Laboratoire Kastler Brossel
Physique quantique et applications

# Precise determination of the fine-structure constant

Pierre Cladé

# Fine structure constant and QED

- 1916 : Relativistic correction to the Bohr Model (Sommerfeld):
  $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$
  (Spectroscopy of hydrogen : $hcR_\infty = \frac{1}{2}mc^2\alpha^2$)
- 1928 : Dirac Equation
  - Energy level of hydrogen atoms
  - Electron spin and magnetic moment
- 1947 : Experiment:
  - Lamb shift in hydrogen (Lamb and Retherford 1947)
  - Anomalous magnetic moment of the electron (Kush and Foley, 1947)
- 1949 : Birth of QED (Schwinger, Feynman, Tomonaga ...)

The fine structure constant is the only parameter of QED

Electron in a Penning trap.

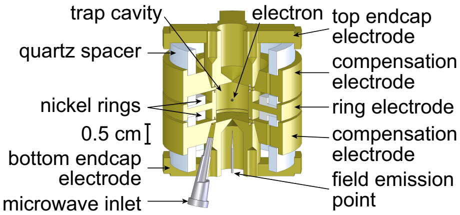
- Larmor frequency
  $\omega_{\mathrm{lar}} = \gamma B$
- Cyclotron frequency
  $\omega_{\mathrm{cyc}} = \frac{eB}{m}$

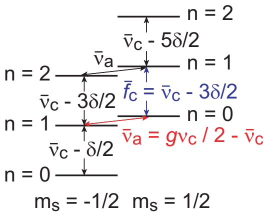
$$g_{\mathrm{e}} = \frac{\omega_{\mathrm{lar}}}{\omega_{\mathrm{cyc}}}$$

$$g/2 = 1.001\,159\,652\,180\,73(28)$$
$$[0.28 \times 10^{-12}]$$

Hanneke et al. PRL 120801 (2008)

One Electron Quantum Cyclotron



Fan, X. et al., arxiv:2209.13084.

Improvement by a factor of 2.

## Measurement of the Electron Magnetic Moment

X. Fan,[1,2,*] T. G. Myers,[2] B. A. D. Sukra,[2] and G. Gabrielse[2,†]

[1] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2] *Center for Fundamental Physics, Northwestern University, Evanston, Illinois 60208, USA*

(Dated: September 28, 2022)

The electron magnetic moment in Bohr magnetons, $-\mu/\mu_B = 1.001\,159\,652\,180\,59\,(13)\,[0.13\text{ ppt}]$, is consistent with a 2008 measurement and is 2.2 times more precise. The most precisely measured property of an elementary particle agrees with the most precise prediction of the Standard Model (SM) to 1 part in $10^{12}$, the most precise confrontation of all theory and experiment. The SM test will improve further when discrepant measurements of the fine structure constant $\alpha$ are resolved, since the prediction is a function of $\alpha$. The magnetic moment measurement and SM theory together predict $\alpha^{-1} = 137.035\,999\,166\,(15)\,[0.11\text{ ppb}]$

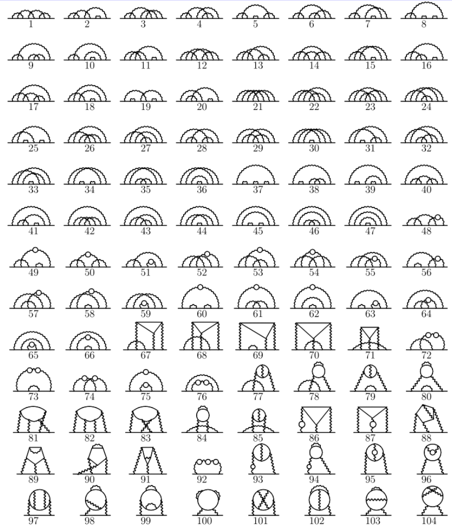
# The anomalous magnetic moment of the electron

- QED calculation

$$a_e(QED) = \frac{g_e - 2}{2} = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{(2n)}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

# The anomalous magnetic moment of the electron

## Feynman diagrams




- Analytical calculation up to $A^8$ (Laporta)
- Calculation of the $A^{10}$ term : 12672 Feynman diagrams.

