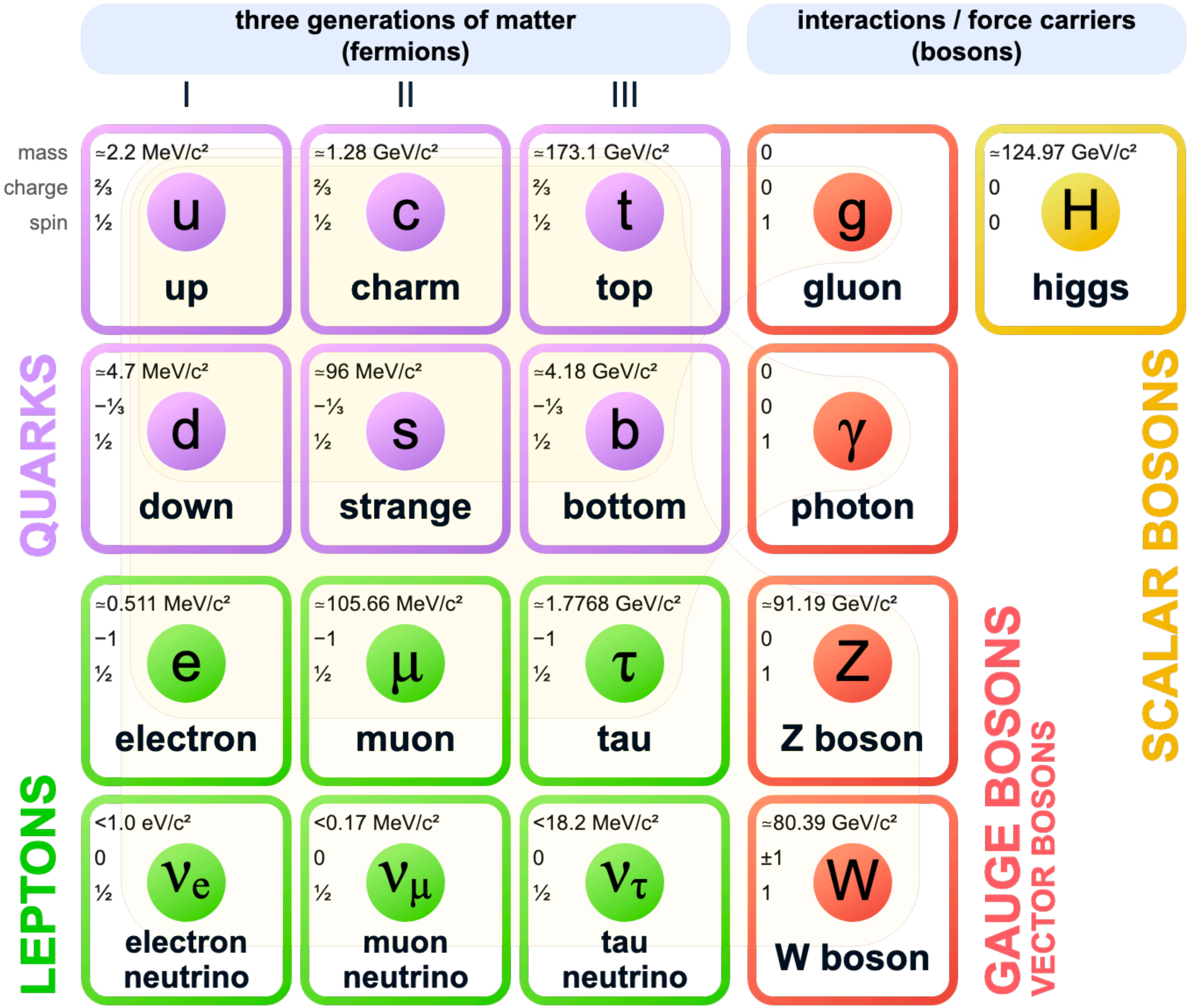
The Standard Model

CSU-NUPAX/CERN IRES Program

Johan S Bonilla

March 1st and 3rd, 2022

Standard Model of Elementary Particles



Standard Model Symmetries

$U(1) \times SU(2)_L \times SU(3)$ **->** $U(1)_{QED} \times SU(3)_C$

- $U(1) \times SU(2)_L$ represents Electroweak symmetry
  — 4 degrees of freedom -> electroweak symmetry breaking -> 3 leftover
  — Goldstone bosons -> physical bosons
  — Higgs ‘eats’ a degree of freedom

- $SU(3)$ represent Quantum Chromo Dynamics (QCD, aka strong force)
  — No symmetry breaking
  — Decoupled from Electroweak sector

Lorentz Group

Lorentz boost (*x direction*)

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x' = \gamma\,(x - vt)$$
$$y' = y$$
$$z' = z$$

- Set of all Lorentz transformations
