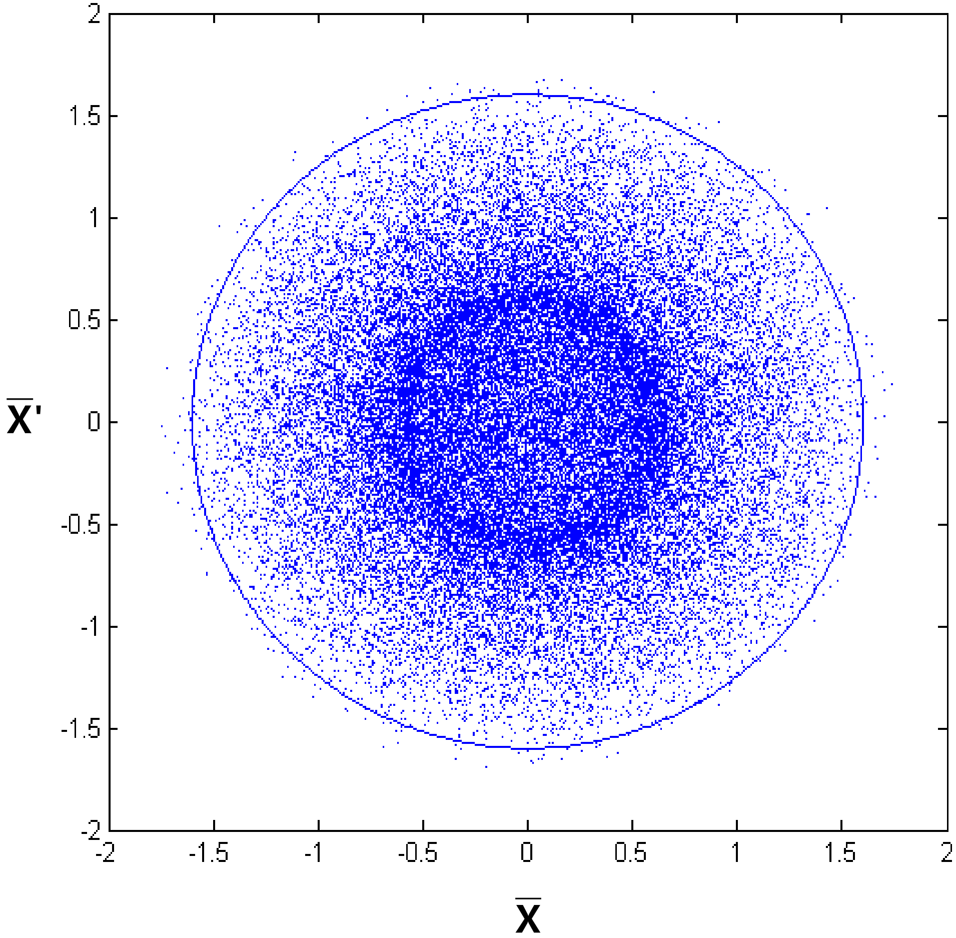
Filamentation



Filamentation



Filamentation

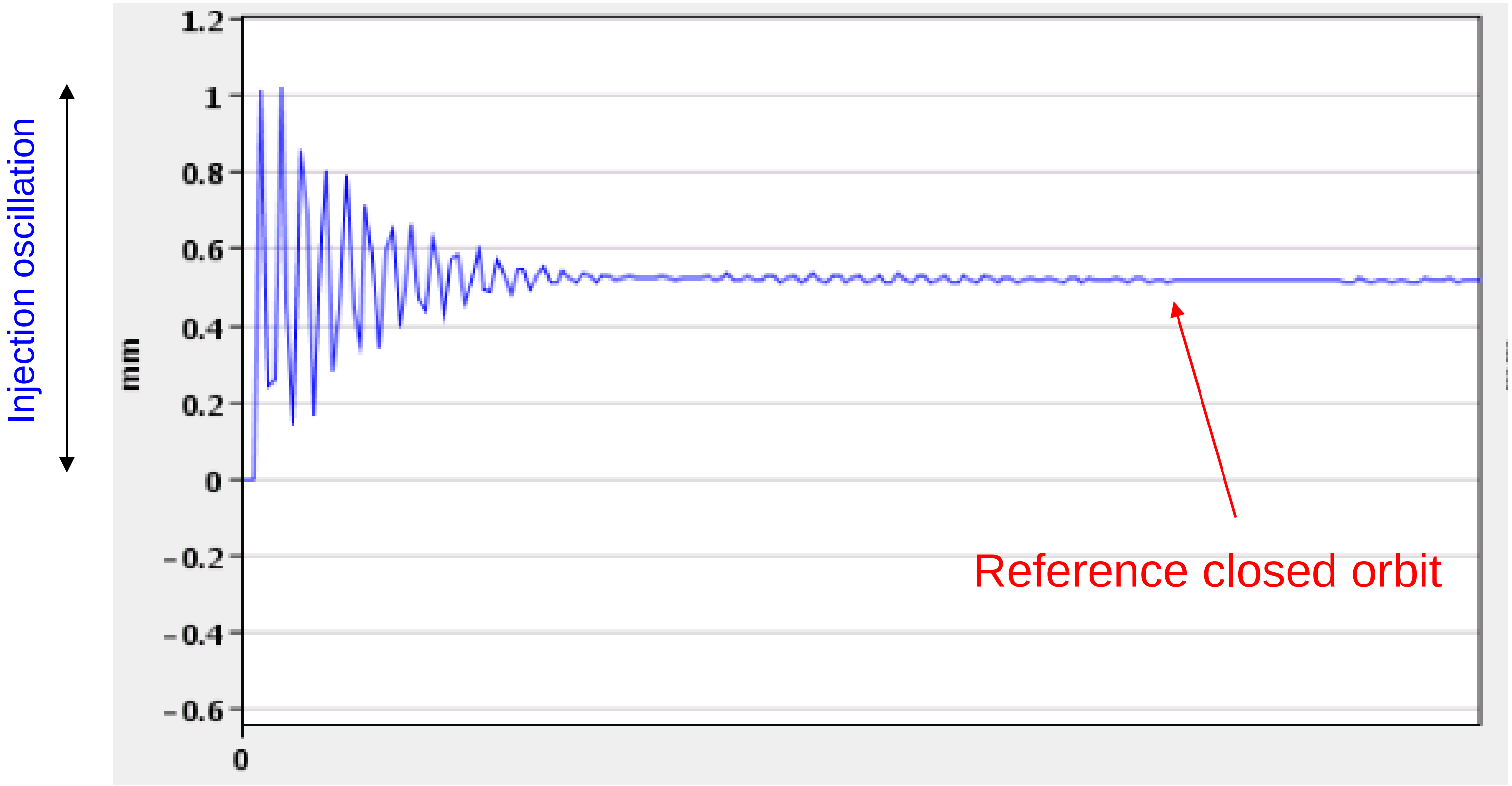


Filamentation

- Non-linear effects (e.g. higher-order field components) introduce amplitude-dependent effects into particle motion
- Over many turns, a phase-space oscillation is transformed into an emittance increase
- So any residual transverse oscillation will lead to an emittance blow-up through filamentation
  - "Transverse damper" systems are used to damp injection oscillations - bunch position measured by a pick-up, which is linked to a kicker
  - Chromaticity coupled with a non-zero momentum spread at injection can also cause filmentation, often termed *chromatic decoherence*
- See appendix for derivation of the emittance increase

Filamentation - Decoherence

- Residual transverse oscillations lead to an *effective* emittance blow-up through filamentation.
- Due to tune spread and energy spread, the oscillation will not be seen for long on a BPM signal:

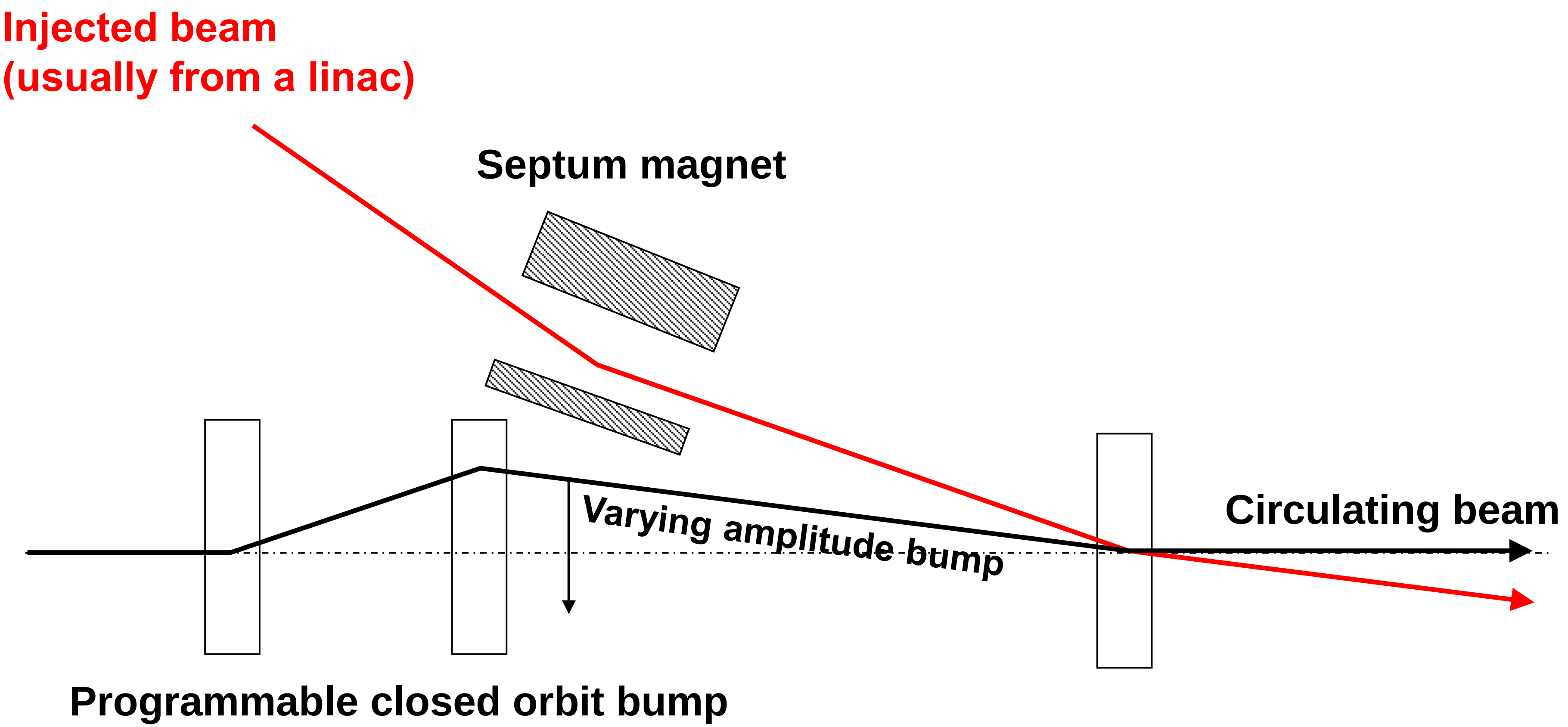


Multi-turn injection

- For hadrons the beam density at injection can be limited either by space charge effects or by the injector capacity

- If we cannot increase charge density, we can sometimes fill the horizontal phase space to increase overall injected intensity.
  - Cannot inject into same phase space area, as we would kick out the beam located there
  - If the acceptance of the receiving machine is larger than the delivered beam emittance we can accumulate intensity

Multi-turn injection for hadrons

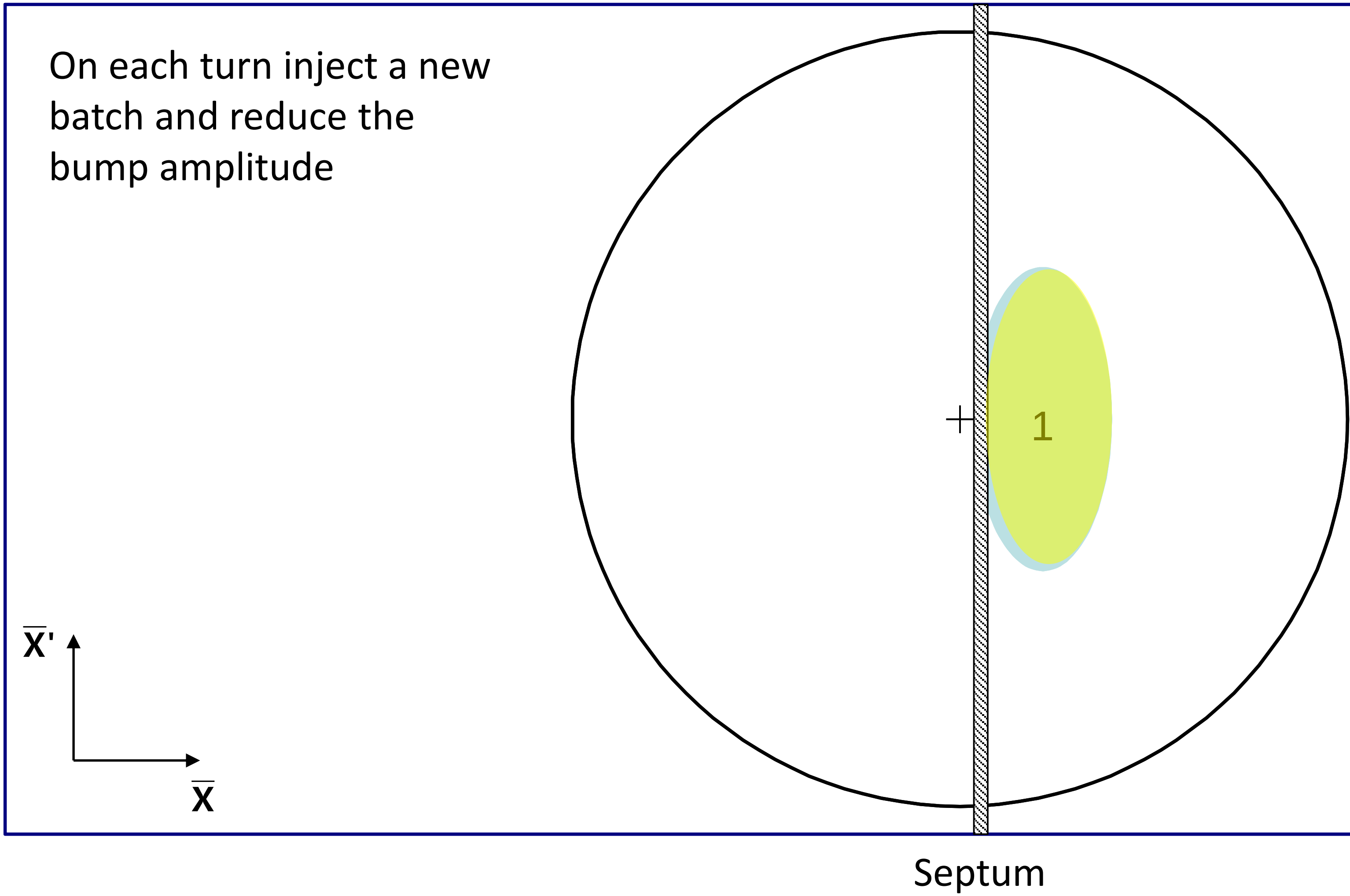


- No kicker but fast programmable bumpers
- Bump amplitude decreases and a new batch injected turn-by-turn
- Phase-space “painting”

Multi-turn injection for hadrons

Example: CERN PSB injection from Linac 2, fractional tune $Q_h \approx 0.25$
Beam rotates π/2 per turn in phase space

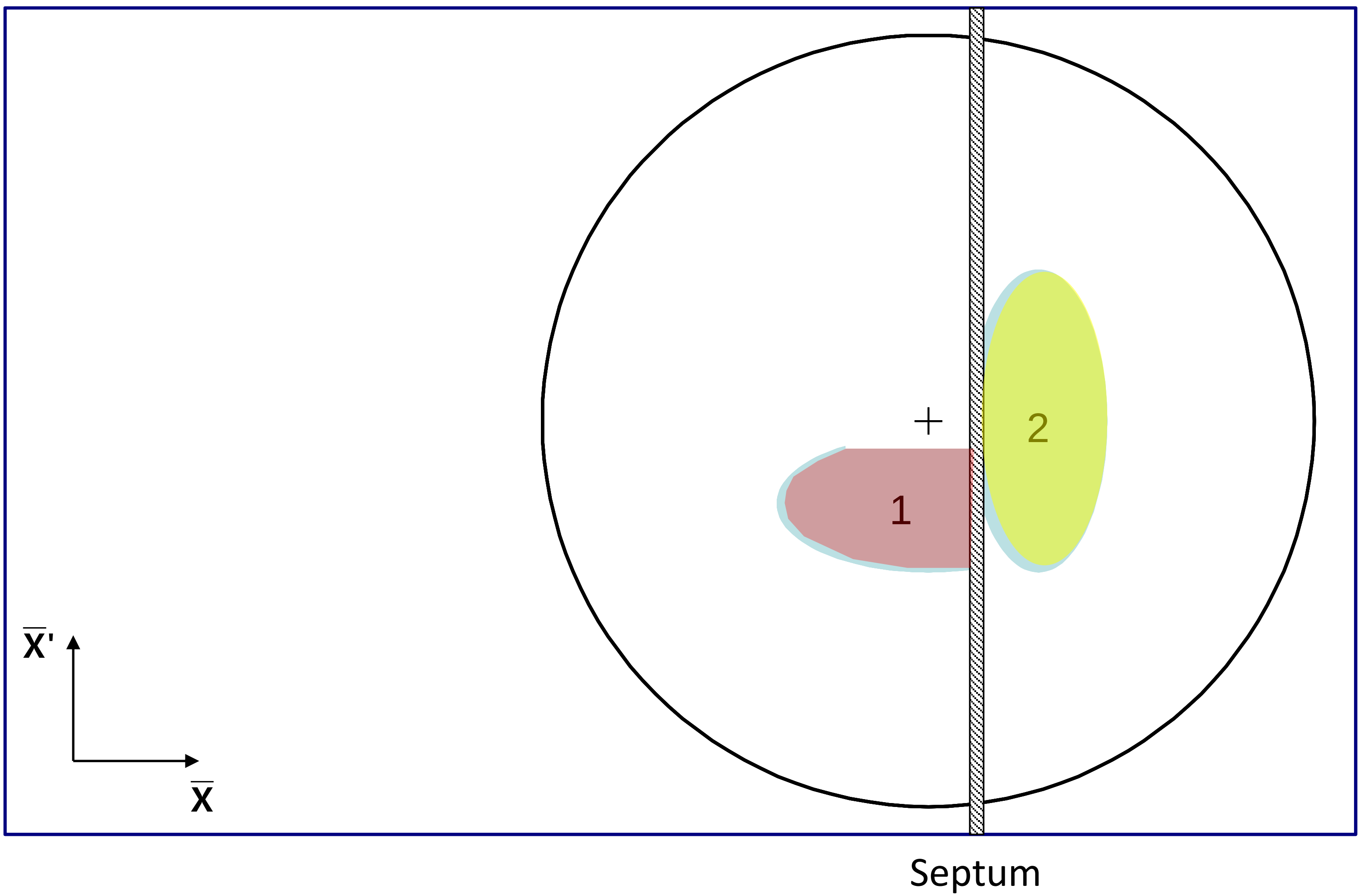
Turn 1



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates π/2 per turn in phase space

Turn 2



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
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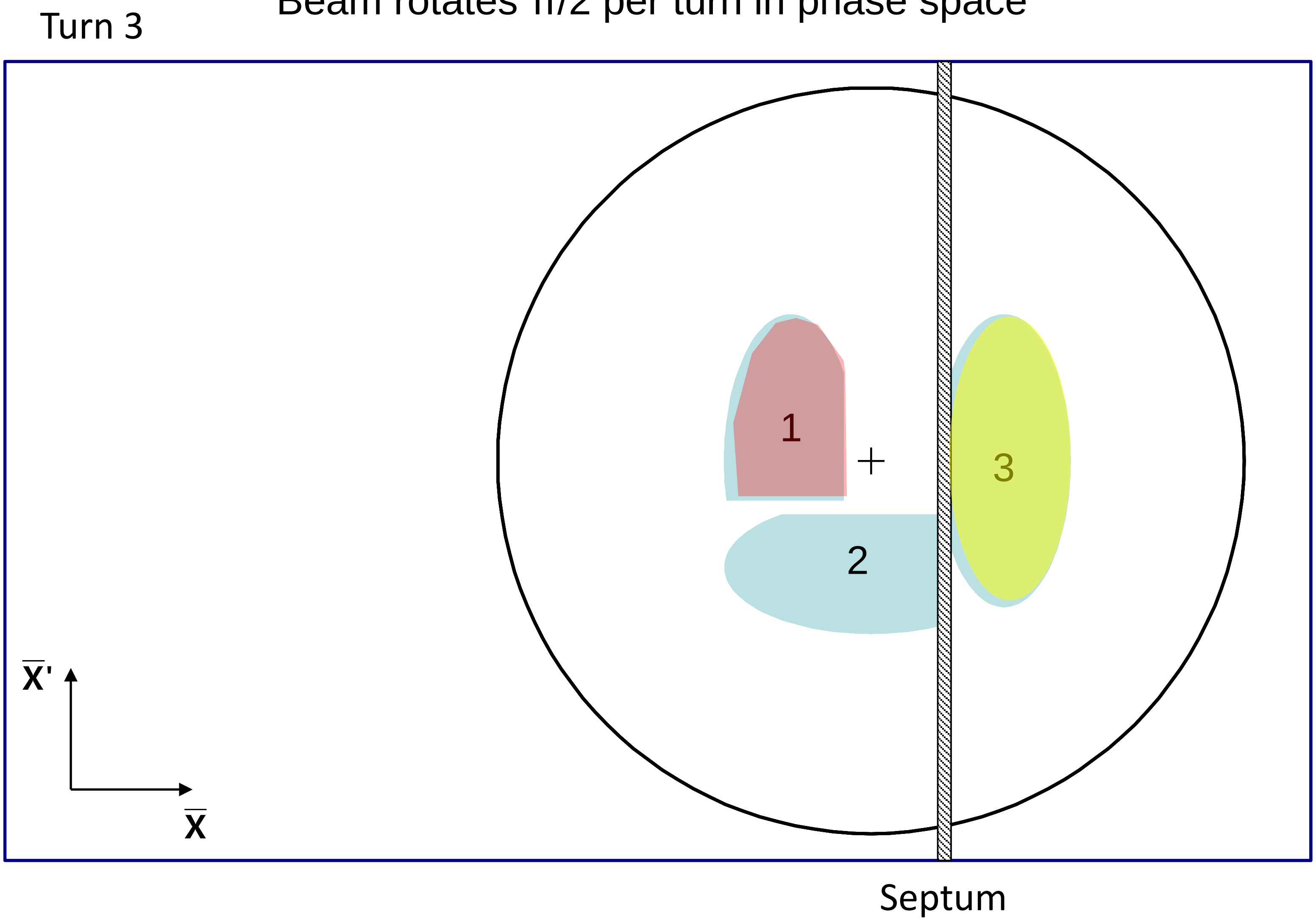
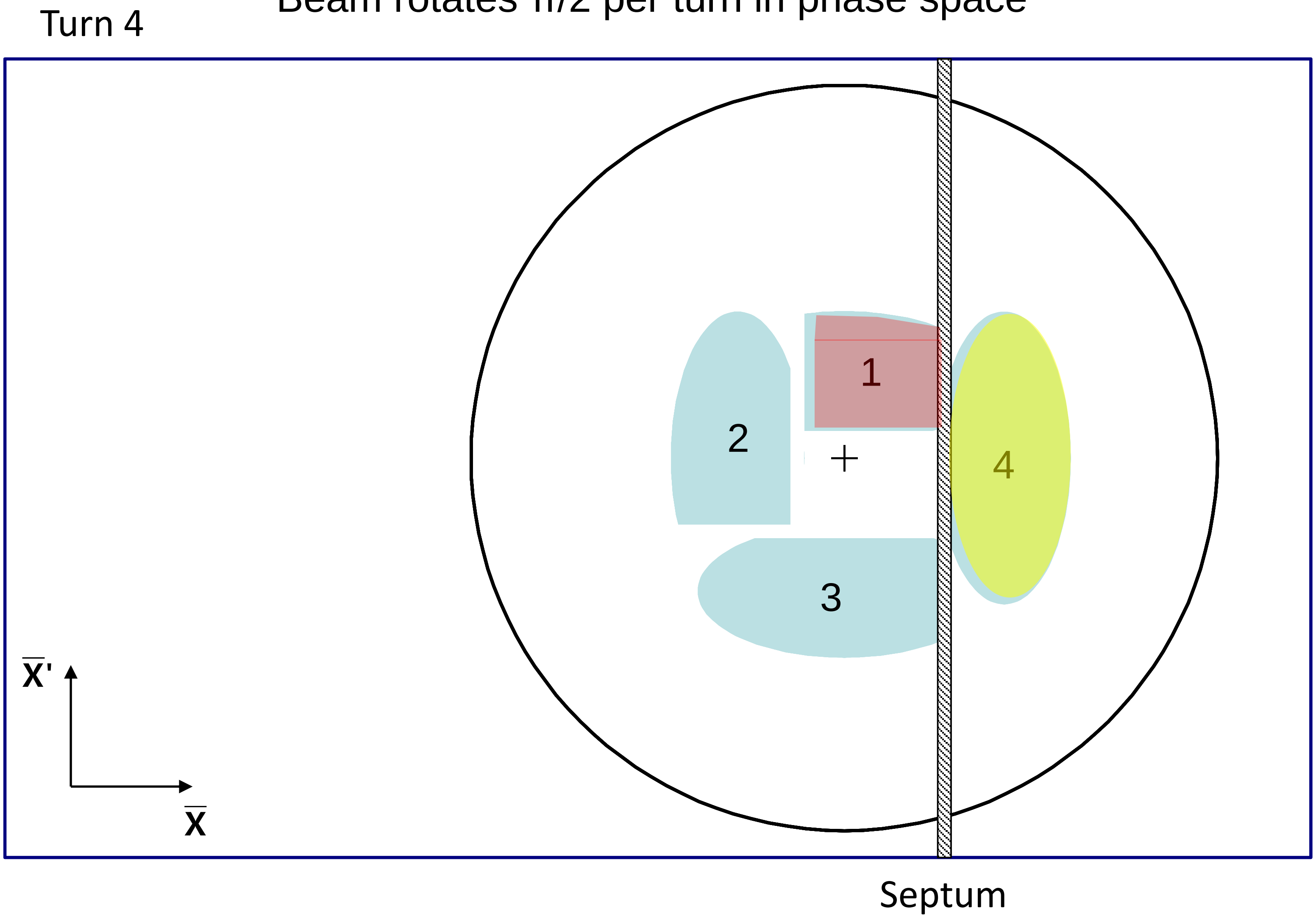


Multi-turn injection for hadrons

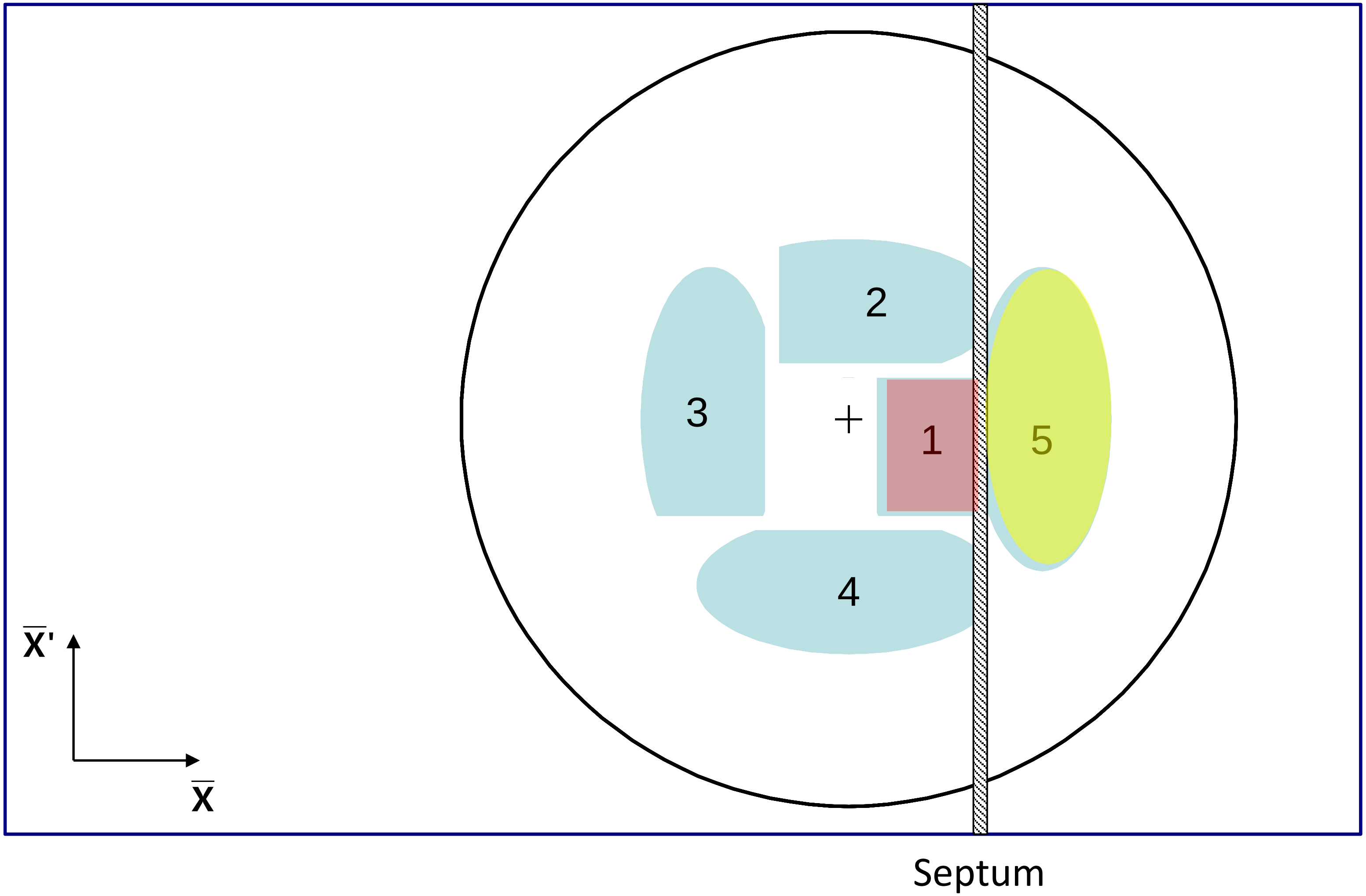
Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space

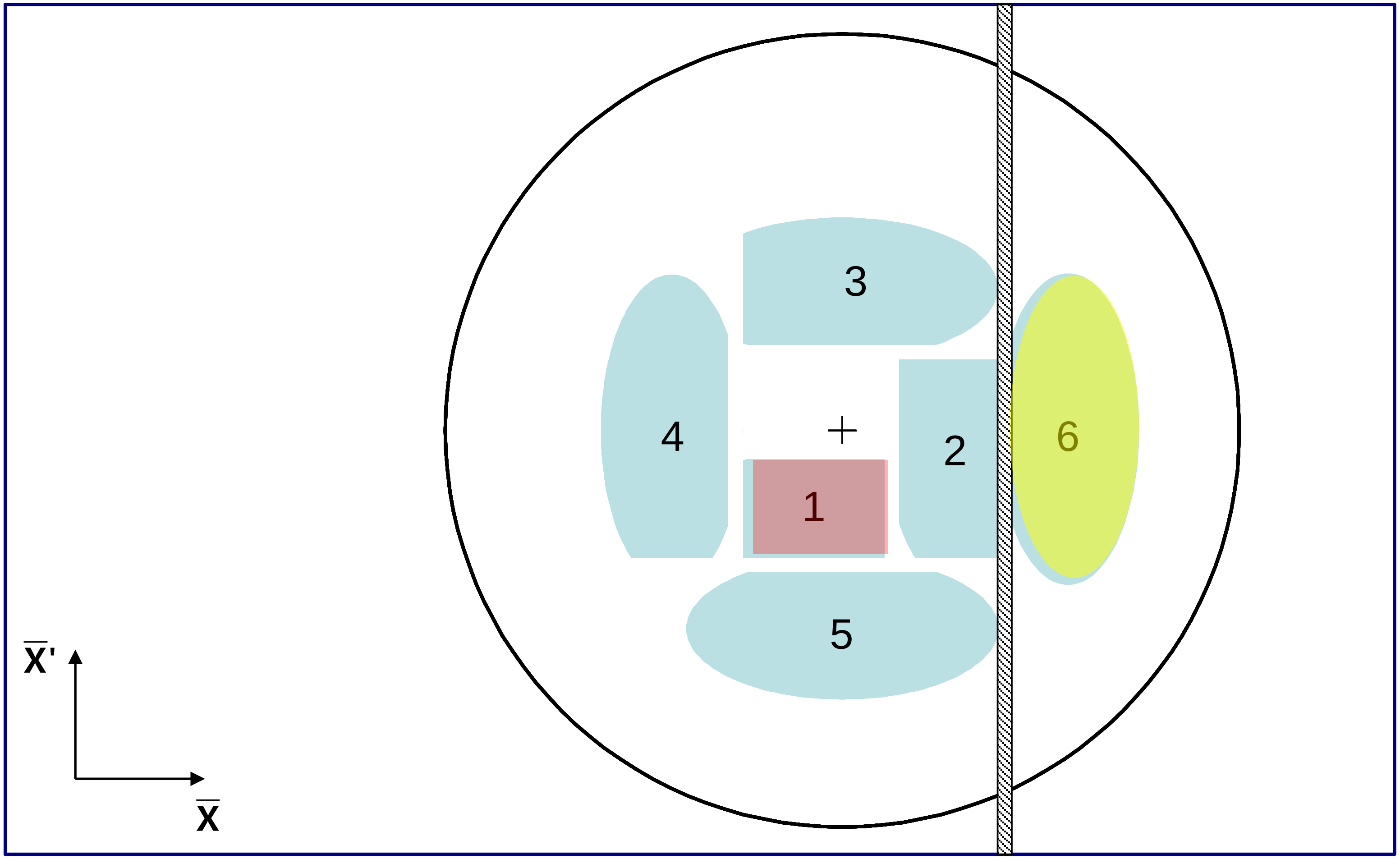
Turn 5



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates π/2 per turn in phase space

