Bethe Center for Theoretical Physics

UNIVERSITÄT BONN

The dawn of N^3LO

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based on work in collaboration with Julien Baglio, Falko Dulat, Bernhard Mistlberger and Robert Szafron.

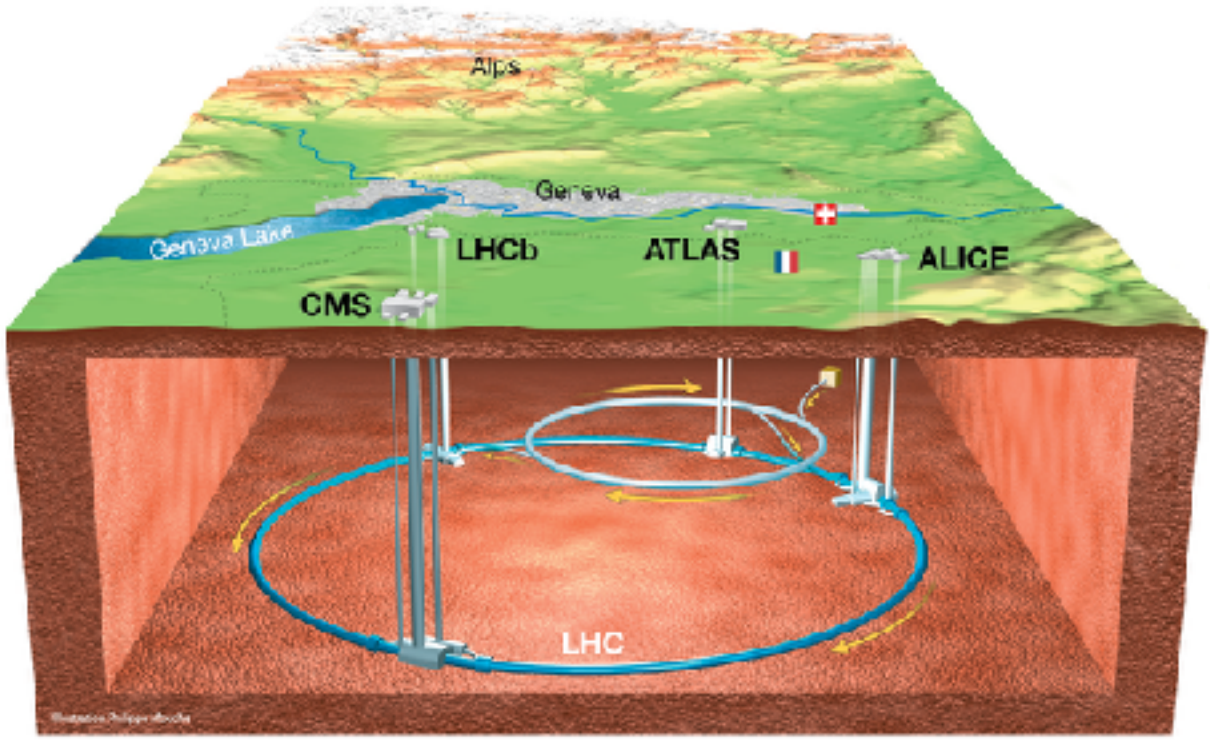
CERN TH Colloquium
09 March 2022

QCD at the LHC

- The Run III of the LHC is about to start!

  ➡ Huge increase in statistics.

  ➡ Experimentally, we may reach a precision of 1% (e.g., luminosity, JES, ..)



We need to make sure that our theory predictions also reach this standard of 1%!

➡ It's time to critically assess and revisit our theory tools.

QCD at the LHC

- Standard approach to LHC computations: QCD factorisation

$$\mathrm{d}\sigma_{pp\to X} = \sum_{i,j}\int_0^1 \mathrm{d}x_1\,\mathrm{d}x_2\, f_i(x_1)\, f_j(x_2)\, \mathrm{d}\hat{\sigma}_{ij\to X} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

Sum over parton species

Parton density functions (PDFs)

Partonic cross sections

Higher-twist effects

- The partonic cross sections are expanded in perturbation theory:

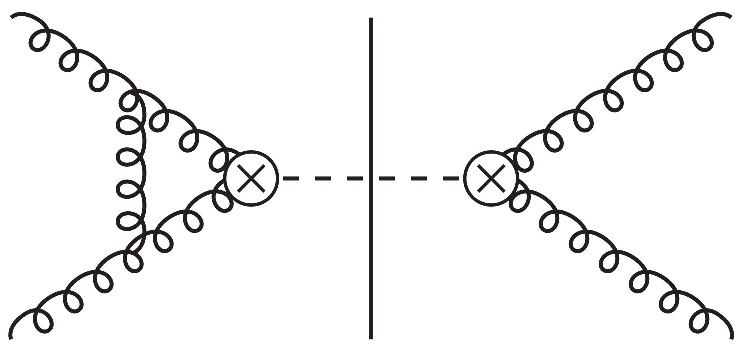
$$\mathrm{d}\hat{\sigma}_{ij\to X} = \mathrm{d}\hat{\sigma}_0 + \alpha_s(\mu_R)\,\mathrm{d}\hat{\sigma}_1 + \alpha_s(\mu_R)^2\,\mathrm{d}\hat{\sigma}_2 + \ldots$$

$$\alpha_s(m_Z) \simeq 0.118$$

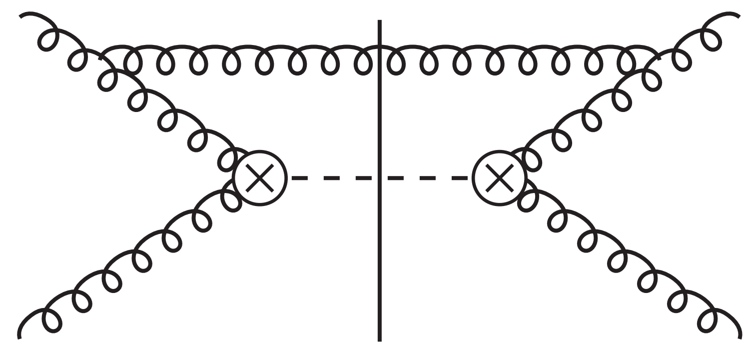
- Naive counting: NLO $\longrightarrow$ 10% NNLO $\longrightarrow$ 1%

NLO & NNLO

- The NLO cross section:

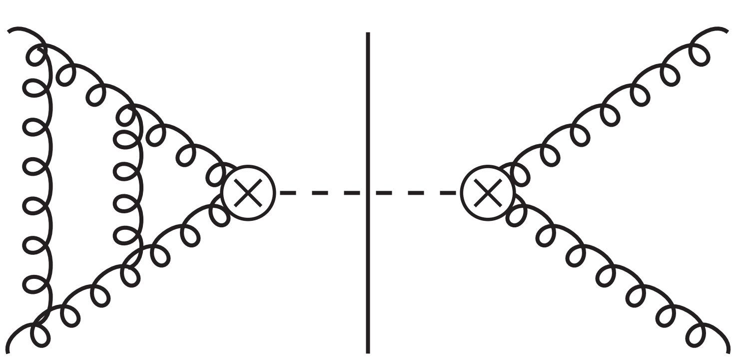


Virtual corrections ('loops')

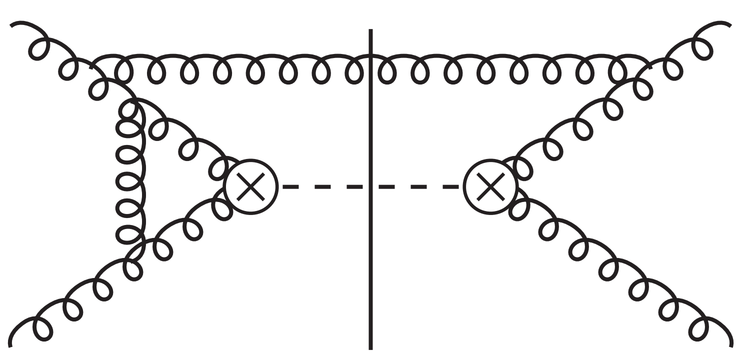


Real emission

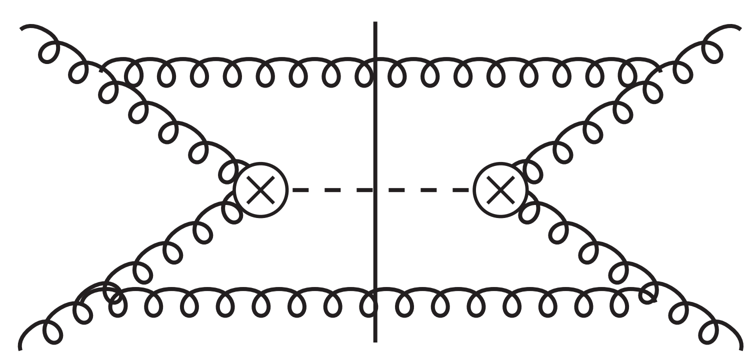
- The NNLO cross section:



Double virtual



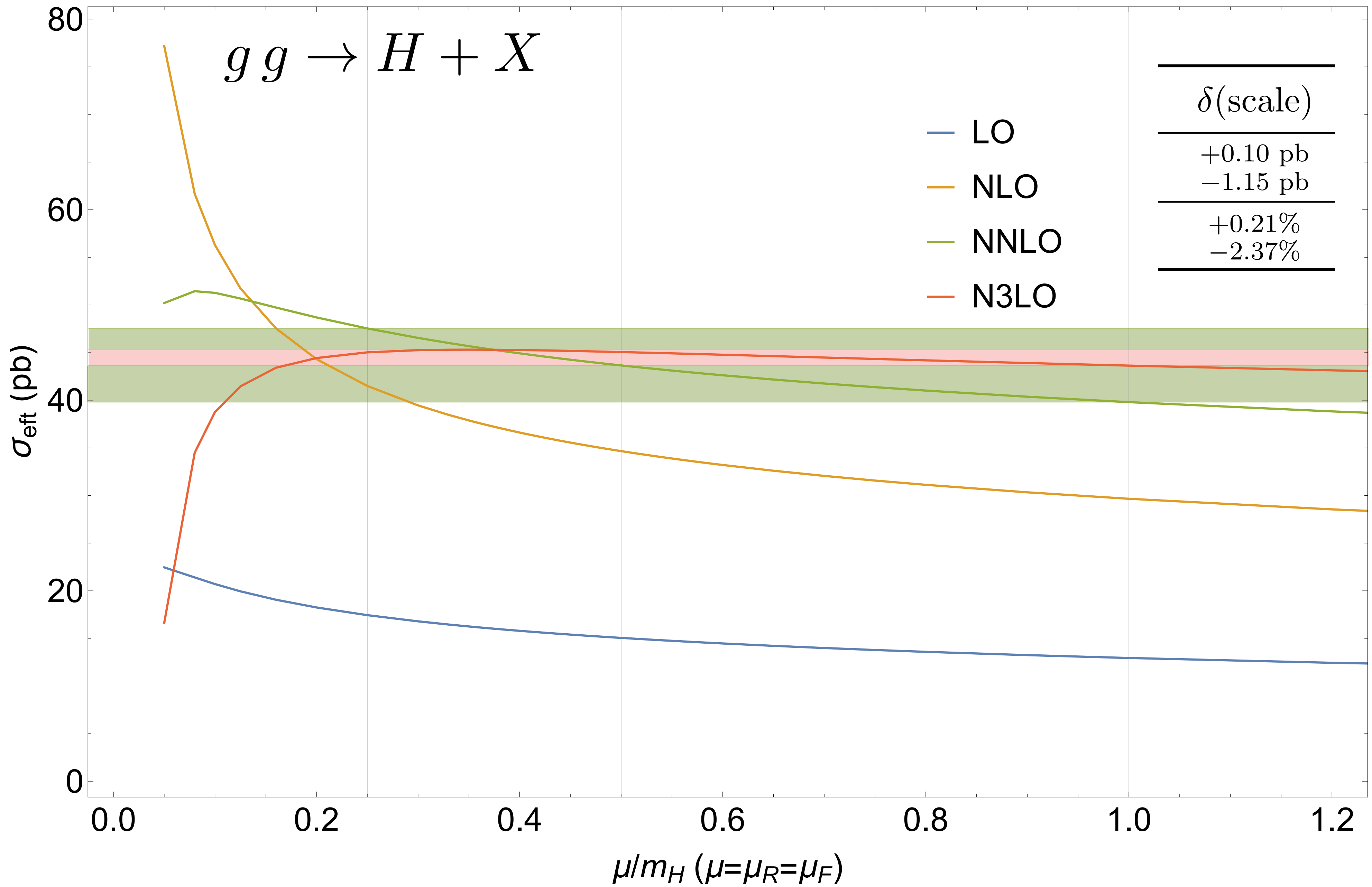
Real-virtual



Double real

The dawn of N^3LO

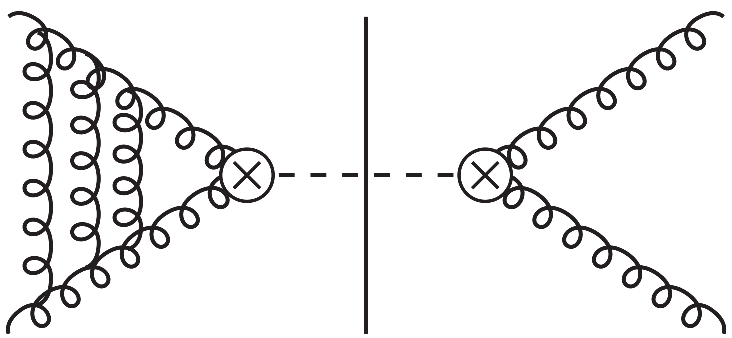
- Naive counting: NLO $\longrightarrow$ 10% NNLO $\longrightarrow$ 1%
  - ➡ Sometimes the naive counting fails!