  — Invariant physics in 4-dimensional spacetime
  — 3 degrees of freedom for rotations
  — 3 degrees of freedom for translations (boosts)
  — 3+3=6 *generators* of group

- SO(4): Special Ordinary Group of Rank 4
  — Happens to have 6 generators
  — Turns out SO(4)=SU(2)xSU(2) -> Left/Right-Handed
  — Useful since respects Lorentz Invariance, and lines up with evidence of ‘handed-ness’ of physics (see Wu and Goldhaber experiments in 1950’s)

Writing Down a Lagrangian

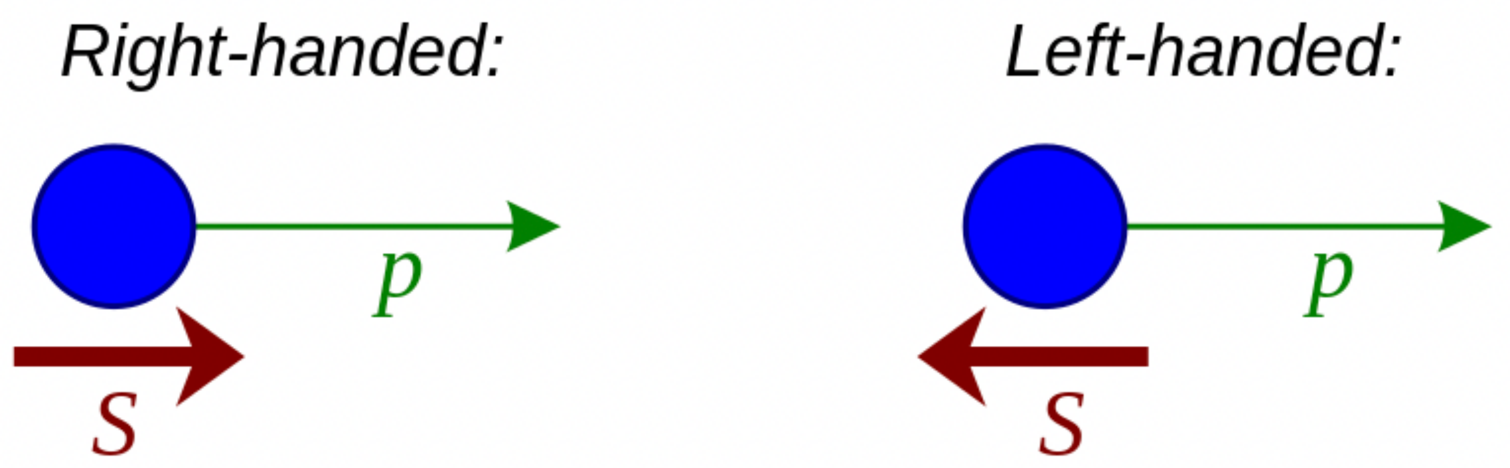
- For each Gauge-Symmetry -> Field Tensor $F^{\mu\nu}$
  - SU(3) -> G_μ^a (x8)
  - SU(2) -> W_μ^i (x3)
  - U(1) -> B_μ (x1)
- Total derivative involves all field tensors
- Introduce chiral (handed) fermions
  - LH: Doublets; RH: Singlets (Y==0)