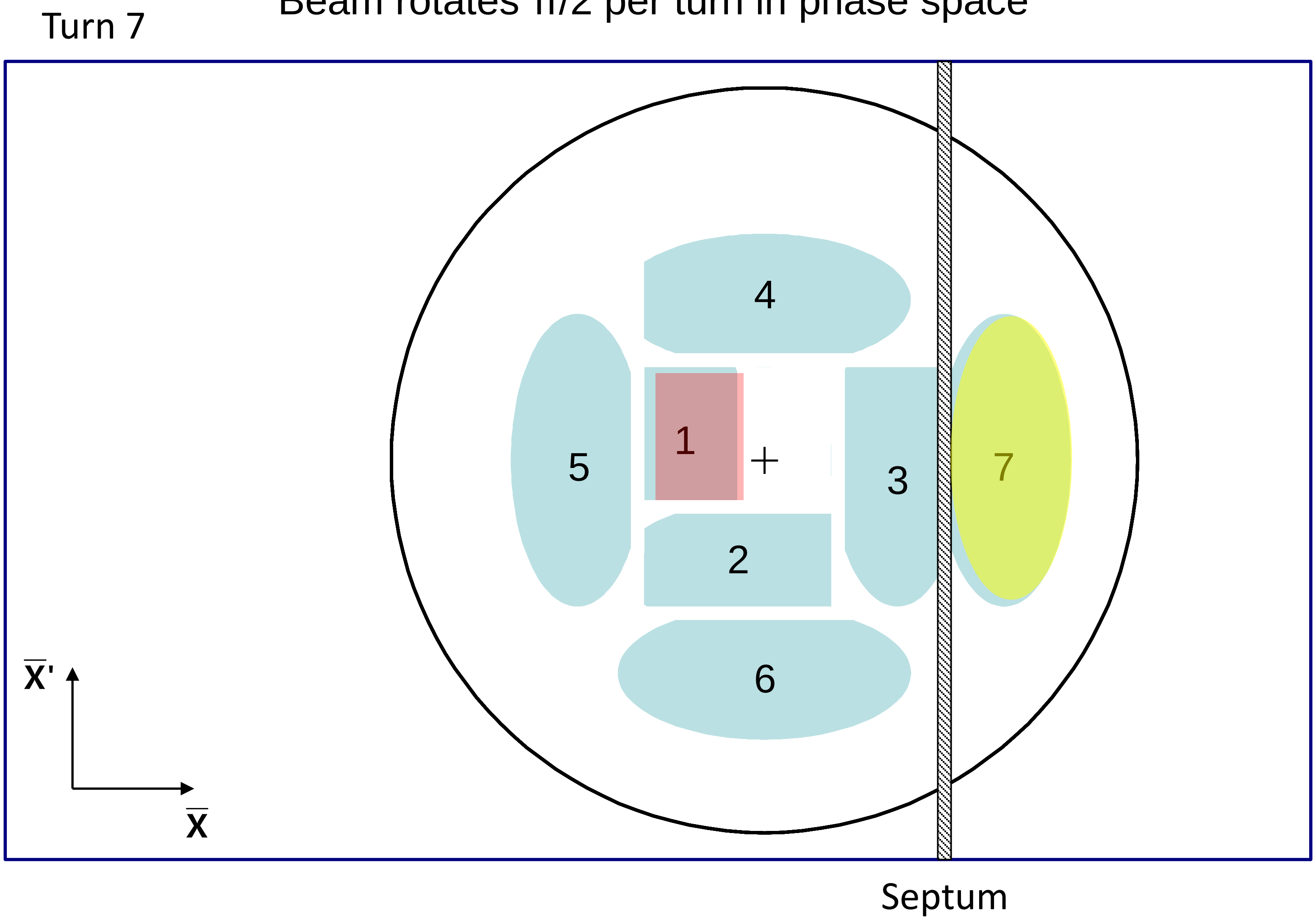
Turn 6



Septum

Multi-turn injection for hadrons

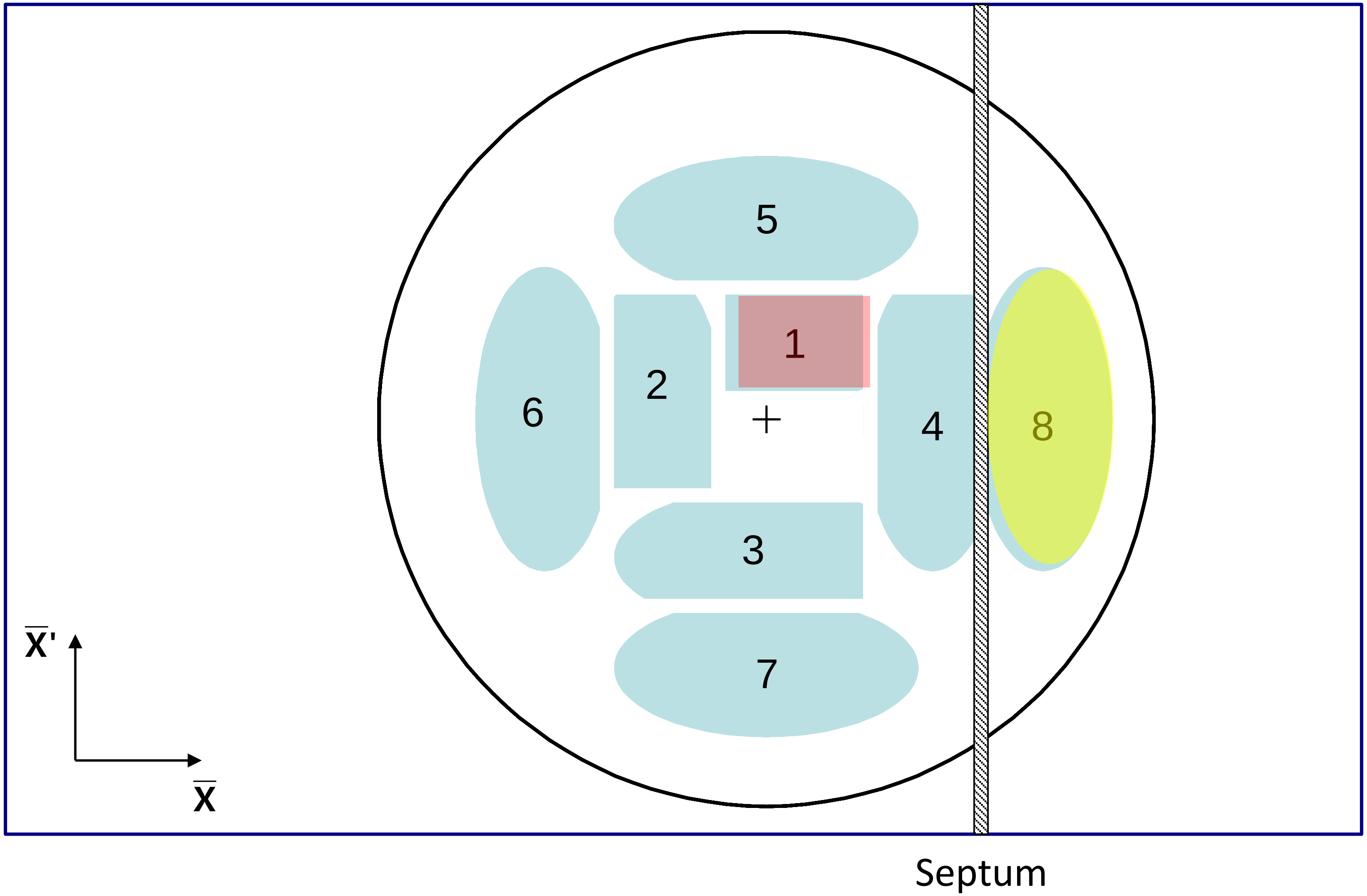
Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space



Multi-turn injection for hadrons

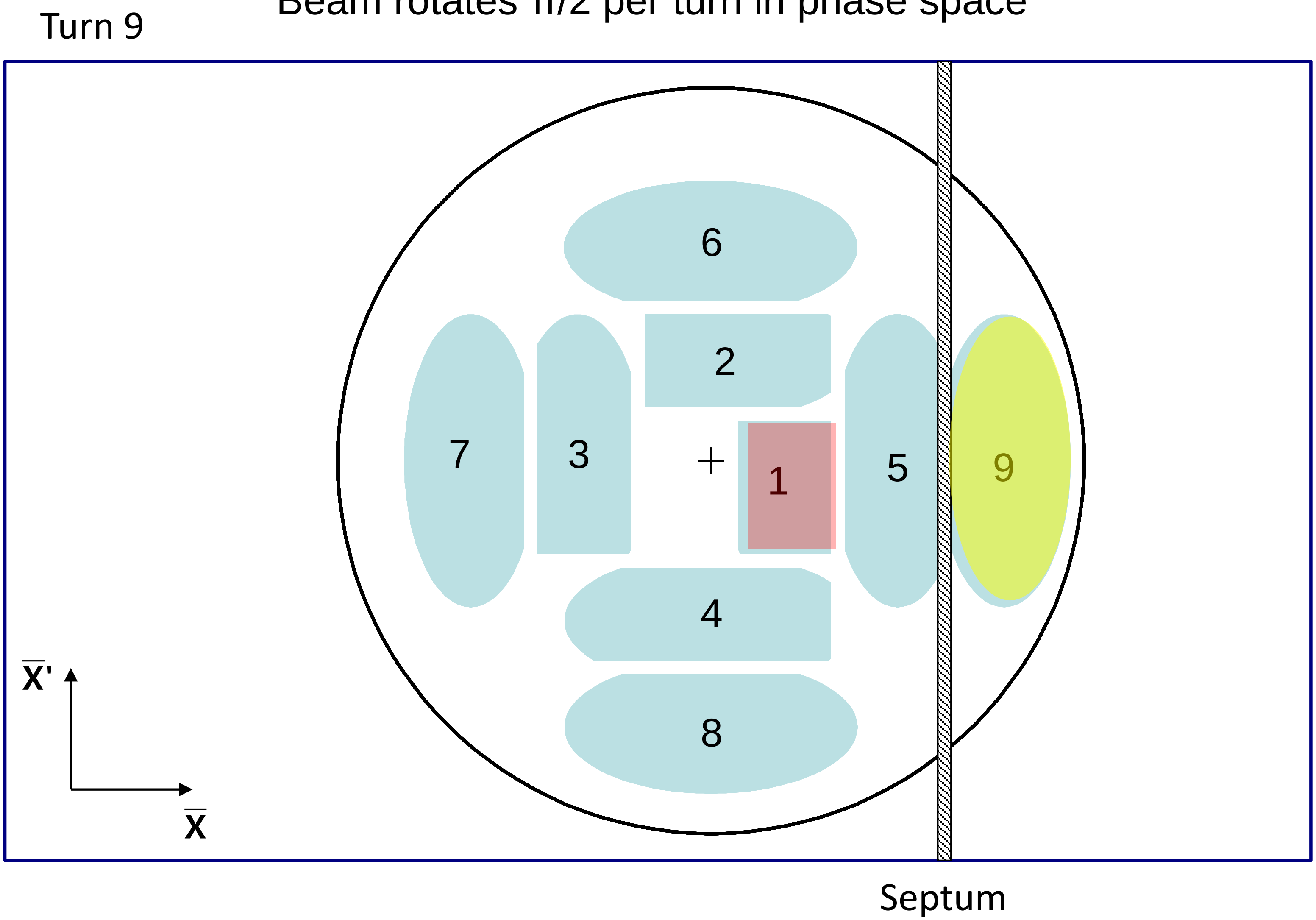
Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space

Turn 8



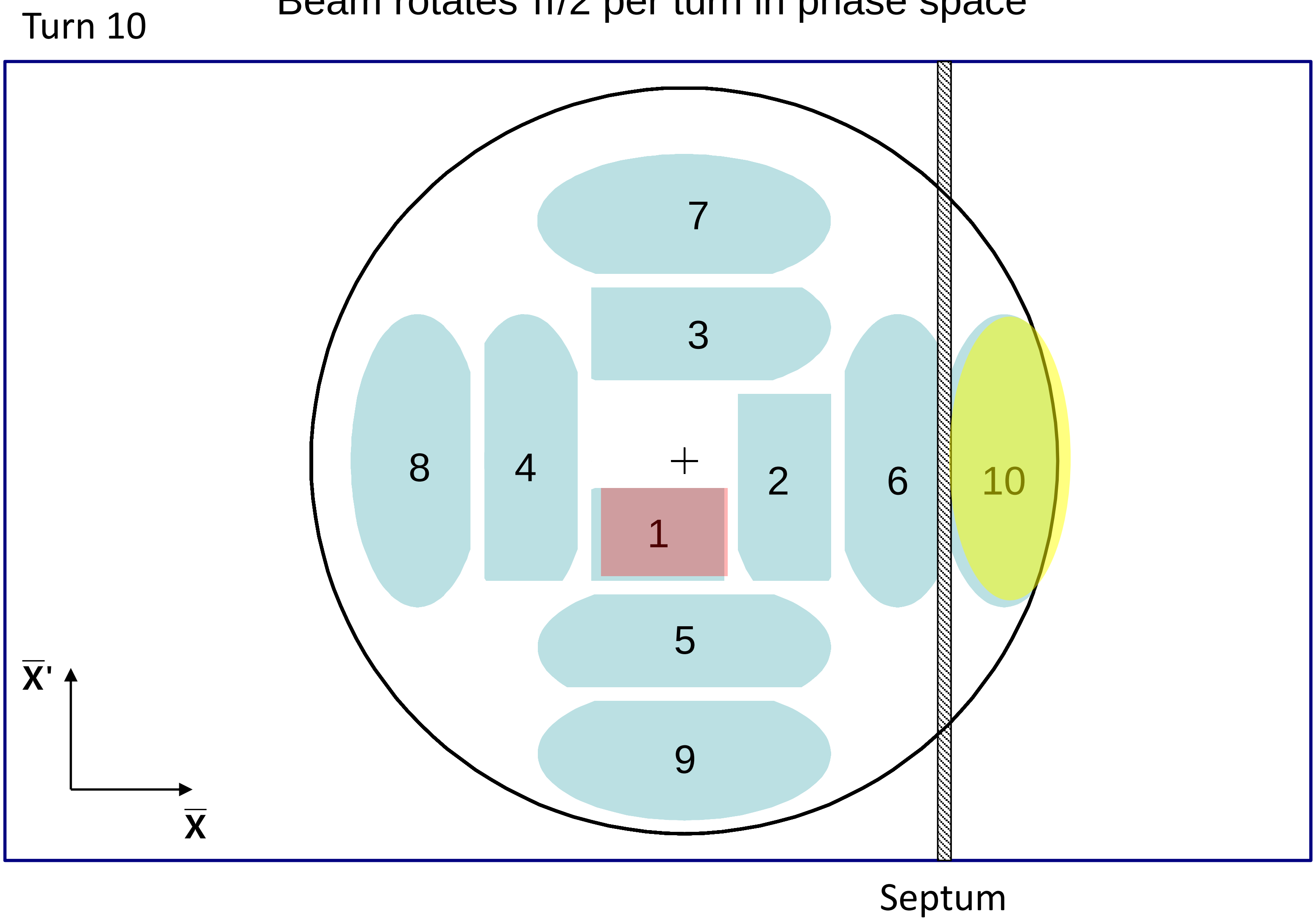
Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space



Multi-turn injection for hadrons

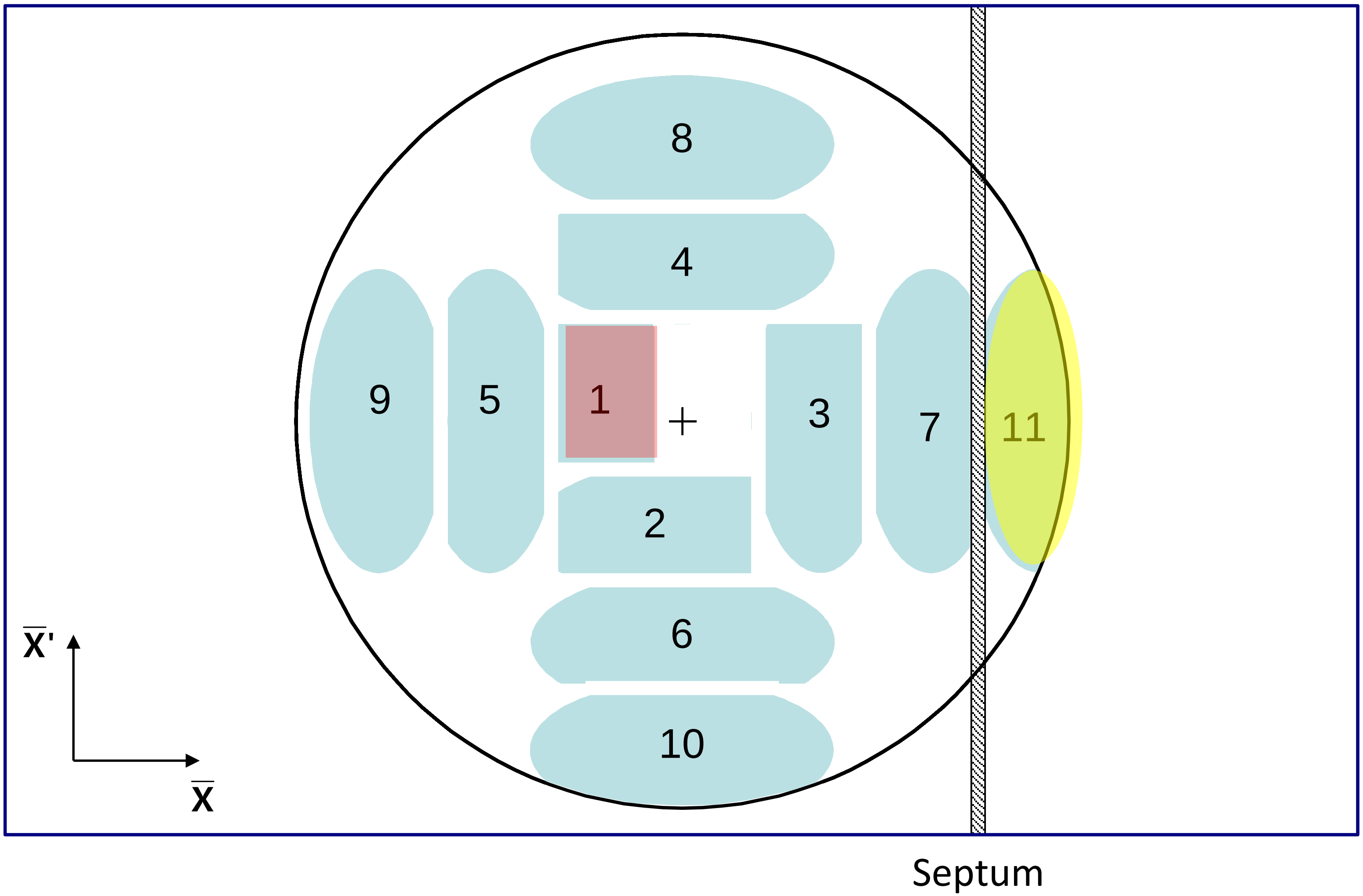
Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates π/2 per turn in phase space



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space

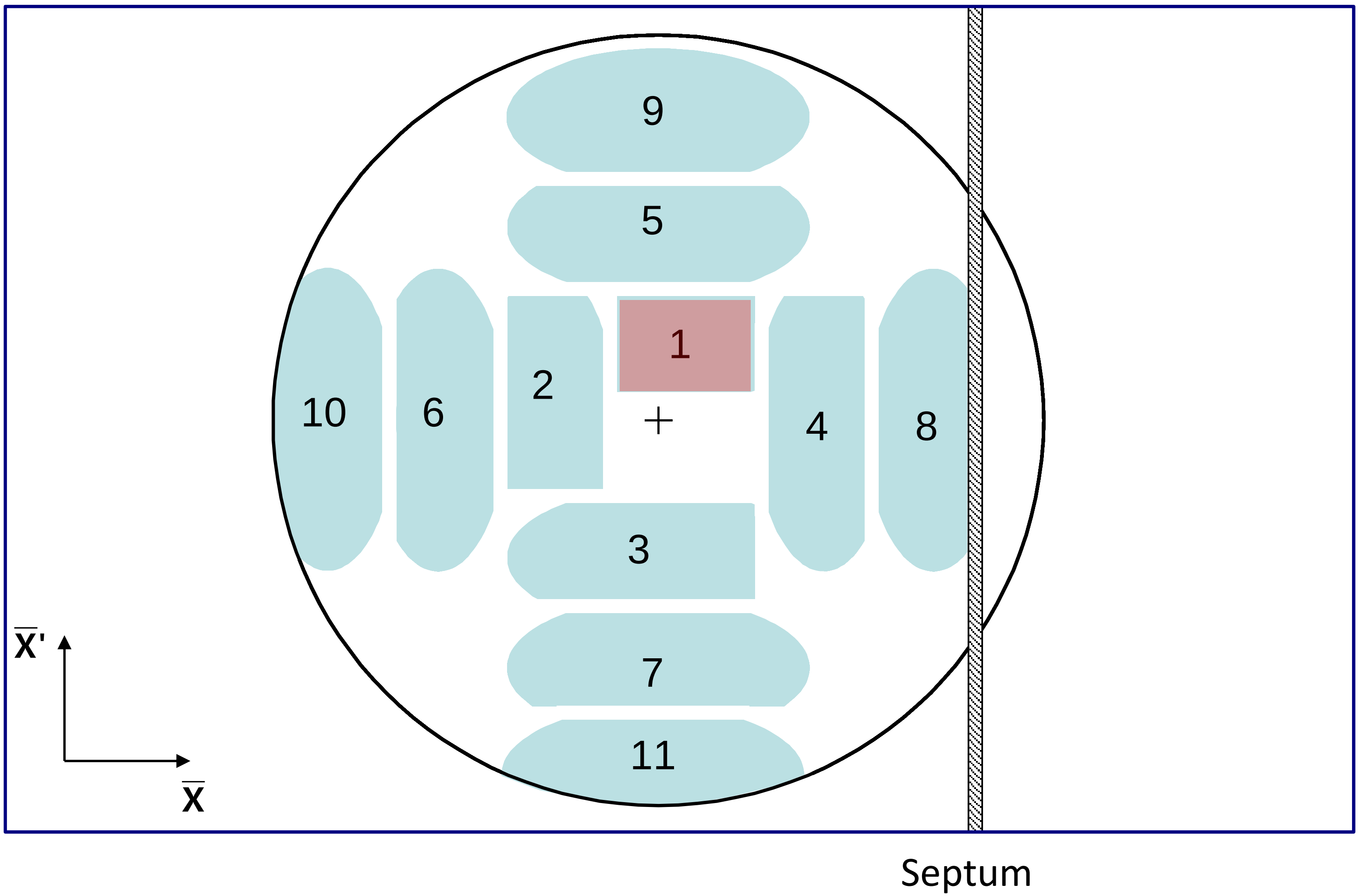
Turn 11



Multi-turn injection for hadrons

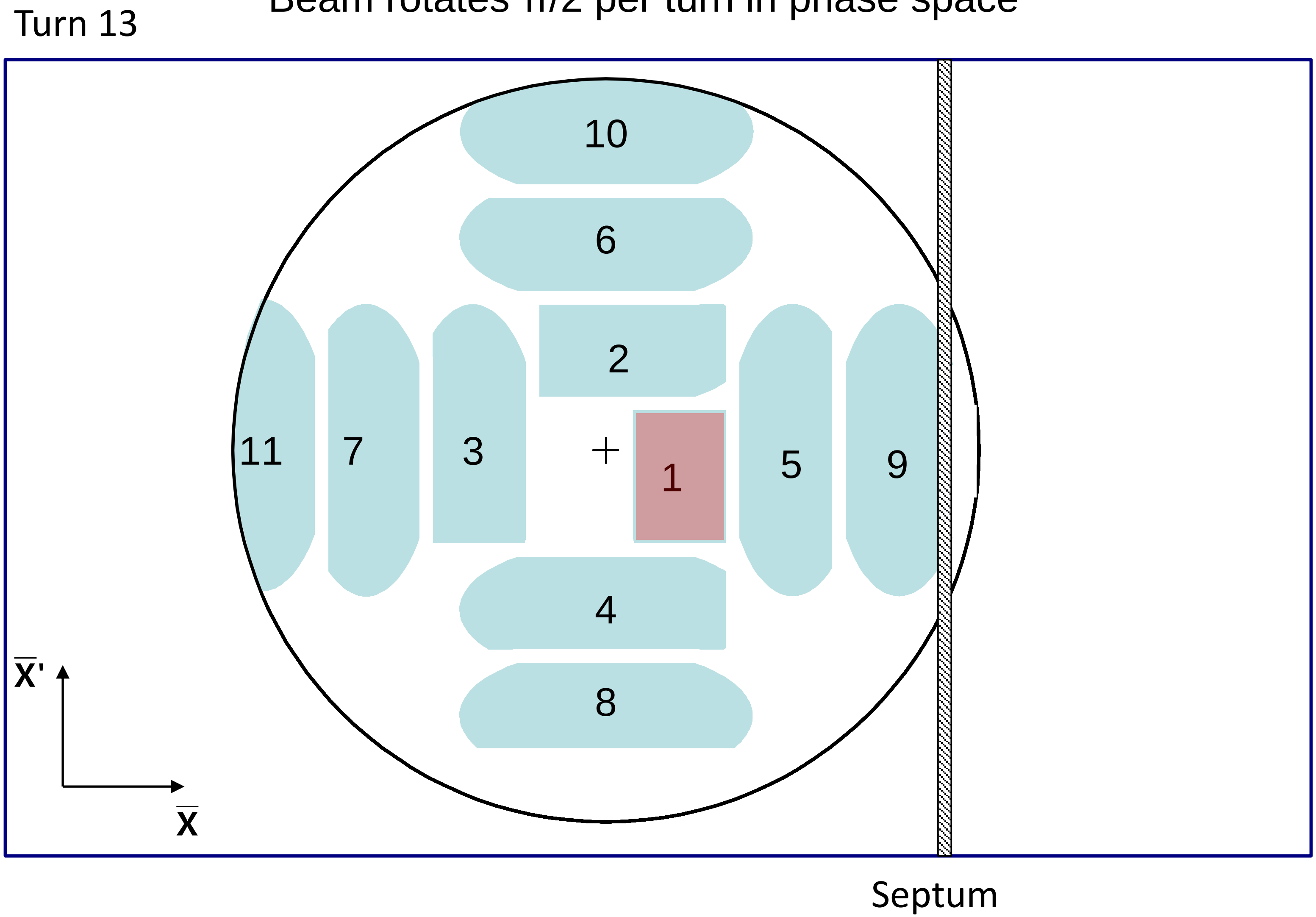
Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space

Turn 12



Multi-turn injection for hadrons

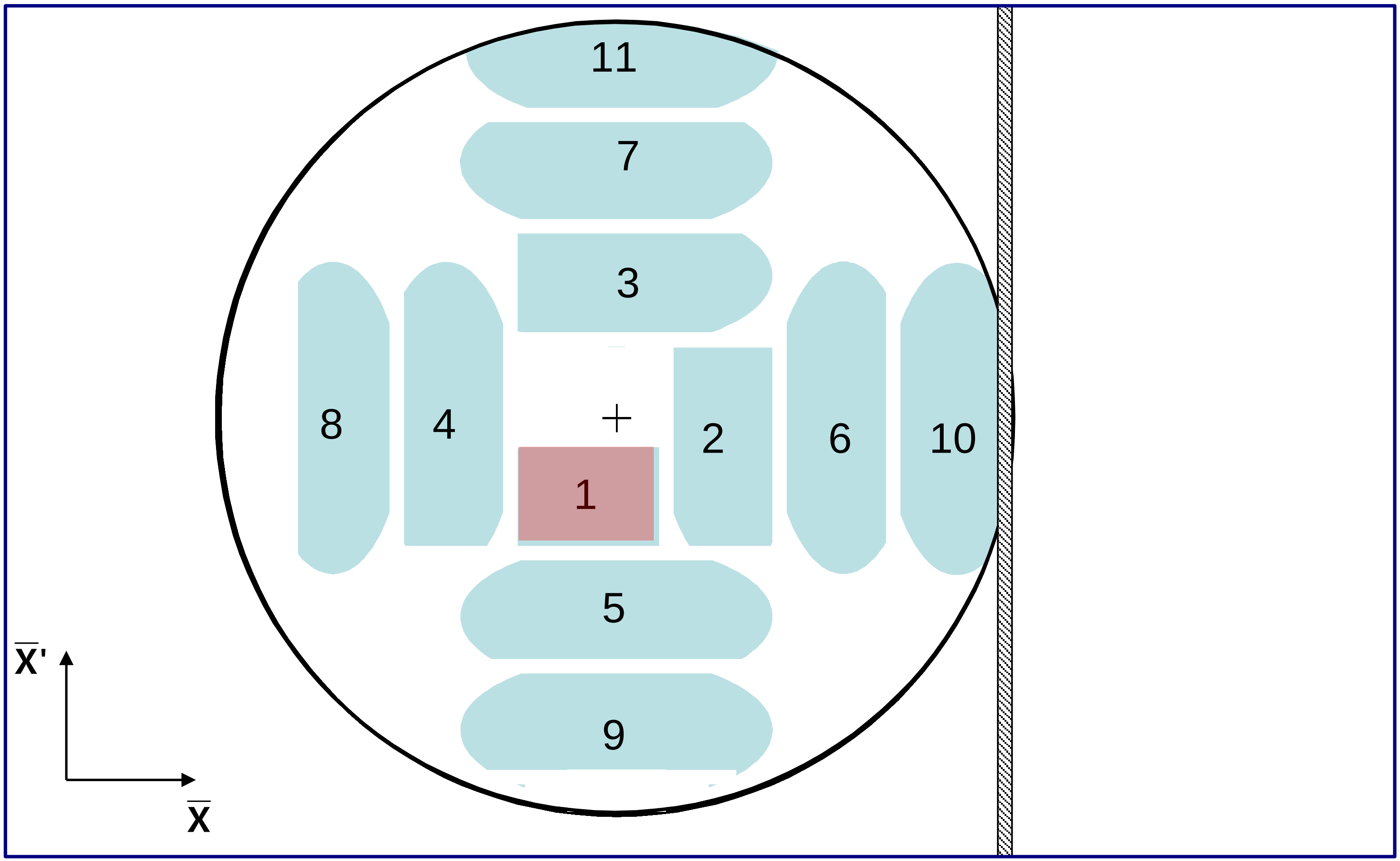
Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates $\pi/2$ per turn in phase space



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams, fractional tune $Q_h \approx 0.25$
Beam rotates π/2 per turn in phase space

Turn 14

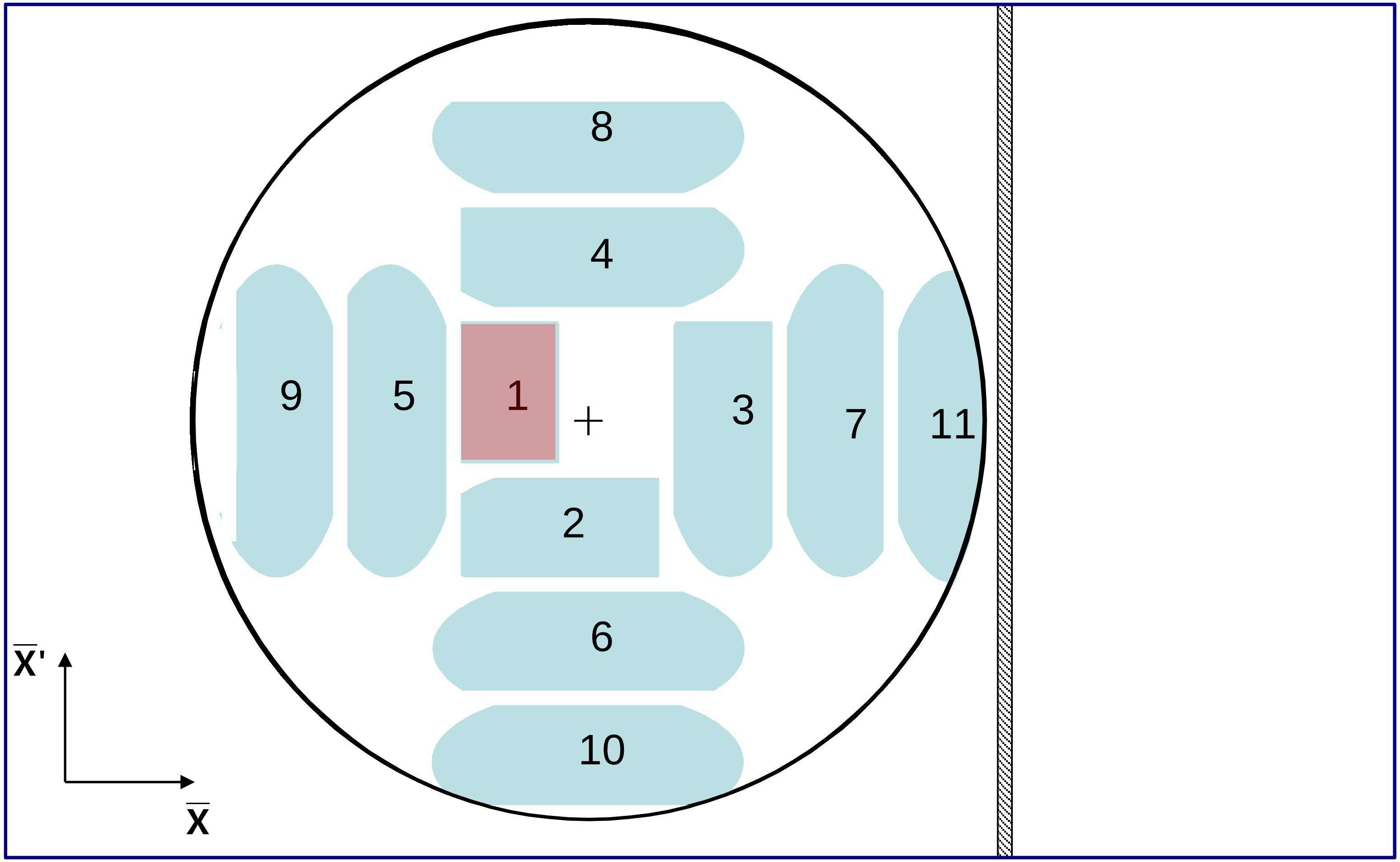


Septum

Multi-turn injection for hadrons

Phase space has been "**painted**"