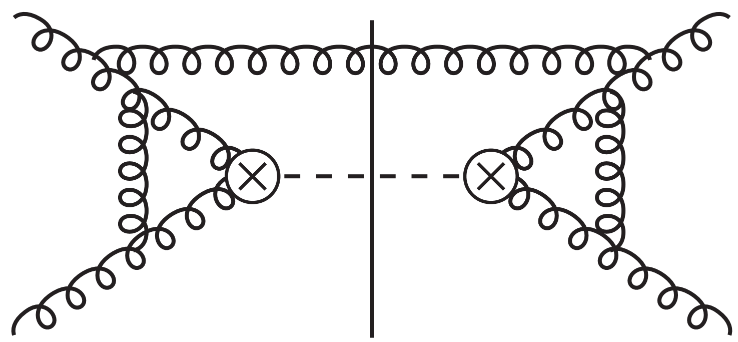
[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger]

The dawn of N^{3}LO

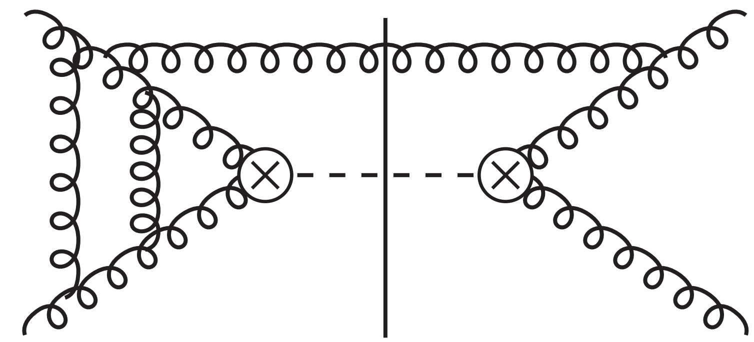
- The N3LO cross section:



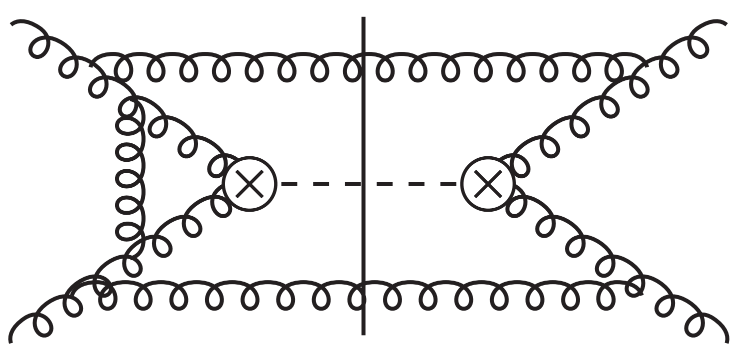
Triple virtual



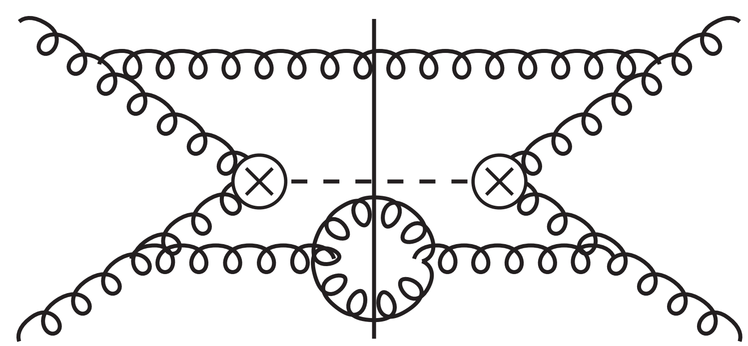
Real-virtual squared



Double virtual real

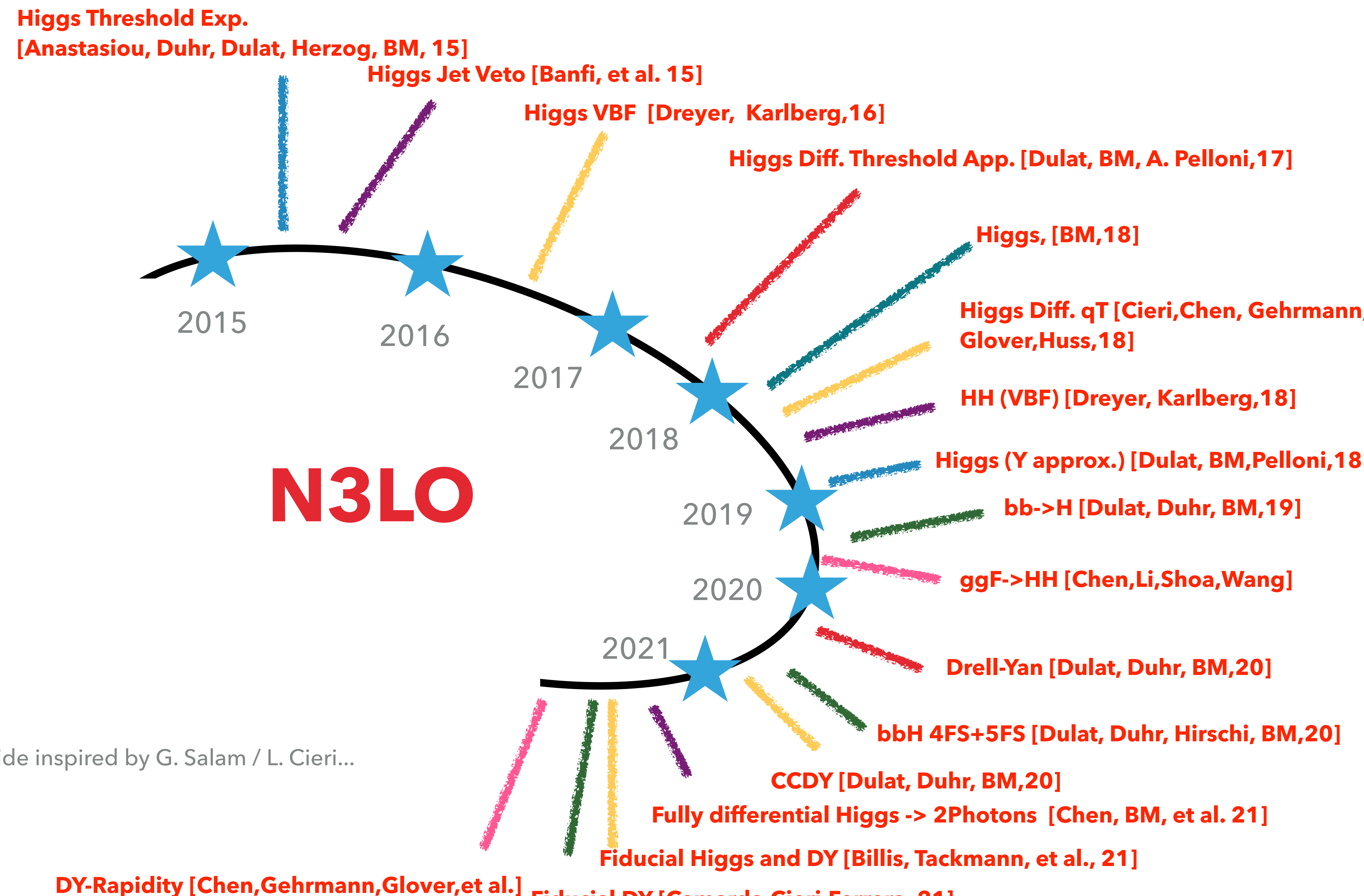


Double real virtual



Triple real

The dawn of N^3LO



Slide inspired by G. Salam / L. Cieri...

[Picture by B. Mistlberger]

The dawn of N^3LO

- N3LO computations are <u>extremely</u> challenging!
- Focus of this talk: Inclusive cross sections for $2 \to 1$ processes.

$$g\,g \to H \qquad b\,\bar{b} \to H \qquad q\,\bar{q} \to \gamma^*/Z \to \ell^+\ell^- \qquad q\,\bar{q}' \to W^\pm \to \ell^\pm\nu_\ell$$

- Cons:
  - ➡ Idealised theoretical observables (no cuts, no differential info)
  - ➡ Simple color-singlet final states.
- Pros:
  - ➡ Can be computed analytically.
  - ➡ Can serve as a template to understand N3LO phenomenology.
  - ➡ Historically, inclusive cross sections were also the first milestones for NNLO computations.

Outline

- Inclusive N3LO cross sections:
  - ➡ Review of computational strategy.
  - ➡ Neutral-current Drell-Yan production and γ^5.
- Phenomenological results:
  - ➡ Scale dependence.
  - ➡ PDF dependence.
- Outlook and conclusion.

Inclusive N3LO cross sections

Inclusive cross sections

- Inclusive cross sections can be cast in the form ($\tau = Q^2/S$):

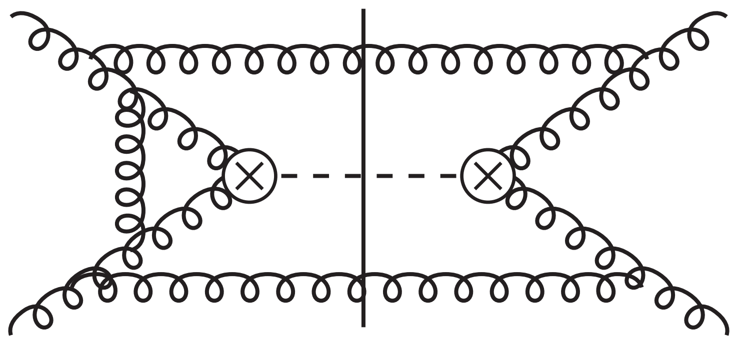
$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \frac{\hat{\sigma}_{ij}(z)}{z} \qquad \mathcal{L}_{ij}(\tau/z) = \int_{\tau/z}^1 \frac{dx}{x} f_i(x) f_j(\tau/(xz))$$

Partonic luminosity

➡ Partonic cross sections only depend on $z = Q^2/\hat{s}$.

- Reverse Unitarity: interpret phase space integrals as loop integrals with a cut.

$$\frac{1}{p^2 - m^2} \longrightarrow \delta_+(p^2 - m^2)$$



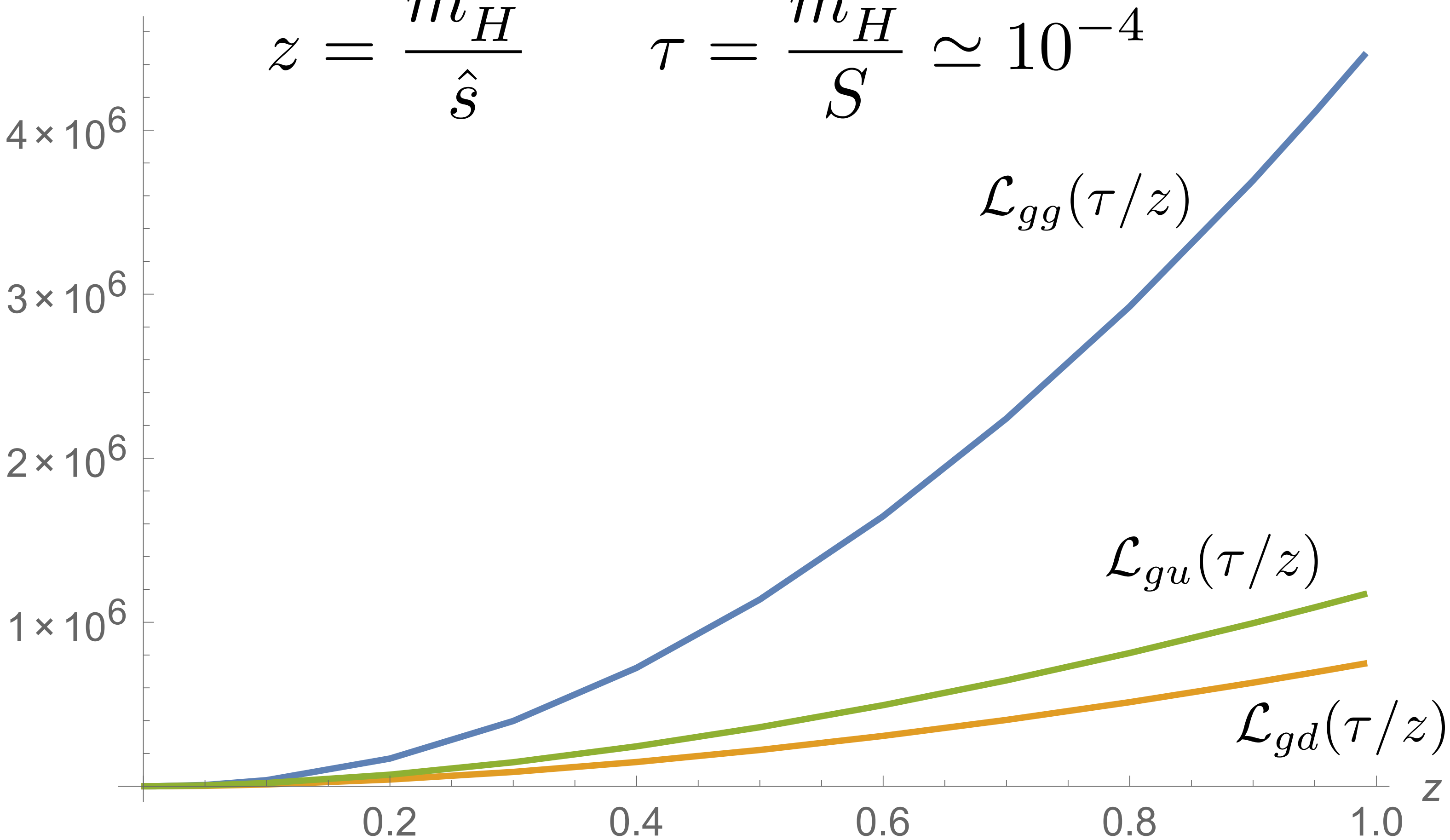
➡ We can use multi-loop technology to evaluate inclusive phase space integrals.

The gluon fusion cross section

- In 2015, we solved the differential equations as a series

$$\hat{\sigma}(z) = \sigma_{-1} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

$$z = \frac{m_H^2}{\hat{s}} \qquad \tau = \frac{m_H^2}{S} \simeq 10^{-4}$$

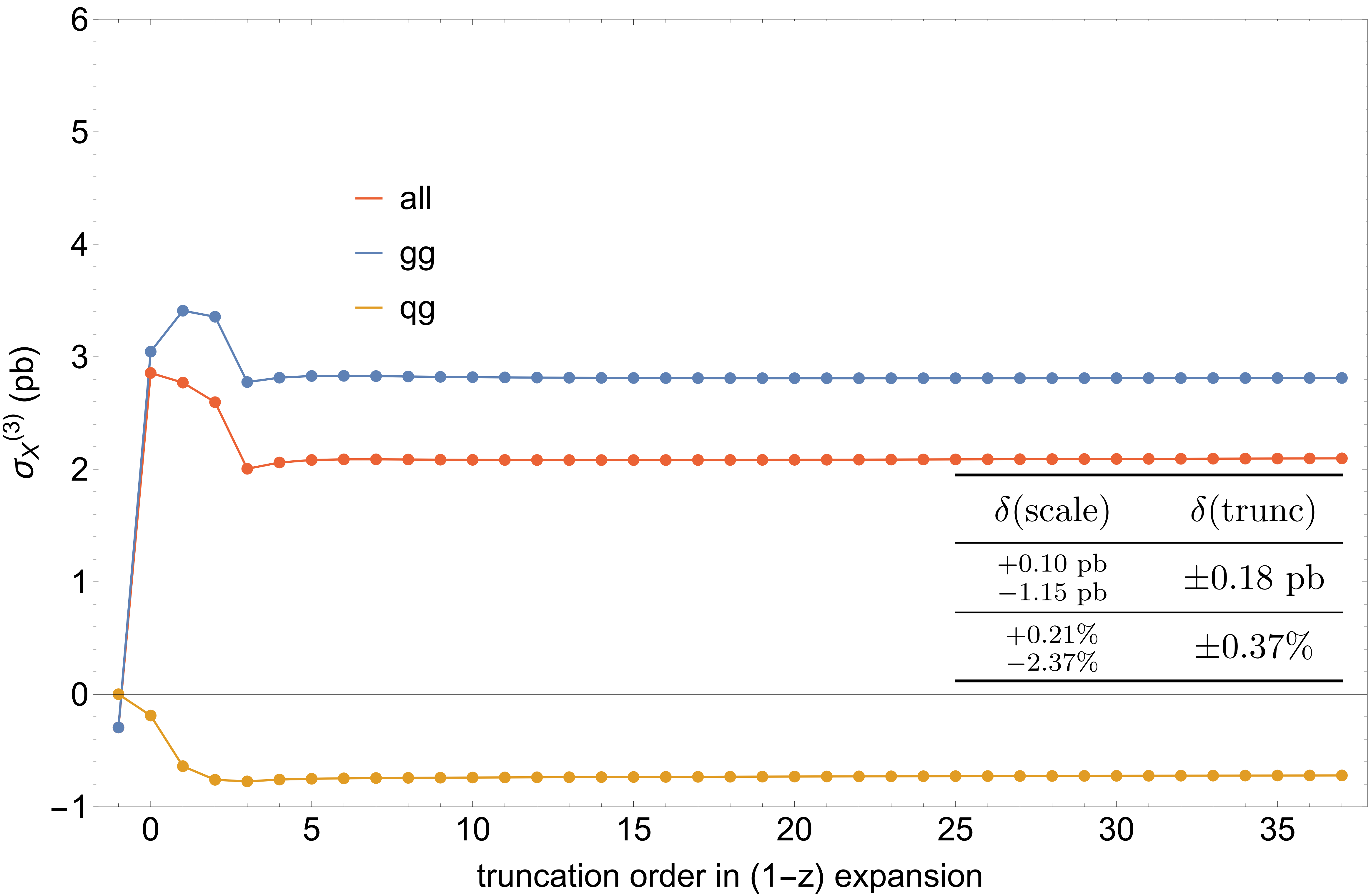


➡ For Higgs: Main contribution from region where $z \simeq 1$.

