The Standard Model

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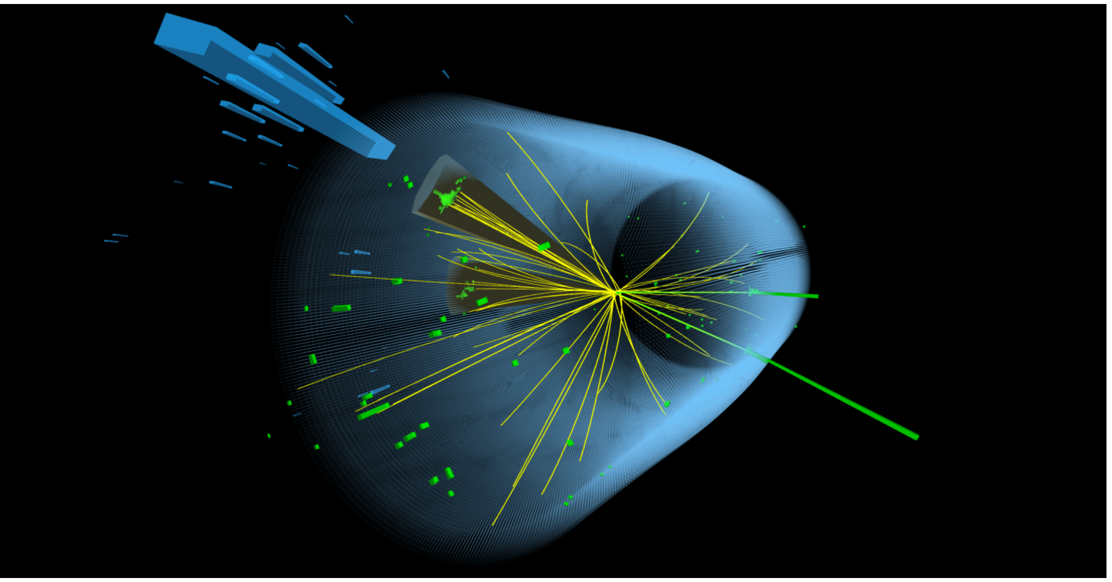
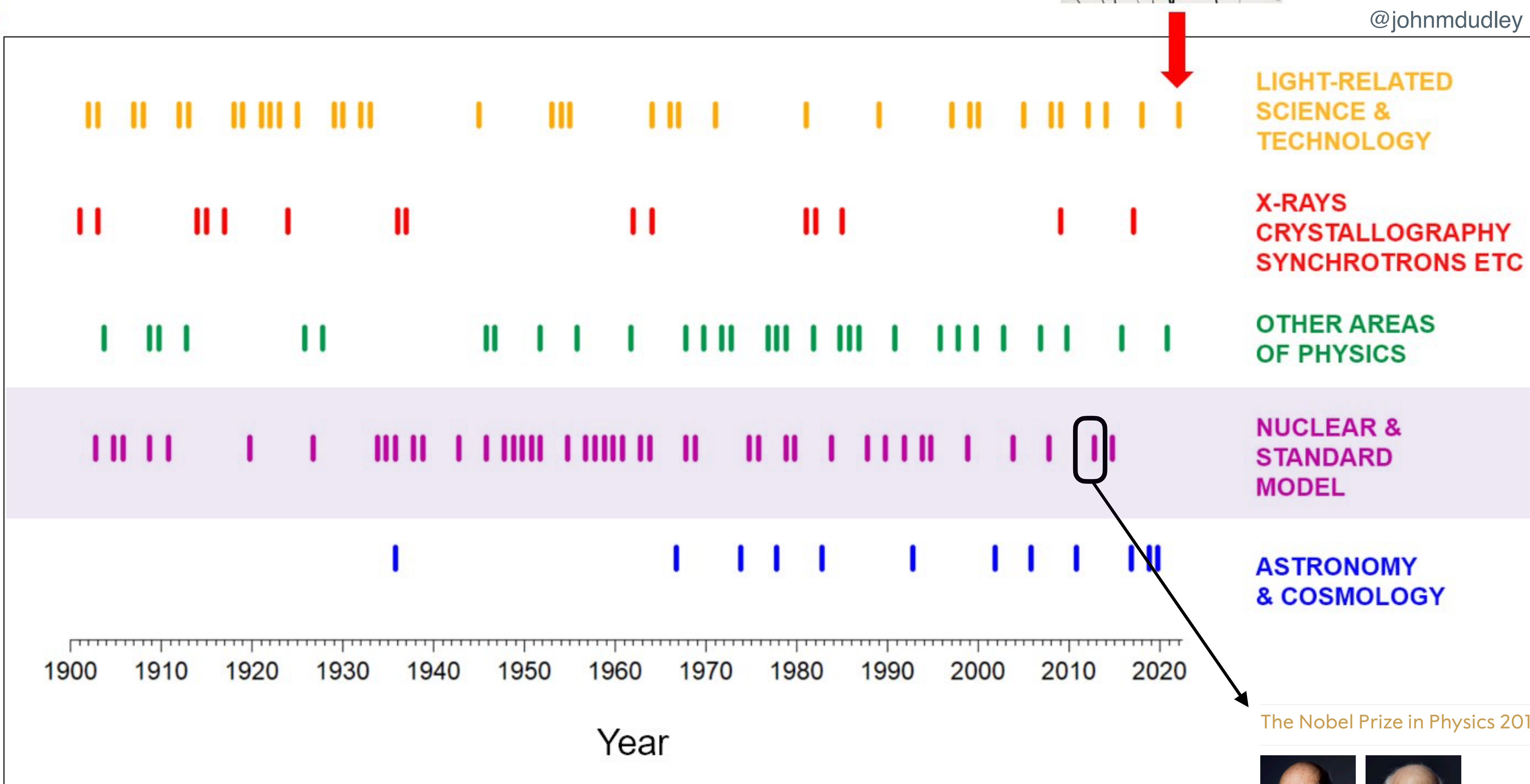


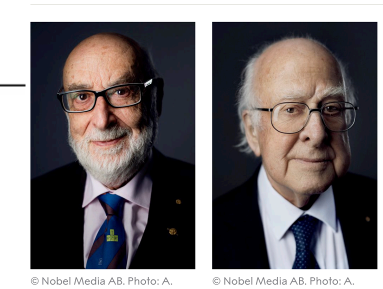
Higgs boson discovery

CERN 2012

A timeline of the Nobel Prize



The Nobel Prize in Physics 2013



François Englert
Peter W. Higgs

Quantum fields

- The Basic Building Blocks of the Universe

Operator on the Hilbert space of particle states

$$\hat{\phi}(x)$$

Function of spacetime

Quantum + Fields = Particles are **ripples (excitations)** of fields tied into little parcels of energy due to quantum mechanics.

All electrons in the universe are identical copies of each other. They are excitations of a single electron field.

Quantum fields

- Free quantised Dirac field: $\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$

$$\Psi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\mathbf{p}}}} \sum_{s=1,2} \left(a_{\mathbf{p},s} u^s(p) e^{-ipx} + b^\dagger_{\mathbf{p},s} v^s(p) e^{ipx} \right)$$

$$\bar{\Psi} = \Psi^\dagger \gamma^0 \qquad E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2} \qquad spin = 1/2$$

- Particle state

$$a^\dagger_{\mathbf{p},s}|0\rangle$$

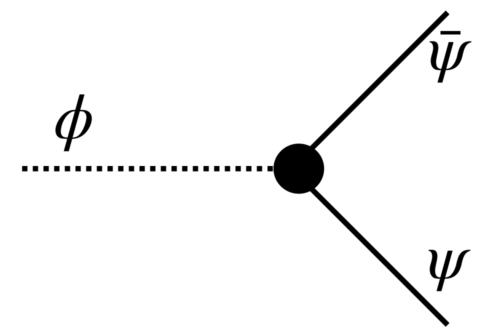
- Antiparticle state

$$b^\dagger_{\mathbf{p},s}|0\rangle$$

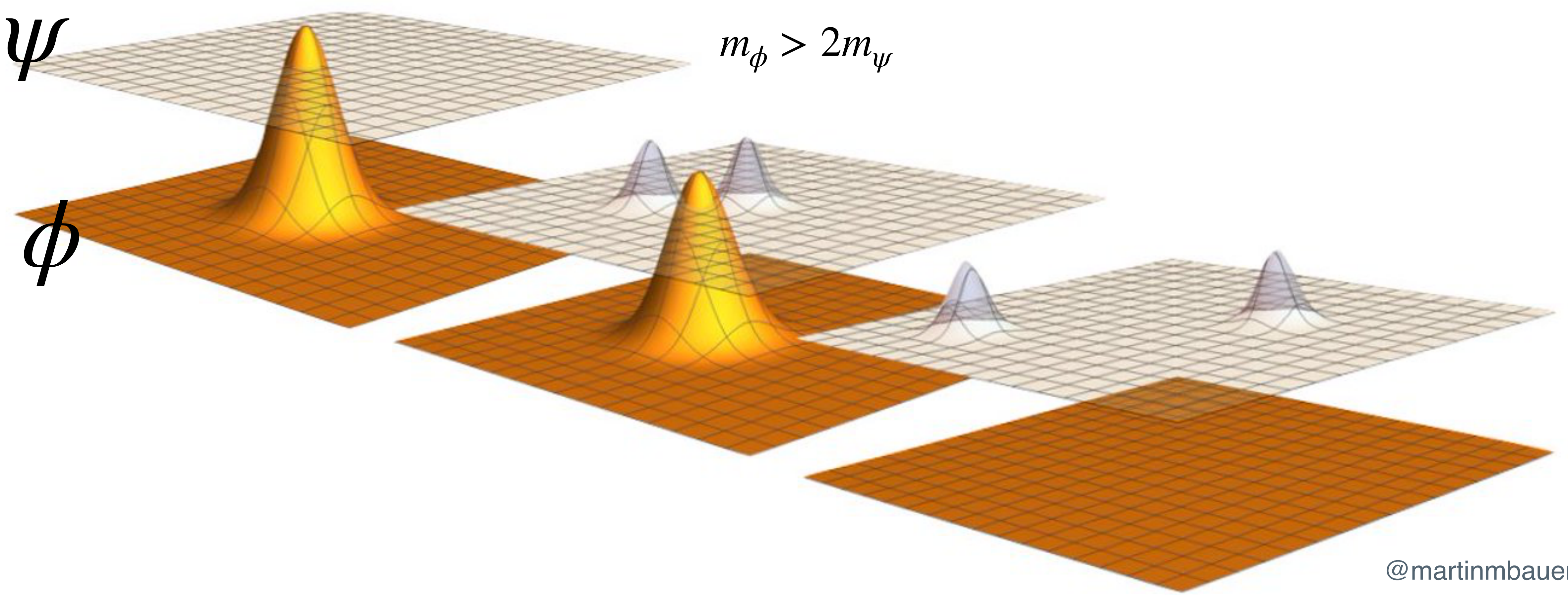
Quantum fields

- Local interactions:

$$\mathscr{L}(x) \supset y\,\phi(x)\bar{\psi}(x)\psi(x)$$

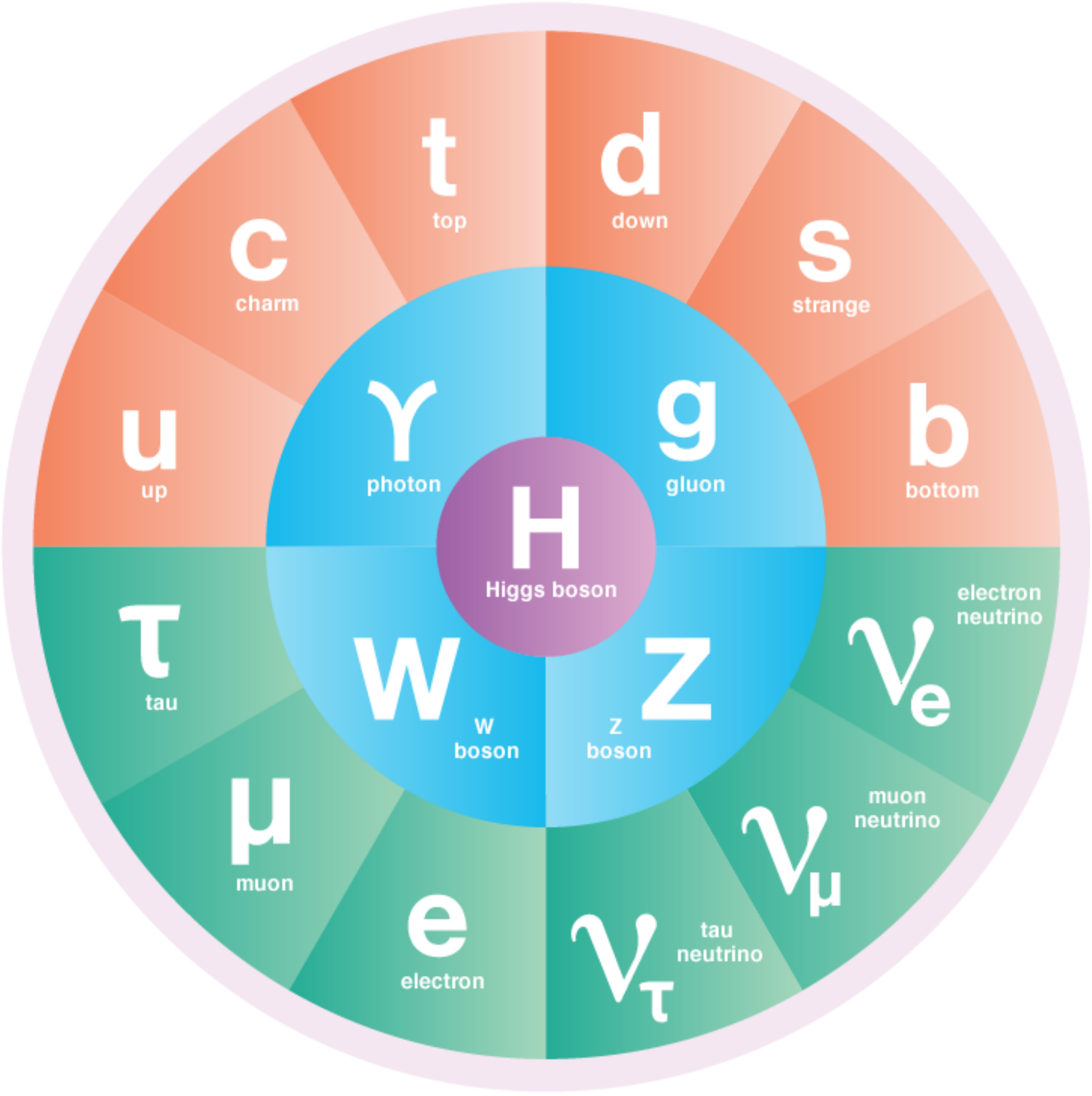


$\propto y$



Decay: The ripple of the ϕ field excites ψ and $\bar{\psi}$ fields

The Standard Model fields



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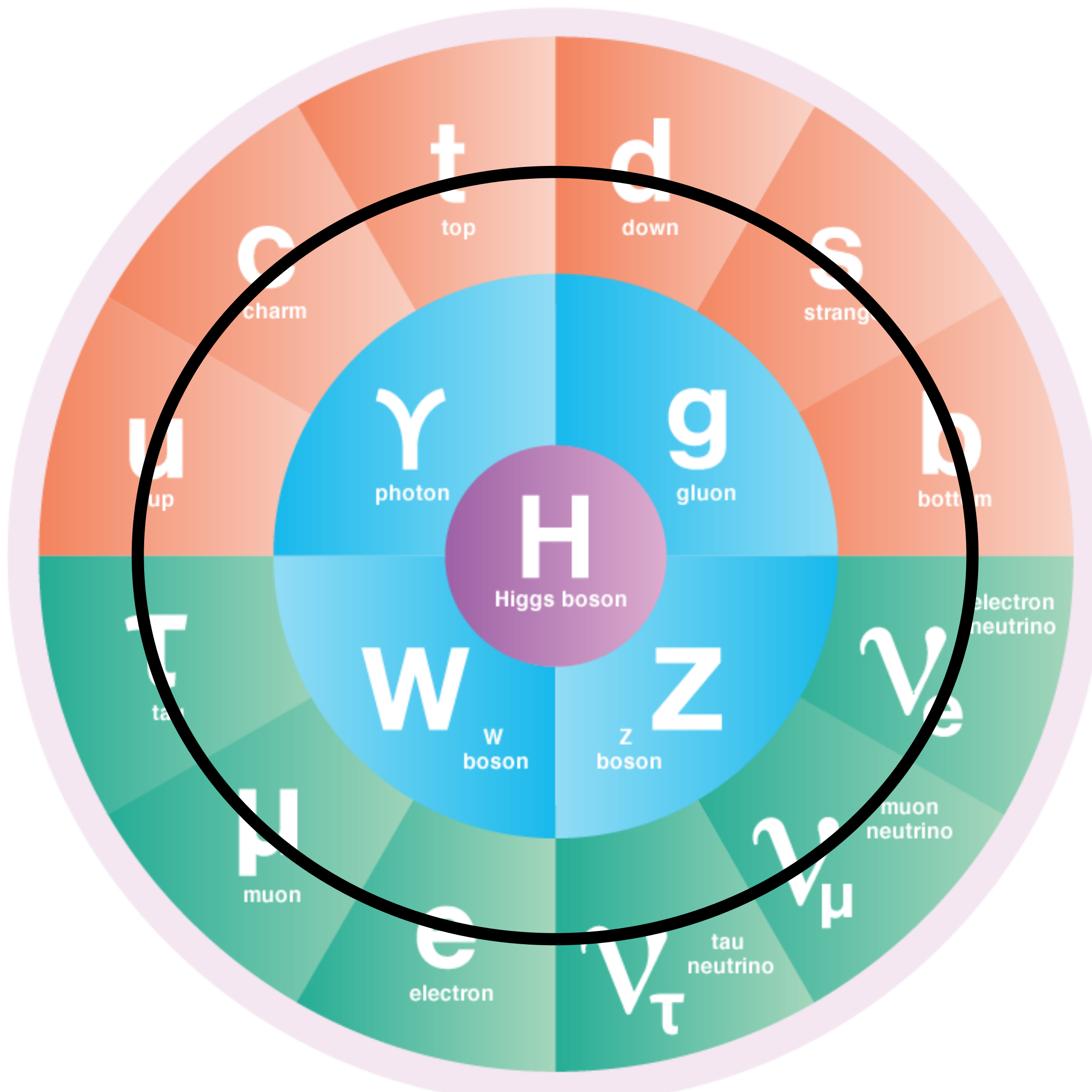
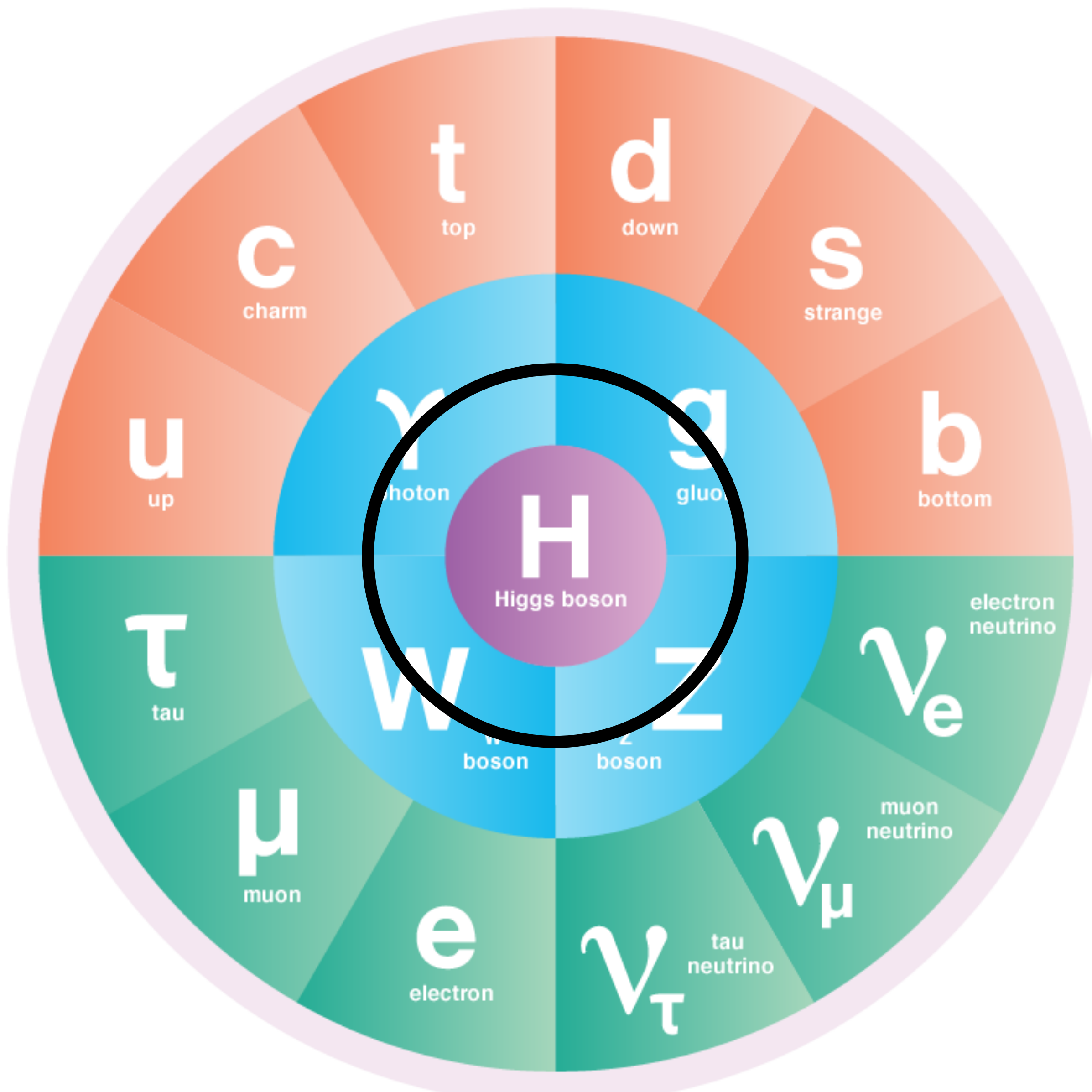


