Vlasov Solvers for e-cloud driven Instabilities
PhD Progress Report

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Introduction

Simulation Model

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Impedance
Electron clouds

Depends on:
- Beam Chamber
- Beam Configuration
- Magnetic fields

Unwanted effects:
- Transverse instabilities
- Transverse emittance blowup
- Particle losses
- Heat Loads
- Vacuum Degradation

Motivation

• Electron clouds cause unwanted effects that can lead to instabilities

• Conventional simulation methods are limited to short time scales

• The Vlasov equation offers an alternative but development is needed to simulate a realistic accelerator
Objectives

Develop the Vlasov method to

- reach timescales not currently available
- reach the degree of complexity needed to simulate a realistic accelerator by including the effects of chromaticity, multiple electron cloud distributions, effects of transverse feedback, interplay with impedance, effects of accelerator optics
- Benchmark simulations with empirical measurements

Outline

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  The Vlasov Equation
  Macroparticle simulations

Closer look at the Quadrupolar forces

Impedance
Linear model of e-cloud - Dipolar Forces

Choose a set of sinusoid beam distortions, $h_n(z)$ where $z$ is the position along the bunch. The sinusoid test functions satisfy the orthogonality condition:

$$\int h_n(z)h_{n'}(z) = H_n^2 \delta_{n,n'}$$

(1)

Each distortion, $h_n$, corresponds to a response function $k_n$ calculated from the interaction with e-cloud using single-pass PIC simulations.

Linear model of e-cloud - Dipolar Forces

Describe the transverse centroid along the bunch, $\bar{x}(z)$, as a linear combination of test functions $h_n$:

$$\bar{x}(z) = \sum_{n=0}^{\infty} a_n h_n(z); \quad a_n = \frac{1}{H_n^2} \int \bar{x}(z) h_n(z) dz \quad (2)$$

$k_n$ is the resulting electron cloud kick from a bunch distortion $h_n$.

Assume linear behaviour such that the kick of arbitrary distribution $\bar{x}(z)$ is:

$$\Delta x'(z) = \sum_{n=0}^{\infty} a_n k_n(z) \quad (3)$$
Linear model of electron clouds - Quadrupolar forces

Model detuning using a polynomial

\[ \Delta Q(z) = \sum_{n=0}^{N_p} A_n z^n \]  \hspace{1cm} (4)

Generalize by adding chromaticity

\[ \Delta Q(z, \delta) = \sum_{n=0}^{N_p} A_n z^n + B_n \delta^n \]  \hspace{1cm} (5)

Including only \textit{linear} chromaticity:

\[ \Delta Q(z, \delta) = Q' \delta + \sum_{n=0}^{N_p} A_n z^n \]  \hspace{1cm} (6)

---

The Linerized Vlasov Equation

- The Vlasov equation describes a distribution $\psi$ of particles
- $\psi$ is a sum of the stationary distribution $\psi_0$ and a distortion $\Delta \psi$
- Each individual particles obeys a Hamiltonian $H_0$
- Introduce a first order perturbation of the Hamiltonian, $\Delta H$
- which leads to the Linearized Vlasov Equation, expressed with Poisson brackets:
  \[
  \frac{\partial \Delta \psi}{\partial t} + [\Delta \psi, H_0] = -[\psi_0, \Delta H]
  \] (7)
- The electron cloud forces are contained in $\Delta H$
- $\Delta \psi$ is the impact of the perturbation and the unknown

The Linerized Vlasov equation

We look for perturbations in the form:

\[ \Delta \psi(J_x, \theta_x, r, \phi, t) = e^{i(\Omega t - \Delta \Phi(r, \phi))} f_1(J_x) e^{i\theta_x} \sum_{l=-\infty}^{\infty} e^{-jl\phi} W_l \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r) \]  

(8)

where the unknown are contained in \( \Omega \) and \( b_{lm} \)

The linerized Vlasov Equation now becomes an eigenvalue problem:

\[ b_{lm}(\Omega - Q_{x0}\omega_0 - l\omega_s) = \sum_{l' m'} (M_{lm, l'm'} + \tilde{M}_{lm, l'm'}) b_{l'm'} \]  

(9)

Unkowns in red and terms including electron cloud forces in green

Solved by computing the matrices \( M_{lm, l'm'} \) and \( \tilde{M}_{lm, l'm'} \) and solve for "eigenvalue" \( \Omega \) and mode \( b_{lm} \) using standard linear algebra packets

Vlasov, linear e-cloud, Qp = 0

\[
\Delta \psi(J_x, \theta_x, r, \phi, t) = e^{i(\Omega t - \Delta \Phi(r, \phi))} f_1(J_x) e^{i\theta_x} \sum_{l=-\infty}^{\infty} e^{-jl\phi} W_l \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r) \tag{10}
\]

\[
\text{Re}(\Omega) = Q\omega_0 \text{ is the oscillation frequency of the mode.}
\]

\[
-\text{Im}(\Omega) \text{ is the instability growth rate of the mode}
\]

The Vlasov equation is solved for one e-cloud strength at a time, and this is repeated for many strengths to build these plots.
Vlasov, linear e-cloud, $Q_p = 10$

$\text{Re}(\Omega) = Q\omega_0$ is the oscillation frequency of the mode.

