Impact of material properties of Nb_3Sn coil on its thermo-mechanical behavior

Methods and preliminary results

CAMPAGNA Guillaume

13/10/2022

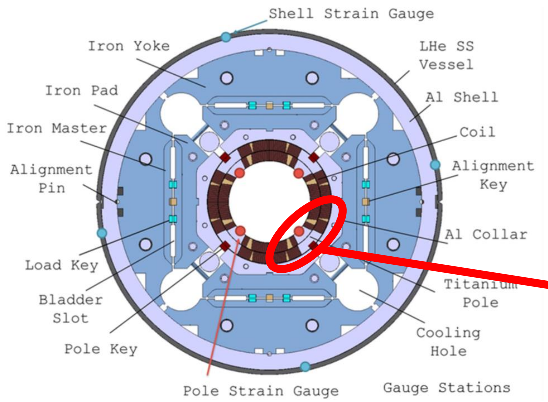
Introduction – *Who am I?*

- *2021:* **CEA Paris-Saclay** for a research internship (6 months)**.** Supervisors: **E. Rochepault and S. Perraud.**

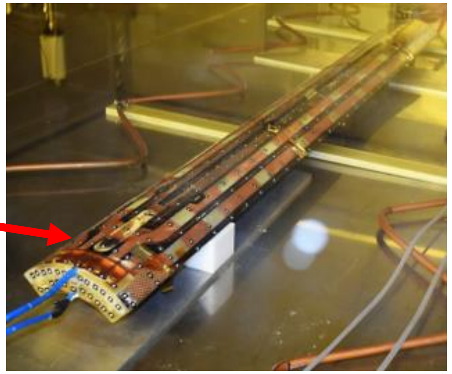
- *2021-2022:* **CERN** as a technical student (11 months)**.** Supervisor: **H. Felice**.

- *Now:* **CEA Paris Saclay** for a PhD. Supervisors: **K. Lavernhe, E. Rochepault.**

Introduction: Nb_3Sn based magnets

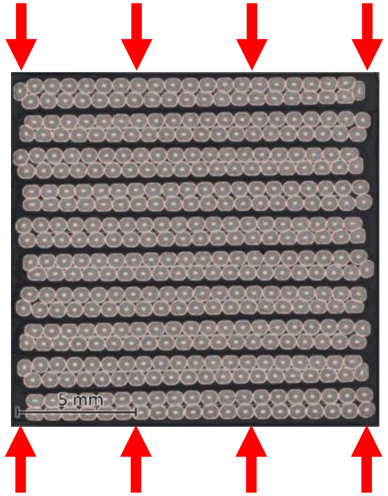


MQXF quadrupole for HL-LHC

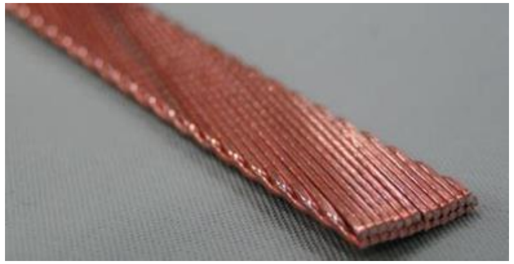


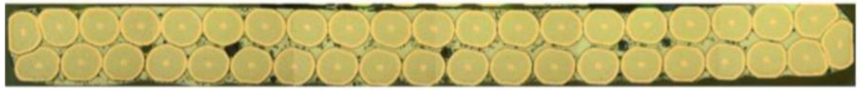
MQXFS coil

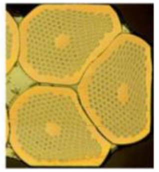
Mechanical characterization (courtesy F. Wolf)



Courtesy FERMILAB







Rutherford cable, a composite material (courtesy L. Oberli)

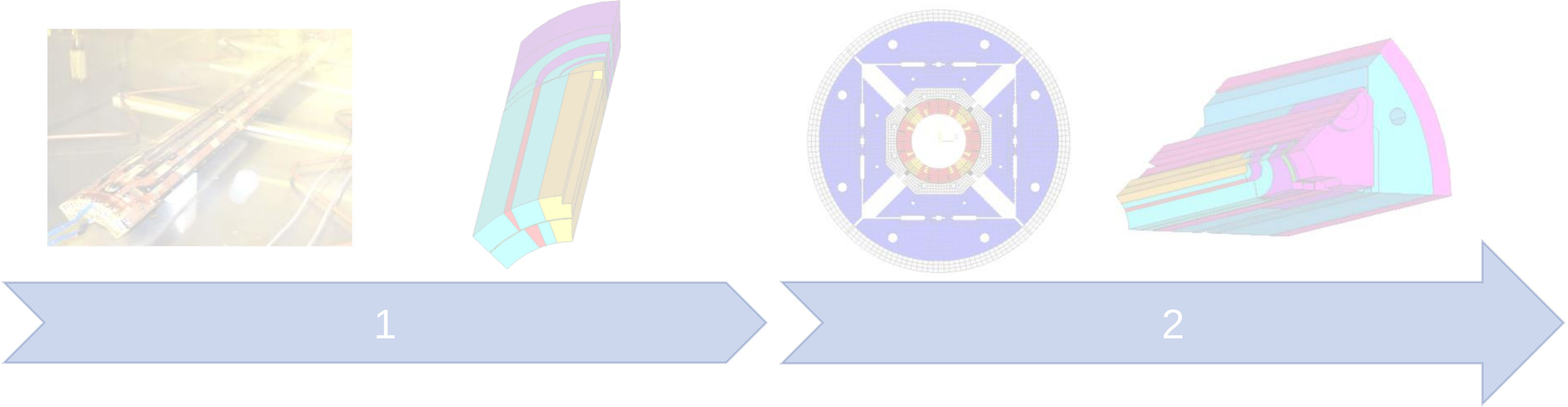
To ensure **magnets performances**, there is a need to:

- **pre-stress the coils** to counteract the Lorenz forces,
- **control the preload** (brittle material).

Problem and outline

How could the thermomechanical behavior of Nb_3Sn superconducting coil be accurately reproduced in finite element models of magnets?

What level of complexity do we need in our FE models?



Thermomechanical study of a self standing coil: correlation between models and experiment

Method development for an orthotropic coil parametric study in a full magnet structure

Problem and outline

How could the thermomechanical behavior of Nb_3Sn superconducting coil be accurately reproduced in finite element models of magnets?

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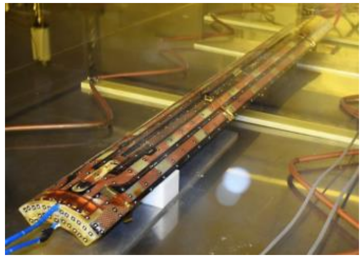
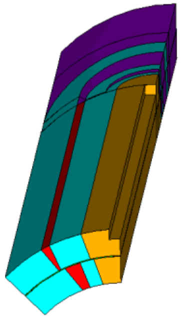
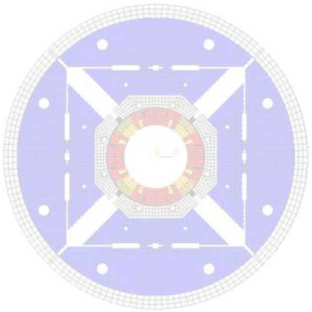
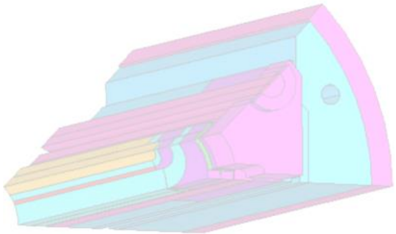








Experiment was done with 11 T

1

Thermomechanical study of a self standing coil: correlation between models and experiment

2

Method development for an orthotropic coil parametric study in a full magnet structure

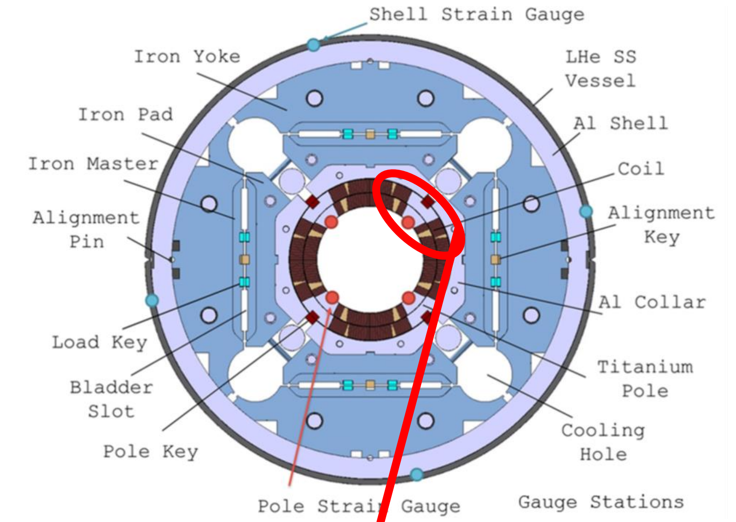
Experimental set-up in cryolab

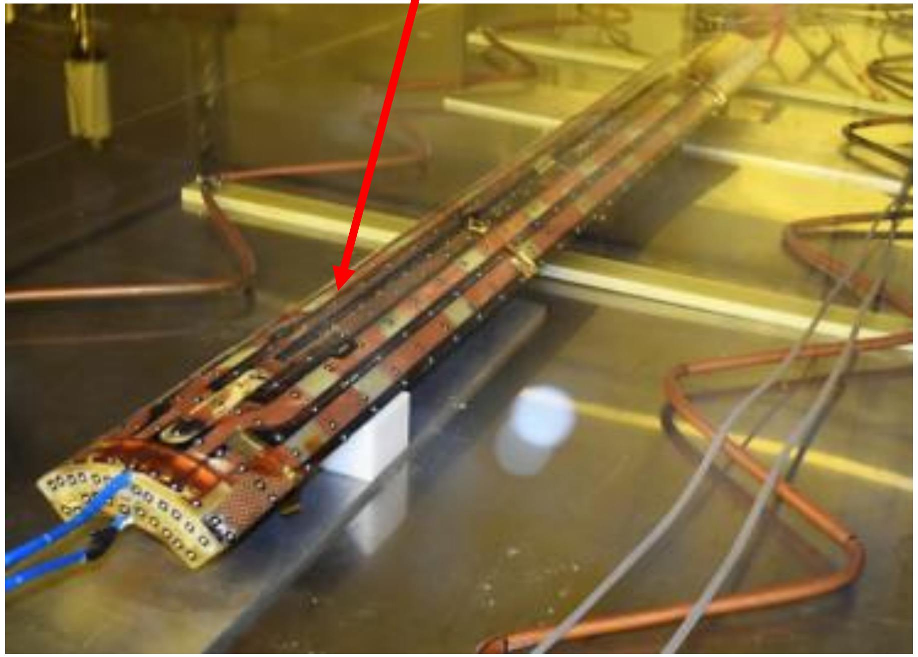
- **Protocol:**
  - Coil in Cryogenic pool on isostatic supports,
  - Cooling down at 77K (thermal gradient controlled),
  - Drain LN2 as fast as possible, open the pool, measurements.

  Cryolab: J. Bremer, L. Dufay-Chanat, T. Koettig.

- **Two acquisition systems:**
  - ~~Strain Gauges (SG), 12 thermally compensated,~~
  - Photogrammetry by BE-GM *(D. Mergelkuhl and K. Nikolitsas).*

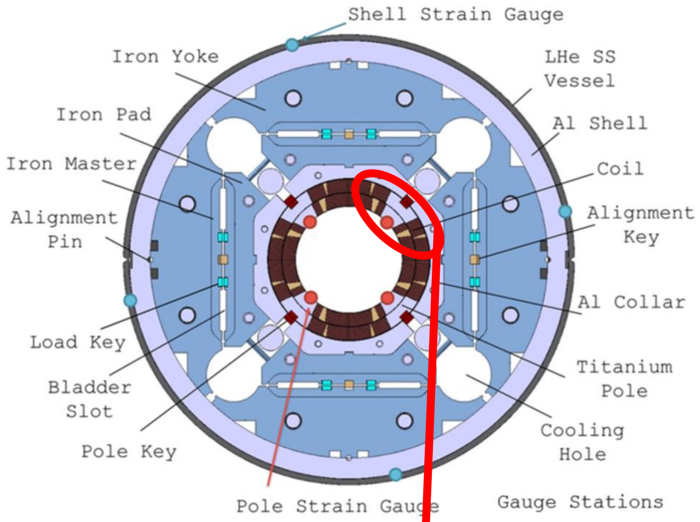
<u>Warning from the cryolab</u>: this may not be representative of real magnet conditions (water solidification into coil's cracks).





Report and data from the acquisition team: https://edms.cern.ch/document/2659563/1

Experimental set-up in SM18

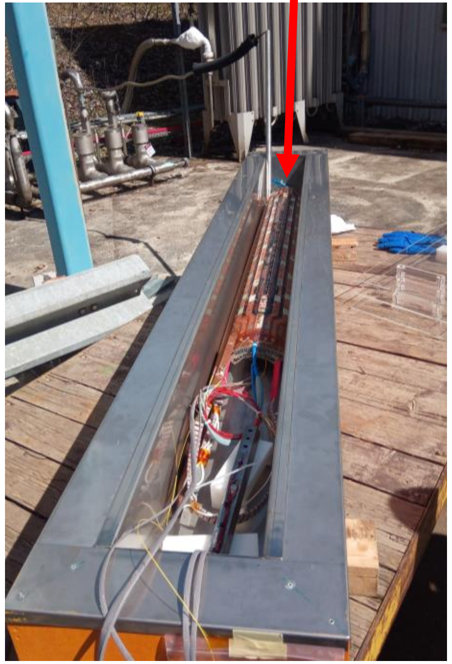


- **Protocol:**
  - Coil in a small cryostat on isostatic supports,
  - Quick cooldown at 77K (not controlled),
  - Slow warmup.

  SM18: G. Willering, P. Viret.

- **Two acquisition systems**
  - Strain Gauges (SG), 12 thermally compensated,
  - Optical fiber.

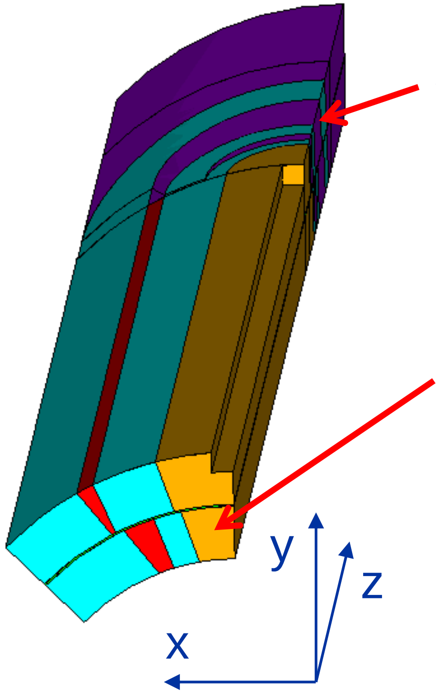
  EN-MME: S. Mugnier, K. Kandemir and M. Guinchard.



Coil instrumented with strain gauges and optical fiber

Report and data from the acquisition team: https://edms.cern.ch/document/2598050/1

Finite element model of MQXFS self-standing coil

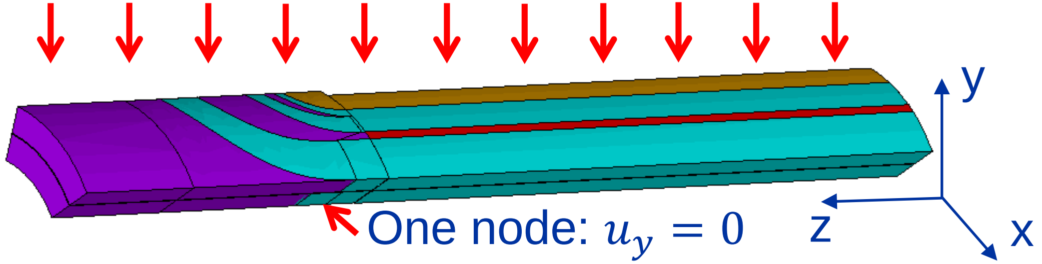


Symmetric BC:
$u_x = 0$
$ROT_y = 0$
$ROT_z = 0$

Symmetric BC:
$u_z = 0$
$ROT_x = 0$
$ROT_y = 0$

- **Same coil as in the FE model of the full MQXFS magnet.**
- **All parts are bounded as a starting point.**
- **Two loading step :**
  - Acceleration on all nodes (gravity),
  - Temperature change: 77 K (cool down).

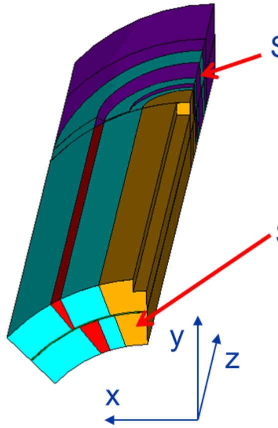
Acceleration on all nodes : $a_y = 9.8\, m.s^{-2}$



One node: $u_y = 0$

Thank you to J. Ferradás Troitiño for providing the FE model.

