Introduction to Cosmology

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1) **The Big Bang model**
2) **Content of the Universe**
3) Cosmological probes
4) Large Scale Structure: from SDSS to DESI
5) The Hubble constant tension

The rise of the Big Bang model

Orders of magnitude. The three pillars of the Big Bang model

- Distances, cosmological scales
- Redshift
- Expansion of the Universe, the metric
- The rise of the Big Bang model

1. Distances

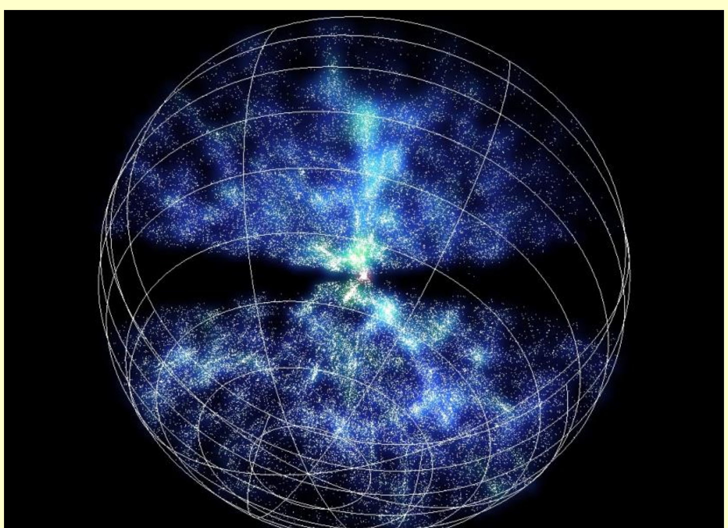
- solar system: Earth-Sun ~ 150. 10^6 km ~ 1 AU

- galaxies, eg the Milky Way:

  ∅ ~100,000 lyr ~ 30kpc Sun-Gal. center ~ 10kpc

1lyr=63,240 AU   1pc=3.26 lyr   1Mpc=3.3 10^6 lyr

- galaxy clusters: largest and most massive gravitationally bound structures, 50 to 1,000's of galaxies, ∅ ~ 2 to 10 Mpc

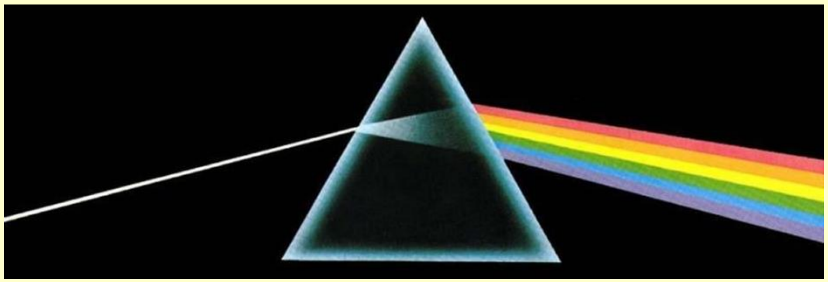


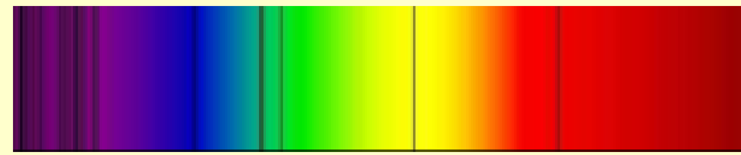
- large scale structures: galaxies → clusters → superclusters (15 - 100Mpc) making a network of voids (25 - 125Mpc) and filaments (90 - 300 Mpc)

  ⇒ beyond 100Mpc, homogeneous and isotropic Universe

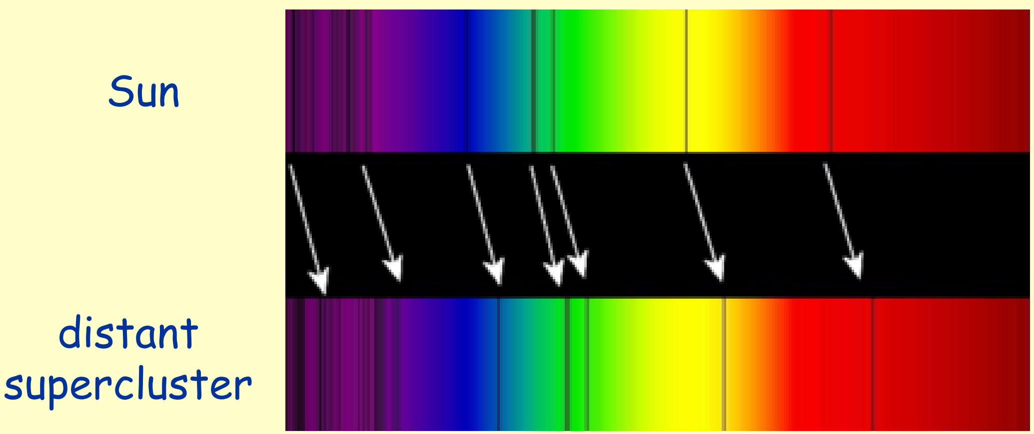
6dF Galaxy Redshift Survey, (2009)

- Emitted light spectrum ⇒ spectral lines ⇒ astro. object composition, environment … and motion relative to Earth.





- Redshift : the object moves away from us ⇒ λ_γ increases



$$z \equiv \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

or

$$1 + z = \frac{\lambda_{observed}}{\lambda_{emitted}}$$

See http://astro.unl.edu/classaction/animations/cosmology/galacticredshift.html

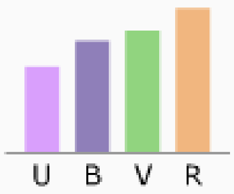
Galactic Redshift   reset   about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.00

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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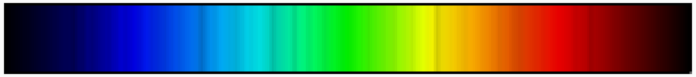
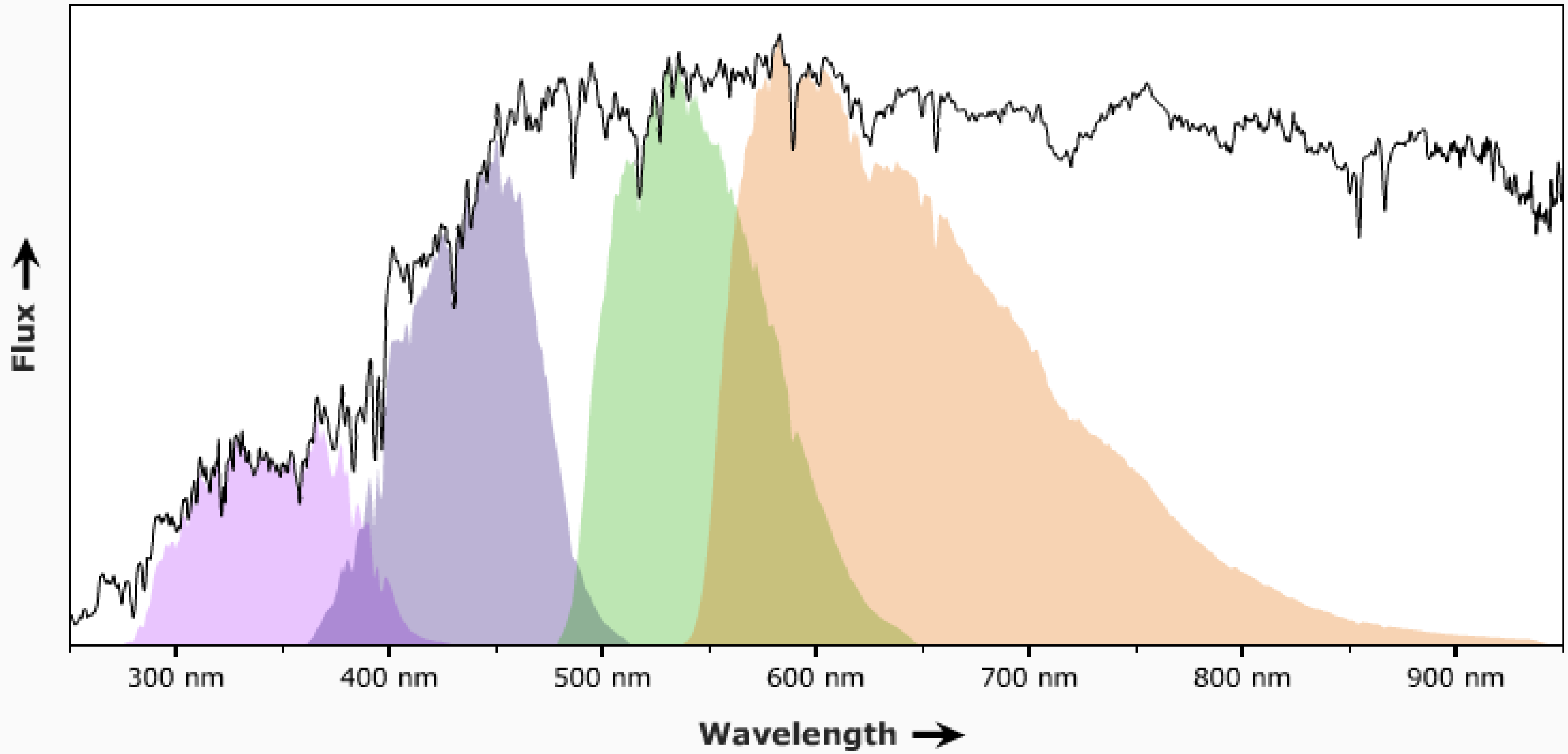
Visible Spectrum





Galactic Redshift

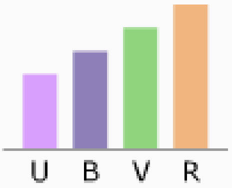
reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.10

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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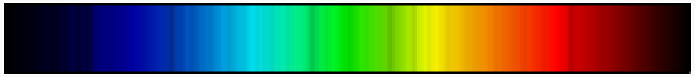
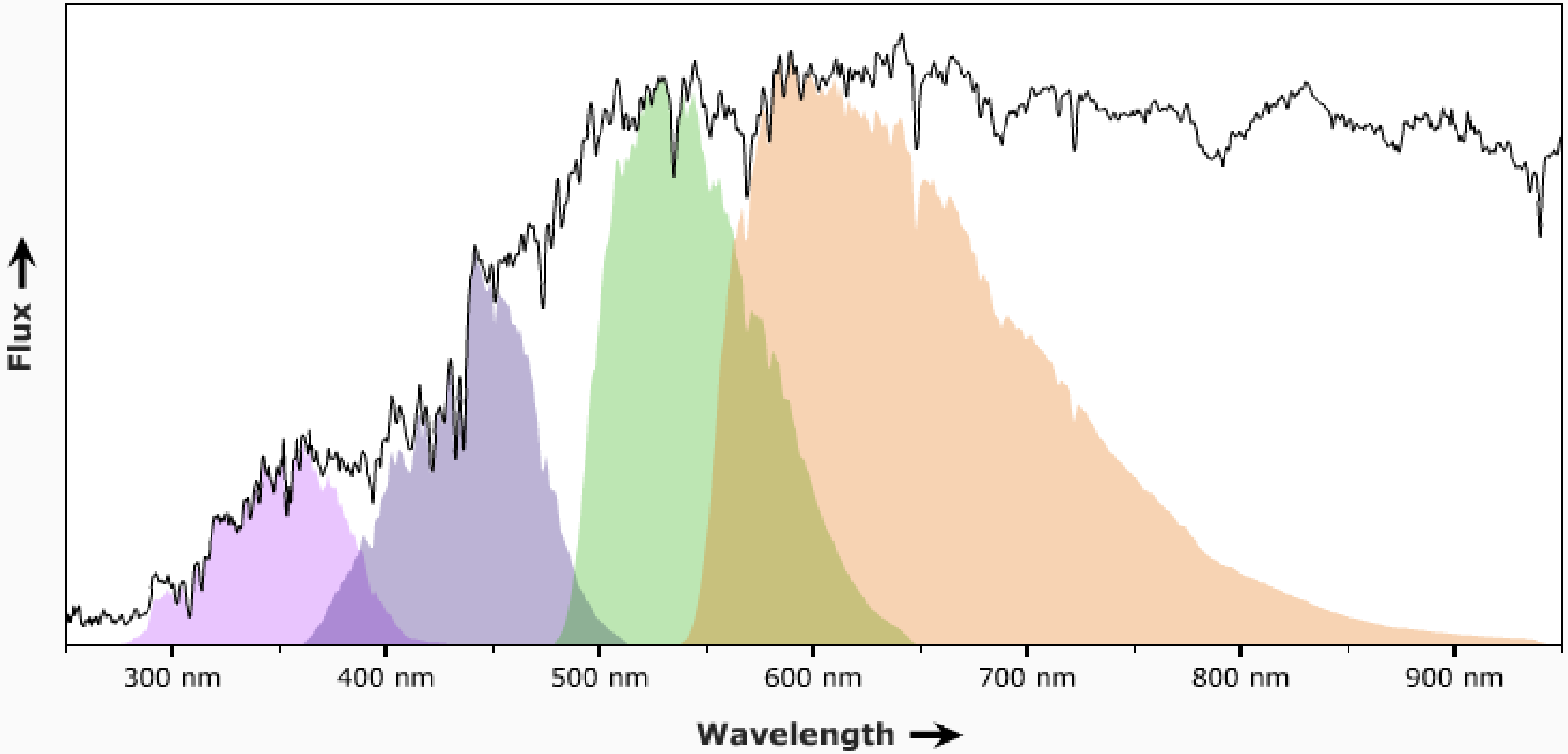
Visible Spectrum





Galactic Redshift

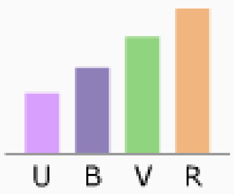
reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.20