$-\text{Im}(\Omega)$ is the instability growth rate of the mode.

The maximum instability growth rates decrease, however instabilities are awakened sooner.

![Graph showing the relationship between electron cloud strength and oscillation frequency and instability growth rate.](image)
Macroparticle Simulation - Convention

- PyHEADTAIL is a macroparticle simulation code
- Each bunch has $\sim 10^{11}$ particles
- Simulate and track instead $\sim 10^6$ particles where each of the macroparticles have the mass and charge of $\sim 10^5$ protons.
- A one turn map is constructed from accelerator elements (dipoles, quadrupoles, etc)
- Between each one turn map e-cloud forces are acting on the beam
- The conventional simulation method of this interaction is the Particle-In-Cell simulation method
Macroparticle simulation - Linear model

An alternative is to use the linear model of e-cloud forces in the macroparticle simulations. This will also benchmark the linear model.

Dipolar forces are modeled as a kick between each one turn map. They are modelled as a matrix of the response functions $k_n$.

$$\Delta x'(z) = \sum_{n=0}^{\infty} a_n k_n(z)$$ (11)

The coefficients $a_n$ comes from the projection of the transverse centroid position $x(z)$ along $z$ on the test functions $h_n$

$$a_n = \frac{1}{H_n^2} \int \bar{x}(z) h_n(z) dz$$ (12)

The transverse centroid position is given by the one turn map.
Macroparticle simulation - Linear model

Quadrupolar forces are modeled as $z$ dependent quadrupoles placed between each one turn map

The strength of each quadrupole is calculated from the set of coefficients $A_n$

$$\Delta Q(z, \delta) = \sum_{n=0}^{N_p} A_n z^n$$

$$\Delta Q = \int \Delta k \frac{\beta(s)}{4\pi}$$

$\Delta k$ is the quadrupole strength of the new $z$ dependent quadrupole

The average $\langle \beta \rangle$ is used for these calculations

The chromaticity, $Q'$, is not included in this "quadrupole" but implemented separately in PyHEADTAIL
Vlasov and Macroparticle results, $Q_p = 10$

The two simulations models use the same linearized model or e-cloud $\Rightarrow$ they should have the same simulation results
Discrepancy in instability growth rates

Compare instability growth rates from macroparticle simulations (dashed lines) and Vlasov simulations (whole lines).

Both use the same linearised description of the electron cloud forces.
Remove Quadrupolar forces

It is unclear why there is a discrepancy between the results of the two simulations models

→ Try simulating a simpler case by removing quadrupolar forces.

Remove Quadrupolar forces

Keep only dipolar forces

Linear model of electron clouds - Quadrupolar forces

Model detuning using a polynomial

\[ \Delta Q(z) = \sum_{n=0}^{N_p} A_n z^n \]  

(4)

Generalize by adding chromaticity

\[ \Delta Q(z, \delta) = \sum_{n=0}^{N_p} A_n z^n + B_n \delta^n \]  

(5)

Including only linear chromaticity:

\[ \Delta Q(z, \delta) = Q \delta \sum_{n=0}^{N_p} A_n z^n \]  

(6)

Linear model of electron clouds - Dipolar Forces

Response functions from sinusoid beam distortions, \( h_n(z) \), using single-pass PIC simulations. \( z \) is the position along the bunch and \( h_n \) satisfy the orthogonality condition:

\[ \int h_n(z) h_{n'}(z) = H^2 \delta_{n,n'} \]  

(1)

Each distortion, \( h_n \), corresponds to a response function \( k_n \).
Instability Growth rates, only dipolar e-cloud forces

Good agreement between macroparticle and Vlasov simulations, even for high chromaticity

→ The problem lies in the quadrupolar forces.
Understanding the discrepancy

Several convergence checks were made, the error is not due to numerical convergence.