Couplings

$$D_\mu = \partial_\mu - ig_C \lambda_\alpha G_\mu^\alpha - ig_L T_j W_\mu^j - \frac{i}{2} g_Y Y \ B_\mu$$

Weak Hyperspin

Weak Hypercharge



$$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \quad Q_{L_a} = \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L$$

Lagrangian of the Standard Model

No mass terms!! Everything is massless!!!!

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + H.C.\ \text{(Kinetics and Gauge-Interactions)}$$

$$+ |D_\mu\phi|^2 + V(\phi)\ \text{(Higgs)}$$

$$+ \psi_i y_{ij}\psi_j\phi + H.C.\ \text{(Yukawa)}.$$

Higgs Sector -> Leads to effective mass terms (Yukawa)

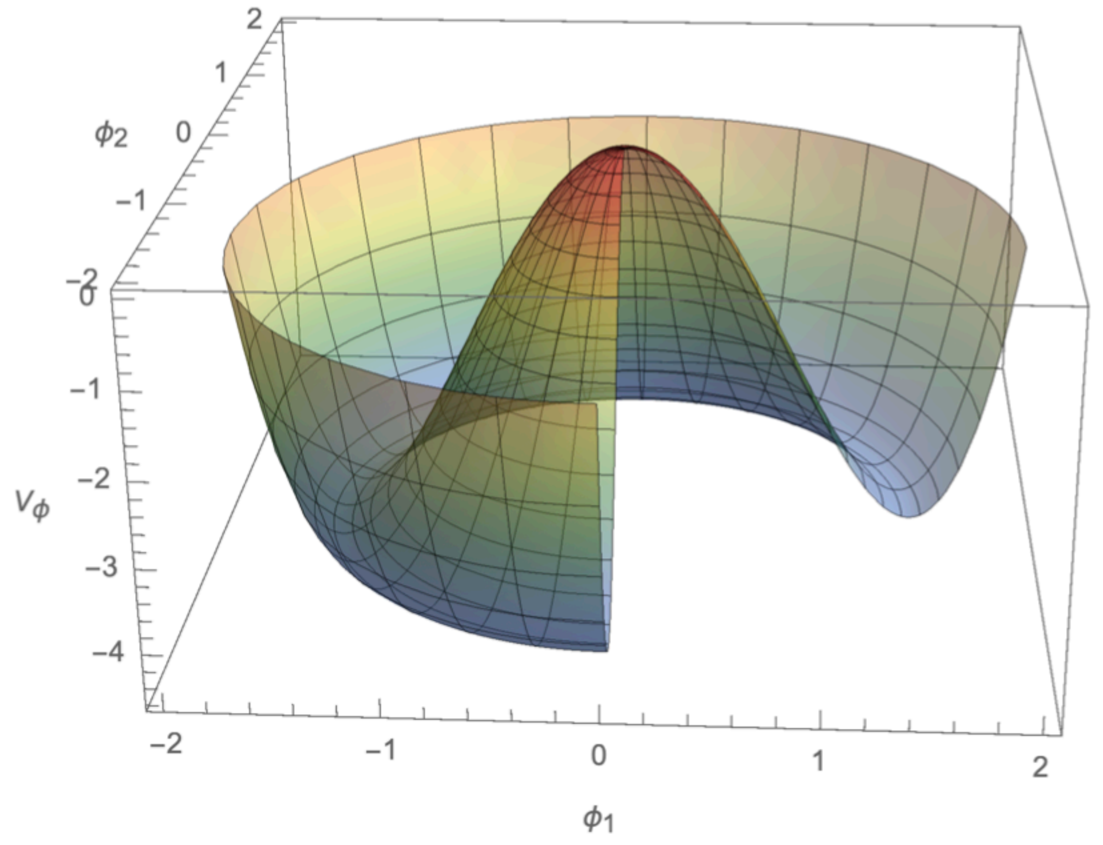
The Higgs Potential

- Where we start: $SU(2)_L \otimes U(1)_Y$ => $(W_\mu^{1,2,3}, B_\mu)$ three generators
- Higgs potential: $V(\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + \frac{1}{4}\lambda(\phi^\dagger\phi)^2$
- Minimum defines Vacuum Expectation Value

$$\text{VEV} \equiv v \equiv \frac{\mu^2}{\lambda} \text{ such that } \langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Choose representation of scalar field as

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$



$|\lambda| > 0$ and $|\mu| < 0$

Electroweak Symmetry Breaking

Expanding the Total Derivative

$$(D^\mu\phi)^\dagger(D_\mu\phi) = \frac{v^2}{8}\left|\begin{pmatrix} g_L W^1_\mu - i g_L W^2_\mu \\ -g_L W^3_\mu + g_Y B_\mu \end{pmatrix}\right|^2 = \frac{v^2}{8}\left(g_L^2\left(\left(W^1_\mu\right)^2 + \left(W^2_\mu\right)^2\right) + g_Y\left(W^3_\mu - g_Y B_\mu\right)^2\right)$$

We can measure VEV

v=246 GeV

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right) \text{ such that } m_W = \frac{g_L v}{2},$$

$$Z_\mu \equiv \frac{1}{\sqrt{g_L^2 + g_Y^2}}\left(g_Y W^3_\mu + g_L B_\mu\right) \text{ such that } m_Z = \frac{v}{2}\sqrt{g_L^2 + g_Y^2},$$

$$A_\mu \equiv \frac{1}{\sqrt{g_L^2 + g_Y^2}}\left(g_Y W^3_\mu + g_L B_\mu\right) \text{ such that } m_A = 0.$$