Finite element model of MQXFS self-standing coil



Symmetric BC:
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Symmetric BC:
$u_z = 0$
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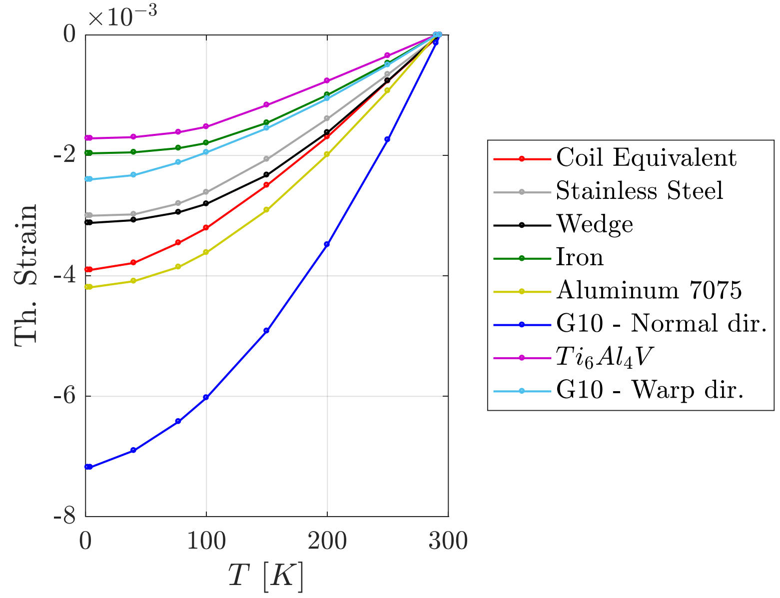
Starting point:

- **Same thermo-mechanical material parameters as in the full MQXFS magnet FE model.**
- **Coil block is an isotropic linear elastic material.**

	Coil Block	Stainless steel		Copper	G10	Titanium
	293-77 K	293 K	77 K	293-77 K	293-77 K	293-77 K
E (GPa)	20	193	197.3	120	30	130
ν (-)	0.3	0.28	0.28	0.3	0.3	0.3
ρ (kg/m^3)	7000	7900	7900	8300	1948	4620

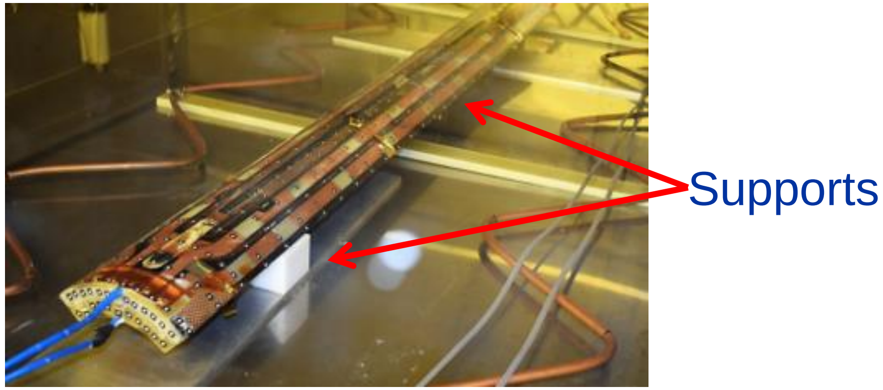
Inherited material properties used in the full magnet model

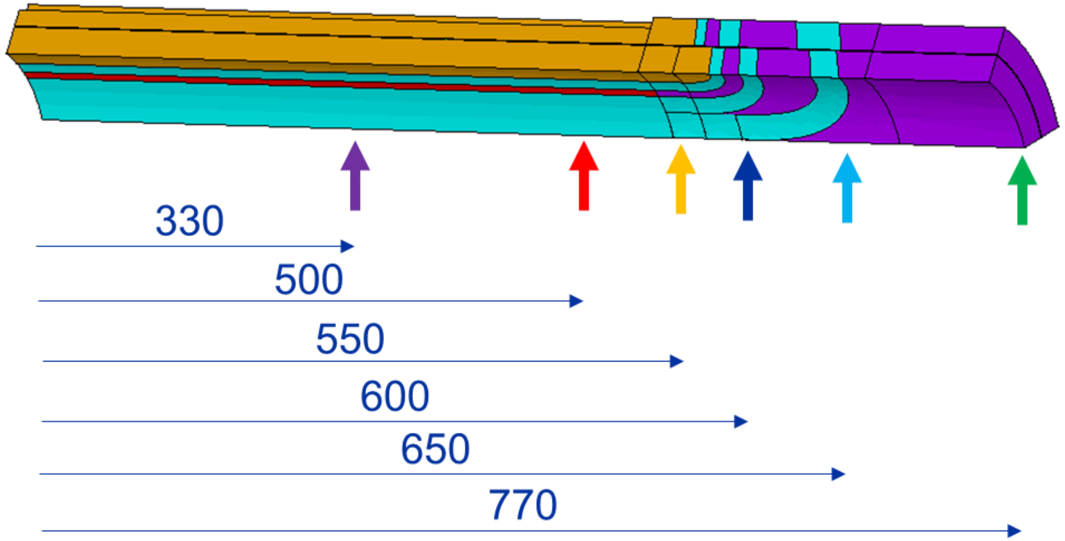


Inherited CTE used in the full magnet model (thermally dependent)

Thank you to J. Ferradás Troitiño for providing the FE model.

A possible experimental bias: the supports location





Multiple support positions tested numerically

Problem: the cryostat is contracting during cool down and the coil was displaced between tests. **This can change the support location**.

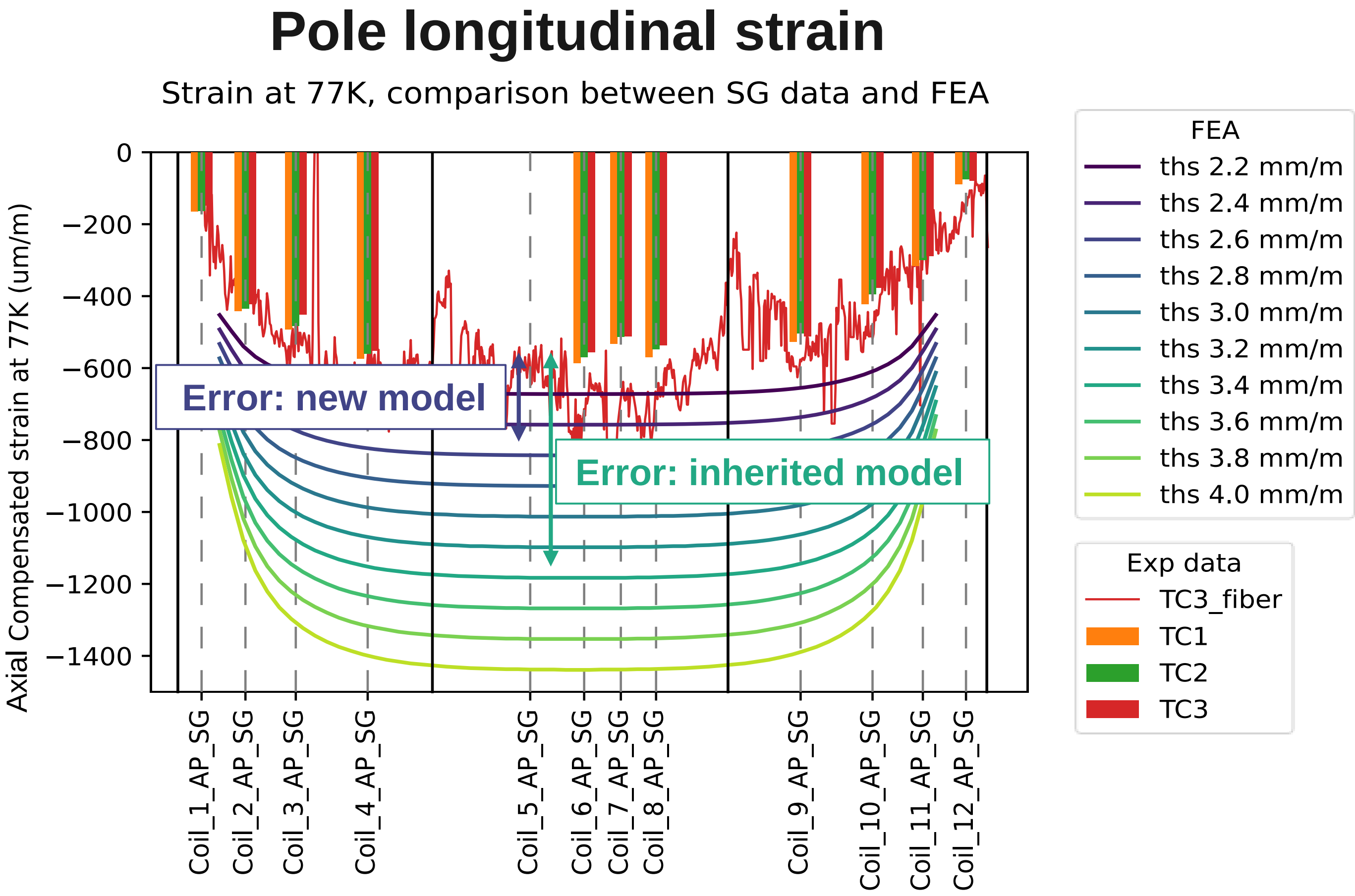
Effect of the supports position on FE models:

- **Negligible** on **strain gauges and optical fiber**.
- **Negligible** on **photogrammetry**.

This was also **checked experimentally** (room temperature only) with strain gauges measurements with multiple support positions.

Dominating parameter: the Coefficient of Thermal Expansion (CTE) of the coil block 1/2

E isotropic
CTE isotropic



Due to **differential thermal contraction**, **the coil bends** and the **pole is under compression**.

Inherited model: error of ~500 µm/m on the pole longitudinal strain.

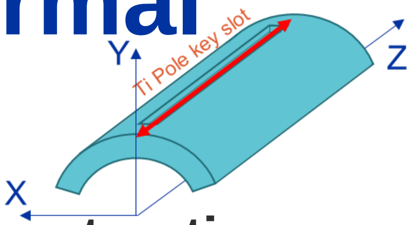
- ➢ **Lowering the CTE improve the model**.
- ➢ Yet, ongoing thermal contraction studies suggests a **CTE around -2.5 mm/m at 77 K** [1].

New model: error of ~200 µm/m on the pole longitudinal strain.

- ➢ Improved model, but still some discrepancies with measurements.

[1] M. Guinchard, ongoing thermal contraction on cuts taken from MQXF coil.

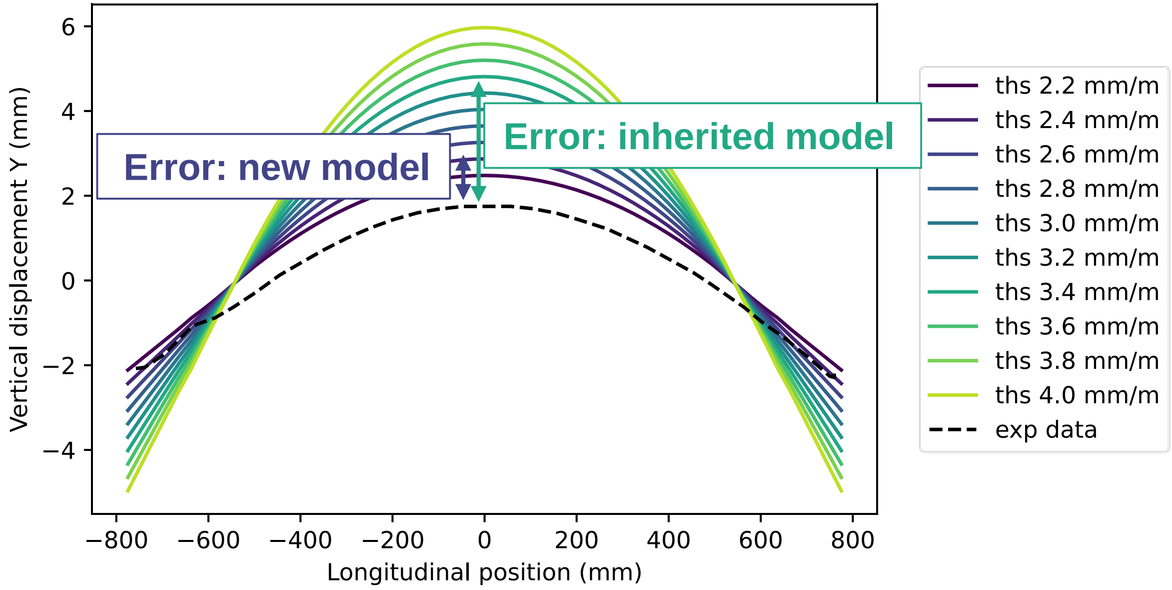
Dominating parameter: the Coefficient of Thermal Expansion (CTE) of the coil block 2/2



E isotropic
CTE isotropic

Coil vertical bending (photogrammetry)

CTE parametric study compared with experimental data (vertical displacement at 77K)



Due to **differential thermal contraction**, **the coil bends by about 4 mm** according to photogrammetry measurement.

Inherited model: error of ~4 mm on the coil vertical bending.

- **Lowering the CTE improve the model**.
- Yet, ongoing thermal contraction studies suggests a **CTE around -2.5 mm/m at 77 K** [1].

New model: error of ~1 mm on the coil vertical bending.

- Improved model, but still some discrepancies with measurements.

→ **Same conclusion** as with mechanical instrumentation

[1] M. Guinchard, ongoing thermal contraction on cuts taken from MQXF coil.

Effect of other isotropic mechanical parameters

Parametric studies were conducted on the properties of the coil block:

- **Elastic modulus** (1 to 120 GPa),
- **Poisson ratio** (0.2 to 0,35).

} **Isotropic behavior**

Effects on the coil bending and the pole longitudinal strain in the FE models are low or negligible (less than 50 µm/m for ±20 GPa of elastic modulus change).

The model will not get closer to the measurements when changing these isotropic mechanical parameters.

→ Need to look for other model features.

Development of a code allowing the orthotropy of the coil block in ANSYS APDL

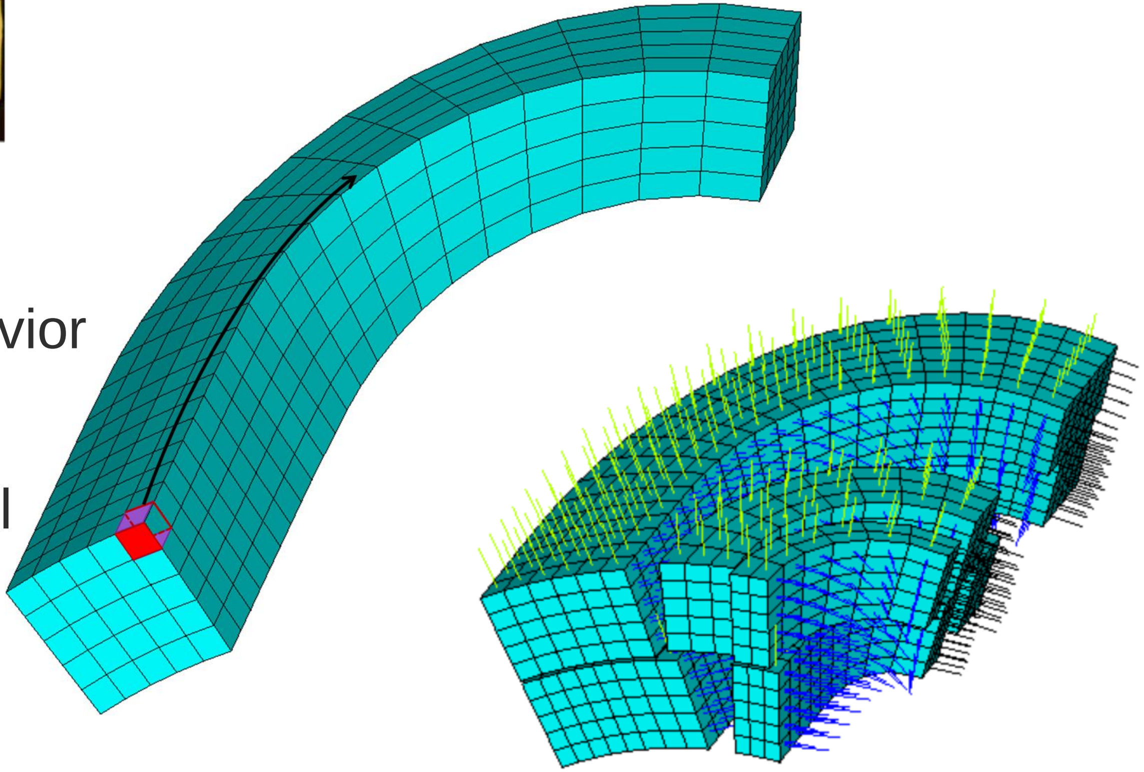
Today: the coil block is modeled as **isotropic linear elastic** material → heavy hypothesis.