# The anomalous magnetic moment of the electron

- QED calculation

$$a_e(QED) = \frac{g_e - 2}{2} = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{(2n)}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

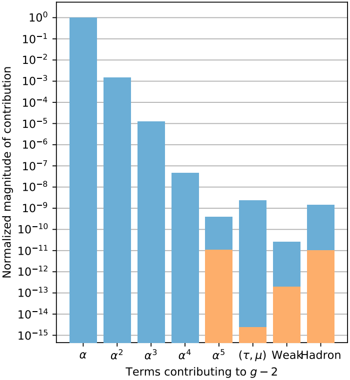
# The anomalous magnetic moment of the electron

- QED calculation

$$a_e(QED) = \frac{g_e - 2}{2} = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{(2n)}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

- Other (known) contributions

$$a_e\,(\text{theo}) = a_e\,(\text{QED}) + a_e\,(\text{Weak}) + a_e\,(\text{Hadron})$$



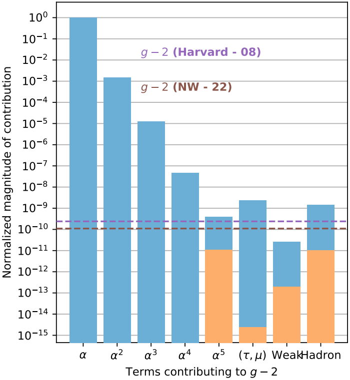
# The anomalous magnetic moment of the electron

- QED calculation

$$a_e(QED) = \frac{g_e - 2}{2} = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{(2n)}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

- Other (known) contributions

$$a_e\,(\text{theo}) = a_e\,(\text{QED}) + a_e\,(\text{Weak}) + a_e\,(\text{Hadron})$$




Experiments :

- Gabrielse 2008 → 2022
- 0.24 → 0.11 ppb

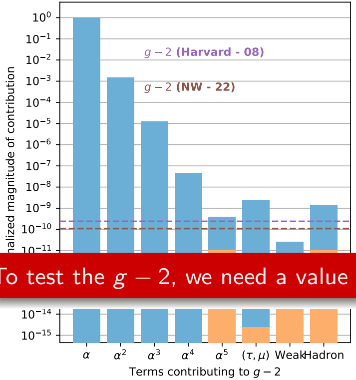
# The anomalous magnetic moment of the electron

- QED calculation

$$a_e(QED) = \frac{g_e - 2}{2} = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{(2n)}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

- Other (known) contributions

$$a_e\,(\text{theo}) = a_e\,(\text{QED}) + a_e\,(\text{Weak}) + a_e\,(\text{Hadron})$$




Experiments :

- Gabrielse 2008 $\rightarrow$ 2022
- 0.24 $\rightarrow$ 0.11 ppb

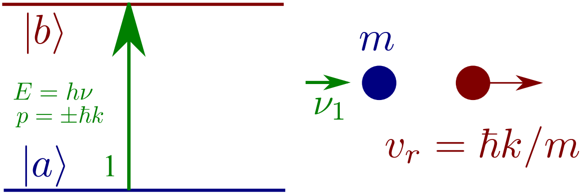
To test the $g-2$, we need a value of $\alpha$.

# Fine structure constant and recoil velocity

- Hydrogen : $hcR_\infty = \frac{1}{2} m_\mathrm{e} c^2 \alpha^2$

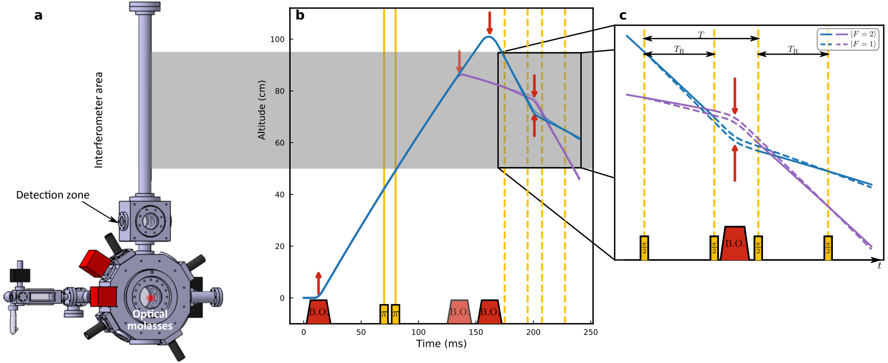
$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_\mathrm{e}} = \frac{2R_\infty}{c}\frac{A_\mathrm{r}(\mathrm{X})}{A_\mathrm{r}(\mathrm{e})}\frac{h}{m_\mathrm{X}}$$