➡ Physically: production at threshold + emission of soft partons.

➡ Fails for quark-initated processes.

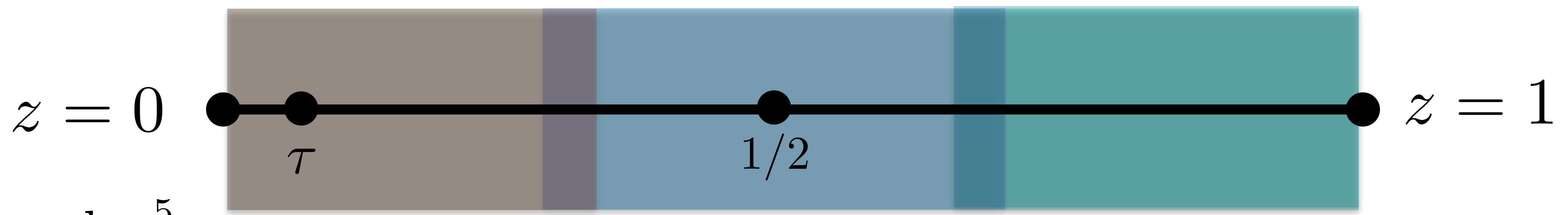
Convergence of the expansion



δ(scale)	δ(trunc)
$^{+0.10\ \mathrm{pb}}_{-1.15\ \mathrm{pb}}$	± 0.18 pb
$^{+0.21\%}_{-2.37\%}$	$\pm 0.37\%$

The threshold expansion

Slow convergence 10^{-15} agreement on overlap Initial condition soft limit



$$\frac{\hat{\sigma}_{ij}(z)}{z} \sim \frac{\log^5 z}{z}$$

$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z}\, \mathcal{L}_{ij}(\tau/z)\, \frac{\hat{\sigma}_{ij}(z)}{z} \qquad \tau = \frac{m_H^2}{S} \simeq 10^{-4}$$

- Exact numerical results! [Mistlberger]
  - ➡ Removes truncation uncertainty.
  - ➡ Opens the way to extend to quark-initiated processes.

Quark-initiated processes

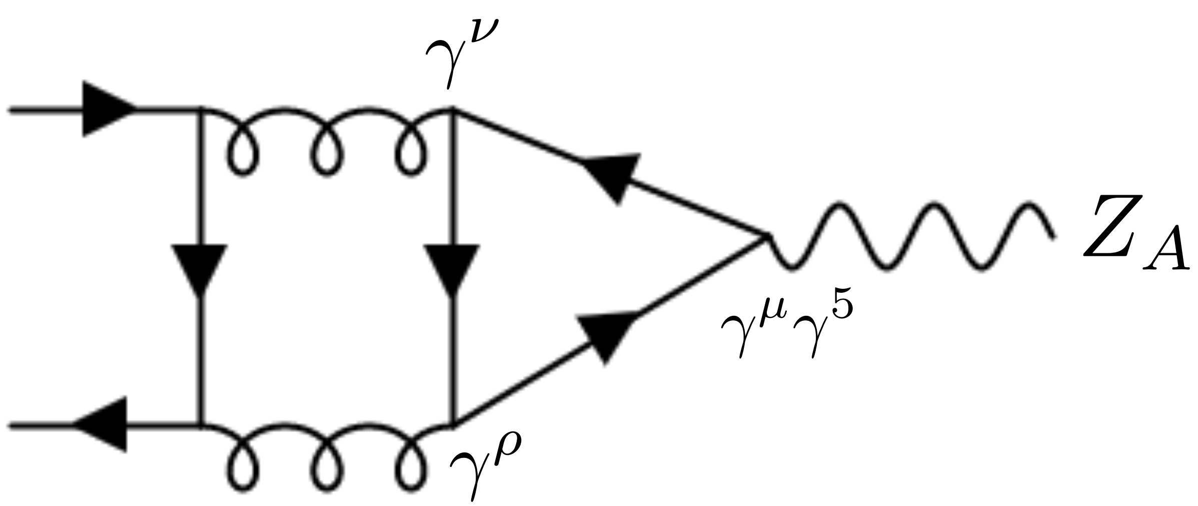
- We have recently completed the N3LO cross sections for

$$b\,\bar{b} \to H \qquad q\,\bar{q} \to \gamma^*/Z \to \ell^+\ell^- \qquad q\,\bar{q}' \to W^\pm \to \ell^\pm\nu_\ell$$

[CD, Dulat, Mistlberger]

- Spin-off: First independent confirmation of all 3-loop splitting functions. [Moch, Vermaseren, Vogt]

- All computations are done in massless QCD with $N_f = 5$ flavours.
  - ➡ γ^5 is ambiguous in dimensional regularisation.
  - ➡ The axial current is anomalous in QCD.
  - ➡ The anomaly cancels for an even number of flavours.



Treatment of γ^5

- We use the Larin-scheme to work with γ^5.
  - ➡ Write $\gamma^\mu \gamma_5 = \frac{1}{2}\{\gamma^\mu, \gamma_5\} = -\frac{i}{3!}\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_\rho\gamma_\sigma$.
  - ➡ Evaluate Dirac traces in D dimensions.
  - ➡ Replace

$$\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4} = \det \begin{pmatrix} g^{\mu_1\nu_1} & g^{\mu_1\nu_2} & g^{\mu_1\nu_3} & g^{\mu_1\nu_4} \\ g^{\mu_2\nu_1} & g^{\mu_2\nu_2} & g^{\mu_2\nu_3} & g^{\mu_2\nu_4} \\ g^{\mu_3\nu_1} & g^{\mu_3\nu_2} & g^{\mu_3\nu_3} & g^{\mu_3\nu_4} \\ g^{\mu_4\nu_1} & g^{\mu_4\nu_2} & g^{\mu_4\nu_3} & g^{\mu_4\nu_4} \end{pmatrix}$$

- The result is UV-divergent, and the divergence is removed by renormalising the axial current.

$$J^\mu_{A,f} = \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}\bar{q}_f\gamma_\nu\gamma_\rho\gamma_\sigma q_f$$

$$\left[J^\mu_{A,f}\right]_R = Z_{ns}\,J^\mu_{A,f} + Z_s \sum_{f'=1}^{N_f} J^\mu_{A,f'} = Z_{ns}\,J^\mu_{A,f} + Z_s\,J^\mu_{A,S}\,;$$

Axial anomaly

- The relevant 3-loop counterterms have recently been computed.

  [Ahmed, Gehrmann, Mathews, Rana, Ravindran; Ahmed, Chen, Czakon; Chen, Czakon, Niggetiedt]

- Important point: It is not a pure $\overline{\text{MS}}$-counterterm!

  ➡ Finite renormalisation to restore Adler-Bell-Jackiw anomaly equation:

$$\partial_\mu \left[J^\mu_{A,S}\right]_R = \frac{\alpha_S(\mu^2)}{8\pi}\, N_f \left[F\widetilde{F}\right]_R$$

- After the renormalisation of the axial current, we obtain a finite result.

- However: This is not the result we want!

  ➡ Example: The result depends on $\log \mu_R^2$ coming from the cancellation of the UV-pole from the axial anomaly… but in the SM there is no anomaly!

Axial anomaly

We want this!

$$\sigma_{\mathrm{QCD}_6}(m_t) = \sigma_{\mathrm{QCD}_5} + \mathcal{O}(1/m_t)$$

$$\mathcal{L}_{\mathrm{QCD}_6} + g \sum_{f=1}^{6} A_f \, Z_{A\mu} \, J^{\mu}_{A,f} \quad \overset{m_t \to \infty}{\not\longrightarrow} \quad \mathcal{L}_{\mathrm{QCD}_5} + g \sum_{f=1}^{5} A_f \, Z_{A\mu} \, J^{\mu}_{A,f}$$

Anomaly free

Anomalous!

$m_t \to \infty$

$$\mathcal{L}_{\mathrm{QCD}_5} + g \sum_{f=1}^{5} C_f(\mu_R/m_t) \, Z_{A\mu} \, [J^{\mu}_{A,f}]_R$$

Wilson coefficient - captures non-decoupling top-quark effects

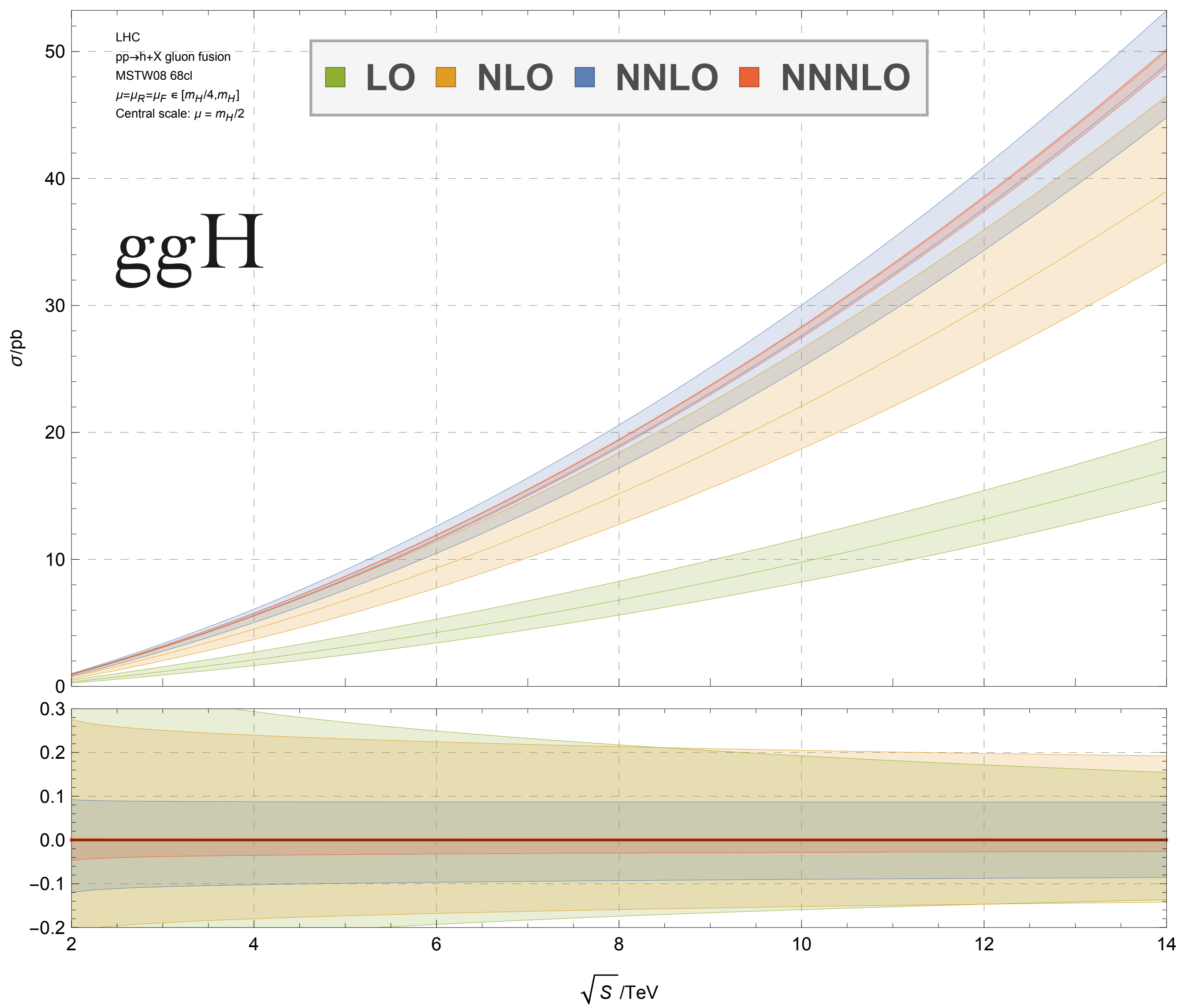
- ➡ 3-loop Wilson coefficient was recently computed. [Ju, Schönherr; Chen, Czakon, Niggetiedt]
- ➡ RGE for Wilson coefficient compensates 'wrong' log μ_R^2 dependence.

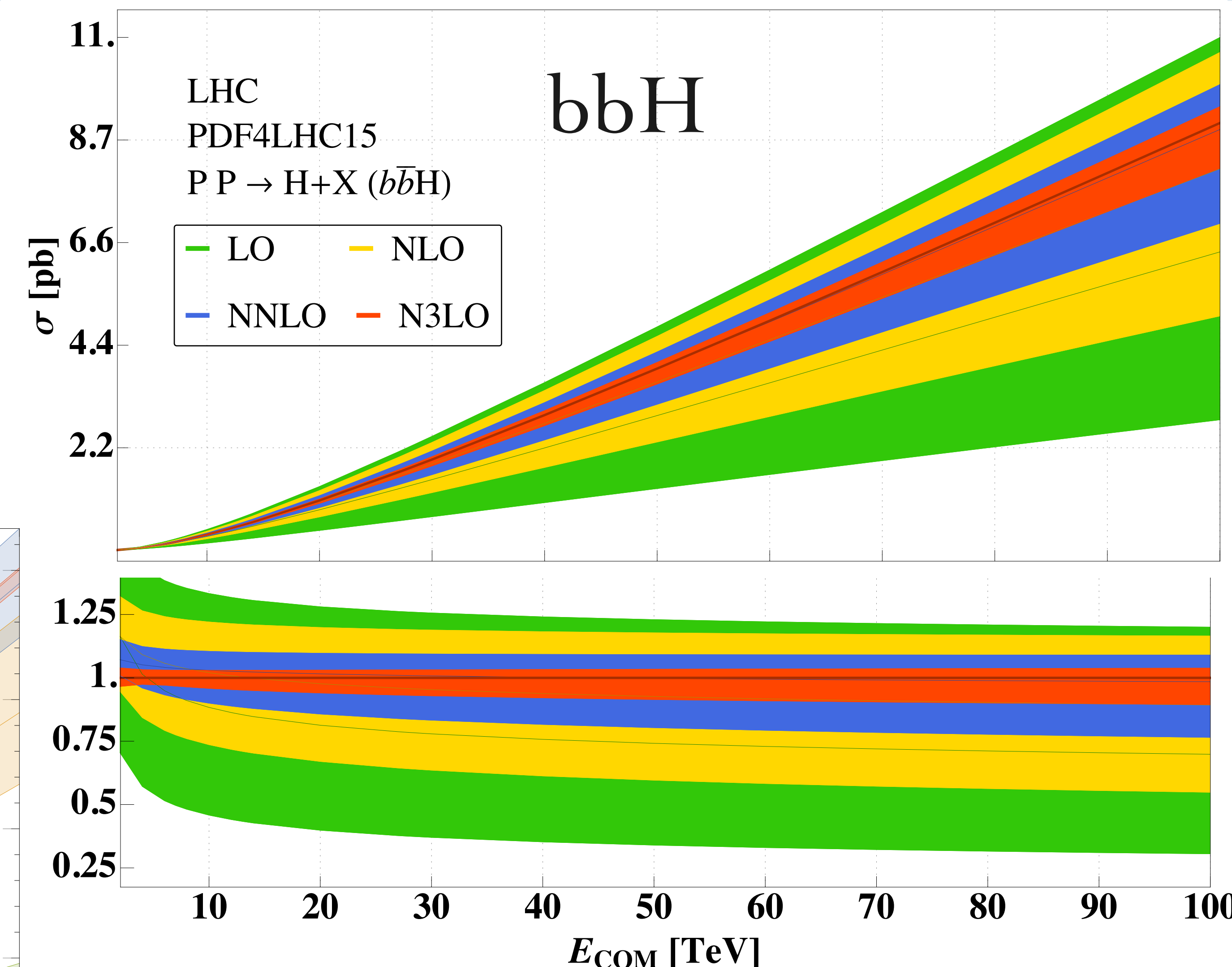
Phenomenological results

Energy variation

Higgs production:

Nice convergence of perturbative expansion.