Matter fields

Quarks and Leptons

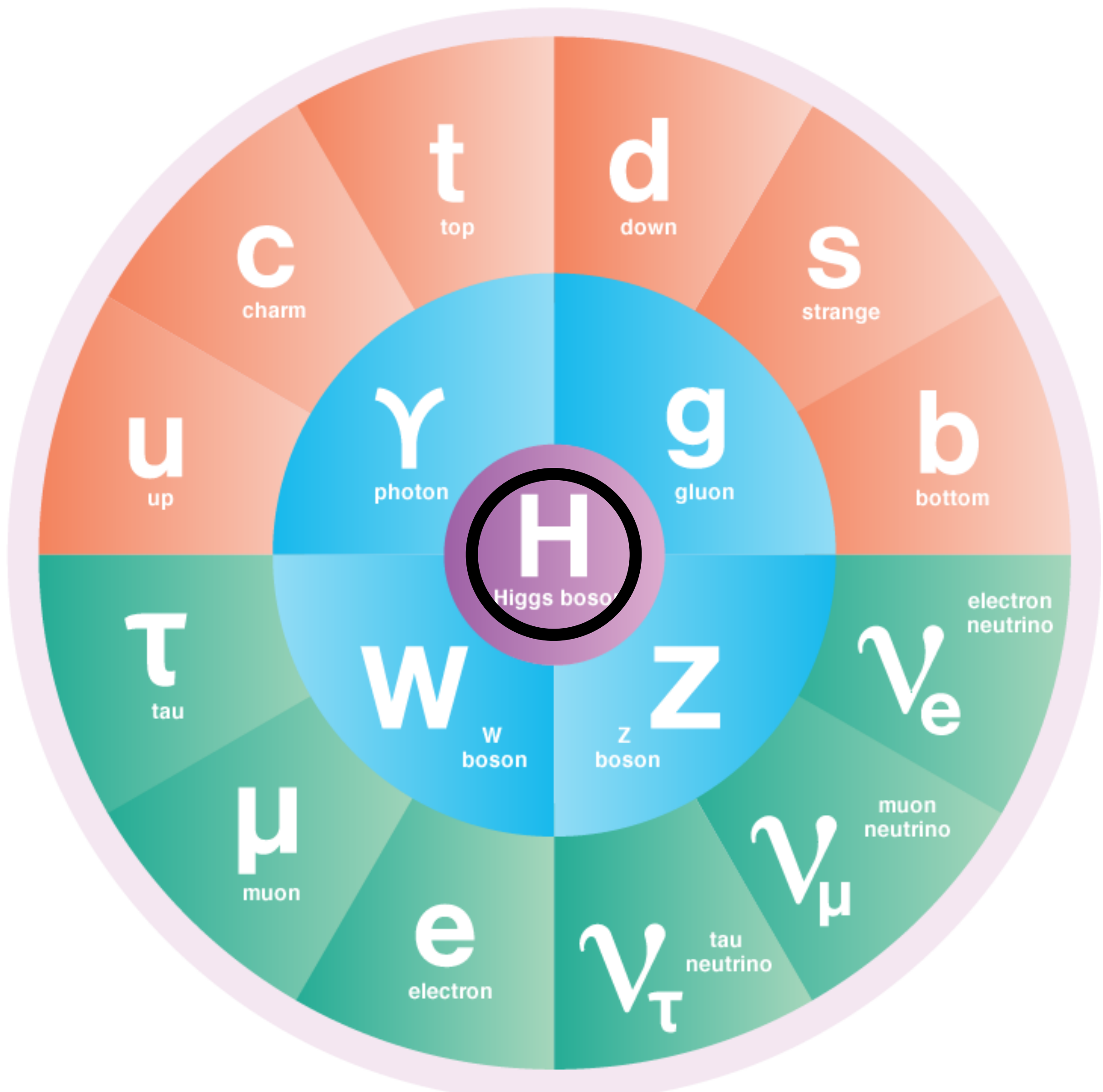
Fermions / spin-1/2

The Standard Model fields



Force carrier fields
Vector bosons / spin-1

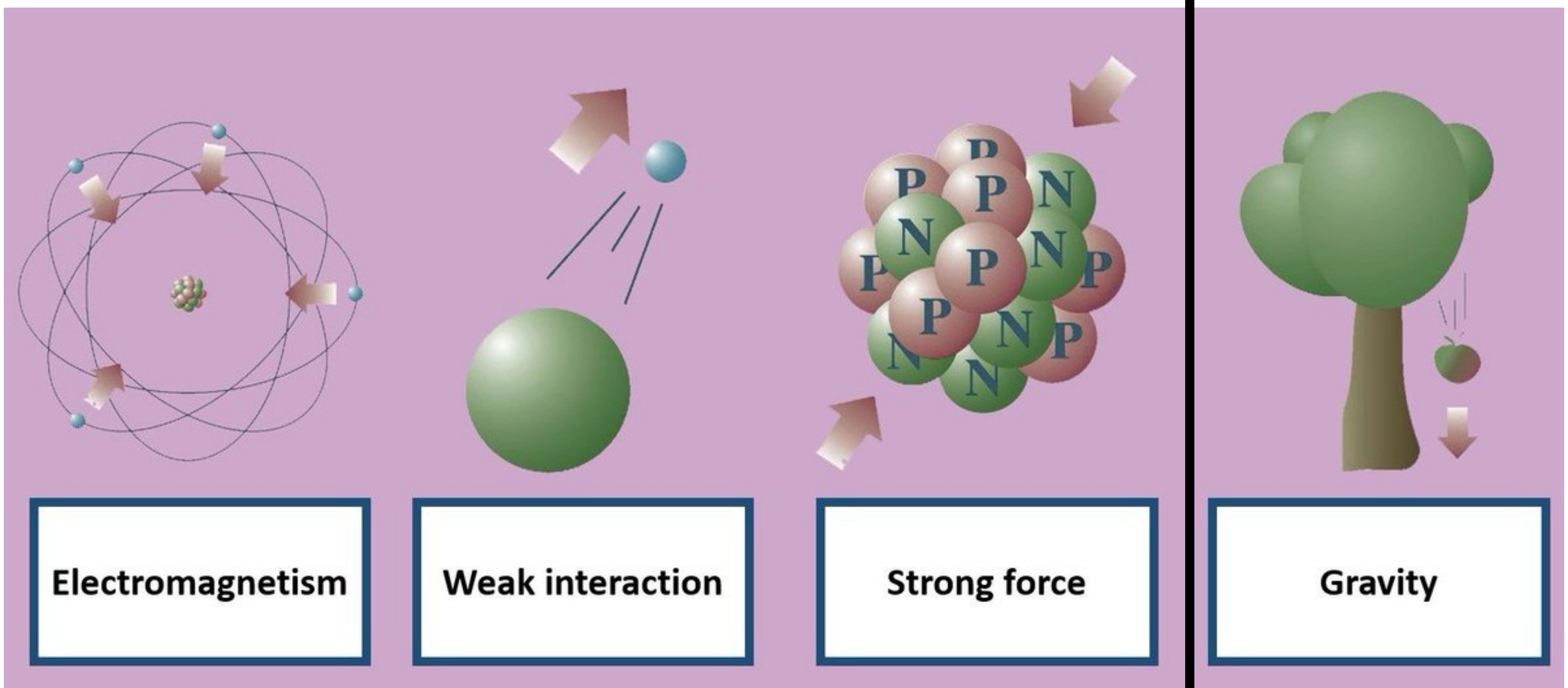
The Standard Model fields



The Higgs field
Scalar boson / spin-0

Fundamental forces

The Standard Model



γ $\quad W, Z \quad$ g

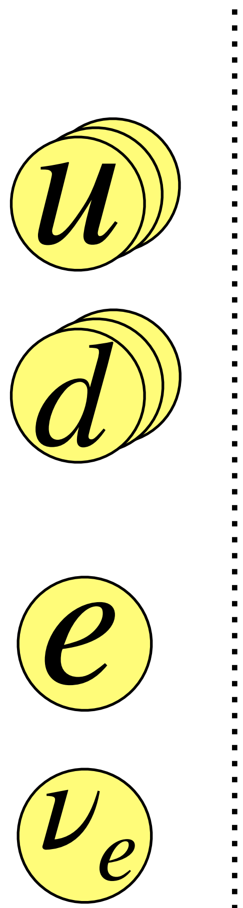
$e^+ \quad e^-$

$F_{EM}/F_{GR} \approx \mathcal{O}(10^{42})$

Elementary Particles of Matter

Quarks

Leptons



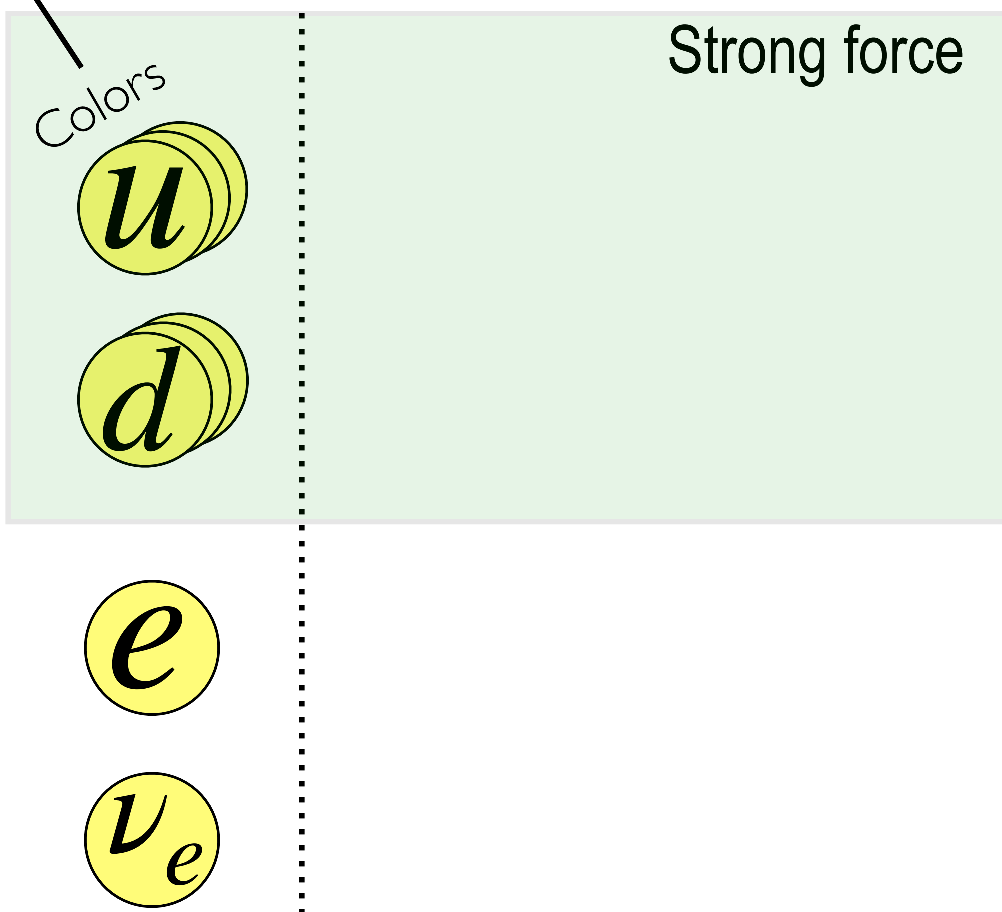
Elementary Particles of Matter

g Long-range ($m_g = 0$) but confining

The charge of the strong force

Quarks

Leptons

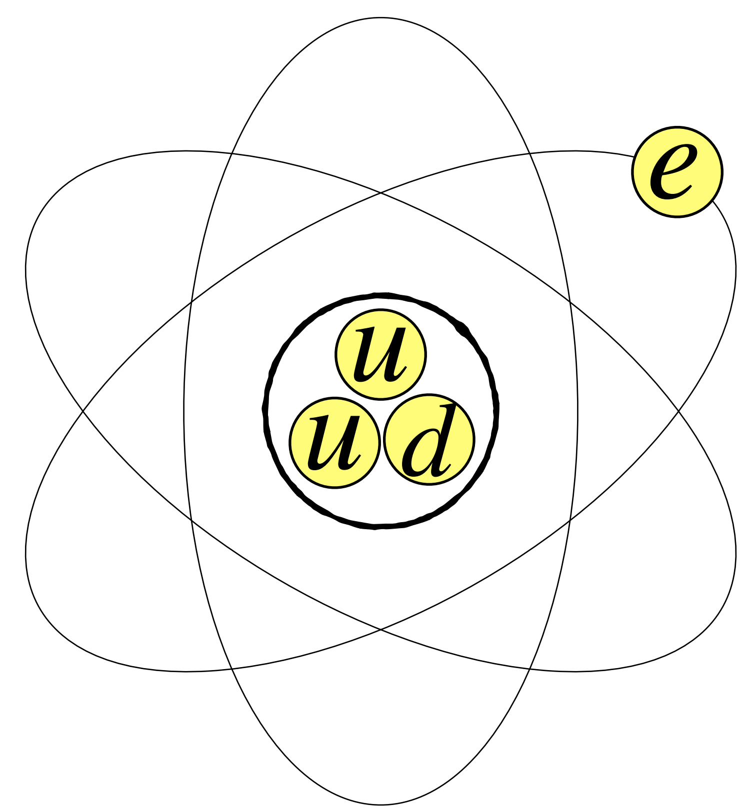


Proton

Elementary Particles of Matter

g γ Long-range ($m_\gamma = 0$)

		Strong force	EM
Quarks	u		$+\frac{2}{3}$
	d		$-\frac{1}{3}$
Leptons	e		-1
	ν_e		0



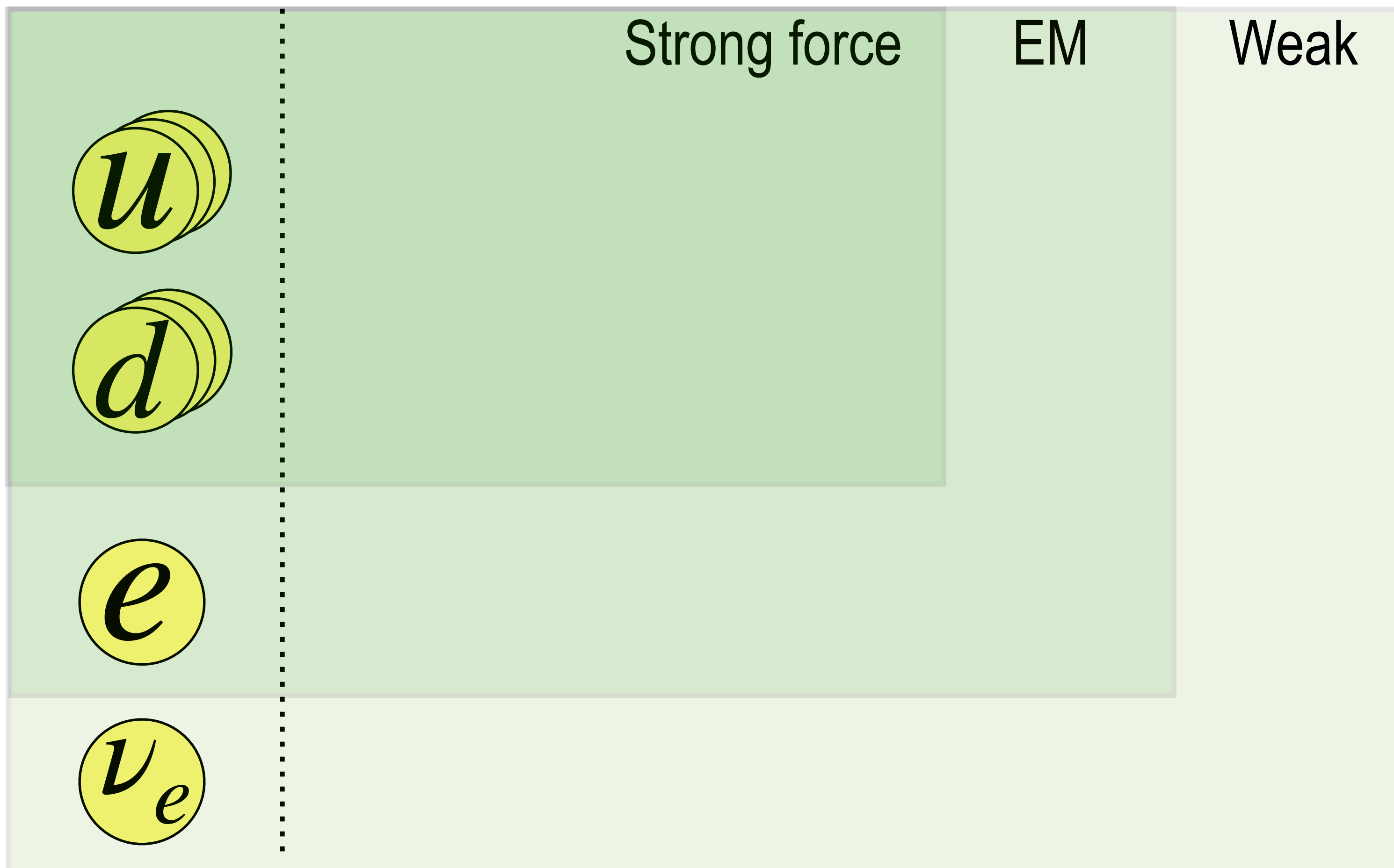
Hydrogen atom

Elementary Particles of Matter

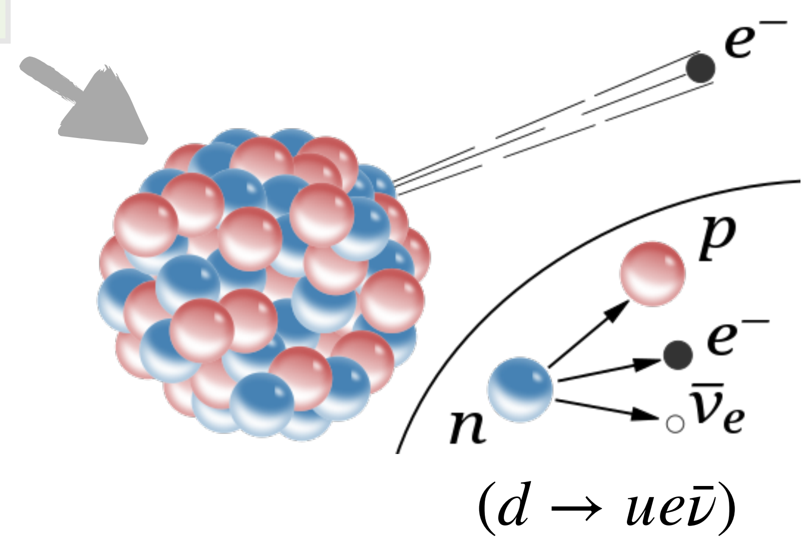
$g \quad \gamma \quad W, Z$ Short-range ($m_{W,Z} \neq 0$)