- Limitation : determination of $\frac{h}{m_\mathrm{X}}$ (or $m_\mathrm{X}$ in the new SI)
- Recoil velocity :



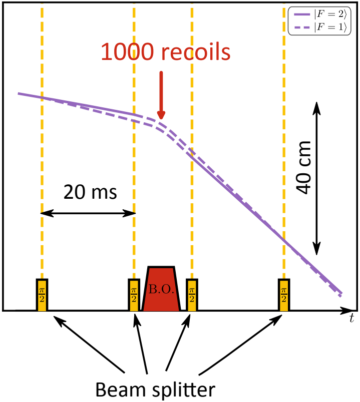

Rubidium atoms : $v_r = 6\,\mathrm{mm\,s^{-1}}$

# The experiment



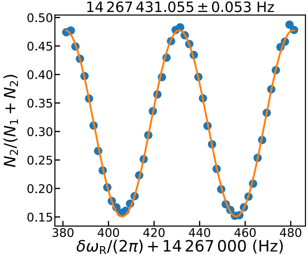

- Laser cooled rubidium atoms (4 μK, about 10 cm s$^{-1}$)
- Atom interferometer

# Atom interferometer



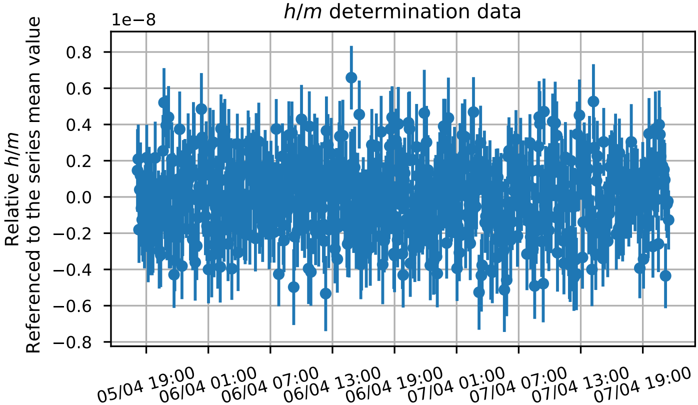

- Beamsplitter using laser transition: coherent superposition
- Acceleration of atoms : absorption and stimulated emission of photons (500 times)
- Phase ⇔ kinetic energy ⇔ Doppler effect