Composite Nb_3Sn Rutherford cables

Starting point: Code defining **quasi-isotropic** behavior in the coil block (provided by G. Vallone, LBNL).

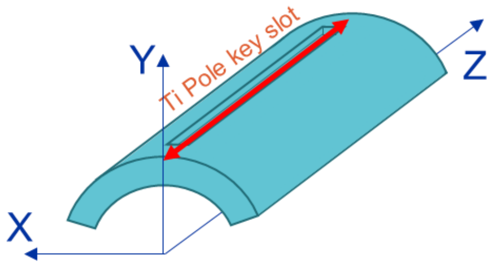
My contribution: code defining **orthotropic** material behavior in the coil block (based on each element surfaces and its neighbors).

Ongoing writing of a technical note.



Working principle (left) and result (right) of the code defining the orthotropic material behavior in the coil block

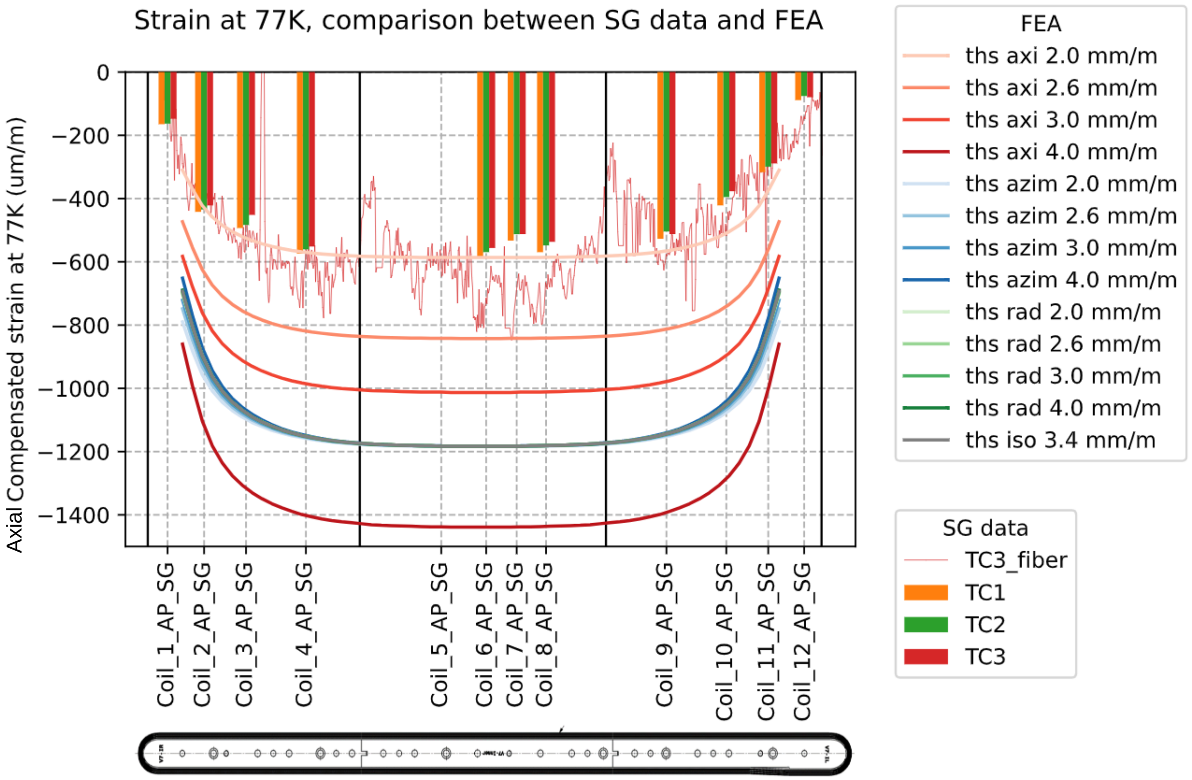
Orthotropic parametric study on CTE



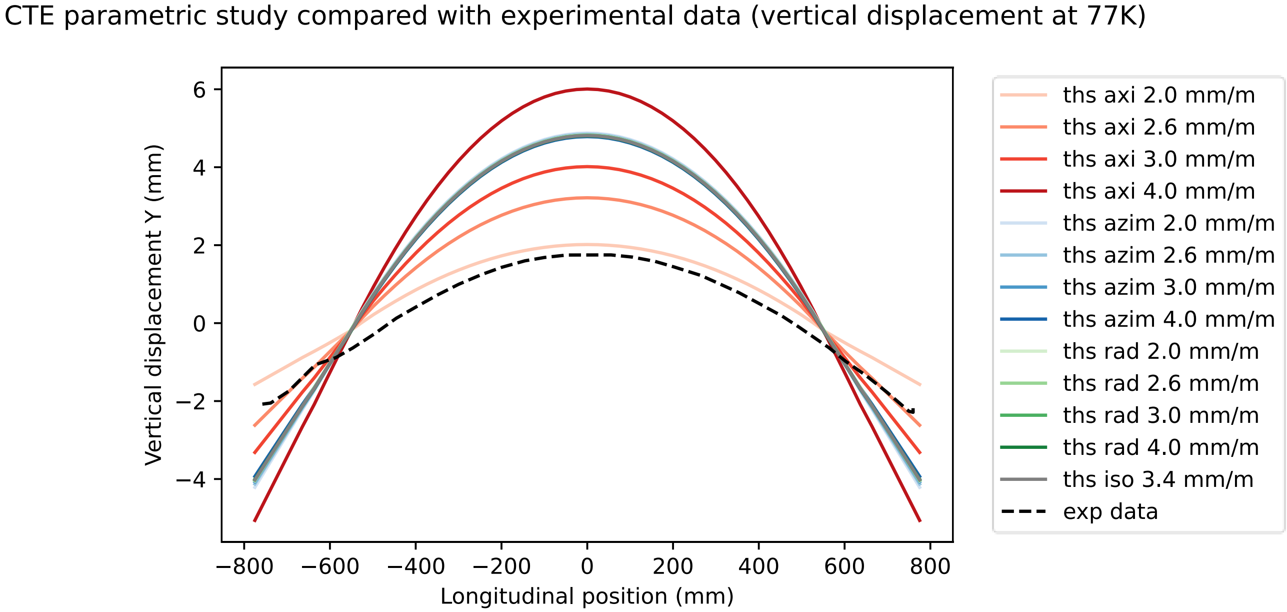
One parameter change at a time

E, ν isotropic
CTE orthotropic

Pole longitudinal strain



Coil vertical bending



In this experiment, **the dominating direction** is without surprise **the longitudinal direction.**

Effect of other orthotropic mechanical parameters

Parametric studies were conducted on the properties of the coil block:

- **Elastic modulus** (1 to 120 GPa),
- **Poisson ratio** (0.2 to 0,35),
- **Shear modulus** (1 to 40 GPa).

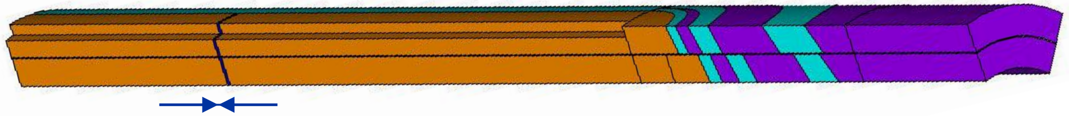
} **Orthotropic behavior**

Effects on the coil bending and the pole longitudinal strain in the FE models are low or negligible (less than 50 µm/m for ±20 GPa of elastic modulus change).

The model will not get closer to the measurements when changing these mechanical parameters.

→ **Again, need to look for other model features.**

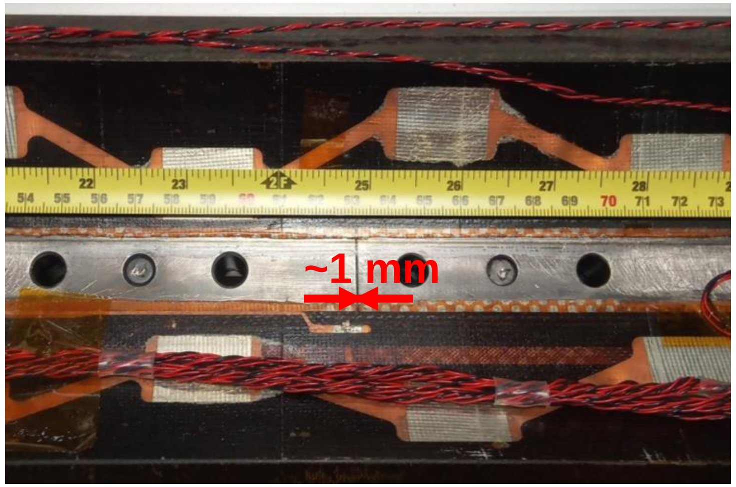
Other contribution to the mechanical behavior: Pole separations



2 mm, 4 mm and 6 mm

E, ν isotropic
CTE isotropic

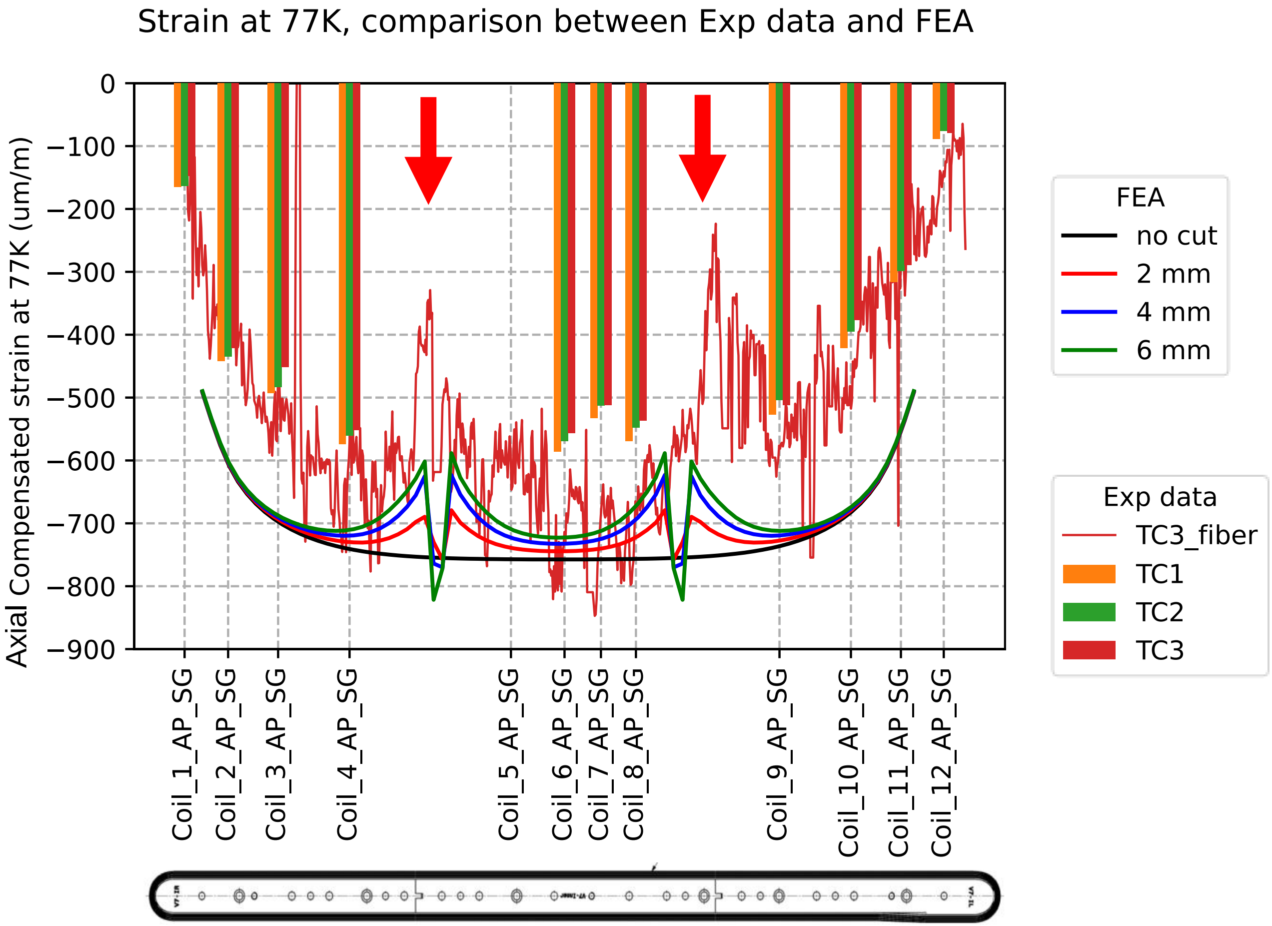
→-2.4 mm/m



Inner coil surface: separation between the poles

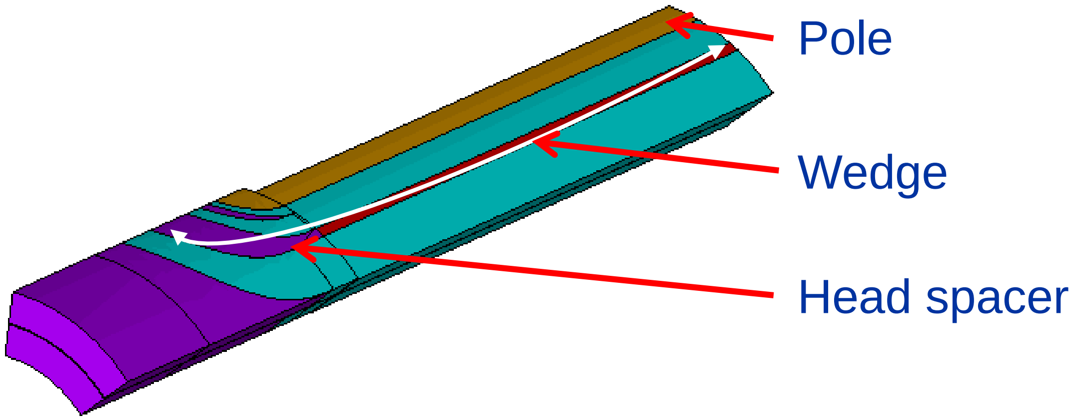
- **Pole separation effect** is visible thanks to the **optical fiber**.
- The FE model reproduce the effect, but with lower values

→ **Need to look for other model features.**



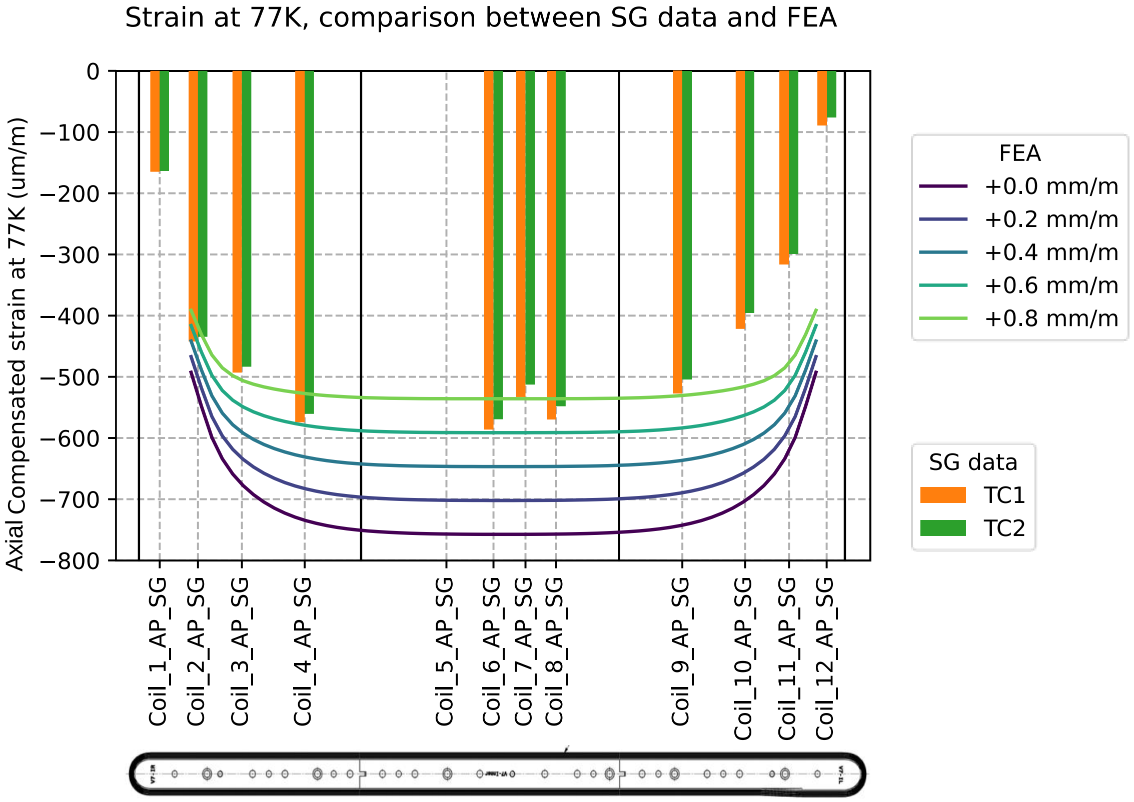
Other contribution to the mechanical behavior: Wedge CTE

E, ν isotropic
CTE isotropic → **-2.4 mm/m**



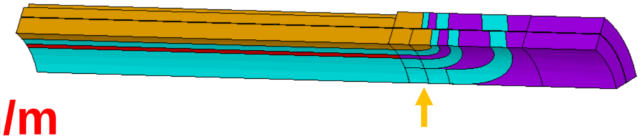
A **loop is formed by the wedge and the spacer** (everything bounded in FEA) that contributes to the pole compression.