Quarks

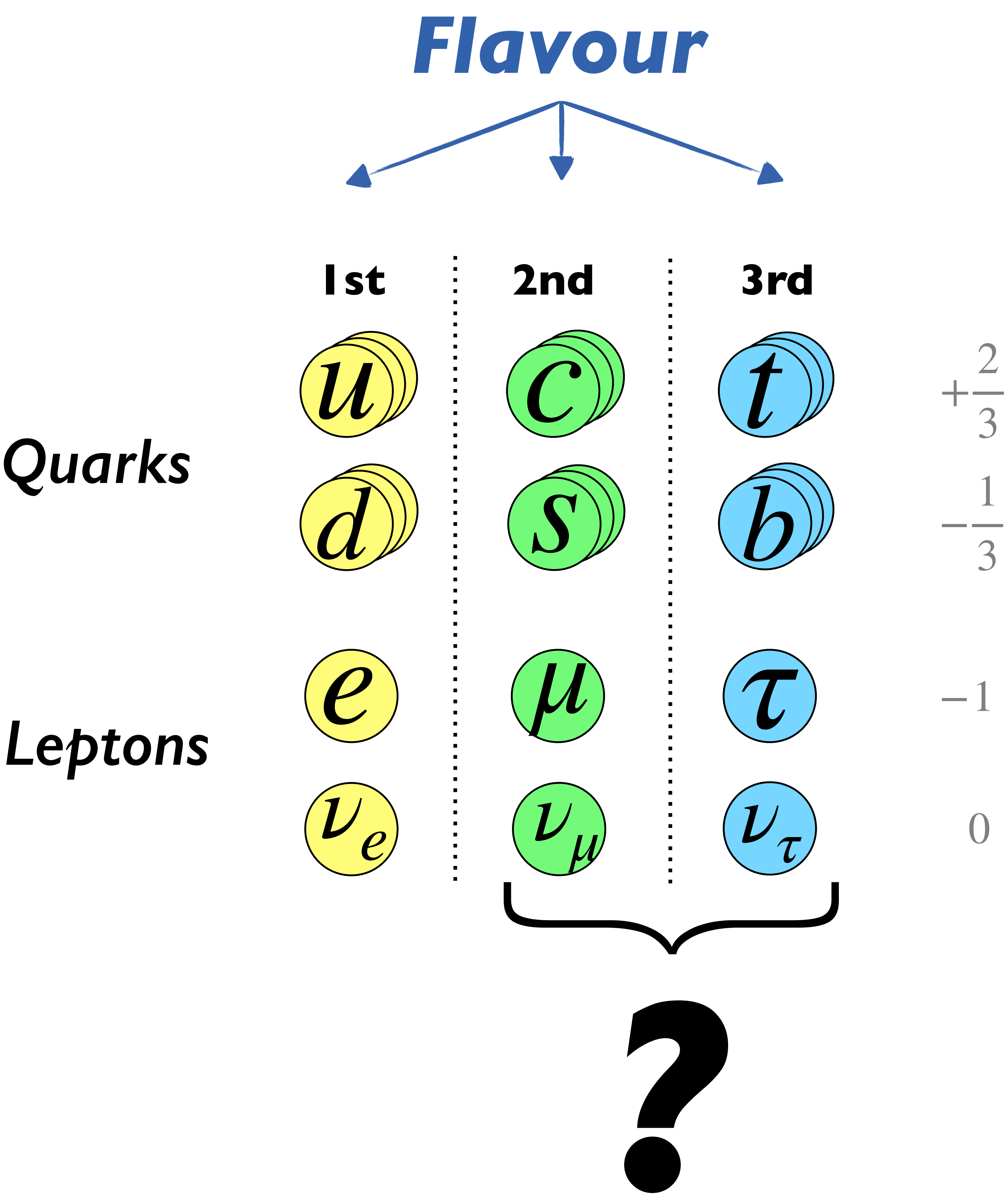
Leptons



✔ Scientific Reductionism



Radioactivity

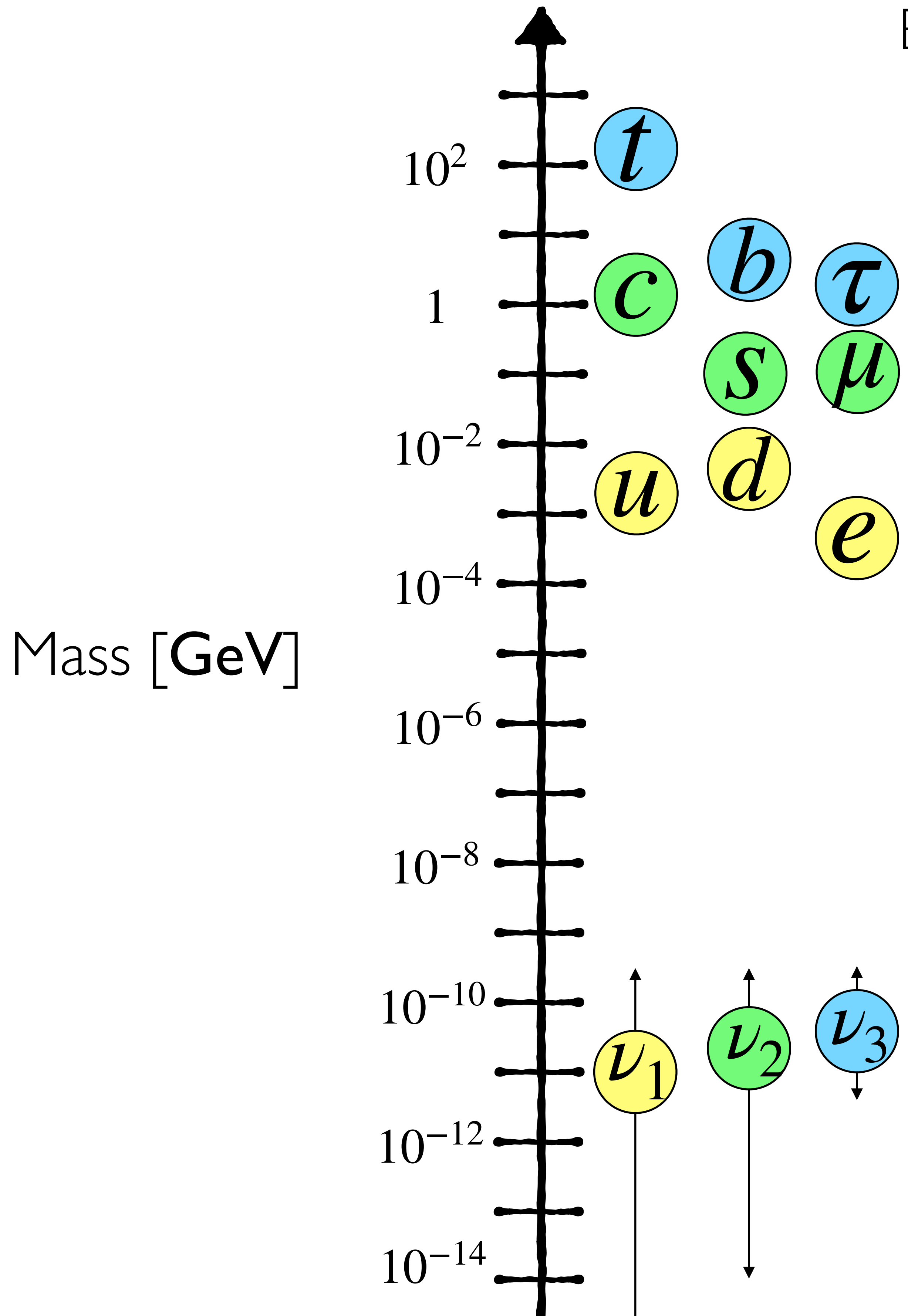


- Generations:

  Mysterious property of matter!

Flavour Puzzle

Empirical



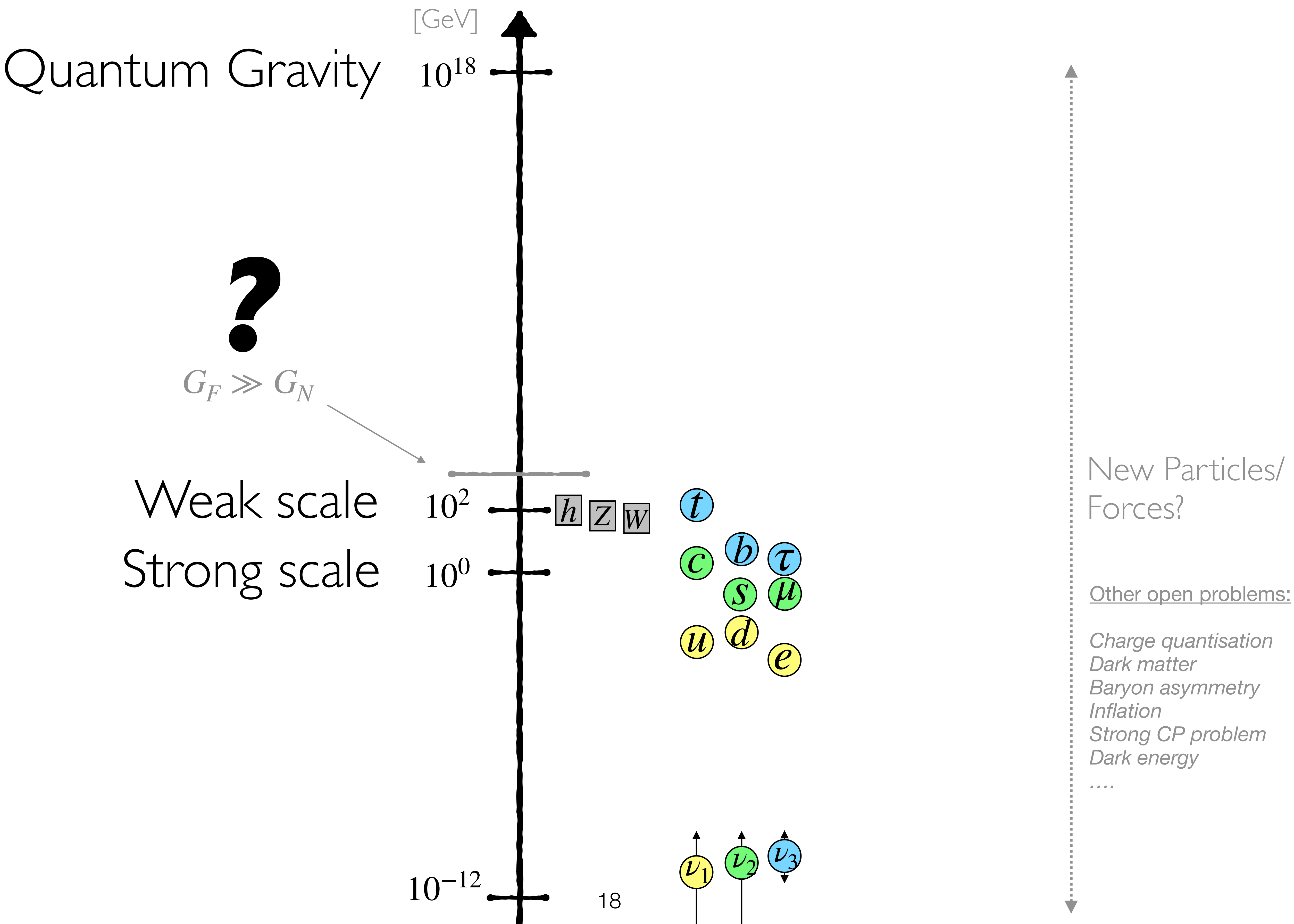
Parameterised in the SM, but not explained!

?

Analogy:
The periodic table of elements

SM predicts massless neutrinos!

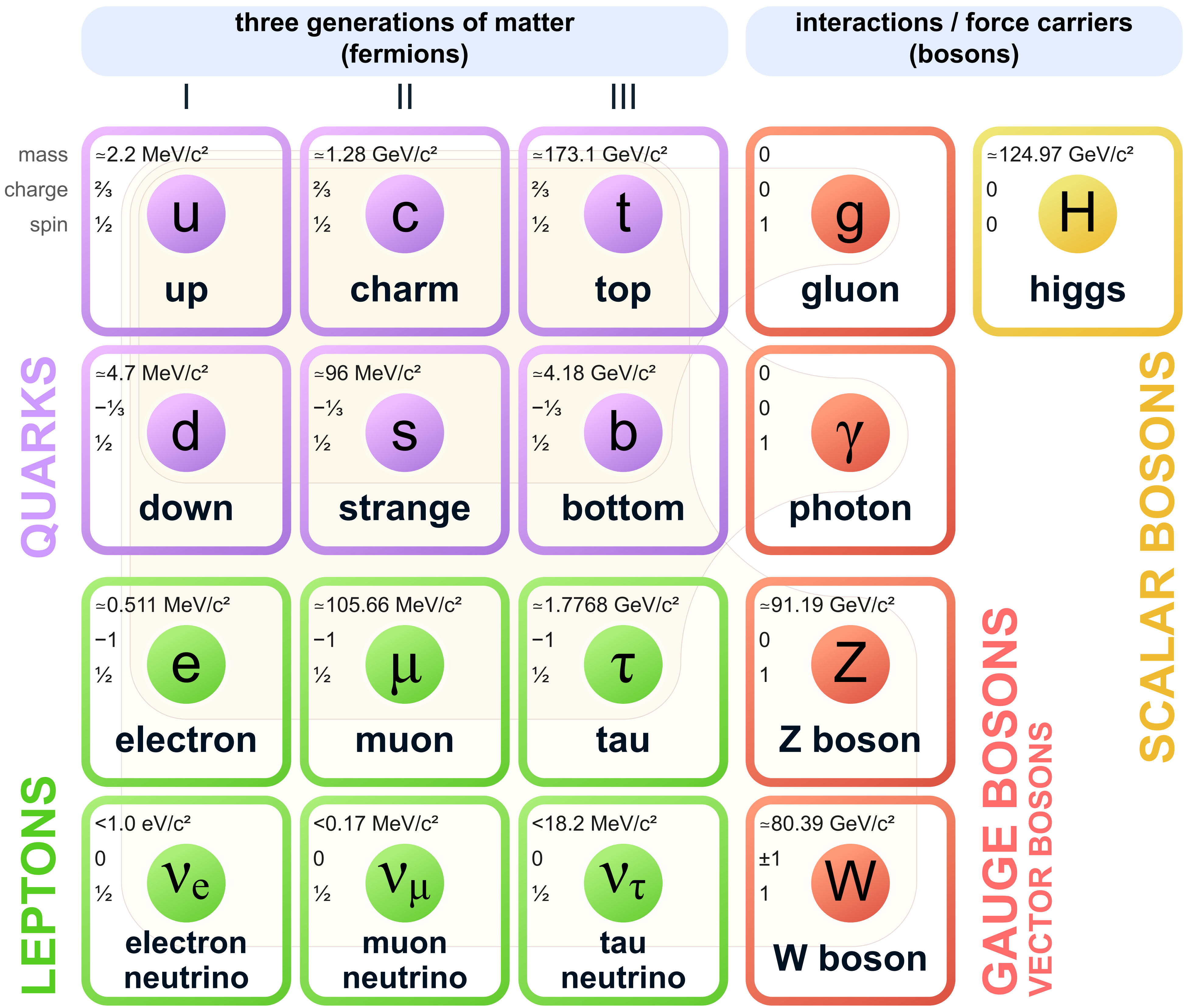
The hierarchy of scales?



Other open problems:

Charge quantisation
Dark matter
Baryon asymmetry
Inflation
Strong CP problem
Dark energy
....

Standard Model of Elementary Particles



Gauge symmetry

Gauge symmetry

- Example: Electrodynamics has a $U(1)$ gauge (or local) symmetry

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- The phase is an arbitrary function over spacetime $\theta(x)$

$$\phi(x) \to e^{i\theta(x)}\phi(x)$$

$$\cancel{\mathscr{L}_{kin} = \partial_\mu \phi^\dagger \partial^\mu \phi}$$

?

Gauge symmetry

- Example: Electrodynamics has a $U(1)$ gauge (or local) symmetry

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- Solution: Introduce a gauge field! Transformation

$$A^\mu(x) \to A^\mu(x) - \frac{1}{g}\partial^\mu\theta(x)$$

- The covariant derivative $D^\mu = \partial^\mu + igA^\mu$

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- The invariant Lagrangian:

$$\mathscr{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mathscr{V}(\phi)$$

- The field strength tensor:
  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

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- The $m^2 A^\mu A_\mu$ is forbidden!

- The field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Quantum electrodynamics

- $U(1)$ gauge theory for a Dirac fermion

$$\mathcal{L}_{\text{QED}} = \bar{\Psi}(i\not{\partial} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - eA_\mu\bar{\Psi}\gamma^\mu\Psi$$

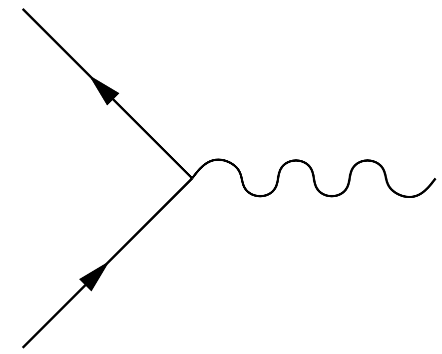
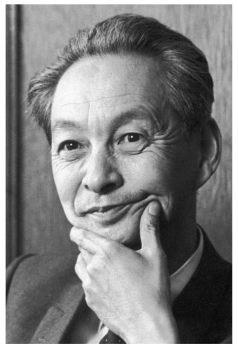


Fig. 5.15 The QED vertex: the solid lines represent the fermions and the wavy line the photon.

The Nobel Prize in Physics 1965



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Sin-Itiro Tomonaga

Photo from the Nobel Foundation archive.
Julian Schwinger

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Richard P. Feynman

"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"

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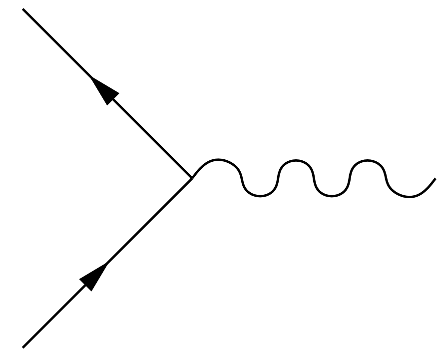

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- Example: Anomalous magnetic moment $\vec{\mu} = g\frac{e}{2m}\vec{S}$

Dirac: $g = 2$

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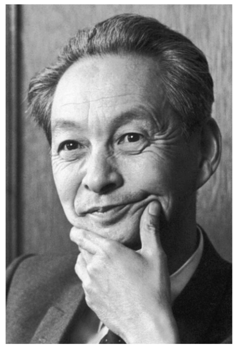

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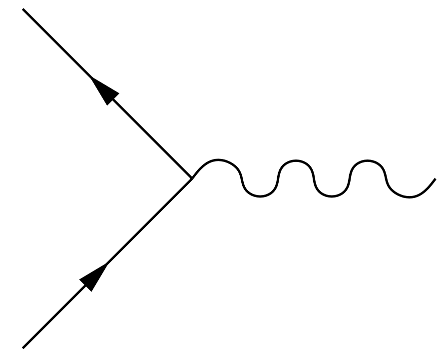

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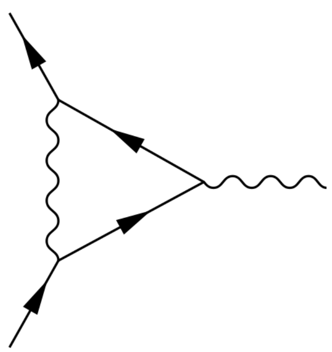

$\alpha = e^2/4\pi$

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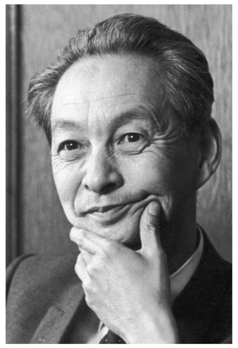

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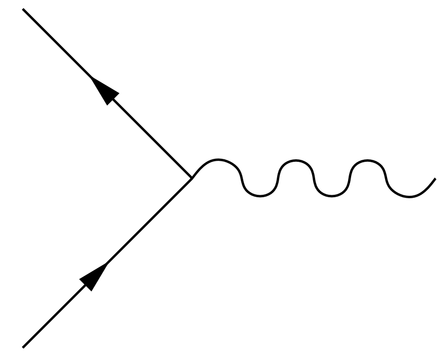

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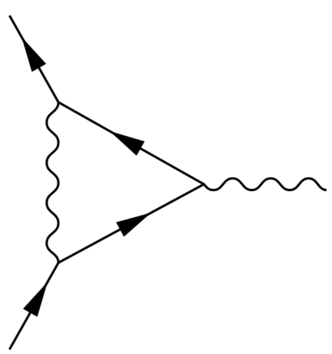

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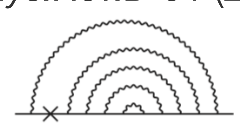
Kinoshita et al $a_e = 0.001\,159\,652\,181\,643(764)$ (5 loops) *Phys.Rev.D* 91 (2015)

Experiment $a_e = 0.001\,159\,652\,180\,73(28)$



6354 diagrams

Quantum chromodynamics

- $SU(3)$ non-Abelian gauge theory
- Quark: Dirac fermion in 3 of $SU(3)$

$$\mathcal{L}_{QCD} = i\bar{\Psi}^{\alpha,A}\not{\partial}\,\Psi^{\alpha,A} - m_A\bar{\Psi}^{\alpha,A}\Psi^{\alpha,A} - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}$$
$$+gA^a_\mu\,\bar{\Psi}^{\alpha,A}\gamma^\mu T^a_{\alpha\beta}\Psi^{\beta,A}\,,$$

$\alpha = 1,2,3$ the color

$A = u,d,c,s,t,b$ the flavor

$a = 1,\ldots,8$ Gluons

NEW!

