Amplitude techniques for EFTs

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with Teppei Kitahara, Camila Machado, Yael Shadmi, Yaniv Weiss

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Amplitude techniques for EFTs

A. On-shell approach to SM EFTs

a. Massless contact terms

b. Massive contact terms

c. Electroweak application

B. Double-copy generalisations for EFTs

A. On-shell approach to SM EFTs

Lagrangians are powerful... but not always

Fields are unphysical and redundant

- redefinitions do not alter the physics

- four-vector/tensor to embed two dofs

Lagrangians are powerful... but not always

Fields are u

- redefiniti
- four-vect

graviton Feynman rules [De Witt '67]

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\
&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\
&\qquad\qquad -P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],
\end{aligned}$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\
&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\
&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\
&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\
&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\
&+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})\\
&\qquad -P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].
\end{aligned}$$

171 & 2850 terms

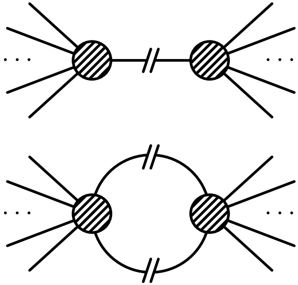
vs.

$$\Big([12]^3/[23][31]\Big)^2 \;\&\; [12]^4\langle 34\rangle^4/stu$$

On-shell bootstrapping

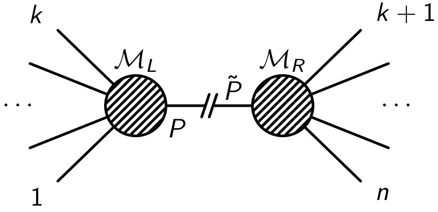
- Trees cut into trees
  - + contact terms
- Loops cut into trees
  - + rational terms
  - (no loops hereafter)

factorization/unitarity



Recently

- applications to EFTs
- covariant massive spinor formalism [Arkani-Hamed, Huang, Huang '17]



$$\mathcal{M}^{\text{tree}}(1, ...k, ...n) = \sum_{\text{channels}} \frac{\mathcal{M}_L^{\text{tree}}(1, ...k, P)\, \mathcal{M}_R^{\text{tree}}(\tilde{P}, k+1, ...n)}{P^2 - m^2}$$

$$+ \mathcal{M}^{\text{contact}}(1, ...k, ...n)$$

Contact terms

massless $\rightarrow$ operator enumeration (unbroken EW symmetry)
massive $\rightarrow$ bootstrap building blocks

massless SMEFT contact terms

[Shadmi, Weiss '18], [Ma, Shu, Xiao '19], [Falkowski '19], [GD, Machado '19],
[Li, Ren, et al. '20, '20, '20]

massive SMEFT contact terms

SM: [Christensen, Field '18], [Bachu, Yelleshpur '19],
SMEFT: [Aoude, Machado '19], [GD, Kitahara, Shadmi, Weiss '19], [GD et al. '20],
[Dong, Ma, Shu, Zheng '21, '22], [De Angelis '22]

SMEFT non-renormalizations, -interferences, anomalous dim.

[Cheung, Shen '15], [Azatov et al. '16], [Bern et al. '19, '20],
[Jiang et al. '20], [Elias Miró et al. '20, '21], [Baratella et al. '20, '20, '21],
[Accettulli Huber, De Angelis '21], [Delle Rose et al. '22]

...

a. Massless contact terms

Helicity spinors

[Mangano, Parke '91]
[Helvang, Huang '13]
[Dixon '13]
[Schwartz '14]
[Cheung '17]

As brackets

$$u_{i^+} = \begin{pmatrix} 0 \\ i] \end{pmatrix}, \qquad u_{i^-} = \begin{pmatrix} i\rangle \\ 0 \end{pmatrix} \qquad \text{for particle } i$$

Rewritting momenta (and polarizations vectors)

$$p_i^\mu \sigma_\mu = i\rangle[i \qquad \left(\varepsilon^\mu_{i^+}\sigma_\mu = \frac{\zeta\rangle[i}{\sqrt{2}\langle\zeta i\rangle}, \qquad \varepsilon^\mu_{i^-}\sigma_\mu = \frac{i\rangle[\zeta}{\sqrt{2}\langle i\zeta\rangle} \right)$$

Trivializing $p_i^2 = \langle ii\rangle[ii]/2 = 0$

$$\langle ii\rangle = \epsilon_{\alpha\beta}\ i\rangle^\alpha i\rangle^\beta = 0, \qquad [ii] = \epsilon^{\dot\alpha\dot\beta}\ i]_{\dot\alpha} i]_{\dot\beta} = 0$$

Little-group covariance

Little-group transformations leave p_i invariant

Little group is $U(1)$ for massless p_i

Spinors $i], i\rangle$ pick up phases proportional $\pm 1/2$

The amplitude picks up a phase proportional to h_i

Massless three-points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h_1+h_2+h_3>0 \\ \langle 12\rangle^{-h_1-h_2+h_3} \langle 23\rangle^{-h_2-h_3+h_1} \langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h_1+h_2+h_3<0 \end{cases}$$

$$\begin{aligned} f^+f^+s \quad & [12] \\ v^+v^+s \quad & [12]^2 \\ f^+f^-v^+ \quad & [13]^2/[12] \\ v^+v^+v^- \quad & [12]^3/[23][31] \\ t^+t^+t^- \quad & \left([12]^3/[23][31]\right)^2 \end{aligned}$$