Turn 15



In reality, filamentation (often space-charge driven) occurs to produce a quasi-uniform beam

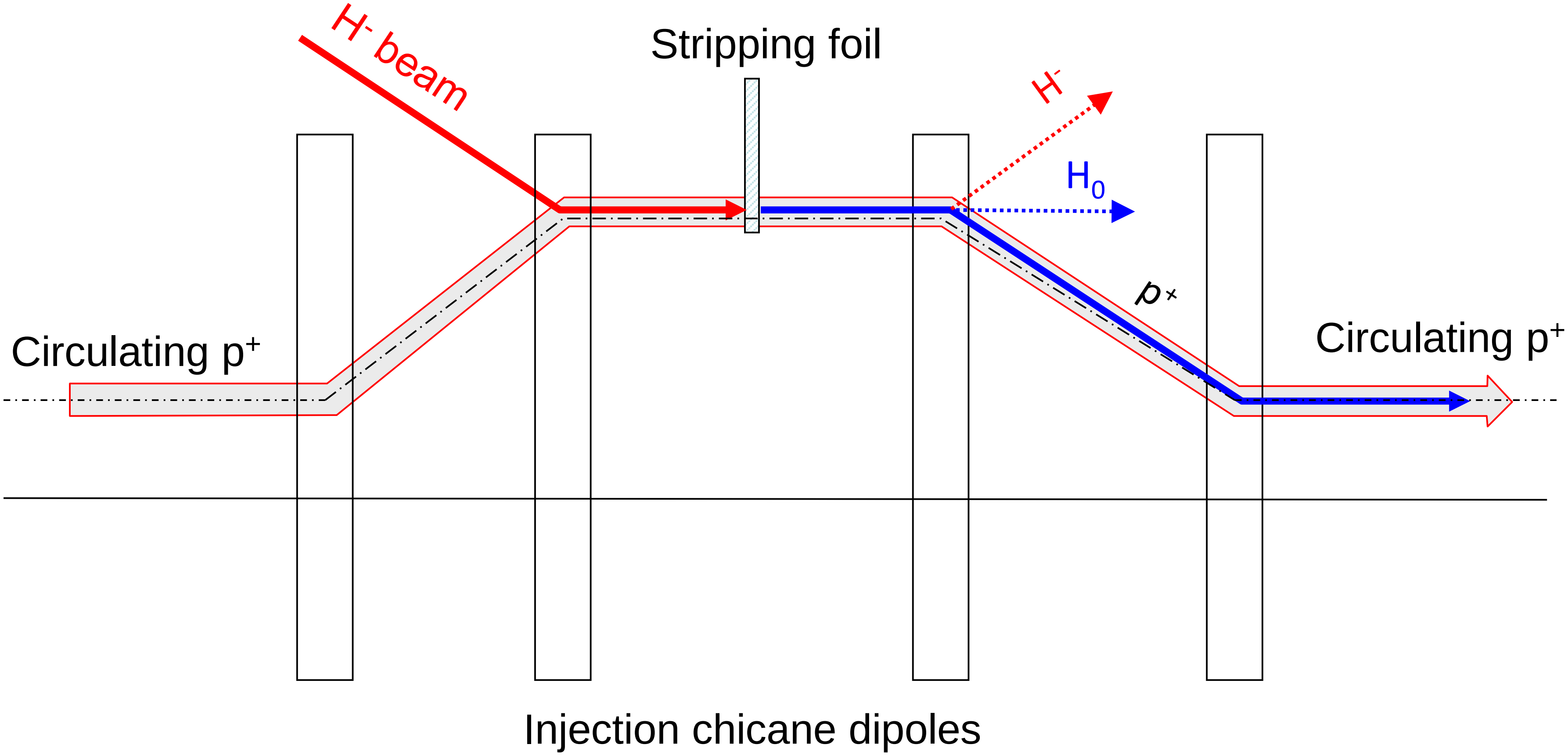
Charge exchange H- injection

- Multi-turn injection is essential to accumulate high intensity
- Disadvantages inherent in using an injection septum:
  - Width of several mm reduces aperture
  - Beam losses from circulating beam hitting septum:
    - typically 30 – 40 % for the CERN PSB injection at 50 MeV
  - Limits number of injected turns to 10 – 20

- **Charge-exchange injection** provides elegant alternative
  - Possible to “cheat” Liouville’s theorem, which says that emittance is conserved….
  - Convert H^- to p^+ using a thin stripping foil, allowing injection into the same phase space area

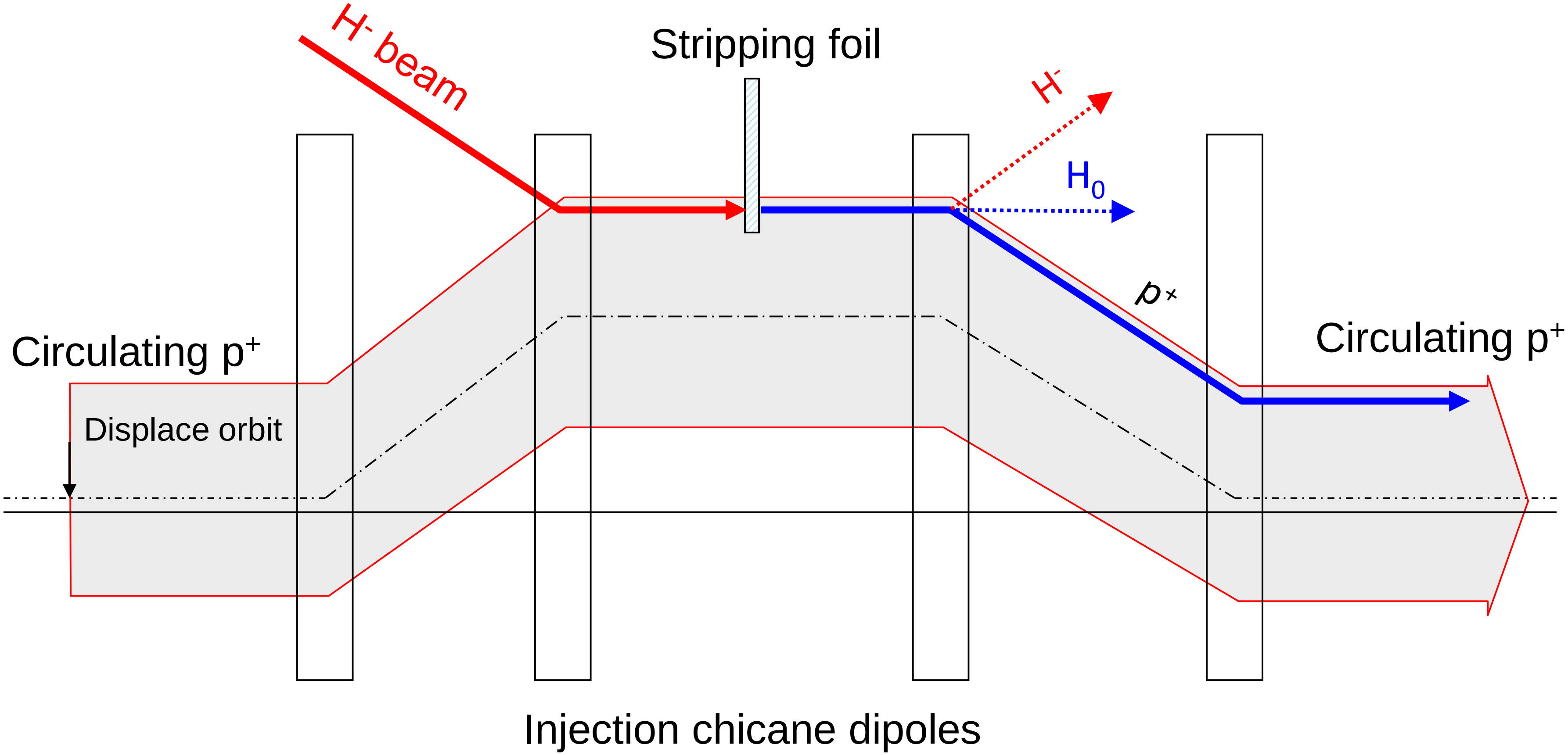
Charge exchange H- injection

Start of injection process



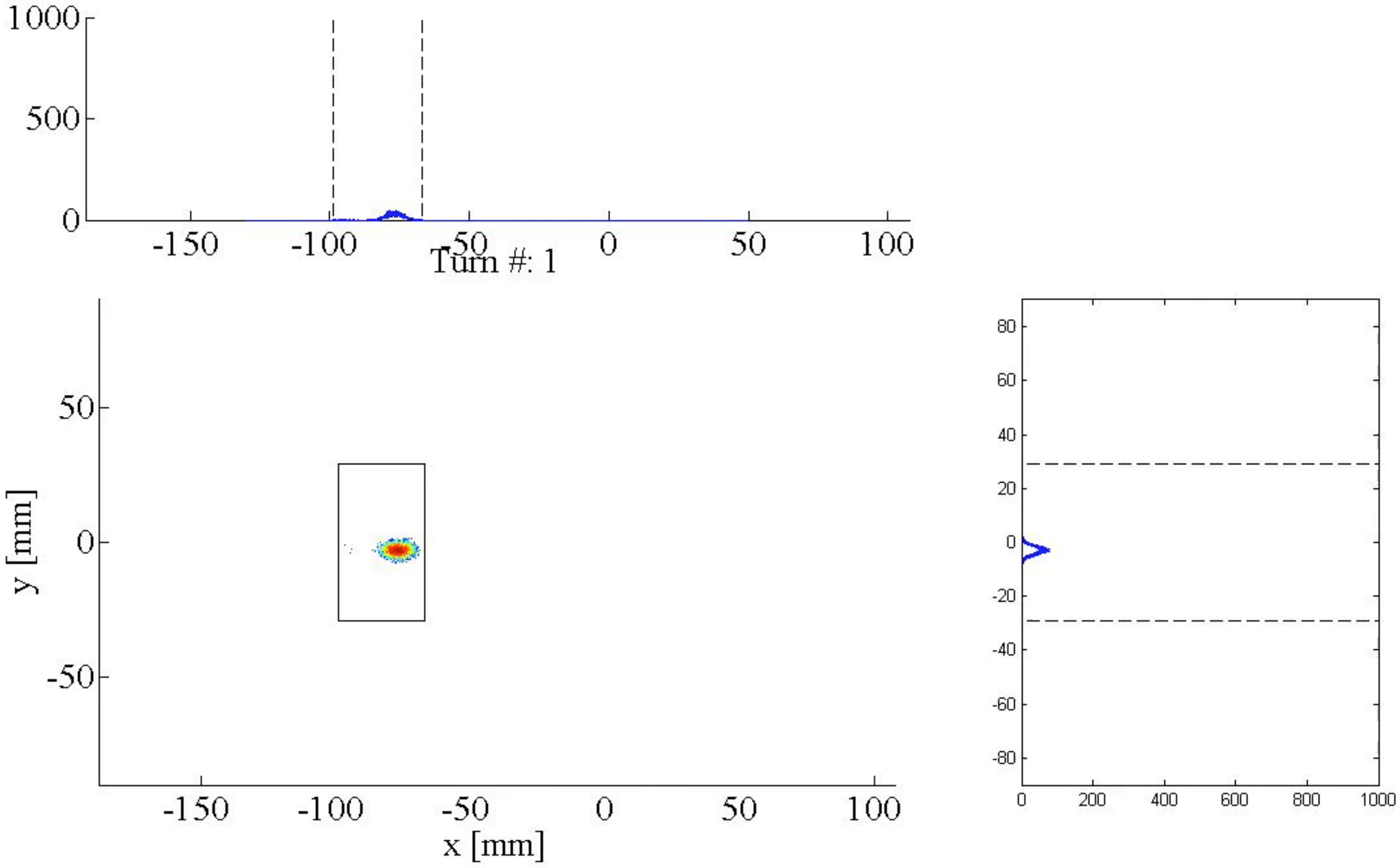
Charge exchange H- injection

End of injection process with painting



Accumulation process on foil

- Linac4 connection to the PS booster at 160 MeV:
  - H^- stripped to p^+ with an estimated efficiency ≈98 % with C foil 200 $\mu g.cm^{-2}$

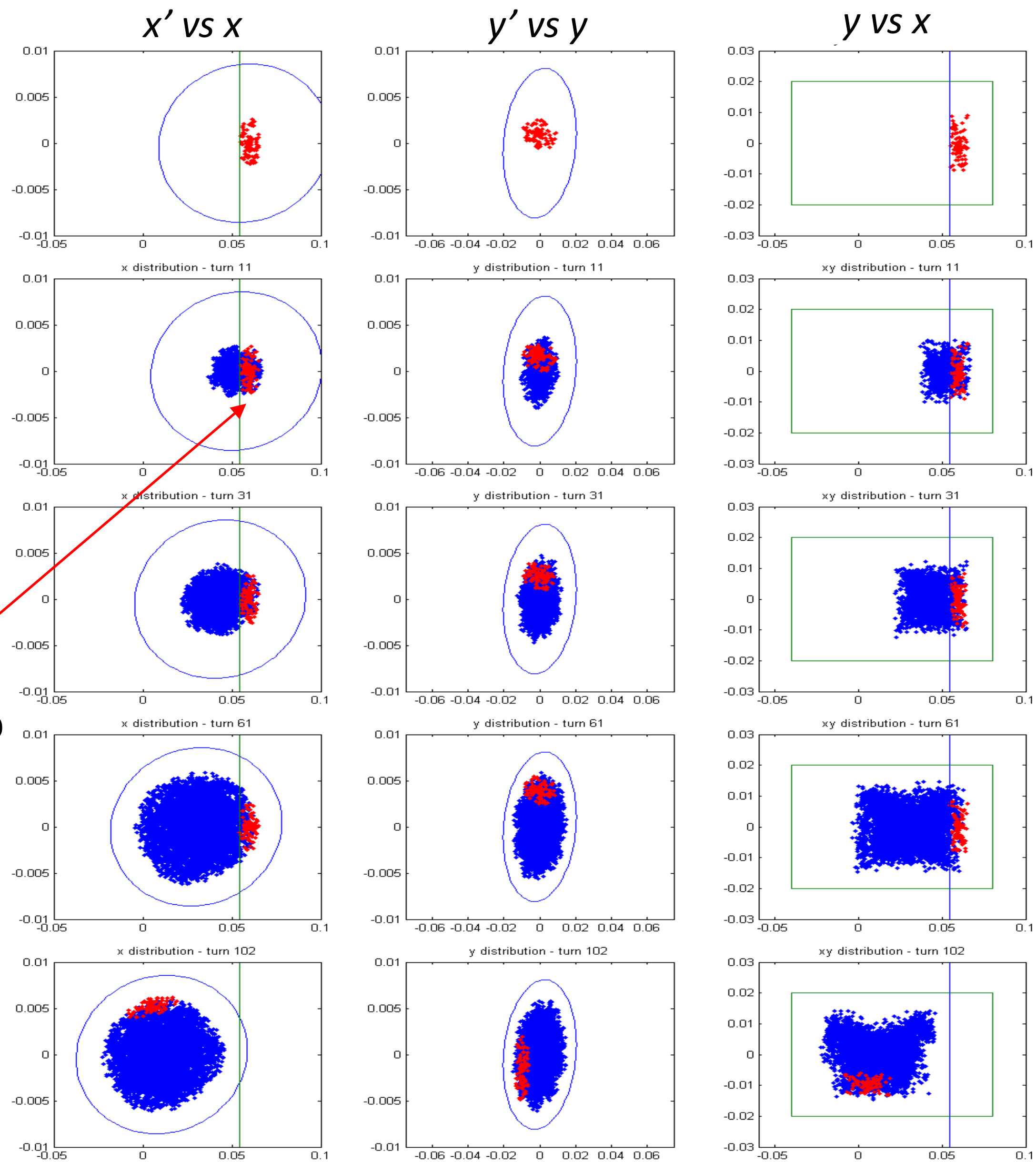


V. Forte, Performance of the CERN PSB at 160 MeV with H- charge exchange injection, PhD thesis – CERN and Université Blaise Pascal

Charge exchange H- injection

- Paint uniform transverse phase space density by modifying closed orbit bump and steering injected beam
- Foil thickness calculated to double-strip most ions (≈99%)
  - 50 MeV – 50 μg.cm^{-2}
  - 800 MeV – 200 μg.cm^{-2} (≈ 1 μm of C!)
- Carbon foils generally used – very fragile
- Injection chicane reduced or switched off after injection, to avoid excessive foil heating and beam blow-up
- Longitudinal phase space can also be painted turn-by-turn:
  - Variation of the injected beam energy turn-by-turn (linac voltage scaled)
  - Chopper system in linac to match length of injected batch to bucket

H- injection - painting



Note injection into same phase space area as circulating beam

Time

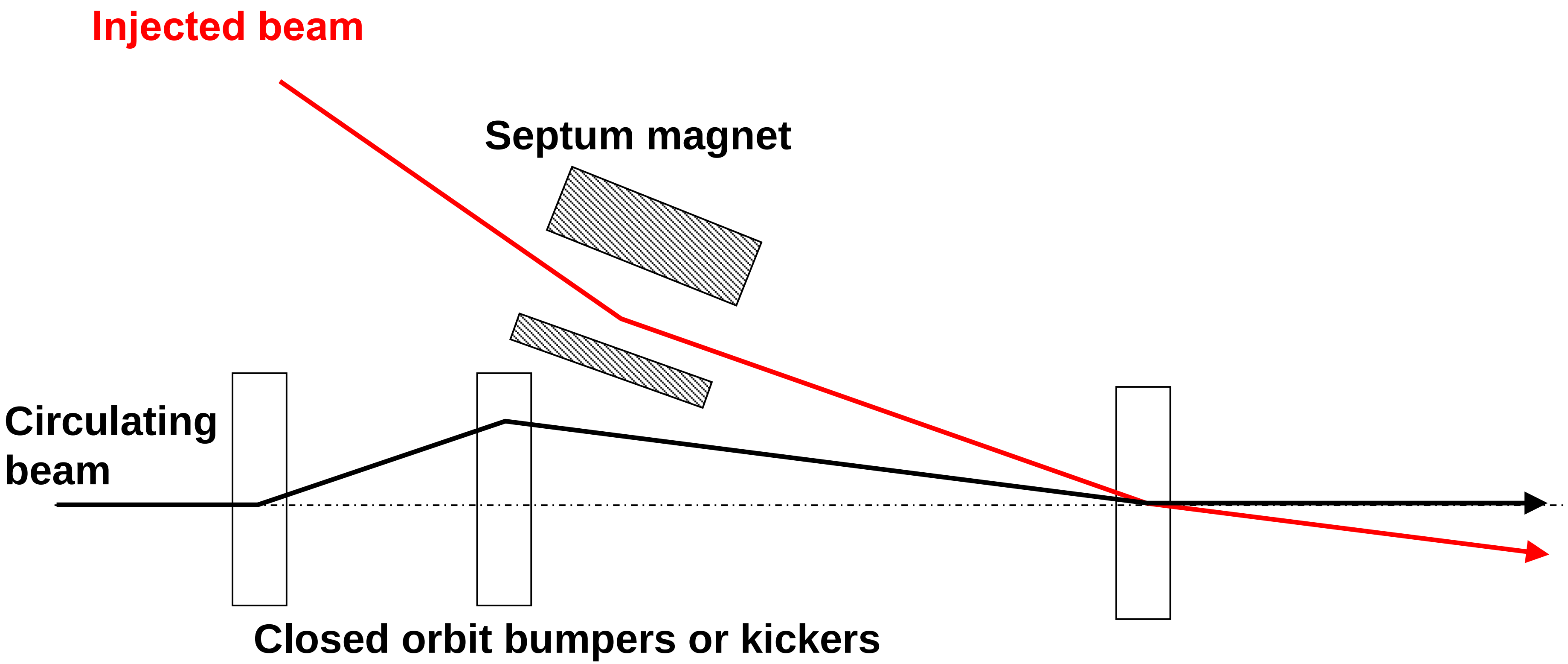
≈100 turns

Lepton injection

- Single-turn injection can be used as for hadrons; however, lepton motion is strongly damped (different with respect to proton or ion injection).
  - Synchrotron radiation
    - see *Electron Beam Dynamics lectures by L. Rivkin*
- Can use transverse or longitudinal damping:
  - Transverse - Betatron accumulation
  - Longitudinal - Synchrotron accumulation (2 x faster than transverse)

- Can be used for top-up injection (keeping constant current)
  - need full-energy injector

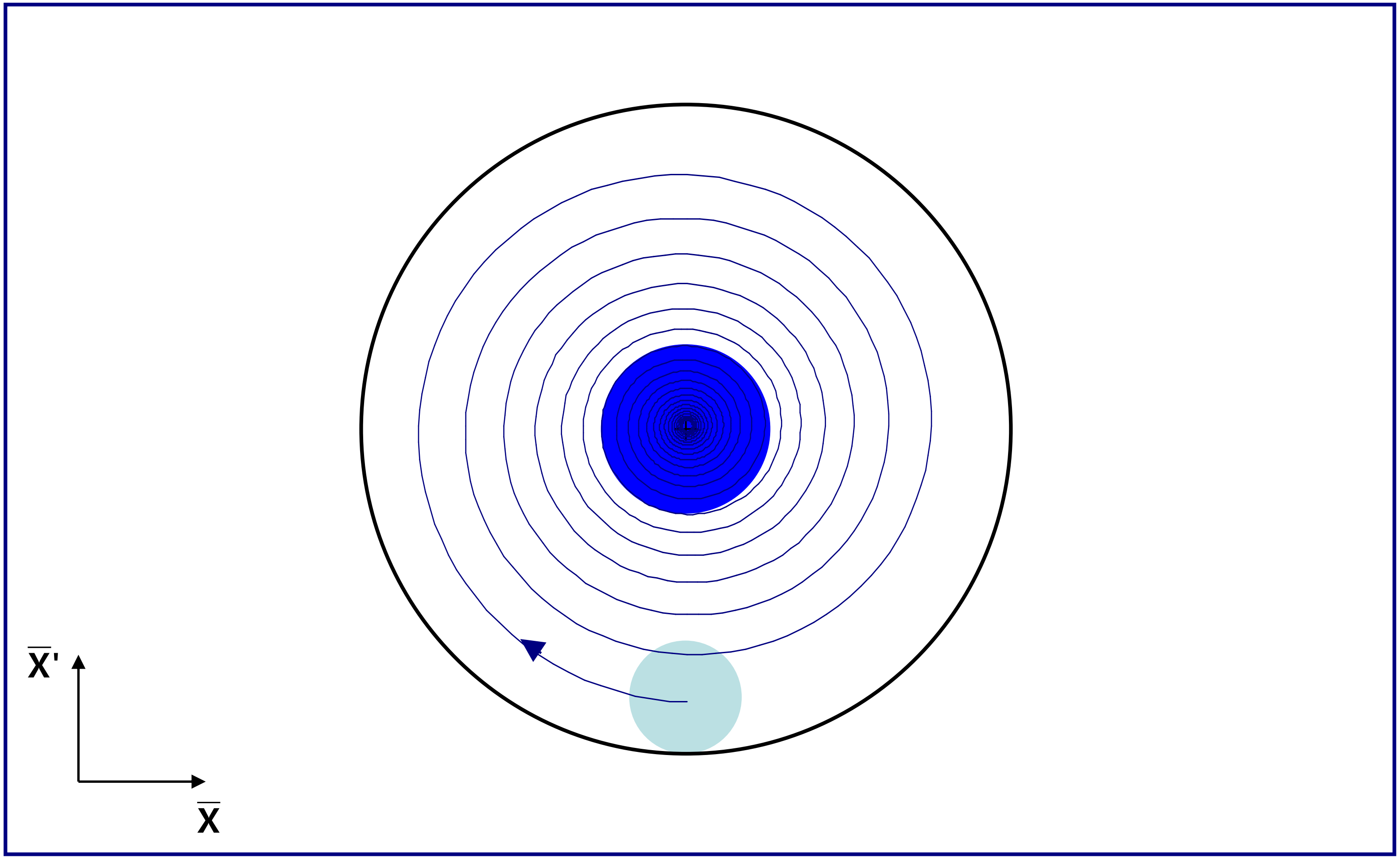
Betatron lepton injection



- Beam is injected with an angle with respect to the closed orbit
- Injected beam performs damped betatron oscillations about the closed orbit

Betatron lepton injection

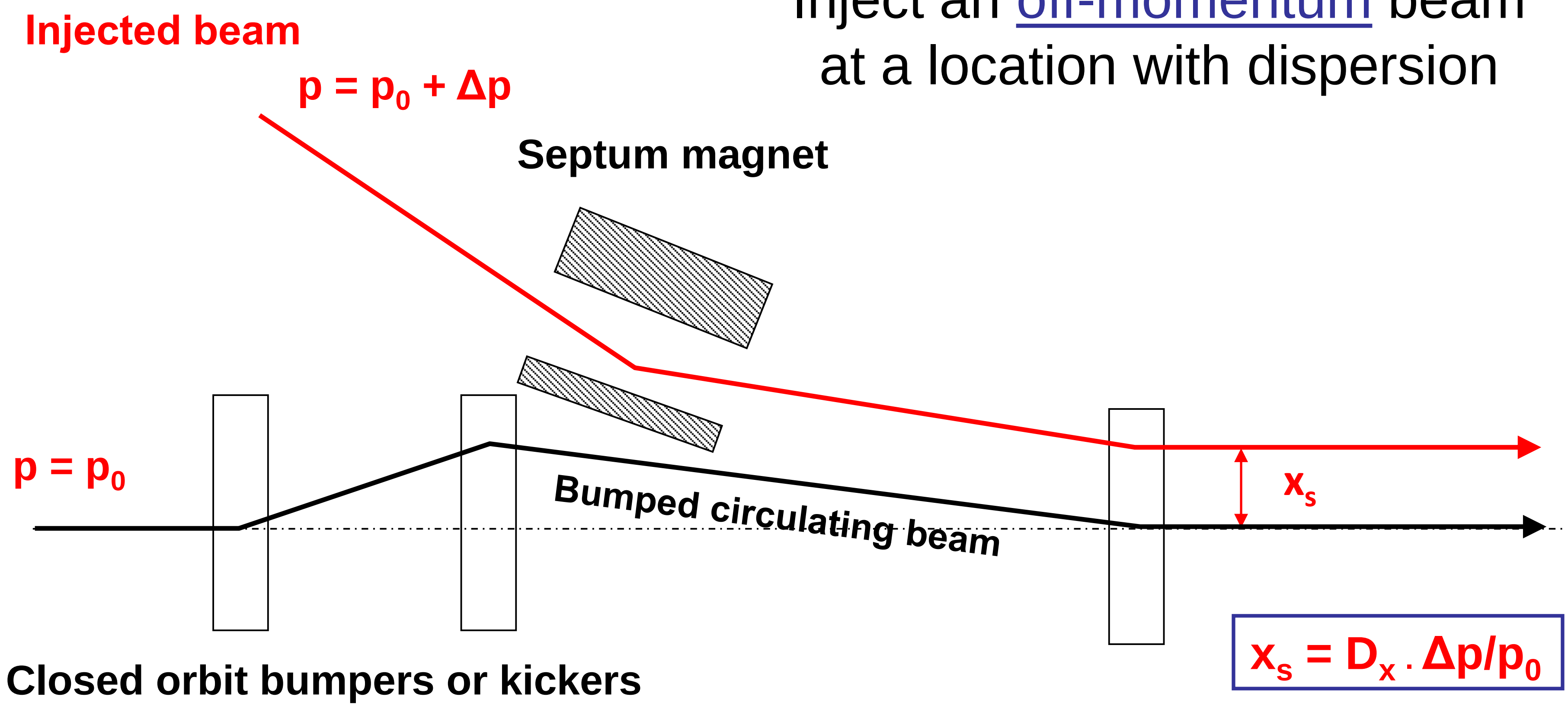
Injected bunch performs damped betatron oscillations



In LEP at 20 GeV, the damping time was about 6'000 turns (0.6 seconds)

Synchrotron lepton injection

Inject an off-momentum beam
at a location with dispersion

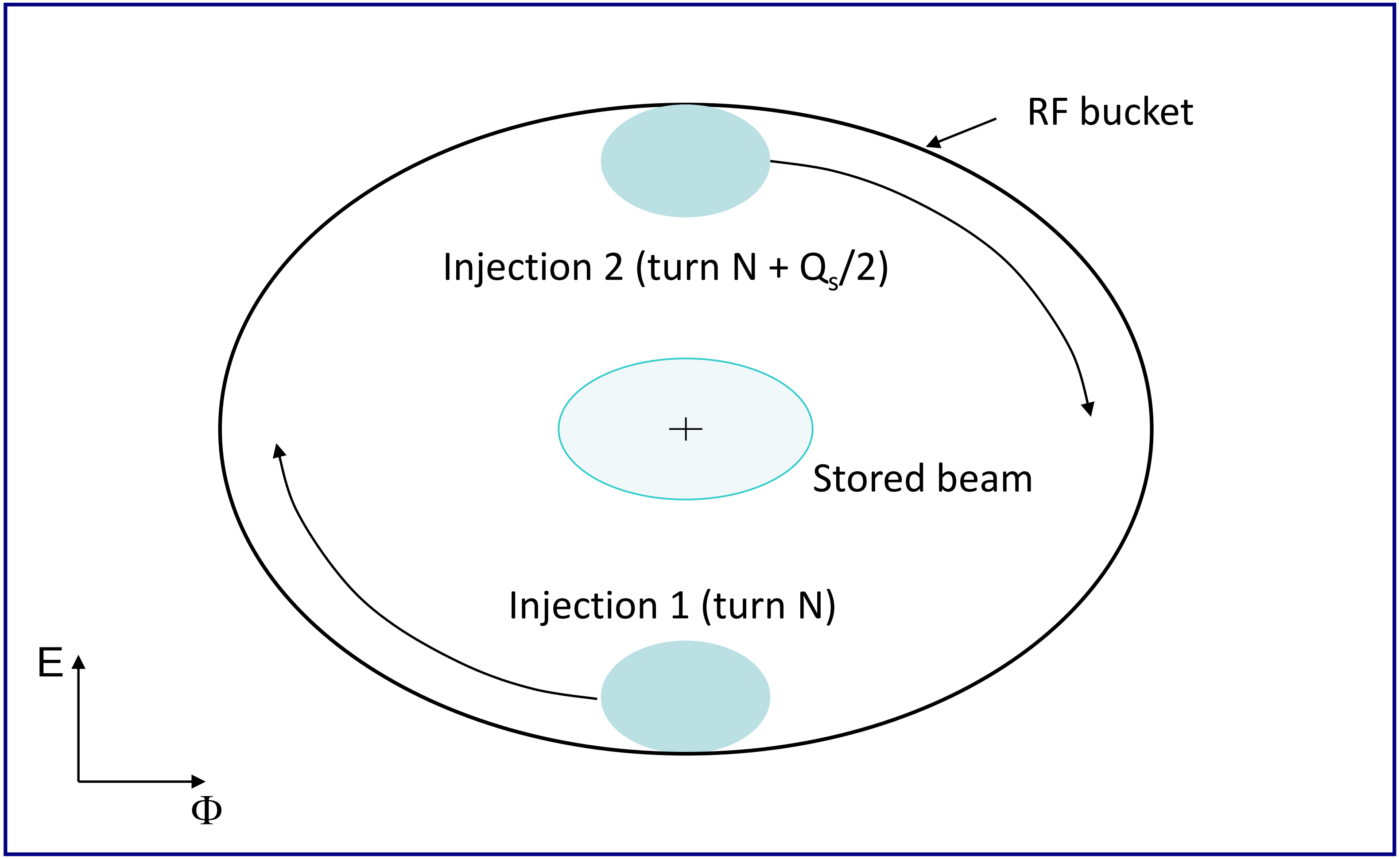


$x_s = D_x \,.\, \Delta p/p_0$

- Beam injected parallel to circulating beam, onto dispersion orbit of a particle having the same momentum offset Δp/p
- Injected beam makes damped synchrotron oscillations at Q_s but does not perform betatron oscillations

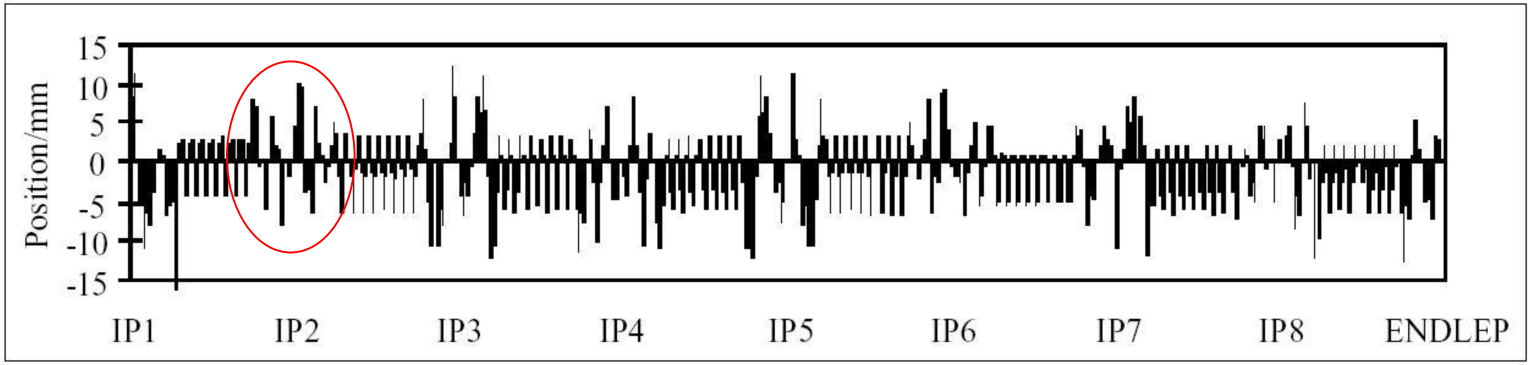
Synchrotron lepton injection

Double batch injection possible….