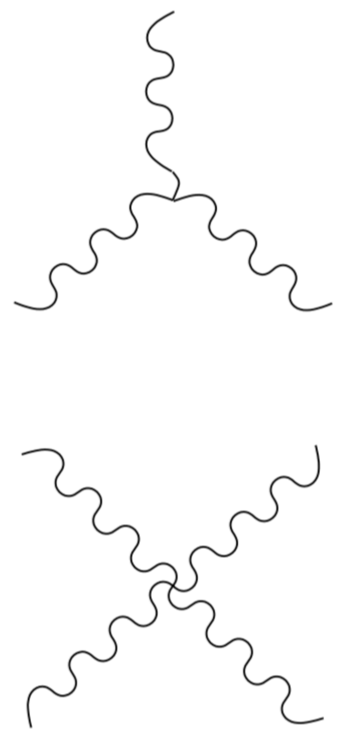
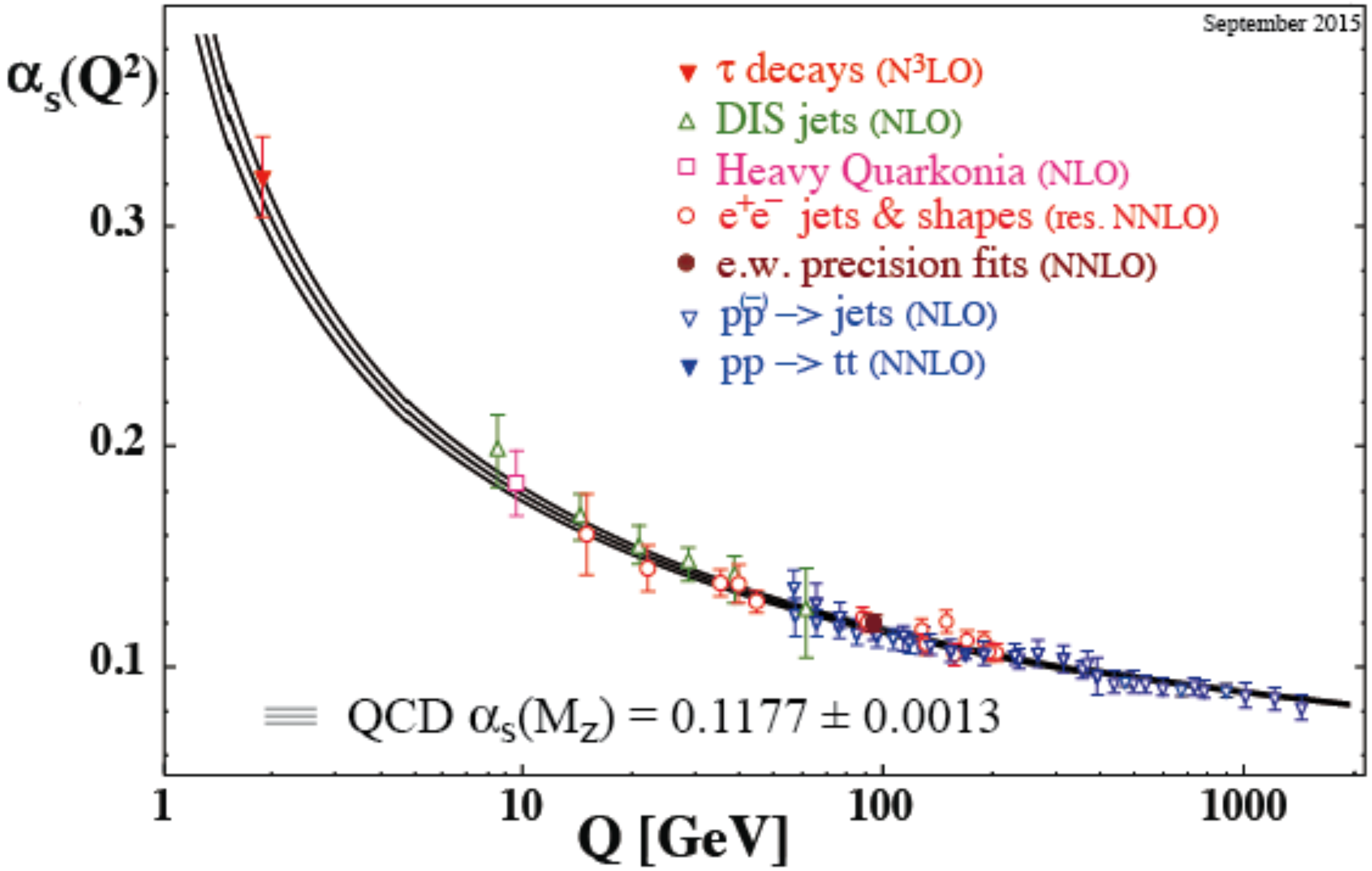


Fig. 10.1 The vertices with three and with four non-abelian gauge bosons.

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

Quantum chromodynamics



The Nobel Prize in Physics 2004

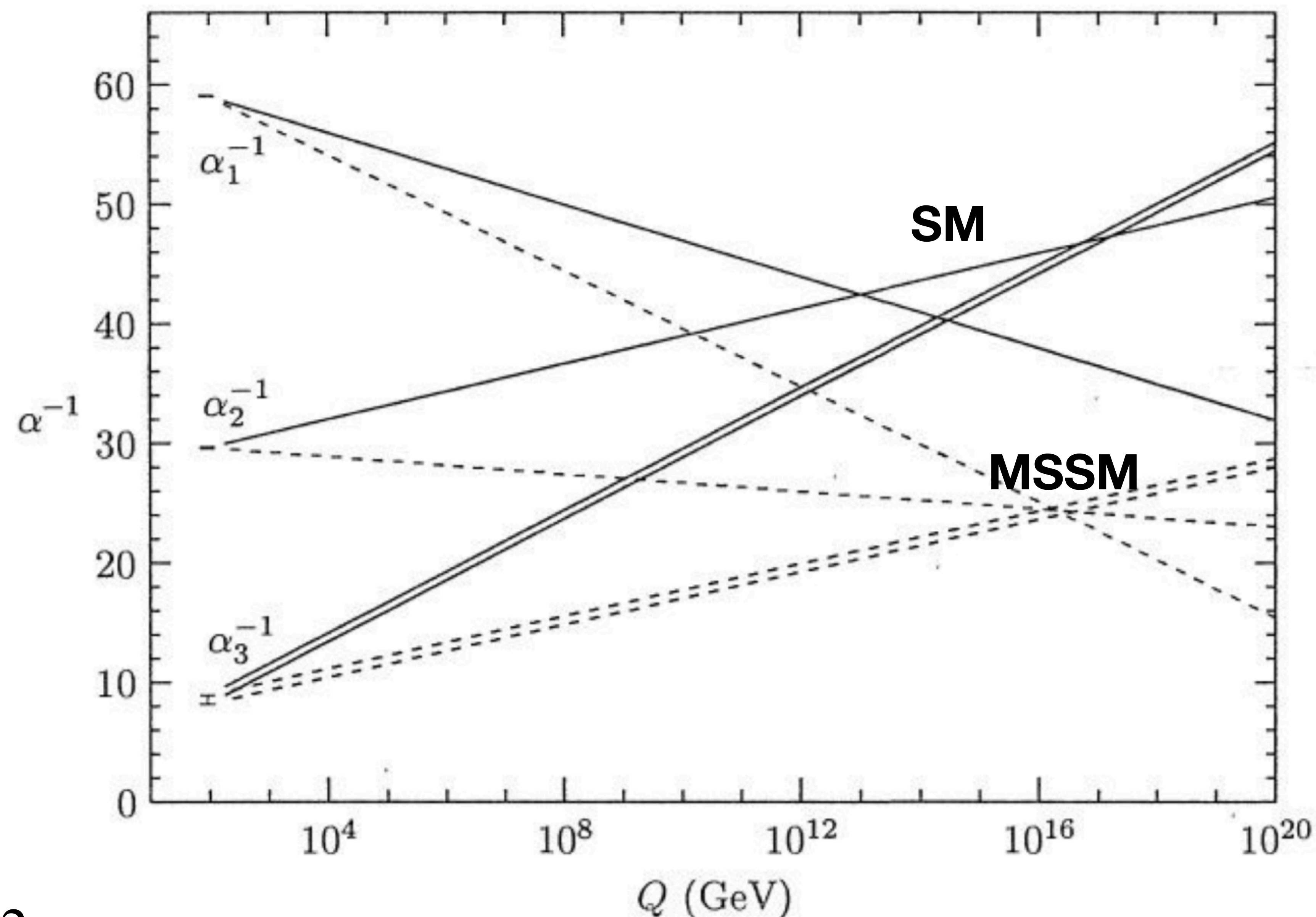
"for the discovery of asymptotic freedom in the theory of the strong interaction"

Confinement ⟷ **Asymptotic freedom**

$$\frac{dg_i}{d\log\mu} = -\frac{b_i}{(4\pi)^2} g_i^3 \qquad b_3 = \frac{33}{3} - \frac{4}{3} N_g$$

The Standard Model running

$$\underbrace{SU(3)_C}_{\text{QCD}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak}}$$



Unification of Forces?

$$\alpha_i = g_i^2/4\pi$$

The Electroweak sector

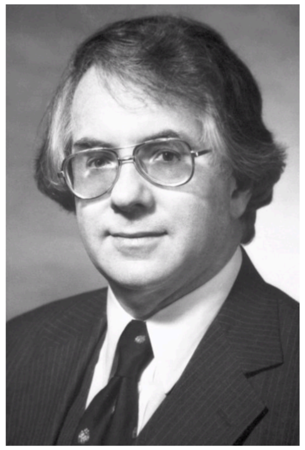
The (Leptonic) Standard Model

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle \phi \rangle \neq 0} U(1)_{EM}$$

$$\mathcal{L}_{\rm kin} = -\frac{1}{4} W_a^{\mu\nu} W_{a\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + i\overline{L_L^i} \not{D} L_L^i + i\overline{E_R^i} \not{D} E_R^i + (D^\mu \phi)^\dagger (D_\mu \phi).$$

$$-\mathcal{L}_\phi = \mu^2 \left(\phi^\dagger \phi\right) + \lambda \left(\phi^\dagger \phi\right)^2 \qquad -\mathcal{L}_{\rm Yuk} = Y_{ij}^e \overline{L_L^i} E_R^j \phi + \text{h.c.}$$

The Nobel Prize in Physics 1979



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Sheldon Lee Glashow

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Abdus Salam

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Steven Weinberg

"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"

(i) The symmetry is a local

$$SU(2)_L \times U(1)_Y.$$

(ii) There are three fermion generations, each consisting of two different representations:

$$L_L^i(2)_{-1/2}, \qquad E_R^i(1)_{-1}, \qquad i = 1, 2, 3.$$

(iii) There is a single scalar multiplet:

$$\phi(2)_{+1/2}.$$

The Higgs field

- How do elementary particles get a mass?

The Higgs mechanism

- The Higgs field plays a key role!
- The Higgs particle is the excitation of the Higgs field.

Spontaneous symmetry breaking

- Complex scalar field: $\mathscr{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mathscr{V}$

Spontaneous symmetry breaking

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- Assume $U(1)$ symmetry: *for the moment GLOBAL
  $\phi(x) \to e^{i\theta}\phi(x)$

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- The potential:

$$\mathscr{V} = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

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  - What about μ^2 ?

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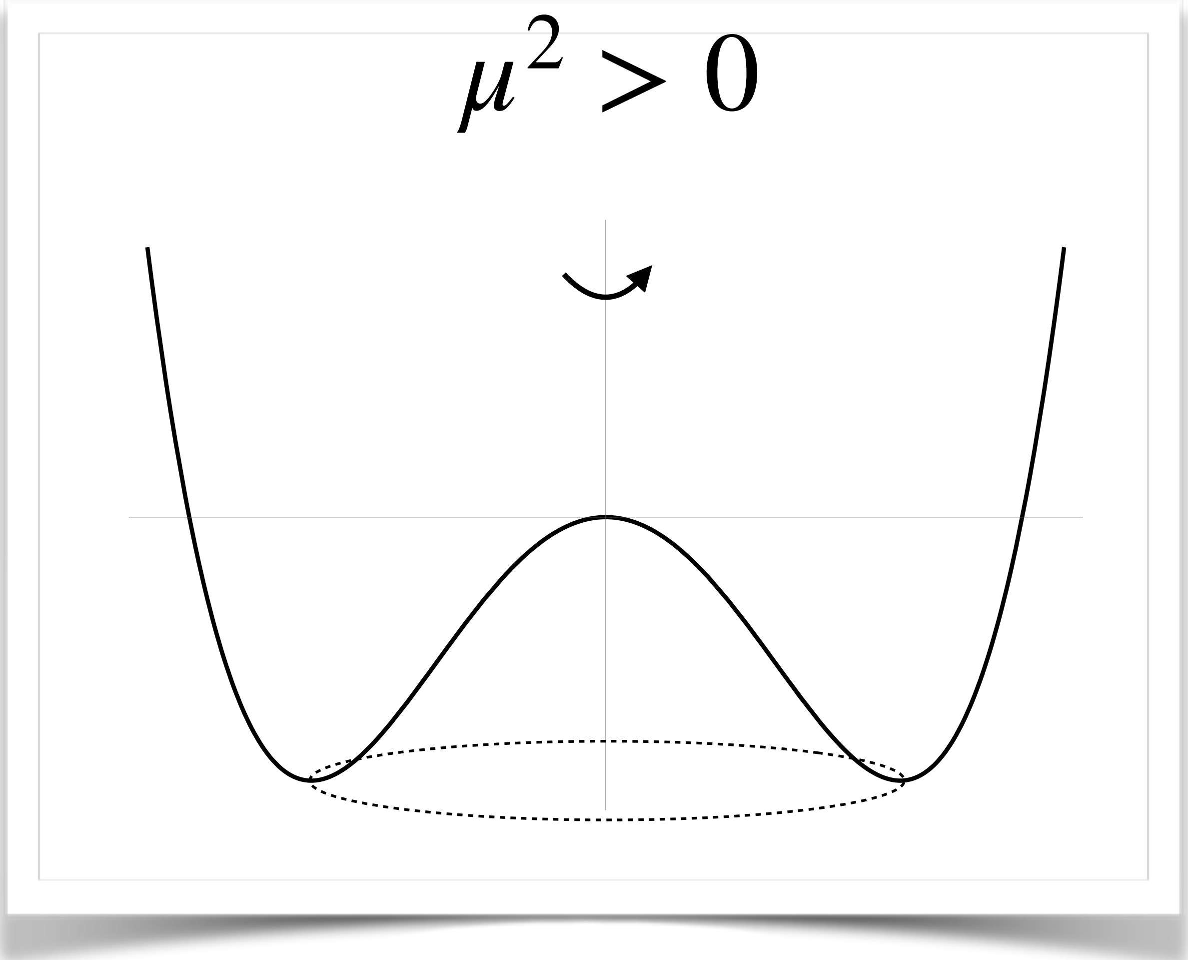

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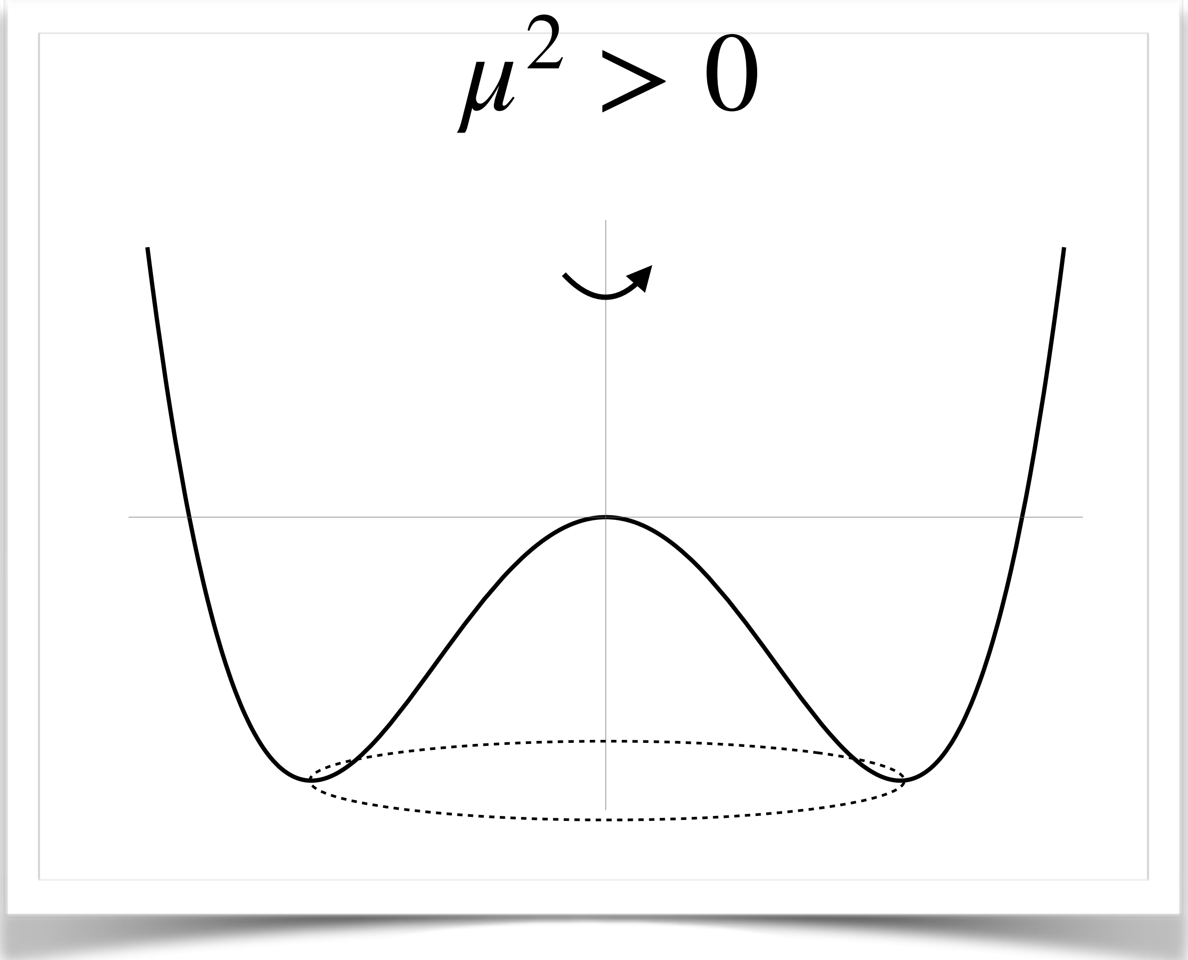
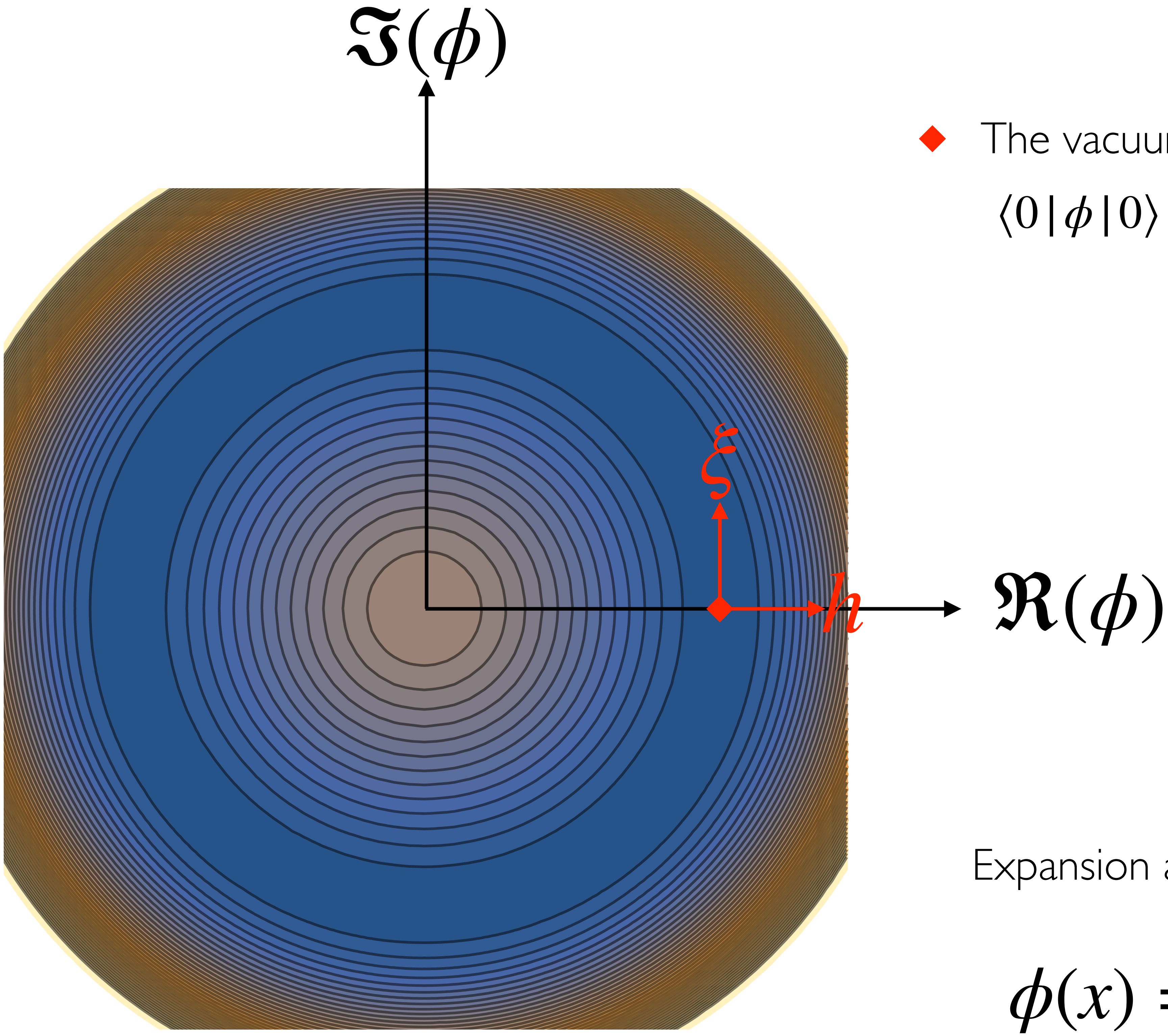


- <u>SSB phenomena</u>:
  Theory has a symmetry but predicts multiple degenerate asymmetrical ground states.