Massless higher-points

Multiple independent structures for given helicities

solving · little group constraints

· momentum conservation

· Schouten identities e.g. $[12][34] - [13][24] + [14][23] = 0$

Construction

- *harmonics* and Young tableaux [Henning, Melia '19]
- *twistors* trivializing momentum conservation [Falkowski '19]
- systematic algorithm and explicit construction [GD, Machado '19] [see also Accettulli Huber, De Angelis '21]

Massless higher-points

Multiple independent structures for given helicities

solving · little group constraints

· momentum conservation

· Schouten identities   e.g. $[12][34]-[13][24]+[14][23]=0$

Construction

e.g. minimal dimension of operators contributing to any helicity amplitude:

$$\dim\{\text{operator}\} \geq n - \sum_i \max(0, \operatorname{ceil}\{|h_i|-1\}) + \sum_i |h_i| + 2\max\begin{bmatrix} \{\sum_{h_i>0} 2h_i\} \mod 2 \\ 2\max\limits_{h_i>0}\{|h_i|\} - \sum\limits_{h_i>0} |h_i| \\ 2\max\limits_{h_i<0}\{|h_i|\} - \sum\limits_{h_i<0} |h_i| \end{bmatrix}$$

[GD, Machado '19]

Massless hig[her points]

Multiple

solving

$4][23] = 0$

Construc[...]

e.g. minim[...] [a]mplitude:

- har[...] [Henning, Melia '19]
- twisto[...] [Falkowski '19]
- system[...] [GD, Machado '19] [...Huber, De Angelis '21]

dim

[...achado '19]

e.g. GR-SMEFT [GD, Machado '19]

mult.	min. dim.	helicity conf.	spinor structures	SM gauge	spin stat.	Hilbert series
3-pt	dim-5	t^+t^+s	$[12]^4$	✗		
	dim-6	$t^+t^+t^+$	$[12]^2[13]^2[23]^2$			C_R^3
		$t^+t^+v^+$	$[12]^3[23][13]$		✗	
		$t^+v^+v^+$	$[12]^2[13]^2$			$(B_R^2, W_R^2, G_R^2)C_R$
4-pt	dim-6	$\underline{t}^+\underline{t}^+ss$	$[12]^4$; $[12]^4 s_{12}$			$HC_R^2H^\dagger$, $HD^2H^\dagger C_R^2$
	dim-7	$t^+t^+t^+s$	$[12]^2[13]^2[23]^2$	✗		
		$t^+t^+v^+s$	$[12]^3[13][23]$	✗		
		$\underline{t}^+\underline{t}^+f^+f^+$	$[12]^4[34]$	✗		
		$t^+t^+f^-f^-$	$[12]^4\langle 34\rangle$	✗		
		$t^+v^+v^+s$	$[12]^2[13]^2$	✗		
		$t^+v^+f^+f^+$	$[12]^2[13][14]$	✗		
	dim-8	$\underline{t}^+\underline{t}^+\underline{t}^+\underline{t}^+$	$[12]^4[34]^4+[13]^4[24]^4+[14]^4[23]^4$			C_R^4
		$t^+t^+t^+v^+$	$[12]^3[13][23][34]^2$, $[12][13]^3[23][24]^2$, $[12][13][23]^3[14]^2$		✗	
		$t^+t^+t^-t^-$	$[12]^4\langle 34\rangle^4$			$C_R^2C_L^2$
		$\underline{t}^+\underline{t}^+v^+v^+$	$[12]^4[34]^2$, $[12]^2[13][14][24][23]$			$2(B_R^2, W_R^2, G_R^2)C_R^2$
		$t^+t^+v^-v^-$	$[12]^4\langle 34\rangle^2$			$(B_L^2, W_L^2, G_L^2)C_R^2$
		$t^+t^+f^+f^-$	$[12]^4[324\rangle$		✗	
		$t^+v^+\underline{v}^+\underline{v}^+$	$[12][13][14]([13][24]+[14][23])$			$(W_R^2, G_R^2)B_RC_R$
		$t^+v^+f^+f^-$	$[12]^2[13][124\rangle$			$(QQ^\dagger, uu^\dagger, dd^\dagger, LL^\dagger, ee^\dagger)DB_RC_R$, $(QQ^\dagger, LL^\dagger)DW_RC_R$, $(QQ^\dagger, uu^\dagger, dd^\dagger)DG_RC_R$
		t^+v^+ss	$[12]^2[1231]$			$(B_R, W_R)HH^\dagger D^2C_R$
		$t^+f^+f^+s$	$[12][13][1231]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)D^2C_R$
	...					
5-pt	dim-7	t^+t^+sss	$[12]^4$	✗		
	dim-8	$t^+t^+t^+ss$	$[12]^2[13]^2[23]^2$			$HH^\dagger C_R^3$
		$t^+t^+v^+ss$	$[12]^3[13][23]$		✗	
		$\underline{t}^+\underline{t}^+f^+f^+s$	$[12]^4[34]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)C_R^2$
		$t^+t^+f^-f^-s$	$[12]^4\langle 34\rangle$			$(QuH, QdH^\dagger, LeH^\dagger)C_R^2$
		$t^+v^+v^+ss$	$[12]^2[13]^2$			$(B_R^2, B_RW_R, W_R^2, G_R^2)HH^\dagger C_R$
		$t^+v^+f^+f^+s$	$[12]^2[13][14]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)(B_R, W_R)C_R$, $(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H)G_RC_R$
		$t^+f^+f^+f^+f^+$	$[12][13][14][15]$			$Q^\dagger Q^\dagger Q^\dagger L^\dagger C_R, d^\dagger e^\dagger u^{\dagger 2}C_R$, $d^\dagger Q^{\dagger 2}u^\dagger C_R, e^\dagger L^\dagger Q^\dagger u^\dagger C_R$
	...					
6-pt	dim-8	t^+t^+ssss	$[12]^4$			$H^2H^{\dagger 2}C_R^2$