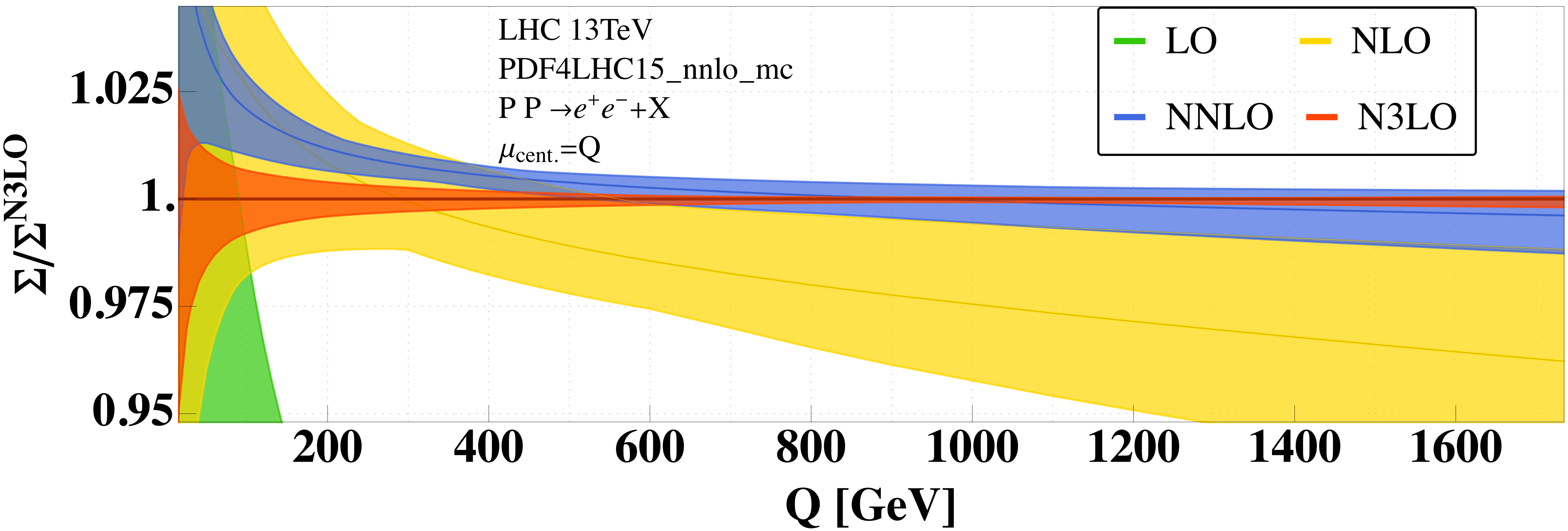


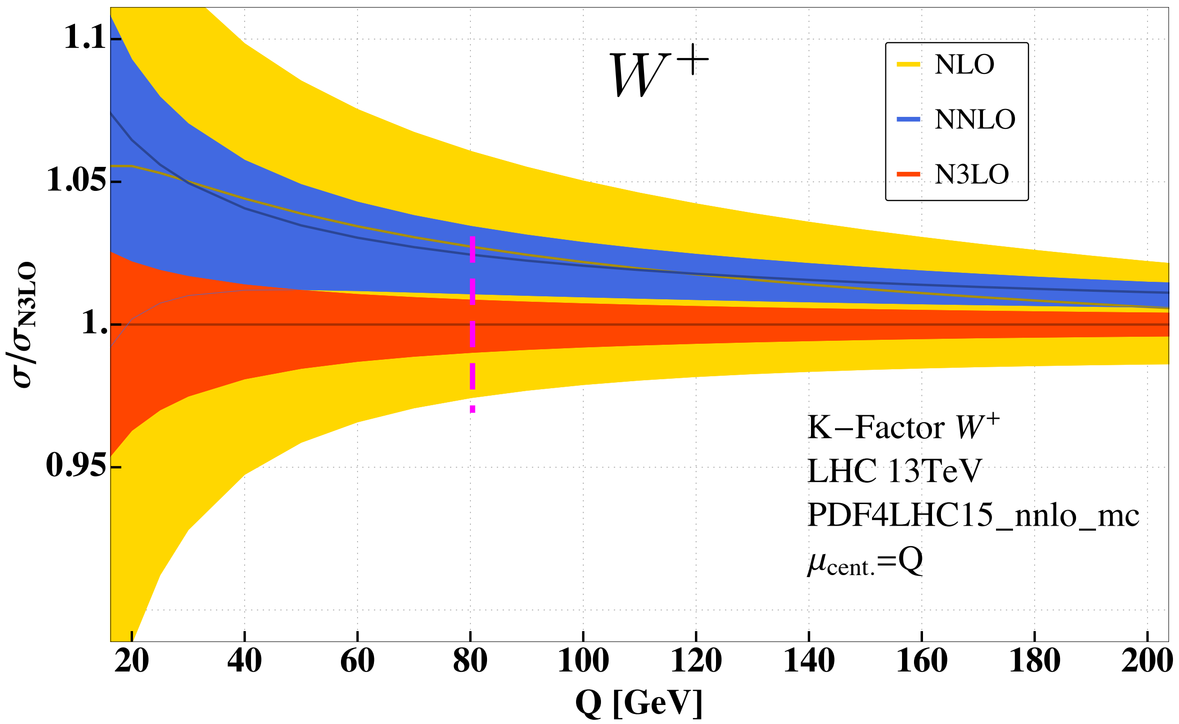
Choice of central scales:

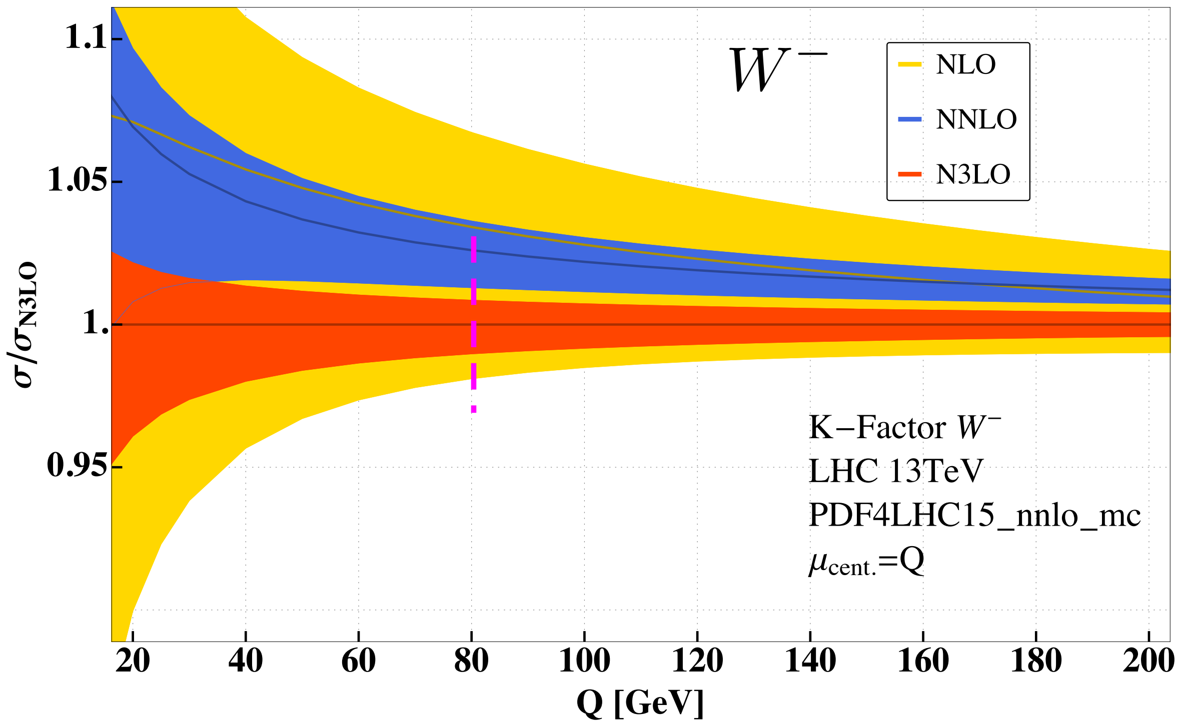
ggH: $\mu_F = \mu_R = m_H/2$

bbH: $\mu_F = m_H/4\,,\ \ \mu_R = m_H$

Q-variation (DY)

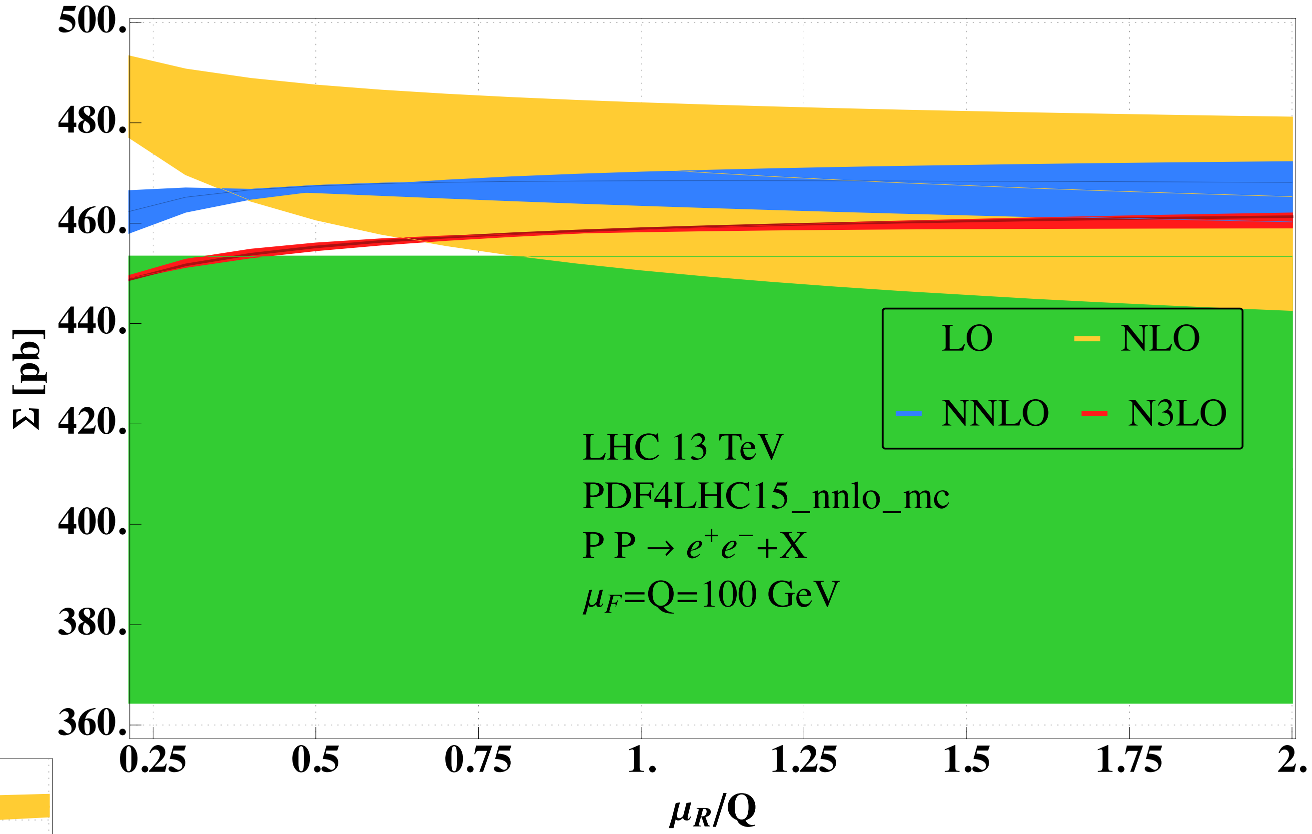




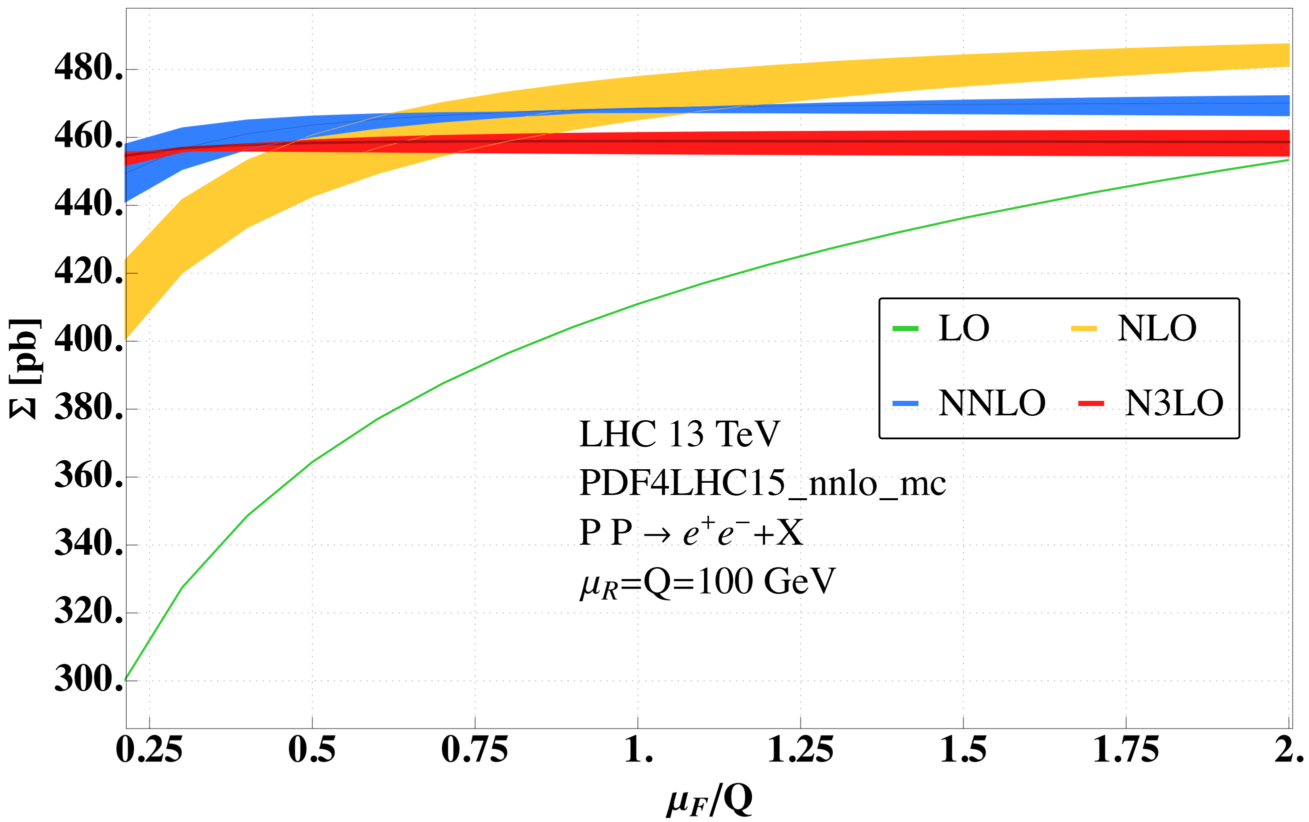


Scale dependence (DY)