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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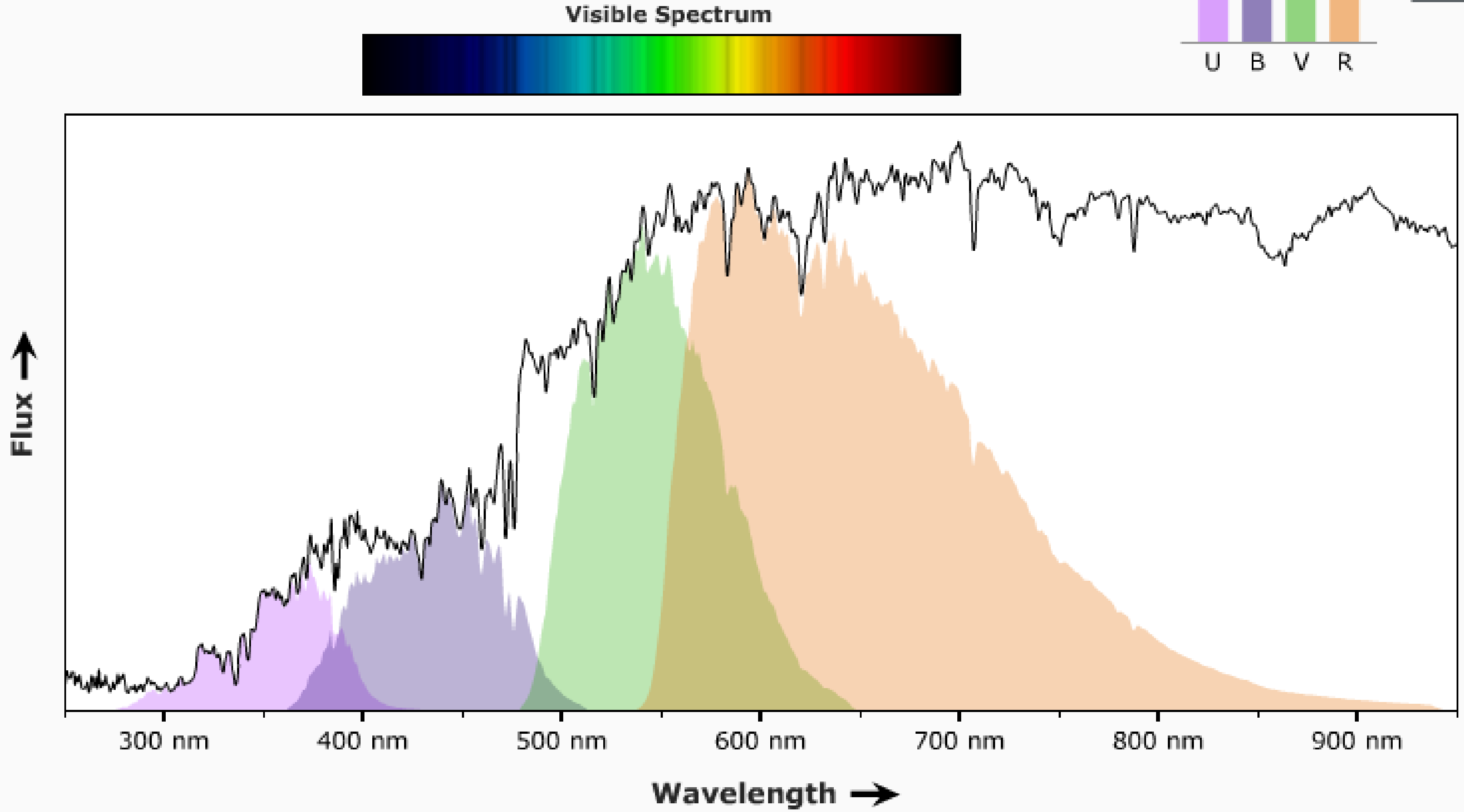


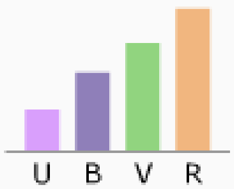
Galactic Redshift reset about

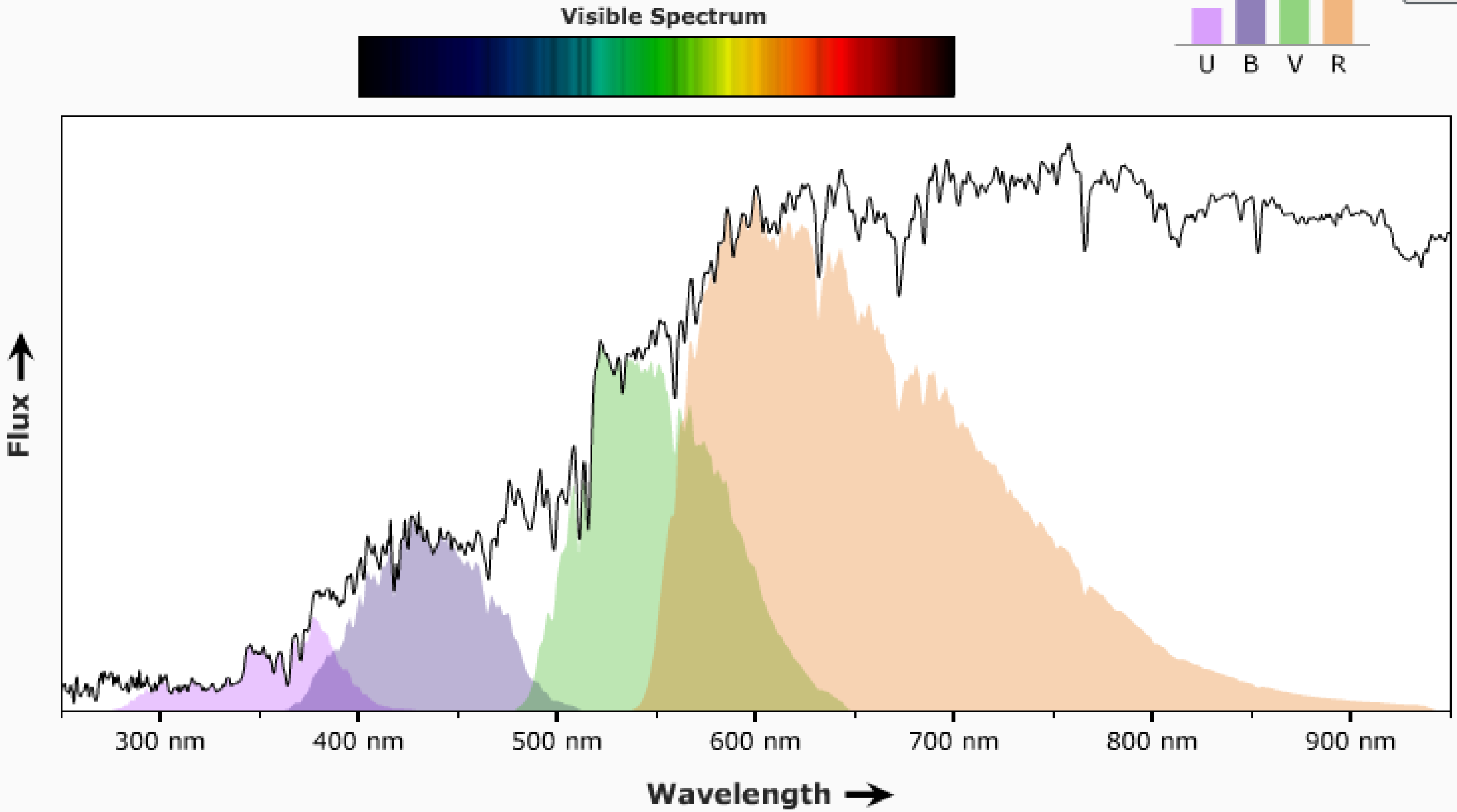
The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.30

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.





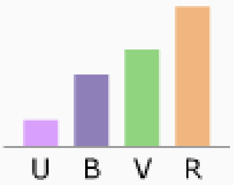
Galactic Redshift   reset   about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.40

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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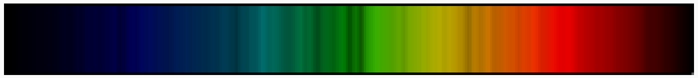
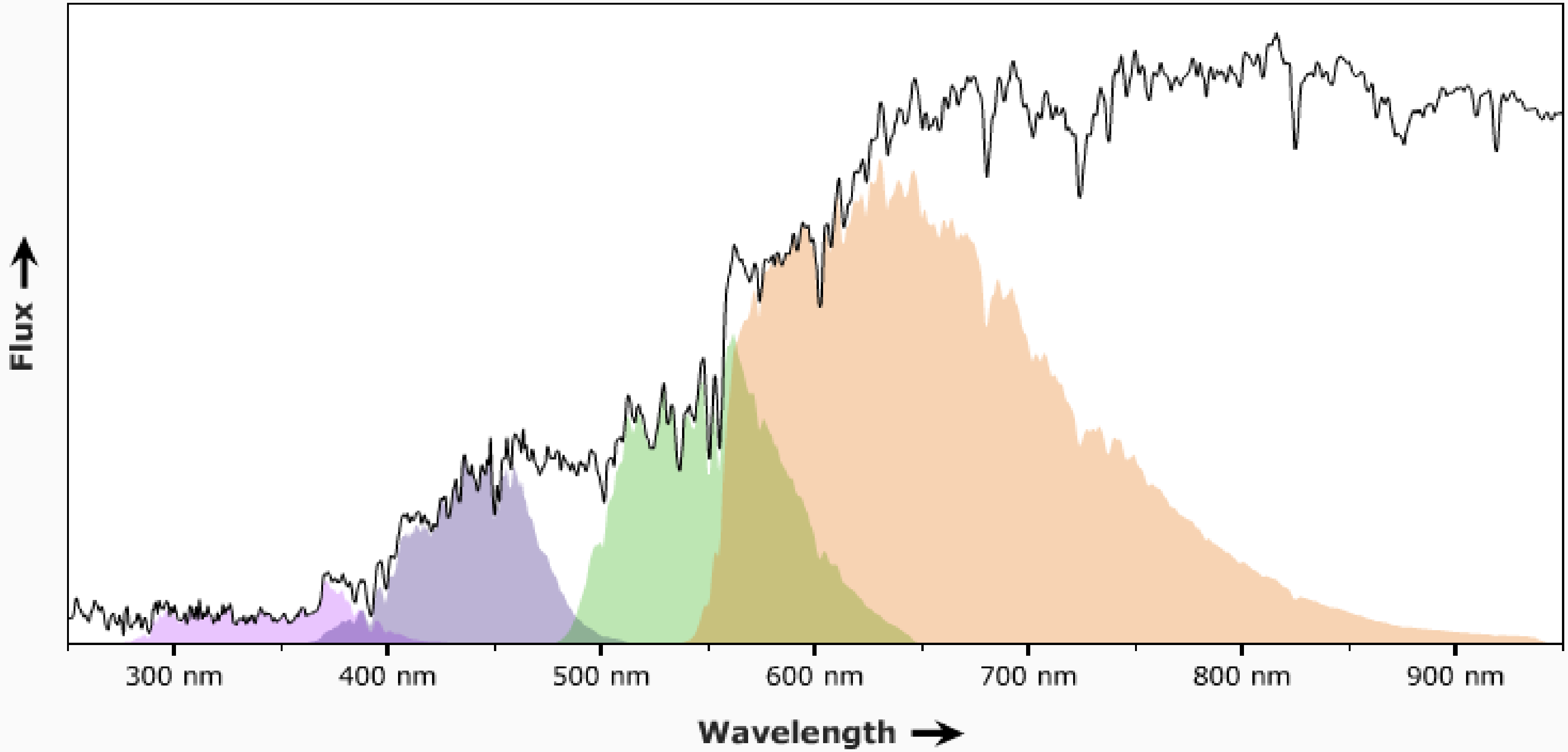
Visible Spectrum





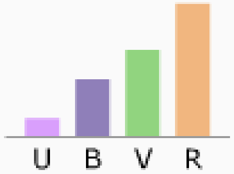
Galactic Redshift   reset   about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.50

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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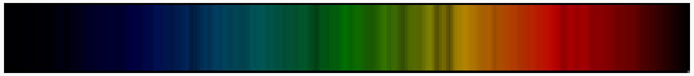
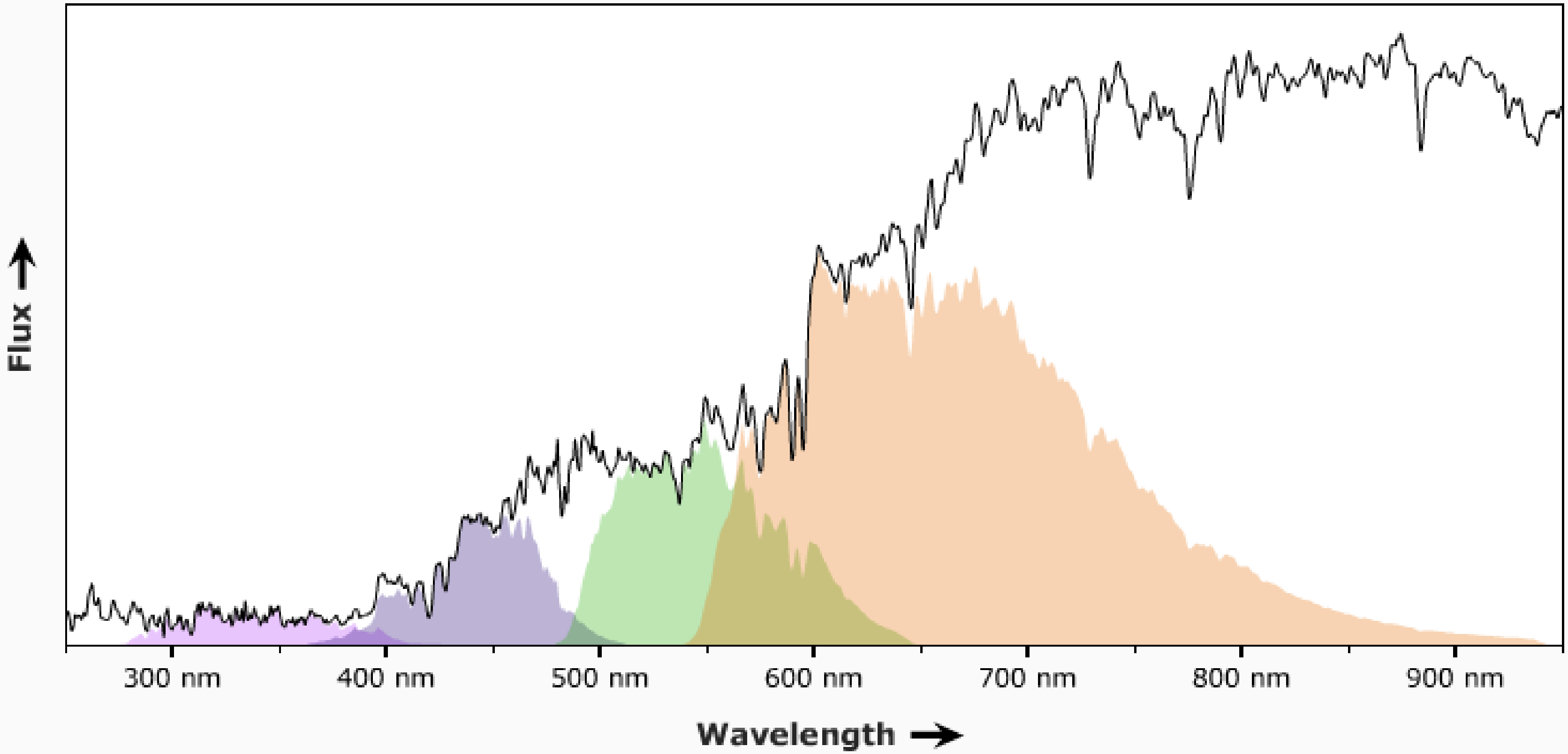
Visible Spectrum





Galactic Redshift

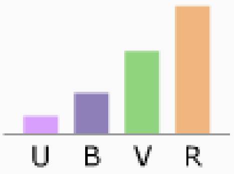
reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.60