[for Hilbert series see Ruhdorfer, Serra, Weiler '19]

b. Massive contact terms

Massive spin spinors

[Arkani-Hamed, Huang, Huang '17]

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \;=\; q^i\rangle[q^i + k^i\rangle[k^i \;=\; i^J\rangle[i_J \qquad \text{with } k_i^2 = 0 = q_i^2,\;\; J=1,2 \quad 2k^i\cdot q^i = m_i^2$$

Little group is now $SO(3) \sim SU(2)$

Spin s from $2s$ symmetrized spin $1/2$

Bolded spinors with implicit symmetrization

e.g. $\langle 1^I 3^J\rangle\langle 2^K 3^{J'}\rangle + (J \leftrightarrow J') \to \langle \mathbf{13}\rangle\langle\mathbf{23}\rangle$

Spin quantisation axis unspecified / little-group covariance

ffs $[\mathbf{12}],\ \langle\mathbf{12}\rangle$

vvs $\langle\mathbf{12}\rangle^2,\ \langle\mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$

ssv $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_\mathbf{1}-p_\mathbf{2})\mathbf{3}\rangle$

ffv $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$

...

Massive three-points

Counting from angular momentum

number of irreps in the spin addition: [Costa, Penedones, Poland, Rychkov '11]

$$(2s_1+1)(2s_2+1)-p(p+1) \quad \text{with} \quad \begin{cases} p \equiv \max\{0, s_1+s_2-s_3\} \\ s_1 \le s_2 \le s_3 \end{cases}$$

Construction by correcting a massless-like ansatz

[GD, Kitahara, Machado, Shadmi, Weiss '20]

$$(\mathbf{12})^{s_1+s_2-\tilde{s}_3}\,(\mathbf{23})^{-s_1+s_2+\tilde{s}_3}\,(\mathbf{13})^{s_1-s_2+\tilde{s}_3}\,[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^{s_3-\tilde{s}_3}$$

with

$$\begin{aligned} &(\boldsymbol{ij})^k \equiv \text{ any } \langle\boldsymbol{ij}\rangle^{k-l}[\boldsymbol{ij}]^l \quad \text{for } l=0,\dots,k \\ &s_1 \le s_2 \le s_3 \\ &\tilde{s}_3 \equiv s_3 - \max\{0, s_3-s_2-s_1\} \end{aligned}$$

removing occurrences of $\epsilon(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3, \boldsymbol{p_1}+\boldsymbol{p_2}+\boldsymbol{p_3})$

$$\begin{aligned} &m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ =\; &m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}] \end{aligned}$$

Massive t

Counti

nu

land, Rychkov '11]