Spontaneous symmetry breaking



- The vacuum expectation value (VEV)
  $\langle 0|\phi|0\rangle \equiv \langle\phi\rangle = v/\sqrt{2} = \mu/\sqrt{2\lambda}$

Expansion around a ground state

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}}\, e^{i\frac{\xi(x)}{v}}$$

Spontaneous symmetry breaking

- Expansion around a ground state

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}}\, e^{i\frac{\xi(x)}{v}}$$

- $h(x)$ - The Higgs

  Massive particle

- $\xi(x)$ - the Goldstone

  Massless particle

$$\mathcal{V} = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

$$m_h^2 = \left.\frac{\partial^2 \mathcal{V}}{\partial h^2}\right|_{h=0}$$

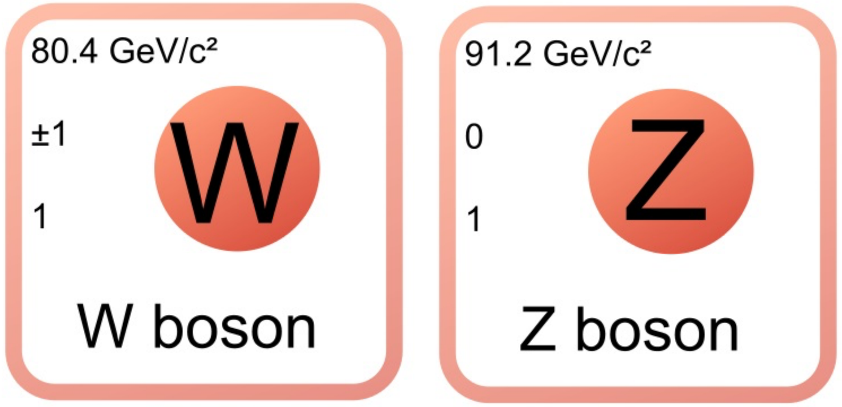
$$m_\xi^2 = 0$$

The Higgs mechanism

- In the SM, the Higgs mechanism gives masses to:
  - Weak force carriers: $W^{\pm}, Z$
  - Matter: Quarks and Leptons

The Higgs mechanism

- The symmetry is gauged when $\theta \to \theta(x)$.
- This introduces a vector field $A_\mu(x)$.
- Gauge theories predict massless $A_\mu(x)$ with 2 d.o.f.
- When SSB happens, the vector field becomes massive (3 d.o.f)!
- The Goldstone boson is the longitudinal polarisation of $A_\mu(x)$.



- $\xi(x)$ - the Goldstone
- Massless particle

The Higgs mechanism

- Start with

$$\mathscr{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathscr{V}(\phi)$$

- And assume $\mathscr{V}(\phi)$ satisfies the SSB condition

The Higgs mechanism

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$$\mathscr{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathscr{V}(\phi)$$

- And assume $\mathscr{V}(\phi)$ satisfies the SSB condition
- Expand around the minimum: $\phi(x) = \dfrac{v + h(x)}{\sqrt{2}} e^{i\frac{\xi(x)}{v}}$
- Fix a gauge: $\theta(x) = -\xi(x)/v$

The Higgs mechanism

- Start with

$$\mathscr{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathscr{V}(\phi)$$

- And assume $\mathscr{V}(\phi)$ satisfies the SSB condition
- Expand around the minimum: $\phi(x) = \dfrac{v + h(x)}{\sqrt{2}} e^{i\frac{\xi(x)}{v}}$
- Fix a gauge: $\theta(x) = -\xi(x)/v$
- The gauge boson eats up the Goldstone boson to become massive!

$$(D_\mu \phi)^\dagger (D^\mu \phi) \longrightarrow \mathscr{L} \supset \frac{1}{2} g^2 v^2 A_\mu A^\mu$$

The covariant derivative: $D^\mu = \partial^\mu + igA^\mu$

[Just a bit more complicated for $SU(2) \times U(1) \to U(1)$]

The Higgs mechanism

- Weak force carriers

Spontaneous symmetry breaking

The Goldstone

- 1 d.o.f.

The Higgs mechanism

- Weak force carriers

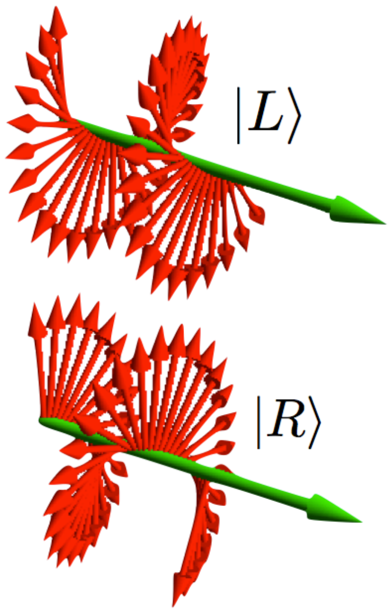
$$\theta \to \theta(x)$$

$\boxed{\text{Spontaneous symmetry breaking}}$ + $\boxed{\text{Gauge symmetry}}$

The Goldstone

- 1 d.o.f.

Massless Vector



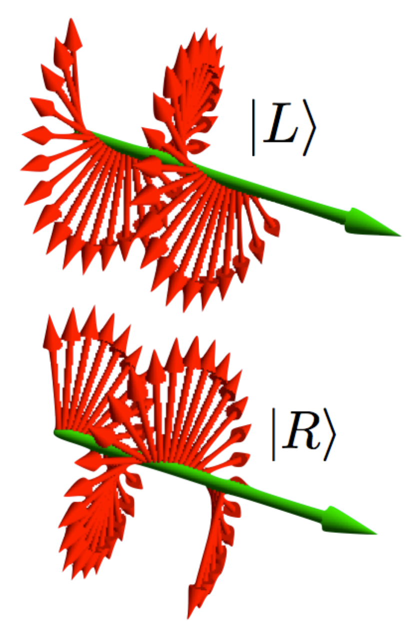
- 2 d.o.f.

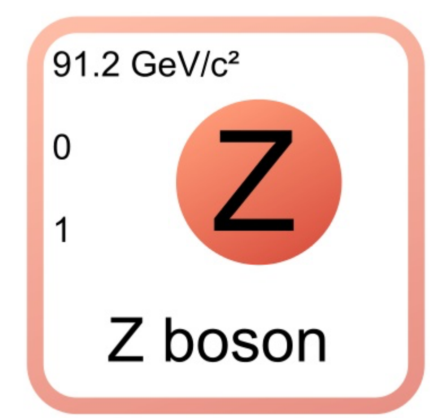
The Higgs mechanism

- Weak force carriers

$$\theta \to \theta(x)$$

Spontaneous symmetry breaking	+	Gauge symmetry	=	The Higgs mechanism
The Goldstone		Massless Vector		Massive Vector

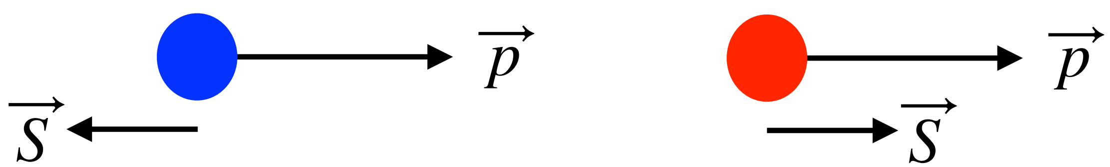




- 1 d.o.f.
- 2 d.o.f.
- 3 d.o.f.

The Higgs mechanism

- Matter: Quarks and Leptons

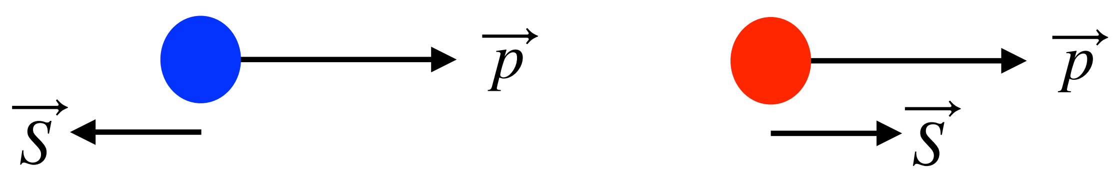


- The *left-handed* and the *right-handed* fields have different $U(1)_Y$ phases:

$$\theta_{f_L} \neq \theta_{f_R} \quad \Longrightarrow \quad \text{The mass } m_f \bar{f}_L f_R \text{ is forbidden!}$$

The Higgs mechanism

- Matter: Quarks and Leptons



- The *left-handed* and the *right-handed* fields have different $U(1)_Y$ phases:

$$\theta_{f_L} \neq \theta_{f_R} \quad \Longrightarrow \quad \text{The mass } m_f \bar{f}_L f_R \text{ is forbidden!}$$

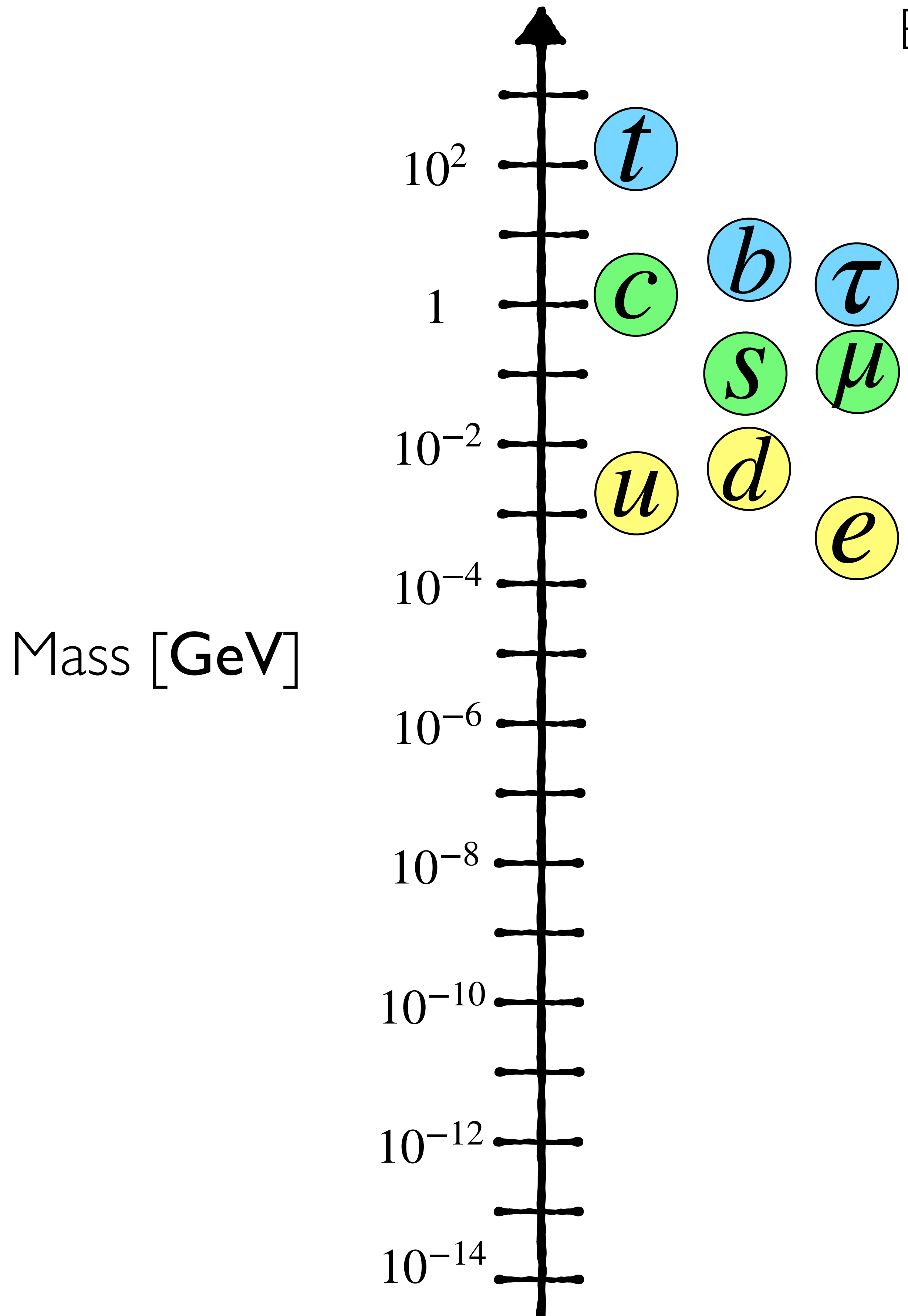
- The Higgs field saves the day, $\theta_H + \theta_{f_R} = \theta_{f_L}$

$$\mathscr{L} \supset -y_f \bar{f}_L f_R \phi \quad \overset{\text{SSB}}{\Longrightarrow} \quad m_f = y_f \langle \phi \rangle$$

- The mass $\propto$ the strength of the interaction with the Higgs field

Flavour Puzzle

Empirical



$$-\mathscr{L} \supset y_f\, \bar{f}_L f_R\, \phi$$

?

Analogy

*Credit to Professor David J Miller
Here is my adaption:

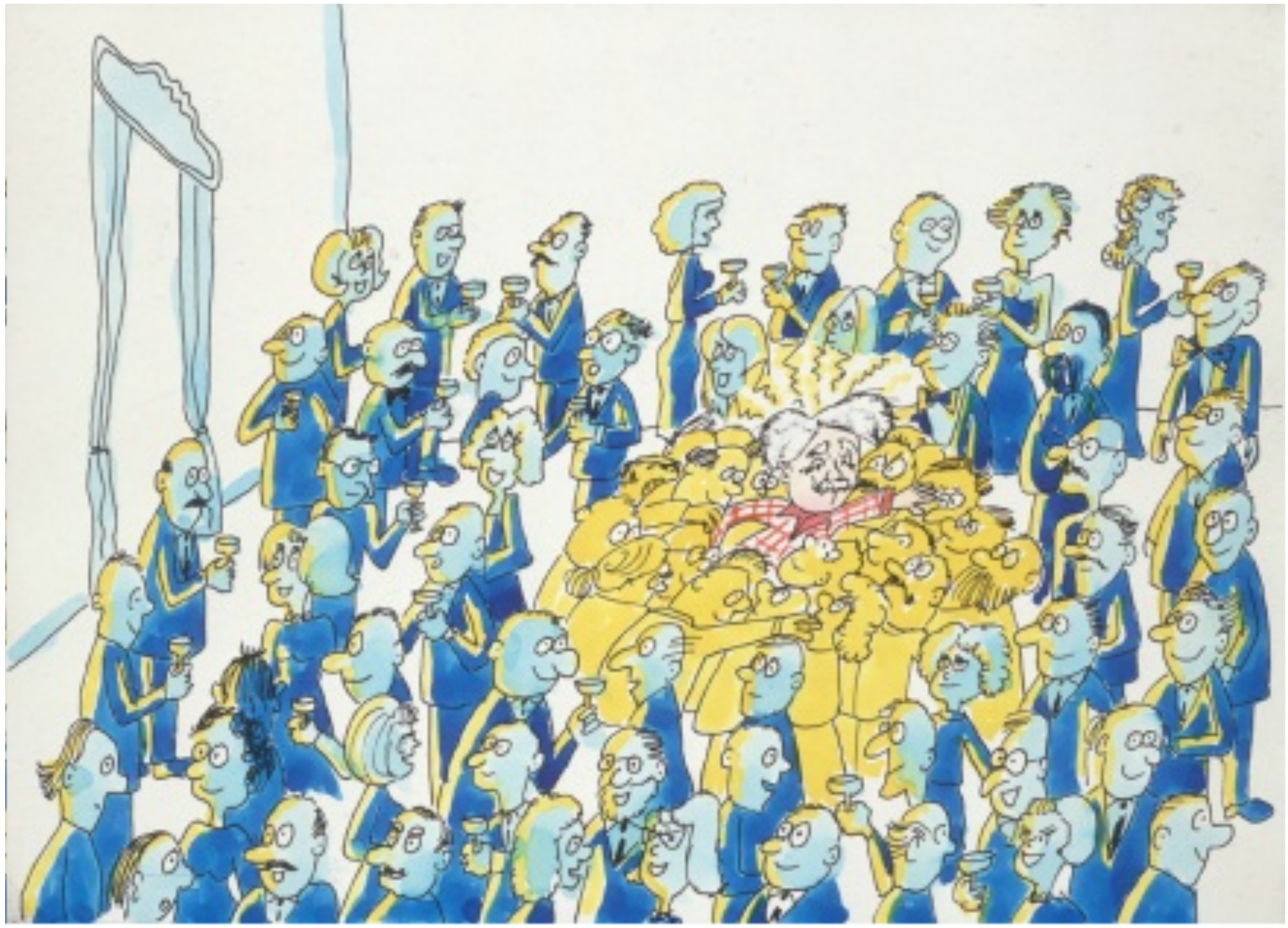
The Higgs field

Analogy

Top quark, $m_t = 173\,\mathrm{GeV}$

Analogy

Top quark, $m_t = 173\,\mathrm{GeV}$



Analogy

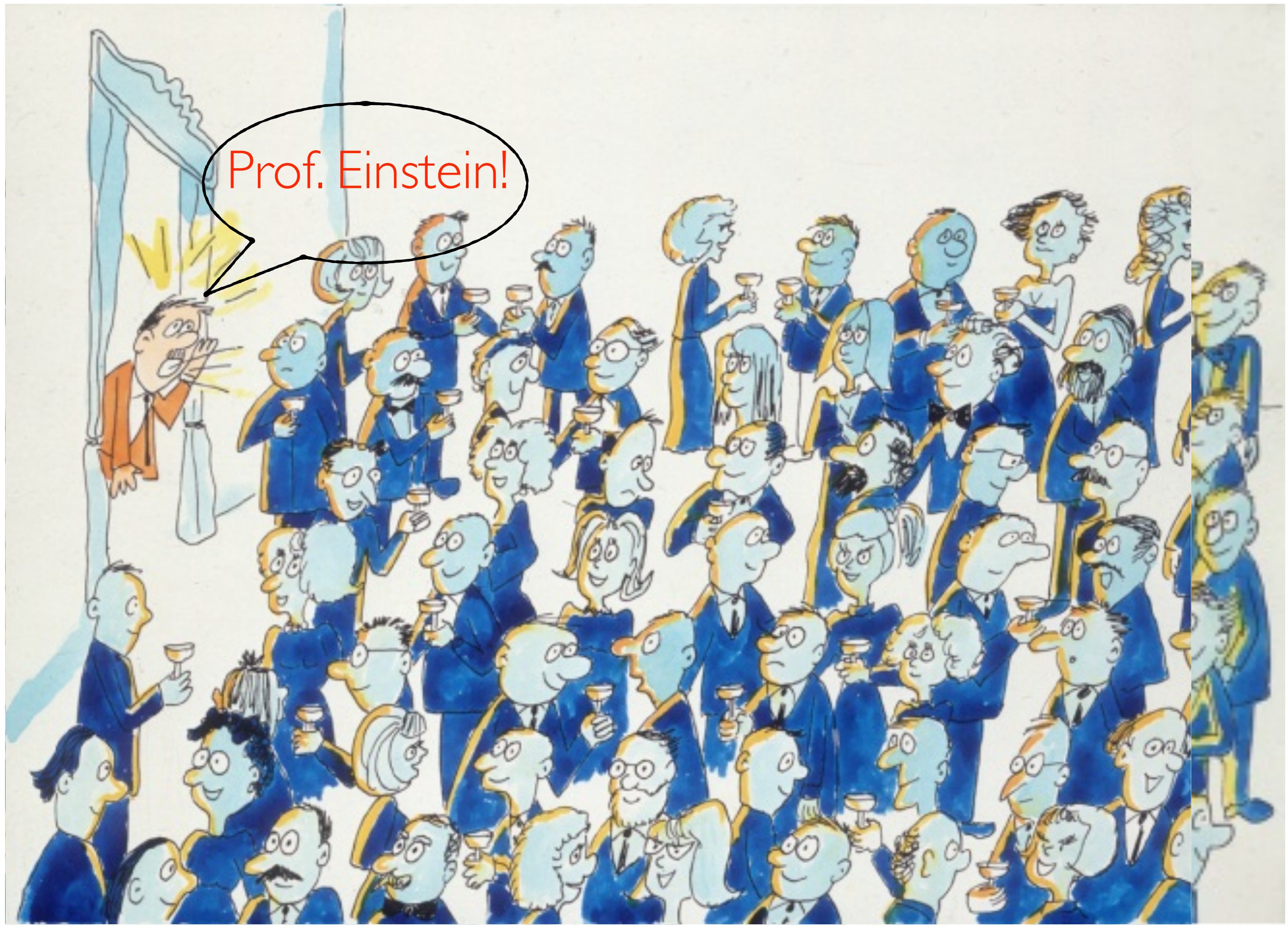
Electron, $m_e = 0.0005\,\mathrm{GeV}$

Analogy

Electron, $m_e = 0.0005\,\mathrm{GeV}$

Analogy

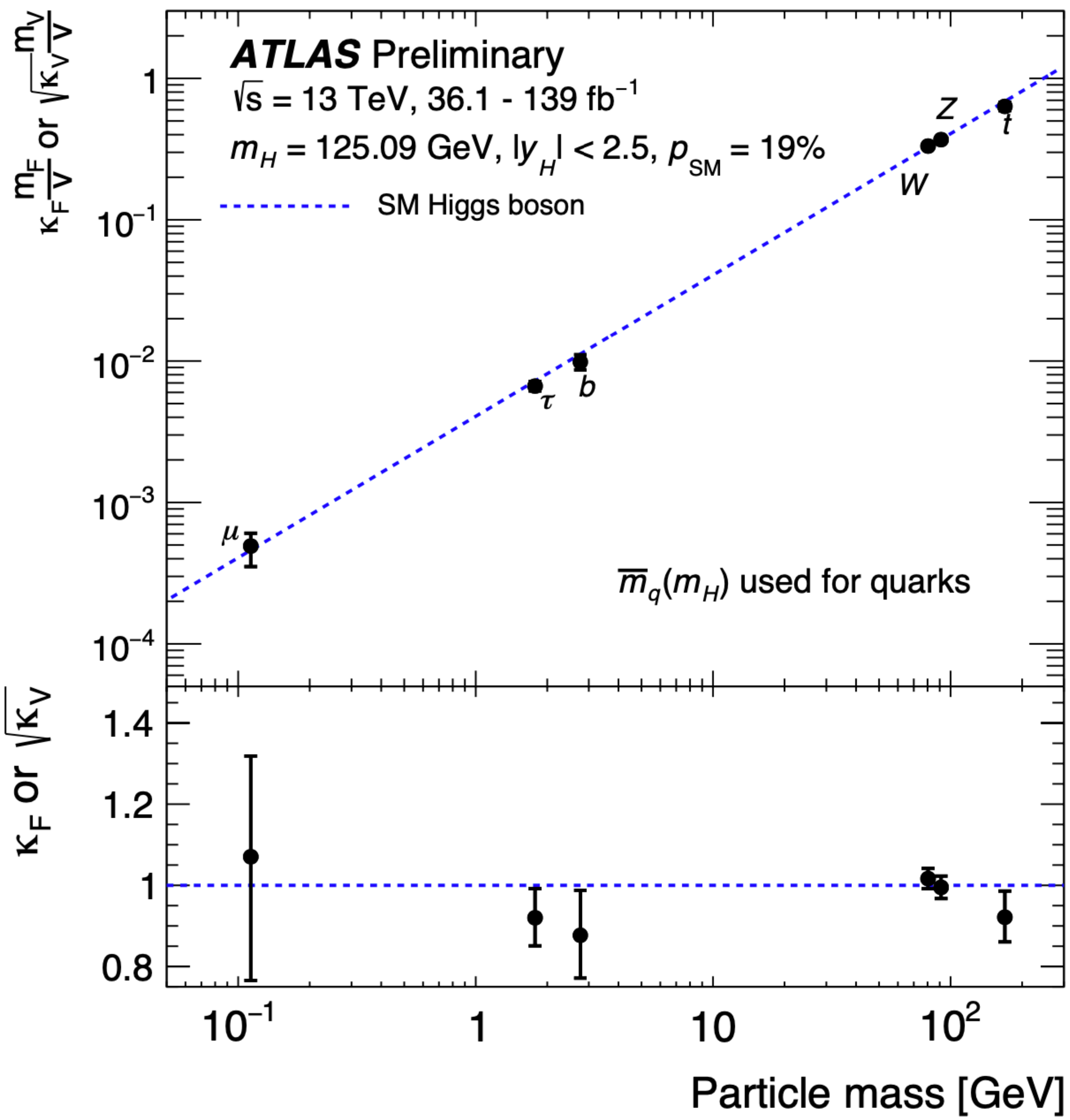
An excitation…



Analogy

The Higgs particle

Experiment



- The Standard Model predicts: *the interaction strength* $\propto$ *the particle mass*
- Confirmed for the weak bosons and 3rd generation or matter with $10\,\%$ precision

<u>Open questions</u>:

1. Higgs interactions with light generations?
2. Do Higgs interactions mix generations?
3. Higgs self-interactions?
4. Is there another Higgs field?
5. …

The Standard Model

(advanced)

The Standard Model

Table 1: The SM particles

particle	spin	color	$Q_{\rm EM}$	mass $[v]$
$W^\pm$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e,μ,τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e,ν_μ,ν_τ	1/2	(1)	0	0
u,c,t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d,s,b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

The Standard Model

– The symmetry is a local

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \quad .$$

– It is spontaneously broken by the VEV of a single Higgs scalar,

$$\phi(1,2)_{+1/2}, \quad (\langle \phi^0 \rangle = v/\sqrt{2}) \quad ,$$

$$G_{\rm SM} \to SU(3)_C \times U(1)_{\rm EM} \quad (Q_{\rm EM} = T_3 + Y) \quad .$$

– There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \; U_{Ri}(3,1)_{+2/3}, \; D_{Ri}(3,1)_{-1/3}, \; L_{Li}(1,2)_{-1/2}, \; E_{Ri}(1,1)_{-1} \quad .$$

Covariant derivative example:

$$D^\mu Q_{Li} \;=\; \left(\partial^\mu + \frac{i}{2} g_s G^\mu_a \lambda_a + \frac{i}{2} g W^\mu_b \tau_b + \frac{i}{6} g' B^\mu \right) Q_{Li}$$

The Standard Model

- $\mathscr{L}_4$ sans Yukawa

$g_S \sim 1, g_W \sim 0.6, g_Y \sim 0.3, \lambda_H \sim 0.2$

$\theta \lesssim 10^{-10}$ - The strong CP problem

ψ : 3 generations of q_i, U_i, D_i, l_i, E_i

<u>Accidental symmetry</u>

$U(3)_q \times U(3)_U \times U(3)_D \times U(3)_l \times U(3)_E$

$$\mathcal{L}_4 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + h.c. + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

The Standard Model

$$\mathcal{L}_4 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ i \bar{\psi} \not{D} \psi + h.c.$$

$$+ \bar{\psi}_i y_{ij} \psi_j \phi + h.c.$$

$$+ |D_\mu \phi|^2 - V(\phi)$$

Yukawas break $U(3)^5$

The Standard Model

- The kinetic Lagrangian (flavor and CP conserving)

$$
\begin{aligned}
\mathcal{L}_{\rm kin}^{\rm SM} \quad = \quad & -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\
& -i\overline{Q_{Li}}\not{D}Q_{Li} - i\overline{U_{Ri}}\not{D}U_{Ri} - i\overline{D_{Ri}}\not{D}D_{Ri} - i\overline{L_{Li}}\not{D}L_{Li} - i\overline{E_{Ri}}\not{D}E_{Ri} \\
& -(D^\mu\phi)^\dagger(D_\mu\phi) \quad .
\end{aligned}
$$

The Standard Model

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$$
\begin{aligned}
\mathcal{L}_{\rm kin}^{\rm SM} \quad = \quad & -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
& - i \overline{Q_{Li}} \not{D} Q_{Li} - i \overline{U_{Ri}} \not{D} U_{Ri} - i \overline{D_{Ri}} \not{D} D_{Ri} - i \overline{L_{Li}} \not{D} L_{Li} - i \overline{E_{Ri}} \not{D} E_{Ri} \\
& - (D^\mu \phi)^\dagger (D_\mu \phi) \quad .
\end{aligned}
$$

- The global symmetry

$$
G_{\rm global}^{\rm SM}(Y^{u,d,e} = 0) = SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)^5
$$

The Standard Model

- The kinetic Lagrangian (flavor and CP conserving)

$$\begin{aligned}\mathcal{L}^{\rm SM}_{\rm kin} = & -\frac{1}{4}G^{\mu\nu}_a G_{a\mu\nu} - \frac{1}{4}W^{\mu\nu}_b W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ & -i\overline{Q_{Li}}\not{D}Q_{Li} - i\overline{U_{Ri}}\not{D}U_{Ri} - i\overline{D_{Ri}}\not{D}D_{Ri} - i\overline{L_{Li}}\not{D}L_{Li} - i\overline{E_{Ri}}\not{D}E_{Ri} \\ & -(D^\mu\phi)^\dagger(D_\mu\phi) \quad .\end{aligned}$$

- The global symmetry

$$G^{\rm SM}_{\rm global}(Y^{u,d,e}=0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5$$

- Reminder:

$$U(1) : \phi \to e^{i\alpha Q}\phi$$

$$\phi^\dagger\phi \to \phi^\dagger e^{-i\alpha Q}e^{i\alpha Q}\phi = \phi^\dagger\phi$$

The Standard Model

- The kinetic Lagrangian (flavor and CP conserving)

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$$U(N) = SU(N)\times U(1)$$

$SU(N)$: group of $\mathrm{N}\times\mathrm{N}$ unitary matrices with $\det = 1$

$$U^\dagger U = 1\ ,\ \det U = 1$$

The Standard Model

- The kinetic Lagrangian (flavor and CP conserving)

$$
\begin{aligned}
\mathcal{L}^{\rm SM}_{\rm kin} \;=\; & -\frac{1}{4} G^{\mu\nu}_a G_{a\mu\nu} - \frac{1}{4} W^{\mu\nu}_b W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
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& - (D^\mu \phi)^\dagger (D_\mu \phi) \quad .
\end{aligned}
$$

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$$G^{\rm SM}_{\rm global}(Y^{u,d,e} = 0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5$$

- Reminder:

$$U(1): \phi \to e^{i\alpha Q}\phi$$

$$\phi^\dagger \phi \to \phi^\dagger e^{-i\alpha Q} e^{i\alpha Q} \phi = \phi^\dagger \phi$$

$$U(N) = SU(N) \times U(1)$$

$SU(N)$: group of N × N unitary matrices with det = 1

$U^\dagger U = 1$, det $U = 1$

$$U = e^{i\alpha^a T^a} \qquad a : 1,\ldots,N^2 - 1$$

$$SU(N): \quad \phi_i \to U_{ij}\phi_j \qquad i, j : 1,\ldots,N$$

$$\phi^\dagger \phi \to \phi^\dagger U^\dagger U \phi = \phi^\dagger \phi$$

The Standard Model

- Flavour and CP violation is in the **Yukawa Lagrangian**

$$-\mathscr{L}_{\text{Yuk}} = \bar{Q} Y^u \tilde{H} U + \bar{Q} Y^d H D + \bar{L} Y^e H E$$

- Flavour breaking **spurions**

$$Y^u \sim (3, \bar{3}, 1)_{SU(3)^3_q} \quad , \quad Y^d \sim (3, 1, \bar{3})_{SU(3)^3_q} \quad ,$$

$$Y^e \sim (3, \bar{3})_{SU(3)^2_\ell}$$

The CKM matrix

$$-\mathscr{L}_{\text{Yuk}} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} U + \bar{q} \hat{Y}^d H D + \bar{l} \hat{Y}^e H E$$

[$U(3)^5$ transformation and a singular value decomposition theorem]

- After EWSB, the CKM matrix can be rotated

$$\mathcal{L}^u_{\text{Yuk}} = (\overline{u_{dL}}\ \overline{u_{sL}}\ \overline{u_{bL}}) V^\dagger \hat{Y}^u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \longrightarrow \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = V \begin{pmatrix} u_{dL} \\ u_{sL} \\ u_{bL} \end{pmatrix}$$

The CKM matrix

$$-\mathscr{L}_{\text{Yuk}} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} U + \bar{q} \hat{Y}^d H D + \bar{l} \hat{Y}^e H E$$

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- $V \mathbf{1} V^\dagger = 1 \implies \bar{u}^i_L \not{Z}\, u^i_L$ universality!
- It only appears in the W interactions, not in γ, g, Z, h

No FCNC at tree-level **!**
They are suppressed in the SM.

The CKM matrix

$$-\mathscr{L}_{\text{Yuk}} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} U + \bar{q} \hat{Y}^d H D + \bar{l} \hat{Y}^e H E$$

$[U(3)^5$ transformation and a singular value decomposition theorem$]$

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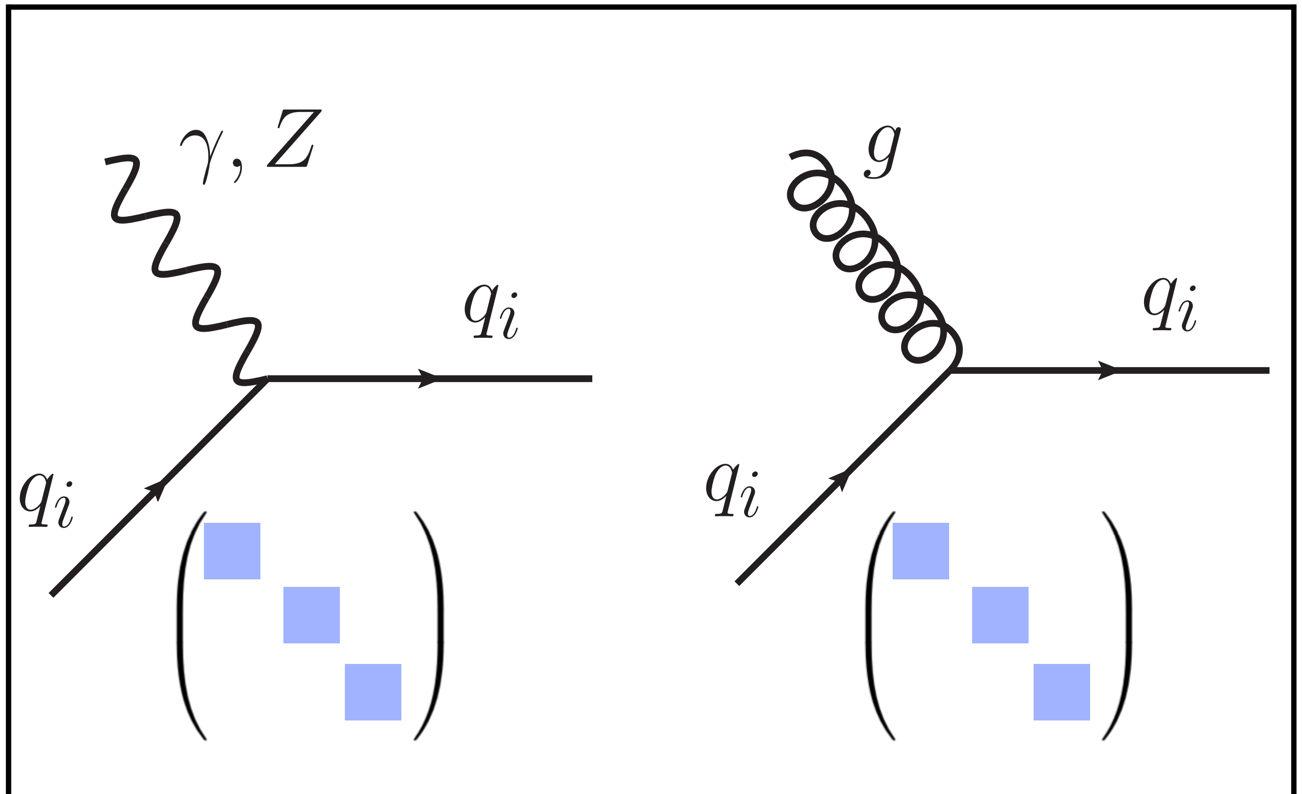
$$\mathcal{L}^u_{\text{Yuk}} = (\overline{u_{dL}}\ \overline{u_{sL}}\ \overline{u_{bL}}) V^\dagger \hat{Y}^u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \longrightarrow \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = V \begin{pmatrix} u_{dL} \\ u_{sL} \\ u_{bL} \end{pmatrix}$$

- $V \mathbf{1} V^\dagger = 1 \implies \bar{u}^i_L \not{Z}\, u^i_L$ universality!
- It only appears in the W interactions, not in γ, g, Z, h

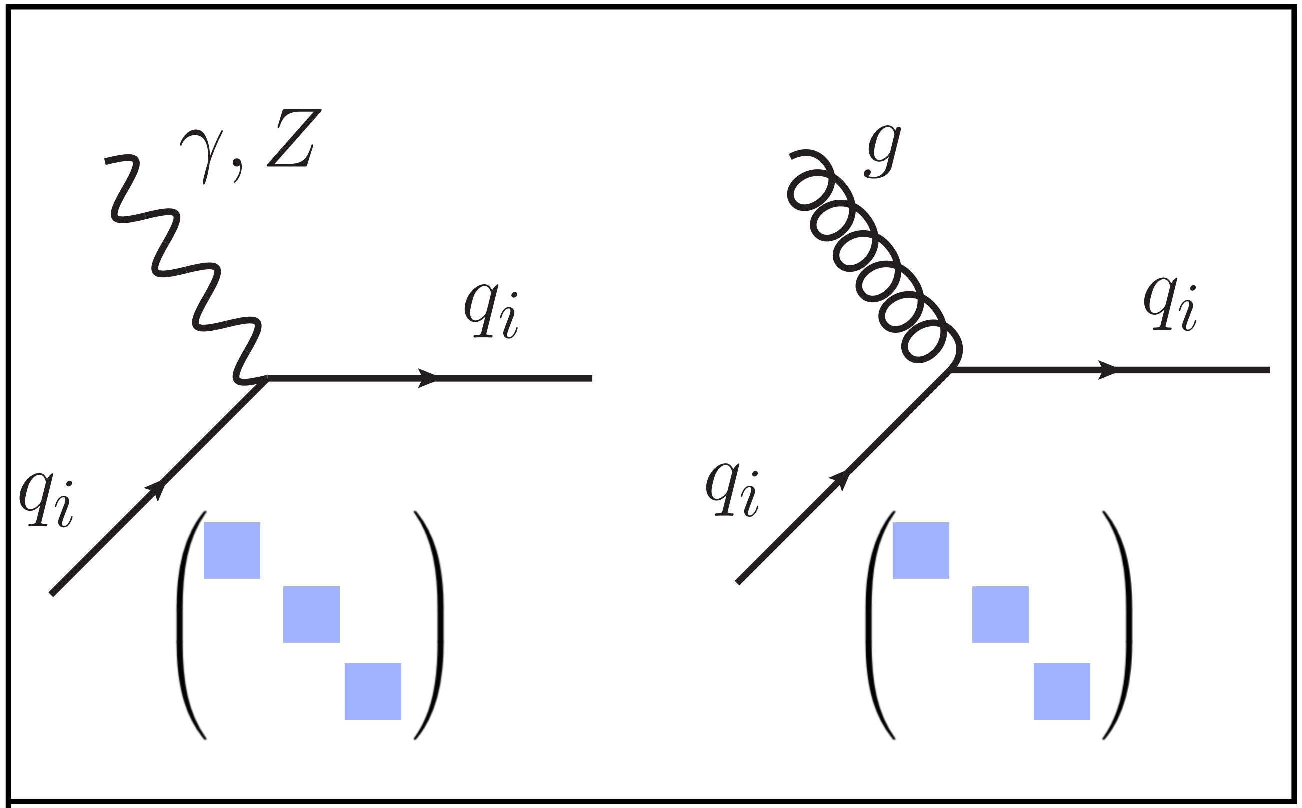
$$\text{FCCC:} \ -\frac{g}{\sqrt{2}} \left(\overline{u_L}\ \overline{c_L}\ \overline{t_L} \right) \underbrace{V}_{\textbf{CKM}} \not{W}^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}$$