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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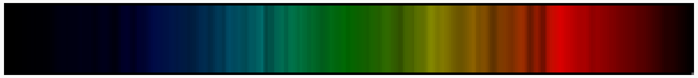
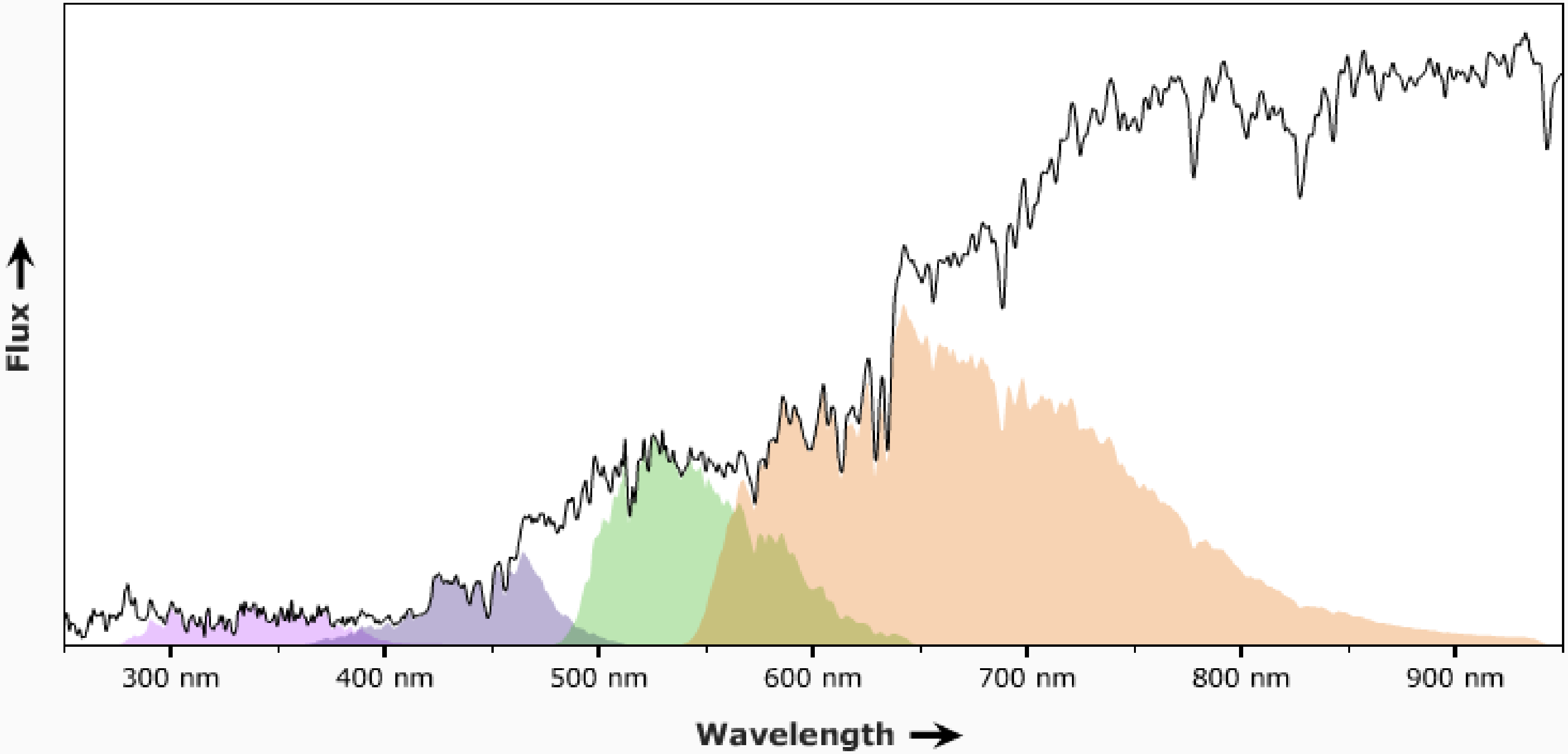
Visible Spectrum





Galactic Redshift

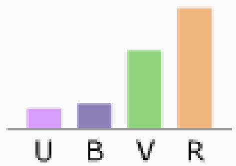
reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.70

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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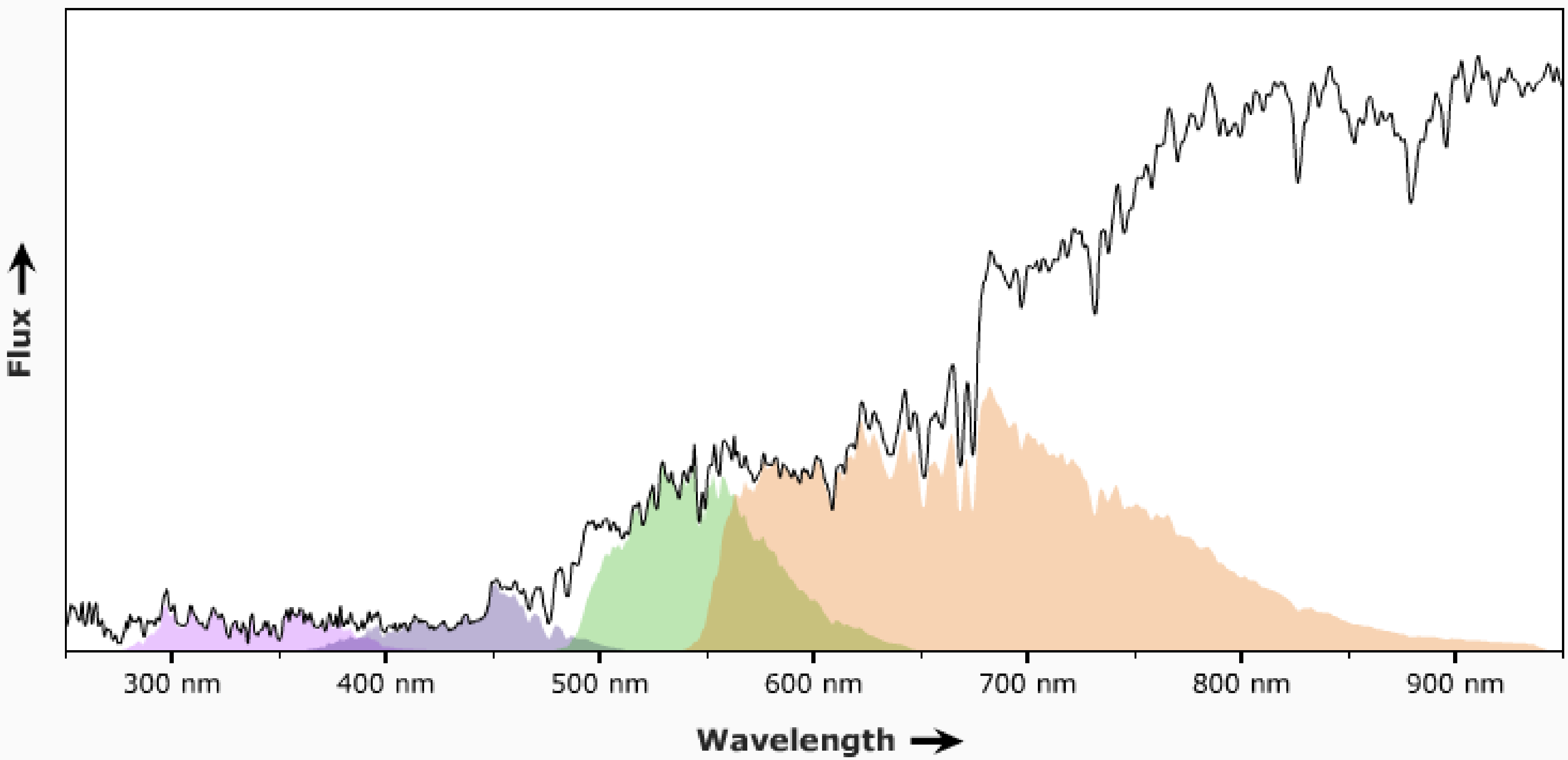


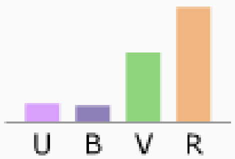
Galactic Redshift reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.80

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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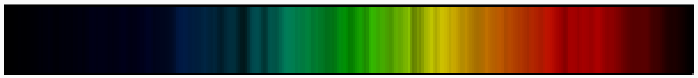
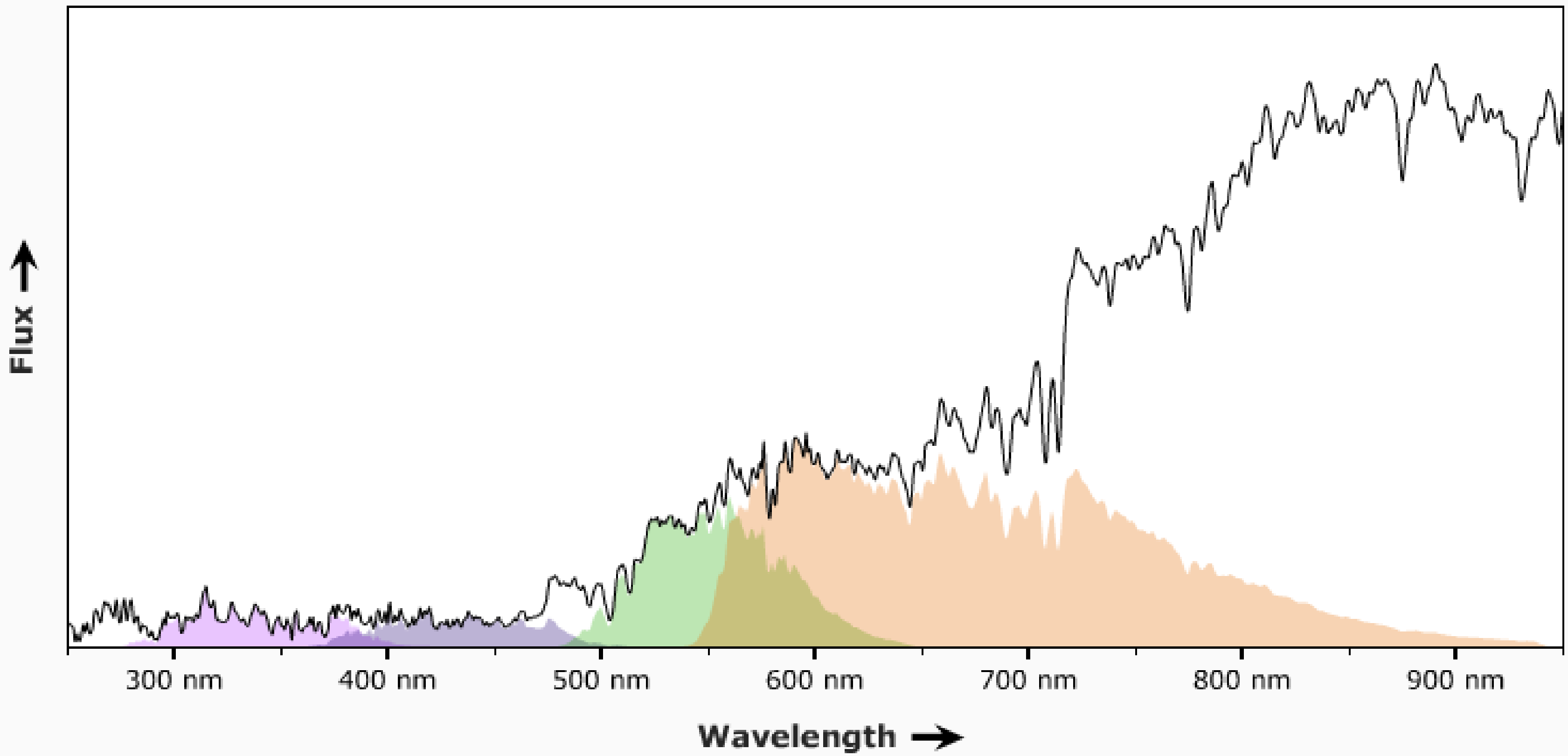
Visible Spectrum





Galactic Redshift

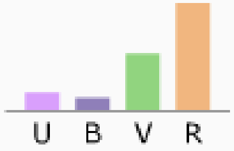
reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 0.90

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.



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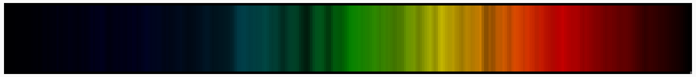

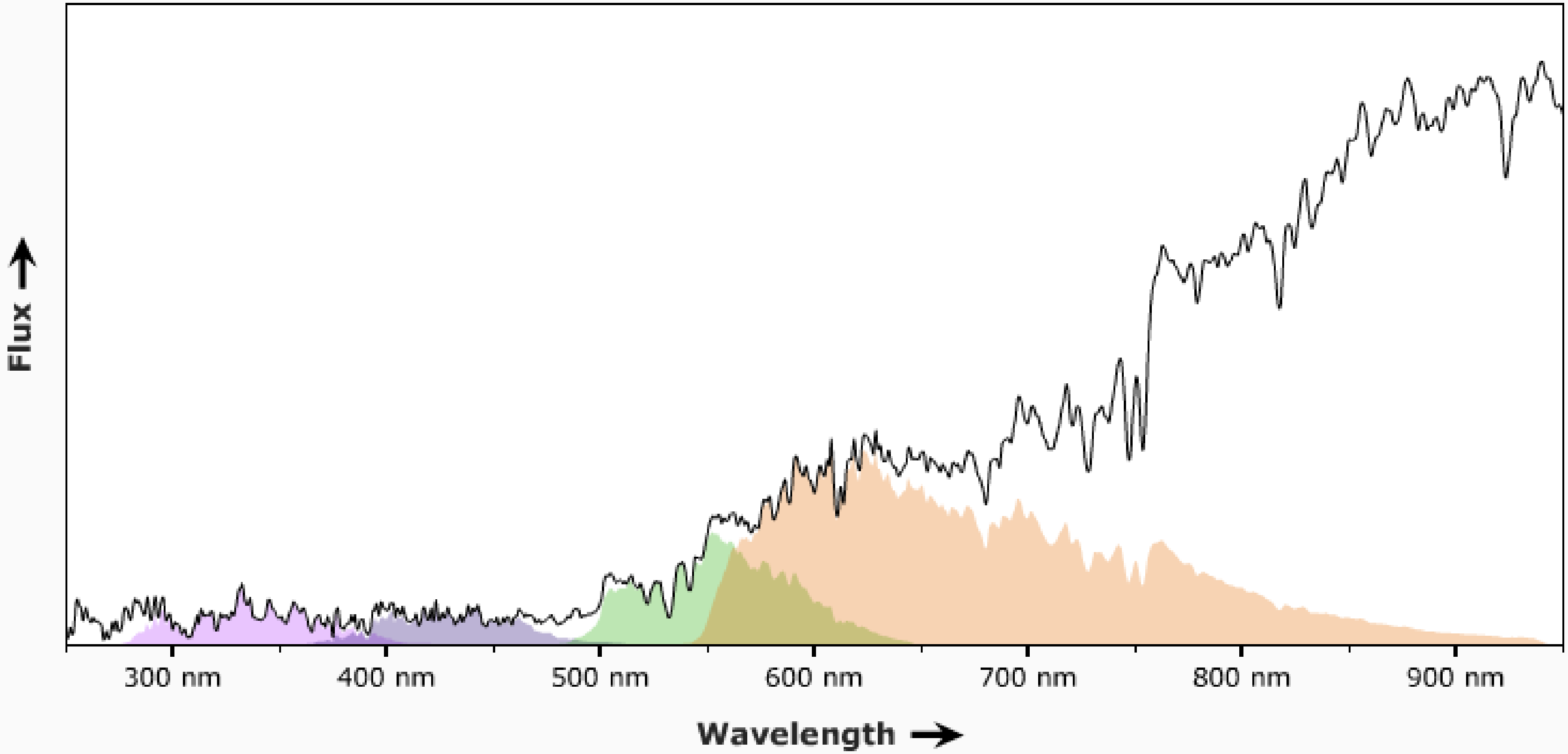

Galactic Redshift

reset about

The spectrum for a galaxy is shown below. As the redshift (z) increases more of the galaxy's light is observed in the infrared and longer wavelengths. Due to the expansion of the universe more distant galaxies have greater redshifts.

z (redshift): 1.00

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

Astronomers observe objects through various filters. As the galaxy's light is redshifted the relative brightness observed through the filters changes.