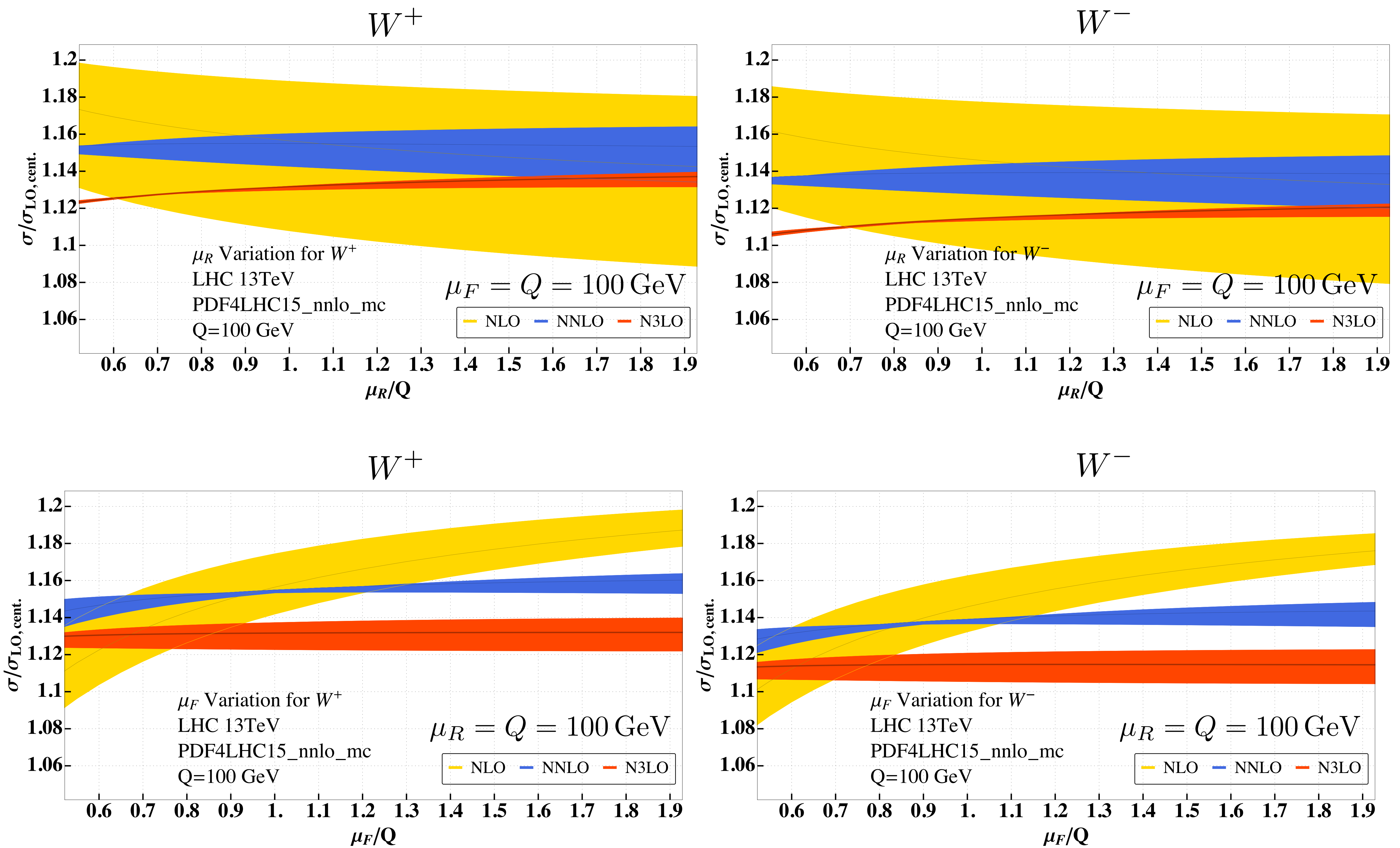
$\mu_F = Q = 100\,\mathrm{GeV}$



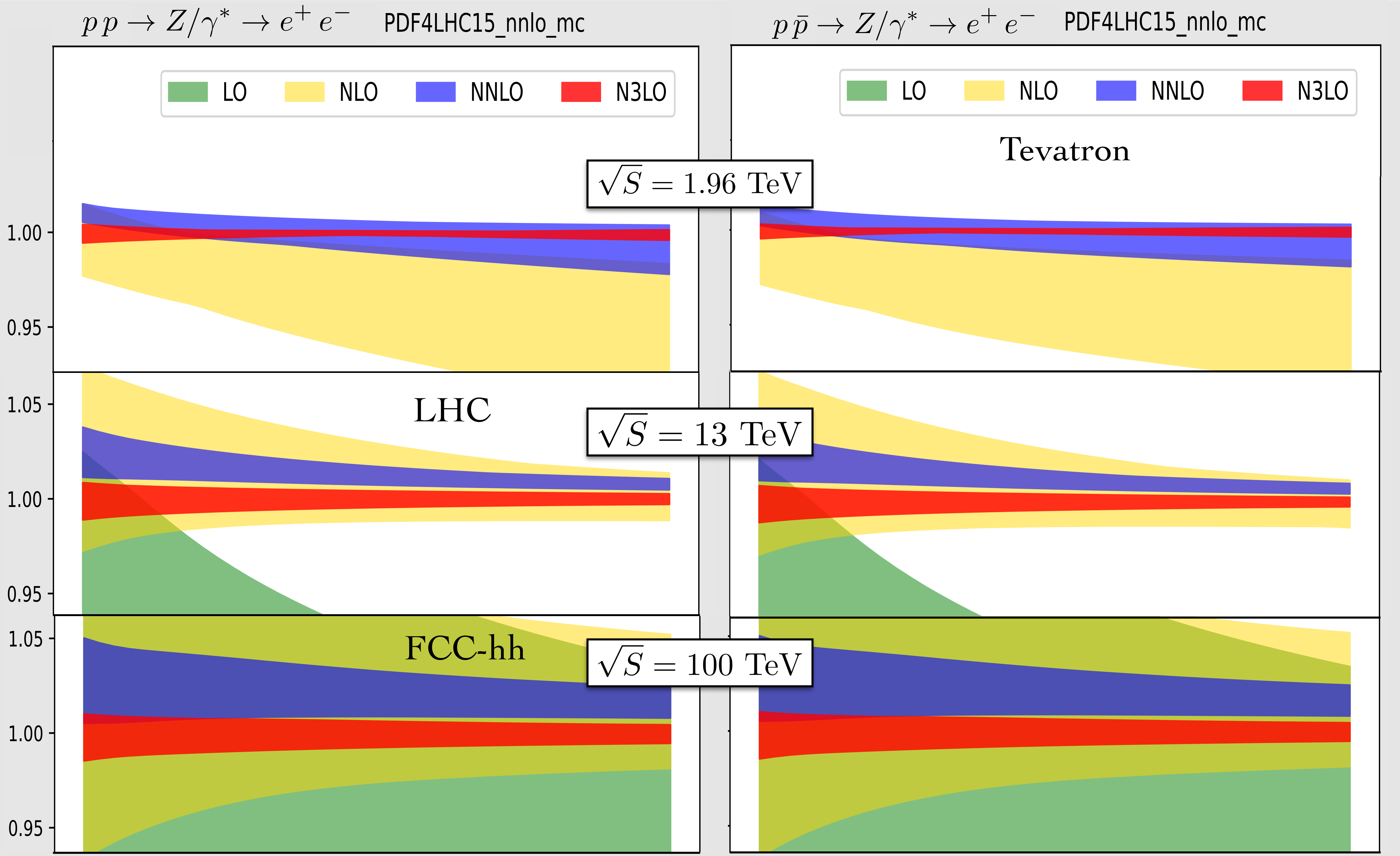
$\mu_R = Q = 100\,\mathrm{GeV}$



Scale dependence (W)

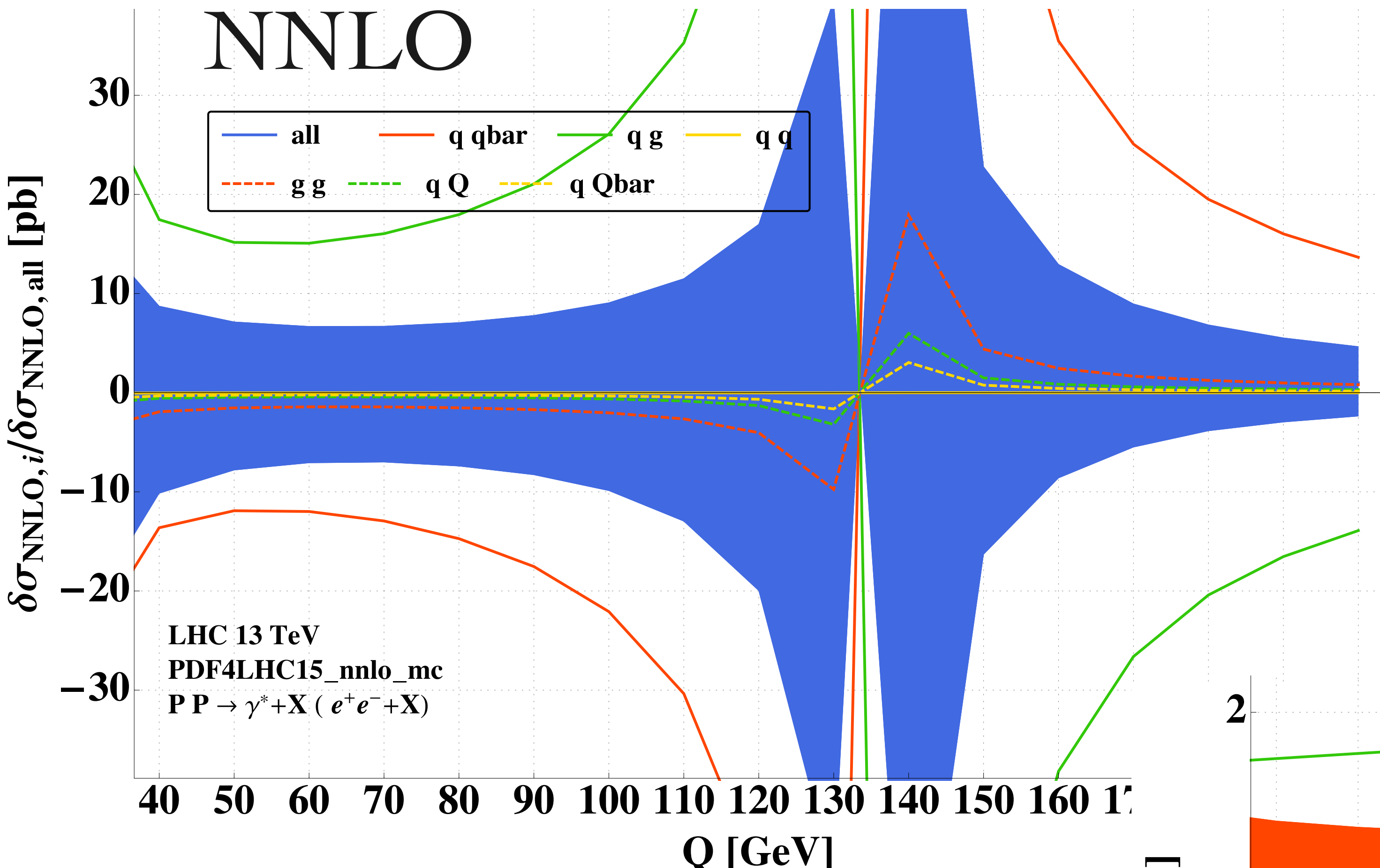


Collider dependence



[Baglio, CD, Mistlberger, Szafron - preliminary]

Variation per channel (DY)

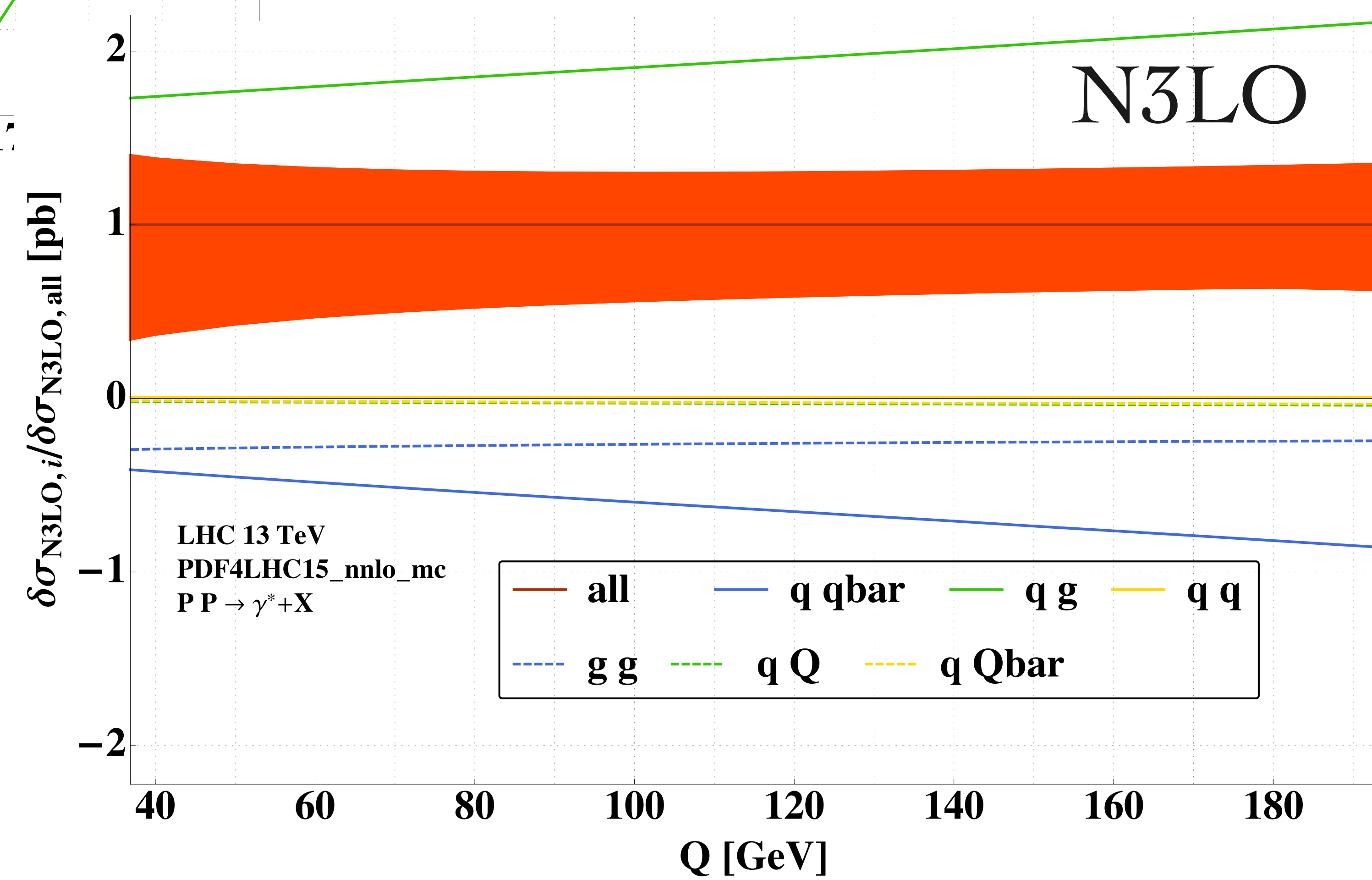


Very large cancellations between channels.