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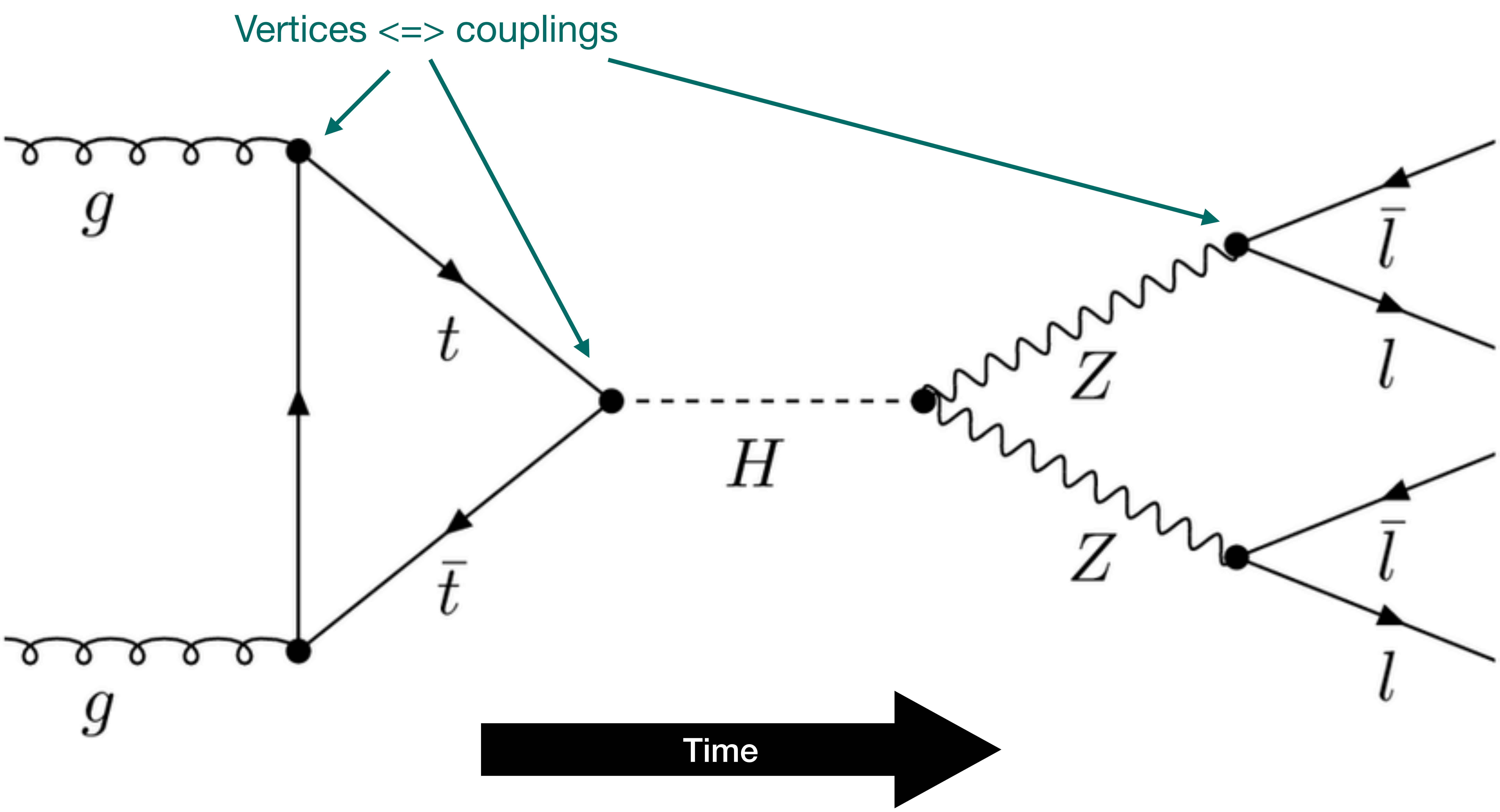
Higgs Sector -> Leads to effective mass terms (Yukawa)

Quantum Numbers of the Standard Model

Type	Mass	Lifetime	Q	J	Y	P	Color
e	0.511 MeV	stable	-1	$1/2$	$-\frac{1}{2}$	$-1/2$	0
ν_e	small	oscillates	0	$1/2$	$-\frac{1}{2}$	$+1/2$	0
μ	106 MeV	$\mathcal{O}(10^{-6})s$	-1	$1/2$	$-\frac{1}{2}$	$-1/2$	0
ν_μ	small	oscillates	0	$1/2$	$-\frac{1}{2}$	$+1/2$	0
τ	1776 MeV	$\mathcal{O}(10^{-13})s$	-1	$1/2$	$-\frac{1}{2}$	$-1/2$	0
ν_τ	small	oscillates	0	$1/2$	$-\frac{1}{2}$	$+1/2$	0
u	2.2 MeV	stable	$+2/3$	$1/2$	$+1/3$	$+\frac{1}{2}$	r/g/b