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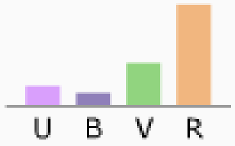
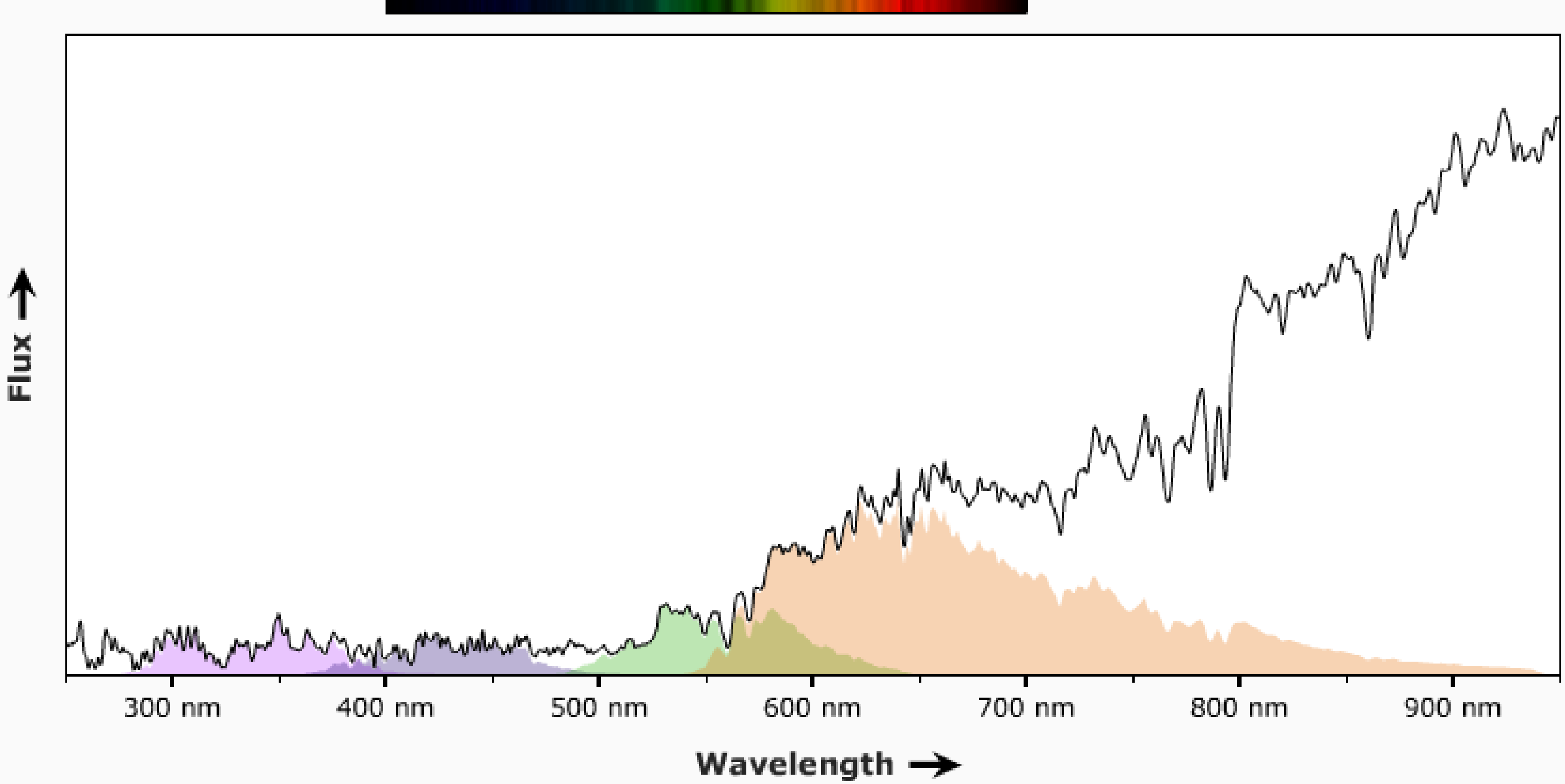
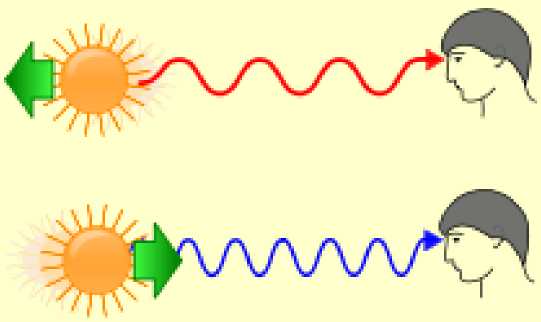




Different origins of redshifts/blueshifts

- Doppler effect : redshift/blueshift due to relative motion



$$1+z=\gamma\left(1+\frac{v_{//}}{c}\right) \qquad z\approx\frac{v_{//}}{c} \quad \text{for small } v_{//}$$

(in Minkowski space i.e. flat spacetime)

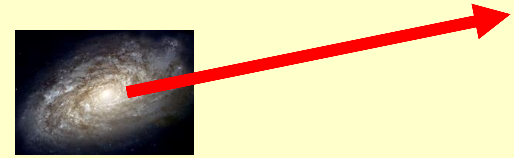
- Cosmological redshift : dominant for distant sources (above tens of Mpc or $z>0.01$). Due to Universe expansion.

- Gravitational red/blue shift : radiation moving out of/into a gravitational field.

  e.g. grav. redshift

$$1+z=\frac{1}{\sqrt{1-2GM/rc^2}}$$

The rise of modern cosmology



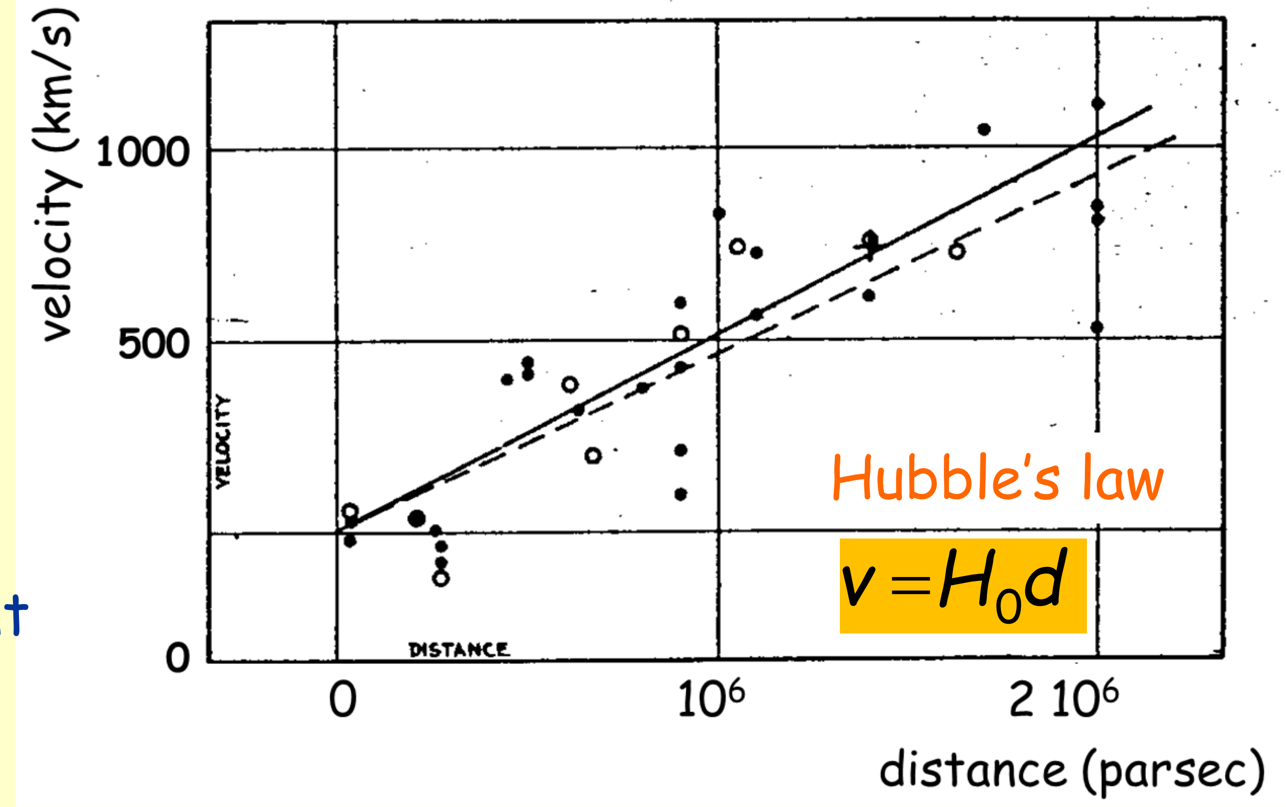
Hubble, 1929

- distant galaxies are receding : the Universe is in expansion
(as predicted in General Relativity by A.Friedmann 1922 and G.Lemaître 1927)

Hubble data

Redshift converted into velocity

$z \sim v/c$



- H_0: Hubble constant
- recent, precise measurements (2011): $H_0 = 73.8 \pm 2.4\ km/s/Mpc$
  (2016): $H_0 = 73.24 \pm 1.74 km/s/Mpc$
  (2022): $H_0 = 73.04 \pm 1.04 km/s/Mpc$
- critical density today

$$\rho_c^0 = 3H_0^2 / 8\pi G = 1.04 h^2 10^{10} eV \cdot m^{-3} \qquad h = H_0 / 100 kms^{-1} Mpc^{-1}$$

⇕

same energy density as 1gal/Mpc^3~5p/m^3

Describing an *expanding* universe

- General Relativity : the simplest relativistic theory of gravitaty consistent with data. Gravity described as a geometric property of spacetime.

- Metric: allows to compute distances between two points

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

$g_{\mu\nu}$: metric tensor ds^2 : line element, invariant

  - Reminder: in special relativity (no gravity):

$$ds^2 \equiv \eta_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$= c^2 dt^2 - \left(dr^2 + r^2 \left(d\theta^2 + \sin\theta^2 d\varphi^2 \right) \right)$$

$\eta_{\mu\nu}$: Minkowski metric

  - in a particle's rest frame:

$$ds = c\ d\tau$$ $d\tau$: particle proper time

- Friedmann-Lemaître-Robertson-Walker (FLRW) metric : metric for a spatially homogeneous and isotropic expanding universe, with scale/expansion factor a(t) and curvature k

$$ds^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right)$$

expansion factored out

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$ expansion rate or Hubble parameter

$$H_0 \equiv H(t_0) \quad t_0 = today$$

r,θ,φ spherical comoving coordinates:

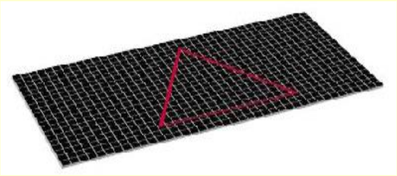
r : dimensionless & stationary wrt expansion

k, gaussian curvature:

closed universe

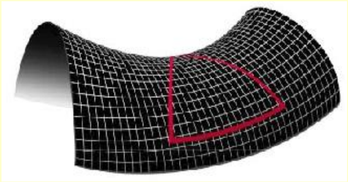
k=+1

flat universe



k=0

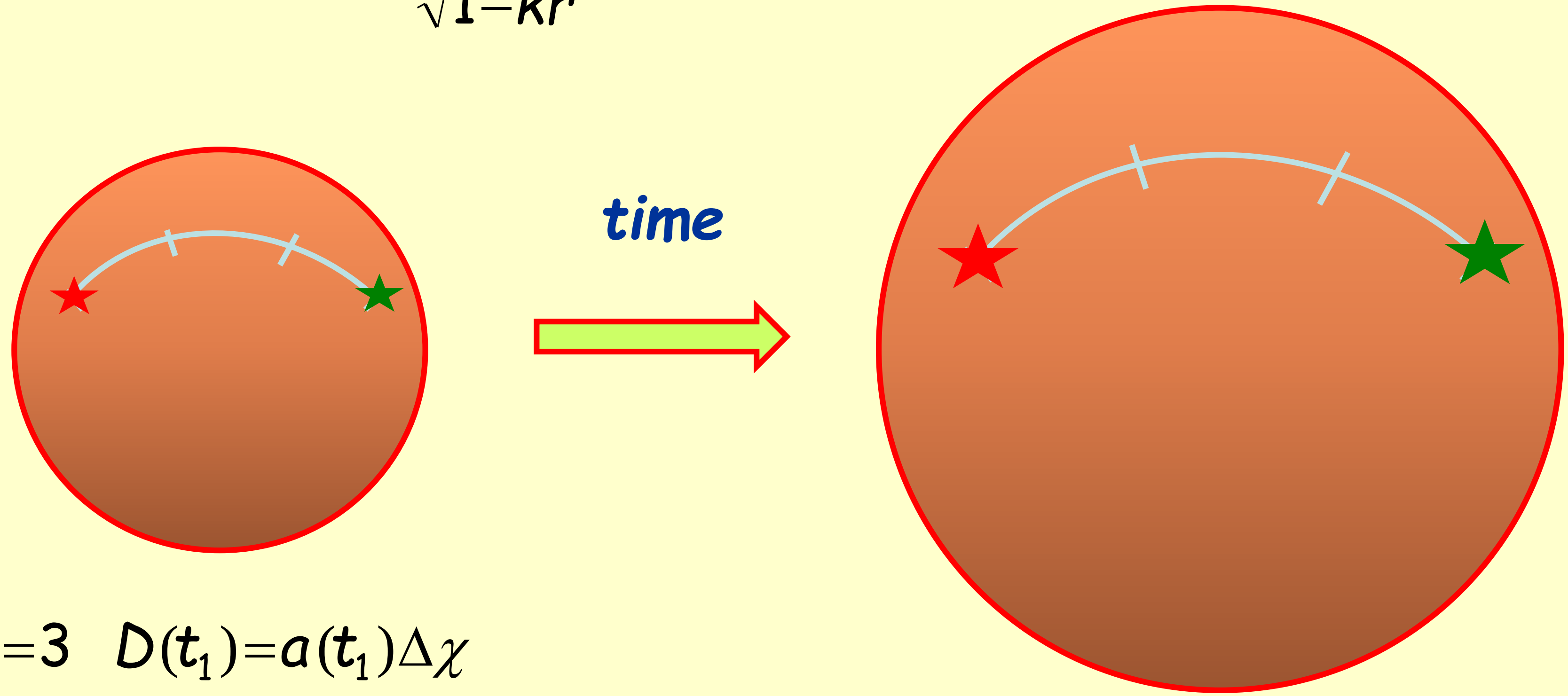
open universe



k=-1

More on FLRW metric

- Introducing $d\chi \equiv \frac{dr}{\sqrt{1-kr^2}}$ to account for curvature :



$\Delta\chi = 3 \quad D(t_1) = a(t_1)\Delta\chi$

$\Delta\chi = 3 \quad D(t_2) = a(t_2)\Delta\chi \geq D(t_1)$

$a(t)$: scale factor

χ: comoving coordinate, stationary wrt expansion

$D(t)$: proper/physical distance $\quad \Delta\chi$: comoving distance

- FLRW metric:

$$ds^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right)$$

$$ds^2 = c^2 dt^2 - a(t)^2 \left(d\chi^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right)$$

$$r = f(\chi) = \begin{cases} \sin\chi & k=1 \\ \chi & k=0 \\ sh\chi & k=-1 \end{cases}$$