$- s_3\}$

s_1	s_2	s_3	$n^{\text{3-pt}}$	n_{rel}	spinor structures
0	0	0	1		constant
0	0	1	1		$[\mathbf{3(1-2)3}\rangle$
0	0	2	1		$[\mathbf{3(1-2)3}\rangle^2$
0	0	3	1		$[\mathbf{3(1-2)3}\rangle^3$
0	1/2	1/2	2		$([\mathbf{23}],\langle\mathbf{23}\rangle)$
0	1/2	3/2	2		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)$
0	1/2	5/2	2		$[\mathbf{3(1-2)3}\rangle^2\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)$
0	1	1	3		$([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)$
0	1	2	3		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)$
0	1	3	3		$[\mathbf{3(1-2)3}\rangle^2\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)$
0	3/2	3/2	4		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)$
0	3/2	5/2	4		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)$
0	2	2	5		$([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)$
0	2	3	5		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)$
0	5/2	5/2	6		$([\mathbf{23}]^5,\langle\mathbf{23}\rangle[\mathbf{23}]^4,\langle\mathbf{23}\rangle^2[\mathbf{23}]^3,\langle\mathbf{23}\rangle^3[\mathbf{23}]^2,\langle\mathbf{23}\rangle^4[\mathbf{23}],\langle\mathbf{23}\rangle^5)$
0	3	3	7		$([\mathbf{23}]^6,\langle\mathbf{23}\rangle[\mathbf{23}]^5,\langle\mathbf{23}\rangle^2[\mathbf{23}]^4,\langle\mathbf{23}\rangle^3[\mathbf{23}]^3,\langle\mathbf{23}\rangle^4[\mathbf{23}]^2,\langle\mathbf{23}\rangle^5[\mathbf{23}],\langle\mathbf{23}\rangle^6)$
1/2	1/2	1	4		$([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1/2	2	4		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1/2	3	4		$[\mathbf{3(1-2)3}\rangle^2\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1	3/2	6		$([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	1	5/2	6		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	3/2	2	8		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	3/2	3	8		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	2	5/2	10		$([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1/2	5/2	3	12		$([\mathbf{23}]^5,\langle\mathbf{23}\rangle[\mathbf{23}]^4,\langle\mathbf{23}\rangle^2[\mathbf{23}]^3,\langle\mathbf{23}\rangle^3[\mathbf{23}]^2,\langle\mathbf{23}\rangle^4[\mathbf{23}],\langle\mathbf{23}\rangle^5)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	1	1	7	1	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}],\langle\mathbf{23}\rangle)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	1	2	9		$([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	1	3	9		$[\mathbf{3(1-2)3}\rangle\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	3/2	3/2	10	2	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	3/2	5/2	12		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	2	2	13	3	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	2	3	15		$([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
1	5/2	5/2	16	4	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
1	3	3	19	5	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^5,\langle\mathbf{23}\rangle[\mathbf{23}]^4,\langle\mathbf{23}\rangle^2[\mathbf{23}]^3,\langle\mathbf{23}\rangle^3[\mathbf{23}]^2,\langle\mathbf{23}\rangle^4[\mathbf{23}],\langle\mathbf{23}\rangle^5)\otimes([\mathbf{13}],\langle\mathbf{13}\rangle)$
3/2	3/2	2	14	4	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
3/2	3/2	3	16		$([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$
3/2	2	5/2	18	6	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
3/2	5/2	3	22	8	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
2	2	2	19	8	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^2,\langle\mathbf{23}\rangle[\mathbf{23}],\langle\mathbf{23}\rangle^2)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
2	2	3	23	9	$([\mathbf{12}],\langle\mathbf{12}\rangle)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$
2	5/2	5/2	24	12	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
2	3	3	29	16	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^4,\langle\mathbf{23}\rangle[\mathbf{23}]^3,\langle\mathbf{23}\rangle^2[\mathbf{23}]^2,\langle\mathbf{23}\rangle^3[\mathbf{23}],\langle\mathbf{23}\rangle^4)\otimes([\mathbf{13}]^2,\langle\mathbf{13}\rangle[\mathbf{13}],\langle\mathbf{13}\rangle^2)$
5/2	5/2	3	30	18	$([\mathbf{12}]^2,\langle\mathbf{12}\rangle[\mathbf{12}],\langle\mathbf{12}\rangle^2)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$
3	3	3	37	27	$([\mathbf{12}]^3,\langle\mathbf{12}\rangle[\mathbf{12}]^2,\langle\mathbf{12}\rangle^2[\mathbf{12}],\langle\mathbf{12}\rangle^3)\otimes([\mathbf{23}]^3,\langle\mathbf{23}\rangle[\mathbf{23}]^2,\langle\mathbf{23}\rangle^2[\mathbf{23}],\langle\mathbf{23}\rangle^3)\otimes([\mathbf{13}]^3,\langle\mathbf{13}\rangle[\mathbf{13}]^2,\langle\mathbf{13}\rangle^2[\mathbf{13}],\langle\mathbf{13}\rangle^3)$

Constr

Kitahara, Machado, Shadmi, Weiss '20]

$(\mathbf{12}$

wit

rem

$+ p_2 + p_3)$

Massive higher-points

New massive redundancies arise

e.g. in *ffvs*:

$$[\mathbf{12}]\langle\mathbf{3123}\rangle = ([\mathbf{12}][\mathbf{313}\rangle\tilde{s}_{23} - [\mathbf{12}][\mathbf{323}\rangle\tilde{s}_{13})/m_3$$
$$- \tilde{s}_{12}[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}]$$

Clear massless origin, but non-trivial 'mass-completions'

Explicit construction for 4-points and spins$\leq$1

[GD, Kitahara, Machado, Shadmi, Weiss '20]

See also very recent proposals: [De Angelis '22]
[Dong, Ma, Shu, Zheng '22]