Already present at NNLO!

Similar cancellations for W.

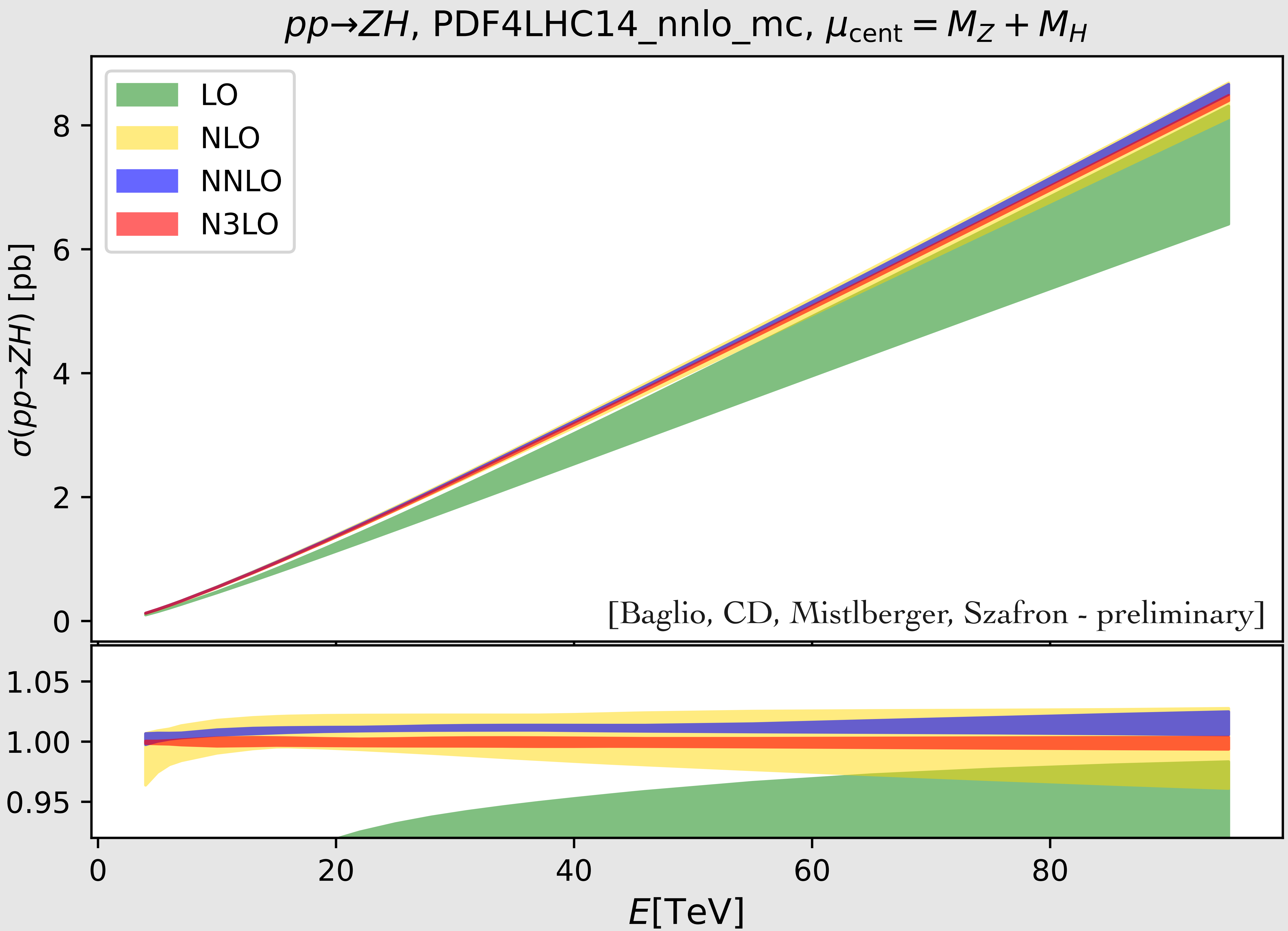
There are no such cancellations for Higgs production.



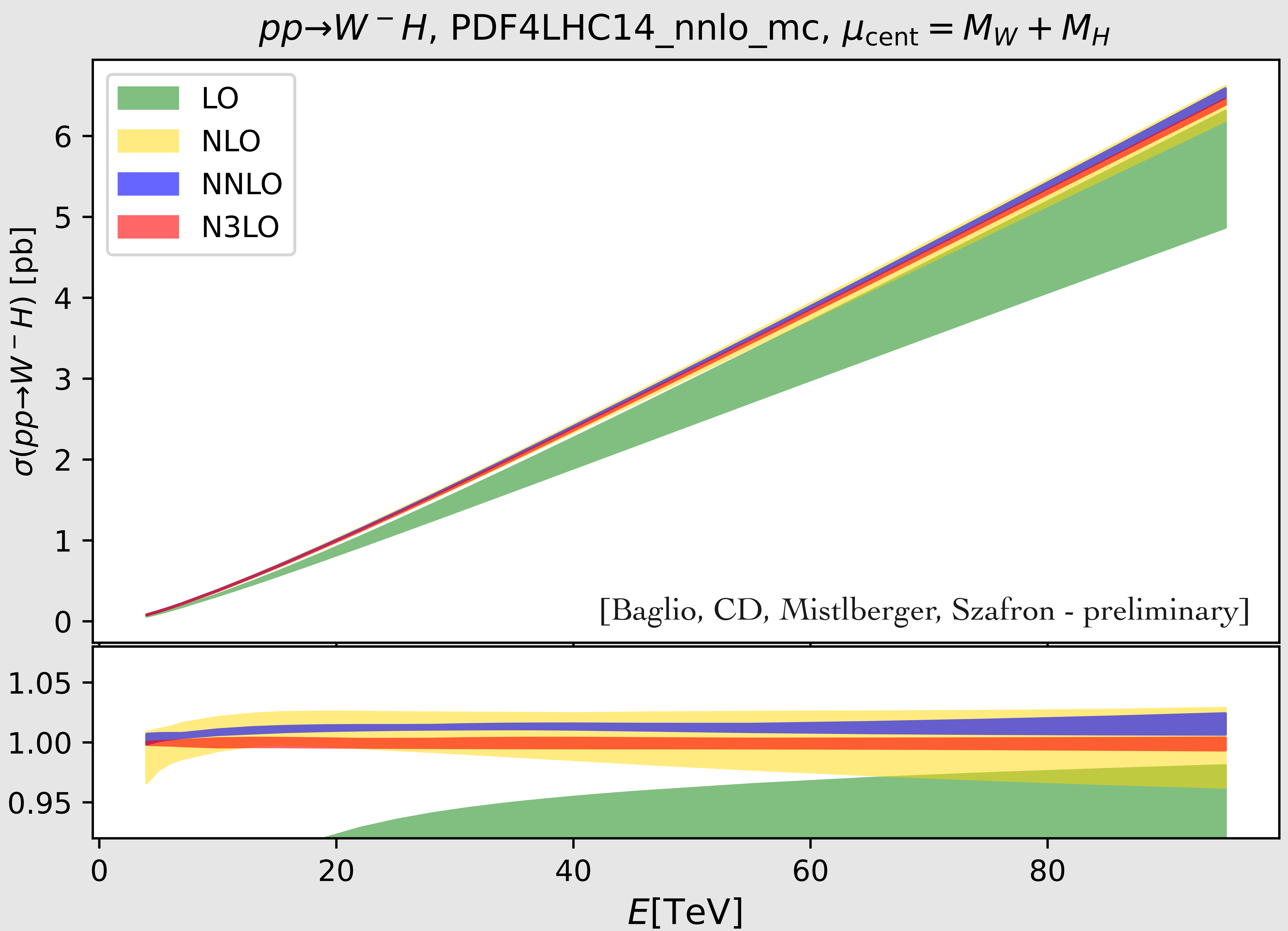
Scale dependence

- For Higgs (ggH & bbH): Scale bands overlap very well (for smallish μ_F).
- For DY (NC & CC): Scale bands do not overlap over a large range of virtualities.
  - ➡ Difference in central values: few %.
  - ➡ For both μ_F and μ_R .
  - ➡ All results obtained with `pdf4lhc_nnlo_mc` (more later).
- Observation: Large cancellation between channels for DY at NNLO and N3LO (both NC and CC).
  - ➡ No cancellation for Higgs.

