Non-perturbative decoupling of massive fermions

Tobias Rindlisbacher*, Ahmed Salami†, Kari Rummukainen†, Kimmo Tuominen†

* AEC, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland

† Department of Physics, University of Helsinki, P.O. Box 64, FI-00014 University of Helsinki, Finland

Contact e-mail: * trindlis@itp.unibe.ch

Introduction

SU(2) gauge theory with $N_f = 24$ massless fermions is expected to have an UV Landau pole and is free at IR. With massive fermions the IR behaviour changes: at energy scales $\ll m_f$ the fermions decouple and the system behaves as confining SU(2) gauge theory. We demonstrate this cross-over on the lattice using gradient flow coupling constant measurements, and compare with perturbative predictions.

Finite mass RG evolution

Running of coupling g^2 and quark mass m under a change of length-scale λ is governed by renormalization group (RG) equations:

$$\frac{dg^2}{d\log(\lambda)} = -\beta(g^2, \lambda m) \ , \quad \frac{d\log(m)}{d\log(\lambda)} = \gamma(g^2, \lambda m)$$

with beta function β and mass-anomalous dimension γ.

We employ the massive BF-MOM scheme from [2] where $m = m_0$ is a pole mass and $\gamma := 0$.

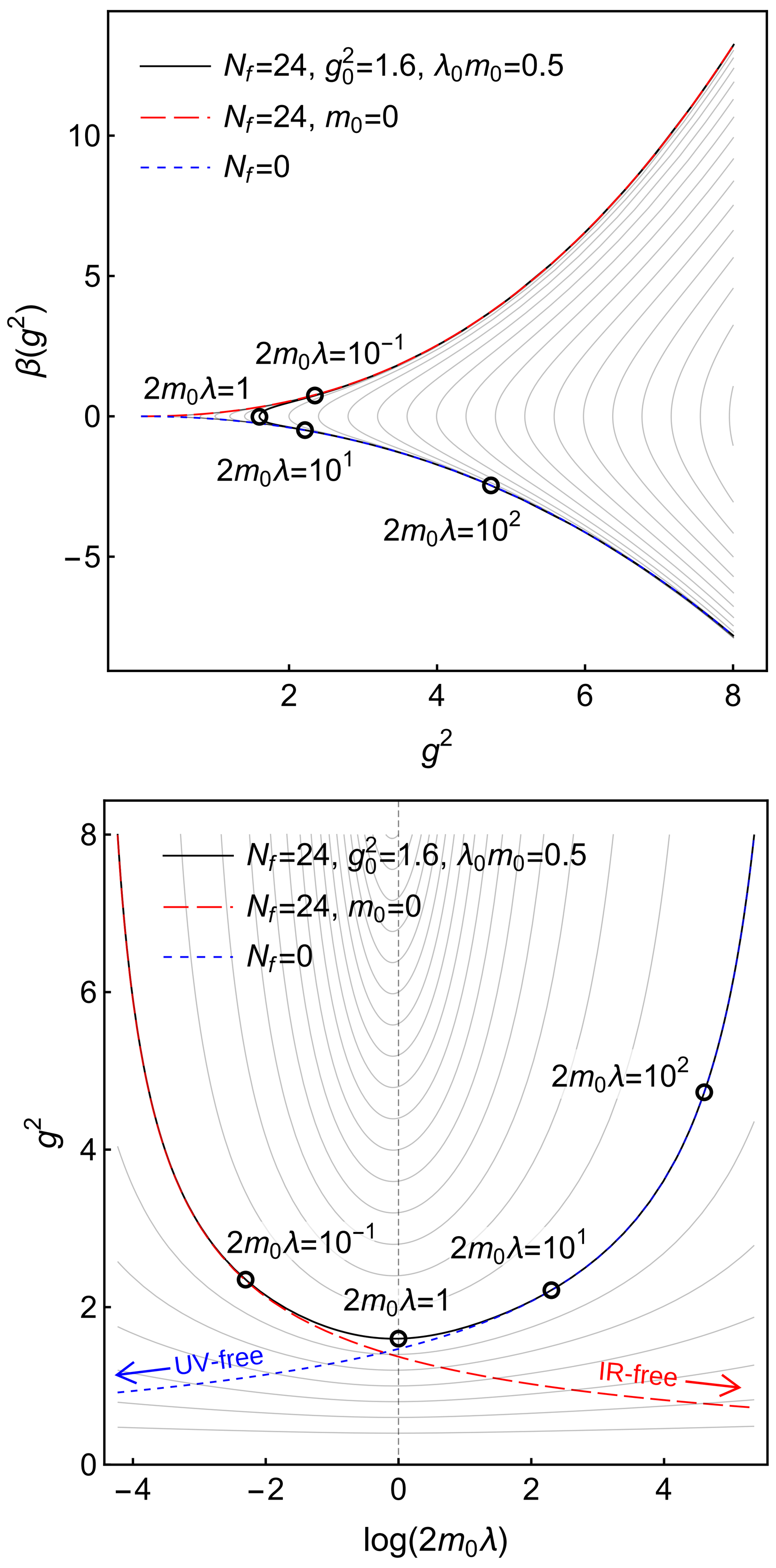

Figure 1: 2-loop running coupling (top) and 2-loop beta function (bottom) in the massive BF-MOM scheme [2] (black and gray). Also shown are the scheme-independent 2-loop curves for $N_f = 24$ massless fermions (long red dashes) and pure gauge (short blue dashes), to which the (non-universal) 2-loop BF-MOM curve for $g_0^2 = 1.6$ converges when $\lambda \ll 1/(2\,m_0)$ resp. $\lambda \gg 1/(2\,m_0)$.

References

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Simulation setup

Lattice action: $S = S_G(U) + S_F(V) + c_{\mathrm{SW}}\,S_{\mathrm{SW}}(V)$ on toroidal L^4 lattices. S_G is the Wilson gauge and S_F the Wilson fermion action with SU(2) gauge link matrix U in fundamental rep. and V a corresponding HEX smeared [3] link. S_{SW} is the clover term with SW coefficient $c_{\mathrm{SW}} = 1$ [6].