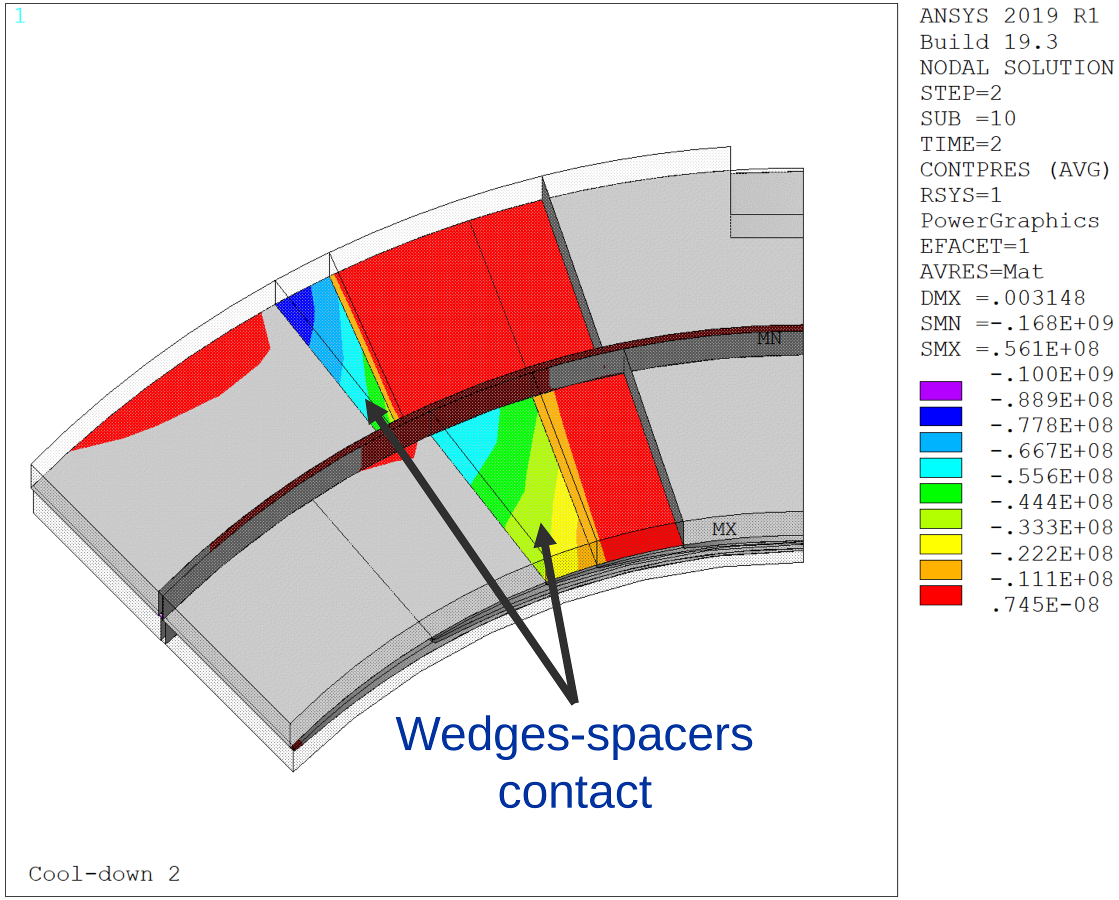
With a 5% of variation on CTE measurement of the wedge could bring the model closer to measurements by 70 µm/m.



Contact pressure wedge-spacer

E, ν isotropic
CTE isotropic → -2.4 mm/m



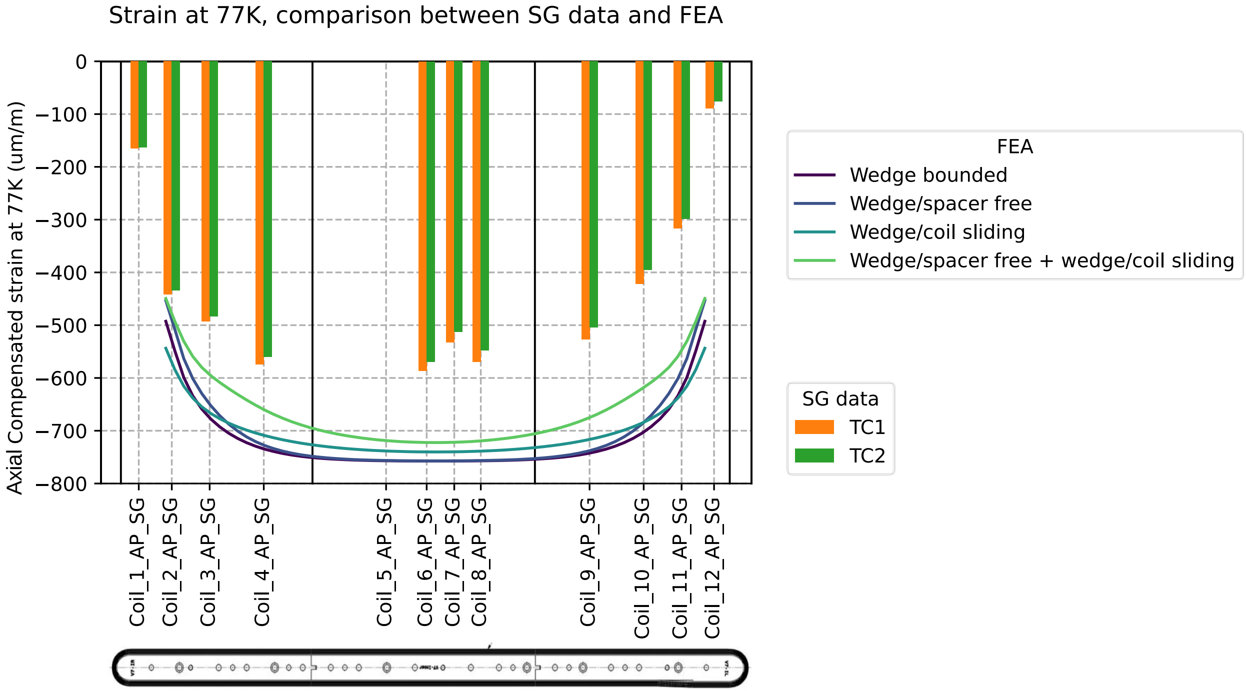


Wedges-spacers contact

Contact wedges-spacers is bounded.

Traction: between -10 to -90 MPa (high!).

Contact study between wedge and its surrounding

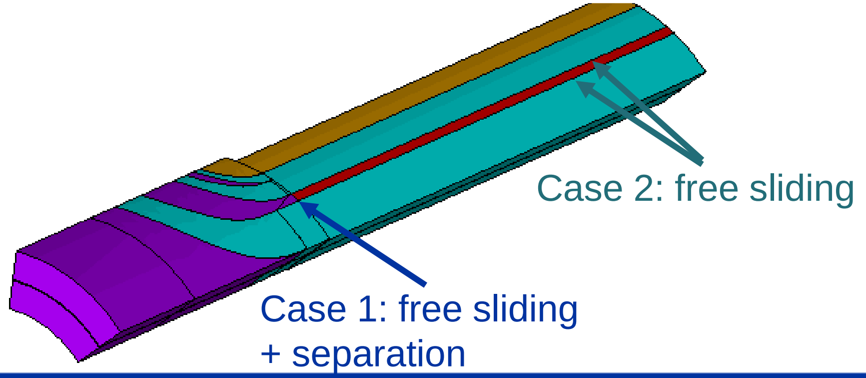


E, ν isotropic
CTE isotropic → **-2.4 mm/m**

Reference: Wedge is bounded with everything.

Three cases:

- **Case 1: Wedge/spacer contact separation and sliding allowed.**
- **Case 2: Wedge/coil contact sliding allowed.**
- **Case 3: case 1 + case 2.**



Does not change a lot the contribution of the wedge (true also for the bending).

Partial conclusion

- An instrumented **self standing coil** was **cooled down at 77 K multiple times**,
- **Comparison** between **experimental data and FE models** suggests that the **coil block CTE should be changed from -3.45 mm/m at 77 K to -2.5 mm/m at 77 K**.
- This change **improves significantly the model**, yet some **differences remains** (150 µm/m of pole longitudinal strain).
- Other mechanical parameters were studied (Elastic modulus, Poisson ratio, shear modulus), but **only the wedge CTE** could **partially explains the small remaining differences**.
- Implementation of orthotropy confirmed that **longitudinal CTE is the driving parameter**, and **in the case of a self standing coil, isotropy and orthotropy lead to very close FE results**.
- **Further investigation** should be led to understand the remaining differences, for instance **contact definition** elsewhere in the coil that were supposed bounded (work was started, not completed by lack of time).

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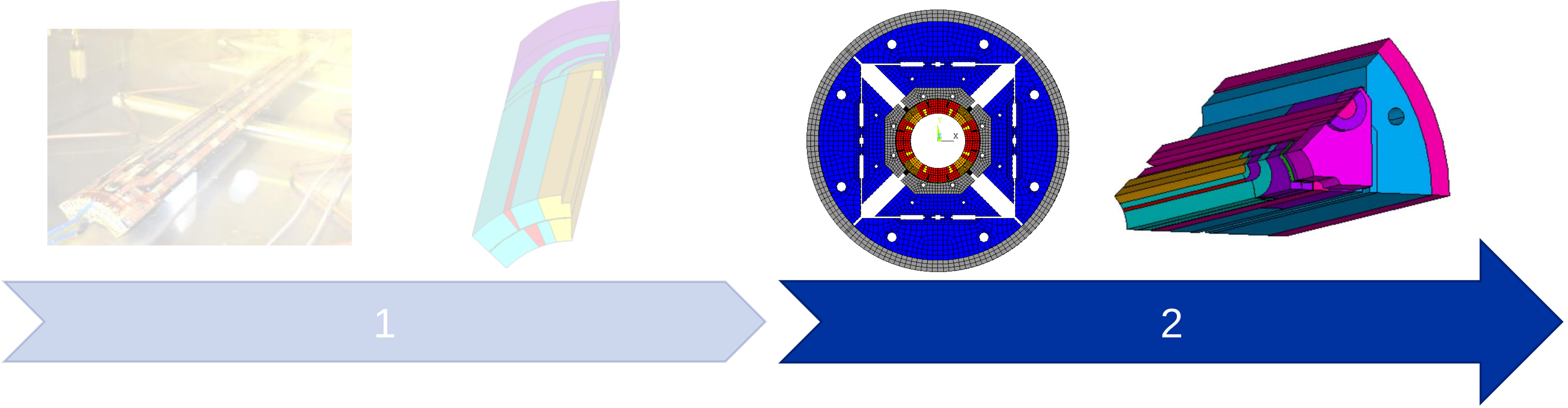
What level of complexity do we need in our FE models?



Thermomechanical study of a self standing coil: correlation between models and experiment

Method development for an orthotropic coil parametric study in a full magnet structure

Method development for an orthotropic parametric study in a magnet structure

Problem: What is the impact of the coil material orthotropy implementation in the full magnet FE results? → Development of numerical tools.

A **parametric study** on the coil block material definition (one parameter at a time):

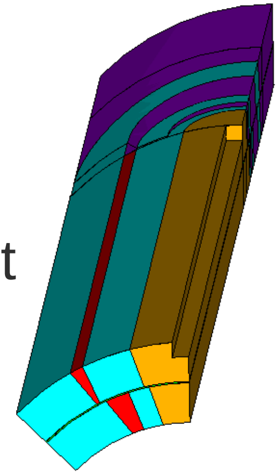
- **Elastic modulus,**
- **Shear modulus,**
- **Poison ratio,**
- **CTE.**

in the **three directions**

→ Very costly on today's FE models (~10 hours each!!)

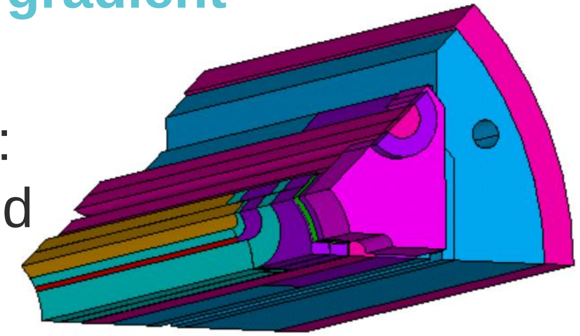
Self standing MQXFS coil in an « infinitely » rigid structure

- Use **additional boundary conditions** to simulate support structure,
- Fast computing (10 minutes).



Full magnet model: High gradient Quadrupole (HQ):

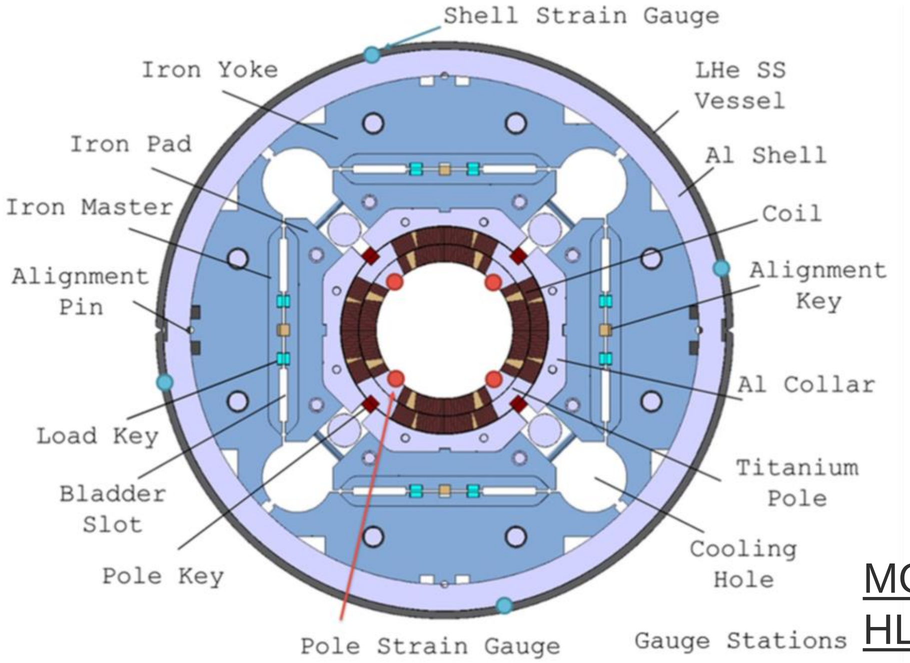
- **True magnet behavior**: FE model was compared to the real magnet,
- Fast computing (15 minutes).