d	4.7 MeV	$\mathcal{O}(10^{-8})$s	$-1/3$	$1/2$	$+1/3$	$-\frac{1}{2}$	r/g/b
s	95 MeV	$\mathcal{O}(10^{-8})$s	$-1/3$	$1/2$	$+1/3$	$-\frac{1}{2}$	r/g/b
c	1.275 GeV	$\mathcal{O}(10^{-12})$s	$+2/3$	$1/2$	$+1/3$	$+\frac{1}{2}$	r/g/b
b	4.18 GeV	$\mathcal{O}(10^{-12})$s	$-1/3$	$1/2$	$+1/3$	$-\frac{1}{2}$	r/g/b
t	173 GeV	$\mathcal{O}(10^{-25})$s	$+2/3$	$1/2$	$+1/3$	$+\frac{1}{2}$	r/g/b
γ	0	stable	0	1	0	0	0
$W^\pm$	80.4 GeV	$\mathcal{O}(10^{-25})$s	±1	1	0	±1	0
Z	91.2 GeV	$\mathcal{O}(10^{-25})$s	0	1	0	0	0
g	0	-	0	1	0	0	double ($c\bar{c}$)
H	125 GeV	$\mathcal{O}(10^{-22})$s	0	0	+1	$-\frac{1}{2}$	0