Hybrid Monte Carlo (HMC) algorithm using leapfrog integrator with unit-length trajectories and the number of leapfrog steps adjusted to yield acceptance rates above 80%.

Gradient flow (GF) is governed by the Lüscher-Weisz Symanzik action [1].

GF running coupling defined as $g^2_{\mathrm{GF}}(\lambda_L, L) = \dfrac{2\,\pi^2\,\lambda_L^4\,\langle E(\lambda_L, L)\rangle}{3\,(N^2-1)\,\big(1+\delta_{L/a}(\lambda_L/L)\big)}$ at constant L ,

with $\langle E(\lambda_L, L)\rangle$ being the (clover) energy of the GF-evolved gauge field after flow time t resp. flow scale $\lambda_L = \sqrt{8\,t}$, and $\delta_N(c)$ is a finite volume and finite lattice spacing correction [7] (cf. also [4]).

Simulations were carried out for L=32,40,48 with inverse bare couplings $\beta = 4/g_0^2 \in \{-0.25, 0.001, 0.25\}$ and the fermion hopping parameter κ chosen to obtain PCAC quark masses $m_q \in [0.01, 0.8]$.

Relating BF-MOM and GF scheme

Leading beta function coefficient for perturbative massive GF scheme obtained from [5, 8]:

$$\beta_{0,\mathrm{GF}}\big(g^2_{\mathrm{GF}}, \lambda\,m\big) \;=\; \beta_0 + \frac{4}{3} T_R\,N_f\,x\,\frac{\mathrm{d}\Omega_{1q}(x)}{\mathrm{d}x} \ ,$$

where β_0 is the leading coefficient of massless $\overline{\mathrm{MS}}$ scheme, $x = -1/(2m\lambda)^2$, and $\Omega_{1q}(x)$ is given in [5].

Relate scales λ_{GF} and λ_{BFM} by relative rescaling $\lambda_{\mathrm{GF}} = \rho_s\,\lambda_{\mathrm{BFM}}$.

Determine ρ_s by requiring that decoupling (beta function changes sign) is physical and should be described equally by both schemes. At the one-loop level, this leads to $\beta_{0,\mathrm{BFM}}(\lambda_0\,m_0) = \beta_{0,\mathrm{GF}}(\lambda_0\,m_0\,\rho_s)$, where $\beta_{0,\mathrm{BFM}}$ is the leading BF-MOM beta function coefficient and $\lambda_0 = 1/(2\,m_0)$ the approximate decoupling scale in the BF-MOM scheme $\Rightarrow \rho_s = 2.5359\ldots$.