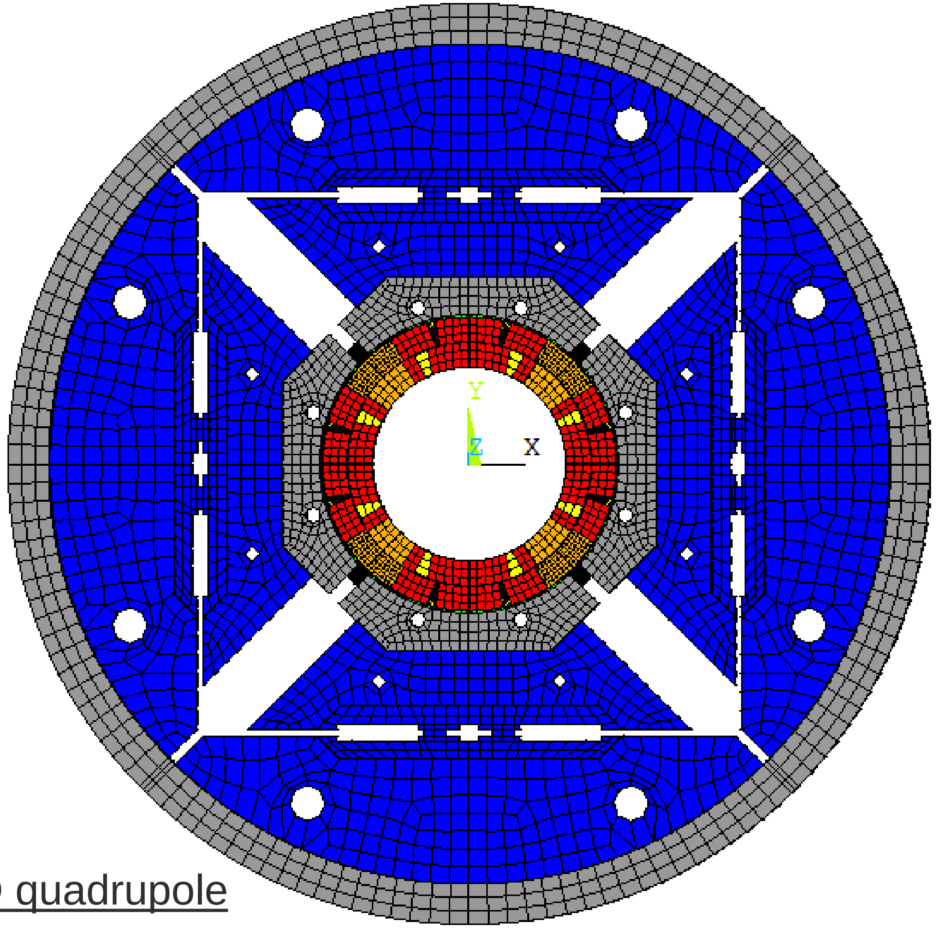
Reminder: HQ finite element model – bladders and keys support structure 1/3

Goal: counterbalance the Lorenz forces by **preloading the coils** to avoid their displacements that would limit the magnet performances.

Magnet at room temperature, without preload.



MQXF quadrupole for HL-LHC

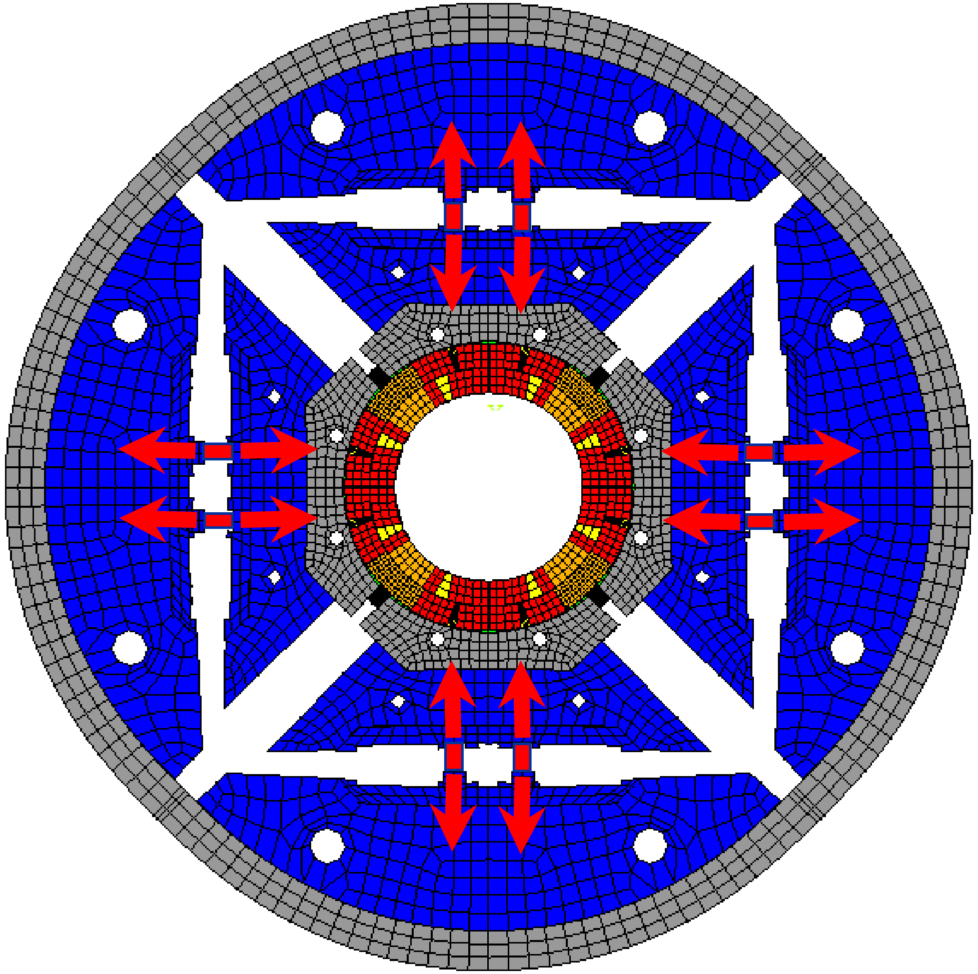


HQ quadrupole

Reminder: HQ finite element model – bladders and keys support structure 2/3

Goal: counterbalance the Lorenz forces by **preloading the coils** to avoid their displacements that would limit the magnet performances.

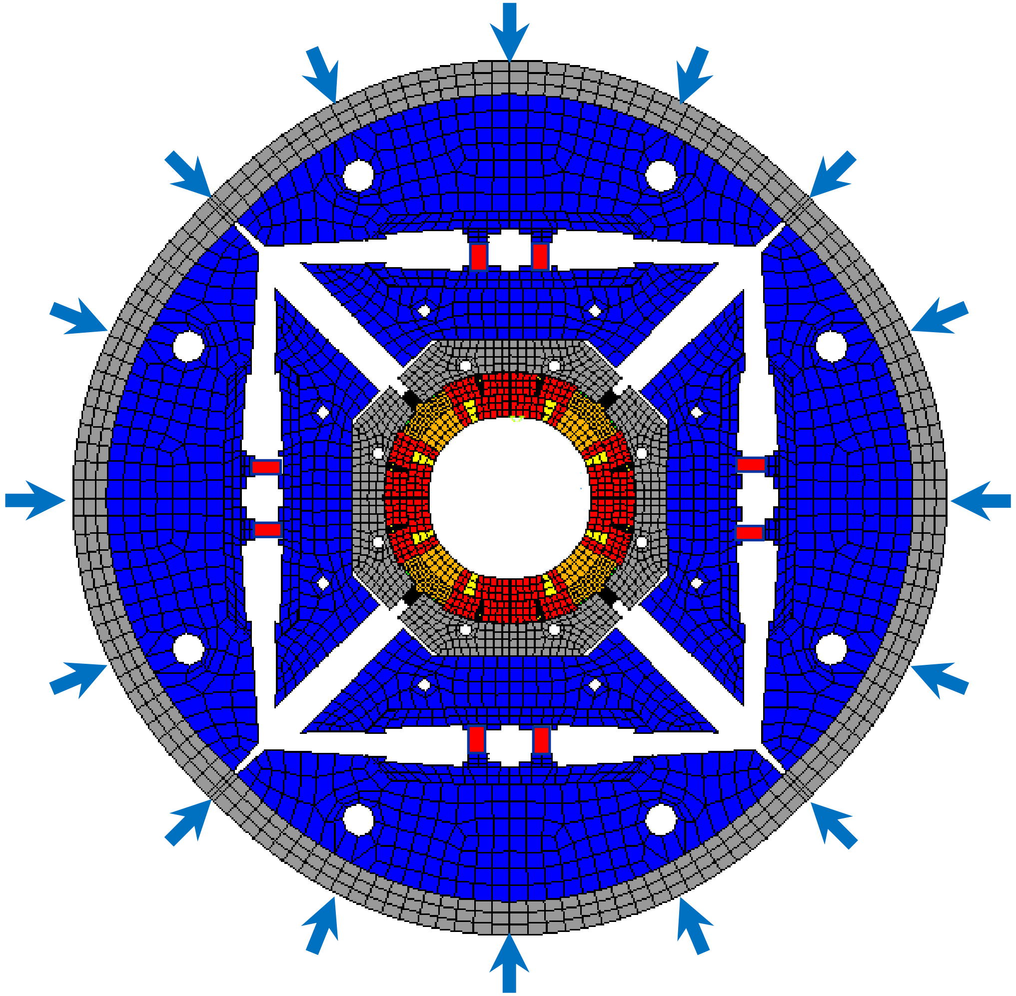
Inserting the keys (metal shims) **to maintain the preload**, at room temperature.



Reminder: HQ finite element model – bladders and keys support structure 3/3

Goal: counterbalance the Lorenz forces by **preloading the coils** to avoid their displacements that would limit the magnet performances.

Magnet at **cryogenic temperature** (~2 K), to provide an **additional preload** with the differential thermal contraction.

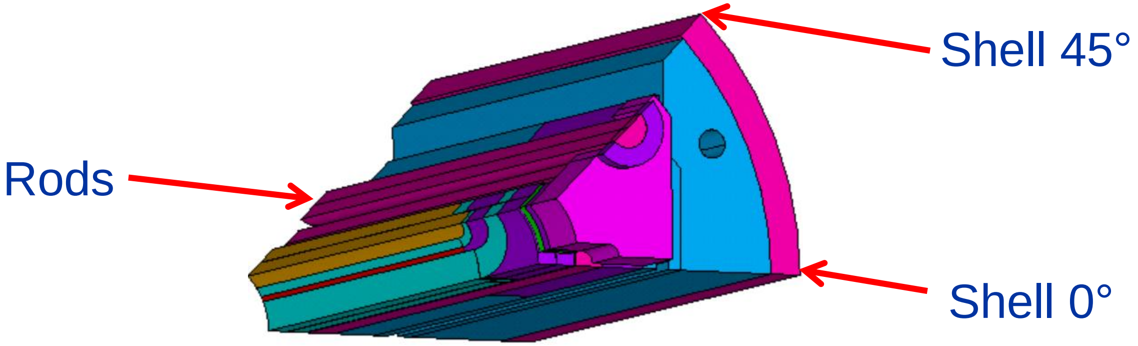


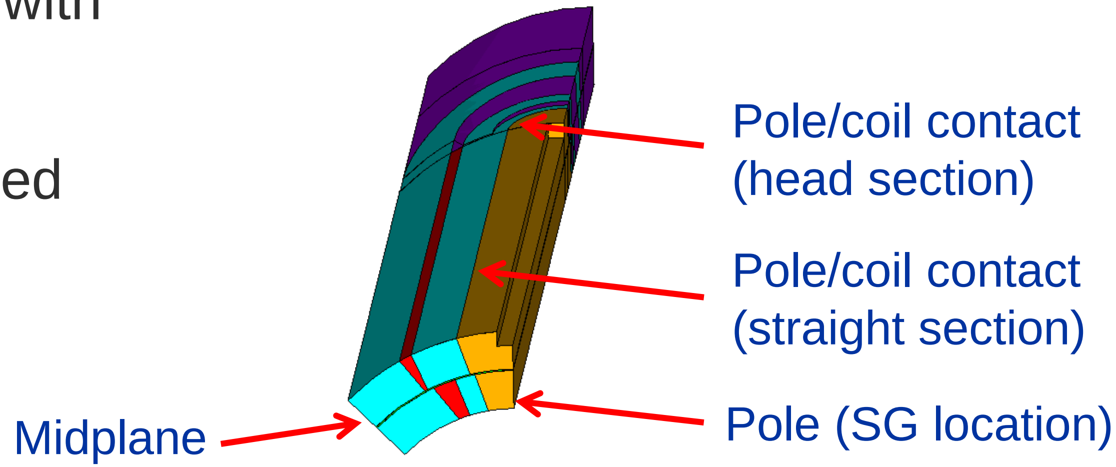
Defining the zones of interest

It is **not possible** to completely compare element by element the **orthotropic and isotropic** models.

So, I extracted from the FE software stresses and displacements in different locations useful to monitor the magnet behavior:

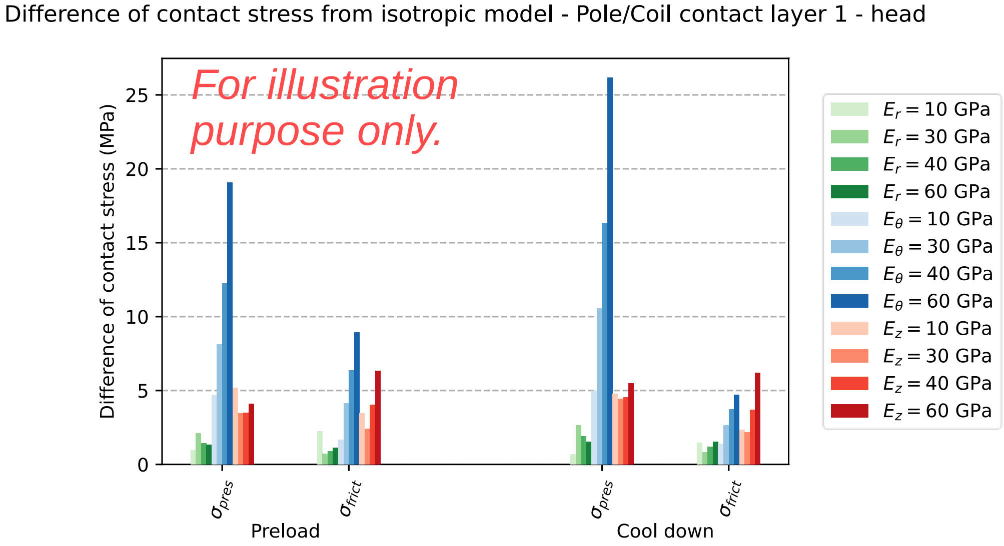
- On the **mechanical structure** (rods and shell) where the magnet is usually instrumented with strain gauges,
- On the **coils** where it is usually instrumented and target zones for “healthy” mechanical behavior.





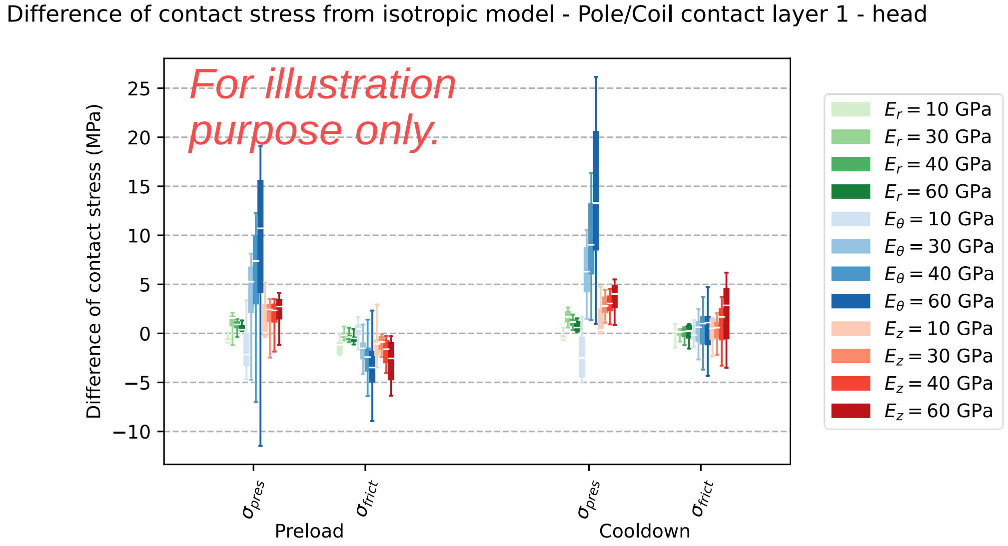
Development of an analysis method through numerical tools 1/2

Bar plots of the maximum of the absolute difference between orthotropic and isotropic



- Easy to read.
- Problem if the difference is only local (numerical singularity)

Box plots of the differences between orthotropic and isotropic

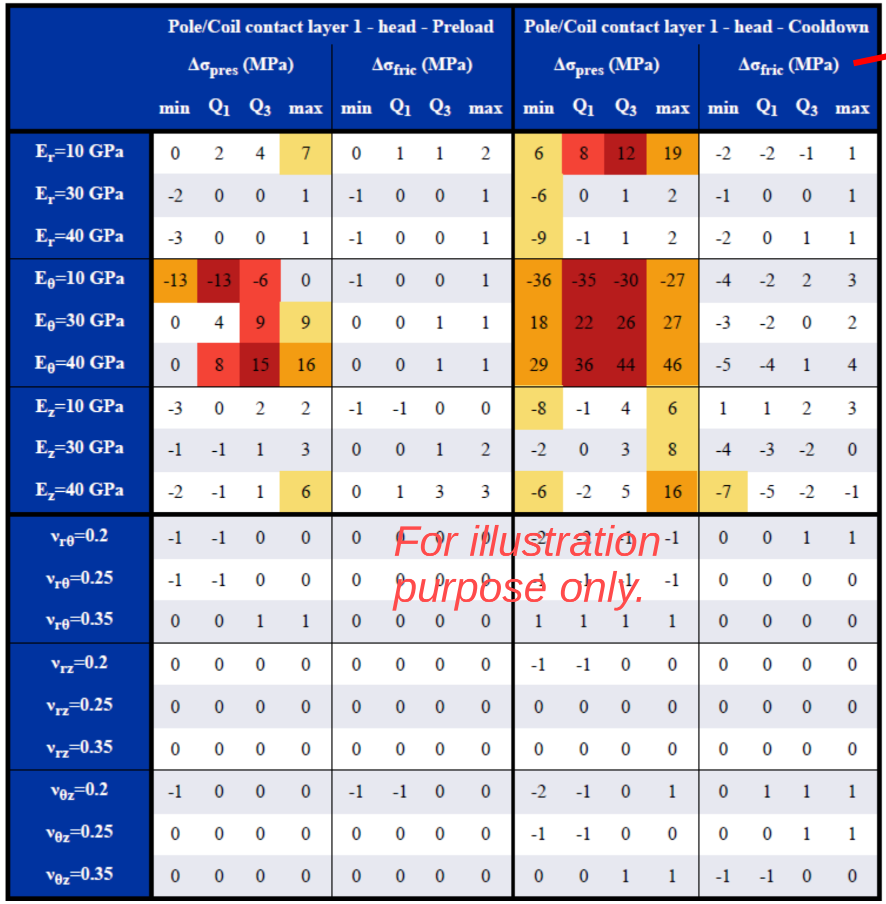


- More complex, and sometimes unreadable
- Give more information on the spread of the differences.