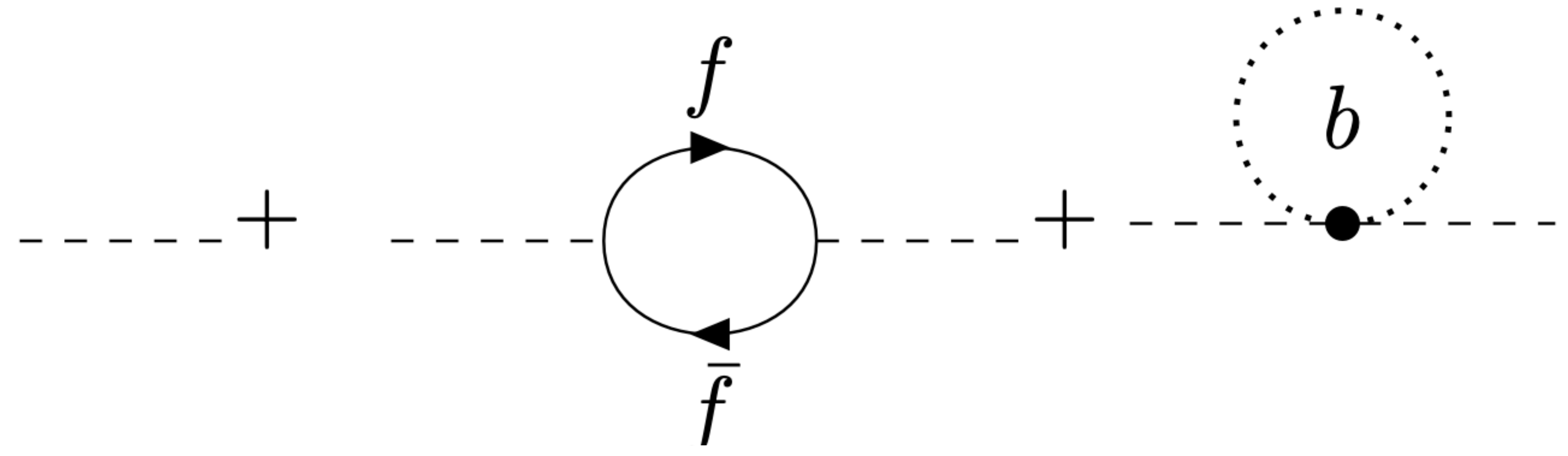
Feynman Diagrams



Naturalness in the Standard Model

$$m^2_{h,phys} = 2\mu^2 = m^2_{h,\ bare} + \sum_{Fermions} k_f m_f^2 + \sum_{Bosons} k_b m_b^2$$

$=$



$$= m^2_{H,bare} - \frac{|\lambda_f|^2}{8\pi^2}\Lambda^2_{UV} + \frac{|\lambda_b|^2}{8\pi^2}\Lambda^2_{UV}$$

Left Side: Measured at 125 GeV

Right Side: (Bare Mass) + (User Defined Scale)

Beyond the Standard Model

Questions to Think About

- Is the Higgs really fundamental?
- Could there be new physics to help with quadratic divergence of Higgs mass?
- Why is gravity so weak? Can it be easily integrated into the SM?
- What about Dark Matter?
- Is lepton universality really true?
- Where did all the anti-matter go?