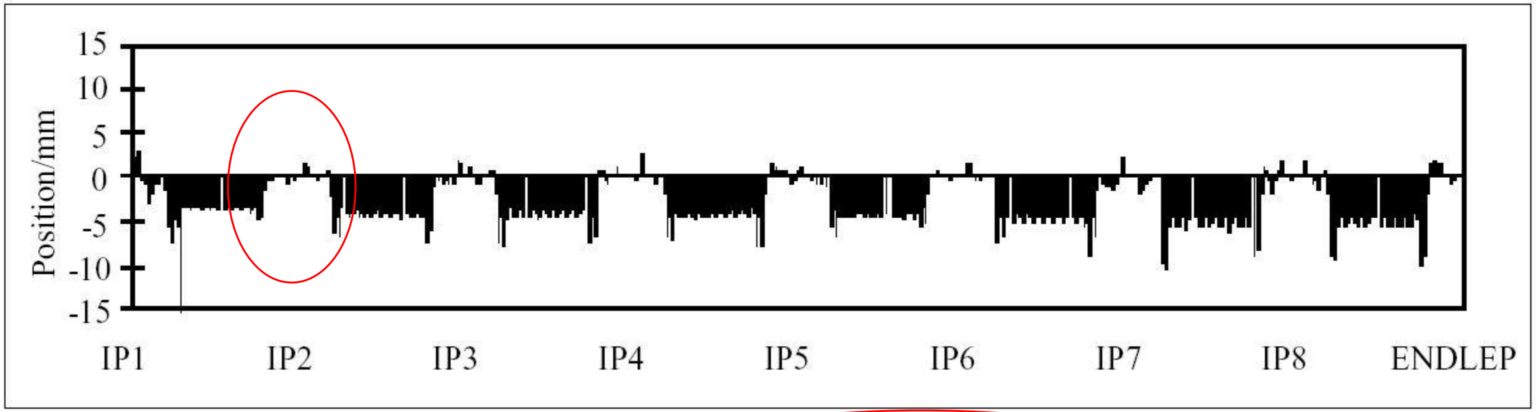


Longitudinal damping time in LEP was ~3'000 turns (2x faster than transverse)

Synchrotron lepton injection in LEP



Optimized Horizontal First Turn Trajectory for Betatron Injection of Positrons into LEP.



Optimized Horizontal First Turn Trajectory for Synchrotron Injection of Positrons with $\Delta P/P$ at -0.6%

Synchrotron injection in LEP gave improved background for LEP experiments due to small orbit offsets in zero dispersion straight sections

Injection - summary

- Several different techniques using kickers, septa and bumpers:
  - **Single-turn injection** for hadrons
    - Boxcar stacking: transfer between machines in accelerator chain
    - Angle / position errors $\Rightarrow$ injection oscillations
    - Uncorrected errors $\Rightarrow$ filamentation $\Rightarrow$ emittance increase
  - **Multi-turn injection** for hadrons
    - Phase space painting to increase intensity
    - H- injection allows injection into same phase space area
  - **Lepton injection**: take advantage of damping
    - Less concerned about injection precision and matching

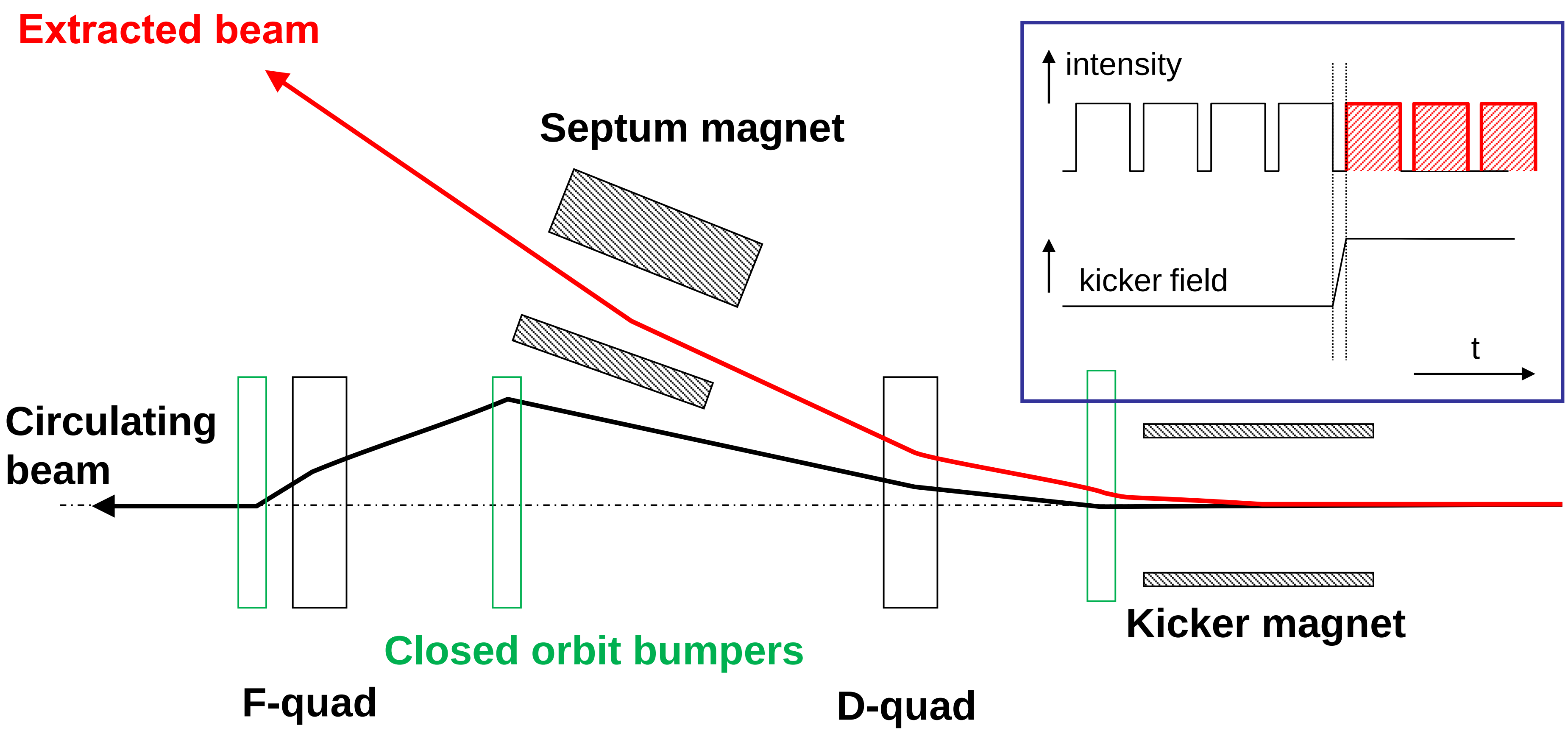
Extraction

- Different extraction techniques exist, depending on requirements
  - **Fast extraction**: ≤1 turn
  - **Non-resonant (fast) multi-turn extraction**: few turns
  - **Resonant low-loss (fast) multi-turn extraction**: few turns
  - **Resonant multi-turn extraction**: many thousands of turns

- Usually higher energy than injection $\Rightarrow$ stronger elements ($\int B.dl$)
  - At high energies many kicker and septum modules may be required
  - To reduce kicker and septum strength, beam can be moved near to septum by closed orbit bump
  - Beam size scales with $1/\sqrt{\gamma}$ => smaller than injection

Fast single turn extraction

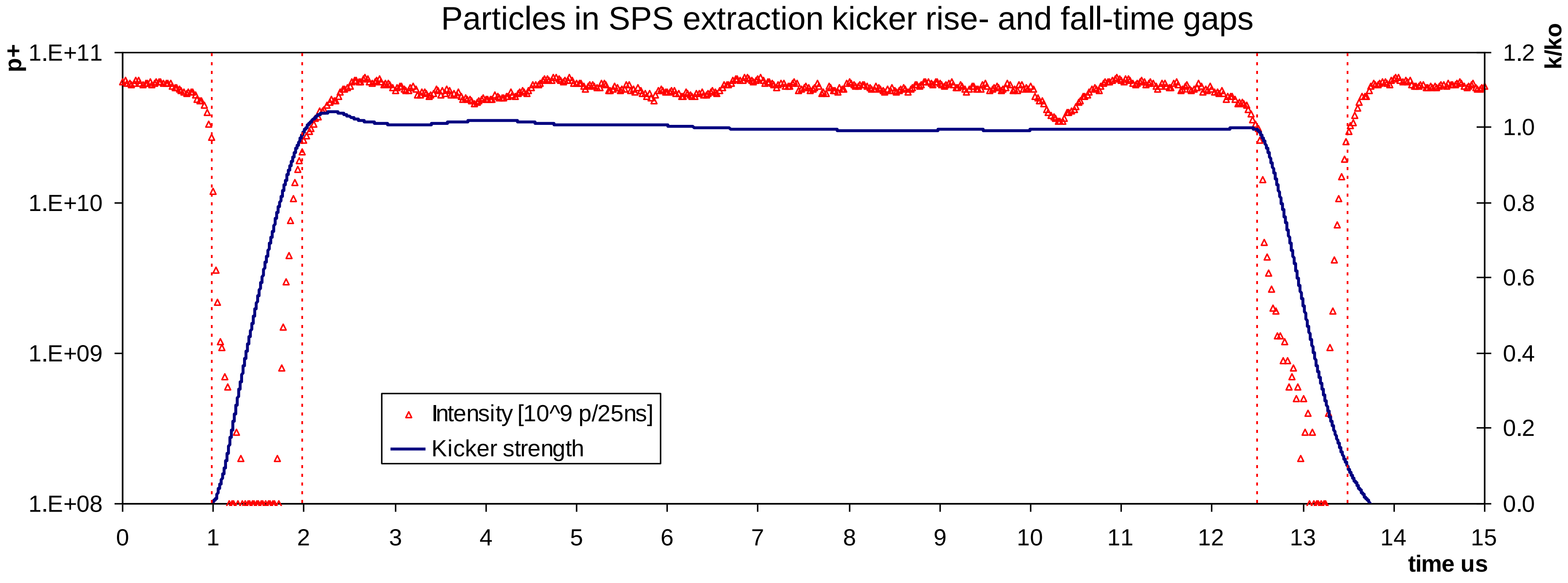
Entire beam kicked into septum gap and extracted over a single turn



- Bumpers move circulating beam close to septum to reduce kicker strength
- Kicker deflects the entire beam into the septum in a single turn
- Most efficient (lowest deflection angles required) for π/2 phase advance between kicker and septum

Fast single turn extraction

- For transfer of beams between accelerators in an injector chain
- For secondary particle production
  - e.g. neutrinos, radioactive beams
- Losses from transverse scraping or from particles in extraction gap:
  - Fast extraction from SPS to CNGS:



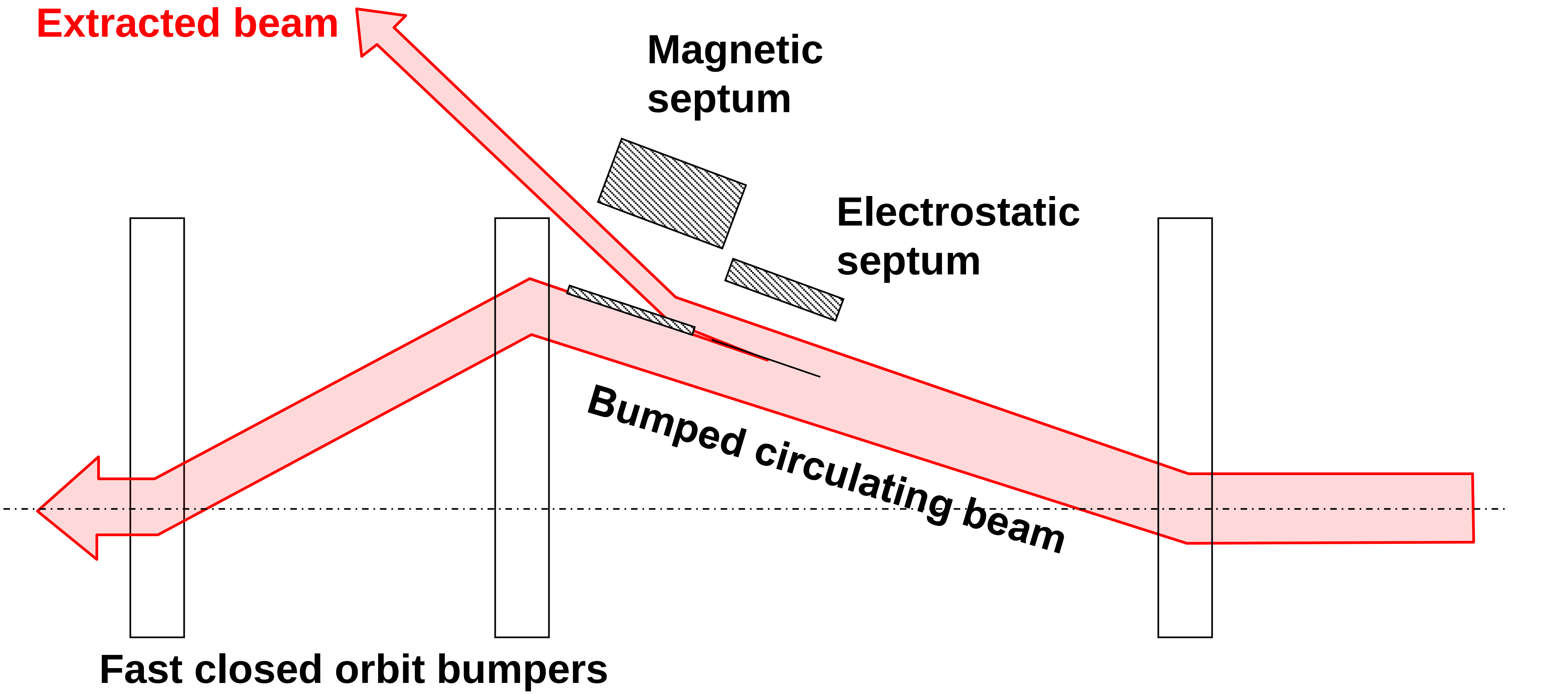
Multi-turn extraction

- Some filling schemes require a beam to be injected in several turns to a larger machine…
- And very commonly Fixed Target physics experiments and medical accelerators often need a quasi-continuous flux of particles…

- Multi-turn extraction…
  - **Fast**: Non-resonant and resonant multi-turn ejection (few turns) for filling
    - e.g. PS to SPS at CERN for high intensity proton beams (>2.5 10^{13} protons)
  - **Slow**: Resonant extraction (ms to hours) for experiments

Non-resonant multi-turn extraction

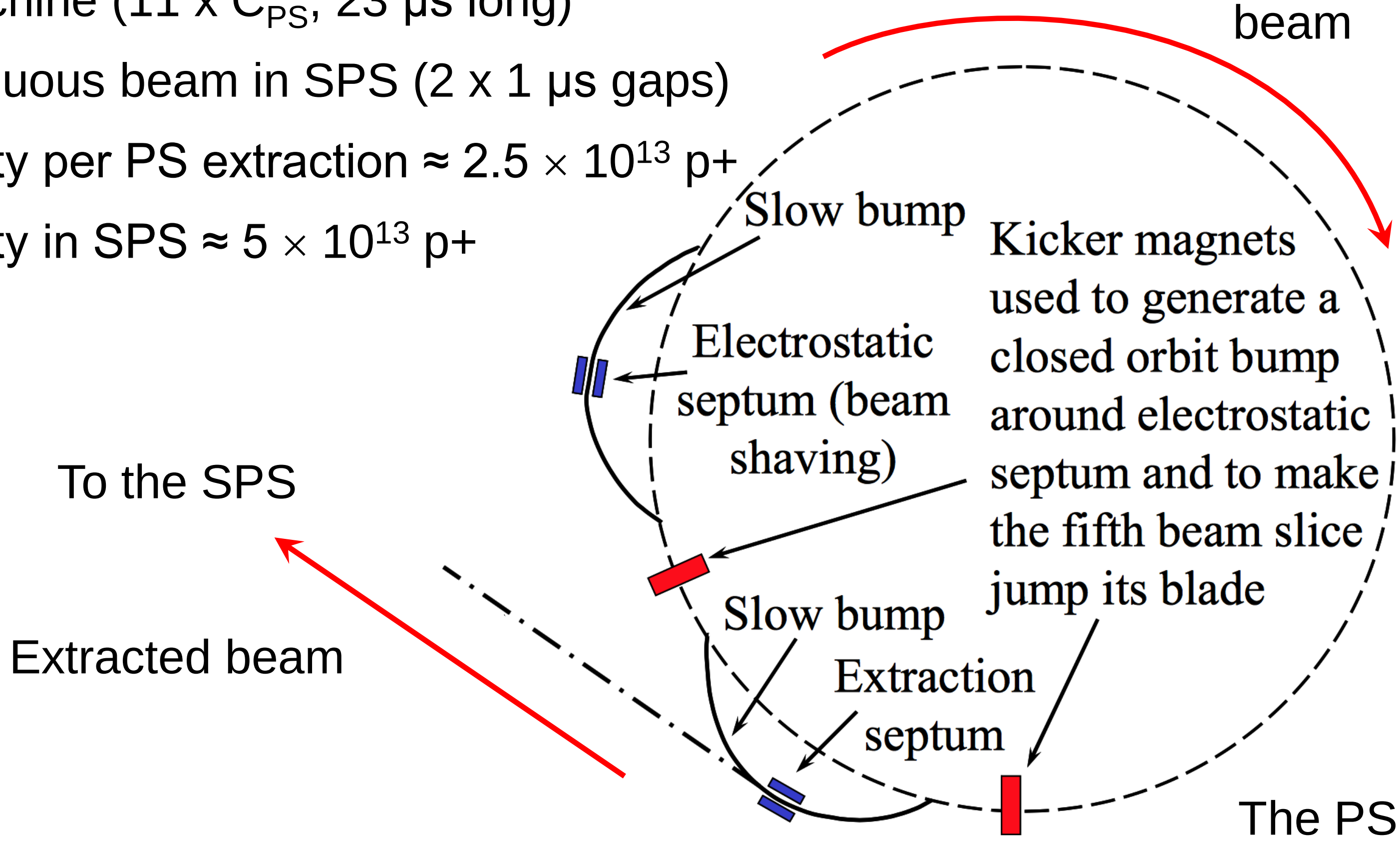
Beam bumped to septum; part of beam ‘shaved’ off each turn



- Fast bumper deflects the whole beam onto the septum
- Beam extracted in a few turns, with the machine tune rotating the beam
- Intrinsically a high-loss process: thin septum essential
- Often combine thin electrostatic septa with magnetic septa

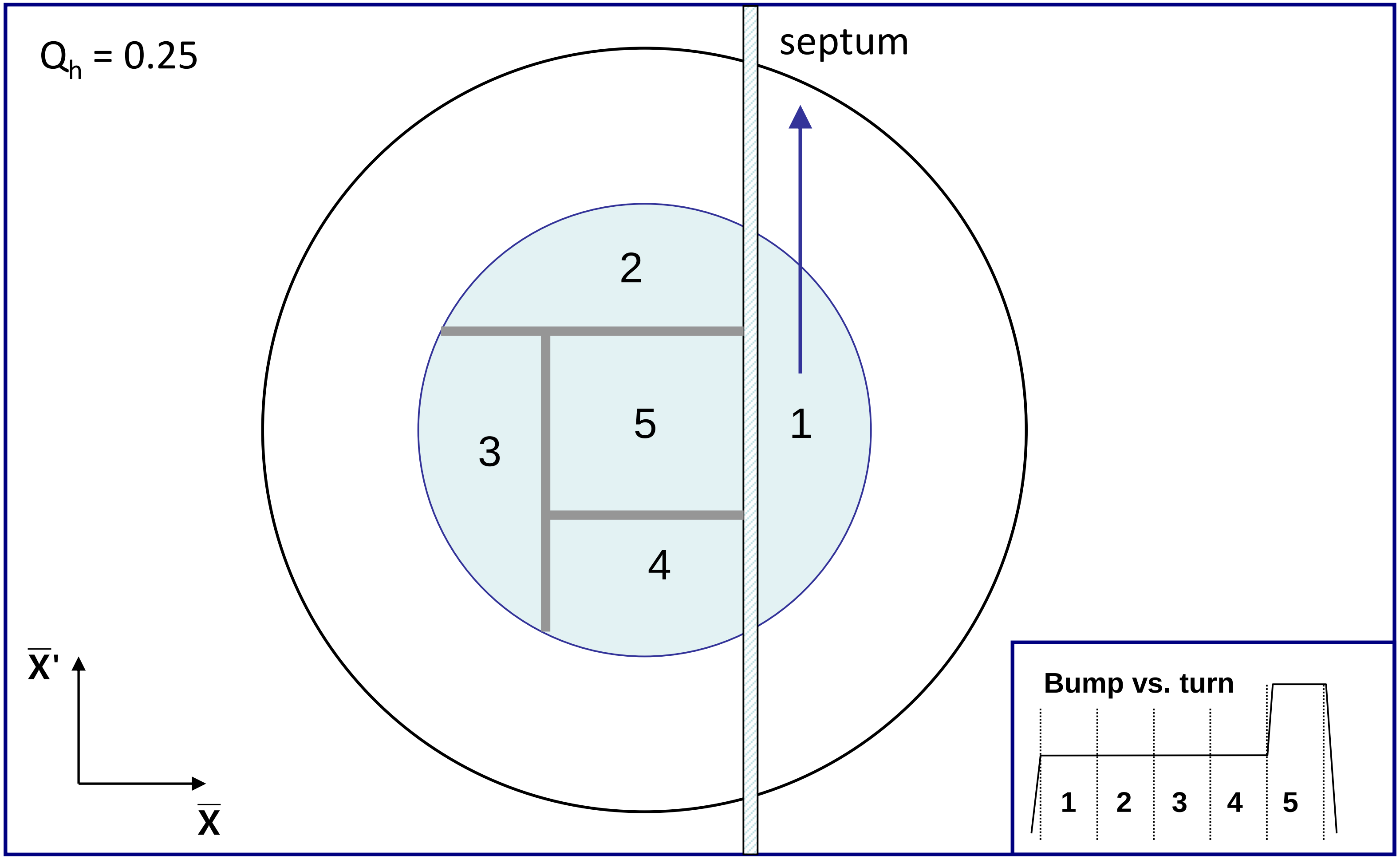
Non-resonant multi-turn extraction

- Example system: CERN PS to SPS Fixed-Target ‘continuous transfer’.
  - Accelerate beam in PS to 14 GeV/c
  - Empty PS machine (2.1 μs long) in 5 turns into SPS
  - Do it again
  - Fill SPS machine (11 x C_{PS}, 23 μs long)
  - Quasi-continuous beam in SPS (2 x 1 μs gaps)
  - Total intensity per PS extraction ≈ 2.5×10^{13} p+
  - Total intensity in SPS ≈ 5×10^{13} p+



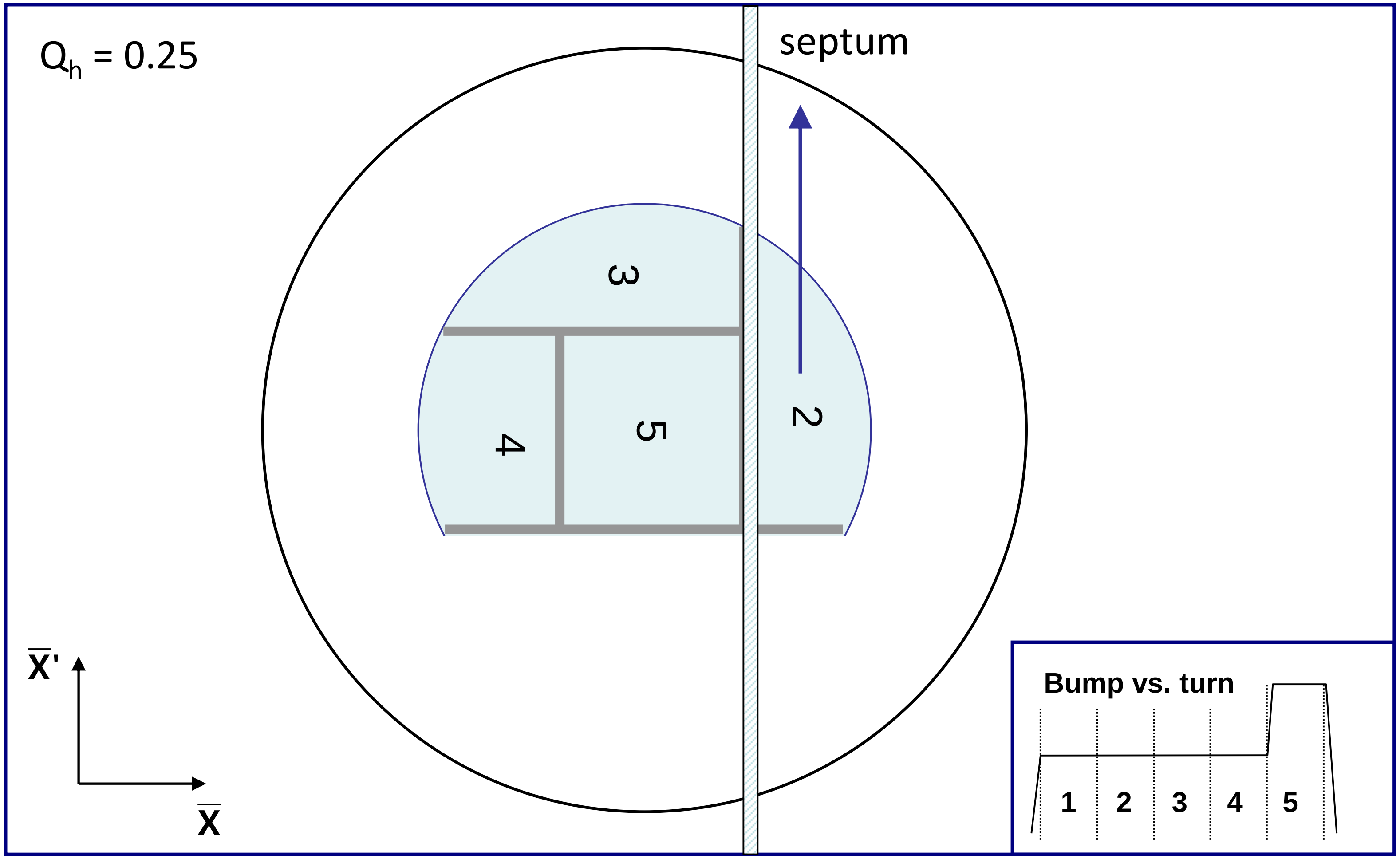
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 1st turn



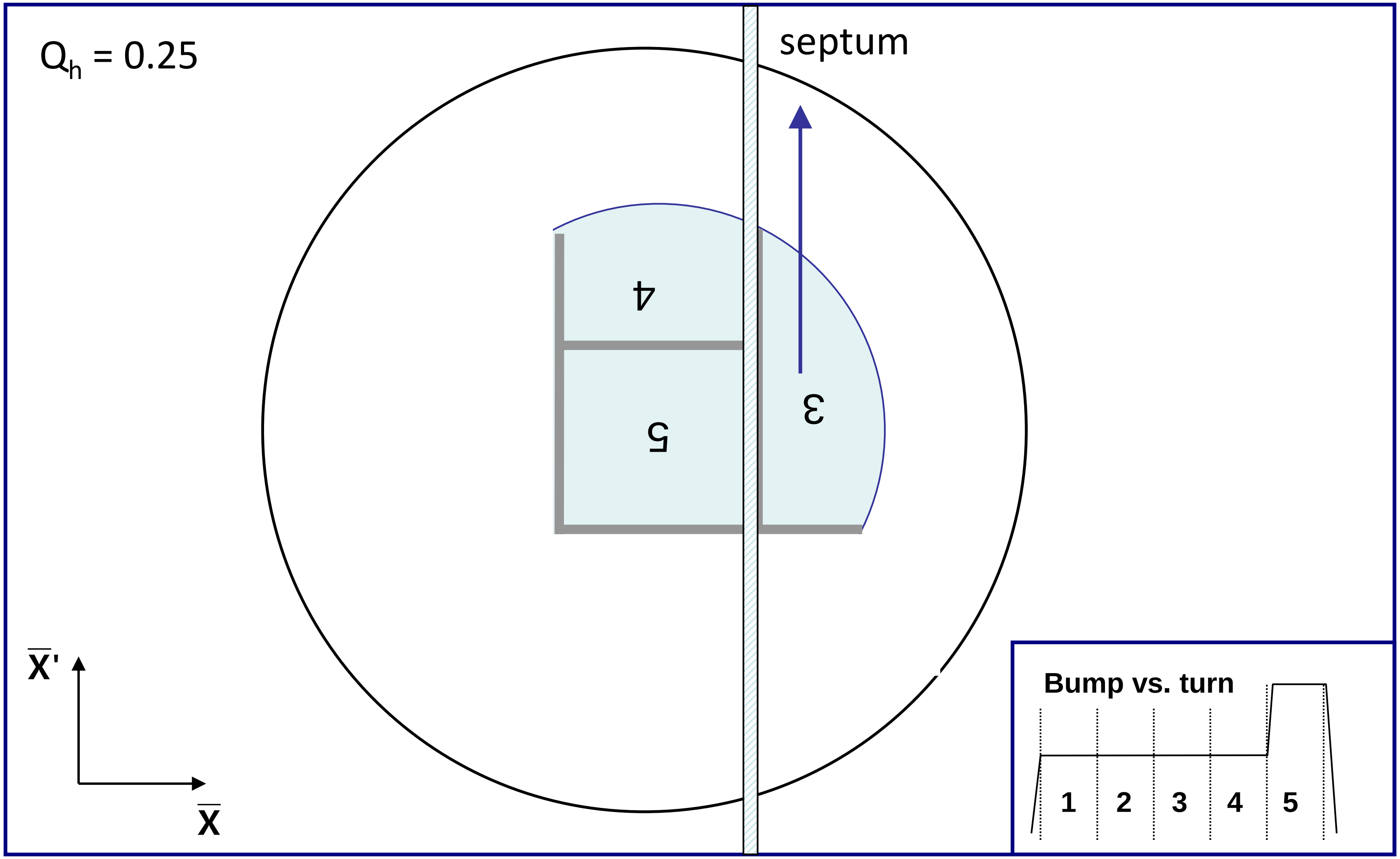
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 2nd turn



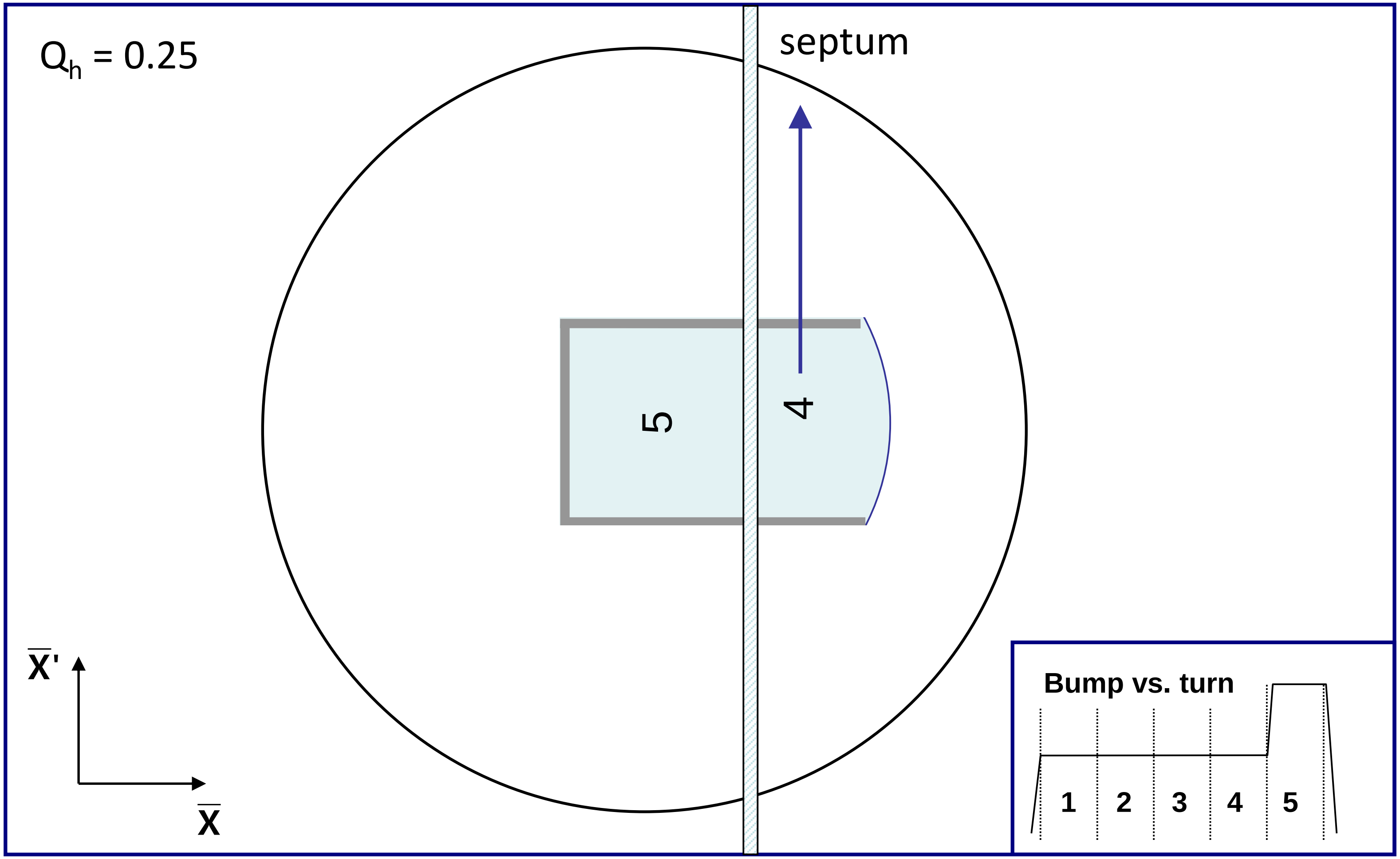
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 3rd turn