Recap: The SM interactions



- Flavour universal / blind

Recap: The SM interactions



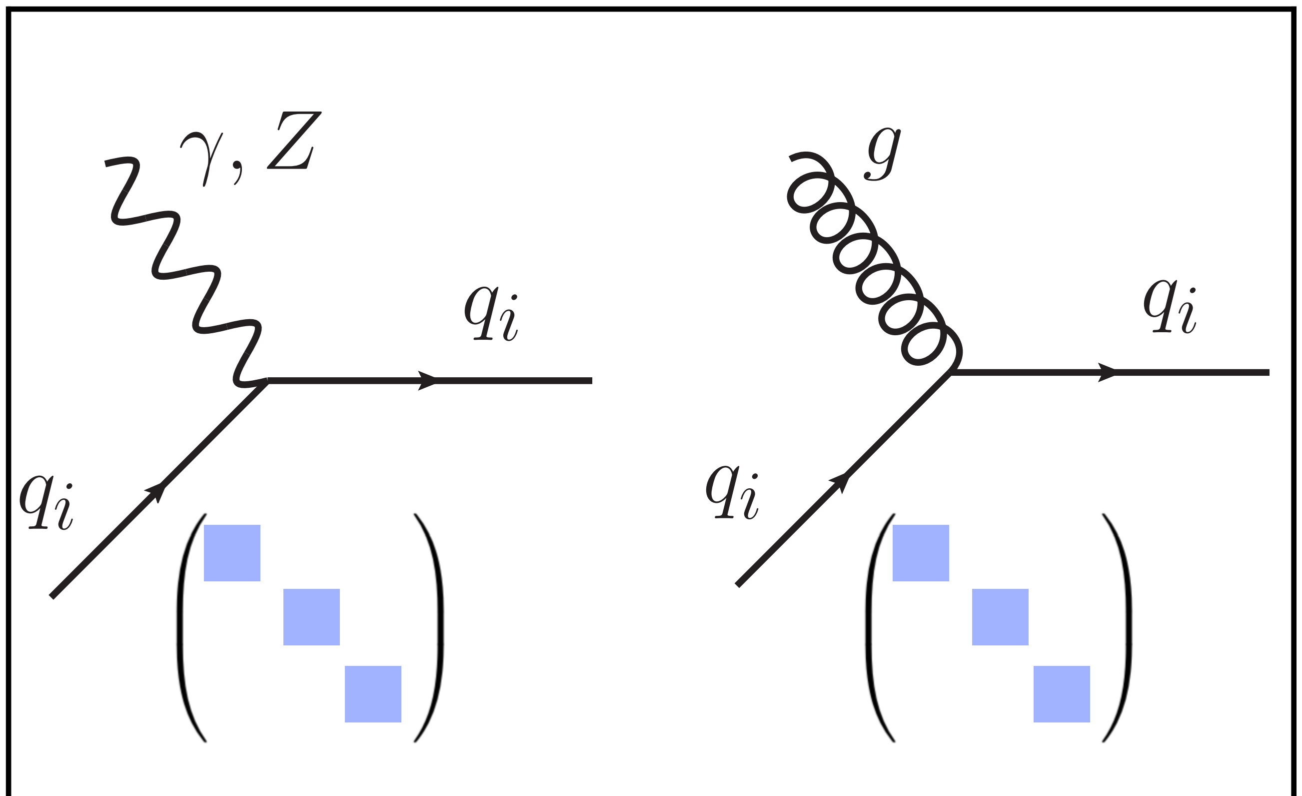
- Flavour universal / blind

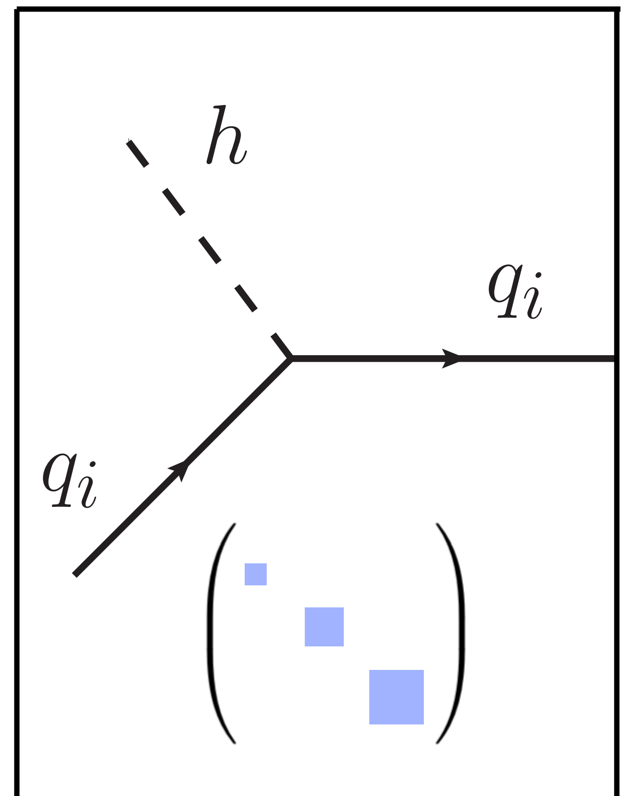
PDG

$$\Gamma(\mu^+\mu^-)/\Gamma(e^+e^-) = 1.0009 \pm 0.0028$$
$$\Gamma(\tau^+\tau^-)/\Gamma(e^+e^-) = 1.0019 \pm 0.0032$$

$$\mathrm{BR}(Z \to e^+\mu^-) < 7.5 \times 10^{-7} ,$$
$$\mathrm{BR}(Z \to e^+\tau^-) < 9.8 \times 10^{-6} ,$$
$$\mathrm{BR}(Z \to \mu^+\tau^-) < 1.2 \times 10^{-5} .$$

Recap: The SM interactions



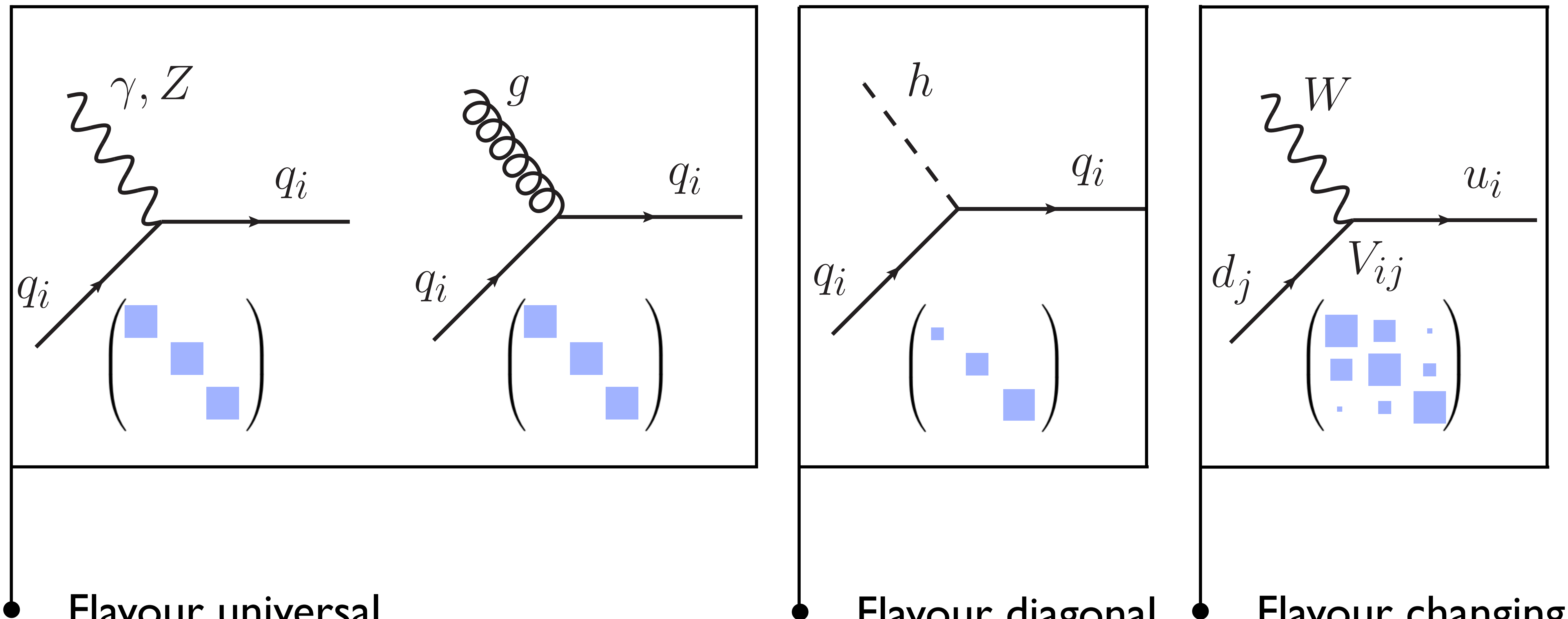


- Flavour universal / blind

- Flavour diagonal non-universal

Recap: The SM interactions

CKM matrix V



- Flavour universal / blind
- Flavour diagonal non-universal
- Flavour changing / violating

Recap: The SM interactions

Table 2: The SM fermion interactions

interaction	fermions	force carrier	coupling	flavor
Electromagnetic	u, d, ℓ	A^0	eQ	universal
Strong	u, d	g	g_s	universal
NC weak	all	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	universal
CC weak	$\bar{u}d/\bar{\ell}\nu$	$W^\pm$	gV/g	non-universal/universal
Yukawa	u, d, ℓ	h	y_q	diagonal

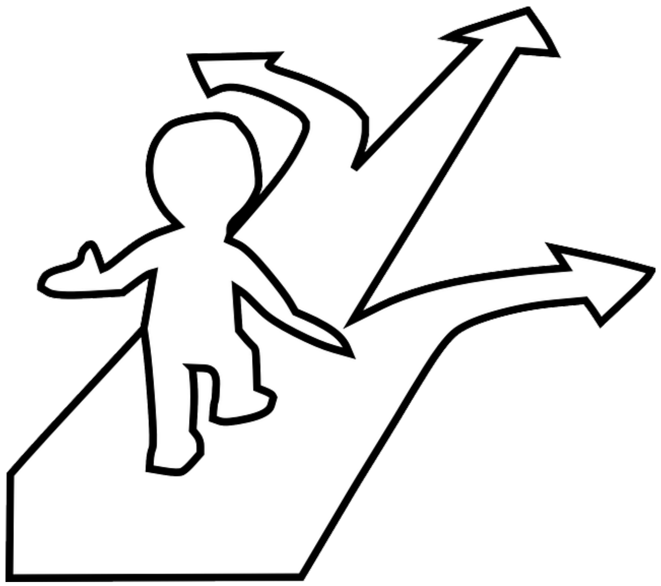
- The list of open question

Hierarchy problem
Flavour puzzle
Strong CP problem
Charge quantisation

Dark matter
Baryon asymmetry
Neutrino masses
Inflation

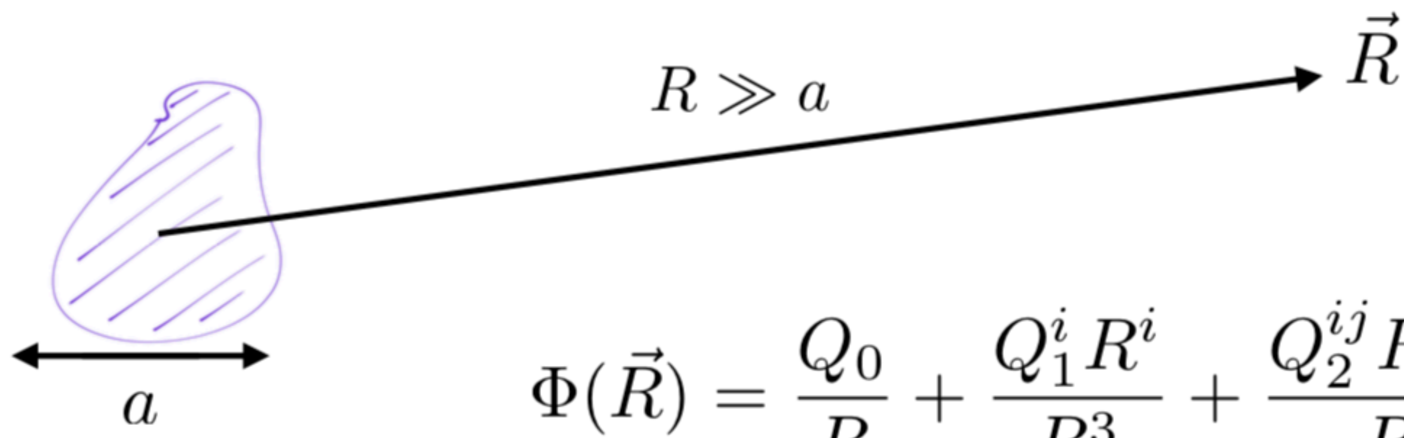
Dark energy
Quantum gravity
….

New Physics



Backup

Effective theory



$$\Phi(\vec{R}) = \frac{Q_0}{R} + \frac{Q_1^i R^i}{R^3} + \frac{Q_2^{ij} R^i R^j}{R^5} + \dots$$

$$1/R \qquad a/R^2 \qquad a^2/R^3$$

n-multipole contribution is of relative size $\left(\frac{a}{R}\right)^n$

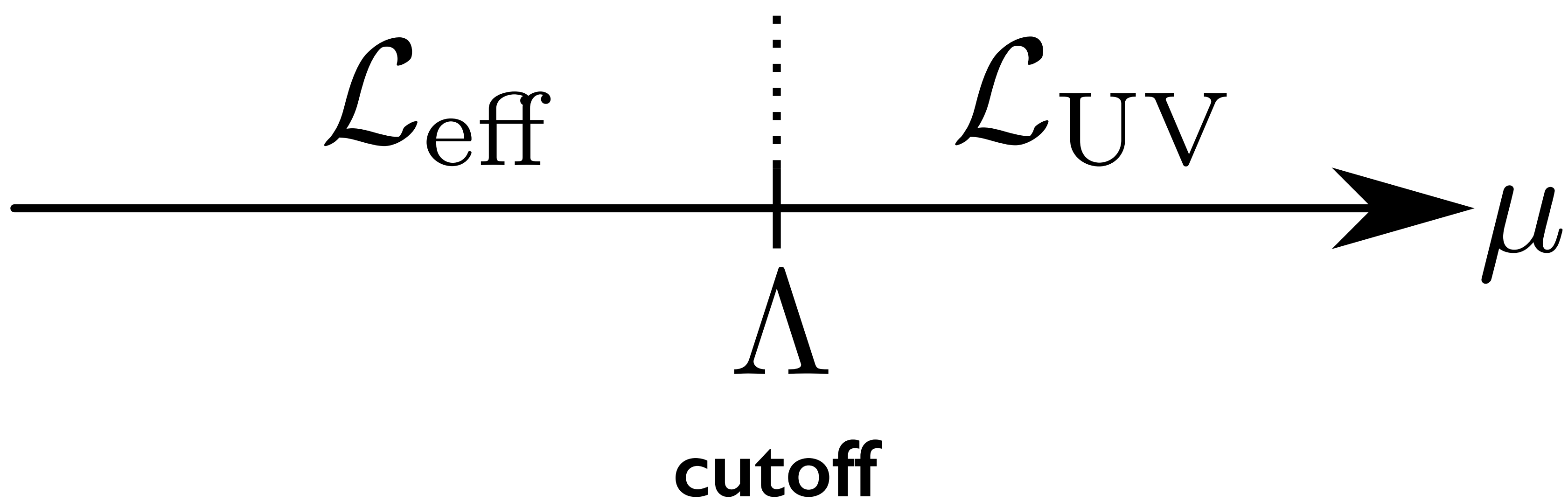
at fixed accuracy

- $R \to$ large: fewer multipoles needed ➔ Universality
- $R \to$ small: more multipoles needed ➔ Reductionism

$R \sim a$ expansion breaks down: ∞ number of parameters needed

Lectures by Rattazzi, GGI 2020

Effective quantum field theory



Infrared,
Long-distance,
Soft

Ultraviolet,
Short-distance,
Hard

Matching

Tree-level example

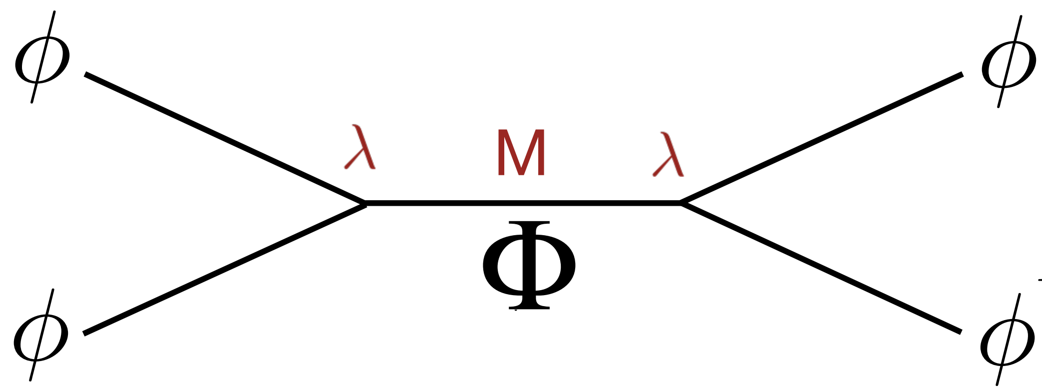


Figure 1: Generating higher dimension operators by integrating out fields.

Matching

Tree-level example

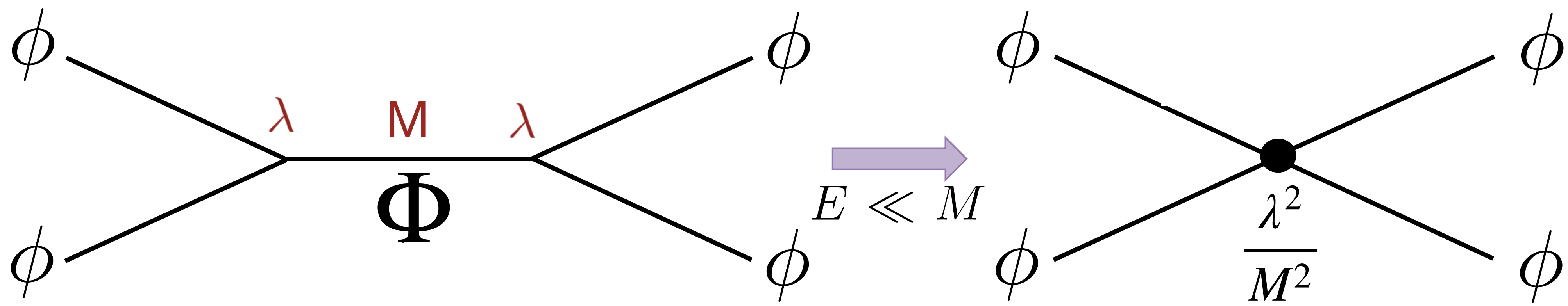


Figure 1: Generating higher dimension operators by integrating out fields.

$$\underbrace{\langle 0|T\{\Phi(0)\Phi(x)\}|0\rangle}_{\text{(Propagator)}} = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{i}{k^2 - M^2}$$

$$k^2 \sim \mathcal{O}(E^2) \ll M^2$$

$$\frac{1}{k^2 - M^2} = -\frac{1}{M^2}\left[1 + \mathcal{O}\left(\frac{k^2}{M^2}\right)\right]$$

Matching

Tree-level example

Local interaction:
The Compton wavelength M^{-1} is very small.