For BOTH: it makes a lot of graphs to analyze (~200 plots!!)

Development of an analysis method through numerical tools 2/2

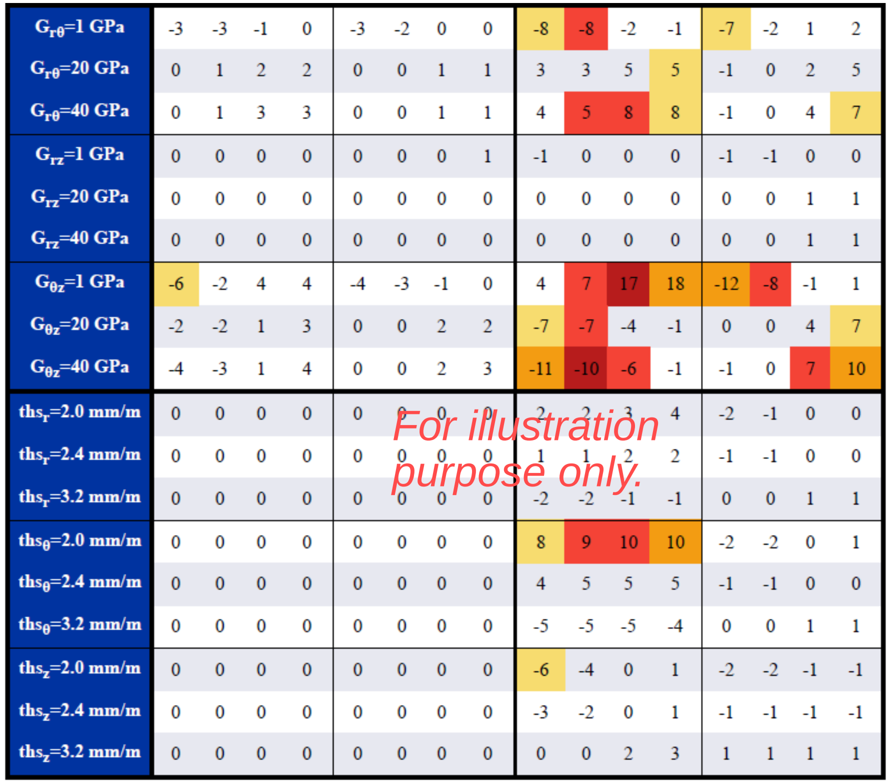
Summarizing tens of box plots in global tables per zone of interest.



	Pole/Coil contact layer 1 - head - Preload								Pole/Coil contact layer 1 - head - Cooldown							
	$\Delta\sigma_{pres}$ (MPa)				$\Delta\sigma_{fric}$ (MPa)				$\Delta\sigma_{pres}$ (MPa)				$\Delta\sigma_{fric}$ (MPa)			
	min	Q_1	Q_3	max	min	Q_1	Q_3	max	min	Q_1	Q_3	max	min	Q_1	Q_3	max
E_r=10 GPa	0	2	4	7	0	1	1	2	6	8	12	19	-2	-2	-1	1
E_r=30 GPa	-2	0	0	1	-1	0	0	1	-6	0	1	2	-1	0	0	1
E_r=40 GPa	-3	0	0	1	-1	0	0	1	-9	-1	1	2	-2	0	1	1
E_θ=10 GPa	-13	-13	-6	0	-1	0	0	1	-36	-35	-30	-27	-4	-2	2	3
E_θ=30 GPa	0	4	9	9	0	0	1	1	18	22	26	27	-3	-2	0	2
E_θ=40 GPa	0	8	15	16	0	0	1	1	29	36	44	46	-5	-4	1	4
E_z=10 GPa	-3	0	2	2	-1	-1	0	0	-8	-1	4	6	1	1	2	3
E_z=30 GPa	-1	-1	1	3	0	0	1	2	-2	0	3	8	-4	-3	-2	0
E_z=40 GPa	-2	-1	1	6	0	1	3	3	-6	-2	5	16	-7	-5	-2	-1
$\nu_{r\theta}$=0.2	-1	-1	0	0	0	0	0	0	-2	-2	-1	-1	0	0	1	1
$\nu_{r\theta}$=0.25	-1	-1	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0
$\nu_{r\theta}$=0.35	0	0	1	1	0	0	0	0	1	1	1	1	0	0	0	0
ν_{rz}=0.2	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0
ν_{rz}=0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ν_{rz}=0.35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\nu_{\theta z}$=0.2	-1	0	0	0	-1	-1	0	0	-2	-1	0	1	0	1	1	1
$\nu_{\theta z}$=0.25	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	1	1
$\nu_{\theta z}$=0.35	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0

For illustration purpose only.

$\Delta\sigma_{friction}$ with respect to the reference case for the various zones of interest and load steps



$G_{r\theta}$=1 GPa	-3	-3	-1	0	-3	-2	0	0	-8	-8	-2	-1	-7	-2	1	2
$G_{r\theta}$=20 GPa	0	1	2	2	0	0	1	1	3	3	5	5	-1	0	2	5
$G_{r\theta}$=40 GPa	0	1	3	3	0	0	1	1	4	5	8	8	-1	0	4	7
G_{rz}=1 GPa	0	0	0	0	0	0	0	1	-1	0	0	0	-1	-1	0	0
G_{rz}=20 GPa	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
G_{rz}=40 GPa	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$G_{\theta z}$=1 GPa	-6	-2	4	4	-4	-3	-1	0	4	7	17	18	-12	-8	-1	1
$G_{\theta z}$=20 GPa	-2	-2	1	3	0	0	2	2	-7	-7	-4	-1	0	0	4	7
$G_{\theta z}$=40 GPa	-4	-3	1	4	0	0	2	3	-11	-10	-6	-1	-1	0	7	10
ths_r=2.0 mm/m	0	0	0	0	0	0	0	0	2	2	3	4	-2	-1	0	0
ths_r=2.4 mm/m	0	0	0	0	0	0	0	0	1	1	2	2	-1	-1	0	0
ths_r=3.2 mm/m	0	0	0	0	0	0	0	0	-2	-2	-1	-1	0	0	1	1
ths_θ=2.0 mm/m	0	0	0	0	0	0	0	0	8	9	10	10	-2	-2	0	1
ths_θ=2.4 mm/m	0	0	0	0	0	0	0	0	4	5	5	5	-1	-1	0	0
ths_θ=3.2 mm/m	0	0	0	0	0	0	0	0	-5	-5	-5	-4	0	0	1	1
ths_z=2.0 mm/m	0	0	0	0	0	0	0	0	-6	-4	0	1	-2	-2	-1	-1
ths_z=2.4 mm/m	0	0	0	0	0	0	0	0	-3	-2	0	1	-1	-1	-1	-1
ths_z=3.2 mm/m	0	0	0	0	0	0	0	0	0	0	2	3	1	1	1	1

For illustration purpose only.

- Highly visual
- Give a large overview,
- Too large for presentations, papers, etc.

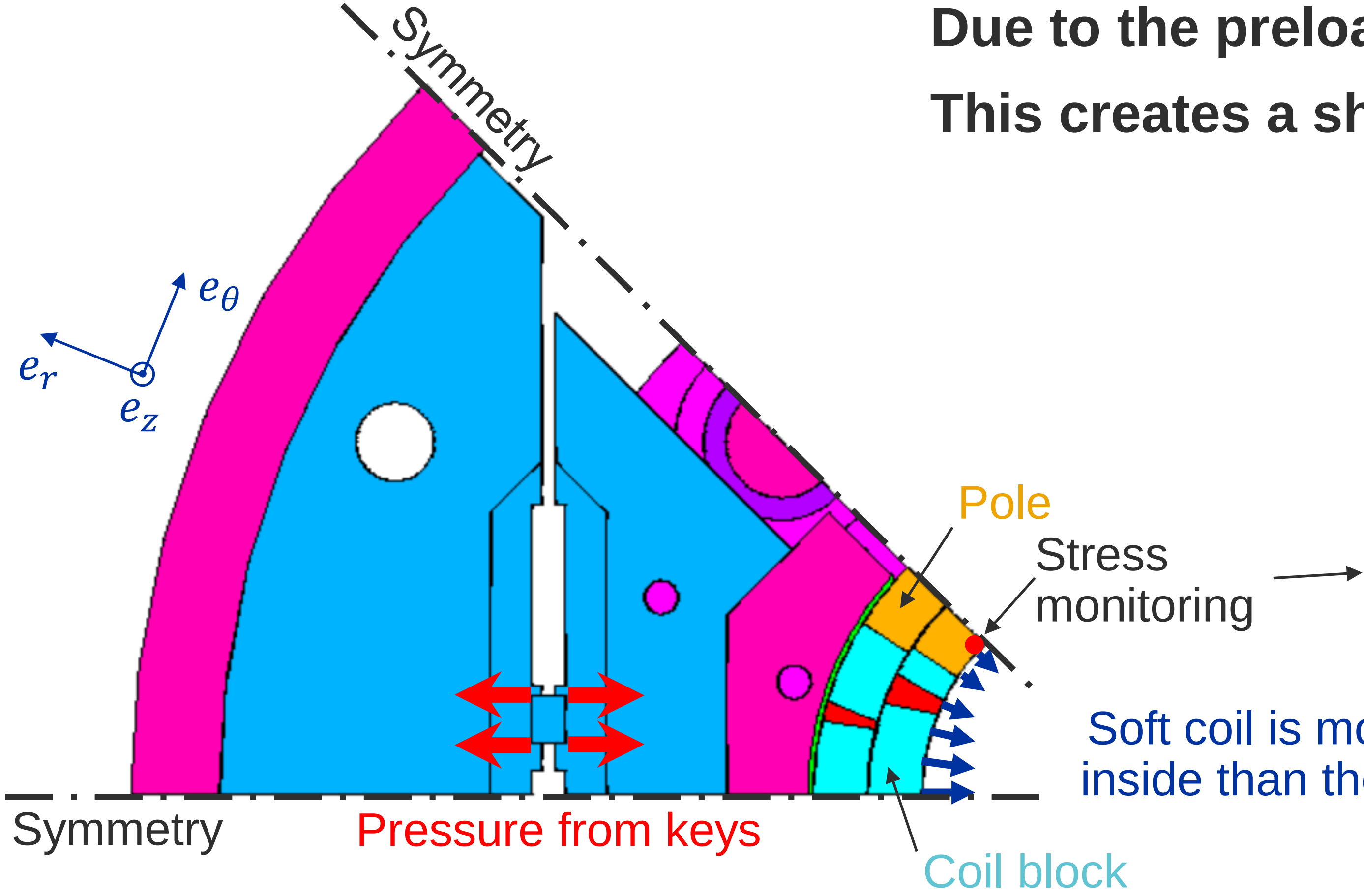
→ Tables are great tools for the analysis.

→ Outcome of the analysis will be presented with boxplots.

Illustration of the approach, unexpected orthotropic parameter effects $G_{r\theta}$ 1/2

Due to the preload, the coil is bending.

This creates a shear stress $\sigma_{r\theta}$ in the coil block.



Soft coil is more pushed inside than the hard pole

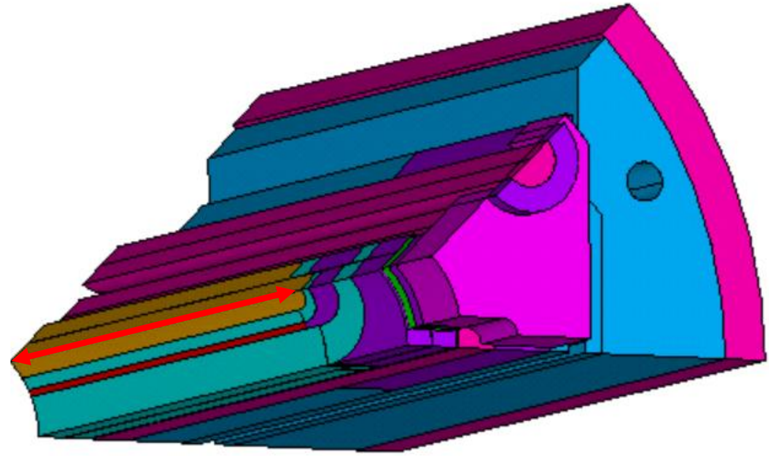


Illustration of the approach, unexpected orthotropic parameter effects $G_{r\theta}$ 2/2

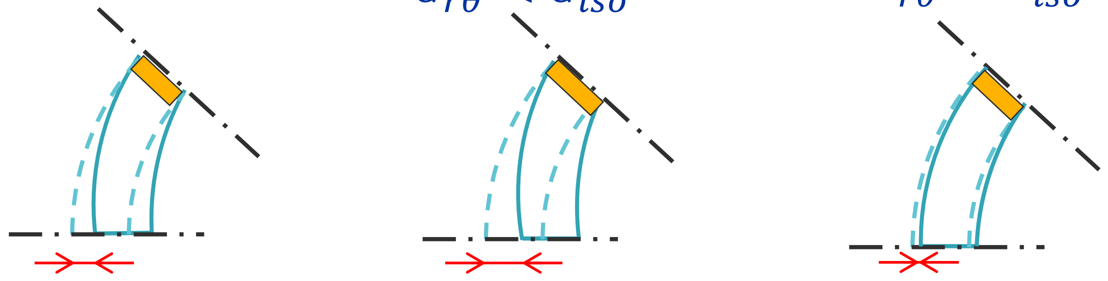
$G_{iso} = 7.7\ GPa$

Softer coil

$G_{r\theta} < G_{iso}$

Stiffer coil

$G_{r\theta} > G_{iso}$



Medium bending

Larger bending

→ coil block absorb more energy

→ decrease the pole compression

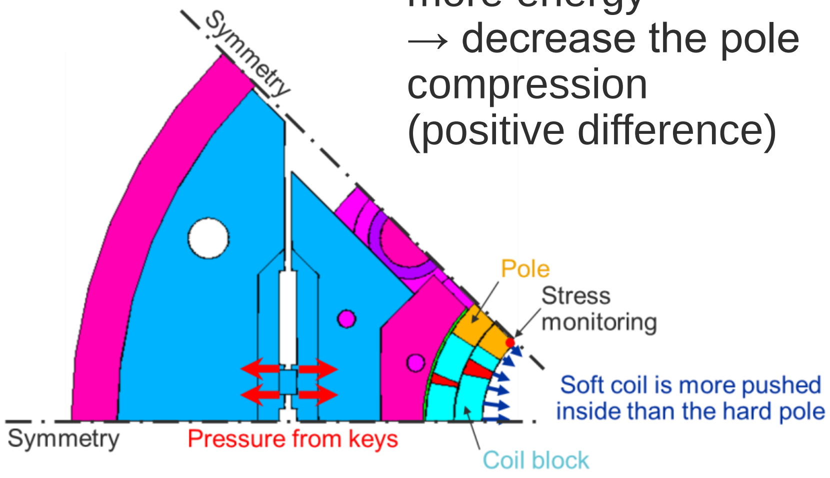
(positive difference)

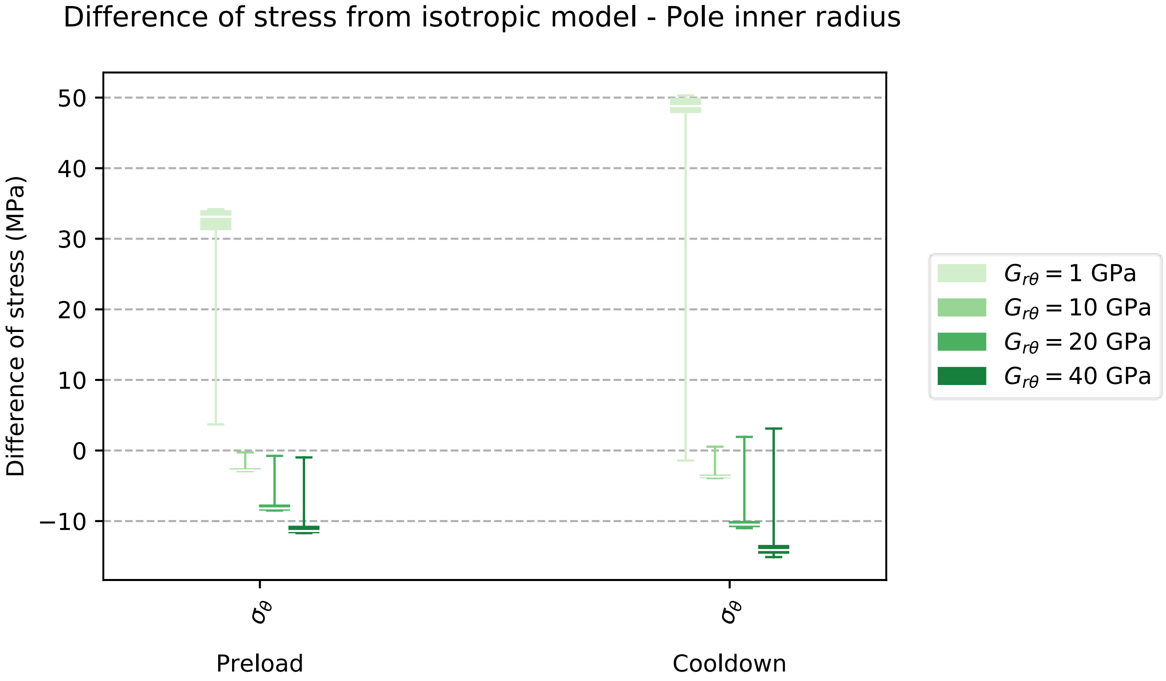
Smaller bending

→ coil block absorb less energy

→ increase the pole compression

(negative difference)





Non negligible impact of $G_{r\theta}$ (more than 10 MPa)

Should **characterization tests** be developed for $G_{r\theta}$?