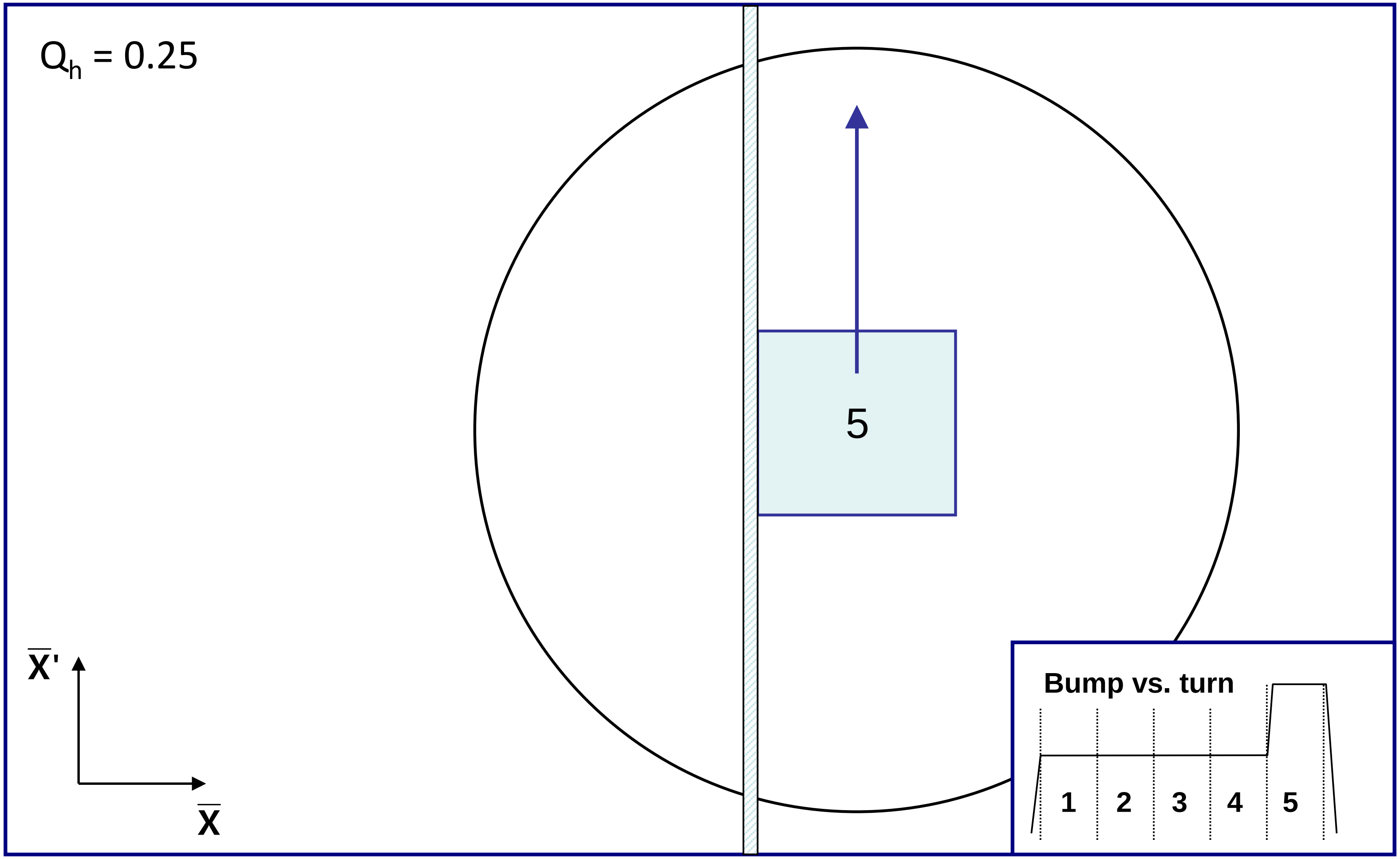
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 4th turn



Non-resonant multi-turn extraction

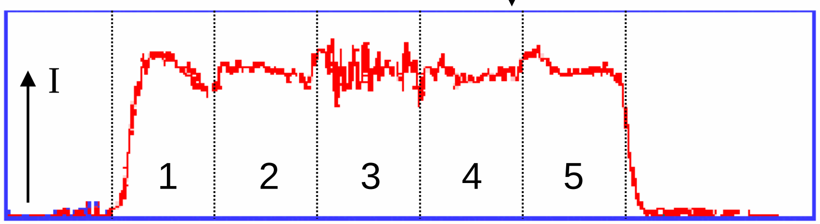
CERN PS to SPS: 5-turn continuous transfer – 5th turn

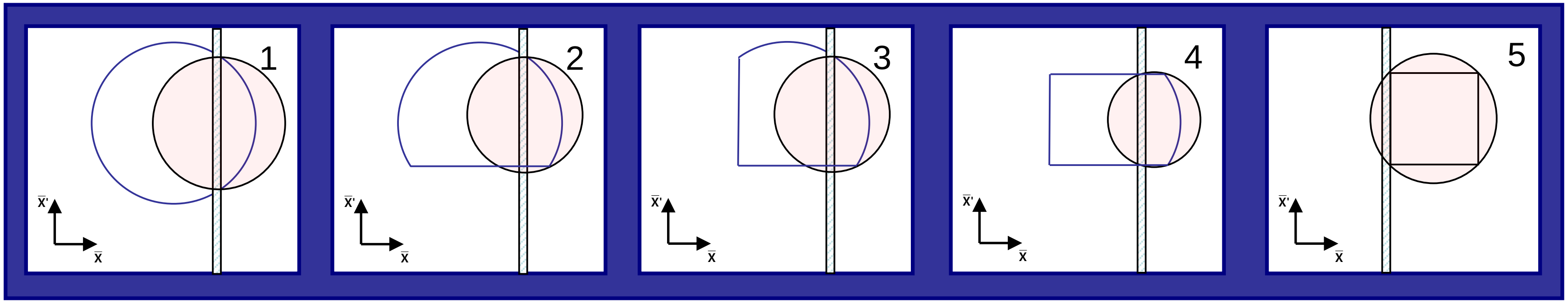


Non-resonant multi-turn extraction

- CERN PS to SPS: 5-turn continuous transfer
  - Losses impose thin septum…

    … an electrostatic septum is needed in addition to the magnetic septum
  - Still about 15 % of beam lost in PS-SPS CT
  - Difficult to get equal intensities per turn
  - Different trajectories for each turn
  - Different emittances for each turn



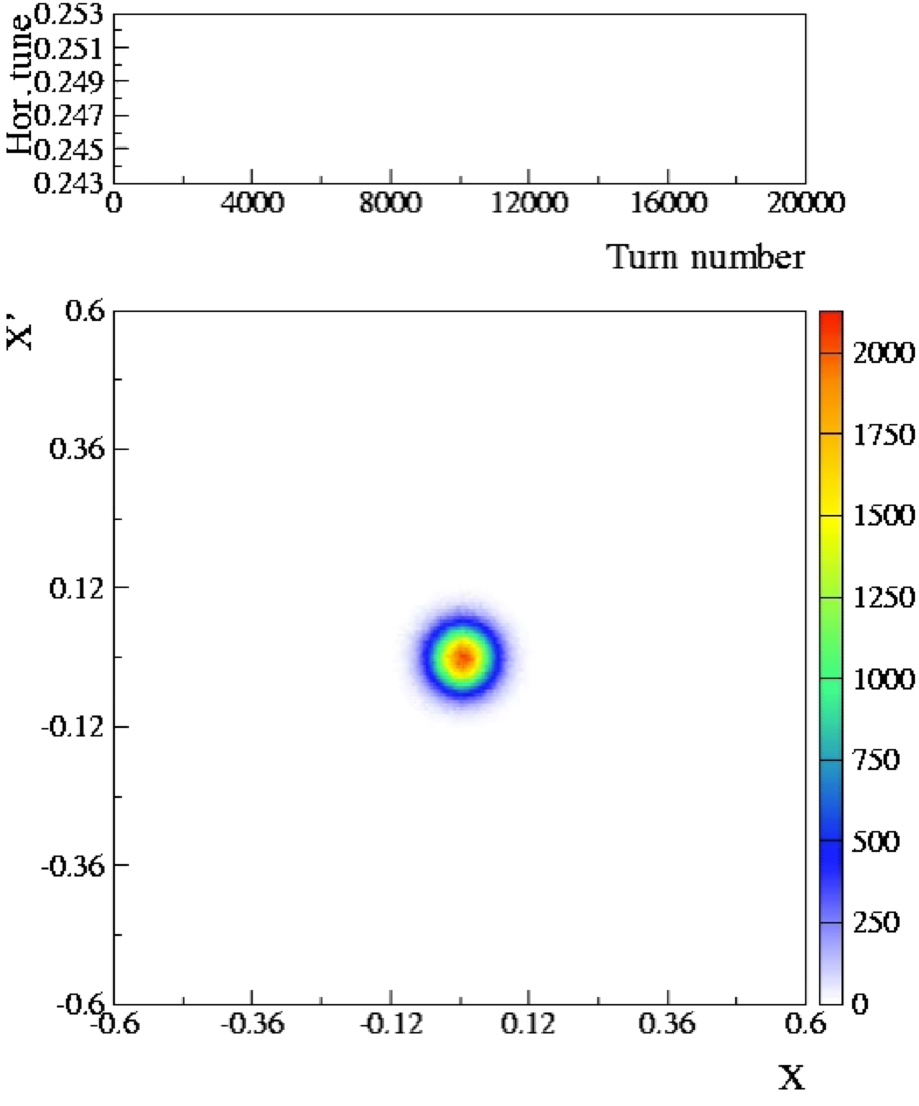


Resonant multi-turn (fast) extraction

- Adiabatic capture of beam in stable “islands”
  - Use non-linear fields (sextupoles and octupoles) to create islands of stability in phase space
  - A slow (adiabatic) tune variation to cross a resonance and to drive particles into the islands (capture) with the help of transverse excitation (using damper)
  - Variation of field strengths to separate the islands in phase space

- Several big advantages:
  - Losses reduced significantly (no particles at the septum in transverse plane)
  - Phase space matching improved with respect to existing non-resonant multi-turn extraction - ‘beamlets’ have similar emittance and optical parameters

Resonant multi-turn (fast) extraction



a. Unperturbed beam

b. Increasing non-linear fields

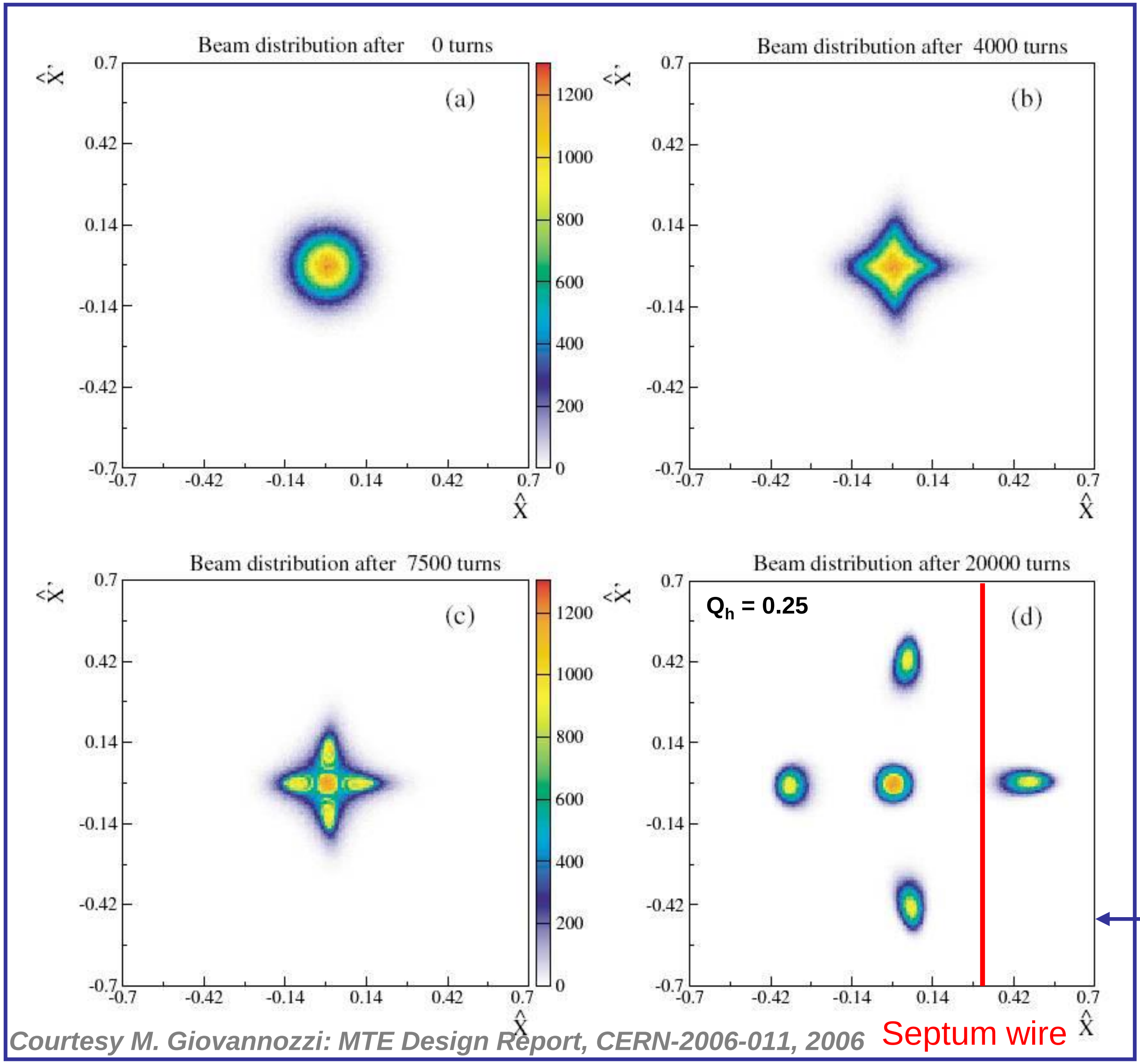
a. Beam captured in stable islands

b. Islands separated and beam bumped across septum – extracted in 5 turns

(see *Non-Linear Beam Dynamics lectures by Hannes Bartosik*)

Courtesy M. Giovannozzi: MTE Design Report, CERN-2006-011, 2006

Resonant multi-turn (fast) extraction



Courtesy M. Giovannozzi: MTE Design Report, CERN-2006-011, 2006

a. Unperturbed beam
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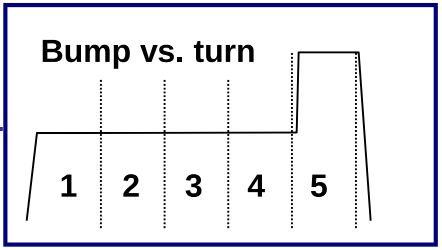
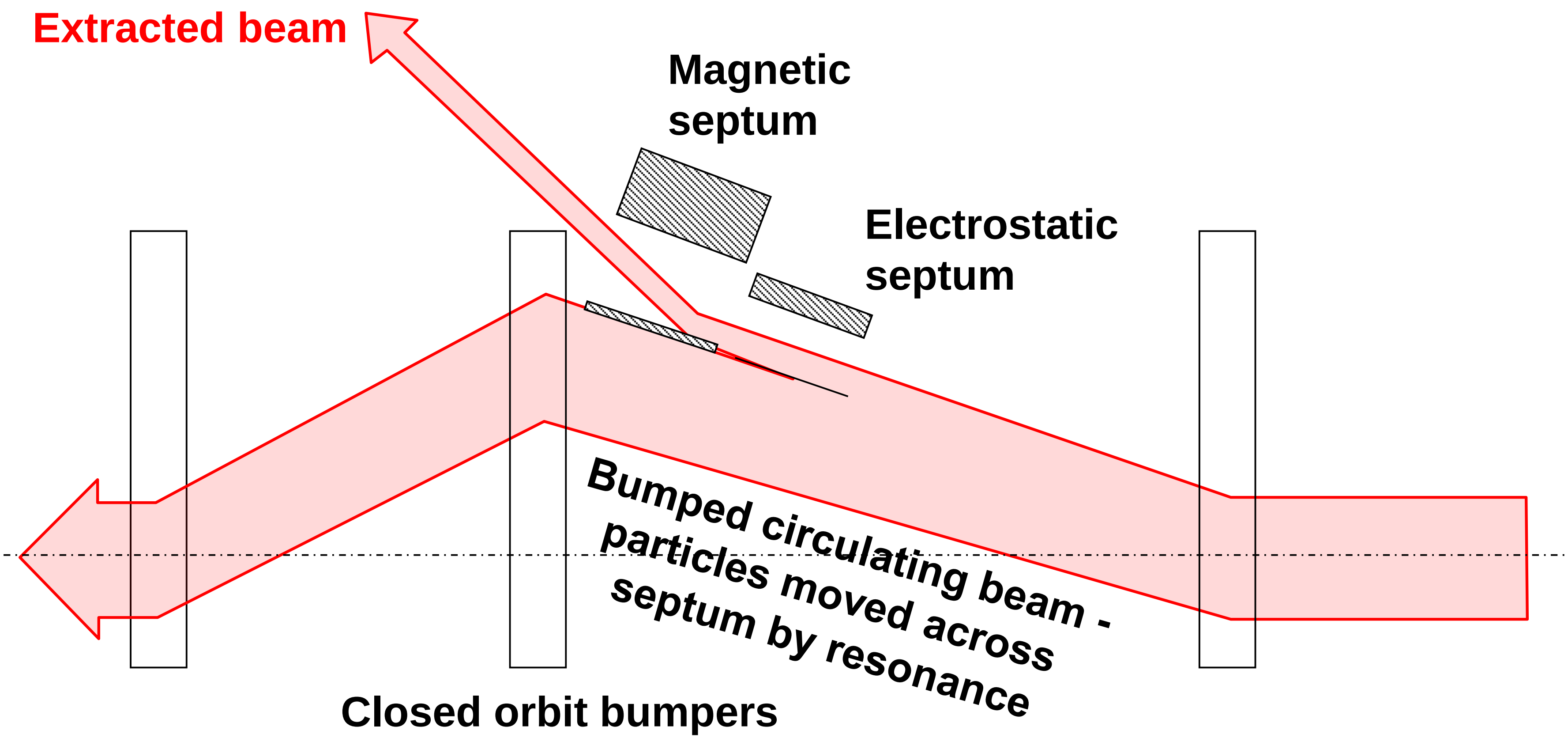


Resonant multi-turn (slow) extraction

Non-linear fields excite resonances that drive the beam slowly across the septum

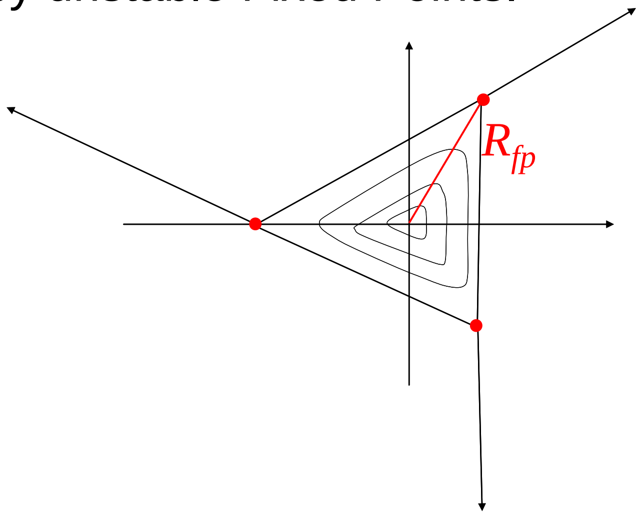


- Slow bumpers move the beam near the septum
- Tune adjusted close to nth order betatron resonance
- Multipole magnets excited to define stable area in phase space, size depends on $\Delta Q = Q - Q_r$

Resonant multi-turn (slow) extraction

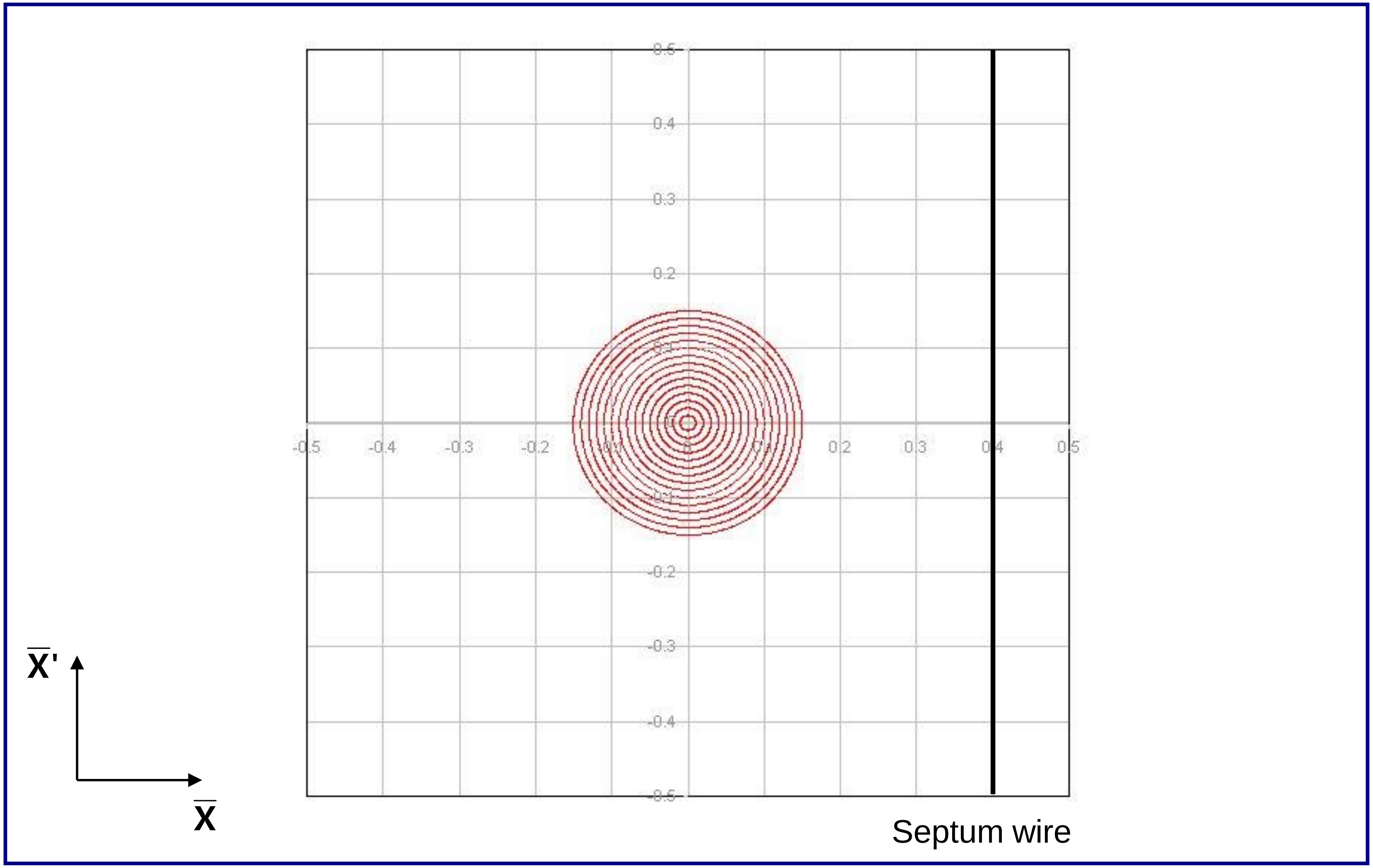
- 3rd order resonances – see *lectures by Hannes Bartosik*
  - Sextupole fields distort the circular normalised phase space particle trajectories.
  - Stable area defined, delimited by unstable Fixed Points.

$$R_{fp}^{\ 1/2} \propto \Delta Q \cdot \frac{1}{k_2}$$



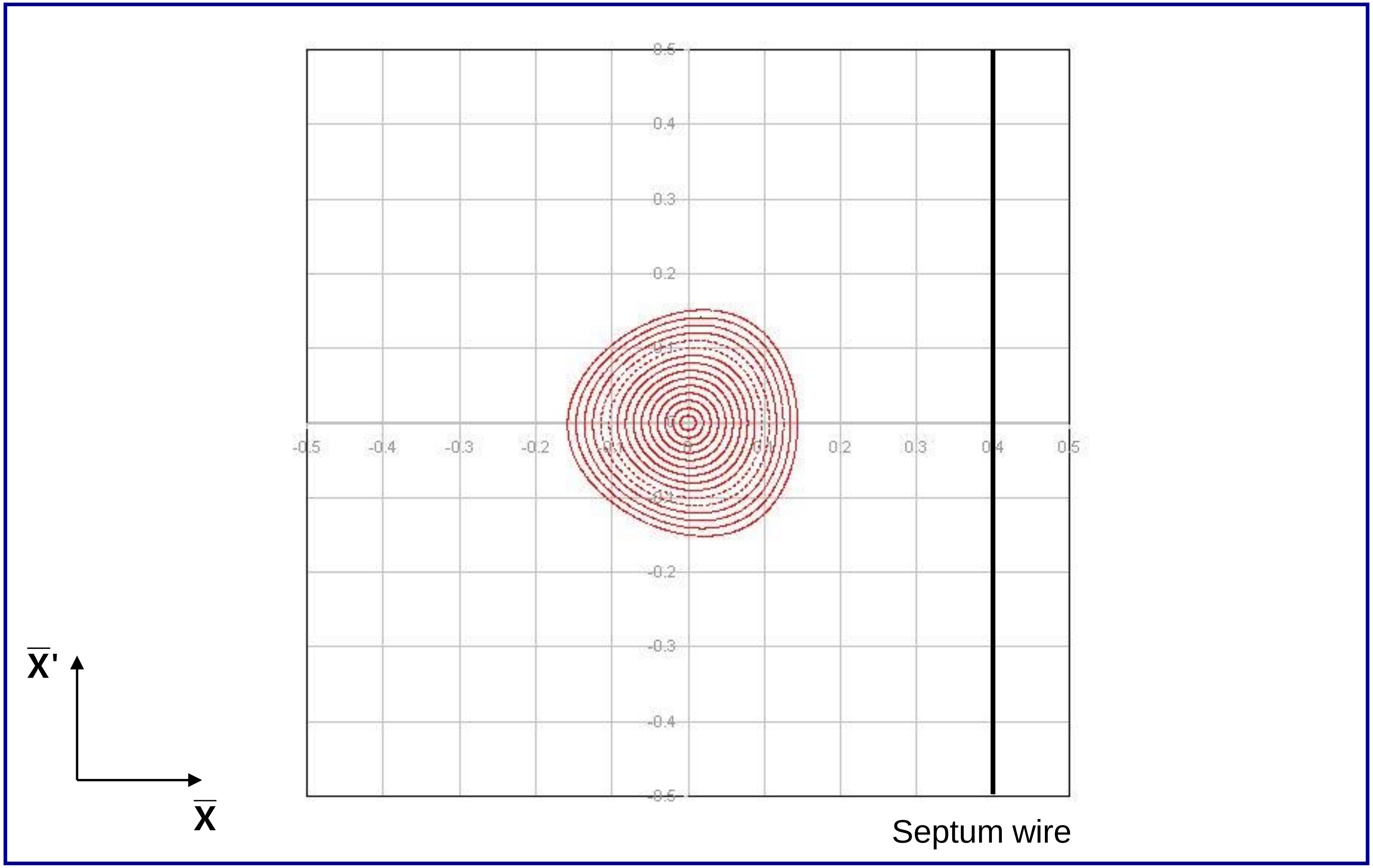
  - Sextupole magnets arranged to produce suitable phase space orientation of the stable triangle at thin electrostatic septum
  - Stable area can be reduced by…
    - Increasing the sextupole strength, or…
    - Fixing the sextupole strength and scanning the machine tune Q_h (and therefore the resonance) through the tune spread of the beam
    - Large tune spread created with RF gymnastics (large momentum spread) and large chromaticity

Third-order resonant extraction



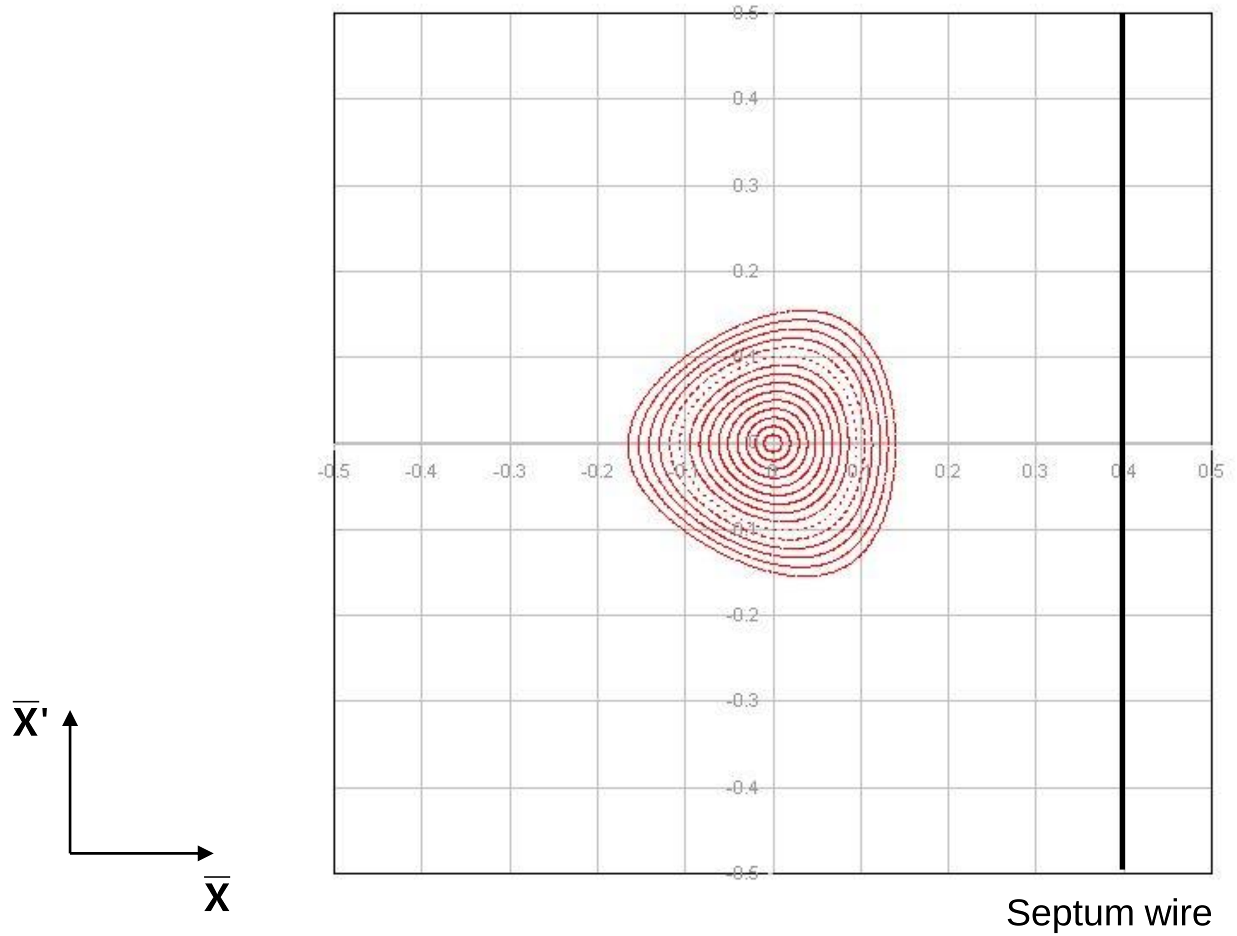
- Particles distributed on emittance contours
- ΔQ large – no phase space distortion

Third-order resonant extraction



- Sextupole magnets produce a triangular stable area in phase space
- ΔQ decreasing – phase space distortion for largest amplitudes

Third-order resonant extraction



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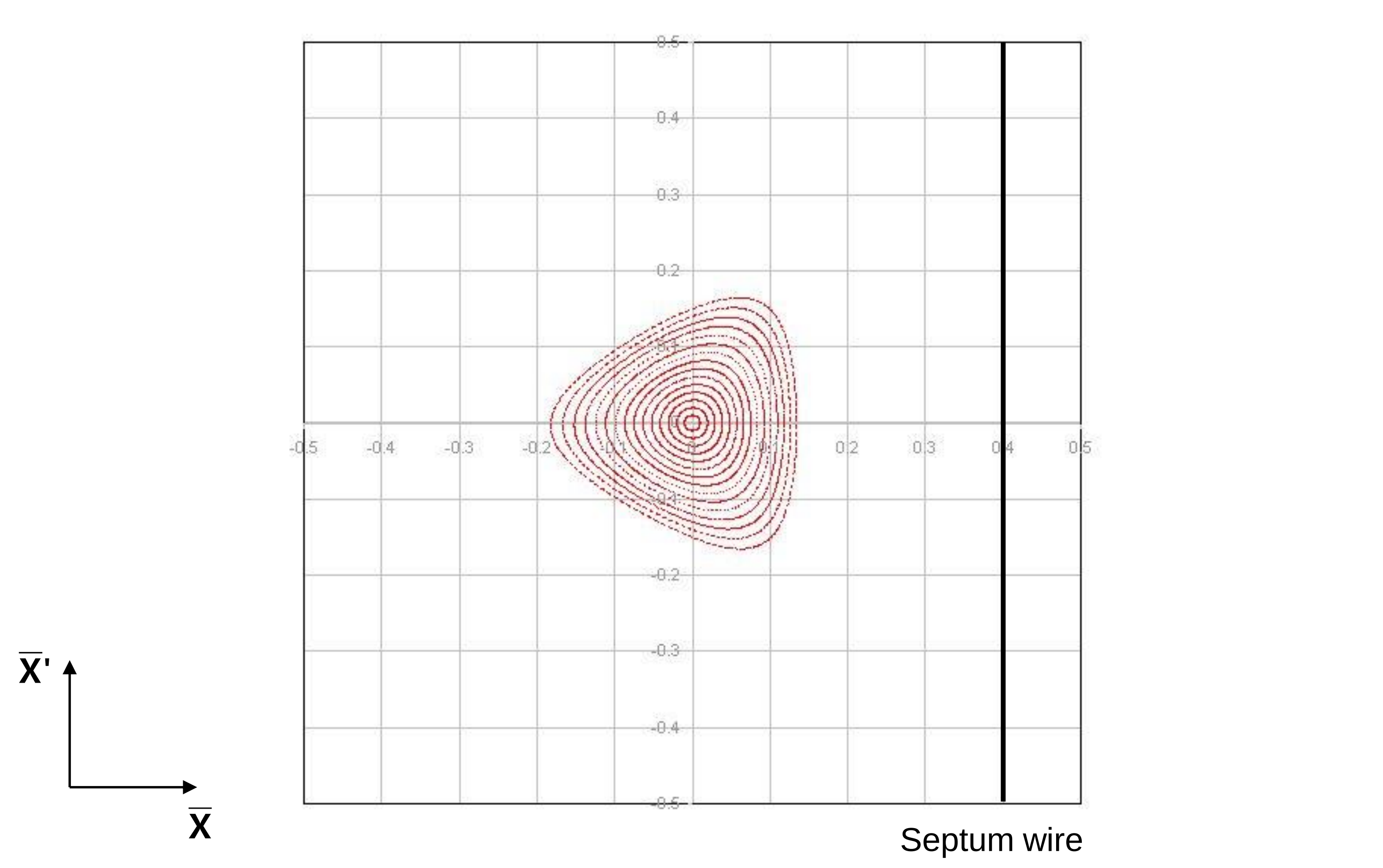
Third-order resonant extraction