$$\Phi \propto kvT$$



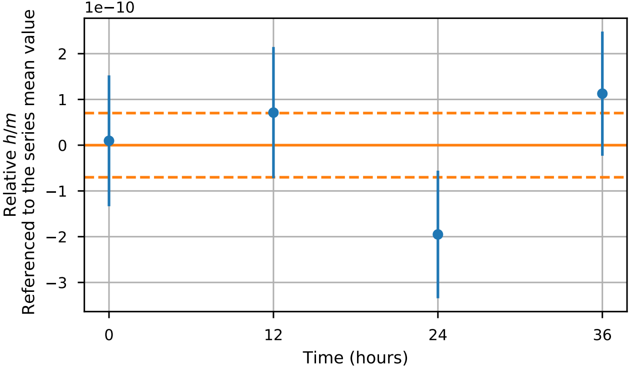
# 48 hours integration

Stable and reliable device ⇒ Long measurement periods




From Friday to ............................... Sunday

# Estimation of statistical uncertainty




48h integration: $8.5 \cdot 10^{-11}$ on $\frac{h}{m}$

$\rightarrow 4.3 \cdot 10^{-11}$ on $\alpha$

# Error budget

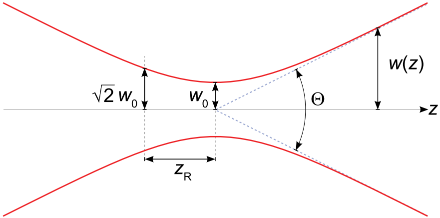
| Source | Correction $[10^{-11}]$ | Relative uncertainty $[10^{-11}]$ |
|---|---|---|
| Gravity gradient | -0.6 | 0.1 |
| Alignment of the beams | 0.5 | 0.5 |
| Coriolis acceleration | | 1.2 |
| Frequencies of the lasers | | 0.3 |
| Wave front curvature | 0.6 | 0.3 |
| Wave front distortion | 3.9 | 1.9 |
| Gouy phase | 108.2 | 5.4 |
| Residual Raman phase shift | 2.3 | 2.3 |
| Index of refraction | 0 | $< 0.1$ |
| Internal interaction | 0 | $< 0.1$ |
| Light shift (two-photon transition) | -11.0 | 2.3 |
| Second order Zeeman effect | | 0.1 |
| Phase shifts in Raman phase lock loop | -39.8 | 0.6 |
| Global systematic effects | 64.2 | 6.8 |
| Statistical uncertainty | | 2.4 |
| Relative mass of $^{87}$Rb [16] : $86.909\,180\,531\,0(60)$ | | 3.5 |
| Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$ | | 1.5 |
| Rydberg constant [14] : $10\,973\,731.568\,160(21)\mathrm{m}^{-1}$ | | 0.1 |
| Total: $\alpha^{-1} = 137.035\,999\,206(11)$ | | 8.1 |

# Atom recoil in a gaussian beam

Electric field $E(\vec{r}, t) = A(\vec{r}, t) e^{i\phi(\vec{r})} \rightarrow \vec{k}_{\text{eff}} = \vec{\nabla}\phi$

Plane wave model: $k = \omega/c$

Gaussian beam:




Correction to the effective wavevector:

$$\frac{\delta k}{k} = -\frac{2}{k^2 w^2(z)} \left(1 - \frac{\langle r^2 \rangle}{w^2(z)}\right) - \frac{\langle r^2 \rangle}{2R^2(z)}$$

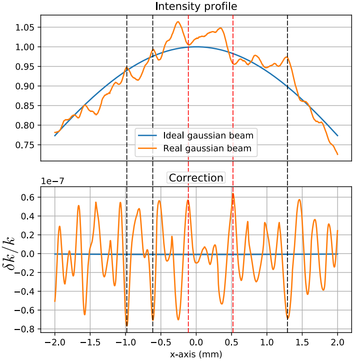
Related to the dispersion of wavevectors $\sim -\frac{\Theta^2}{2}$

# Effect of distortions

Momentum correction due to transverse phase and amplitude fluctuations:

$$\delta k_{\mathrm{rel}} = \frac{\delta k}{k} = -\frac{1}{2}\left|\left|\frac{\vec{\nabla}_{\perp}\phi}{k}\right|\right|^2 + \frac{1}{2k^2}\frac{\Delta_{\perp}A}{A} \qquad (I = A^2)$$

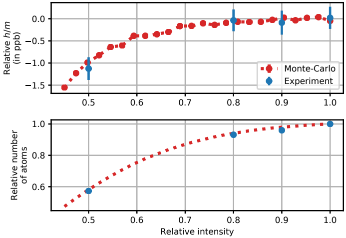
Correlation between $I$ and $\delta k$




During the Bloch Oscillation pulse, the survival propability $P(I)$ is governed by Landau-Zener Losses:

$$\langle \delta k_{\mathrm{rel}} \rangle = \langle \delta k_{\mathrm{rel}} P(I) \rangle \,/\, \langle P(I) \rangle$$