ZH production

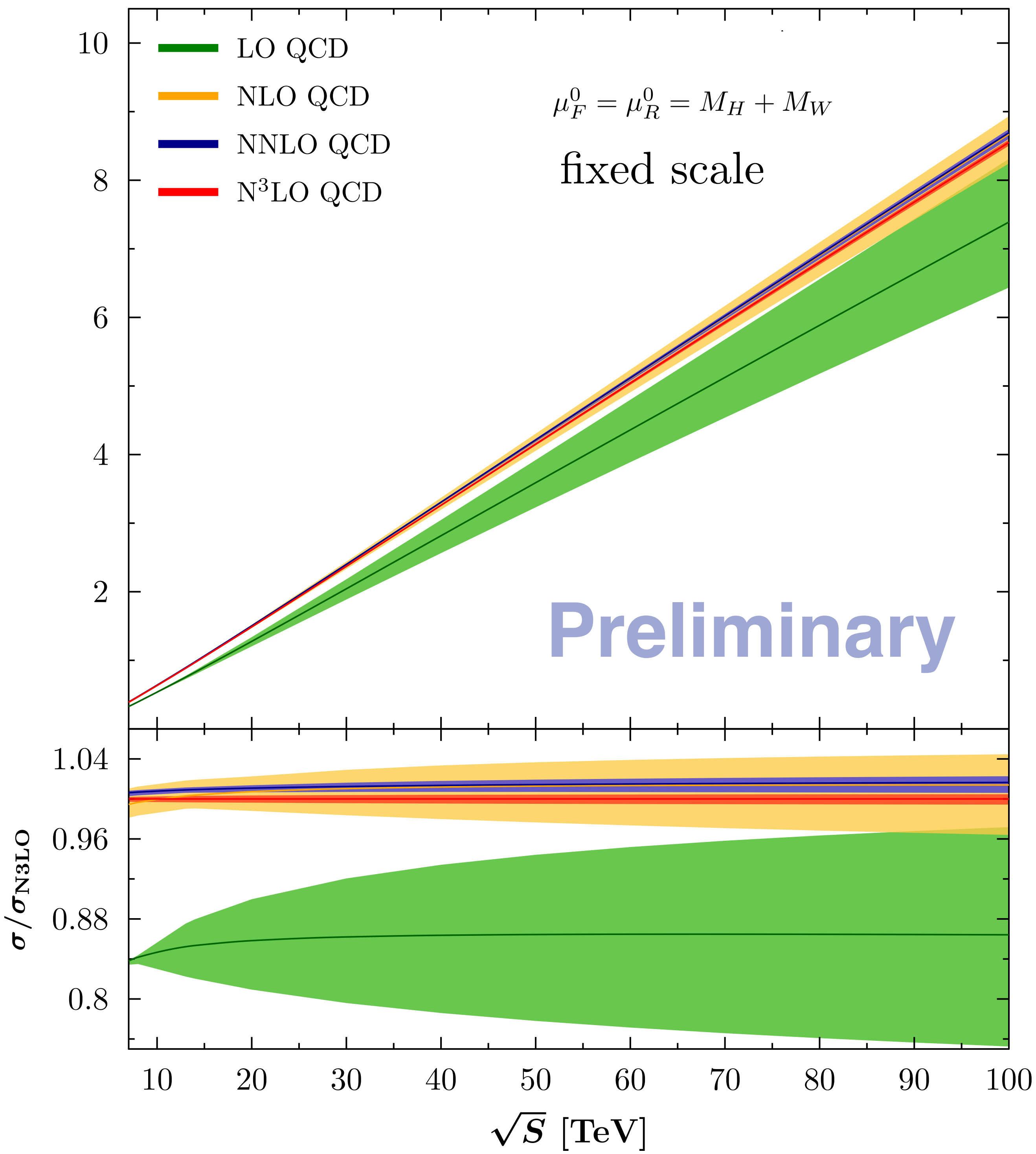


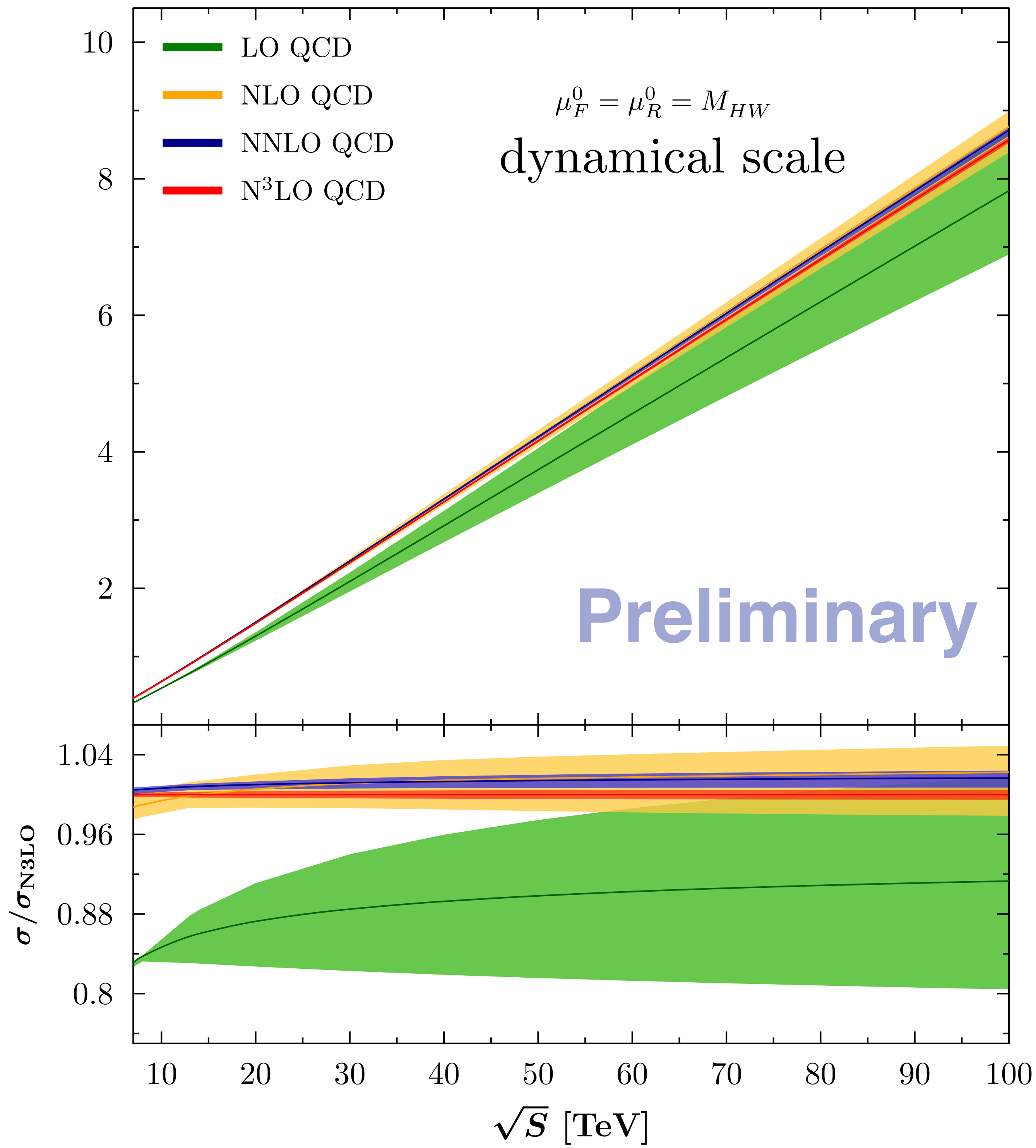
WH production



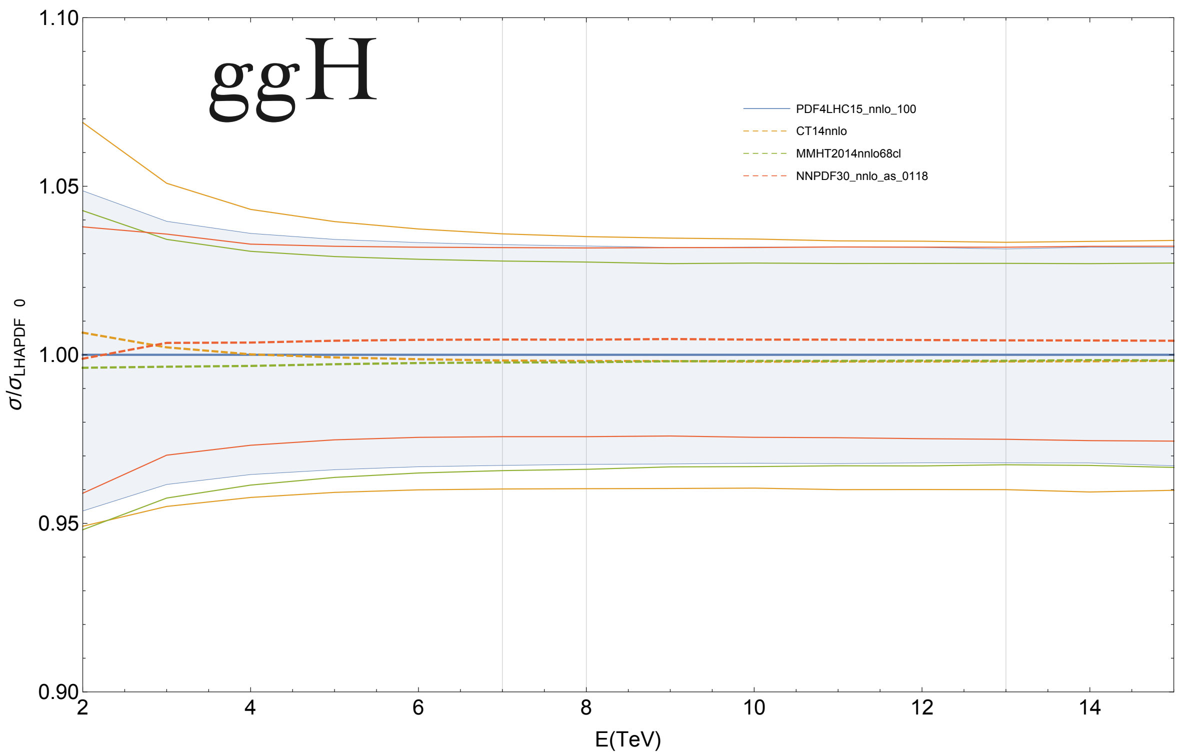
WH production

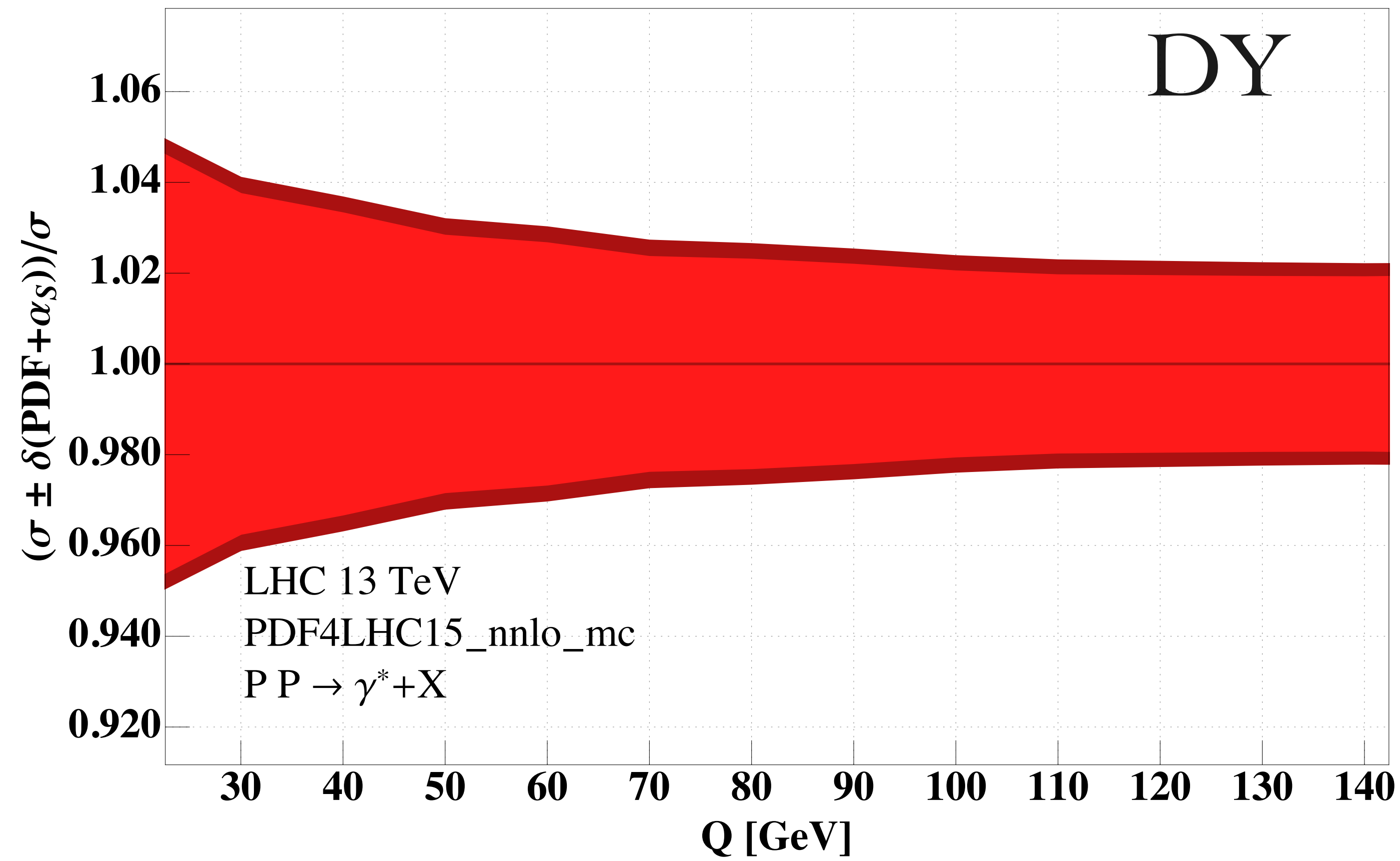
$$p\,p \to W^+H, \text{pdf4lhc}$$





PDF+ α_s -uncertainty





- Dependence of the cross on PDF+ α_s : ~2 — 9% at LHC energies.
  - ➡ Central set: pdf4lhc_nnlo_mc.
  - ➡ Uncertainty band obtained following PDF4LHC recommendation.

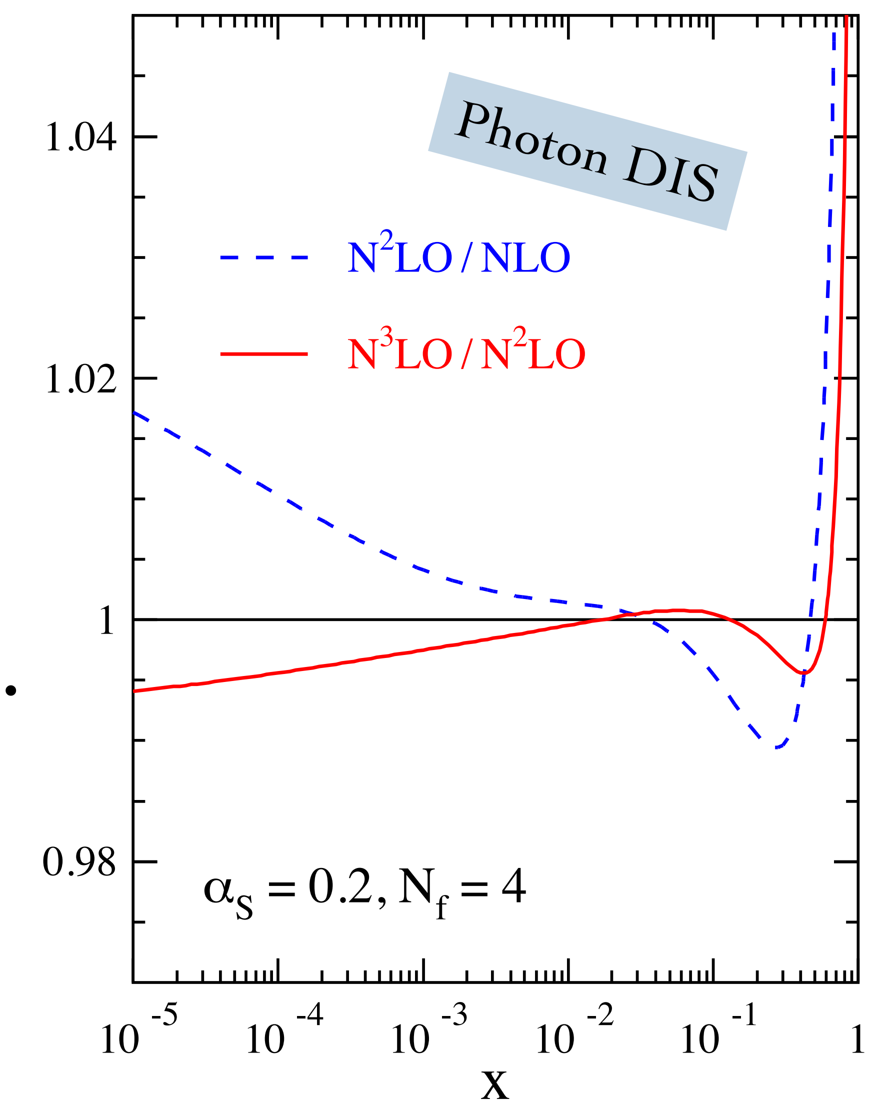
Missing N3LO PDFs

- We do not have N3LO PDFs
- This introduces a mismatch in our calculation.
- Estimate of the uncertainty:

$$\delta_{\text{PDF}}^{\text{N}^3\text{LO}} = \frac{1}{2}\left|\frac{\sigma^{\text{NNLO}}_{\text{NNLO-PDFs}} - \sigma^{\text{NNLO}}_{\text{NLO-PDFs}}}{\sigma^{\text{NNLO}}_{\text{NNLO-PDFs}}}\right|$$

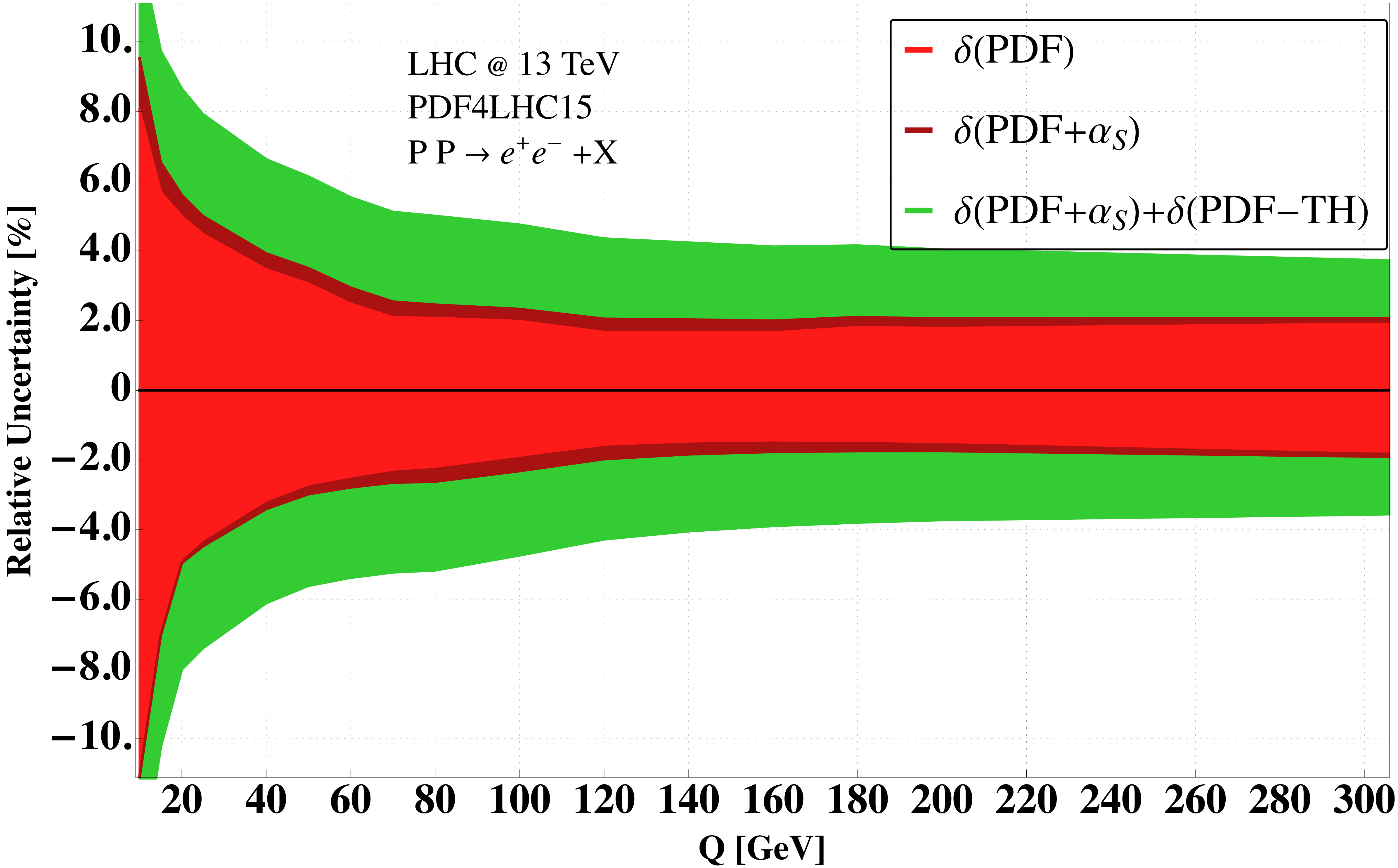
- The factor 1/2 takes into account that this estimate is most likely overly conservative.