Massive high

spins	$n_{\rm SCT}$	$n_{\rm s}$	hel. cat.	spinor structures	$n_{\rm perm}$	$\min\{d_{\rm op}\}$
ssss	1	1	(0000)	constant	1	4
vsss	$4 \to 3$	3	(0000)	$[\mathbf{121}\rangle, [\mathbf{131}\rangle$	1	5
			(+000)	$[\mathbf{1231}] \to [\mathbf{1231}] - \langle\mathbf{1231}\rangle$	$\not{2} \to 1$	7
ffss	4	4	(++00)	$[\mathbf{12}]$	2	5
			(+−00)	$[\mathbf{132}\rangle$	2	6
vvss	$10 \to 9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle, [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2} \to 1$	8
ffvs	$14 \to 12$	12	(++00)	$[\mathbf{12}]\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\}$	2	6
			(+−00)	$[\mathbf{13}]\langle\mathbf{23}\rangle$	2	5
			(+++0)	$[\mathbf{13}][\mathbf{23}]$	2	6
			(++−0)	$[\mathbf{12}]\langle\mathbf{3123}\rangle \to \emptyset$	$\not{2} \to 0$	8
			(+−+0)	$[\mathbf{13}][\mathbf{312}\rangle$	4	7
ffff	18	16	(++++)	$[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]$	2	6
			(++−−)	$[\mathbf{12}]\langle\mathbf{34}\rangle$	6	6
			(+++−)	$[\mathbf{12}][\mathbf{324}\rangle$	8	7
vvvs	$35 \to 29$	27	(0000)	$[\mathbf{12}][\mathbf{343}\rangle\langle\mathbf{12}\rangle, [\mathbf{13}][\mathbf{242}\rangle\langle\mathbf{13}\rangle, [\mathbf{23}][\mathbf{141}\rangle\langle\mathbf{23}\rangle$	1	5
			(+000)	$[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle$	6	5
			(++00)	$[\mathbf{12}]^2\left\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\right\}$	6	7
			(+−00)	$[\mathbf{13}][\mathbf{132}\rangle\langle\mathbf{23}\rangle$	6	7
			(+++0)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]$	2	7
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{3123}\rangle \to \emptyset$	$\not{6} \to 0$	9
vvff	$46 \to 38$	36	(00++)	$\langle\mathbf{12}\rangle \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	5
			(00+−)	$\langle\mathbf{14}\rangle\langle\mathbf{231}][\mathbf{23}], \langle\mathbf{24}\rangle\langle\mathbf{132}][\mathbf{13}]$	2	6
			(0−++)	$\langle\mathbf{12}\rangle[\mathbf{34}]\langle\mathbf{241}] \to \langle\mathbf{12}\rangle[\mathbf{34}](\langle\mathbf{241}]/m_1 - \langle\mathbf{142}]/m_2)$	$\not{4} \to 2$	7
			(0+++)	$\langle\mathbf{132}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	4	7
			(0++−)	$\langle\mathbf{14}\rangle[\mathbf{12}][\mathbf{23}]$	8	6
			(+++−)	$[\mathbf{12}]^2[\mathbf{314}\rangle$	4	8
			(++++)	$[\mathbf{12}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	7
			(−+++)	$\langle\mathbf{1231}\rangle[\mathbf{23}][\mathbf{24}] \to \emptyset$	$\not{4} \to 0$	9
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle$	2	7
			(+−−+)	$[\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle \to [\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle - [\mathbf{24}][\mathbf{231}\rangle\langle\mathbf{13}\rangle$	$\not{4} \to 2$	8
vvvv	$116 \to 85$	81	(0000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times \{\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, \langle\mathbf{13}\rangle\langle\mathbf{24}\rangle\}$	1	4
			(+000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{142}\rangle\langle\mathbf{34}\rangle \to \cdots$	$\not{8} \to 6$	6
			(++00)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{12}]\langle\mathbf{34}\rangle$	12	6
			(+−00)	$[\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle$	12	6
			(+++0)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{23}][\mathbf{134}\rangle$	8	8
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle\langle\mathbf{324}] \to [\mathbf{12}]^2\langle\mathbf{34}\rangle(\langle\mathbf{324}]/m_4 - \langle\mathbf{423}]/m_3) \to \cdots$	$\not{24} \to \not{12} \to 5$	8
			(++++)	$[\mathbf{12}]^2[\mathbf{34}]^2, [\mathbf{12}][\mathbf{13}][\mathbf{24}][\mathbf{34}], [\mathbf{13}]^2[\mathbf{24}]^2$	2	8
			(+++−)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]\langle\mathbf{4124}\rangle \to \emptyset$	$\not{8} \to 0$	10
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle^2$	6	8

New massiv

e.g. in *ffvs*

$[\mathbf{12}]\langle\mathbf{31}$ … $\mathbf{12}\rangle[\mathbf{13}]$

Clear massl … ons'

Explicit cor

… hado, Shadmi, Weiss '20]

[De Angelis '22]

… ong, Ma, Shu, Zheng '22]

c. Electroweak application

[GD, Kitahara, Shadmi, Weiss '19]

Recover electroweak symmetry

[Llewellyn Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

From perturbative unitarity!

