The Standard Model

CSU-NUPAX/CERN IRES Program

Johan S Bonilla
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What is a Particle?

Classical | Quantum Mechanics | Quantum Field Theory
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E/B-Field

\[ \psi_n(x) = \sqrt{\frac{1}{2^n n!}} \cdot \left( \frac{m\omega}{\pi\hbar} \right)^{1/4} \cdot e^{-\frac{m\omega^2}{2\hbar}} \cdot H_n \left( \sqrt{\frac{m\omega}{\hbar}} x \right) , \]

Does NOT play nice with special relativity

\[ \hat{\phi}(x, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left( \hat{a}_p e^{-i\omega_p t + ip \cdot x} + \hat{a}^\dagger_p e^{i\omega_p t - ip \cdot x} \right) . \]

\[ \mathcal{L} = \frac{1}{2} \left( \partial_\mu \phi \right) \left( \partial^\mu \phi \right) - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 , \]

See Particle Data Group at LBNL
# Maxwell’s E&M Equations

<table>
<thead>
<tr>
<th>Differential equations</th>
<th>Formulation</th>
<th>Homogeneous equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} )</td>
<td>Fields</td>
<td>( \nabla \cdot \mathbf{B} = 0 )</td>
</tr>
<tr>
<td>( \nabla \cdot \mathbf{B} = 0 )</td>
<td>3D Euclidean space + time</td>
<td>( \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 )</td>
</tr>
<tr>
<td>( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} )</td>
<td>Potentials (any gauge)</td>
<td>( \mathbf{B} = \nabla \times \mathbf{A} )</td>
</tr>
<tr>
<td>( \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) )</td>
<td>Potentials (Lorenz gauge)</td>
<td>( \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} )</td>
</tr>
</tbody>
</table>

3D Euclidean space + time
Gauge Transformations

\[ \varphi \rightarrow \varphi - \frac{\partial \psi}{\partial t} \]

\[ \mathbf{A} \rightarrow \mathbf{A} + \nabla \psi \]

\[ \partial_\mu A^\mu = 0 \ (\mu = 0,1,2,3), \quad \text{Lorenz gauge} \]

\[ \nabla \cdot \mathbf{A} = \partial_j A_j = 0 \ (j = 1, 2, 3), \quad \text{Coulomb gauge or radiation gauge} \]

\[ n^2 A^\mu = 0 \ (n^2 = 0), \quad \text{light cone gauge} \]

\[ A_0 = 0, \quad \text{Hamiltonian or temporal gauge} \]

\[ A_3 = 0, \quad \text{axial gauge} \]

\[ x_\mu A^\mu = 0, \quad \text{Fock-Schwinger gauge} \]

\[ x_j A_j = 0, \quad \text{Poincaré gauge} \]

Identities

\[ \nabla \cdot (\nabla \times \mathbf{A}) = 0 \]

\[ \nabla \times (\nabla \varphi) = 0 \]

\[ \mathbf{E} = -\varphi - \frac{\partial \mathbf{A}}{\partial t} - \nabla \frac{\partial \psi}{\partial t} = -\nabla \left( \varphi + \frac{\partial \psi}{\partial t} \right) - \frac{\partial \mathbf{A}}{\partial t} \]

\[ \mathbf{B} = \nabla \times (\mathbf{A} + \nabla \psi) = \nabla \times \mathbf{A}. \]
Standard Model of Elementary Particles

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Mass</th>
<th>Charge</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>2.2 MeV/c²</td>
<td>1/3</td>
<td>1/2</td>
</tr>
<tr>
<td>c</td>
<td>1.28 GeV/c²</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>t</td>
<td>173.1 GeV/c²</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>d</td>
<td>4.7 MeV/c²</td>
<td>-1/3</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>96 MeV/c²</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>b</td>
<td>4.18 GeV/c²</td>
<td>-1/3</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Leptons</th>
<th>Mass</th>
<th>Charge</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>&lt;1.0 eV/c²</td>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>μ</td>
<td>&lt;0.17 MeV/c²</td>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>τ</td>
<td>&lt;18.2 MeV/c²</td>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>ν_ε</td>
<td>&lt;1.0 eV/c²</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>ν_μ</td>
<td>&lt;0.17 MeV/c²</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>ν_τ</td>
<td>&lt;18.2 MeV/c²</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bosons</th>
<th>Mass</th>
<th>Charge</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>~125 GeV/c²</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>124.97 GeV/c²</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>γ</td>
<td>~91.19 GeV/c²</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Z</td>
<td>~105.66 MeV/c²</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>Z boson</td>
<td>~18.2 MeV/c²</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>W</td>
<td>~80.39 GeV/c²</td>
<td>±1</td>
<td>1</td>
</tr>
</tbody>
</table>
CKM Matrix

\[
\begin{bmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{bmatrix}
= \begin{bmatrix}
0.97370 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\
0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\
0.0080 \pm 0.0003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030
\end{bmatrix}
\]
PMNS Matrix

\[
\begin{bmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{bmatrix}
= 
\begin{bmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix}
\]
Particle Detection
General Collider Detector
Detecting Particles with ATLAS/CMS

Electron: Charged, EM
Photon: Neutral, EM
Muon: Charged, MIP
Jet: Calorimeter Object