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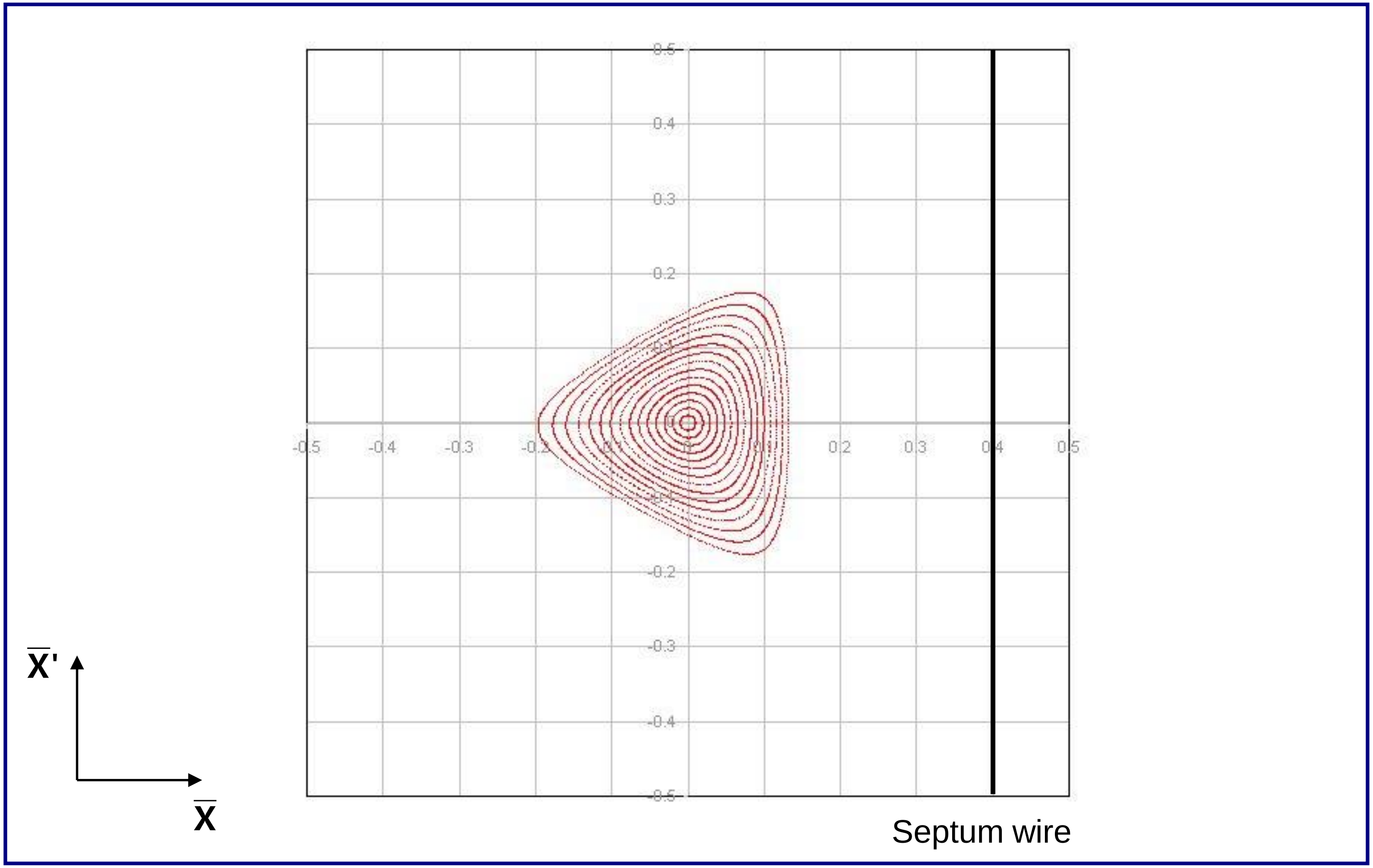

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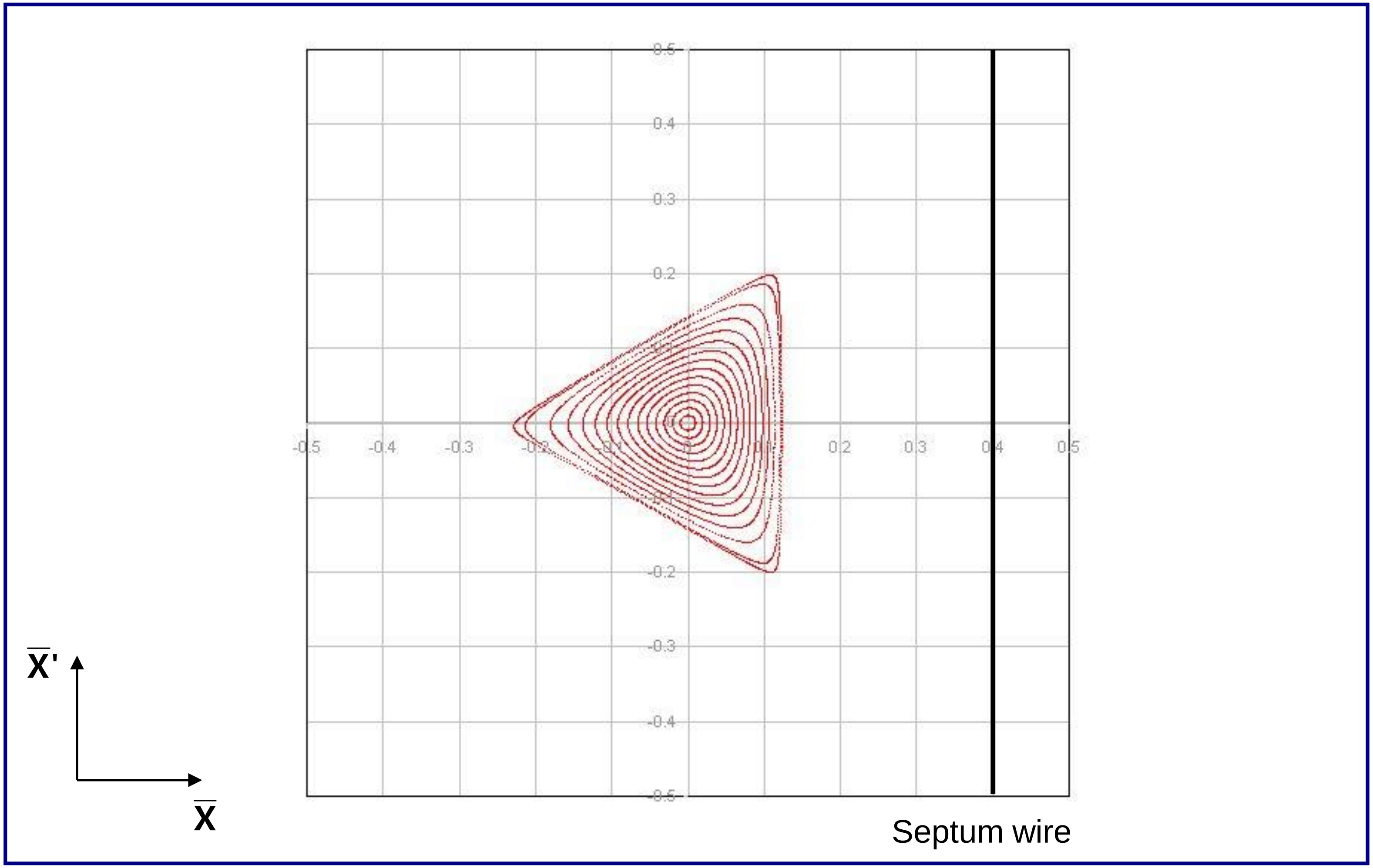
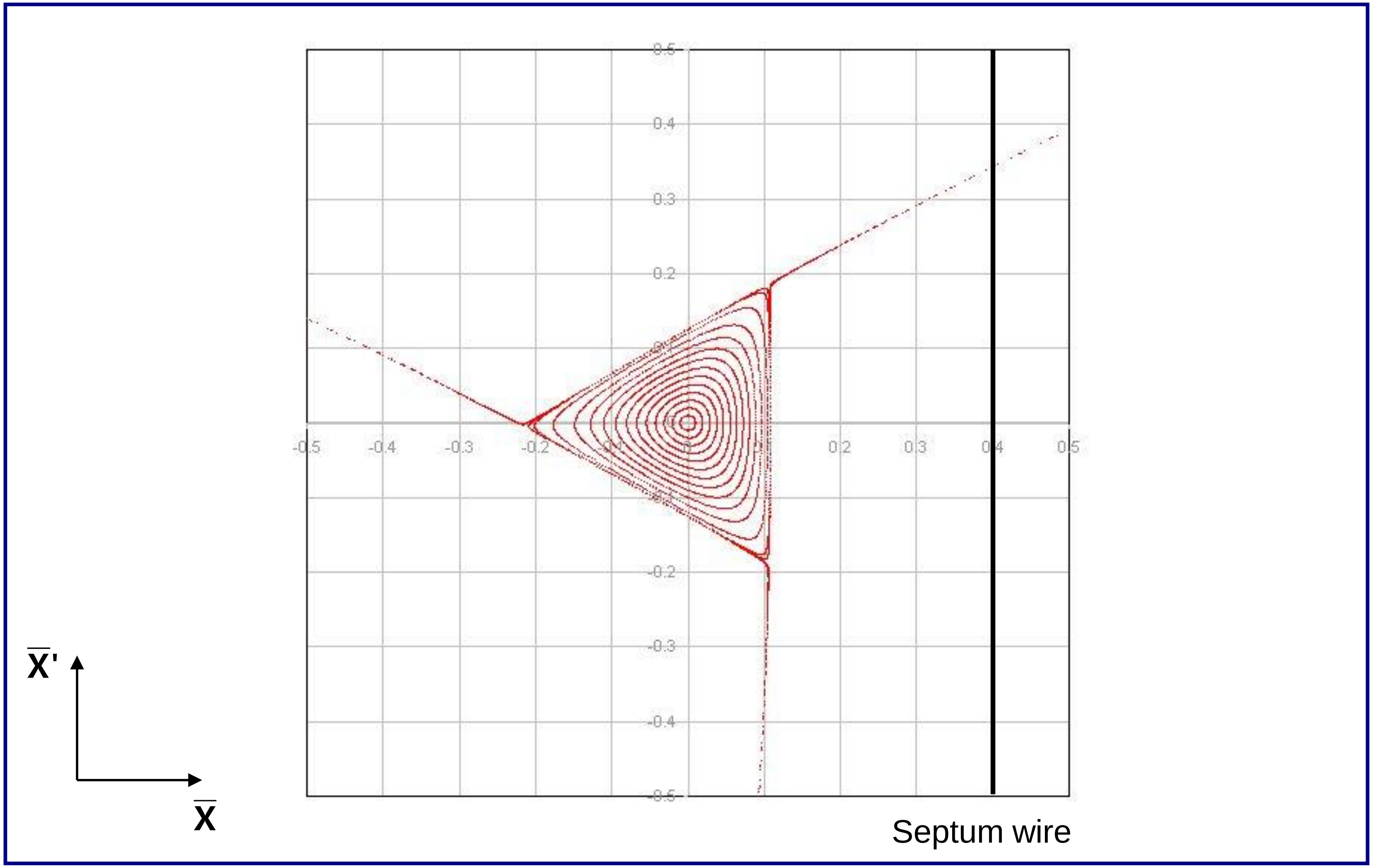
Third-order resonant extraction



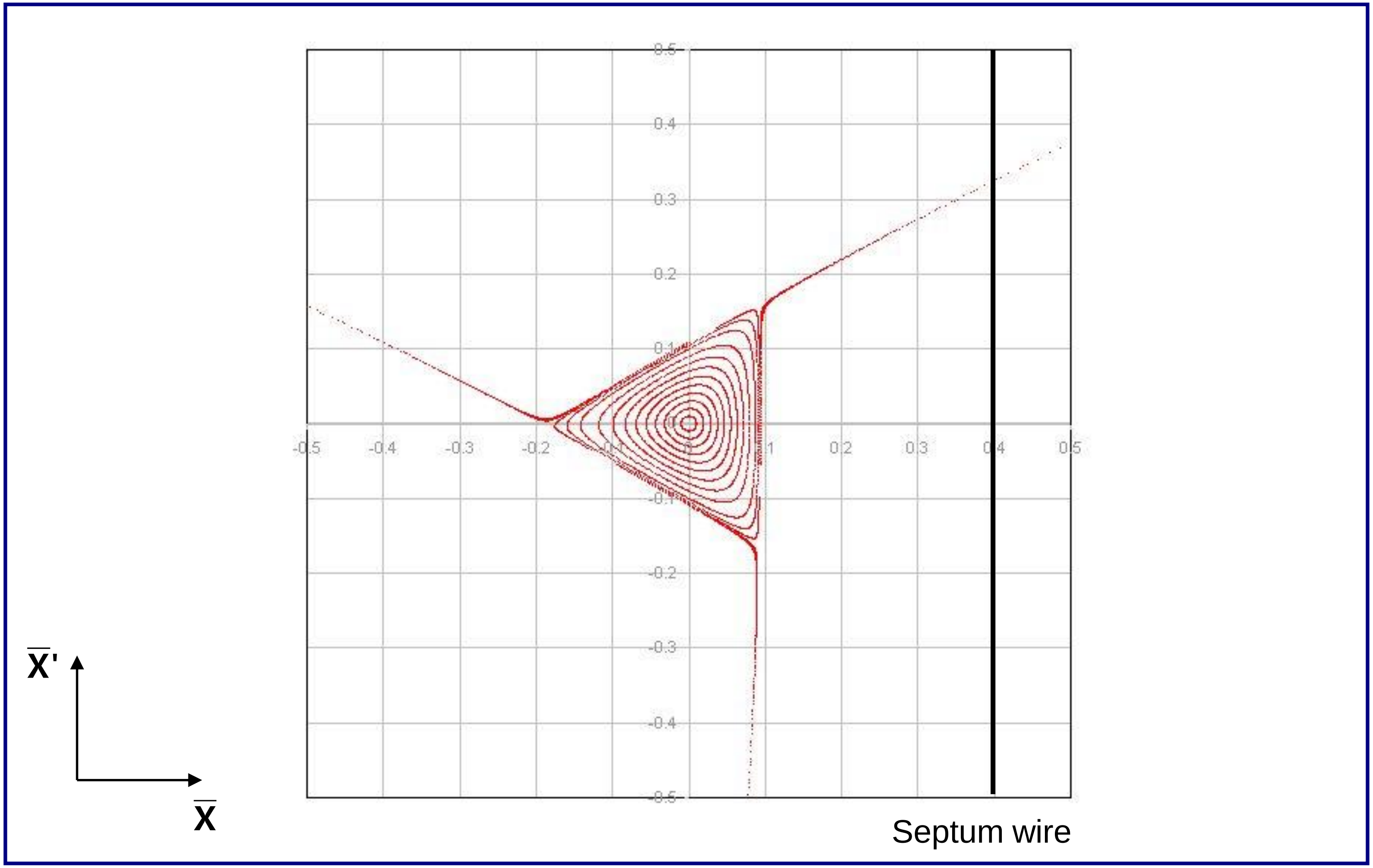
- Largest amplitude particle trajectories are significantly distorted
- Locations of fixed points noticeable at extremities of phase space triangle

Third-order resonant extraction



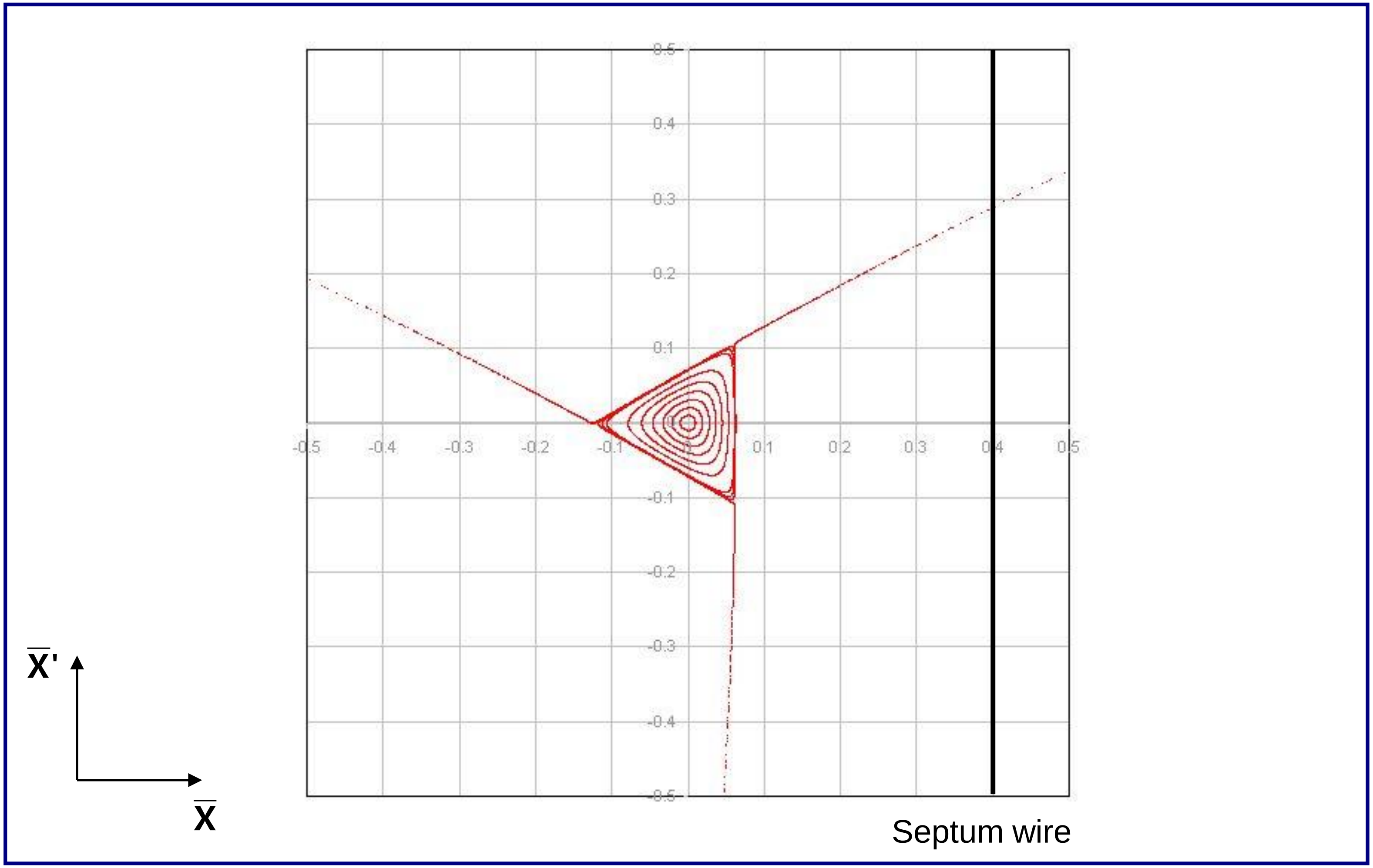
- ΔQ small enough that largest amplitude particle trajectories are unstable
- Unstable particles follow separatrix branches as they increase in amplitude

Third-order resonant extraction



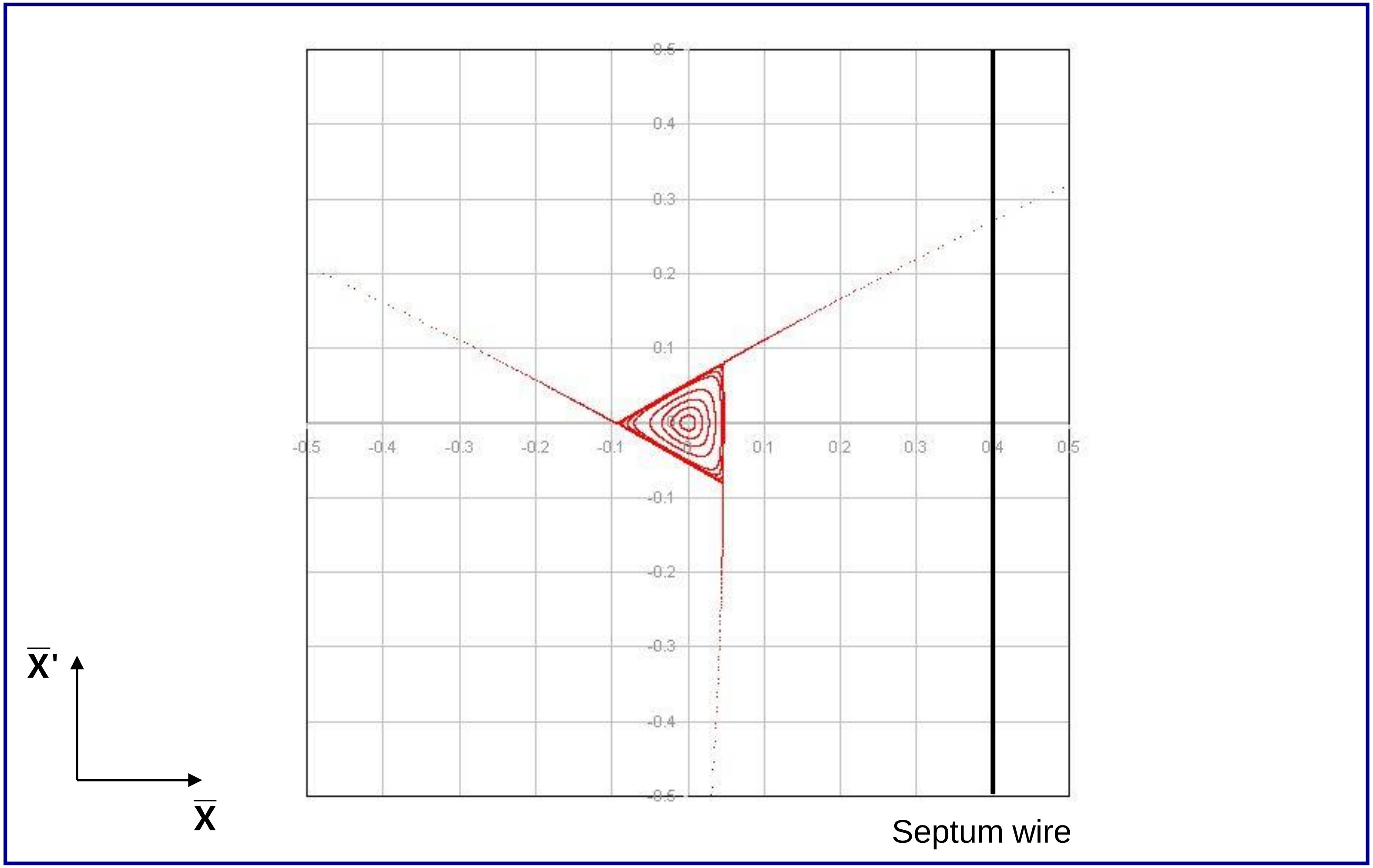
- Stable area shrinks as ΔQ becomes smaller

Third-order resonant extraction



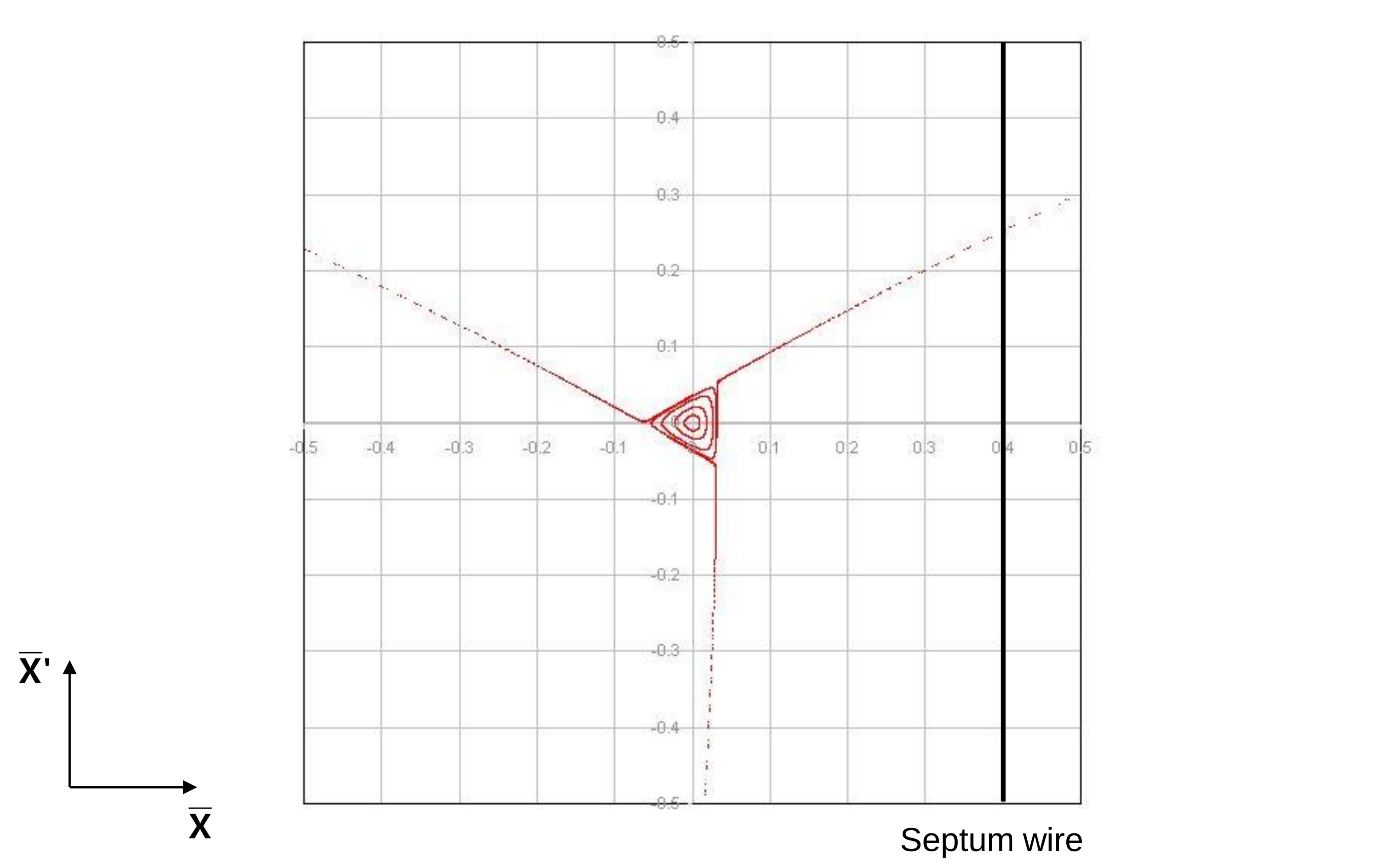
- Separatrix position in phase space shifts as the stable area shrinks

Third-order resonant extraction



- As the stable area shrinks, the circulating beam intensity drops since particles are being continuously extracted

Third-order resonant extraction



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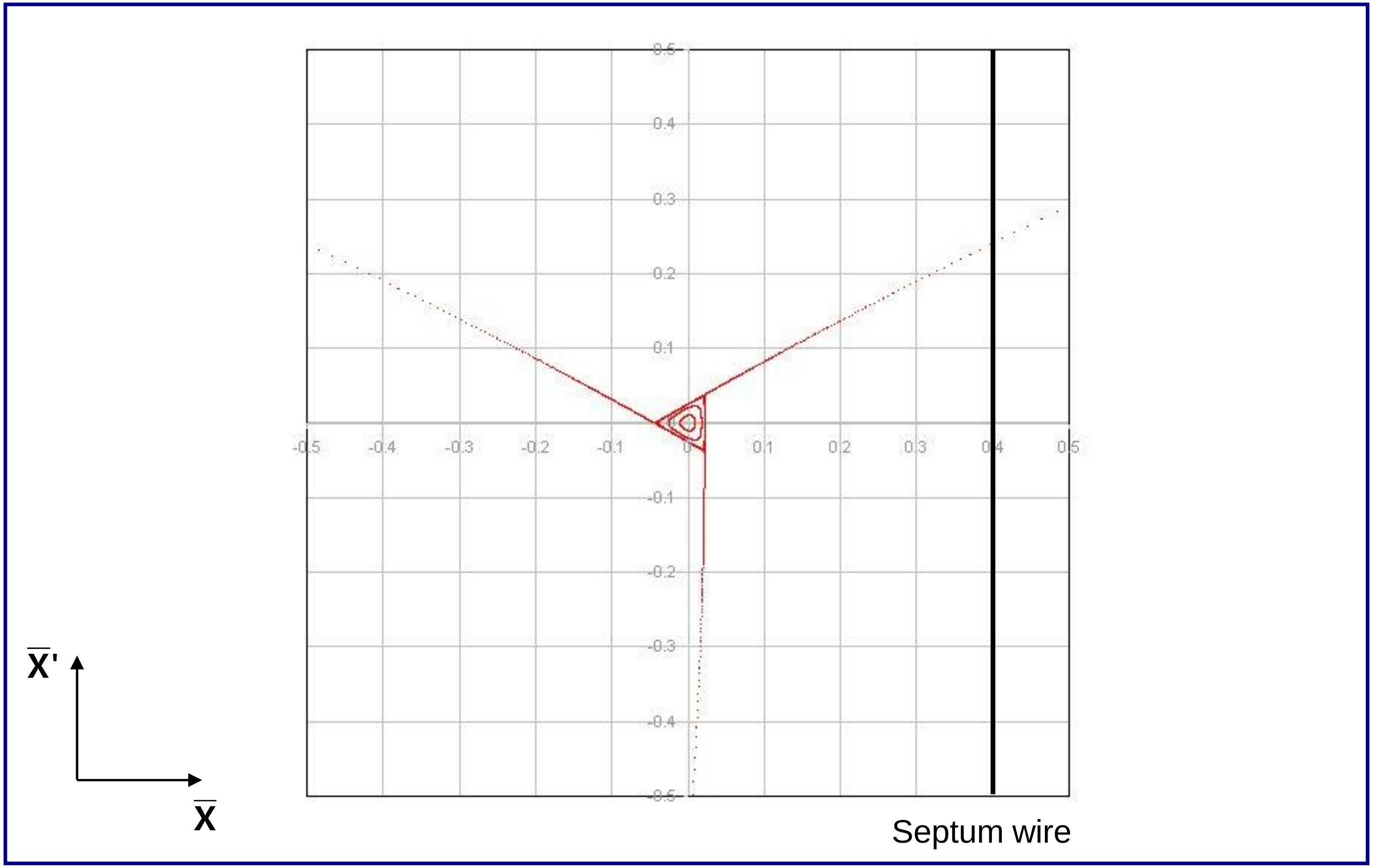
Third-order resonant extraction



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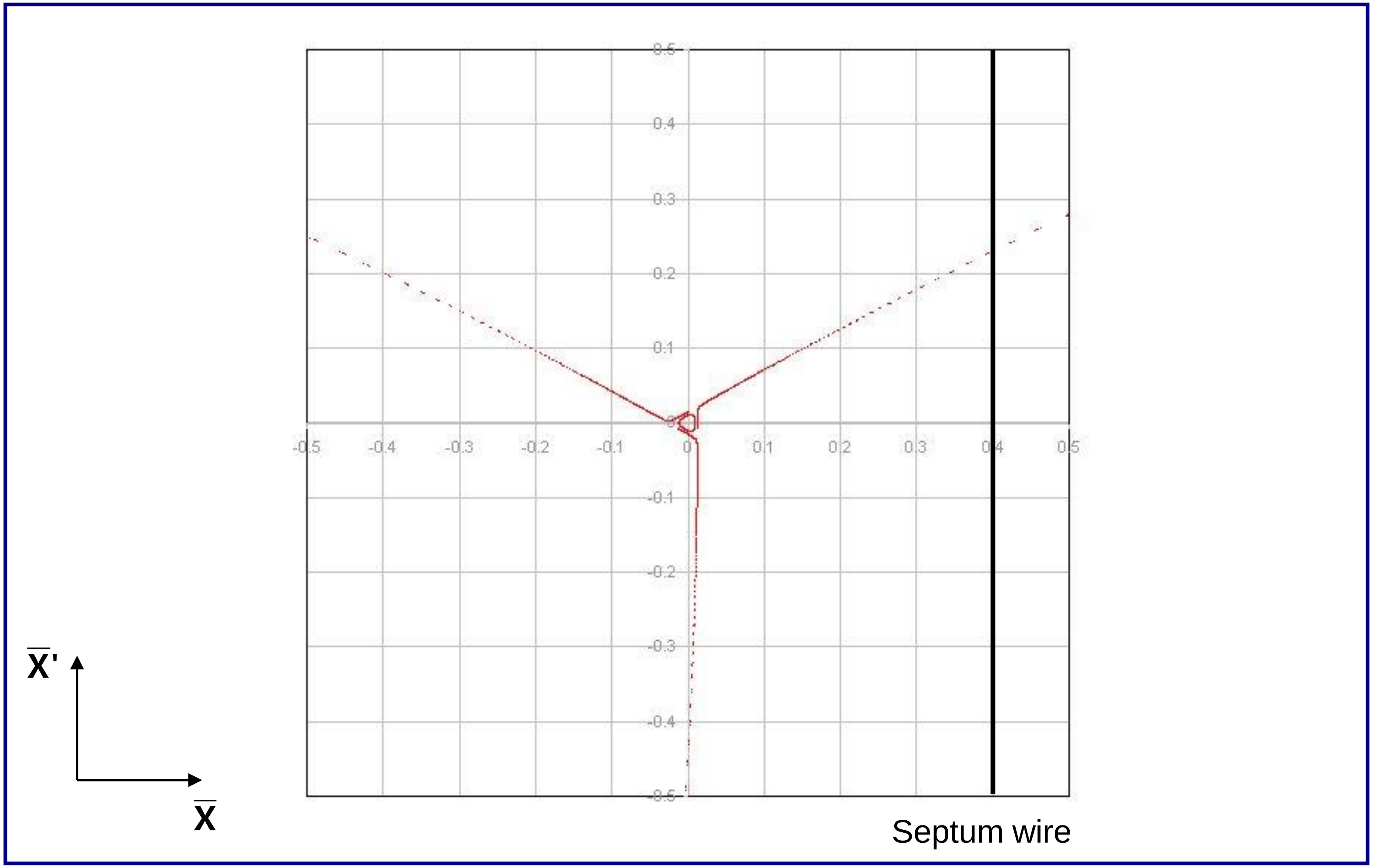
Third-order resonant extraction