<table>
<thead>
<tr>
<th></th>
<th>$Q' = 0$</th>
<th>$Q' = 10$</th>
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<tbody>
<tr>
<td>quadrupolar and dipolar forces</td>
<td>good agreement</td>
<td>bad agreement</td>
</tr>
<tr>
<td>only dipolar forces</td>
<td>good agreement</td>
<td>good agreement</td>
</tr>
</tbody>
</table>

→ Take a closer look at the detuning forces.
Outline

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Simulation Model

Closer look at the Quadrupolar forces

Impedance
Quadrupolar forces in Vlasov simulations

In the ansatz of $\Delta \psi$:

$$\Delta \psi(J_x, \theta_x, r, \phi, t) = e^{i(\Omega t - \Delta \Phi(r, \phi))} f_1(J_x) e^{i \theta_x} \sum_{l=-\infty}^{\infty} e^{-jl\phi} W_l \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r)$$

(13)

The detuning forces are contained in $\Delta \Phi$. This term is also present in the expression of the matrices $M_{lm,l'm'}$ and $\tilde{M}_{lm,l'm'}$.

$\Omega$, the complex frequency of the mode and the modes themselves $b_{lm}$ are the unknowns.

The rest of the terms in the equation comes from expansions in the transverse and longitudinal planes and are basis functions.
Quadrupolar forces in Vlasov simulations

The detuning from the e-cloud forces and chromaticity are as mentioned:

$$\Delta Q(z, \delta) = Q'\delta + \sum_{n=0}^{N_p} A_n z^n$$ \hspace{1cm} (14)

$\Delta \Phi$ is chosen as:

$$\frac{\partial \Delta \Phi}{\partial \phi} = -\frac{\omega_0}{\omega} \Delta Q_\phi (r, \phi), \hspace{0.5cm} ; \Delta Q_\phi = \Delta Q(r, \phi) - \frac{1}{2\pi} \int_0^{2\pi} \Delta Q$$ \hspace{1cm} (15)

where $(r, \phi)$ are polar coordinates in the longitudinal plane.

This choice can be done without loss of generality and this choice is beneficial since it cancels some terms in the linearized Vlasov equation.
Quadrupolar forces in Vlasov simulations

\[
\frac{\partial \Delta \Phi}{\partial \phi} = -\frac{\omega_0}{\omega} \Delta Q_\Phi(r, \phi), \quad \Delta Q_\Phi = \Delta Q(r, \phi) - \frac{1}{2\pi} \int_0^{2\pi} \Delta Q
\]

(16)

The expression for \( \Delta \Phi \) comes by integrating \( \Delta Q_\Phi \)

\[
\Delta \Phi = -\int_0^\phi \frac{\omega_0}{\omega} \Delta Q_\Phi
\]

(17)

In other references [4], a zero is included in the integration limits. If the mathematics is correct, this should have no effect. However, since it only affects the \( \Delta \Phi \) when chromaticity is non-zero, it is an interesting sanity check.

Quadrupolar forces in Valsov simulations

Old expression $\Delta \Phi$

New Expression $\Delta \Phi$

Significant change of $\Delta \Phi$ but continuous in both cases.

$$\Delta \psi(J, \theta, r, \phi, t) = e^{i(\Omega t - \Delta \Phi(r, \phi))} f_1(J) e^{i\theta} \sum_{l=-\infty}^{\infty} e^{-jl\phi} W_l \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r)$$

(18)
Quadrupolar forces in Vlasov Simulations

The simulation results do not change while using the new expression of $\Delta \Phi$.

This means that the constant in $\Delta \Phi$ introduced by the alternative integration can be absorbed in $b_{lm}$.

$$\Delta \psi(J_x, \theta, r, \phi, t) = e^{j(\Omega t - \Delta \Phi(r, \phi))} f_1(J_x) e^{j\theta} \sum_{l=-\infty}^{\infty} e^{-jl\phi} W_l \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r)$$

(19)
Outline

Introduction

Simulation Model

Closer look at the Quadrupolar forces

Impedance
Short introduction of impedance

Motivation for study of impedance:

- Impedance has been studied more thoroughly and is more understood than e-cloud
- Quadrupolar forces have been modeled successfully
- Quadrupolar forces have not yet been implemented using the Vlasov method
- Implementing Quadrupolar forces in using the Vlasov method will improve understanding of how Quadrupolar forces behave in the Vlasov method.
Short introduction of impedance

- Each particle in the bunch is a source of electromagnetic fields
- These fields interact with the surrounding pipe and induce currents
- The induced currents also generate electromagnetic fields, Wake fields
- in the ultrarelativistic regime the induced fields exclusively effect trailing particles, test particles.
- the fourier transform of a Wakefield is the Impedance
Similar to e-cloud simulations, there is agreement for low chromaticity and NOT for high chromaticity.