HIGH ENERGY BEHAVIOUR AND GAUGE SYMMETRY

C.H. LLEWELLYN SMITH
CERN, Geneva, Switzerland

Received 13 May 1973

The imposition of unitarity bounds is shown to lead to a Yang-Mills structure in a wide class of theories involving vector mesons. Scalar fields are needed and, at least in simple cases, the unique unitary theory is of the Higgs type

S-Matrix Derivation of the Weinberg Model[1]

SATISH D. JOGLEKAR

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Received June 18, 1973

Uniqueness of Spontaneously Broken Gauge Theories*

John M. Cornwall,† David N. Levin, and George Tiktopoulos
Department of Physics, University of California, Los Angeles, California 90024
(Received 25 April 1973)

We have made a systematic search for theories of interacting heavy vector mesons which have unitarily bound trees. In simple cases (four vector mesons and one scalar particle) the *only* unitarily bound models are spontaneously broken gauge theories. Evidently, a unitarity bound, which controls high-energy behavior, imposes internal symmetry on heavy-vector-boson interactions.

Three-points

High-energy limit

- find traces of EW symmetry
- determine renormalizability

e.g. WWZ:
$$\begin{aligned}&+[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\; m_Z/m_W\\&+\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle\\&+\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}]\\&+\langle\mathbf{12}\rangle\,[\mathbf{13}]\,[\mathbf{23}]\; m_Z/m_W\\&+[\mathbf{12}]\,\langle\mathbf{13}\rangle[\mathbf{23}]\\&+[\mathbf{12}]\,[\mathbf{13}]\,\langle\mathbf{23}\rangle\end{aligned}$$

Neutral *vvv* & *ssv* vanish identically

Non-trivial massless $Z \to \gamma$ limit

$\psi^c\psi Z$

$\psi^c\psi\gamma$

$\psi^c\psi' W$

$\psi^c\psi h$

ZZh, WWh

$\gamma\gamma h$, γZh

hhZ, $hh\gamma$

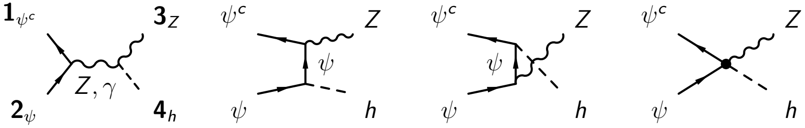
hhh

WWZ

$WW\gamma$

ZZZ, $ZZ\gamma$, $Z\gamma\gamma$, $\gamma\gamma\gamma$

Four-point



Leading high-energy amplitudes $\sim E/m$:

$$(--0): \quad -\frac{\langle 12\rangle}{\sqrt{2}m_Z}\left(c^R_{\psi^c\psi Z}-c^L_{\psi^c\psi Z}\right)\left(c_{ZZh}\frac{m_\psi}{2m_Z}-c^L_{\psi^c\psi h}\right)$$

$$(++0): \quad +\frac{\langle 12\rangle}{\sqrt{2}m_Z}\left(c^R_{\psi^c\psi Z}-c^L_{\psi^c\psi Z}\right)\left(c_{ZZh}\frac{m_\psi}{2m_Z}-c^R_{\psi^c\psi h}\right)$$

Perturbative unitarity up to $\Lambda \gg m$ requires:

either vector-like fermion: $c^R_{\psi^c\psi Z} = c^L_{\psi^c\psi Z}$

or Higgs mechanism: $c^L_{\psi^c\psi h} = c_{ZZh}\frac{m_\psi}{2m_Z} = c^R_{\psi^c\psi h}$

up to $\mathcal{O}(m/\Lambda)$

On-shell approach to SM EFTs

On-shell amplitudes are physical and economical.

Trees are bootstrapped, up to contact terms.

Systematic contact-term construction has been achieved.

Electroweak symmetry emerges from perturbative unitarity.

Ready for massive SM EFTs applications!

B. Double-copy generalisations for EFTs

Double copy

Product operation between amplitudes & theories

$$\mathrm{TH}_1 \times \widetilde{\mathrm{TH}}_2 = \mathrm{TH}_3$$

Forms a *web* of somewhat special theories

$\times$	$\widetilde{\mathrm{BAS}}$	$\widetilde{\mathrm{NLSM}}$	$\widetilde{\mathrm{YM}}$
BAS	BAS	NLSM	YM
NLSM		sGal	BI
YM			GR

e.g. $f^{abc}\,\frac{[12]^3}{[13][23]} \to \left(\frac{[12]^3}{[13][23]}\right)^2$

Facilitates computations

EFT deformations?

What EFTs are valid inputs?

$\equiv$ *single copies* $(\mathcal{A})$

What EFTs obtained as outputs?