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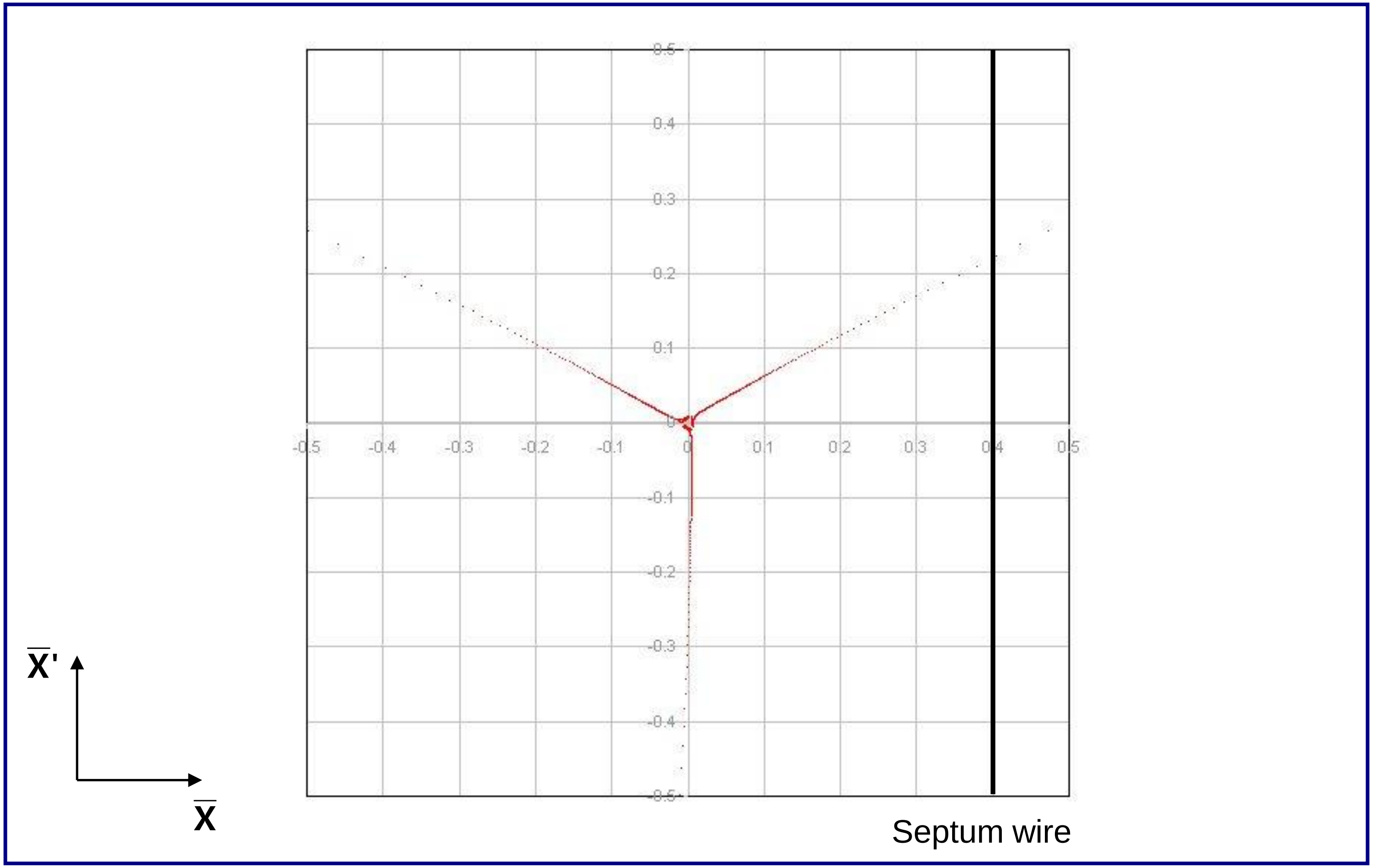
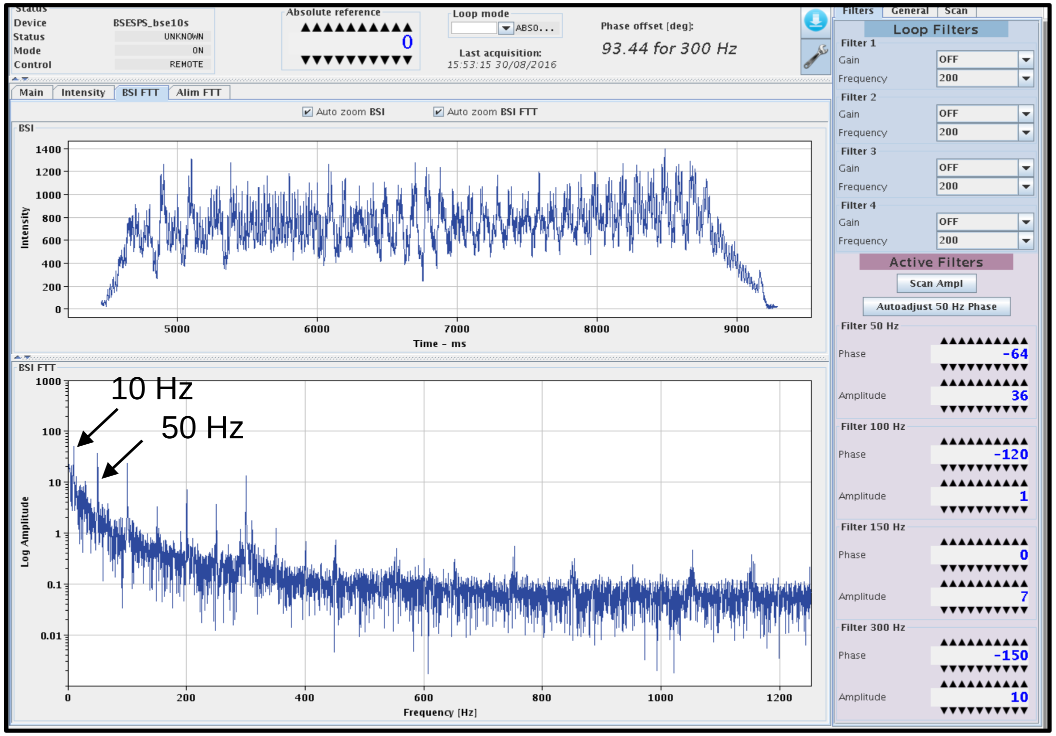
Third-order resonant extraction



- As ΔQ approaches zero, the particles with very small amplitude are extracted

Slow extracted spill quality

- The slow-extraction is a resonant process and it amplifies the smallest imperfections in the machine:
  - e.g. spill intensity variations can be explained by ripples in the current of the quads (mains: *n* x 50 Hz) at the level of a few ppm!
  - Injection of *n* x 50 Hz signals in counter-phase on dedicated quads can be used to compensate



An example of a spill at SPS to the North Area with large n x 50 Hz components and another noise source at 10 Hz

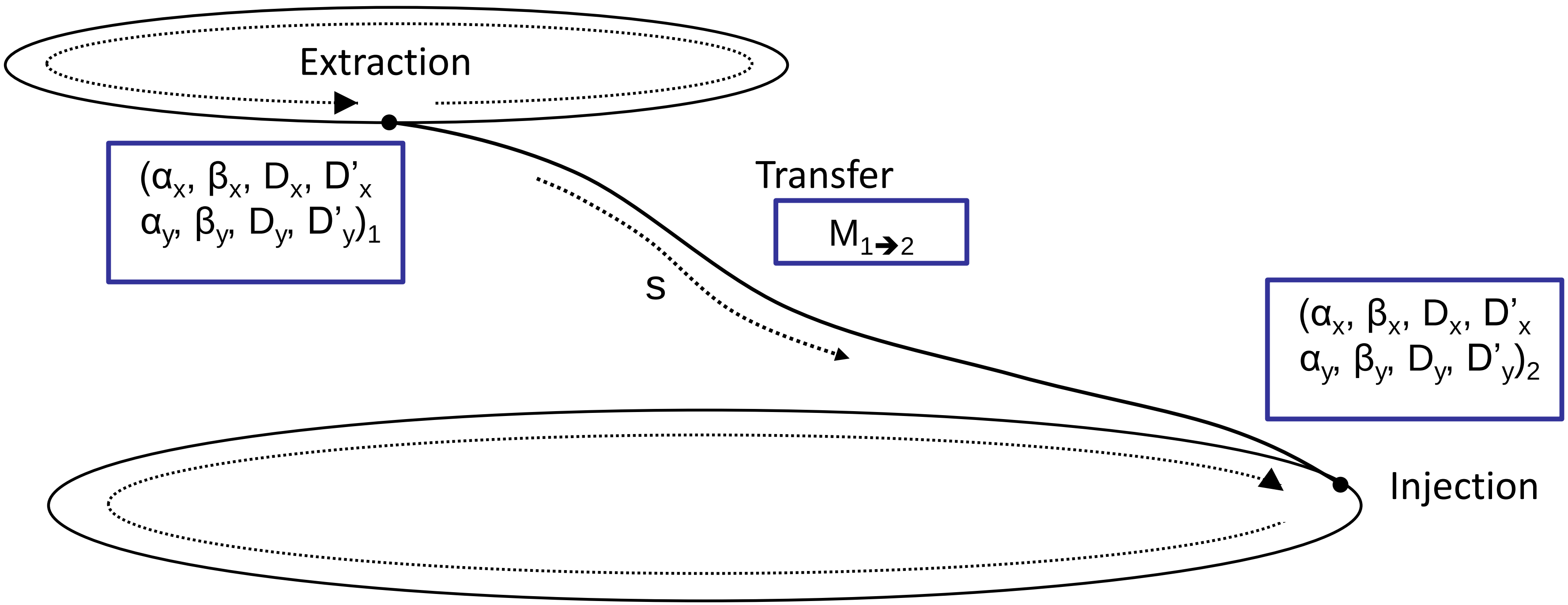
Extraction - summary

- Several different techniques:
  - **Single-turn fast extraction**:
    - for transfer between machines in accelerator chain, beam abort, etc.
  - **Non-resonant (fast) multi-turn extraction**
    - slice beam into equal parts for transfer between machine over a few turns.
  - **Resonant low-loss (fast) multi-turn extraction**
    - create stable islands in phase space: slice off over a few turns.
  - **Resonant (slow) multi-turn extraction**
    - create stable area in phase space $\Rightarrow$ slowly drive particles into resonance $\Rightarrow$ long spill over many thousand turns.

Linking Machines

1. **Extract** a beam out of one machine ➔ initial beam parameters
2. **Transport** this beam towards the following machine (or experiment)
3. **Inject** this beam into a following machine with a predefined optics

➔ Transfer line optics has to produce required beam parameters for matching



Linking Machines

- Beams have to be transported from extraction of one machine to injection of the next machine:
  - Trajectory must be matched in all 6 geometric degrees of freedom (x,y,z,θ,Φ,ψ)

- Linking the optics is a complicated process:
  - Parameters at start of line have to be propagated to matched parameters at the end of the line (injection to another machine, fixed target etc.)
  - Need to “match” 8 variables (α_x, β_x, D_x, D'_x and α_y, β_y, D_y, D'_y)
  - Done with number of independently power (“matching”) quadrupoles
  - Maximum β and D values are imposed by magnetic apertures
  - Other constraints exist:
    - Phase conditions for collimators
    - Insertions for special equipment like stripping foils
    - …
- Matching with computer codes and relying on mixture of theory, experience, intuition, trial and error.

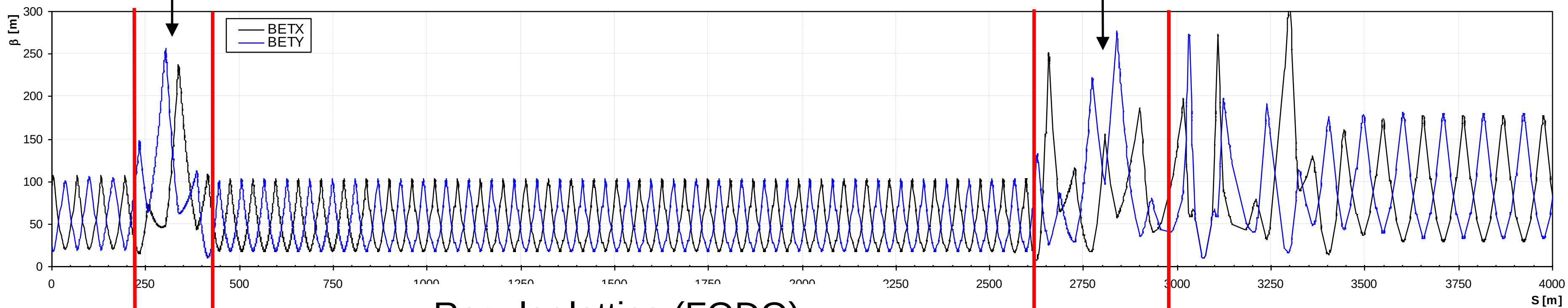
Optics Matching example

Initial matching section

Independently powered (tuneable) quadrupoles

Final matching section

Independently powered (tuneable) quadrupoles and (in this case) passive protection devices



Regular lattice (FODO)
(elements all powered in series
with same strengths)

SPS

LHC

SPS to LHC Transfer Line (3 km)

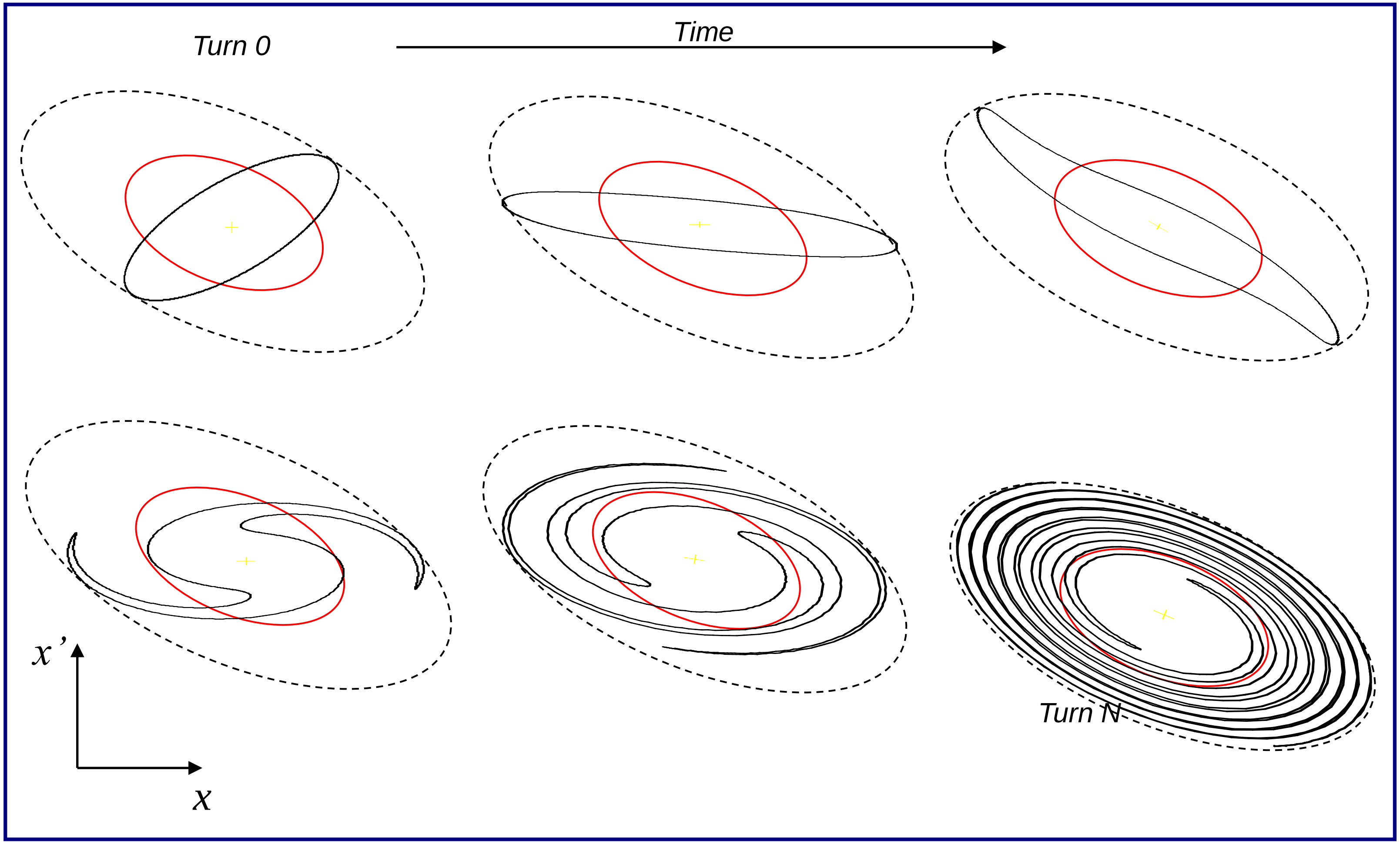
Extraction point

Injection point

Need to match 8 variables (TWISS parameters) for propagation from start to end of a transfer

Optical Mismatch at Injection

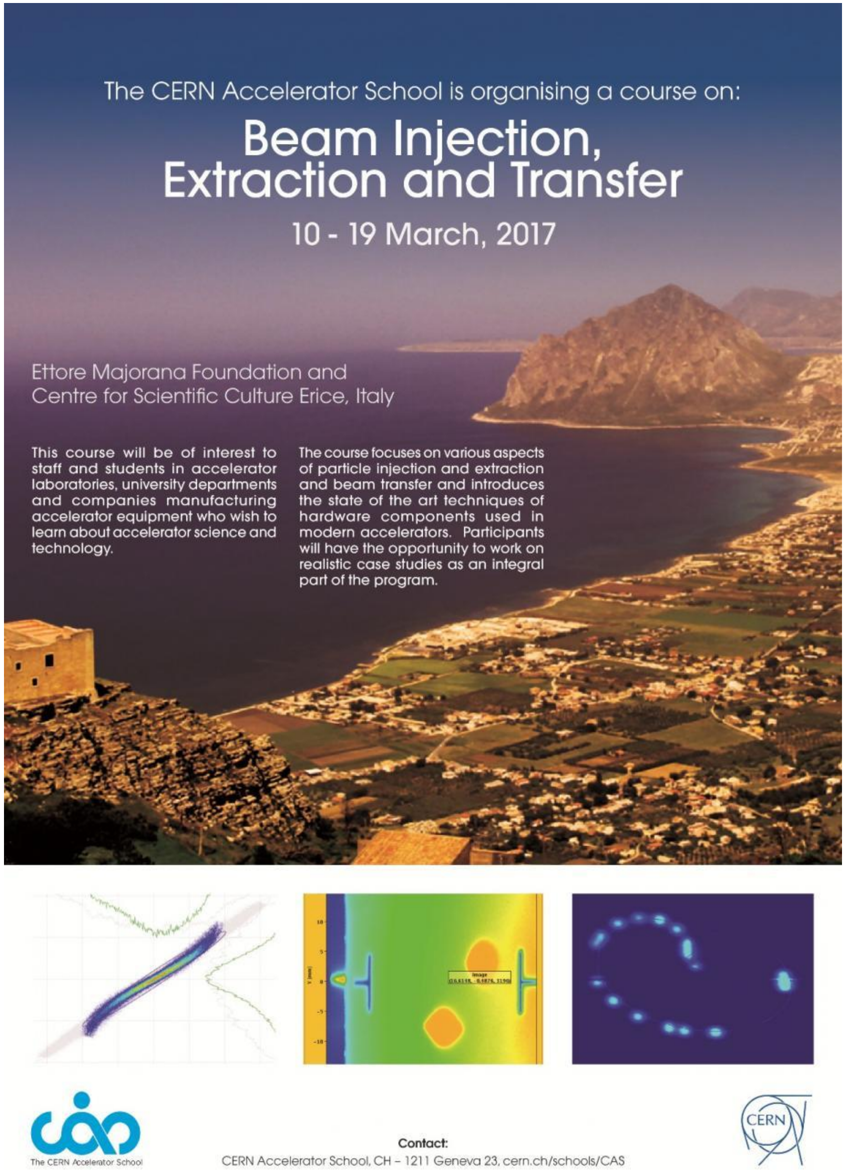
- Filamentation fills larger ellipse with same shape as matched ellipse



- Dispersion mismatch at injection will also cause emittance blow-up

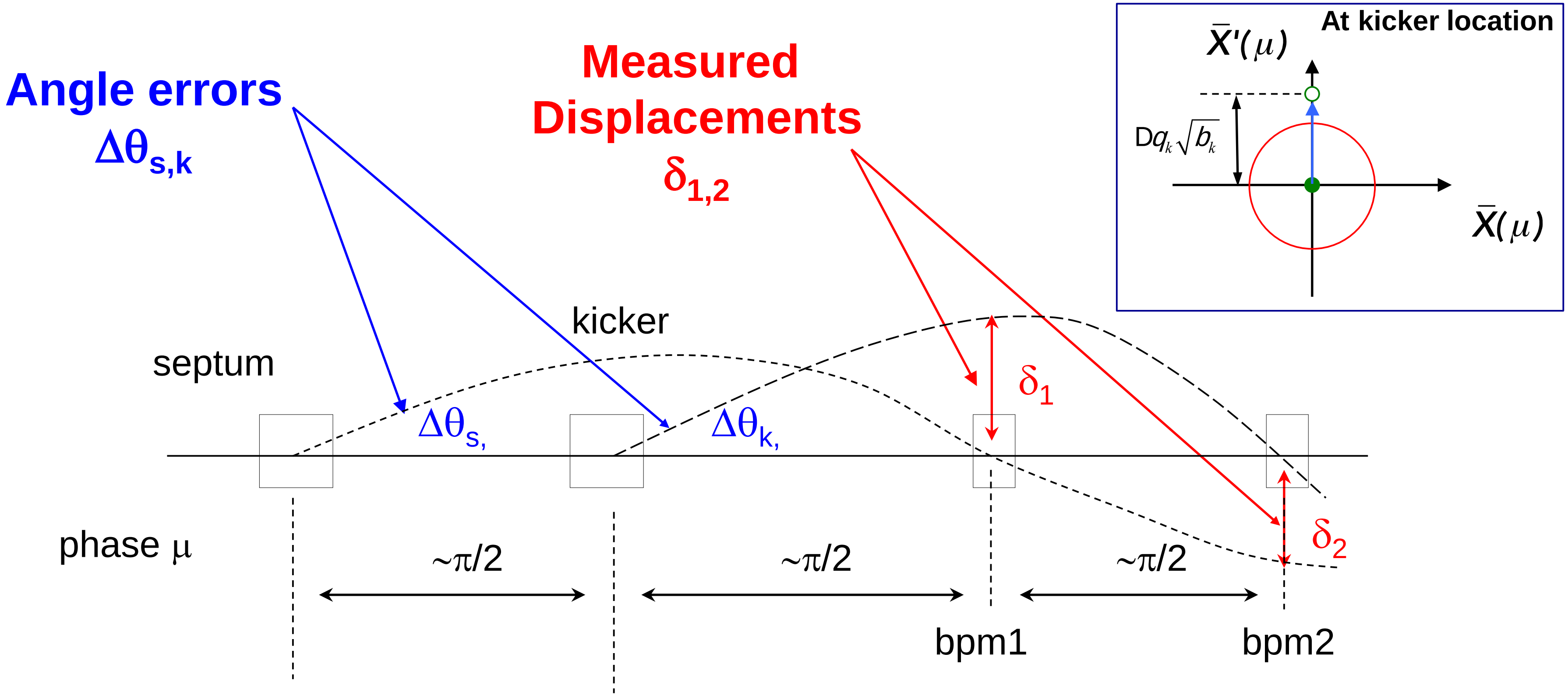
Further reading and references

- Lots of resources presented at the specialised CAS School:
- Beam Injection, Extraction and Transfer, 10-19 March 2017, Erice, Italy
- https://cas.web.cern.ch/schools/erice-2017

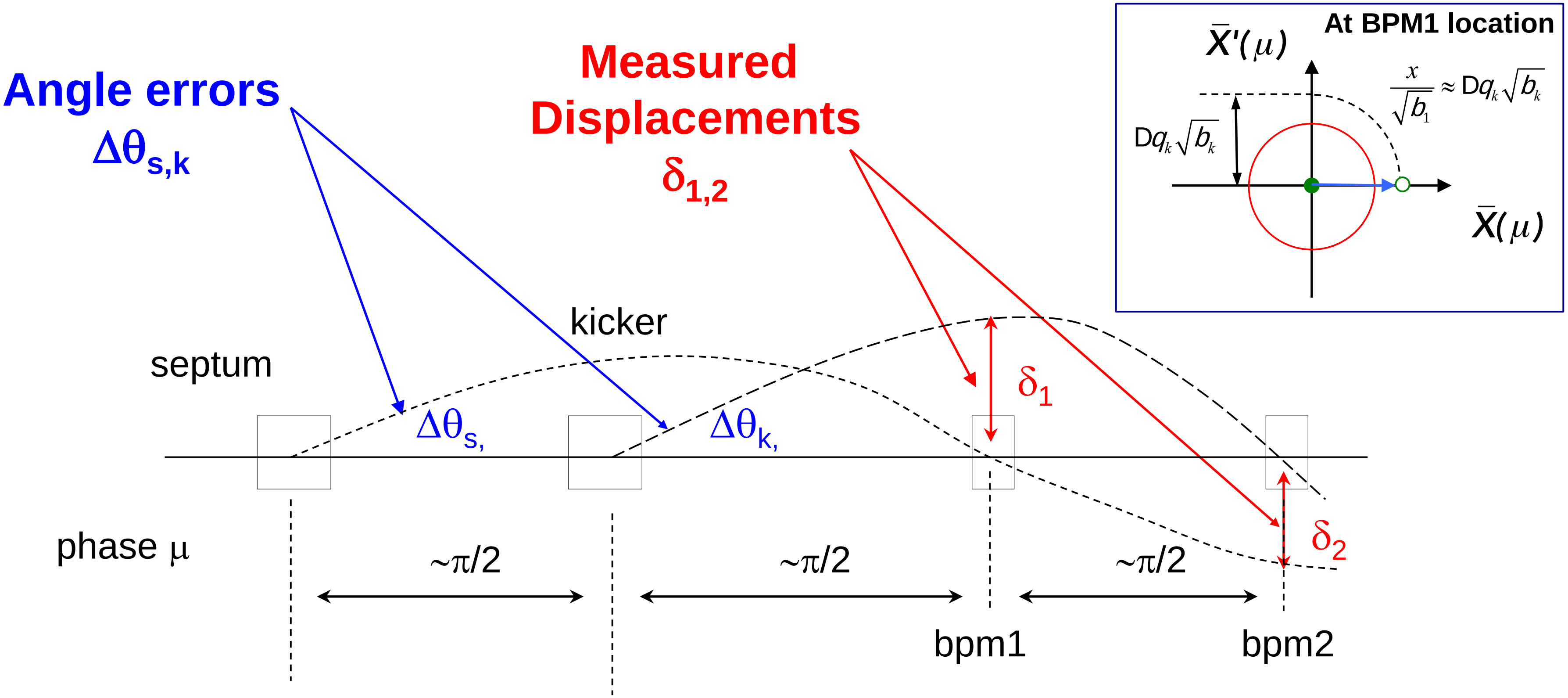


Appendix

Injection errors



Injection errors



$$\delta_1 = \Delta\theta_s \sqrt{(\beta_s\beta_1)} \sin(\mu_1 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_1)} \sin(\mu_1 - \mu_k)$$

$$\approx \Delta\theta_k \sqrt{(\beta_k\beta_1)}$$

$$\delta_2 = \Delta\theta_s \sqrt{(\beta_s\beta_2)} \sin(\mu_2 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_2)} \sin(\mu_2 - \mu_k)$$

$$\approx -\Delta\theta_s \sqrt{(\beta_s\beta_2)}$$

Blow-up from steering error

- The new particle coordinates in normalised phase space are:

$$\overline{X}_{error} = \overline{X}_0 + L\cos\theta$$

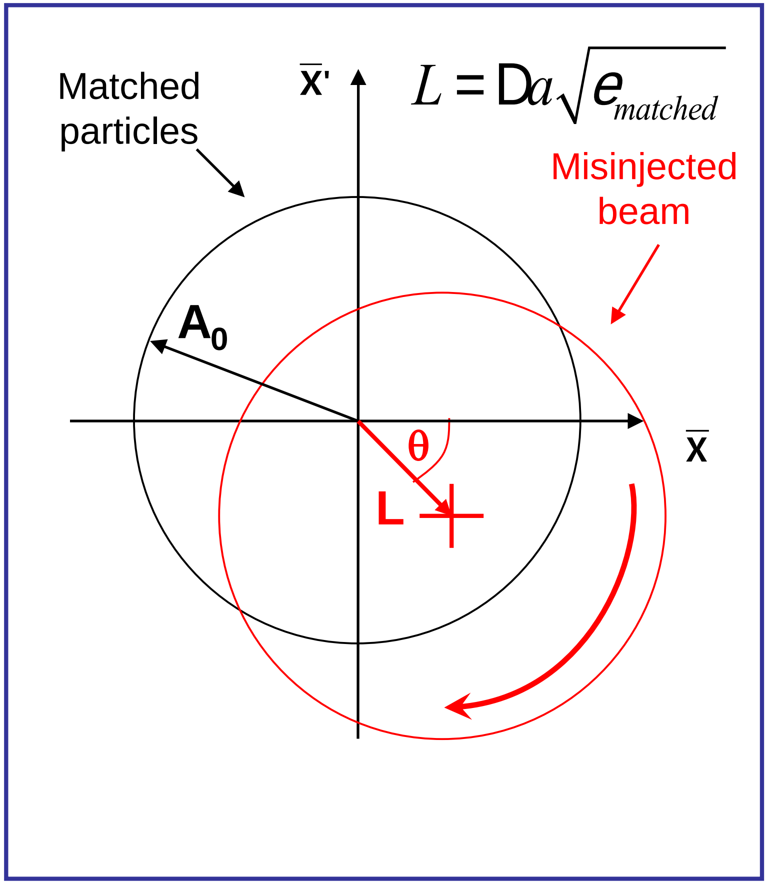
$$\overline{X}'_{error} = \overline{X}'_0 + L\sin\theta$$

- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$\boldsymbol{A}_i^2 = \overline{X}_{0,i}^2 + \overline{X}'^2_{0,i}$$

- The emittance of the distribution is:

$$\varepsilon_{matched} = \left\langle \boldsymbol{A}_i^2 \right\rangle / 2$$



Blow-up from steering error

- So we plug in the new coordinates:

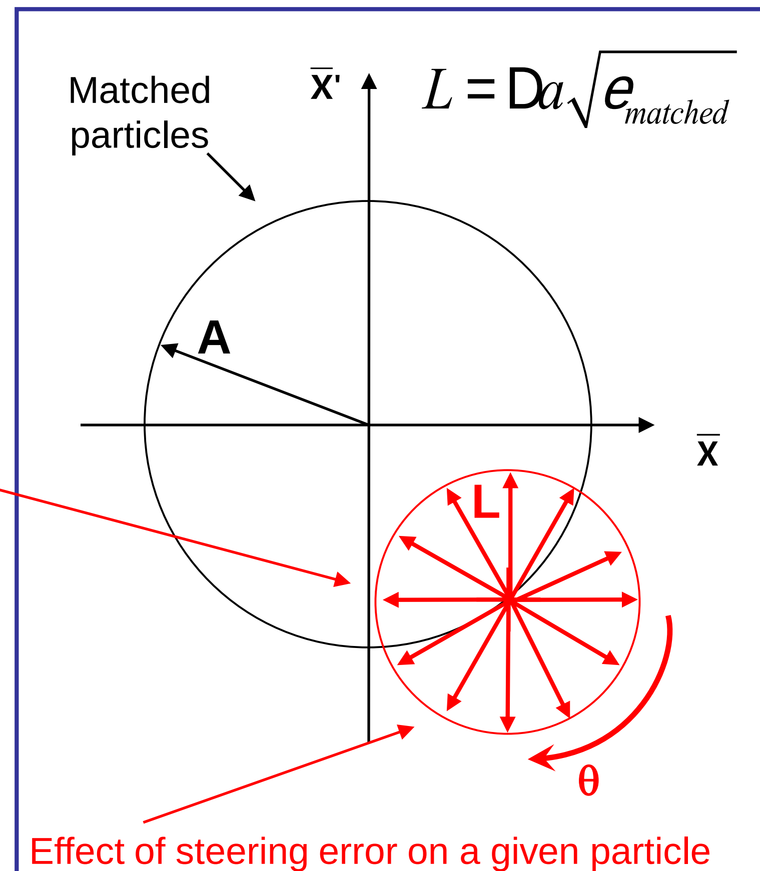
$$
\begin{aligned}
A^2_{error} &= \bar{X}^2_{error} + \bar{X}'^2_{error} \\
&= (\bar{X}_0 + L\cos\theta)^2 + (\bar{X}'_0 + L\sin\theta)^2 \\
&= \bar{X}^2_0 + \bar{X}'^2_0 + 2L(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta) + \overbrace{L^2}^{\cos^2\theta+\sin^2\theta=1}
\end{aligned}
$$

- Taking the average over distribution:

$$
\begin{aligned}
\langle A^2_{error}\rangle &= \langle A^2_0\rangle + 2L(\overbrace{\langle \bar{X}_0\cos\theta\rangle}^{0} + \overbrace{\langle \bar{X}'_0\sin\theta\rangle}^{0}) + \langle L^2\rangle \\
&= 2\varepsilon_{matched} + L^2
\end{aligned}
$$