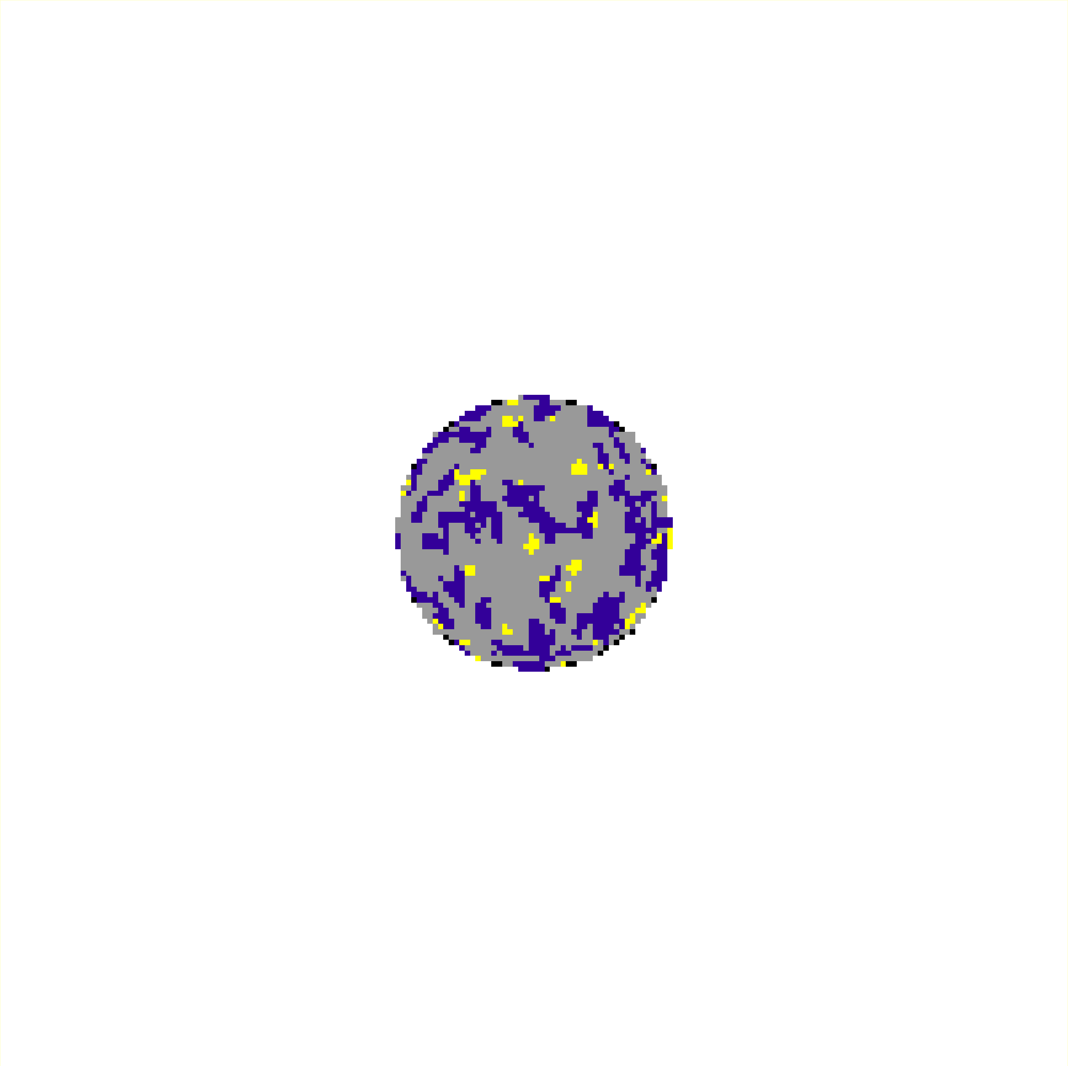
- Hubble's law:

$$D(t) = a(t)\Delta\chi \Rightarrow \frac{dD}{dt} = \dot{a}(t)\Delta\chi = \frac{\dot{a}(t)}{a(t)} D(t) \Rightarrow \frac{dD}{dt} = H(t)D(t)$$

- Particle trajectories :

$ds^2 = 0$ geodesic equation (shortest path in three-space and maximum proper time)

The cosmological redshift

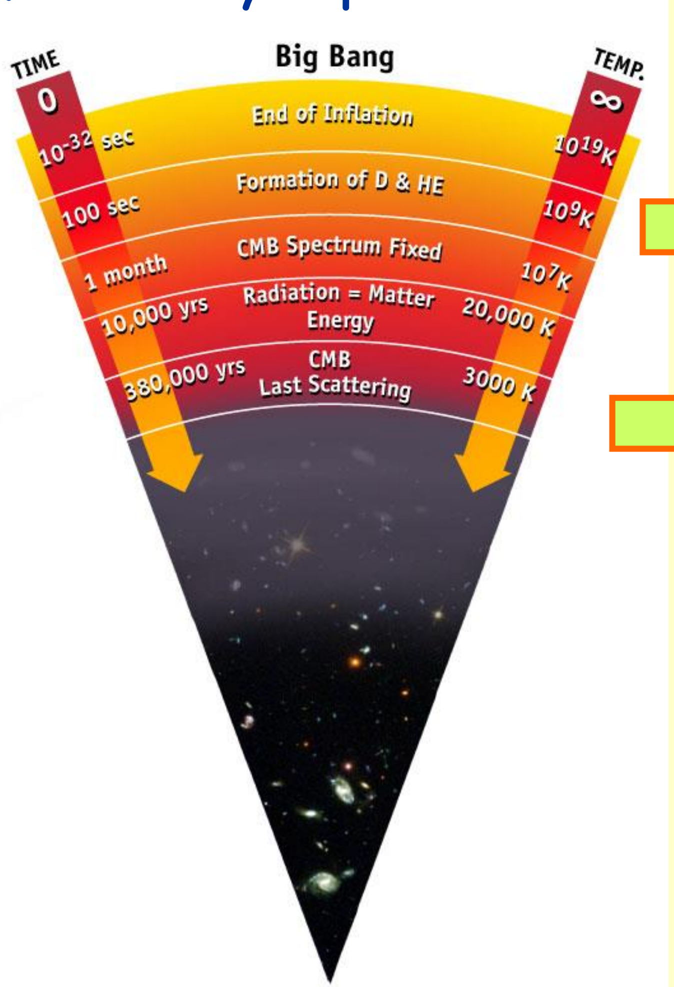


Universe in expansion

light from distant sources is redshifted

$$\frac{\lambda_{observed}}{\lambda_{emitted}} \equiv 1 + z = \frac{a(t_{observation})}{a(t_{emission})}$$

Initial dense and hot phase followed by expansion and cooling



13,7 billions of years after Big Bang

More confirmations of the Big Bang model

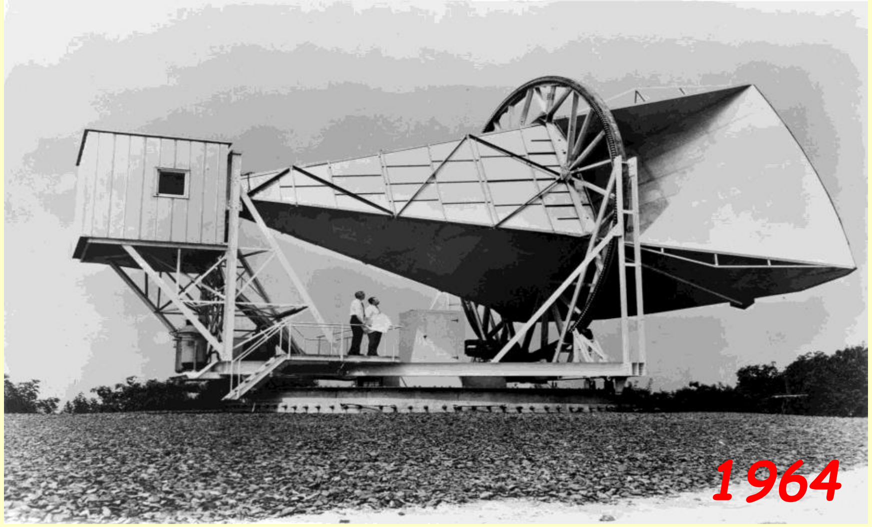
Primordial nucleosynthesis (He, D): T~1MeV predicted light element abundances = data

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-10}$$

1948: G.Gamow, R.Alpher

Matter-radiation decoupling: z~1100 T~3000K
Cosmic **M**icrowave **B**ackground, relic radiation predicted and observed (T~2.725K 1992)

1948: G.Gamow, R.Alpher, R.Herman
1964: A.Penzias, R.Wilson

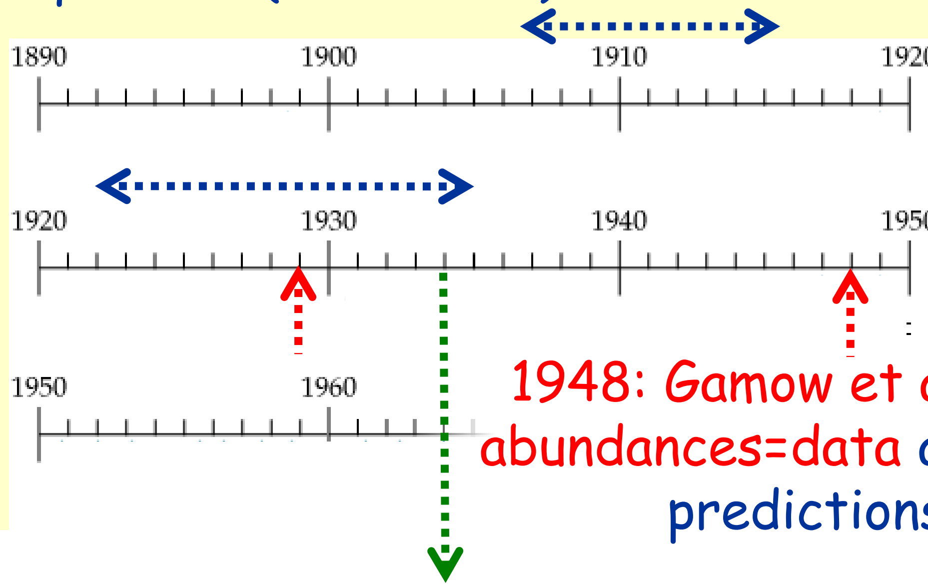


The rise of the Big Bang model

General Relativity, Einstein's equations (1907-1915)

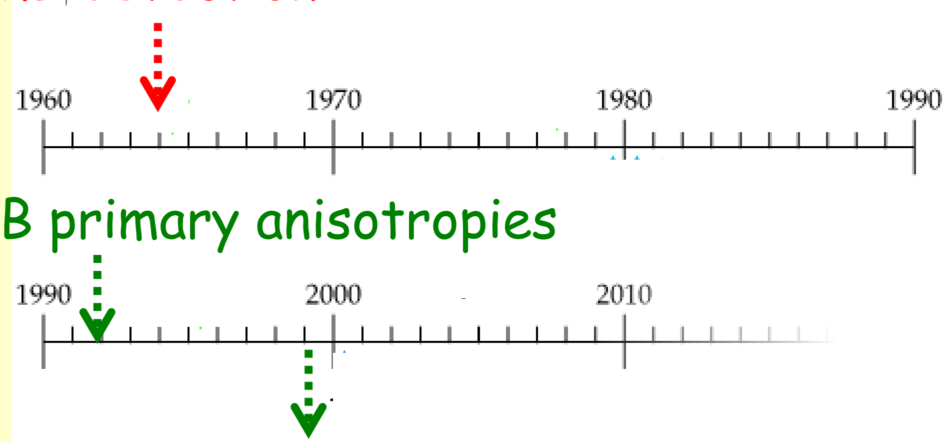
1922-1935 FLRW solutions
(expanding, homogeneous and isotropic Universe)

1929 Hubble's law



1948: Gamow et al: BBN abundances=data and CMB predictions

CMB detection



CMB primary anisotropies

1934: 'missing mass' in orbital velocities of galaxies in clusters → dark matter postulated (F.Zwicky)

Accelerated expansion: dark energy (or modified Einstein's gravity)

CONCLUSIONS (1)

- The Universe is spatially homogeneous and isotropic on cosmological scales (above 100Mpc)

- The Universe, initially in a hot, dense phase, is expanding

$$H_0 \equiv H(t_{today}) \approx 70 km/s/Mpc$$

  - cosmological redshift: due to the Universe expansion

$$\frac{\lambda_{observed}}{\lambda_{emitted}} \equiv 1+z = \frac{a(t_{observation})}{a(t_{emission})}$$

- General relativity and FLRW metric are the basics of the Big Bang model

- Observations: expansion, primordial abundances, CMB

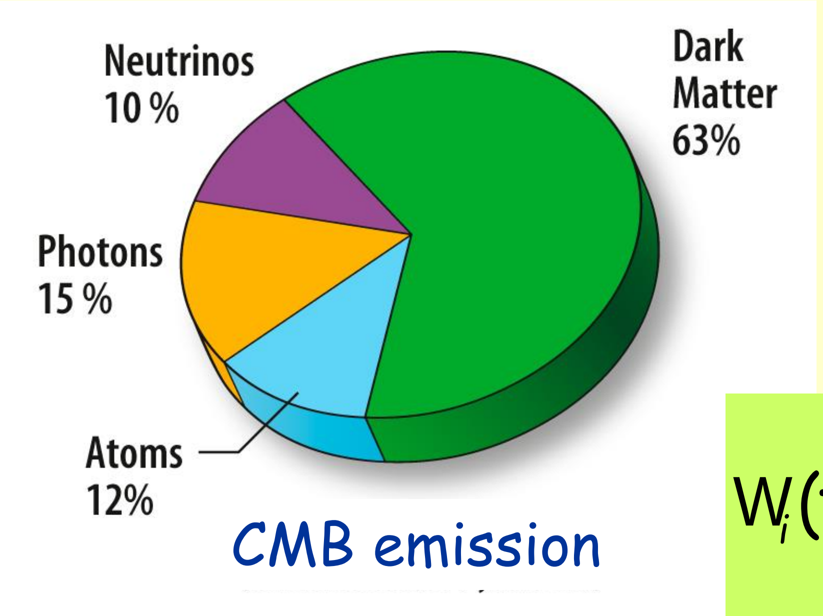
Content of the Universe

From General Relativiy to the matter-energy content of the Universe

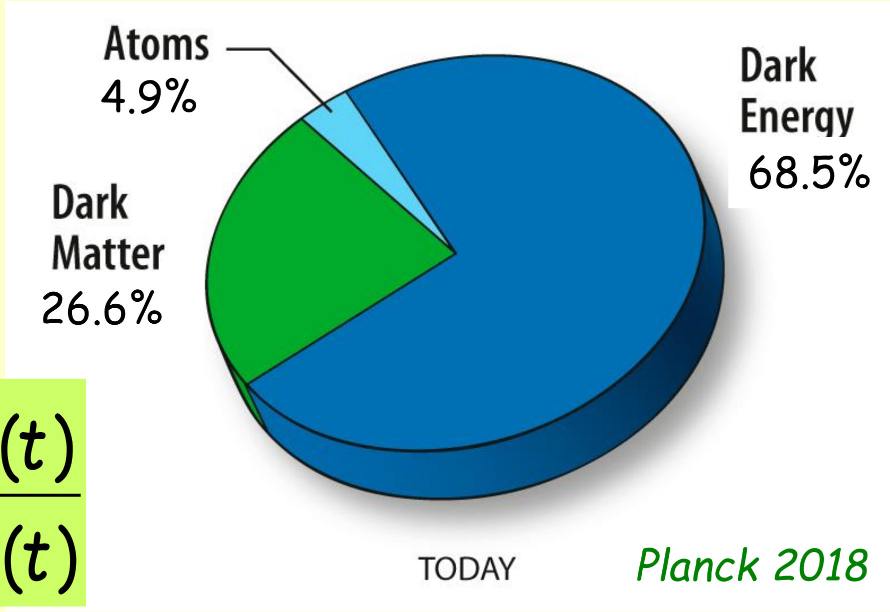
- The Ω_i parameters and the expansion history
- Thermal history of the Universe