Results: gradient flow running coupling

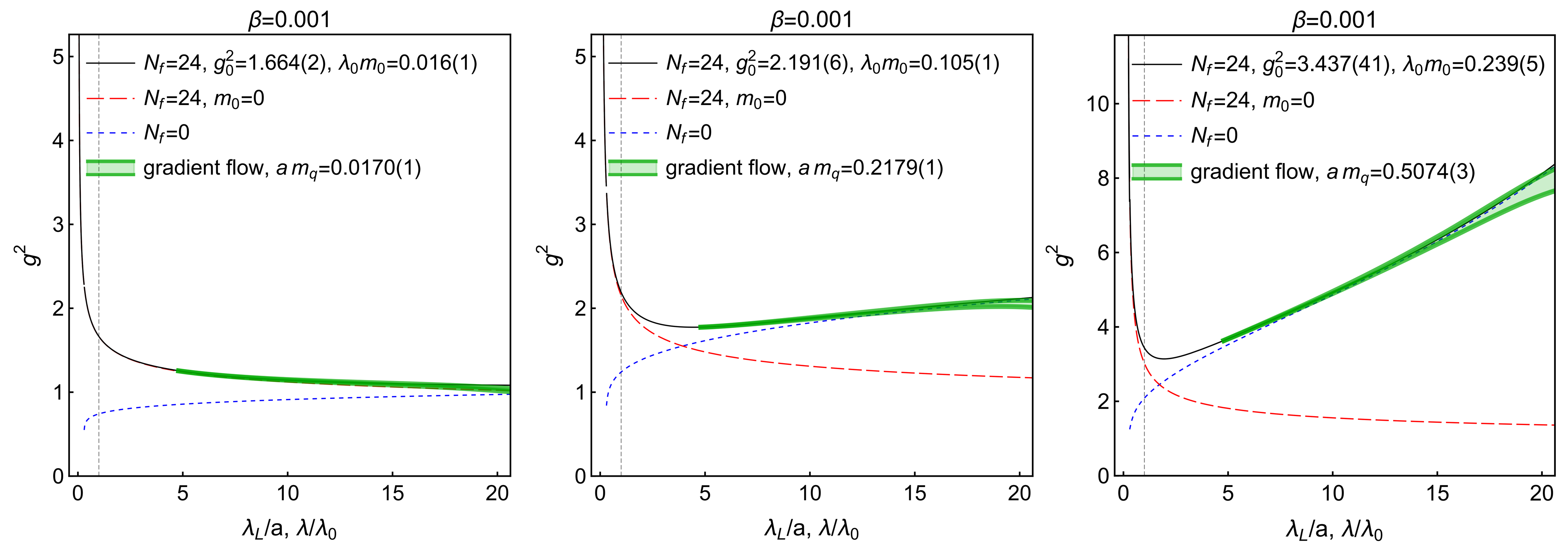

Figure 2: Examples for fits of perturbative two-loop running coupling (black solid line) to corresponding gradient flow running coupling (green band) for $V = 48^4$. Also shown are the curves for the corresponding asymptotic cases of zero-mass (long red dashes) and pure gauge (short blue dashes). Lattice data is shown for $4.8 \le \lambda_L \le 20$ to avoid strong UV- and IR-cutoff effects.

Fig. 2 above shows results obtained by fitting the perturbative running coupling discussed in Fig. 1 to our lattice GF running coupling g^2_{GF}. The quark mass dependent change of slope of g^2_{GF} matches the expectations from perturbation theory: for small m_q the gradient flow coupling is on the decreasing, quasi-IR-free massless $N_f = 24$ branch (left), while for large m_q, it is on the rapidly increasing, confining pure gauge branch (right).

In Fig. 3 **no fits** are performed; the non-perturbative lattice GF running coupling (after analytic scale matching at 1-loop) is directly compared to massive 2-loop BF-MOM and asymptotic (massless) 3-loop GF scheme predictions.