$\Rightarrow$ Effect on the measured recoil velocity




S. Bade *et al.* PRL **121** 073603 (2018)

# Error budget

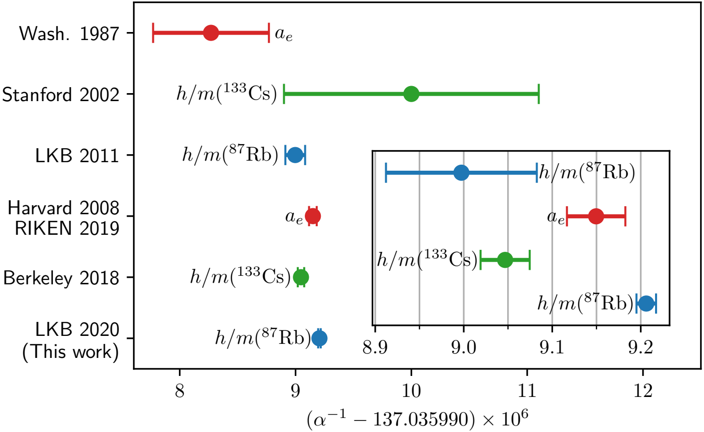
| Source | Correction $[10^{-11}]$ | Relative uncertainty $[10^{-11}]$ |
|---|---|---|
| Gravity gradient | -0.6 | 0.1 |
| Alignment of the beams | 0.5 | 0.5 |
| Coriolis acceleration | | 1.2 |
| Frequencies of the lasers | | 0.3 |
| Wave front curvature | 0.6 | 0.3 |
| Wave front distortion | 3.9 | 1.9 |
| Gouy phase | 108.2 | 5.4 |
| Residual Raman phase shift | 2.3 | 2.3 |
| Index of refraction | 0 | $< 0.1$ |
| Internal interaction | 0 | $< 0.1$ |
| Light shift (two-photon transition) | -11.0 | 2.3 |
| Second order Zeeman effect | | 0.1 |
| Phase shifts in Raman phase lock loop | -39.8 | 0.6 |
| Global systematic effects | 64.2 | 6.8 |
| Statistical uncertainty | | 2.4 |
| Relative mass of $^{87}$Rb [16] : $86.909\,180\,531\,0(60)$ | | 3.5 |
| Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$ | | 1.5 |
| Rydberg constant [14] : $10\,973\,731.568\,160(21)\mathrm{m}^{-1}$ | | 0.1 |
| Total: $\alpha^{-1} = 137.035\,999\,206(11)$ | | 8.1 |

L. Morel *et. al.*, Nature, 02/12/2020. DOI:10.1038/s41586-020-2964-7

# Most precise determinations (2020)

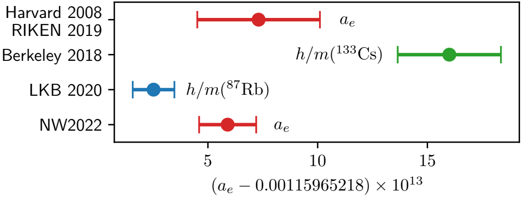
Fine structure constant

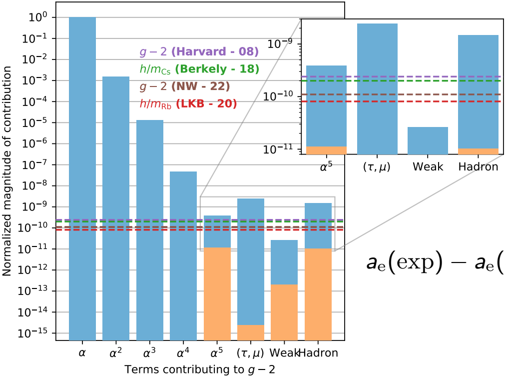
- Recoil based measurement
- $g-2$ based measurement



# Determinations of $a_e$

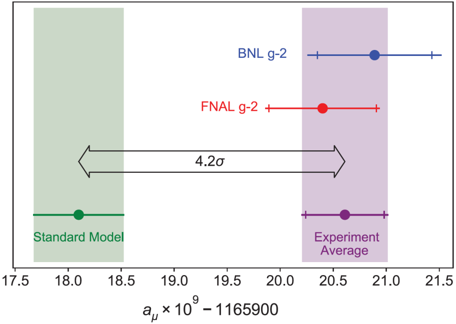
- Recoil based measurement
- Direct measurement of $a_e$




$$a_e(\exp) - a_e(\alpha) = (3.3 \pm 1.6) \times 10^{-13} (2.1\sigma)$$

# The muon $a_\mu$ discrepancy



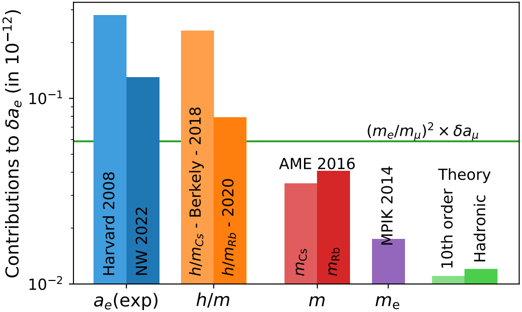

$$\delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{theo}) = 2.51(0.59) \cdot 10^{-9}\ (4.2\sigma)$$