$\equiv$ *double copies* $(\mathcal{M})$

Traditional KLT

[Kawai, Lewellen, Tye '86]

- Field-theory limit of a string-theory relation $\quad$ (closed) $=$ (open) $\times$ $\widetilde{\text{(open)}}$

- Inputs are vectors of *colour-ordered* amplitudes A

  with $\mathrm{Tr}\{T^{a_1}T^{a_2}\ldots\} \equiv (12\ldots)$

  for $\quad \mathcal{A} = A \cdot c^{\mathrm{tr}}$

$$c^{\mathrm{tr}}_{n=4} = \begin{pmatrix} (1324)+(4231) \\ (1234)+(4321) \\ (1243)+(3421) \end{pmatrix}$$

  satisfy *BCJ relations*, leaving a basis of $(n-3)!$

$$A_{n=4}[1234] = \tfrac{t}{u} A_{n=4}[1243]$$

- Proceeds through a *kernel* matrix

$$\otimes_{n=4} = \begin{pmatrix} \frac{tu}{s} & u & t \\ u & \frac{su}{t} & s \\ t & s & \frac{st}{u} \end{pmatrix}$$

$$\mathcal{M} = A \otimes \tilde{A}$$

  sums over any two BCJ bases

Traditional CK duality

- Inputs are *adjoint numerators* from $\mathcal{A} = c^{\text{adj}} \cdot P \cdot n^{\text{adj}}$

Sums run over trivalent graphs/topologies.

colour

$c^{\text{adj}} \ni f^{abx} f^{x\ldots}$

propagators

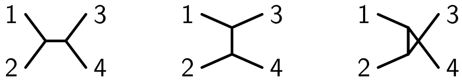
$P_{n=4} = \begin{pmatrix} 1/s & & \\ & 1/t & \\ & & 1/u \end{pmatrix}$

kinematics

$s_{ij},\ p_i \cdot \epsilon_j, \ldots$

- n^{adj} must satisfy the same algebraic properties as c^{adj}

e.g. Jacobi identities, for a $f^{abc} A^a_\mu \phi^b \partial^\mu \phi^c$ theory at four points



$$
\begin{aligned}
c^{\text{adj}}: &\quad f^{12x} f^{x34} + f^{13x} f^{x42} + f^{14x} f^{x23} = 0 \\
n^{\text{adj}}: &\quad (t-u) + (u-s) + (s-u) = 0
\end{aligned}
$$

- Exchange colour for kinematics: $\mathcal{M} = n^{\text{adj}} \cdot P \cdot \tilde{n}^{\text{adj}}$

Traditional approaches and EFTs

The field-theory limit of the string KLT contains a specific tower of operators.

Certain operators are double-copyable out of the box

e.g. $\mathrm{YM} + F^3 \longrightarrow \mathrm{GR} + R^3$ [Brödel, Dixon '12]

Others are not

e.g. $\mathrm{YM} + F^4 \longrightarrow \emptyset$

... double-copy generalisation for EFTs?

The bi-adjoint scalar (BAS)

CK: Exchange kinematics for colour (*zeroth copy*)

$$\mathcal{A}^{\text{BAS}} = c^{\text{adj}} \cdot P \cdot \tilde{c}^{\text{adj}}$$

amplitudes of a $f^{abc} f^{\tilde{a}\tilde{b}\tilde{c}}\, \phi^{a\tilde{a}} \phi^{b\tilde{b}} \phi^{c\tilde{c}}$ theory

KTL: Identity theory $\mathcal{A}^{\text{BAS}} = A^{\text{BAS}} \cdot c^{\text{tr}} = c^{\text{tr}} \cdot m \cdot c^{\text{tr}}$

singly colour ord. (A^{BAS}) — doubly colour ord. (m)

· Kernel satisfying $A^{\text{BAS}} \otimes A^{\text{BAS}} = \mathcal{A}^{\text{BAS}}$

$$= c^{\text{tr}} \cdot m \otimes m \cdot c^{\text{tr}}$$

$\rightarrow$ $\otimes = m^{-1}$ after restriction to a BCJ basis

· BCJ relations encoded in $A \otimes A^{\text{BAS}} = \mathcal{A}$

Generalised KLT

[Chi, Elvang, Herderschee, Jones, Paranjape '21]

- Construct most general higher-derivative m_{hd} and obtain $\otimes_{\text{hd}} = m_{\text{hd}}^{-1}$ (just BAS particle cont.)

- Impose that m_{hd} has same rank as m 'minimal rank' $(n-3)$!
  - observed to be necessary for a sane double copy
  - complicated beyond four points $(n = 4)$

- Find double-copyable EFT amplitudes satisfying

$$\mathcal{A}_{\text{hd}} = A_{\text{hd}} \otimes_{\text{hd}} A_{\text{hd}}^{\text{BAS}} \qquad \text{generalised BCJ relations}$$

- Observe that the space of double copies remains the same

$$\{A_{\text{hd}} \otimes_{\text{hd}} \tilde{A}_{\text{hd}}\} = \{A \otimes \tilde{A}\}$$

Generalised CK

[Carrasco, Rodina, Yin, Zekioglu '19, '21]

- Promote $c^{\text{adj}} \quad \rightarrow \quad c^{\text{adj}}_{\text{hd}}(\text{colour}, \text{kinematics})$
  with identical adjoint algebraic properties (e.g. Jacobi)