Big Bang cosmology model+ **many** cosmological data

energy balance of the Universe



$$\Omega_i(t) \equiv \frac{\rho_i(t)}{\rho_c(t)}$$



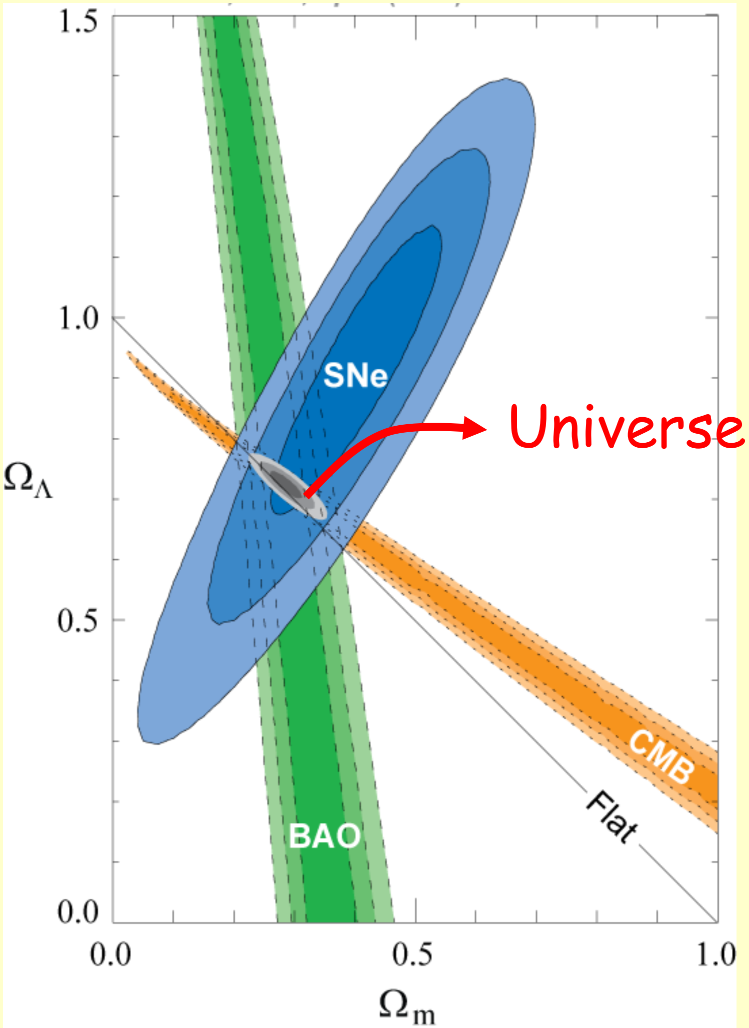
$$1 = \Omega_m(t) + \Omega_r(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

NR matter — radiation — curvature — "dark energy" or cosmological constant Λ

(radiation and curvature: negligible today)

Present *values of the Ω_i's : the Ω_i^0 values*

- CMB + BAO + SNe Ia data :



Universe is flat !

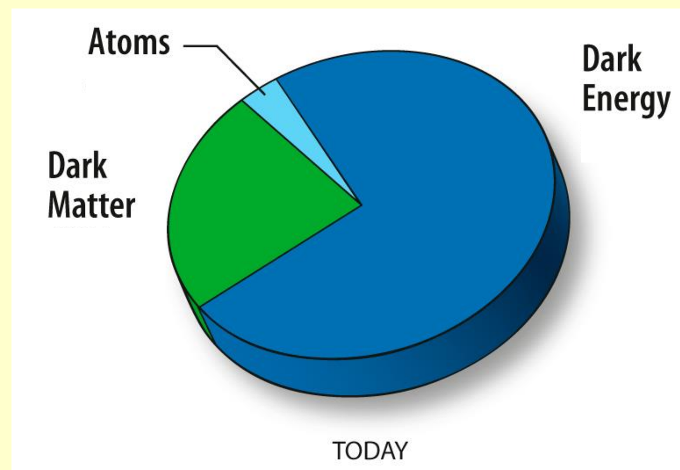
M. Kowalski et al., 2008, ApJ, 686, 749

2.2% precision

$\Omega_m^0 = 0.315 \pm 0.007$

$\Omega_\Lambda^0 = 0.685 \pm 0.007$

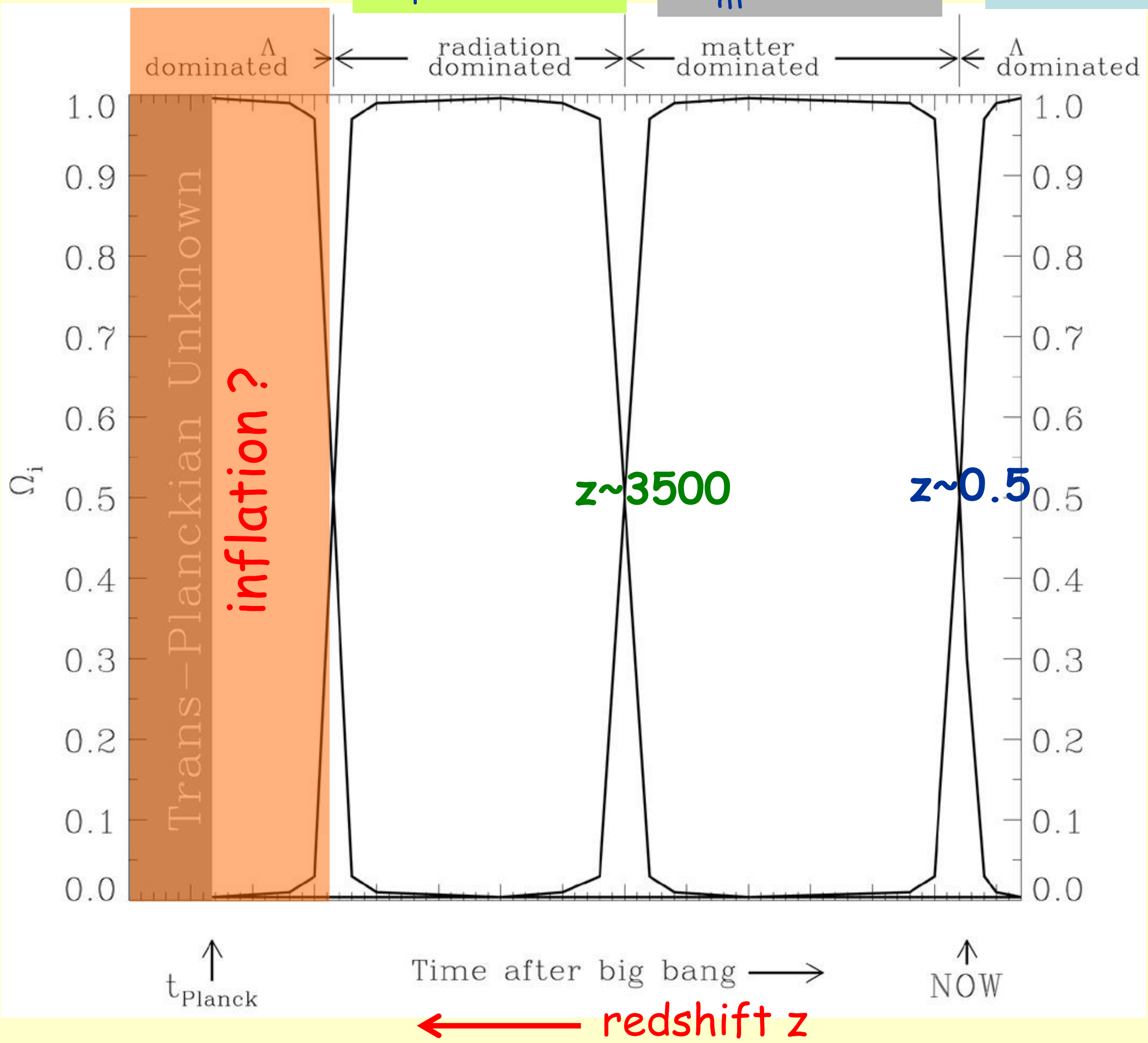
Planck collaboration. 2018, arXiv:1807.06209



atoms: gas (4%), stars (0.4%), ν (0.3%), heavy elements (0.03%)

The rise and fall of the Ω_i's

$\Omega_r \propto a^{-4} H^{-2}$ $\Omega_m \propto a^{-3} H^{-2}$ $\Omega_\Lambda \propto H^{-2}$



Expansion and temperature

- Photons emitted at t (e.g. CMB), received today:

$$1+z \equiv \frac{\lambda_{observed}}{\lambda_{emitted}} = \frac{a_0}{a(t)} = \frac{E_\gamma(t)}{E_\gamma(t_0)} = \frac{T(t)}{T_0}$$

$T(t)$ → at emission; T_0 → today

- Temperature was hotter in the past, expansion implies cooling down
- CMB now: $T_{CMB}^{now} \approx 2.725 K \Rightarrow T_0 \approx 2.35\ 10^{-4} eV \Rightarrow \Omega_\gamma^0 \approx 5.10^{-5}$

  (COBE) $(k = 8.617\ 10^{-5} eV.K^{-1})$ — negligible (today)

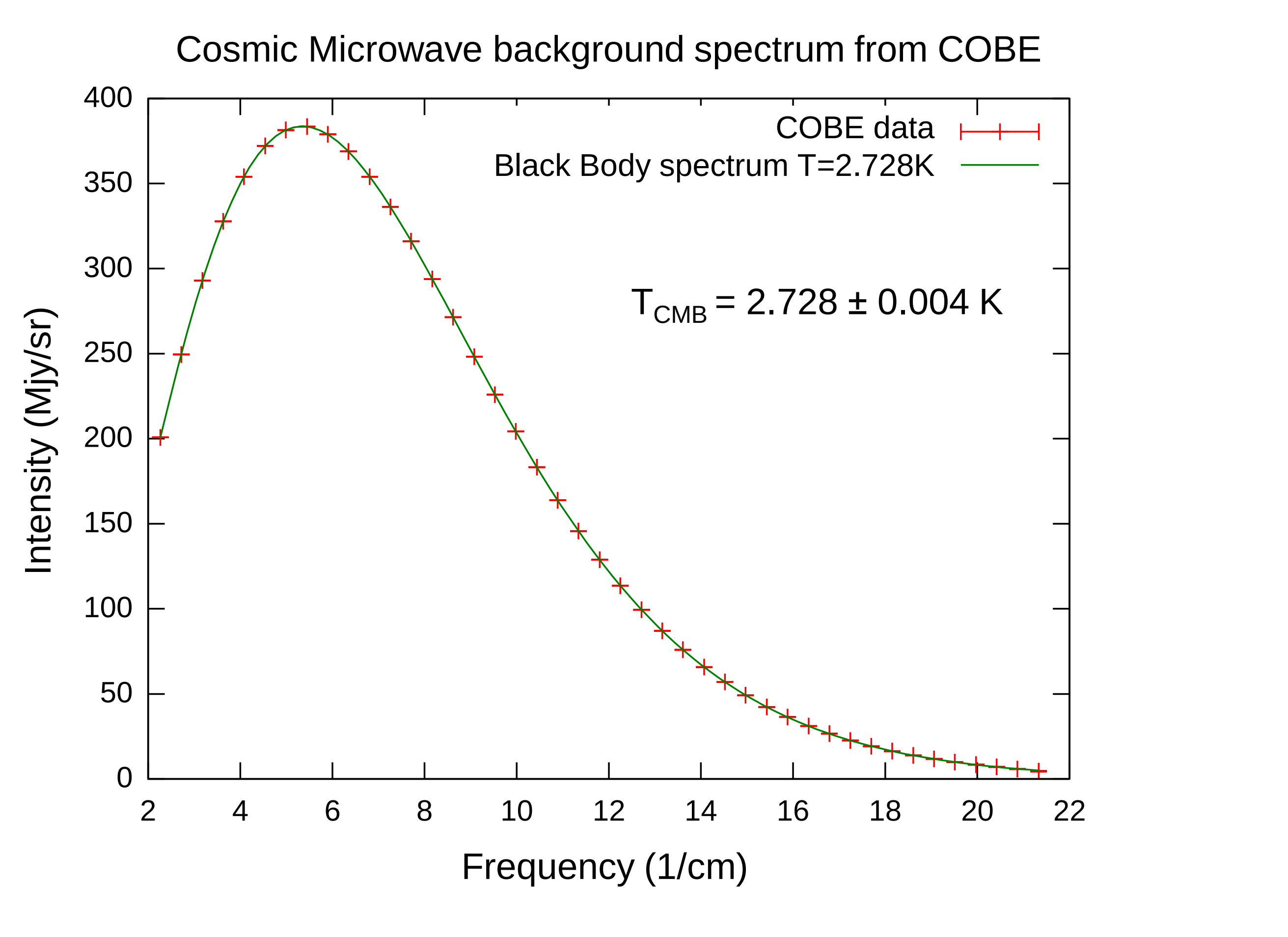
- CMB at emission (from anisotropy measurements):

$$z_{CMB}^{emission} \approx 1100 \Rightarrow T_{CMB}(t_{emission}) \approx 0.26 eV \approx 3{,}000 K$$

- matter-radiation equality:

$$\rho_r(t_{eq}) = \rho_m(t_{eq}) \Rightarrow a_0/a(t_{eq}) \Rightarrow T_{eq} \approx 1 eV \quad z \approx 3500$$

The CMB spectrum as seen by COBE (1989-1993)



D.J. Fixsen et al. 1996, ApJ, 473, 576F

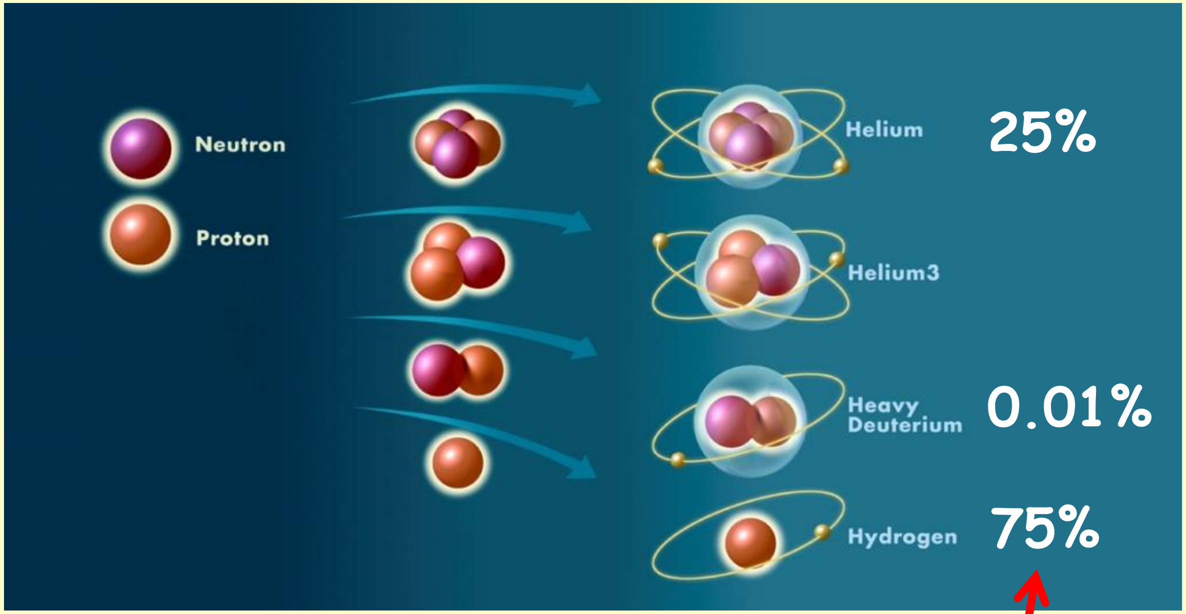
- Final : $T_{CMB} = 2.7255 \pm 0.0006$ K