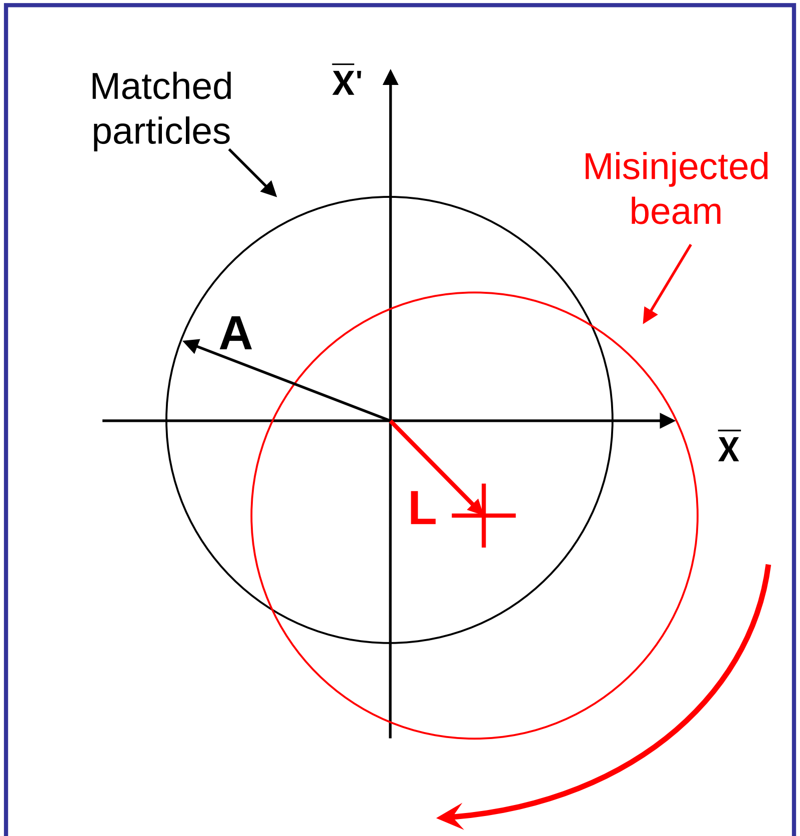
- Giving the diluted emittance as:

$$
\begin{aligned}
\varepsilon_{diluted} &= \varepsilon_{matched} + \frac{L^2}{2} \\
&= \varepsilon_{matched}\left[1 + \frac{\Delta\alpha^2}{2}\right]
\end{aligned}
$$



Blow-up from steering error

- Consider a collection of particles with max. amplitudes A
- The beam can be injected with an error in angle and position
- For an injection error Δa, in units of $\sigma = \sqrt{(\beta\varepsilon)}$, the mis-injected beam is offset in normalised phase space by an amplitude $L = \Delta a\sqrt{\varepsilon}$



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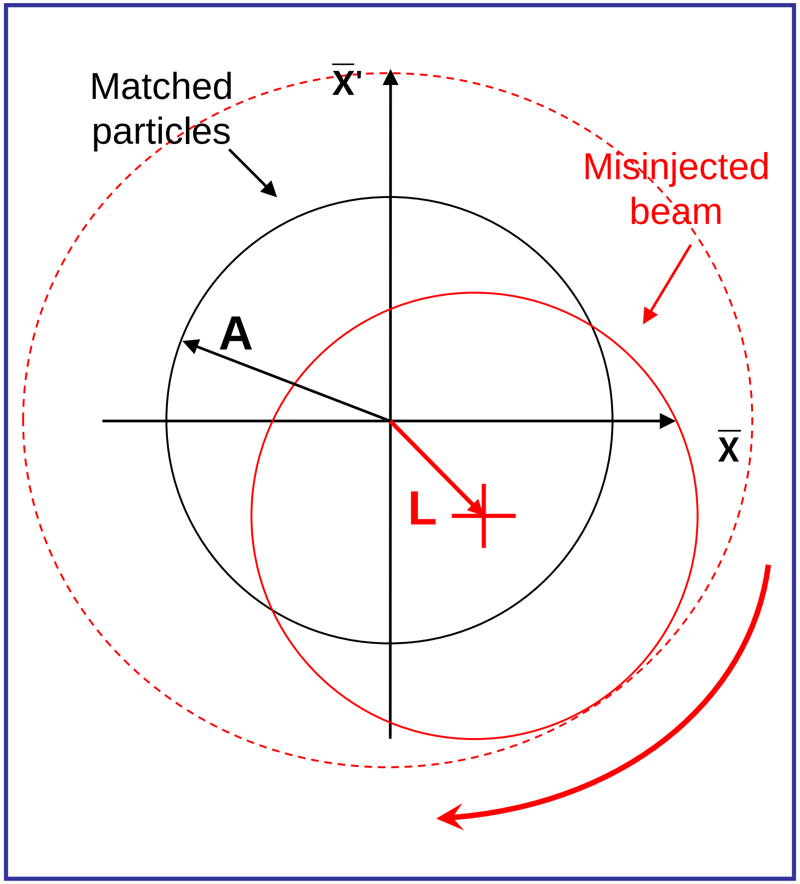

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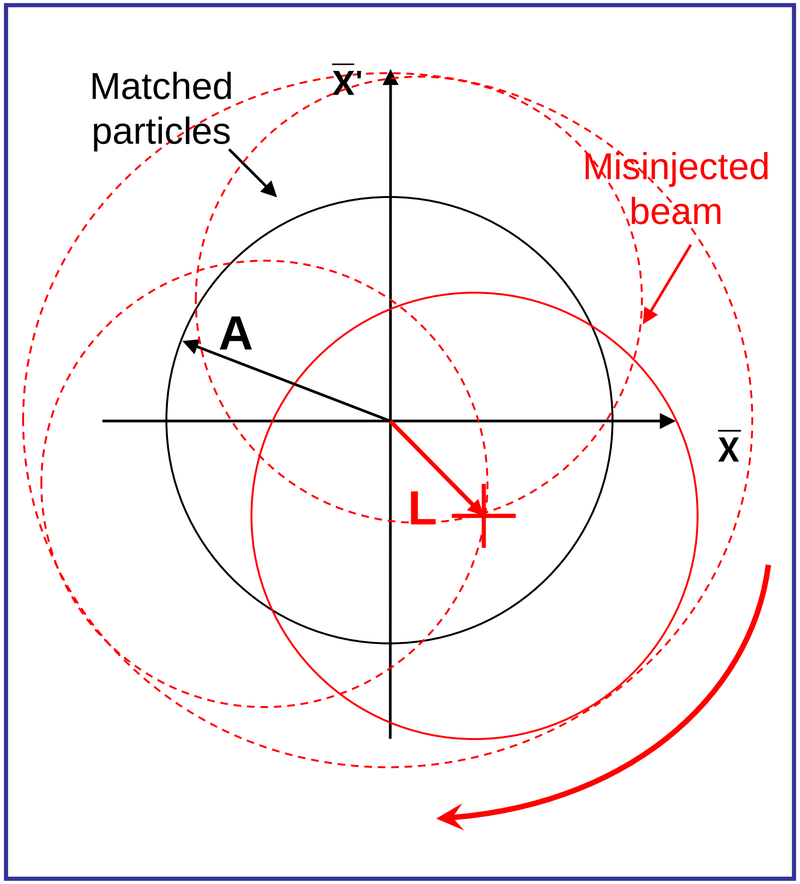

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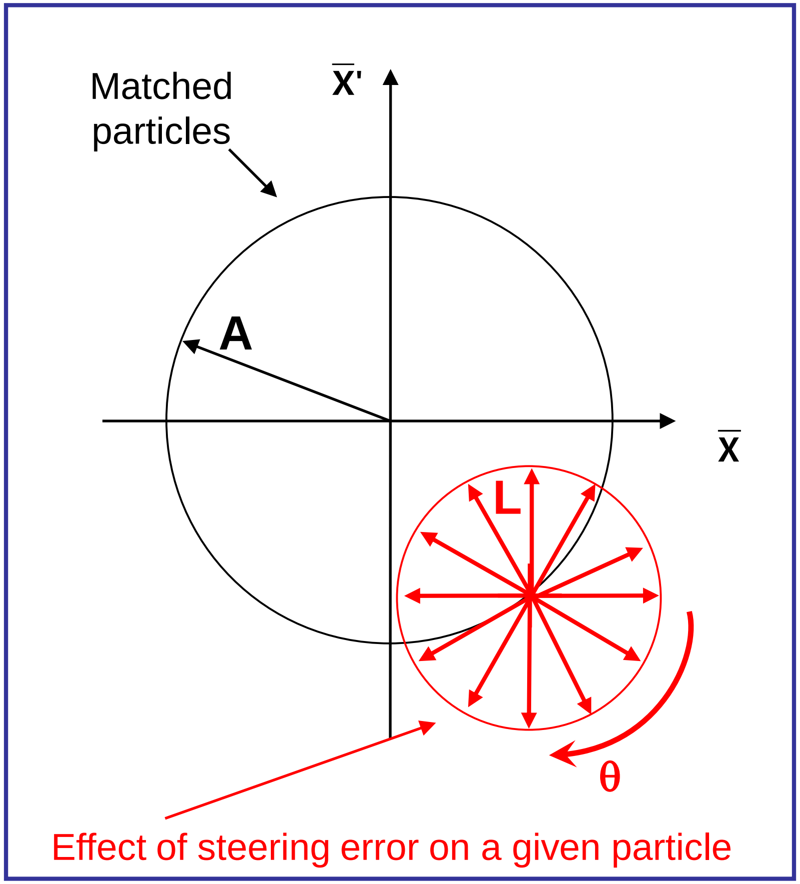
- For an injection error Δa, in units of $\sigma = \sqrt{(\beta\varepsilon)}$, the mis-injected beam is offset in normalised phase space by an amplitude $L = \Delta a\sqrt{\varepsilon}$
- Any given point on the matched ellipse is randomised over all phases after filamentation due to the steering error:



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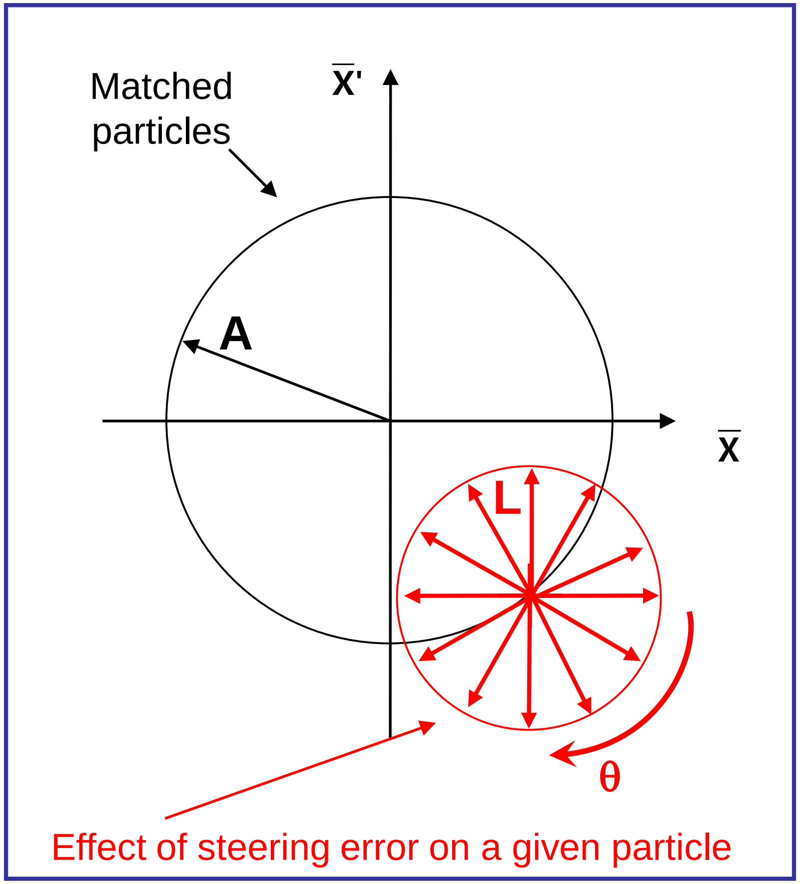
- Any given point on the matched ellipse is randomised over all phases after filamentation due to the steering error
- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$\varepsilon_{matched} = \left\langle A_i^2 \right\rangle / 2$$



Blow-up from steering error

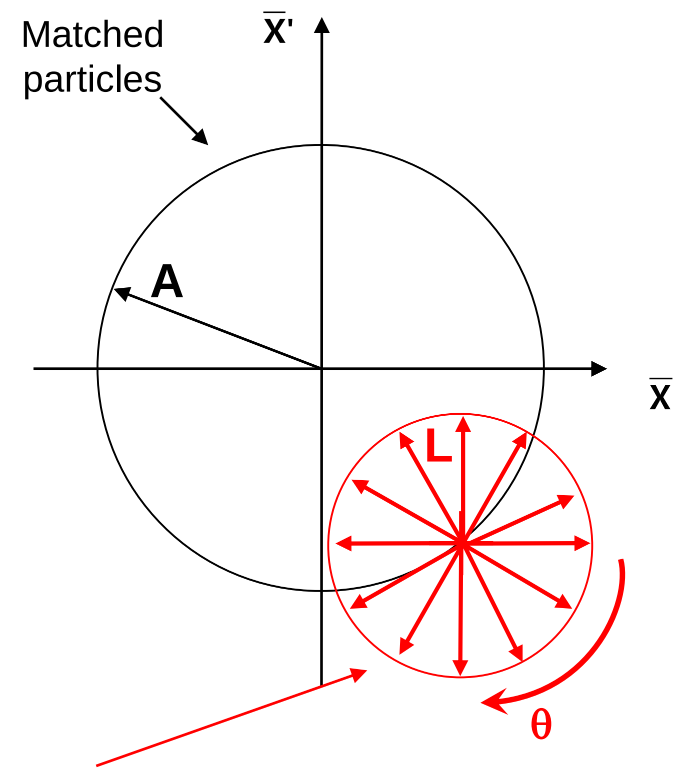
- Consider a collection of particles with max. amplitudes A
- The beam can be injected with an error in angle and position.
- For an injection error Δa, in units of σ = √(βε), the mis-injected beam is offset in normalised phase space by an amplitude L = Δa√ε
- Any given point on the matched ellipse is randomised over all phases after filamentation due to the steering error
- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$\varepsilon_{matched} = \left\langle A_i^2 \right\rangle / 2$$

- After filamentation:

$$e_{diluted} = e_{matched} + \frac{L^2}{2}$$

See appendix for derivation



Blow-up from steering error

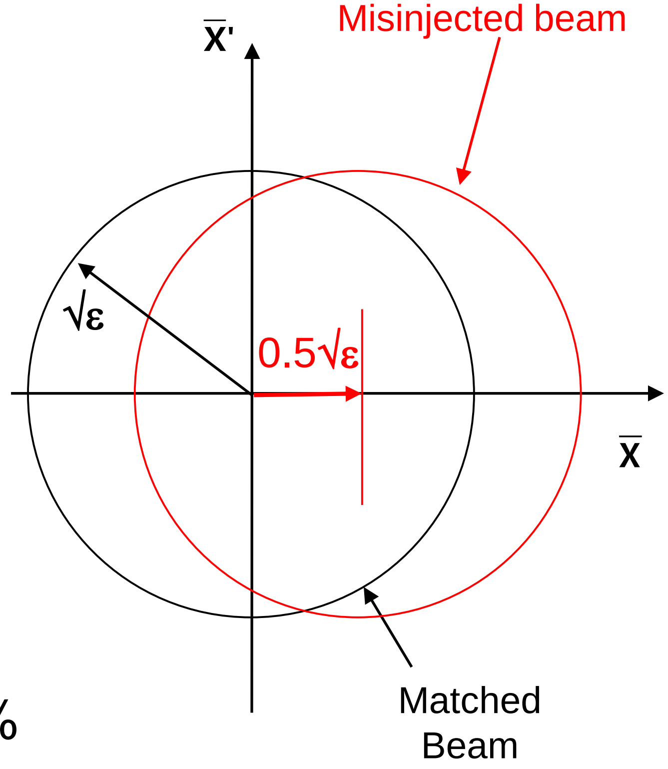
- A numerical example….
- Consider an offset Δa = 0.5σ for injected beam:

$$L = \Delta a \sqrt{\varepsilon_{matched}}$$

$$\varepsilon_{diluted} = \varepsilon_{matched} + \frac{L^2}{2}$$

$$= \varepsilon_{matched}\left[1 + \frac{\Delta a^2}{2}\right]$$

$$= \varepsilon_{matched}\left[1.125\right]$$

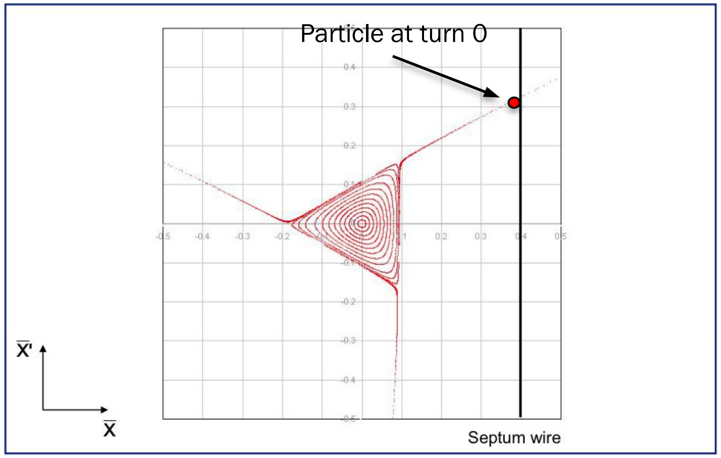


- For nominal LHC beam:

…allowed growth through LHC cycle ~10 %

Third-order resonant extraction

- On resonance, sextupole kicks add-up driving particles over septum



Third-order resonant extraction

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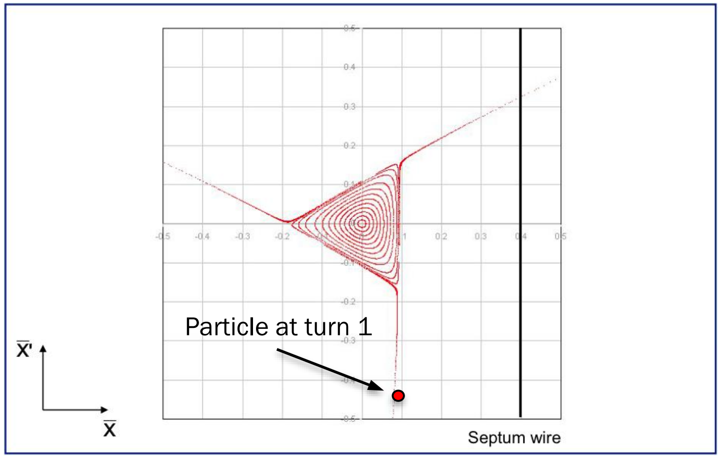

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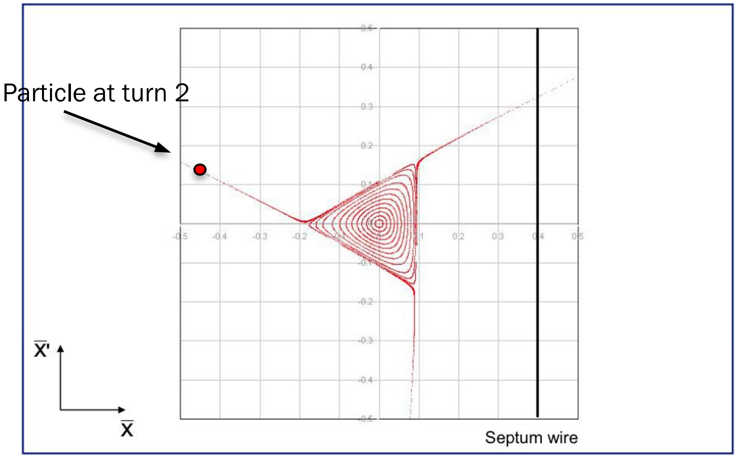

Third-order resonant extraction

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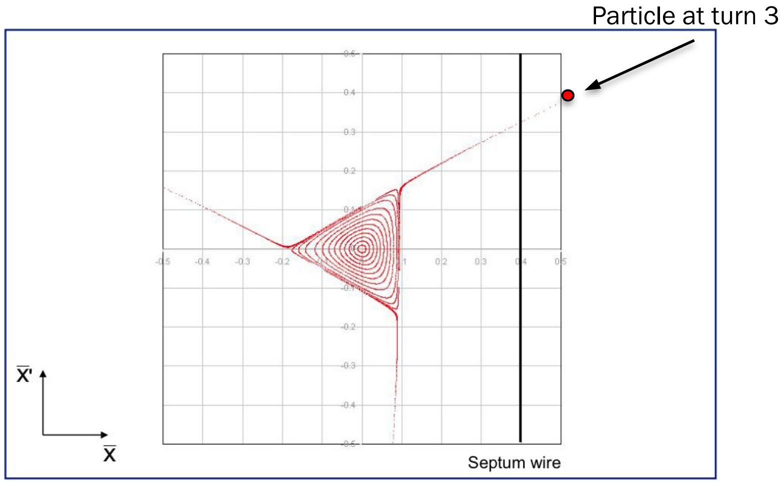
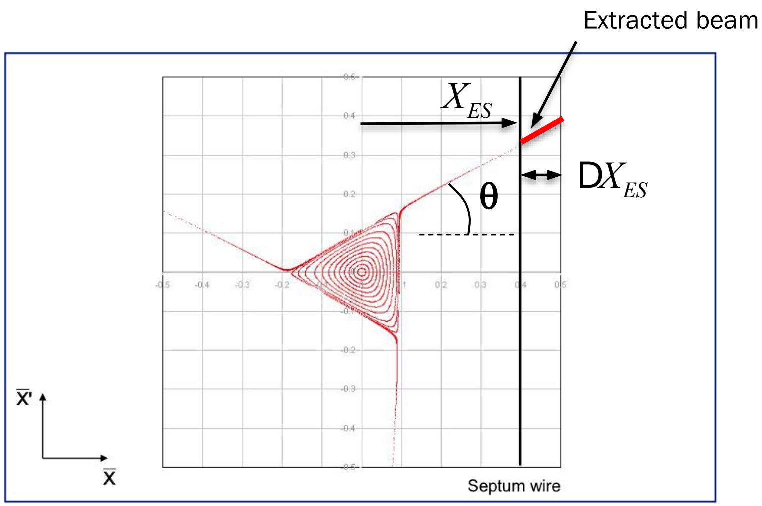


Third-order resonant extraction

- On resonance, sextupole kicks add-up driving particles over septum
  - Distance travelled in these final three turns is termed the "spiral step," ΔX_{ES}
  - Extraction bump trimmed in the machine to adjust the spiral step



$$\Delta X_{ES} \propto |k_2| \frac{X_{ES}^2}{\cos\theta}$$

Third-order resonant extraction

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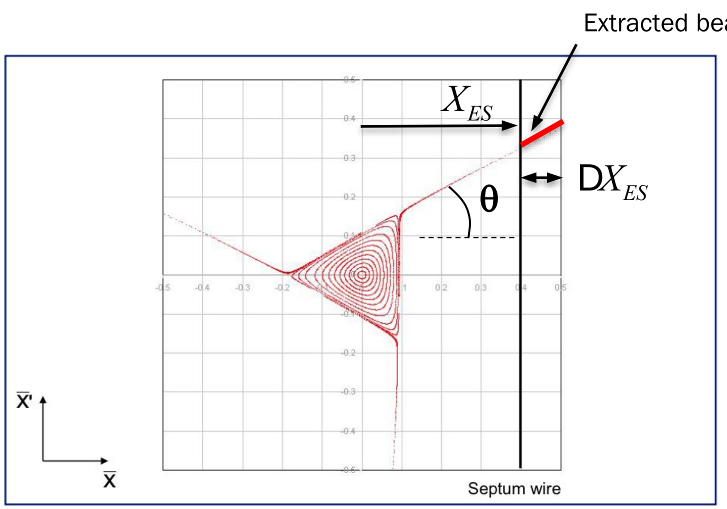
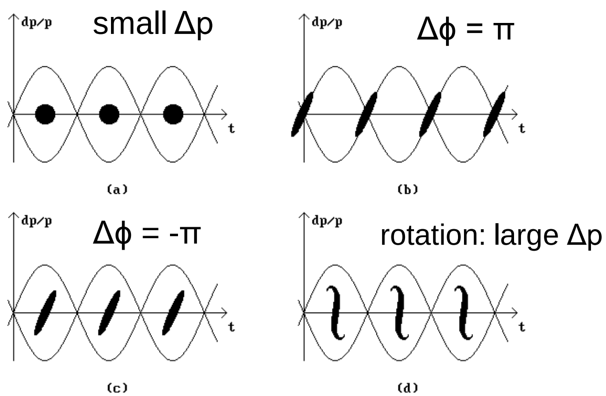
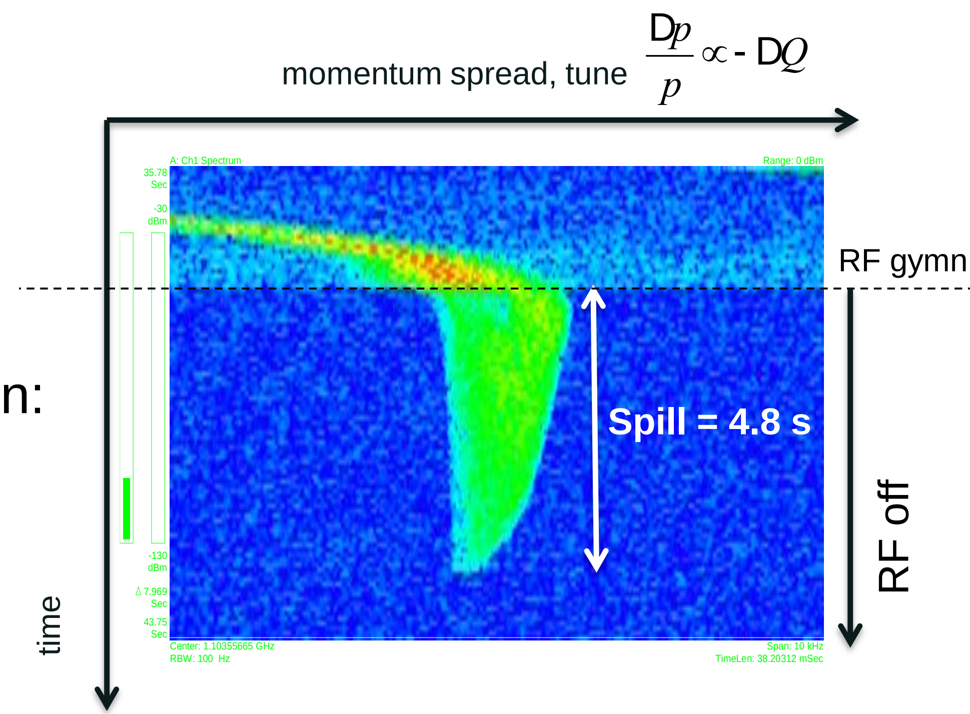


$$\Delta X_{ES} \propto |k_2| \frac{X_{ES}^2}{\cos\theta}$$

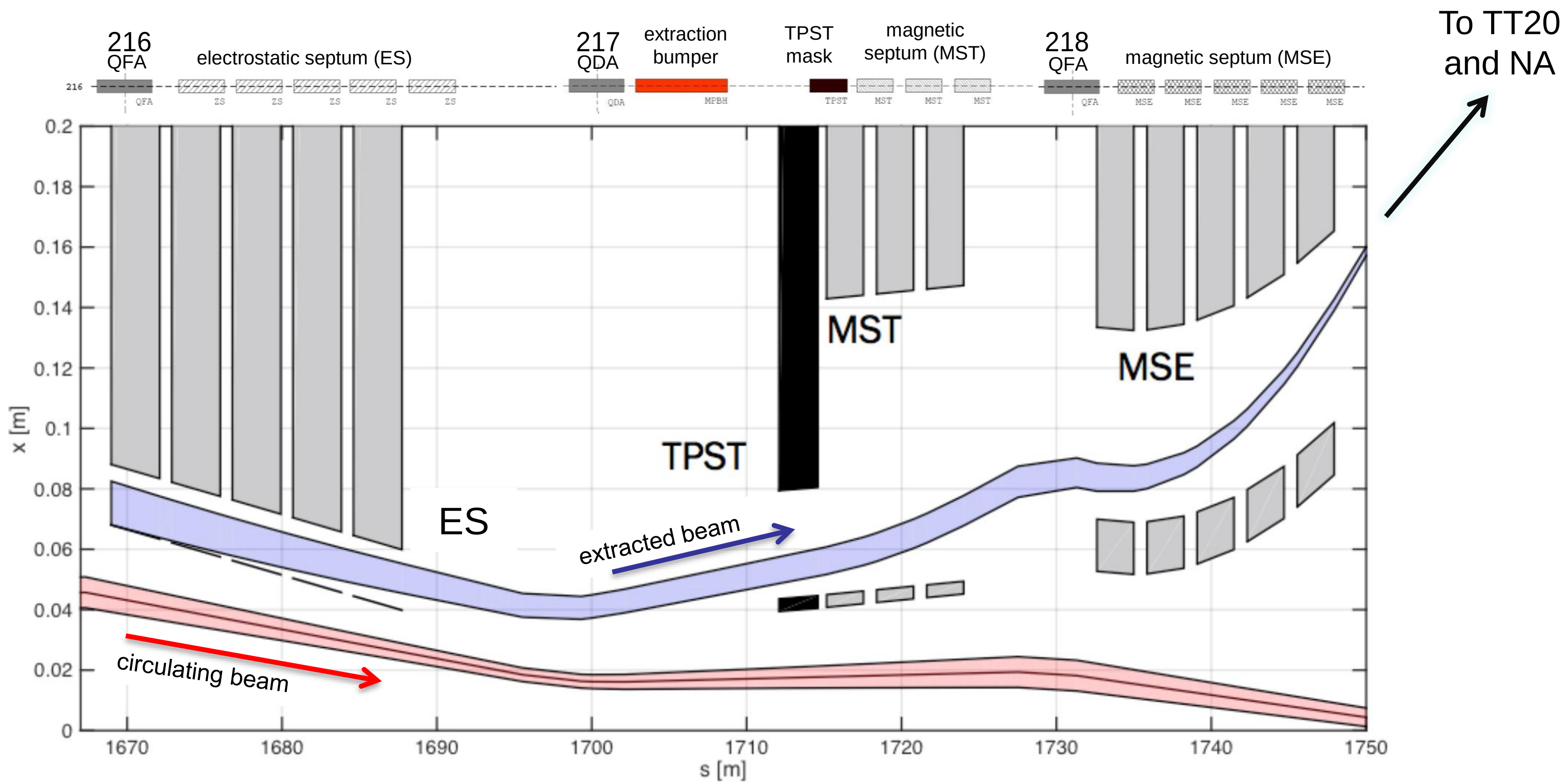
- RF gymnastics before extraction:





Schottky measurement during spill, courtesy of T. Bohl

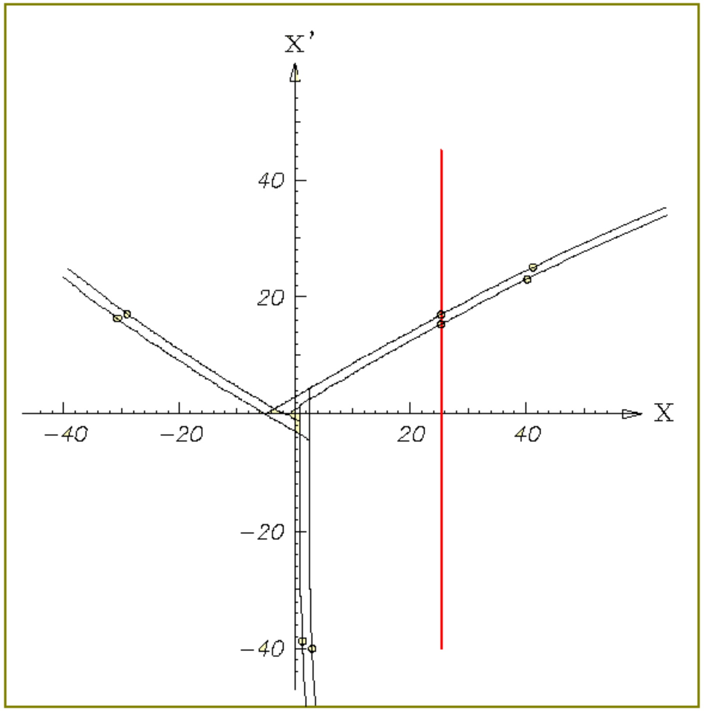
Slow extraction channel: SPS



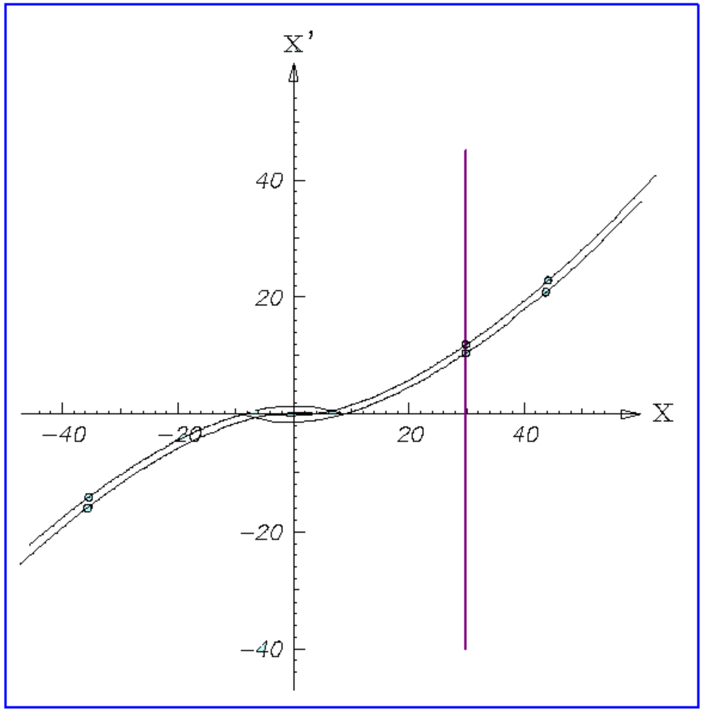
Second-order resonant extraction

- An extraction can also be made over a few hundred turns
- 2nd and 4th order resonances
  - Octupole fields distort the regular phase space particle trajectories
  - Stable area defined, delimited by two unstable Fixed Points
  - Beam tune brought across a 2nd order resonance ($Q \rightarrow 0.5$)
  - Particle amplitudes quickly grow and beam is extracted in a few hundred turns

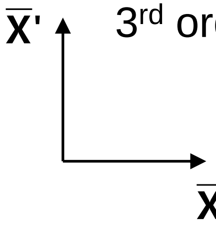
Resonant extraction separatrices



3rd order resonant extraction



2nd order resonant extraction



- Amplitude growth for 2nd order resonance much faster than 3rd – shorter spills (≈milliseconds vs. seconds)
- Used where intense pulses are required on target – e.g. neutrino production