- Build the full tower of $c^{\text{adj}}_{\text{hd}}$ using composition rules

$$c^{\text{adj}}_{\text{hd},1} \circ c^{\text{adj}}_{\text{hd},2} = c^{\text{adj}}_{\text{hd},3}$$

complicated beyond four points

- Obtain double-copyable amplitudes: $\mathcal{A}_{\text{hd}} = n^{\text{adj}} \cdot P \cdot c^{\text{adj}}_{\text{hd}}$
  double copies: $\mathcal{M}_{\text{hd}} = n^{\text{adj}} \cdot P \cdot \tilde{n}^{\text{adj}}$

... which are not generalised

Numerator seeds

[Bonnefoy, Durieux, Grojean, Machado, Roosmale Nepveu '21]

→ Simplify the construction of double-copyable amps (above four points)

→ Relate generalised KLT & CK

· Define the map between colour representations: $c^{\text{adj}} = J \cdot c^{\text{tr}}$

e.g. for $n = 4$:

$$\begin{pmatrix} f^{12x} f^{x34} \\ f^{13x} f^{x42} \\ f^{14x} f^{x23} \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} (1324) + (4231) \\ (1234) + (4321) \\ (1243) + (3421) \end{pmatrix}$$

which encodes Jacobi relations

· Construct generalised adjoint numerators $c^{\text{adj}}_{\text{hd}} = J \cdot c^{\text{tr}}_{\text{hd}}$

from *seeds* satisfying just *trace* algebraic properties

Four-point scalar results

- Removing redundancies, the most general seed: (single colour trace)

$$c^{\text{tr}}_{\text{hd}} = \overbrace{\begin{pmatrix} g(s,t) & 0 & 0 \\ 0 & g(t,s) & 0 \\ 0 & 0 & g(u,s) \end{pmatrix}}^{G_{\text{hd}}} \cdot c^{\text{tr}} \quad \text{with} \quad g(s,t) = \sum_{i,j} a_{ij} \frac{s^i (tu)^j}{\Lambda^{2i+4j}}$$

$u \equiv -s - t$

$$\text{and thus} \quad c^{\text{adj}}_{\text{hd}} = J \cdot c^{\text{tr}}_{\text{hd}} = J \cdot G_{\text{hd}} \cdot c^{\text{tr}}$$

- Most general kinematic numerator: $n^{\text{adj}} = J \cdot \tilde{G}_{\text{hd}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

→ Most general double-copyable amplitude: $\mathcal{A}_{\text{hd}} = n^{\text{adj}} \cdot P \cdot c^{\text{adj}}_{\text{hd}}$

Four-point KLT from seeds

- CK-generalised BAS amplitudes (more than BAS particle content)

$$\mathcal{A}^{\text{BAS}}_{\text{hd}} = c^{\text{tr}}_{\text{hd}} \cdot m \cdot \tilde{c}^{\text{tr}}_{\text{hd}}$$
$$= c^{\text{tr}} \cdot \underbrace{G_{\text{hd}} \cdot m \cdot \tilde{G}_{\text{hd}}}_{\text{rank of } m} \cdot \tilde{c}^{\text{tr}}$$

- Tentative generalised kernel

$$\otimes_{\text{hd}} = \tilde{G}^{-1}_{\text{hd}} \cdot m^{-1} \cdot G^{-1}_{\text{hd}}$$
$$= \tilde{G}^{-1}_{\text{hd}} \otimes G^{-1}_{\text{hd}}$$

on a BCJ basis
$(n-3)! = 1$

indeed contains the generalised KLT solution,
to all EFT orders!

Four-point generalised KLT double copies

- Generalised double copies formally re-written with traditional kernel

$$\mathcal{M}_{\text{hd}} = \tilde{A}_{\text{hd}} \otimes_{\text{hd}} A_{\text{hd}}$$
$$= (\tilde{A}_{\text{hd}} \cdot \tilde{G}_{\text{hd}}^{-1}) \otimes (G_{\text{hd}}^{-1} \cdot A_{\text{hd}})$$

- $G_{\text{hd}}^{-1} \cdot A_{\text{hd}}$ satisfy traditional BCJ relations if A_{hd} satisfies generalised ones.

- $G_{\text{hd}}^{-1} \cdot A_{\text{hd}}$ can be rescaled to a well-behaved BAS amplitudes if A_{hd} is itself well behaved.

$\rightarrow$ Four-point generalised KLT does not generalise double copies.

Higher points

- n^{adj} constructed from seeds up to six points.
  Five-point count matches [Carrasco et al. '21].

- Five-point seeds shown to reproduce generalised KLT kernel
  at all EFT orders provided in [Chi et al. '21].

- Five-point $G_{\text{hd}}^{-1} \cdot A_{\text{hd}}$ shown to be well behaved
  for a subset of (diagonal) G_{hd}.

Double-copy generalisations for EFTs

New approach to double-copy generalisations for EFTs
using *numerator seeds*.

Simpler construction, for all multiplicities,
with redundancies that are easily removed.

It relates existing generalisations and
helps understanding observed features.

Outlook: KLT correspondence at five points, vector
amplitudes, double traces, extended particles content, etc.

Amplitude techniques provide

new opportunities to learn about EFTs!