D.J. Fixsen, 2009, AJ,707,916

Primordial nucleosynthesis

- 1948: G.Gamow, R.Alpher "The origin of Chemical Elements"

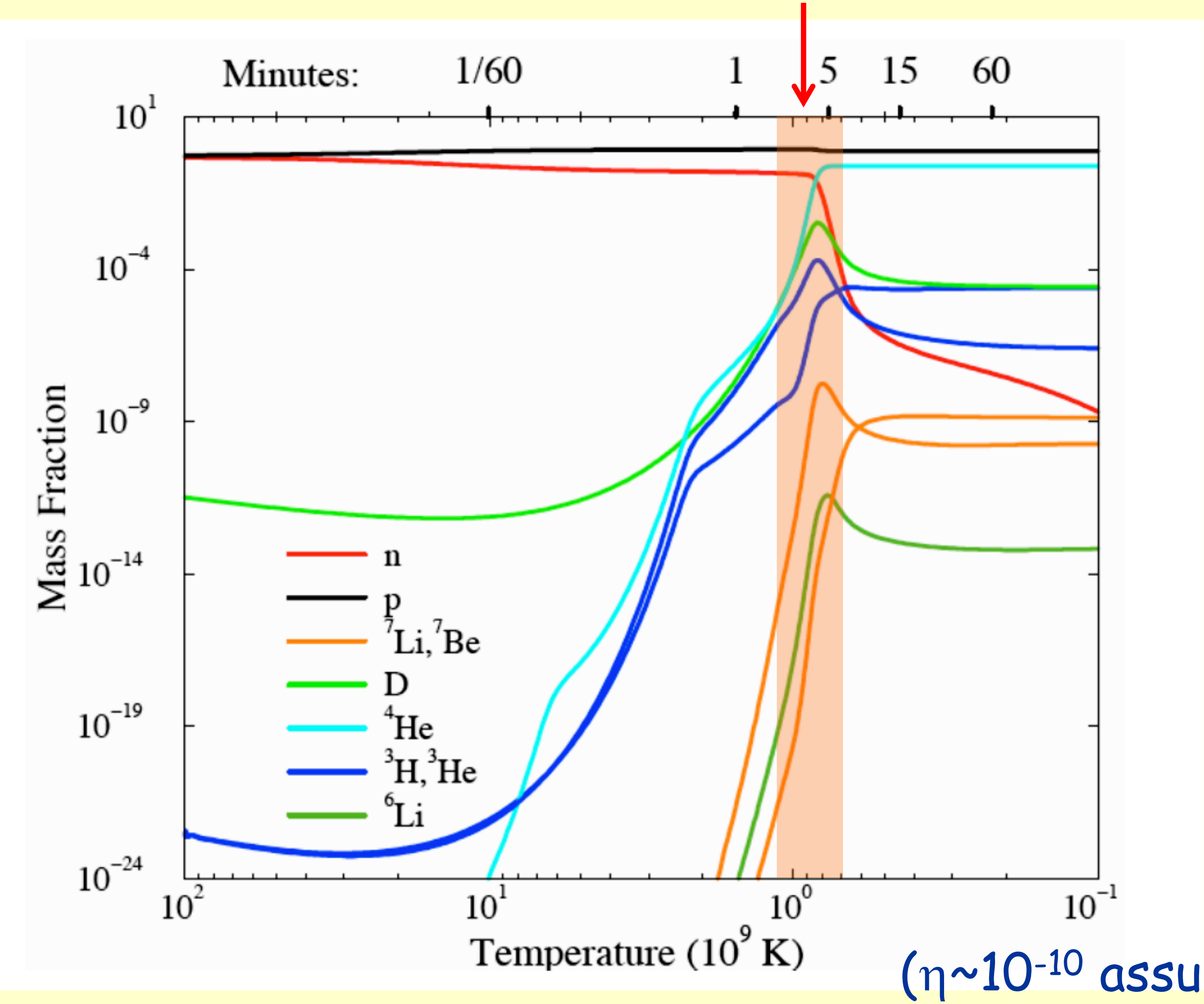
Expanding universe: deuterium and helium nuclei are formed by nuclear reactions inside the primordial plasma of p,n,e,γ when temperature and densities are adequate, leading to light element abundances as measured.



measured primordial abundances

The evolution of abundances

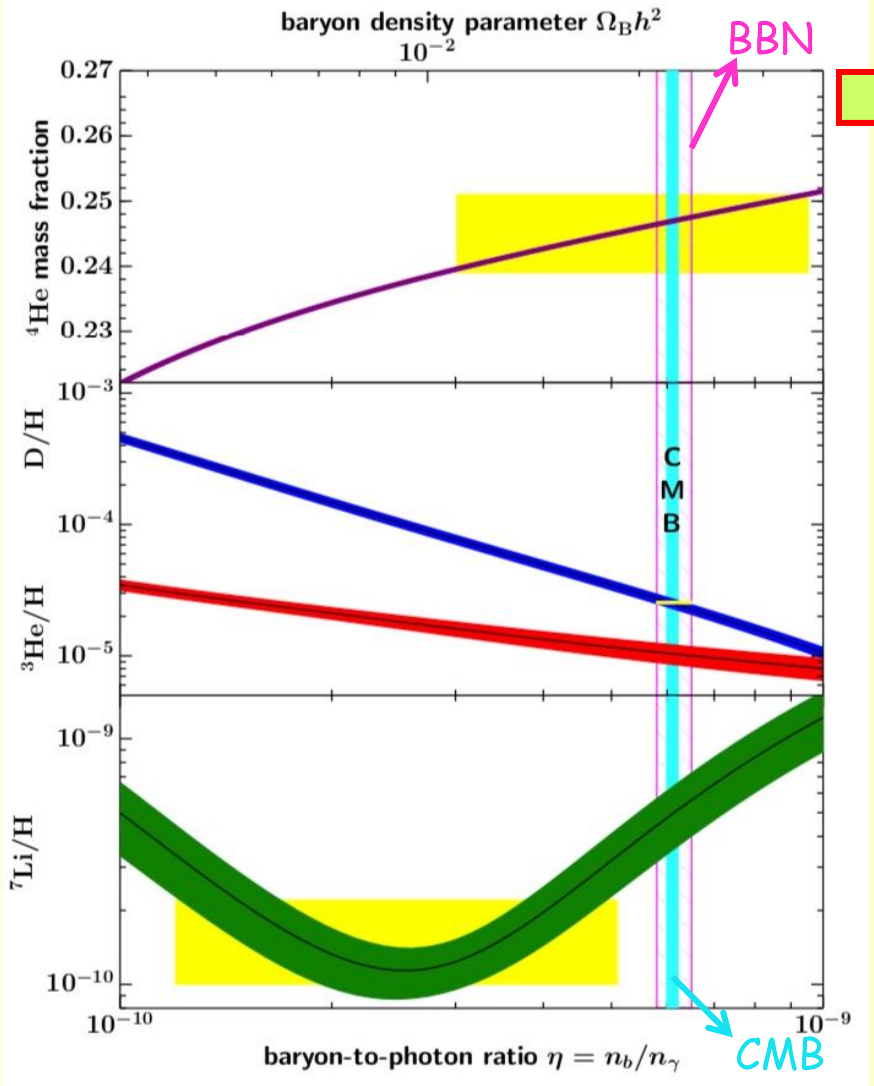
t_{BBN}~3mn : abundances frozen



(η~10^{-10} assumed)

Present values of the Ω_i's

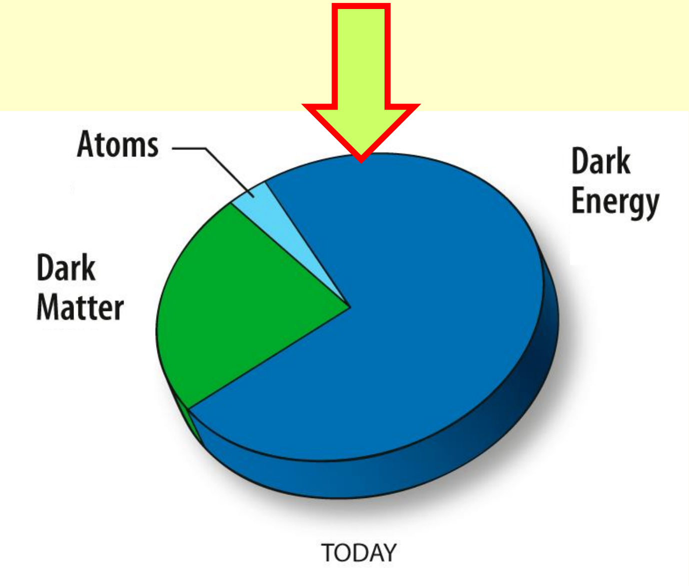
- Measurements of light element abundances vs predictions:



$\Omega_b^0 h^2$ = 0.02244±0.00069

$\Omega_b^0 \sim$ 0.046 (with h=0.7)

(confirmed independently by CMB - Planck 2018)



Most matter in the Universe is not ordinary atoms : dark matter !

R.L. Workman et al. (PDG), Prog. Theor. Exp. Phys., 083C01 (2022)

CONCLUSIONS (2)

- Einstein GR equations + FLRW metric:

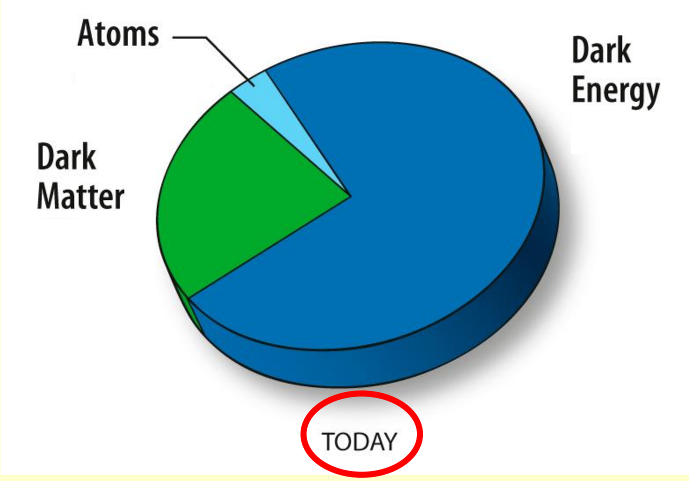
$$1 = \Omega_m(t) + \Omega_r(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

- COBE measurement: radiation today is negligible
- Cosmological measurements vs predictions:
  - CMB+SNeIa+BAO:

    $W_m^0 = 0.315 \pm 0.007$

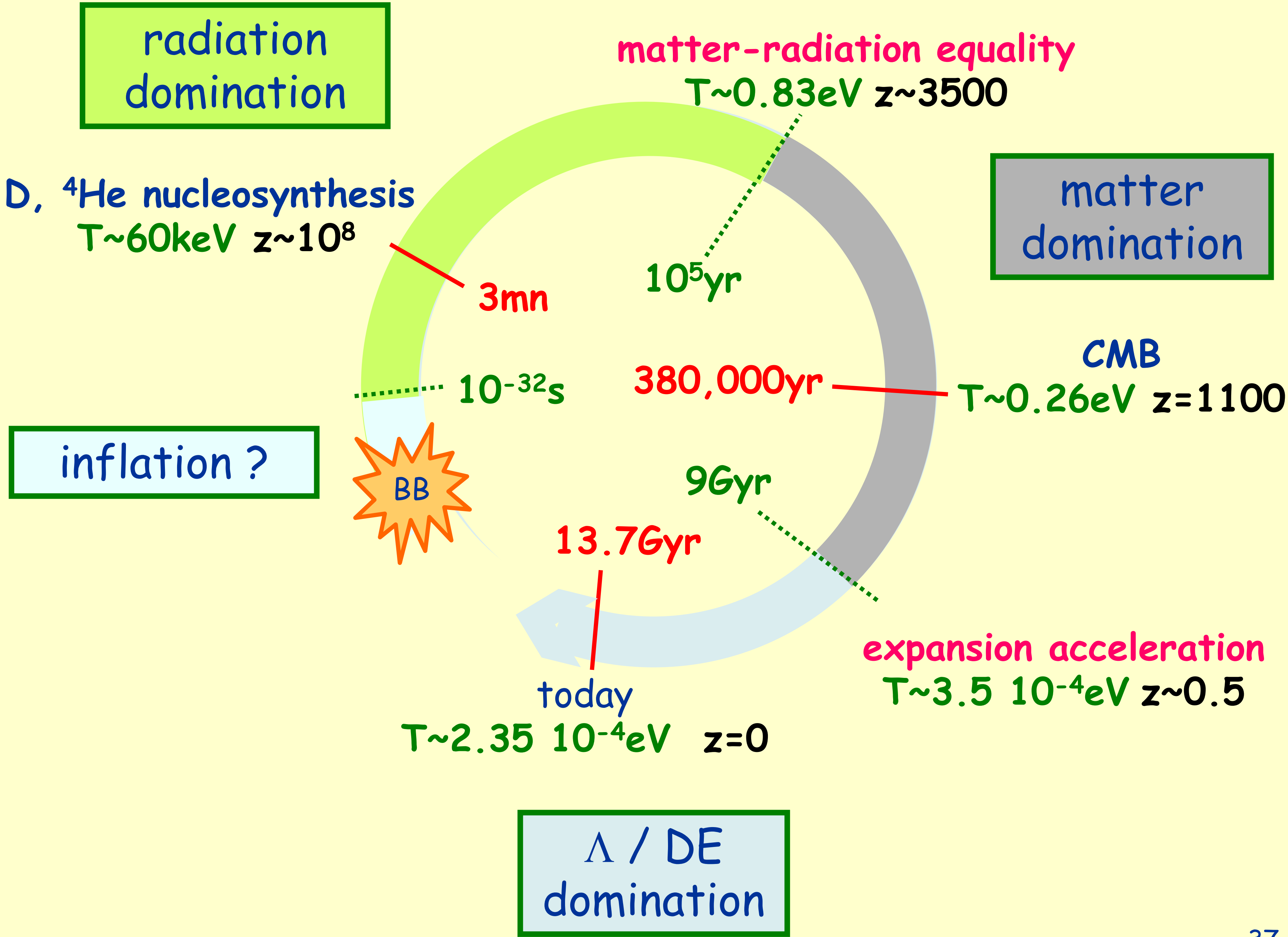
    $W_L^0 = 0.685 \pm 0.007$

    ➔ flat universe (Ω_k^0 ~0)



  - Primordial abundances:

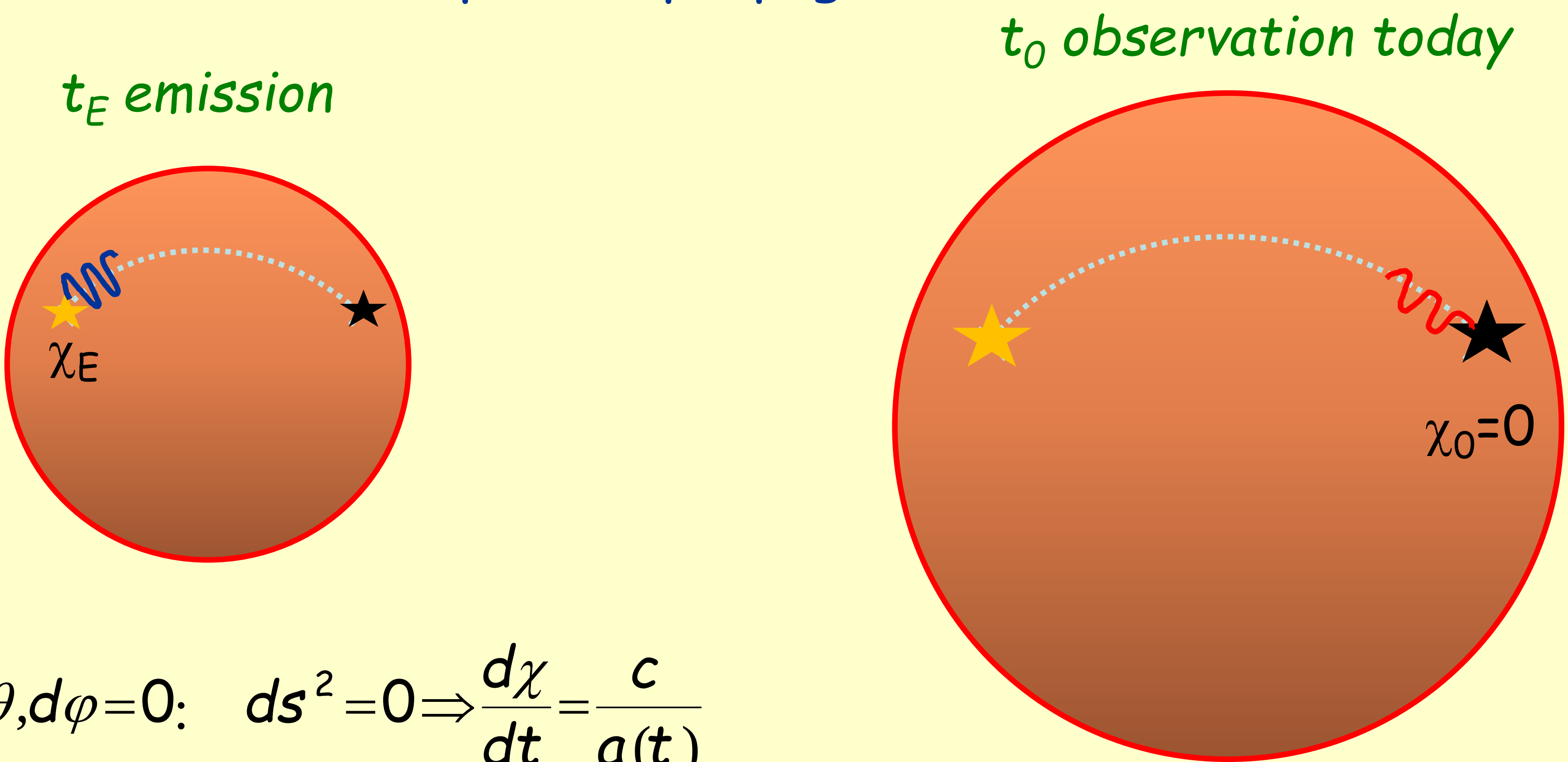
    $\Omega_b^0 = 0.046 \pm 0.001$ (with h=0.7)



radiation domination
matter-radiation equality
T~0.83eV z~3500
matter domination
D, 4He nucleosynthesis
T~60keV z~10^8
3mn
10^5yr
CMB
T~0.26eV z=1100
380,000yr
10^-32s
inflation ?
BB
9Gyr
13.7Gyr
expansion acceleration
T~3.5 10^-4eV z~0.5
today
T~2.35 10^-4eV z=0
Λ / DE domination

BACK UP SLIDES

- Particular case : photon propagation



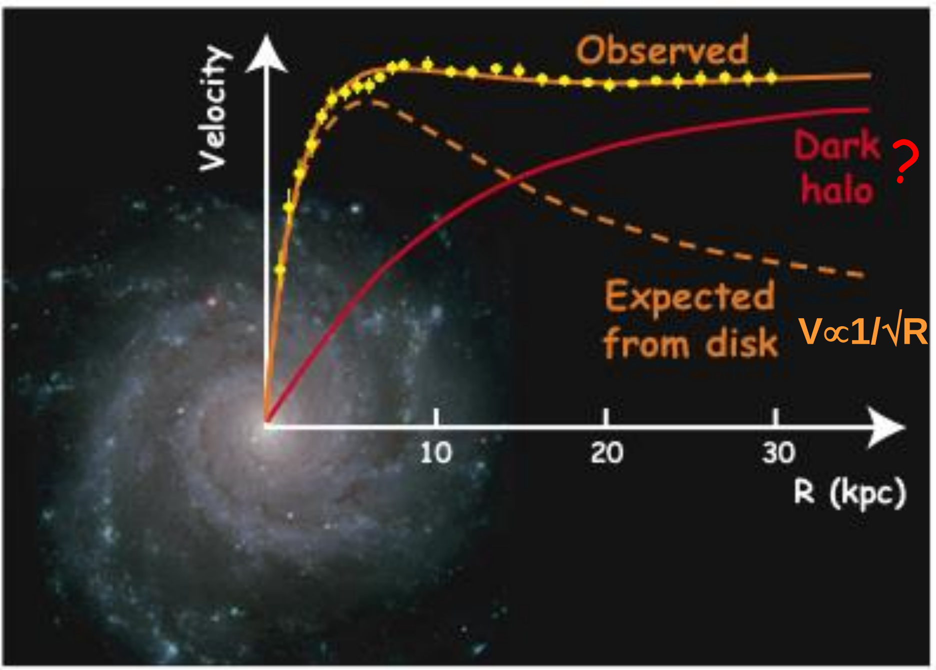
$$d\theta, d\varphi = 0: \quad ds^2 = 0 \Rightarrow \frac{d\chi}{dt} = \frac{c}{a(t)}$$

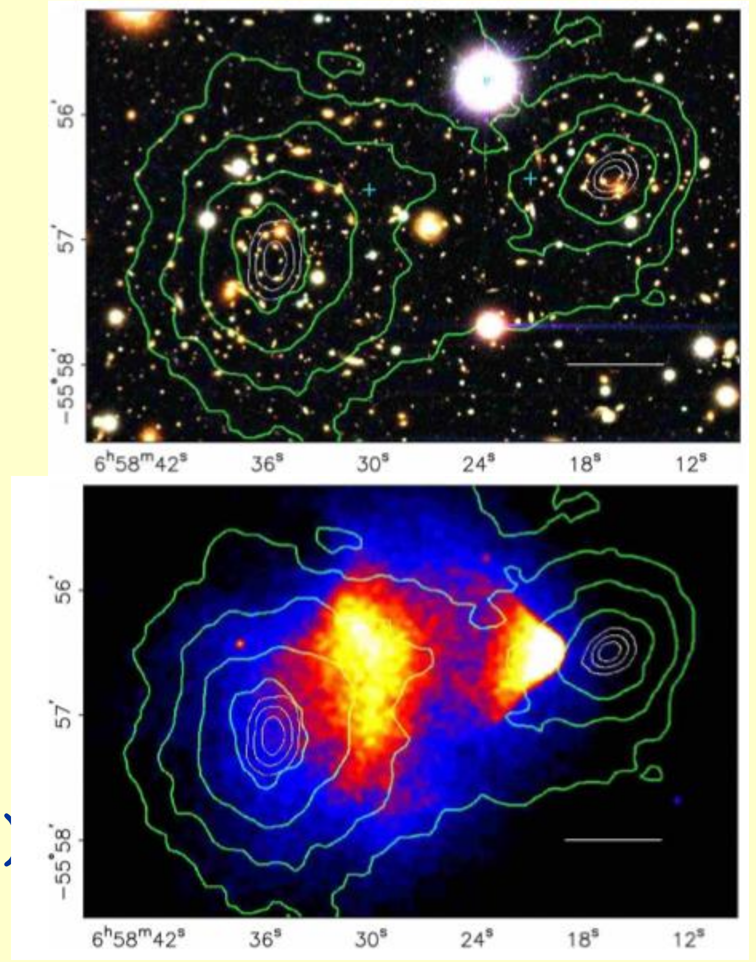
$$\chi_E = \int_{t_E}^{t_0} \frac{cdt}{a(t)} = \int_{t_E+\lambda_E/c}^{t_0+\lambda_0/c} \frac{cdt}{a(t)} = \int_{t_E+\lambda_E/c}^{t_E} \frac{cdt}{a(t)} + \int_{t_E}^{t_0} \frac{cdt}{a(t)} + \int_{t_0}^{t_0+\lambda_0/c} \frac{cdt}{a(t)} = \int_{t_E}^{t_0} \frac{cdt}{a(t)} + \frac{\lambda_0}{a_0} - \frac{\lambda_E}{a_E}$$

$$\Rightarrow \quad \frac{\lambda_0}{\lambda_{emitted}} \equiv 1 + z = \frac{a_0}{a(t_{emission})}$$

light from distant sources is redshifted

Observational evidence





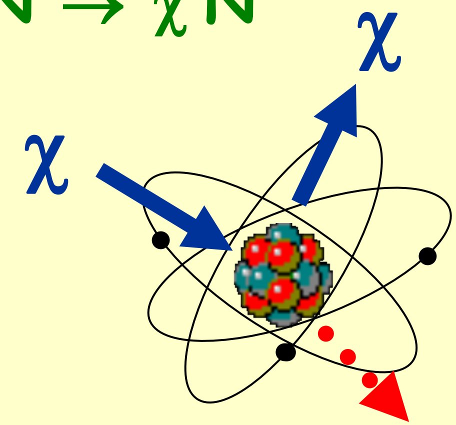
- Examples (beside cosmology):
  - Rotation curves of spiral galaxies (1959..)
  - Colliding clusters (2006)

dark matter (baryonic or new particles)
or modified Newtonian dynamics ?

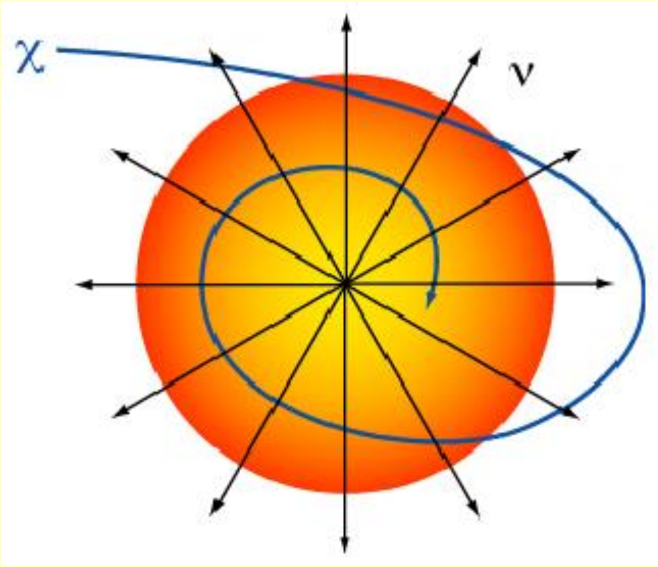
still an open question !

Search for non-baryonic dark matter

$\chi N \rightarrow \chi N$



- **Direct search**: detect nuclear recoil in cryogenic detectors
  $10GeV<m_\chi<1TeV$



- **Indirect search:** gravitational capture and co-annihilation

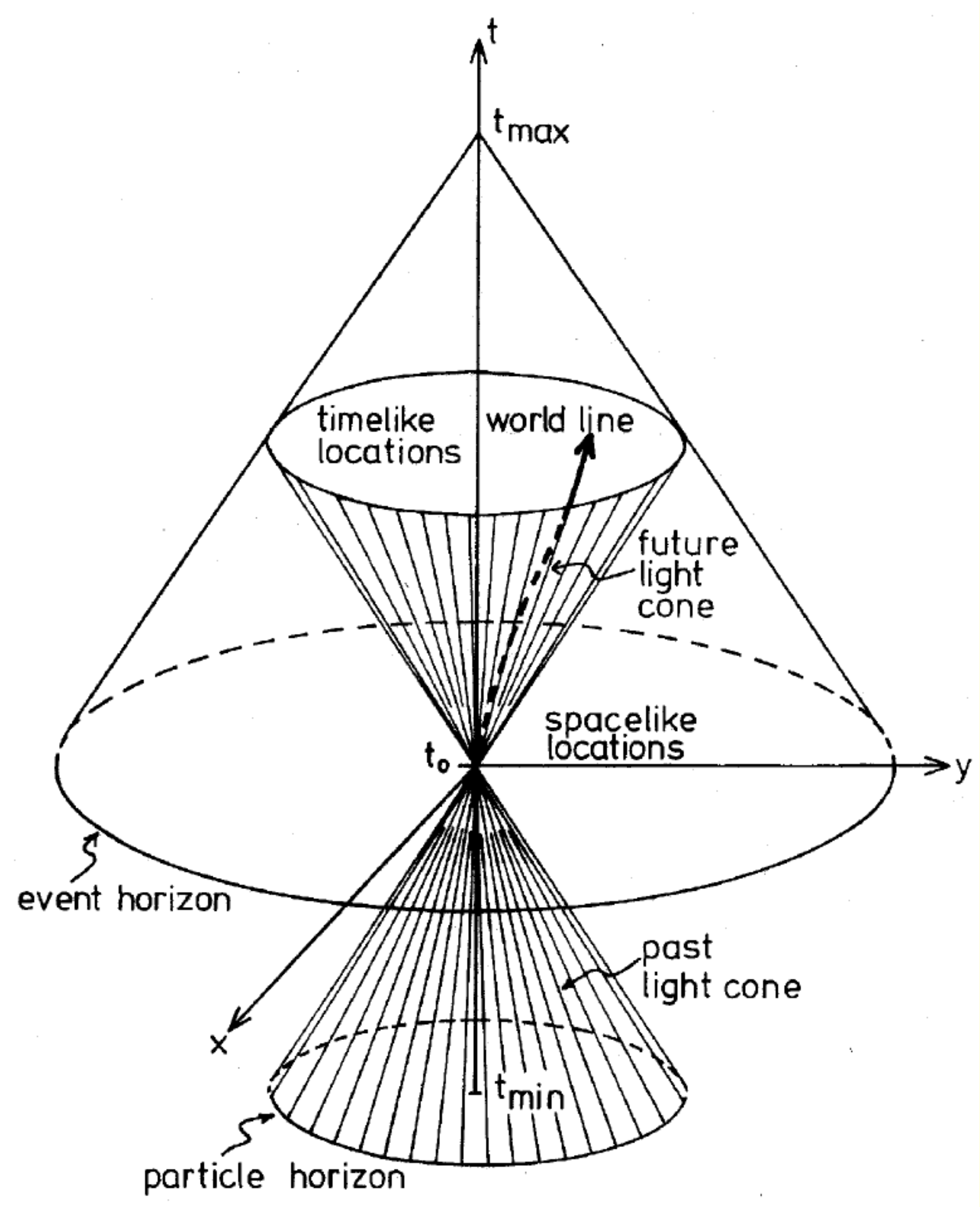
$\chi\chi \rightarrow \gamma\gamma$ γ–ray astronomy
$100GeV<m_\chi<10TeV$

$\chi\chi \rightarrow \nu+X$ ν telescopes
$100GeV<m_\chi<3TeV$

- Complementary to exotic particle searches **at the LHC** ($100GeV<m_\chi<3TeV$)

No undisputed signal so far, stay tuned !

- Les objets de z les plus grands ?
- Horizon, temps de Hubble, distance de Hubble
- Lien redshift/âge de l'Univers/distance



Horizon

- Two seperate events can be causally connected provided |dl/dt|<c : past/future light cones
- particle horizon: largest comoving spatial distance from which a signal could have reached us at t_0
- event horizon at tmax: most distant event today that will come in causal contact with us at t_{max}