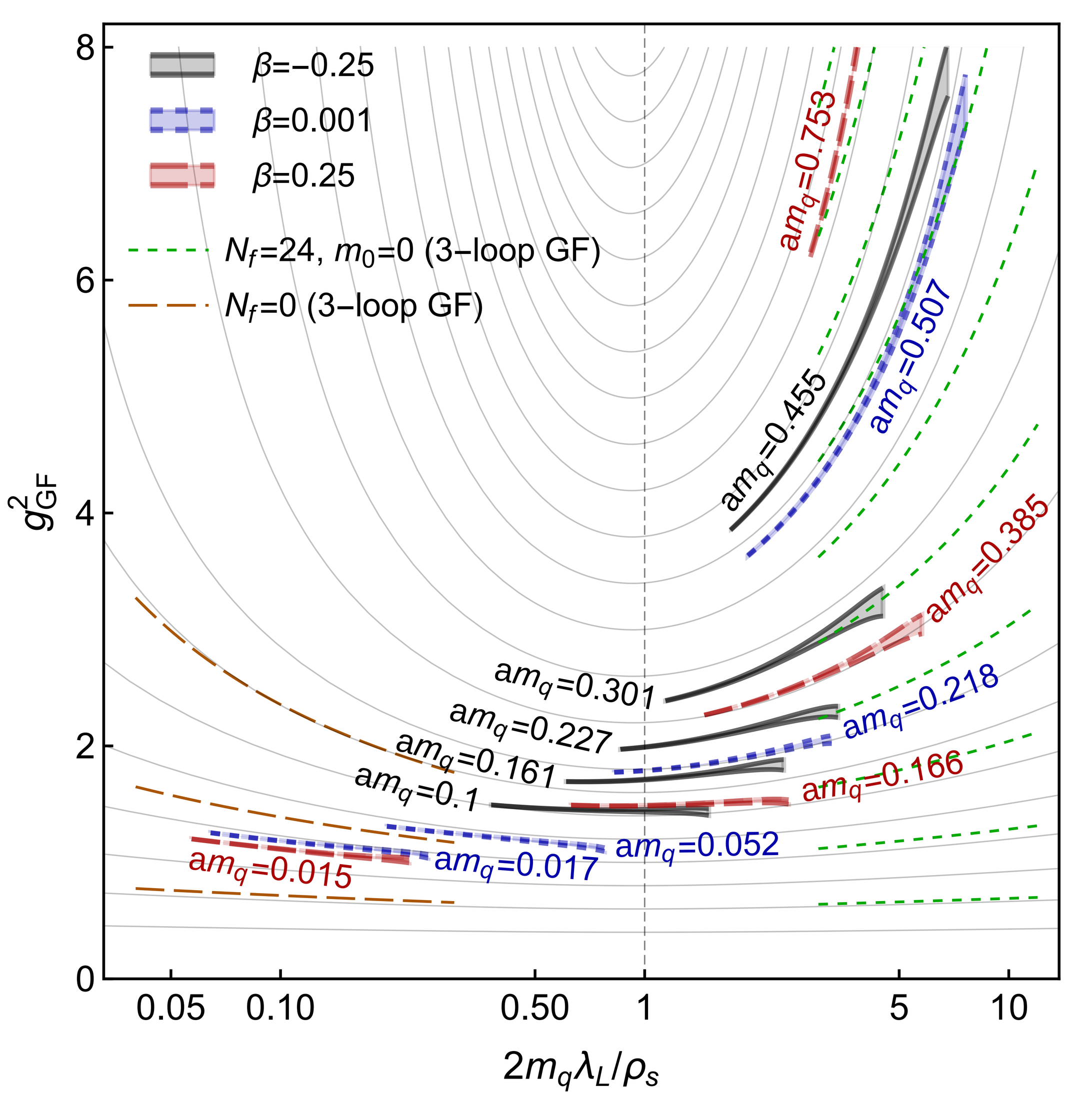

Figure 3: The lattice GF running coupling (black, blue and red error bands) as function of $2\,m_q\,\lambda_L/\rho_s$, with PCAC quark mass m_q and ρ_s from above. The lattice data is compared with the massive 2-loop running coupling from Fig. 1 (gray), and 3-loop GF-scheme running couplings for $N_f = 24$ (short green dashes) and $N_f = 0$ (long red dashes) massless quarks.

Conclusions

We have reviewed the running coupling of a SU(2) gauge theory with $N_f = 24$ massive fermions in BF-MOM perturbation theory [2] and demonstrated that the predicted transition from behaving as the massless theory in the UV (UV-Landau pole and IR-triviality) to behaving like pure gauge in the IR (confining) can be well observed on the lattice in the evolution of the gradient flow running coupling.