Transverse (\(\phi\)) plane

\(E_T^{\text{miss}}\): Missing Energy (Transverse)
ATLAS Tracker

See Video
Silicon (Semiconductor) Strip Detectors

The diagram illustrates a cross-sectional view of a silicon strip detector. It consists of an n-type bulk material with a p-type implantation layer that forms a p-n junction. The depletion region, characterized by the depletion width $X_D$, extends between the n-type bulk and the p-type implantation layer. Metal contacts are present at both the front and rear sides of the detector, with an oxide layer at the front edge. The diagram also shows the direction of charge carriers moving through the depletion region.
Liquid Argon Calorimeter
Hadronic Calorimeter

See Video
The Compact Muon Solenoid

- High resolution silicon tracking in $|\eta| < 2.4$
- PbWO$_4$ EM Calorimetry
- Brass Hadron Calorimeter
  - Provides excellent energy resolution for strongly-coupled parton showers
- Excellent, Robust Muon System
  - Superconducting solenoid creates 3.8$T$ magnetic field in tracker and calorimeters, 2$T$ is steel return yoke
- Cost: ~500 MCHF + ~200 MCHF (Upgrades)
What Are Cathode Strip Chambers (CSCs)?

- Muon system employs different technologies
  - Barrel: Drift Tube + Resistive Plate Chamber (RPC)
  - End-Caps: CSC + RPC + Gas Electron Multipliers (GEM)

- CSCs are 6-layers of wires (anodes) and strips (cathodes) in Ar/CO₂/CF₄ gas mixture
  - Traversing muons ionize gas at HV
  - Avalanche signal read by anode and cathode electronics

- CSCs measure 4D position, \(|\eta| \in [0.9,2.4]\)
  - Work great in intense, non-uniform magnetic fields

\[
\begin{align*}
\sigma_i &\sim 2.1\text{ns} & \sigma_{\phi} &\sim 50 - 150\mu\text{m}
\end{align*}
\]
ALICE

1. ITS
2. V0 and T0
3. TPC
4. TRD
5. TOF
6. EMCAL
7. PHOS
8. Muons
9. AD
10. ZDC
FCC-hh Reference Detector

- 4T, 10m solenoid, unshielded
- Forward solenoids, unshielded
- Silicon tracker
- Barrel ECAL LAr
- Barrel HCAL Fe/Sci
- Endcap HCAL/ECAL LAr
- Forward HCAL/ECAL LAr

50m length, 20m diameter similar to size of ATLAS
Intro to Group Theory

• What is a group?
  — What is an example of a group?

• What is a field?
  — How are fields related to groups?
  — What is a vector field? Is that a group?
  — How is a field related to groups?

A group is a set defined by axioms:
1) an associative operation $X$,
2) $G$ must have an identity $g_I$,
3) For any element $g_1$ of $G$ there exists a $g_2$, such that $g_1 X g_2 = g_I$, the identity of the group

A field is a set in which two binary operations (addition/sum and multiplication/product) and axioms:
Associativity, distributivity, commutativity (only abelian), and have unique inverses and identities for both operations
Groups <-> Symmetries

• Groups can by ANYTHING that satisfies the axioms
  — Solutions to equations can be closed groups (think E&M)
    e.g: if $T(f_1) = 0$ and $T(f_2) = 0$, then $T(f_1+f_2)= T(f_1)+T(f_2)=0$
  — Eigenvectors can be a group
  — Rotations are definitely a group...so are rotations that leave a system invariant
  — Turns out groups can be represented by matrices

• $U(N)$ vs $SO(N)$ vs $SU(N)$
  — Unitary: $U^\dagger U = UU^\dagger = UU^{-1} = I$
  — Orthogonal: $Q^TQ = QQ^T = I \rightarrow Q^T = Q^{-1}$
  — Special Unitary/Orthogonal: Unitary/Orthogonal with norm 1
**Symmetries in Physics**

- For a set of transformations $G$ (group), a theory (Lagrangian) is symmetric/invariant under $G$ if all elements of $G$ transform the states/operators in such a way that leaves the FORM of the Lagrangian.

- Unitary Group of degree 1, U(1)
  - Set of single ($n=1 \rightarrow n \times n = 1 \times 1$ matrices) complex numbers with norm 1
  - $e^{i\theta}$ for $\theta \in [0, 2\pi]$
  - Just a circle!

- Global Symmetry $\rightarrow$ for any $\theta$ $\rightarrow$ Conserved quantity (Noether’s theorem)

- Local/gauge symmetry $\rightarrow$ for $\theta(x)$ dependent on $x$ $\rightarrow$ gauge

- Redundancies in a theory $\rightarrow$ same physics
Standard Model Symmetries

\( U(1) \times SU(2)_L \times SU(3) \rightarrow U(1)_{QED} \times SU(3)_C \)

- \( U(1) \times SU(2)_L \) represents Electroweak symmetry
  - 4 degrees of freedom \( \rightarrow \) electroweak symmetry breaking \( \rightarrow \) 3 leftover
  - Goldstone bosons \( \rightarrow \) physical bosons
  - Higgs ‘eats’ a degree of freedom

- \( SU(3) \) represent Quantum Chromo Dynamics (QCD, aka strong force)
  - No symmetry breaking
  - Decoupled from Electroweak sector