  ➡ cf. convergence pattern of DIS.



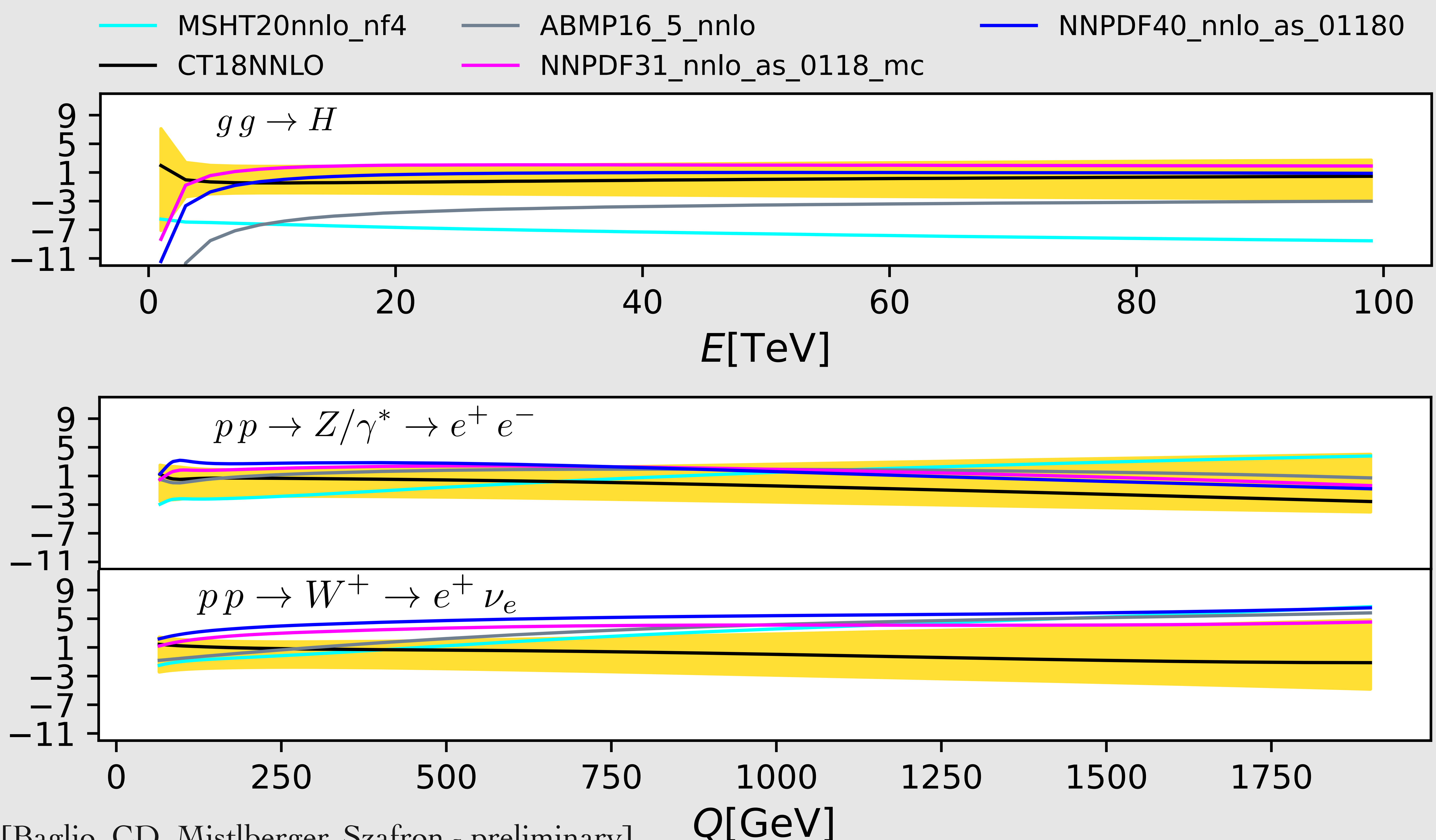
[Moch, Vermaseren, Vogt]

Missing N3LO PDFs



- In all cases we observe $\delta^{\mathrm{N^3LO}}_{\mathrm{PDF}} \sim 1 - 3\%$.

Impact of PDF choice



[Baglio, CD, Mistlberger, Szafron - preliminary]

Summary

	Q [GeV]	K-factor	$\delta(\text{scale})$ [%]	$\delta(\text{PDF} + \alpha_S)$	$\delta(\text{PDF-TH})$
$gg \to \text{Higgs}$	m_H	1.04	$^{+0.21\%}_{-2.37\%}$	$\pm 3.2\%$	$\pm 1.2\%$
$b\bar{b} \to \text{Higgs}$	m_H	0.978	$^{+3.0\%}_{-4.8\%}$	$\pm 8.4\%$	$\pm 2.5\%$
$pp \to e^+ e^-$	30	0.952	$^{+1.53\%}_{-2.54\%}$	$^{+3.7\%}_{-3.8\%}$	$\pm 2.8\%$
	100	0.979	$^{+0.66\%}_{-0.79\%}$	$^{+1.8\%}_{-1.9\%}$	$\pm 2.5\%$
$pp \to e^+ \nu_e$	30	0.953	$^{+2.5\%}_{-1.7\%}$	$\pm 3.95\%$	$\pm 3.2\%$
	150	0.985	$^{+0.5\%}_{-0.5\%}$	$\pm 1.9\%$	$\pm 2.1\%$
$pp \to e^- \bar{\nu}_e$	30	0.950	$^{+2.6\%}_{-1.6\%}$	$\pm 3.7\%$	$\pm 3.2\%$
	150	0.984	$^{+0.6\%}_{-0.5\%}$	$\pm 2\%$	$\pm 2.13\%$

➡ K-factors (N3LO/NNLO) ~ 2-5 %.

➡ Scale dependence ~ few %.

➡ PDF uncertainty: 2 - 9% (+ few percent missing N3LO PDFs)

Already for the simplest hadron collider observables N3LO corrections are important to reach 1% precision!

Outlook

The dawn of N^3LO

- For some simple processes also differential distributions or fiducial cross sections are available at N3LO (mostly DY and ggH).

[Dulat, Mistlberger, Pelloni; Cieri, Chen, Gehrmann, Glover, Huss; Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni; Billis, Dehnadi, Ebert, Michel, Tackmann; Camarda, Cieri, Ferrara; Chen, Gehrmann, Glover, Huss, Yang; Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli]

- More theoretical developments are needed to reach 1% precision for other processes!

$$\mathrm{d}\sigma_{pp\to X} = \sum_{i,j}\int_0^1 \mathrm{d}x_1\,\mathrm{d}x_2\, f_i(x_1)\, f_j(x_2)\, \mathrm{d}\hat{\sigma}_{ij\to X} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

PDFs

Partonic cross sections

Higher-twist effects

N^3LO PDFs

- PDFs are non-perturbative, and need to be extracted by comparing experimental data to theory predictions.
  - ➡ In order to fit N3LO PDFs, we need N3LO predictions!

- The dependence on the factorisation scale is perturbative.
  - ➡ DGLAP evolution equation:

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu_F^2} f_i(x,\mu_F^2) = \frac{\alpha_S(\mu_F^2)}{2\pi}\mathbf{P}_{ij}(x,\alpha_S(\mu_F^2)) \otimes f_j(x,\mu_F^2)$$

- DGLAP anomalous dimensions are known to 3-loop order.
  - ➡ N3LO requires 4-loop ADs! [Moch, Vermaseren, Vogt]
  - ➡ First few Mellin-moments known. [Moch, Ruijl, Ueda, Vermaseren, Vogt]

N^3LO partonic cross sections

$$d\sigma_{pp\to X} = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_i(x_1)\, f_j(x_2)\, d\hat{\sigma}_{ij\to X} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$$

PDFs

Partonic cross sections

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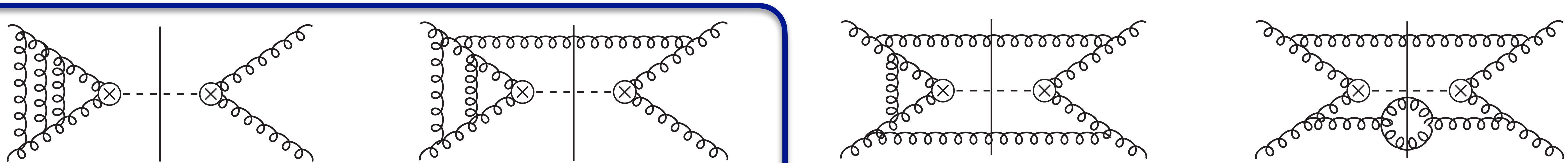


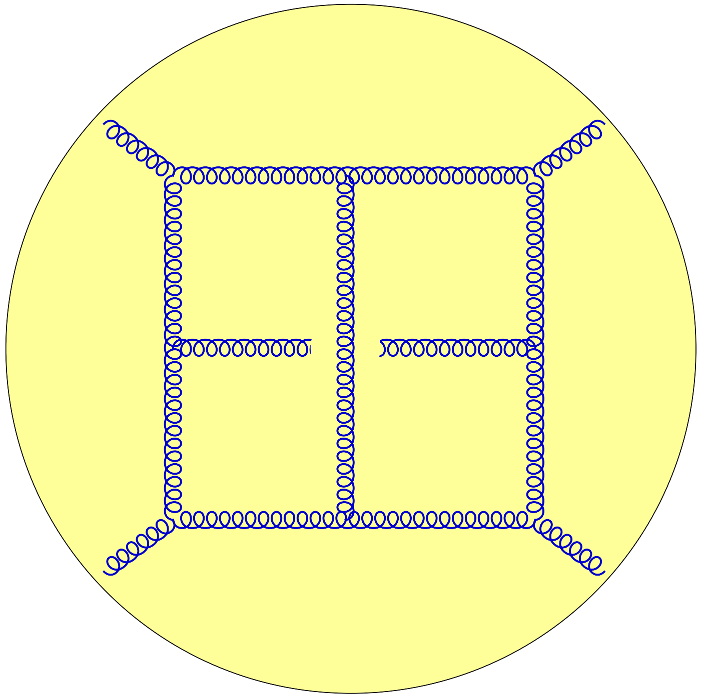
VVV RVV RRV RRR

Require complicated 2 & 3-loop computations

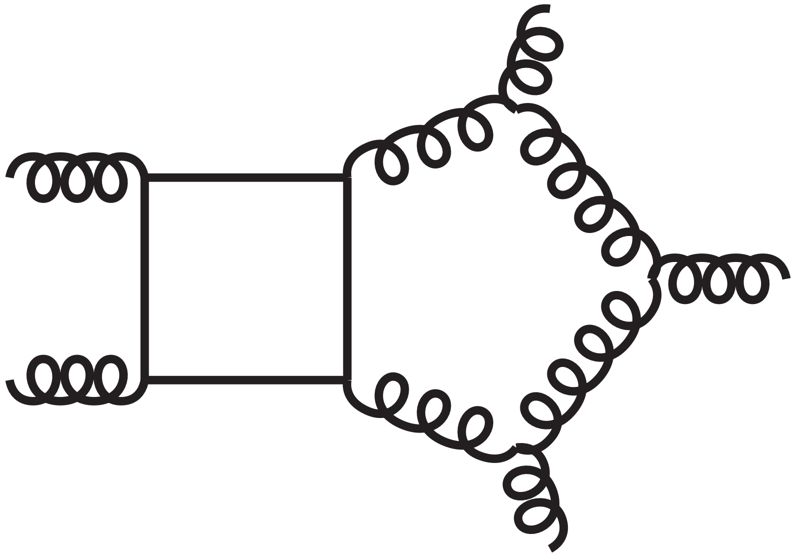
➡ Need to combine all contributions and cancel IR singularities.

N^3LO partonic cross sections

- First amplitudes needed for N3LO computations are becoming available:
  - ➡ 3-loop corrections to 2-to-2 processes.
  - ➡ 2-loop corrections to 2-to-3 processes.



[Bargiela, Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi; …]



[Abreu, Febres-Cordero, Ita, Page, Sotnikov, Tschernow, …; Badger, Chicherin, Gehrmann, Henn, …; Chaudhry, Czakon, Mitov, Poncelet, …; …]

N^3LO partonic cross sections

- At NLO and NNLO, real and virtual corrections are combined using
  - ➡ subtraction methods.
  - ➡ slicing methods.
- Both approaches rely on the factorisation of QCD amplitudes in infrared (soft and collinear) limits.
- IR limits of QCD amplitudes at N3LO starting to be understood.

[CD, Gehrmann; Li, Zhu; Dixon, Herman, Yan, Zhu; Zhu; Catani, Cieri; Catani, Colferai, Torrini; Del Duca, CD, Haindl, Lazopoulos, Michel; ...]

- Caveat: There are indications that for 2-to-4 processes at one-loop, the naive collinear factorisation breaks down! [Catani, de Florian, Rodrigo]

➡ 2-to-4 processes at one-loop are RRV for 2-to-2 processes.

Higher twist

$$\mathrm{d}\sigma_{pp\to X} = \sum_{i,j}\int_0^1 \mathrm{d}x_1\,\mathrm{d}x_2\, f_i(x_1)\, f_j(x_2)\, \mathrm{d}\hat{\sigma}_{ij\to X} + $$

$$+ C_1 \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^1 + C_2 \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^2 + \ldots$$

Higher-twist effects

- For inclusive DY production, $C_1 = 0$. [Beneke, Braun]
  - ➡ Also expected for inclusive ggH.
  - ➡ In general, one expects $C_1 \neq 0$.
- For $\Lambda_{\mathrm{QCD}} \sim 1$ GeV and $Q \sim 100 - 1.000$ GeV, we have

$$\frac{\Lambda_{\mathrm{QCD}}}{Q} \sim 10^{-3} - 10^{-2}$$

Conclusion

- N3LO corrections to key LHC processes are relevant if we want to reach a precision of 1%!

- The computation of inclusive 2-to-1 processes is mature.

$$g\,g \to H \qquad b\,\bar{b} \to H \qquad q\,\bar{q} \to \gamma^*/Z \to \ell^+\ell^- \qquad q\,\bar{q}' \to W^\pm \to \ell^\pm\nu_\ell$$

- There is still a lot to do for more complicated processes
  - ➡ PDFs at N3LO.
  - ➡ Complicated 2 & 3-loop amplitudes.
  - ➡ IR-singularities at N3LO - Factorisation violation?
  - ➡ Higher twist effects?