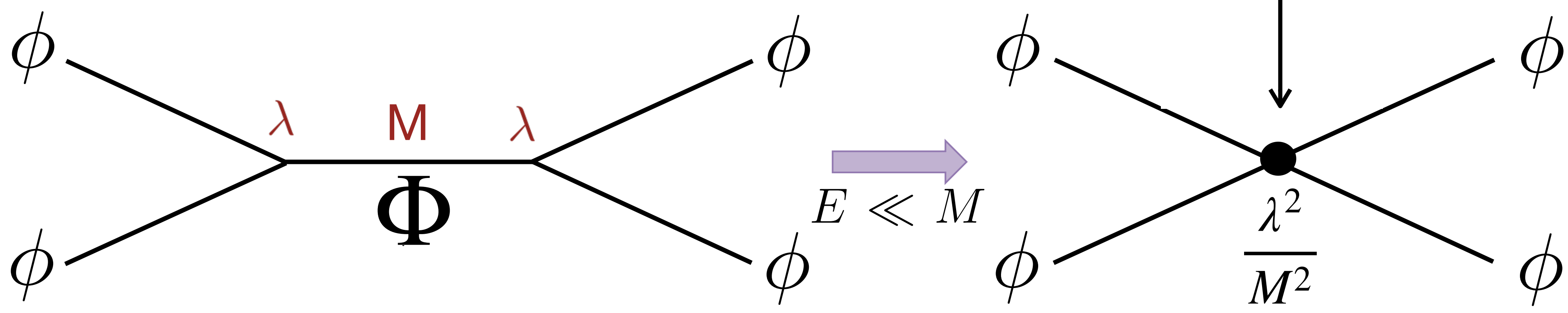


Figure 1: Generating higher dimension operators by integrating out fields.

$$\langle 0|T\{\Phi(0)\Phi(x)\}|0\rangle = \underbrace{\int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{i}{k^2 - M^2}}_{-\frac{i}{M^2}\delta^{(4)}(x)}$$

(Propagator)

$$k^2 \sim \mathcal{O}(E^2) \ll M^2$$

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Effective quantum field theory

The cut-off

$$\boxed{\Lambda \equiv \frac{1}{\tau} \equiv \frac{1}{L}}$$

Example: a theory with just one scalar field φ

Lagrangian is organized in series in inverse powers of Λ:
close analogy with multipole expansion

$$\begin{aligned}
\mathcal{L} \;=\;& \partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2 + \lambda_4\varphi^4 && \Lambda^{\geq 0} \\
&+ \frac{\lambda_6}{\Lambda^2}\varphi^6 + \frac{\eta_4}{\Lambda^2}\varphi^2\partial_\mu\varphi\partial^\mu\varphi && \Lambda^{-2} \\
&+ \frac{\lambda_8}{\Lambda^4}\varphi^8 + \frac{\eta_6}{\Lambda^4}(\partial_\mu\varphi\partial^\mu\varphi)^2 + \cdots && \Lambda^{-4} \\
&+ \cdots && \Lambda^{\leq -4}
\end{aligned}$$

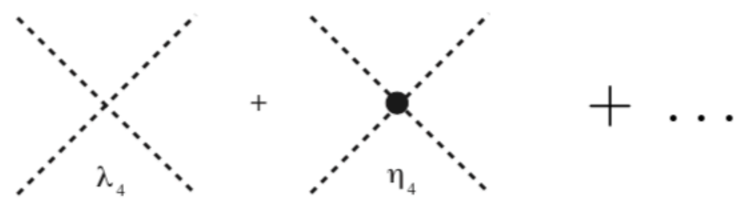
- $\lambda_4,\ \lambda_6,\ \eta_6,\ \ldots$ expected to be < O(1)
- must assume $m^2 \ll \Lambda^2$ otherwise no long wavelength quanta

Lectures by Rattazzi, GGI 2020

Effective quantum field theory

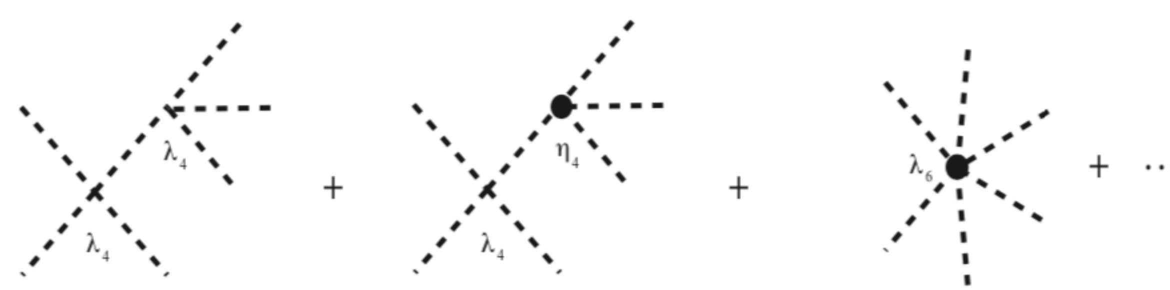
Scattering amplitudes at $E \ll \Lambda$

$$\mathcal{A}_{2\to 2} =$$



$$= \lambda_4 + \eta_4 \frac{E^2}{\Lambda^2} + \ldots \xrightarrow{E \to 0} \lambda_4$$

$$\mathcal{A}_{2\to 4} =$$



$$= \frac{1}{E^2}\left\{ \lambda_4^2 + \lambda_4 \eta_4 \frac{E^2}{\Lambda^2} + \lambda_6 \frac{E^2}{\Lambda^2} + \ldots \right\}$$

at low energy only lowest dimension coupling matters

the infinite set of couplings with negative mass dimension is irrelevant !

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Accidental symmetries in Effective theory

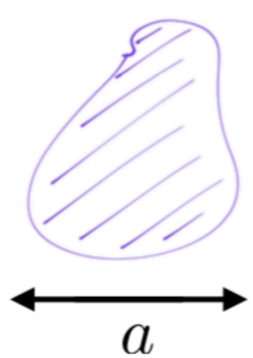
Long Distance Physics: Simplicity & Accidental Symmetries

accidental

•

$SO(3)$

Ex.: electrostatic potential at large distance



$$\begin{array}{cccc} & 1/R & a/R^2 & a^2/R^3 \\ \xrightarrow{R \gg a} \quad \Phi(R) = & \dfrac{Q_0}{R} + & \dfrac{\vec{Q}_1 \cdot \vec{R}}{R^3} + & \dfrac{Q_2^{ij} R_i R_j}{R^5} + \dots \\ & SO(3) \supset & SO(2) \supset & \varnothing \end{array}$$

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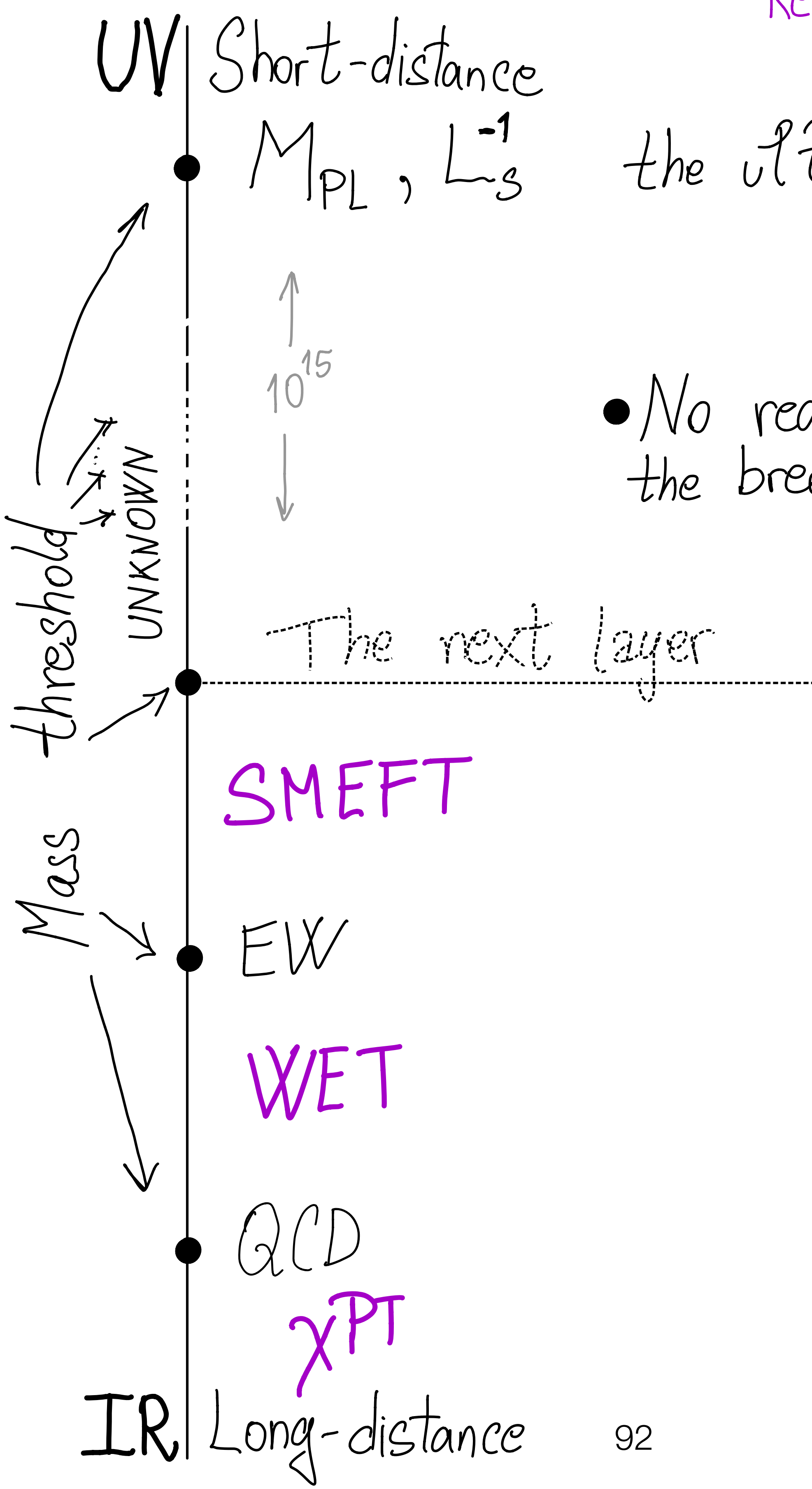
Wilsonian QFT = the HEP paradigm

Reductionism



- No reason to expect the breakdown anytime soon

Wilsonian QFT = the HEP paradigm

Reductionism

UV | Short-distance

M_{PL}, L_s^{-1} the ultimate scale?

10^{15}

- No reason to expect the breakdown anytime soon

threshold

UNKNOWN

The next layer

SMEFT

Theory construction:

1. Space-time & gauge invariance + field content,
2. Lagrangian(x) = infinite polynomial in fields and derivatives,

$$\mathscr{L} = \sum c_{\mathcal{O}} \Lambda_{\mathcal{O}}^{4-\dim \mathcal{O}} \mathcal{O}$$

- IR relevance: $\dim[\mathcal{O}] \leq 4$
- Irrelevant couplings suppressed by $\Lambda_{\mathcal{O}}^{4-\dim \mathcal{O}}$

Mass

EW

WET

QCD

χPT

IR | Long-distance

$\mathscr{L}_2$: *The EW hierarchy puzzle*

- $\mathscr{L}_2 = \mu^2 H^\dagger H$ sets the EW scale.

$$\mu^2 \ll M_P^2 \quad \textbf{?}$$

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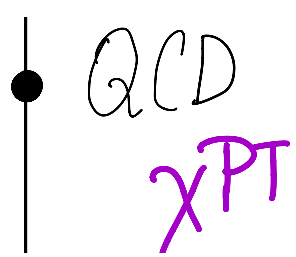

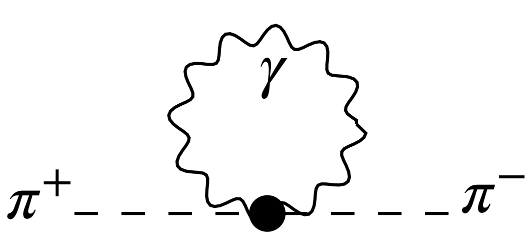

- Pion mass splitting:

$$m_{\pi_+}^2 - m_{\pi_0}^2 = \mathcal{O}(1) \times \frac{e^2}{16\pi^2} m_\rho^2$$

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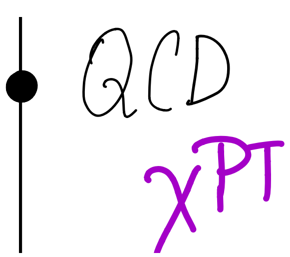

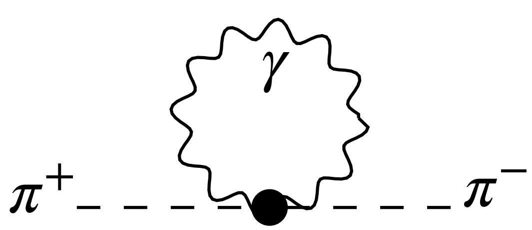

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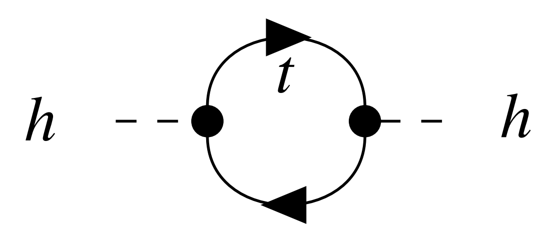

- **Naturalness:** *New mass threshold not far above the EW scale*
- Supersymmetry?
- Composite Higgs / Extra Dimensions?

$\mathscr{L}_4$: ***Accidental symmetries***

$\mathscr{L}_4^{SM}$ sans Yukawa: $\boxed{U(3)_q \times U(3)_U \times U(3)_D \times U(3)_l \times U(3)_E}$

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$$-\mathscr{L}_{\rm Yuk} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} U + \bar{q} \hat{Y}^d H D + \bar{l} \hat{Y}^e H E$$

$\downarrow$

[$U(3)^5$ transformation and a singular value decomposition theorem]

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[$U(3)^5$ transformation and a singular value decomposition theorem]

$\mathscr{L}_4^{SM}$: $\boxed{U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau}$

Prediction: No proton decay nor cLFV

Experiment: $\tau_p \gtrsim 10^{34}$ years, $BR(\mu \to e\gamma) \lesssim 10^{-13}, \ldots$

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- Λ_{NP}^{-1} truncation at the $[\mathscr{L}^{\rm SMEFT}] \leq 4 \Longrightarrow$ **Exact** accidental symmetries
- Peculiar observed values of $Y^{u,d,e} \Longrightarrow$ **Approximate** accidental symmetries

[Mass hierarchy & CKM alignment] [Quark flavour, CP, LFU, etc]

$\mathscr{L}_4$: *Approximate symmetries*

- Approximate Quark Flavor Conservation:

  - When $V = 1$ => $U(1)_{u+d} \times U(1)_{c+s} \times U(1)_{t+b}$

$\mathscr{L}_4$: ***Approximate symmetries***

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  - When $V = 1$ => $U(1)_{u+d} \times U(1)_{c+s} \times U(1)_{t+b}$
  - GIM mechanism: When up or down-quark masses are degenerate, i.e. $\hat{Y}^u \propto 1$ or $\hat{Y}^d \propto 1$, no quark flavour violation.

    $-\mathscr{L}_{\text{Yuk}} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} U + \bar{q} \hat{Y}^d H D + \bar{l} \hat{Y}^e H E$

    $\Longrightarrow$ If $\hat{Y}^d \propto 1$, rotate $q \to V^\dagger q,\ D \to V^\dagger D$, and vice versa

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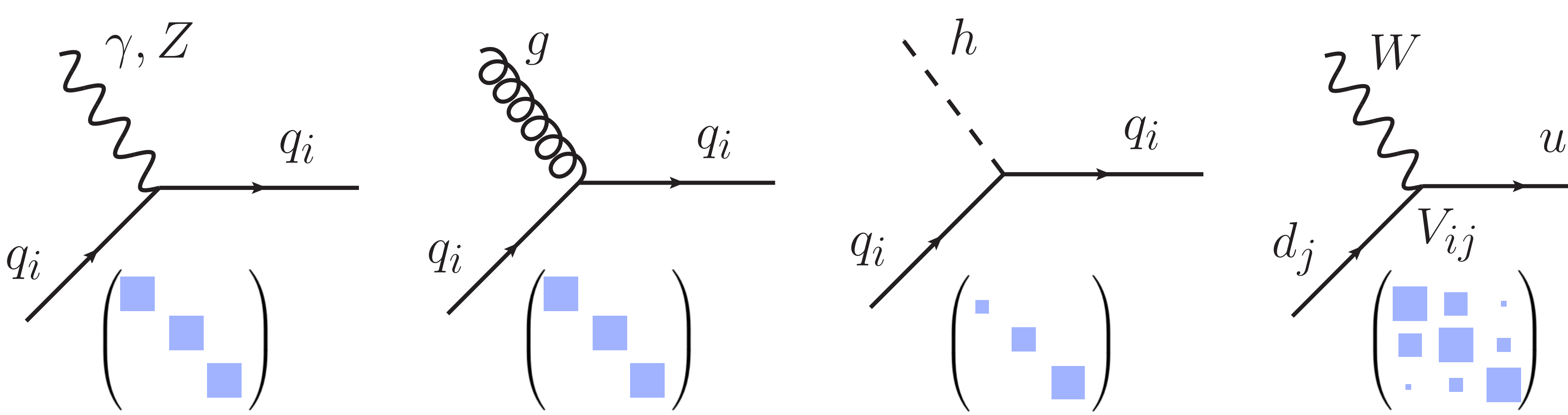
    $\Longrightarrow$ If $\hat{Y}^d \propto 1$, rotate $q \to V^\dagger q$, $D \to V^\dagger D$, and vice versa

- V spurion appears only in $W_\mu^\pm$ interaction $\Longrightarrow$ No tree-level FCNC



$\mathscr{L} \supset \frac{g}{\sqrt{2}} \bar{u}_L^i V_{ij} \gamma_\mu d_L^j W^\mu$

$\mathscr{L}_4$: ***Approximate symmetries***

- Approximate CP

$$\mathscr{L} \supset \frac{g}{\sqrt{2}} \bar{u}_L^i V_{ij} \gamma_\mu d_L^j W^\mu$$

Jarlskog invariant: $V_{ij} \to e^{i(\theta_u^i - \theta_d^j)} V_{ij}$

$$J = \mathrm{Im}(V_{ud} V_{cs} V_{us}^* V_{cd}^*) \sim 3 \times 10^{-5}$$

The CKM alignment

$\mathscr{L}_4$: ***Approximate symmetries***

- Approximate CP

$$\mathscr{L} \supset \frac{g}{\sqrt{2}} \bar{u}^i_L V_{ij} \gamma_\mu d^j_L W^\mu$$

Jarlskog invariant: $V_{ij} \to e^{i(\theta^i_u - \theta^j_d)} V_{ij}$

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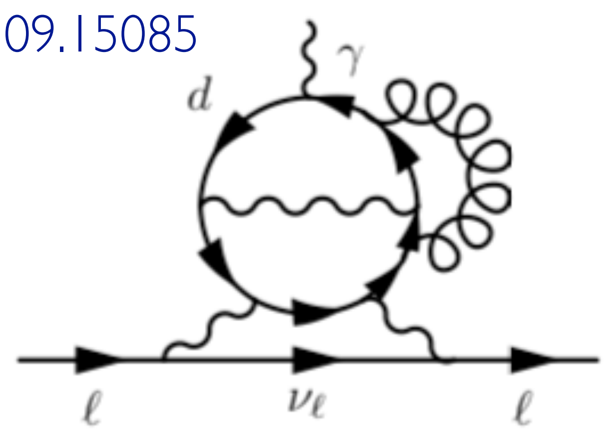
The CKM alignment

Example: *Electron electric dipole moment*

2109.15085



$$d_e \sim e \frac{m_e}{m_W^2} \frac{g^6 g_s^2}{(16\pi^2)^4} \left(\frac{v}{m_W}\right)^{12} \frac{m_b^4 m_s^2 m_c^2}{v^8} J$$

- $J \to$ higher loop suppression
- Chirality flips $\to$ The mass hierarchy suppression

SM: $d_e \sim 10^{-48}\ e \cdot \mathrm{cm}$

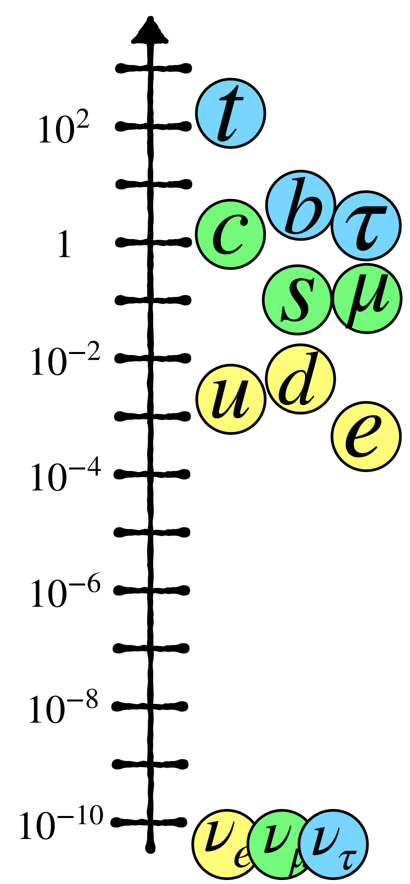
Experiment: $|d_e| < 1.1 \times 10^{-29}\ e \cdot \mathrm{cm}$

- Accidental symmetries (exact and approximate) are broken by the irrelevant couplings / new physics.

- Testing accidental symmetries is an opportunity
$\Longrightarrow$ Efficient probe of high-energy dynamics.

$\mathscr{L}_5$: Neutrino masses

$$\mathscr{L}_5 = \frac{Y^M_{ij}}{\Lambda} L_i L_j H H$$



Large Λ explains tiny m_ν

$\mathscr{L}_5$: Neutrino masses

$$\mathscr{L}_5 = \frac{Y^M_{ij}}{\Lambda} L_i L_j H H$$

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

$$\Big\downarrow \; M_{\nu,ij} = Y^M_{ij} \frac{v^2}{\Lambda}$$

$$\varnothing$$

- LFV

  Neutrino oscillations

$\mathscr{L}_5$: Neutrino masses

$$\mathscr{L}_5 = \frac{Y^M_{ij}}{\Lambda} L_i L_j H H$$

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

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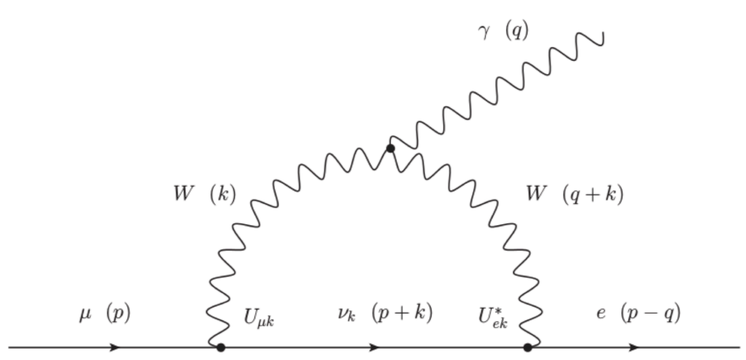
- LFV

  Neutrino oscillations

- cLFV



$\mathscr{B}(\mu \to e\gamma)_{\rm SM} \sim 10^{-54}$

Experiment:
$BR(\mu \to e\gamma) \lesssim 10^{-13}$

Efficient GIM mechanism!