Convergence checks were made to check that the discrepancy was not a numerical error.
Vlasov vs PyHEADTAIL: Standard Impedance

Dipolar forces

Better, but not perfect, agreement for all chromaticities

A discrepancy is still observed for low strengths, this was not expected

→ A closer look at the calculations of the growth rate in the macroparticle simulation.
Simulated centroid position

Plotting the centroid position from the macroparticle simulation at each turn

The Vlasov growth rate agrees with data

exponential fit on macroparticle data appears to be too flat.

→ Improve how growth rate is found
Finding the instability growth rate

Calculated the envelope using a scipy function

Exponential fit of envelope is the same as the fit of the original data

The calculated envelope is still noisy

→ Try low pass filter to reduce noise
Finding the instability growth rate

The fit of the filtered data still does not match the Vlasov mode.

Vlasov mode appears to fit unfiltered data.

The noise is stronger than the instability mode for the first $\sim 3000$ turns, after noise is not a problem.

Try an exponential fit only after the instability is visible $\rightarrow$ introduce threshold
Finding the instability growth rate

Introduce a threshold that is 5x the maximum amplitude of the first 100 turns.

Do an exponential fit on centroid position after it has breached the threshold.

The new threshold fit agrees better with visible instability and with the Vlasov worse mode.
Finding the instability growth rate

A fit on the envelope after the amplitude has breached the threshold gives the same result as not calculating the envelope.

→ Find instability growth rates from doing an exponential fit after the centroid has passed a threshold. Chose a threshold of 5 times the maximum amplitude of the first 100 turns.
Scan of impedance strengths

At low impedance strength no instability is detected since threshold is not passed.
Outlook

Fully understand quadrupolar impedance before going back to electron cloud.

- Develop a more reliable model of fitting the instability growth rate
- Introduce quadrupolar forces for the impedance step by step
- Use the framework for e-cloud forces to model impedance to see if the same results are reached
- Compare with quadrupolar impedance forces implemented in PyHEADTAIL

Measurements of instabilities in Large hadron collider:

- Measure instability for strong e-cloud (before the scrubbing run, a step in the commissioning process)
- Measure instabilities for stabilized e-cloud. Vary parameters such as chromaticity octupole strengths.
- Measure instabilities for varying intensity.
Thank you for your attention!
\[ M_{lm, l'm'} \] and \[ \tilde{M}_{lm, l'm'} \]

\[
M_{l,m,l'm'} = -\frac{N_{b\nu}}{8\pi^2 Q_{x0} F_{lm}} \sum_{n=0}^{\infty} \int \int drd\phi e^{jl\phi} e^{j\Delta \Phi(r,\phi)} w_l(r) f_{lm}(r) \frac{g_0}{W_l(r)} k_n(r \cos \phi) \times \int \int \tilde{r}d\tilde{r}d\tilde{\phi} e^{-j\Delta \Phi(\tilde{r},\tilde{\phi})} f_{l'm'}(\tilde{r}) \frac{W_{l'}(\tilde{r})}{\lambda_0(\tilde{r} \cos \tilde{\phi})} \frac{h_n(\tilde{r} \cos \tilde{\phi})}{H_n^2}
\]

\[
\tilde{M}_{lm, l'm'} = \delta_{l,l'} \frac{\omega_0}{F_{lm}} \int drw_l(r) \Delta Q_R(r) f_{lm}(r) f_{lm'}(r)
\]
## PyHEADTAIL simulation parameters

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<tr>
<th>Simulation parameters</th>
<th>Value</th>
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<td>$\beta_x$</td>
<td>92.7 m</td>
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<td>$\beta_y$</td>
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<td>$Q_x$</td>
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<tr>
<td>$Q_y$</td>
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<td>$l$</td>
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</tr>
</tbody>
</table>
Scan of e-cloud strengths

![Graph showing e-cloud Qp = 10 strength 0.600 with simulated absolute Amplitude, fit, and Vlasov worst mode lines.](image-url)
Scan of e-cloud strengths

![Graph showing e-cloud strengths with Qp = 10 and strength 1.000. The graph plots abs(x_mean) against turns.]
Scan of e-cloud strengths

![Graph showing e-cloud Qp = 10 strength 1.400 over turns](image)

- **Simulated absolute Amplitude**
- **fit**
- **Vlasov worst mode**
Scan of e-cloud strengths
Growth rates from new threshold fit

![Graph showing instability growth rate vs. impedance strength.](image-url)
Growth rates from new threshold fit

The calculated growth rates are closer to the worst mode in Vlasov for certain strengths.

→ further checks are needed