T. Aoyama *et al*, Physics Report **887**, p1-66, (2020)
B. Abi et al. (Muon g-2 Collaboration) Phys. Rev. Lett. 126, 141801 (2021)

# Testing the muon discrepancy in the electron sector

- Naive scaling $\left|\frac{\delta a_e}{\delta a_\mu}\right| = \left(\frac{m_e}{m_\mu}\right)^2 \sim 2.3 \cdot 10^{-5}$
  $\delta_{a_e} =\sim 5.8 \cdot 10^{-14}$ (0.05ppb)

G. F. Giudice et al. JHEP 11, 113 (2012)
F. Terranova et al., PRA 89, 052118 (2014)

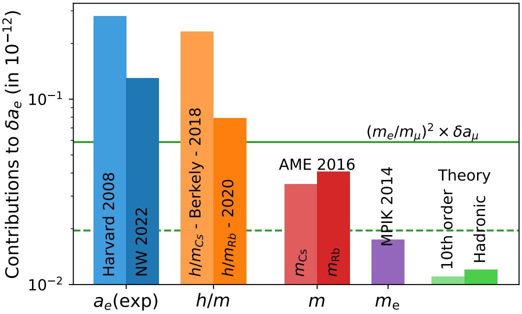



$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e} = \frac{2R_\infty}{c}\frac{A_r(X)}{A_r(e)}\frac{h}{m_X}$$

# Testing the muon discrepancy in the electron sector

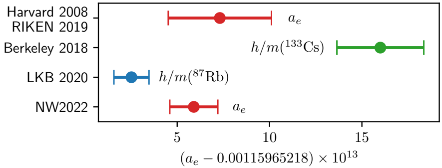
- Naive scaling $\left|\frac{\delta a_e}{\delta a_\mu}\right| = \left(\frac{m_e}{m_\mu}\right)^2 \sim 2.3 \cdot 10^{-5}$
  $\delta_{a_e} =\sim 5.8 \cdot 10^{-14}\,(0.05\text{ppb})$

G. F. Giudice et al. JHEP 11, 113 (2012)
F. Terranova et al., PRA 89, 052118 (2014)




$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e} = \frac{2R_\infty}{c}\frac{A_r(X)}{A_r(e)}\frac{h}{m_X}$$

# Conclusion




Determination of $\alpha$ in Paris

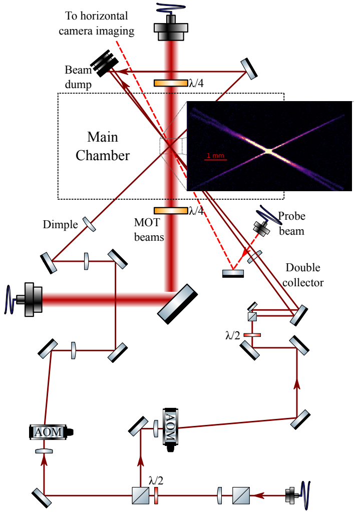
- Most precise determination of the fine structure constant : unprecedented statistics; experimental investigation of many systematics
- Discrepancy with Berkeley measurement needs to be clarified

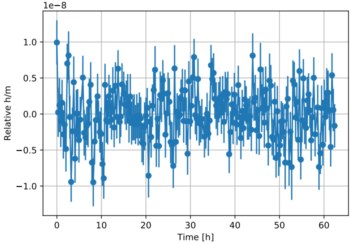
$\rightarrow$ Experiments limited by systematics

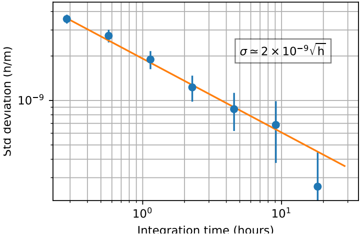
Perspective of the Paris experiment

- New measurement using ultra-cold atoms is in progress (Bose-Einstein condensate)
- New experimental setup : uncertainty on $\alpha$ of $1.5 \times 10^{-11}$

# Perspectives : measurement with a BEC



# Thank you


PhD students (since 2000):

- Z. Yao
- L. Morel
- C. Courvoisier
- M. Andia
- R. Jannin
- R. Bouchendira
- M. Cadoret
- P. Cladé
- R. Battesti

Permanent staff:

- Pierre Cladé
- Saïda Guellati-Khelifa
- François Biraben (emeritus)