Objective of the study will be to identify the relevant parameters that need to be characterized and implemented.

Conclusion

- Proposition of a **new CTE value for the coil block**: -2.5 mm/m at 77 K.
- **Development of a code** implementing **orthotropy in the coil block for ANSYS APDL**.
- **Development of an analysis method** to study the **effect of orthotropy in magnet and to identify the mechanical parameter to be characterized**:
  - ➢ Automatic FE computations,
  - ➢ Automatic post processing drawing plots and tables.
- **Numerical tools are available to go further**.

Next steps: interdependence of material parameters

Study the **interdependence of each parameters** on others, considering **12 variables** (3 elastic modulus, 3 Poisson ratio, 3 shear modulus and 3 CTE)

- **3 values per parameter**

→ 3^{12} ~ 530 000 cases (~3690 days)

- **2 values per parameter**

→ 2^{12} ~ 4 000 cases (~28 days)

Full Factorial design

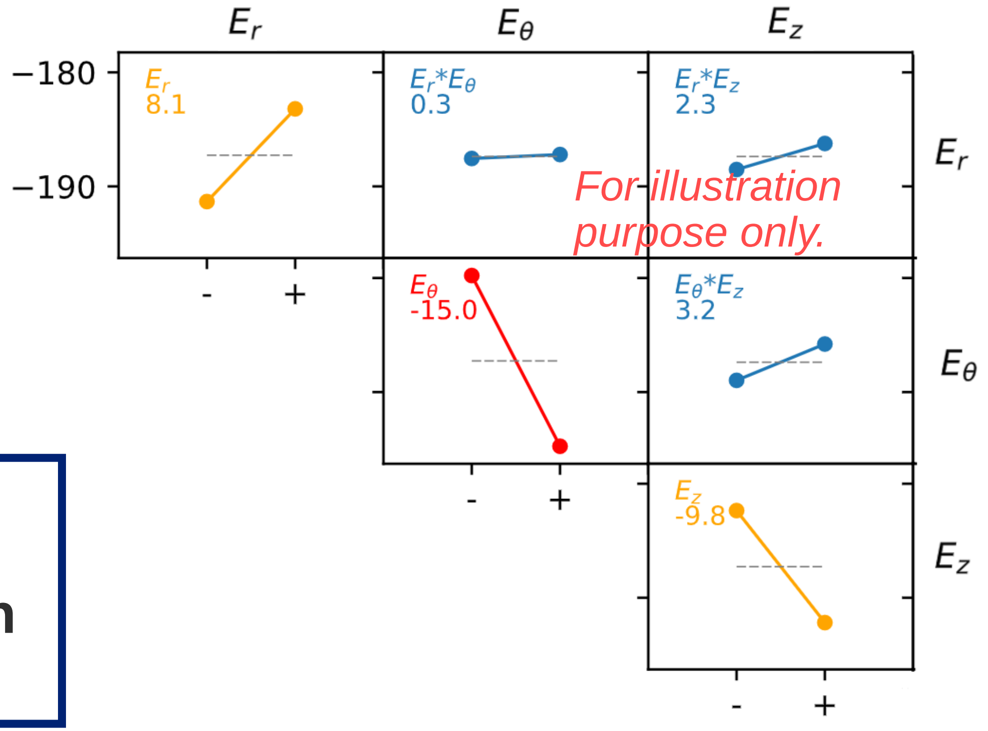
- **2 values per parameter**

→ 2^{12-4} = 256 cases (~1.5 days)

Fractional Factorial design

Interaction effect matrix

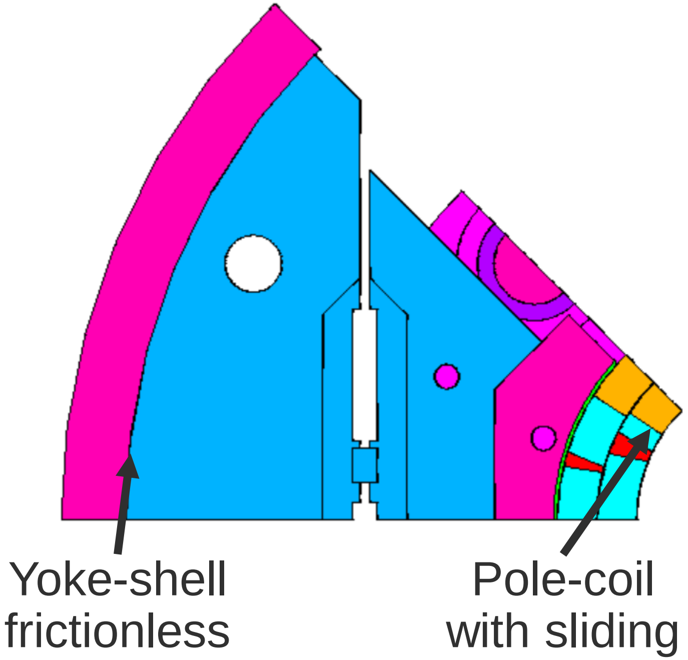
σ_θ of the pole inner radius during cool down



Next steps:

Study the **effect of friction** (same parametric study):

- **Yoke-shell** contact **frictionless,**
- **Pole-coil** contact **sliding allowed,**
- **Full model frictionless.**



Is the **sensitivity** to material properties the same for **different magnet configurations**? Dipoles vs quadrupoles? Block type vs collared structures?

To be continued…

Thank you for your attention

Special thanks to:

- *Hélène Felice,*
- *Emma Gautheron, Gianluca Vernassa, Przemyslaw Wachal,*
- *The complete SMT section (without any exception!!)*
- *Jose Ferradás Troitiño, Susana Izquierdo Bermudez,*
- *EN-MME team, Cryolab team, SM18 team, BE-GM team and many others.*

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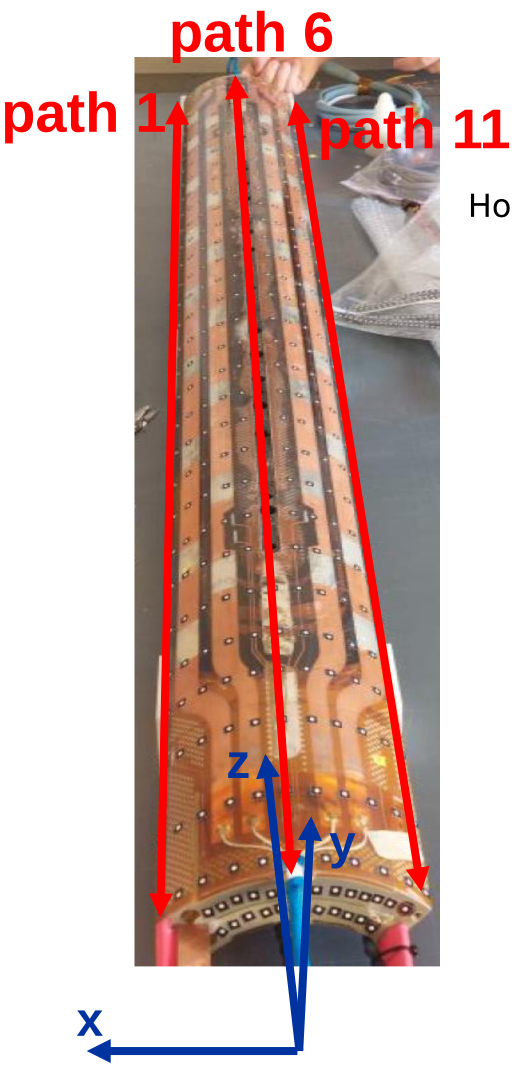
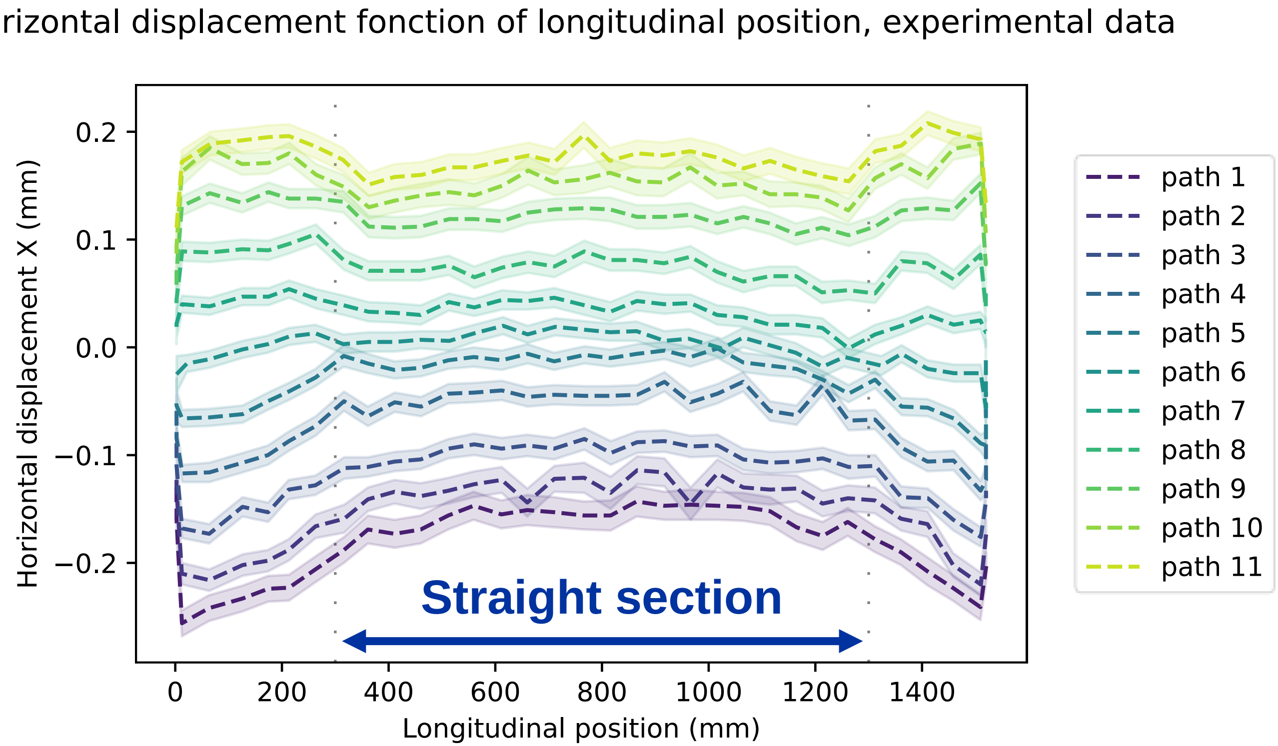
Analysis of photogrammetry data, X displacement

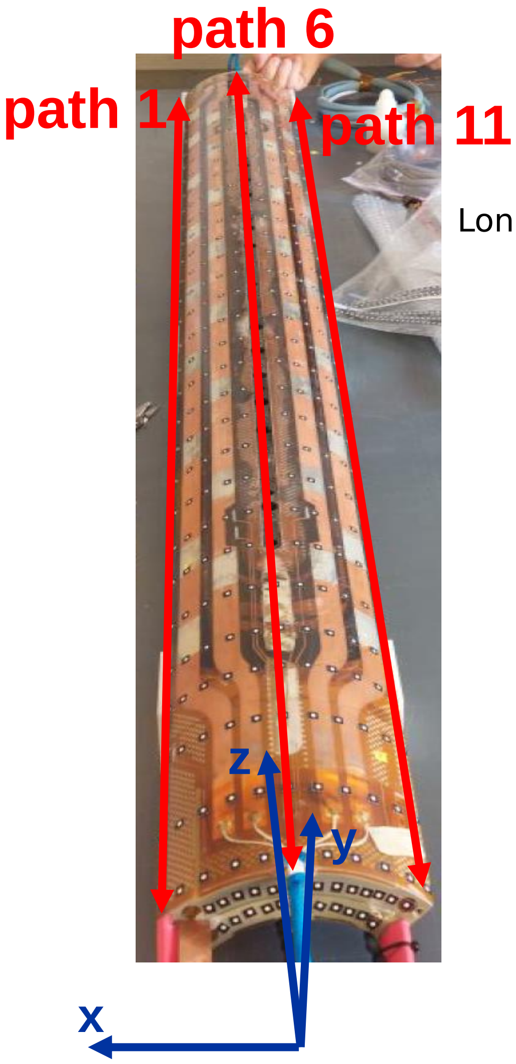


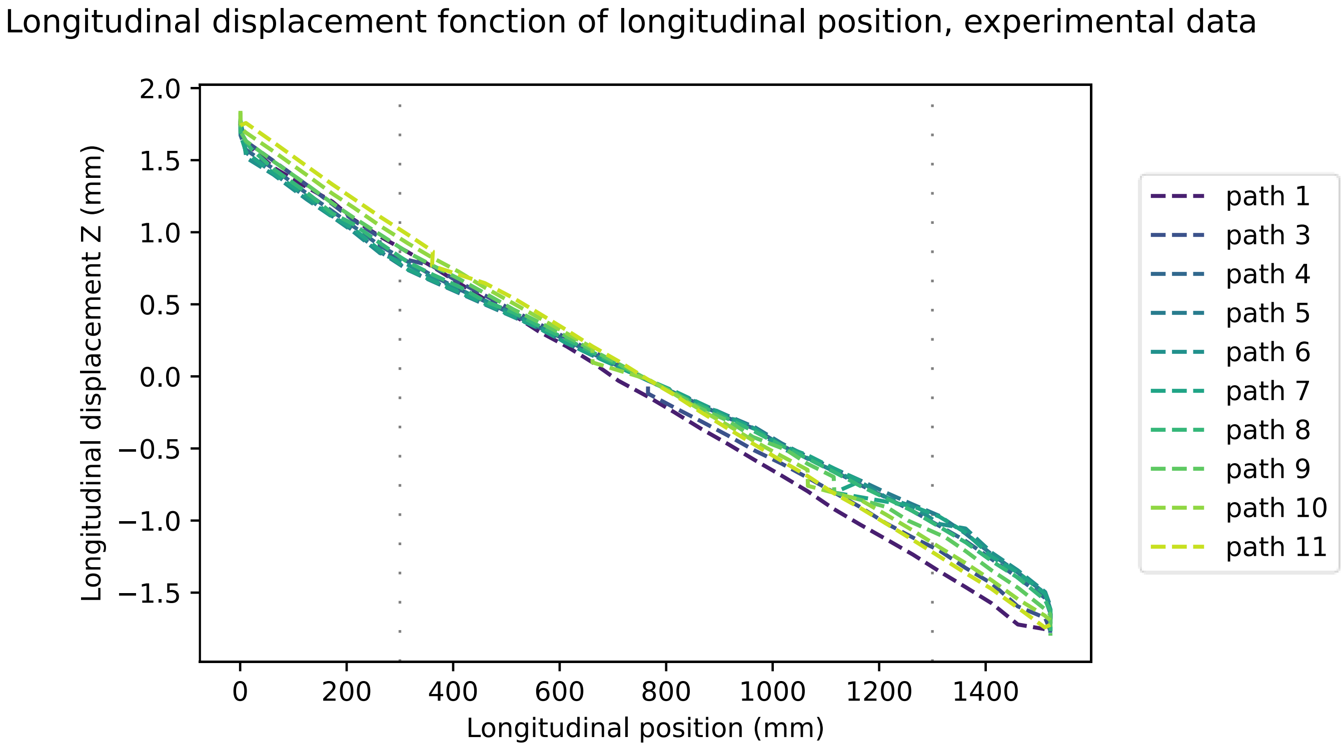


Observations:

- **See effect of coil heads vs straight section.**
- **Asymmetry between path 1 and 11 (bending around Y)**

Analysis of photogrammetry data, Z displacement

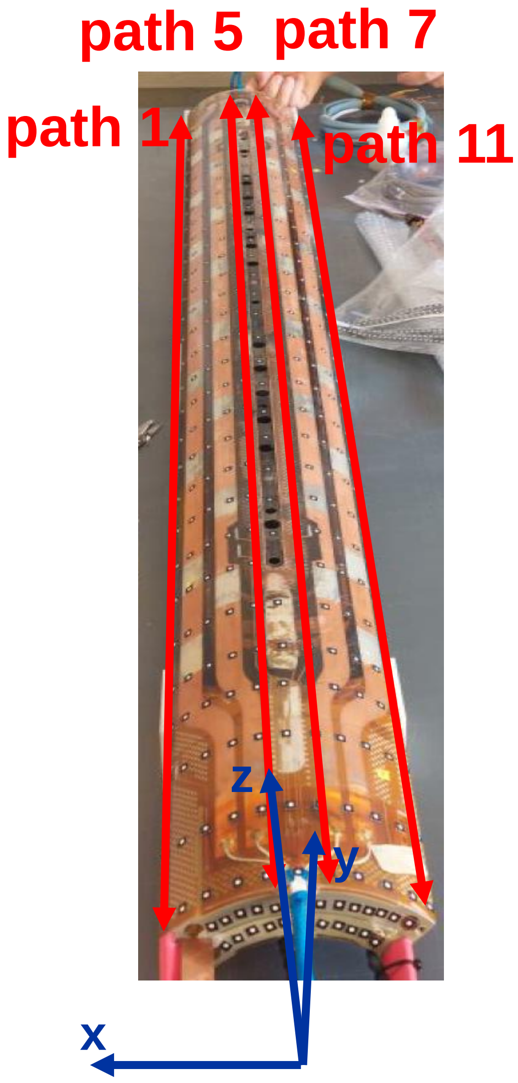




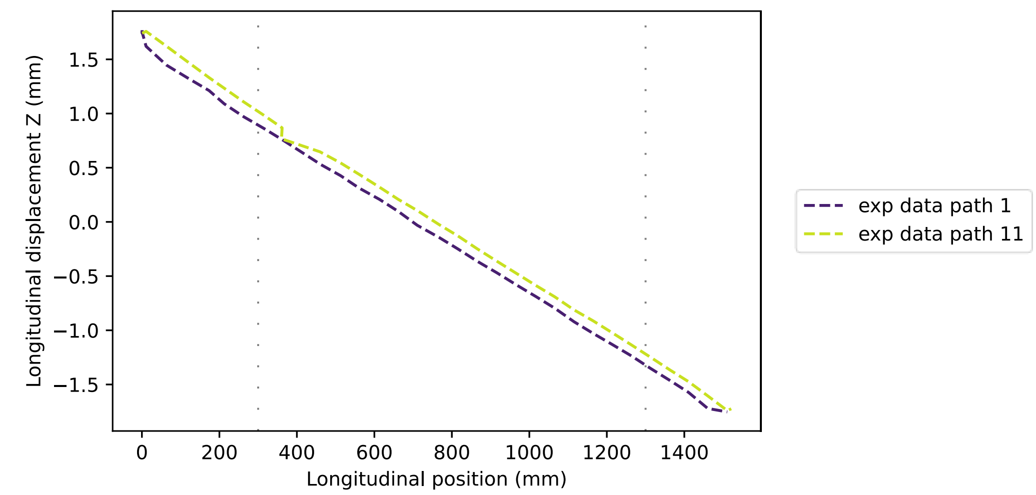
Observations:

- **Dispersion is observed between paths.**
- **Uncertainty is negligible.**

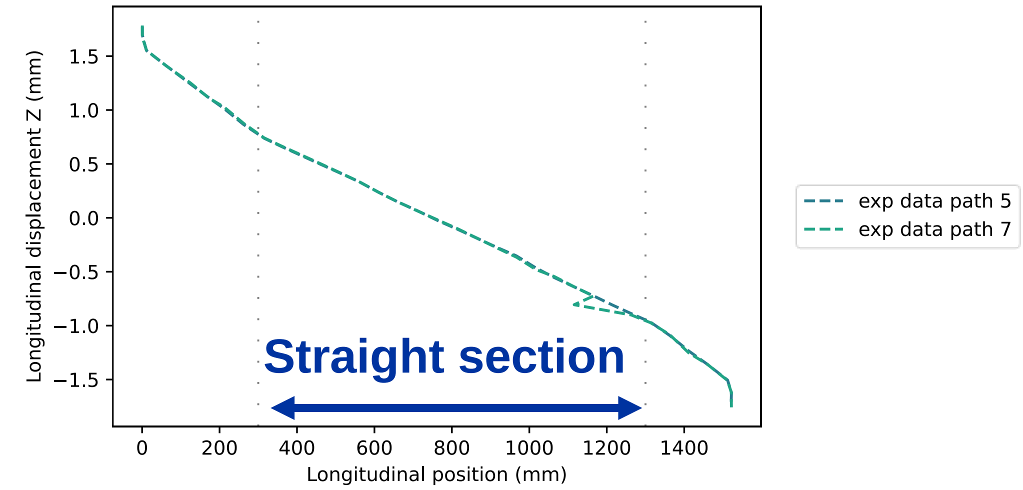
Analysis of photogrammetry data, Z displacement



Longitudinal displacement fonction of longitudinal position, experimental data



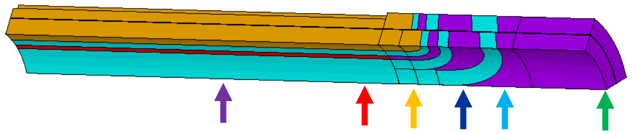
Longitudinal displacement fonction of longitudinal position, experimental data



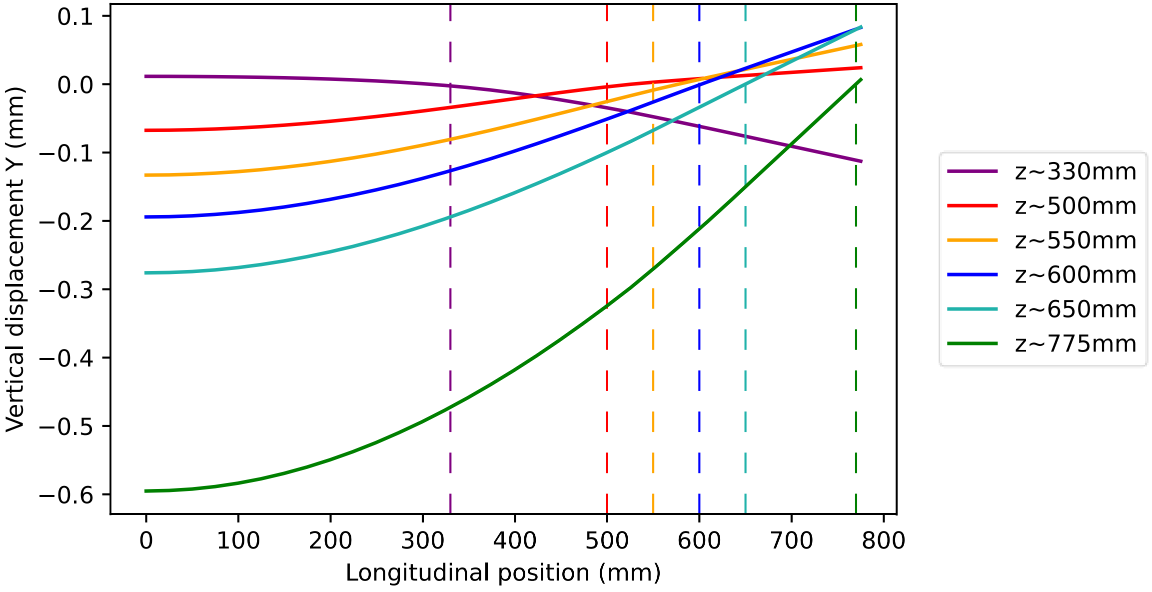
- **Same slopes for symmetric path (1-11, 2-10,…)**
- **See effect of heads vs straight section (path 5 and 7)**

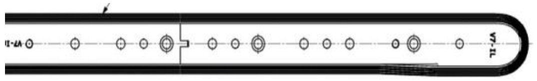
Influence of support position on photogrammetry measurement

E isotropic
CTE isotropic

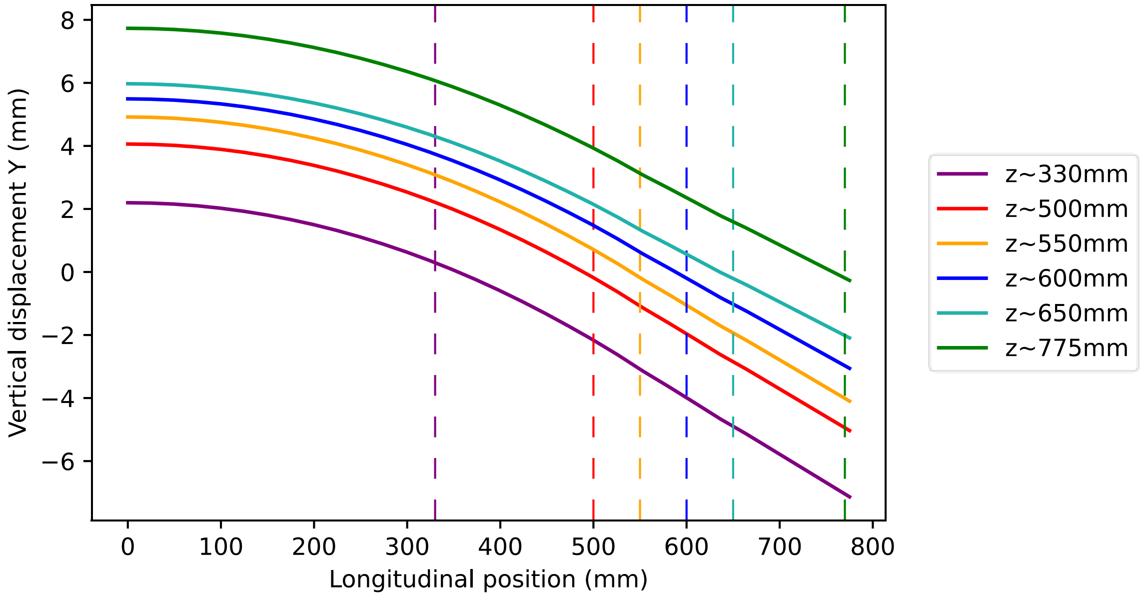


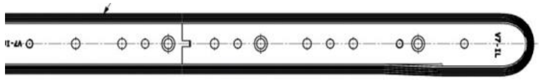
Room temperature, visible but negligible


Vertical displacement fonction of longitudinal position, gravity alone



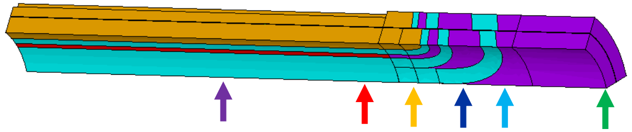
77K, negligible (just offset)


Vertical displacement fonction of longitudinal position, 77K



Influence of support position on SG measurement

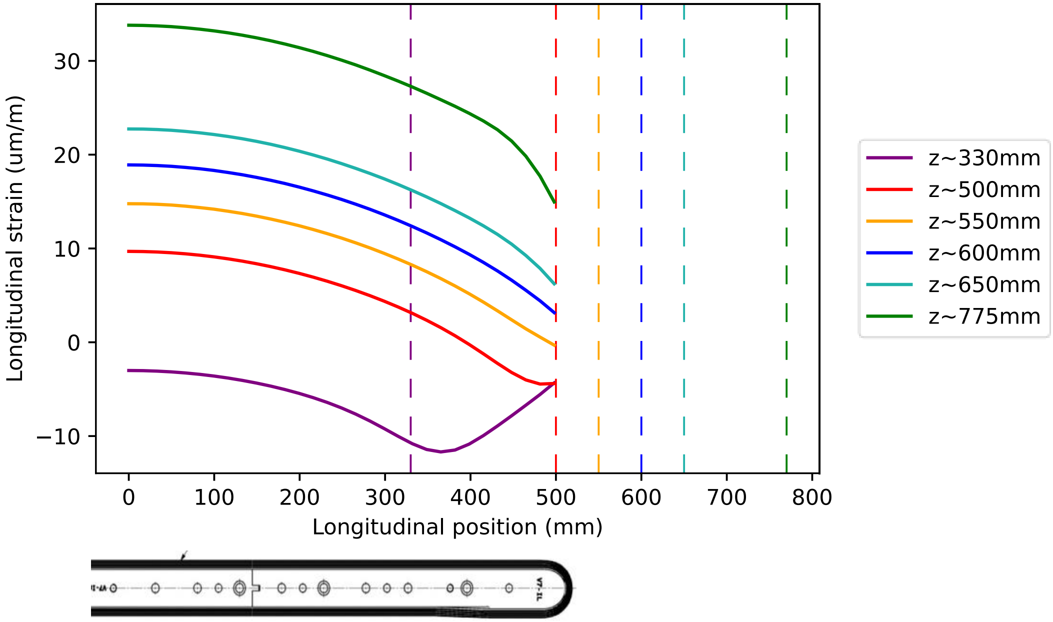
E isotropic
CTE isotropic



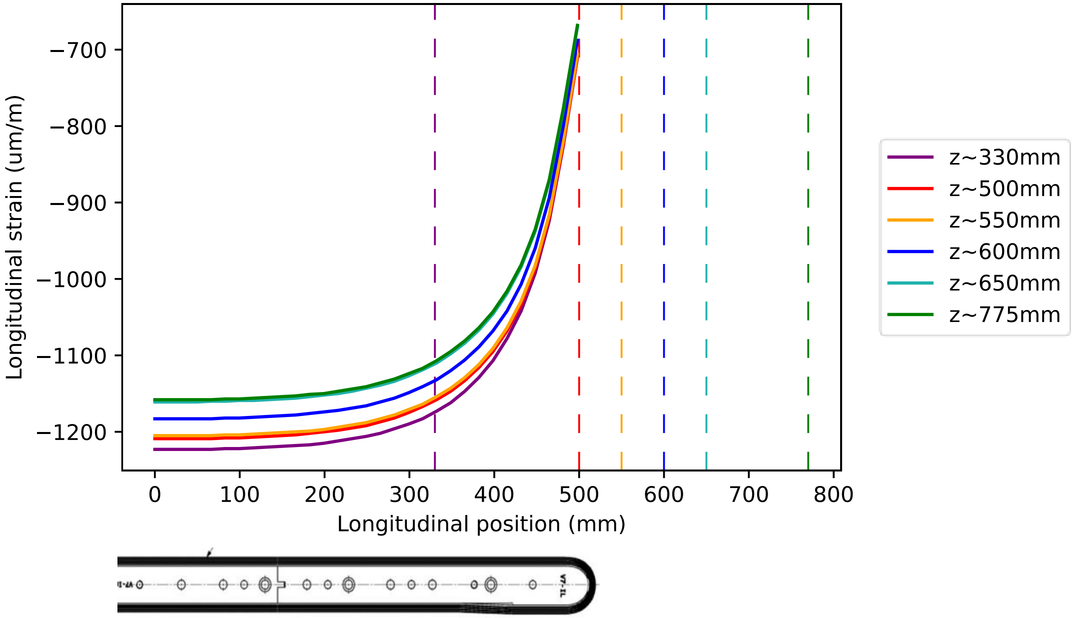
Room temperature, gravity effect is small on SG

At 77K, difference of the support is: on pole or not pole

Longitudinal strain of inner pole (SG) fonction of longitudinal position, gravity alone

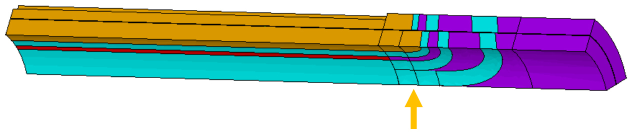


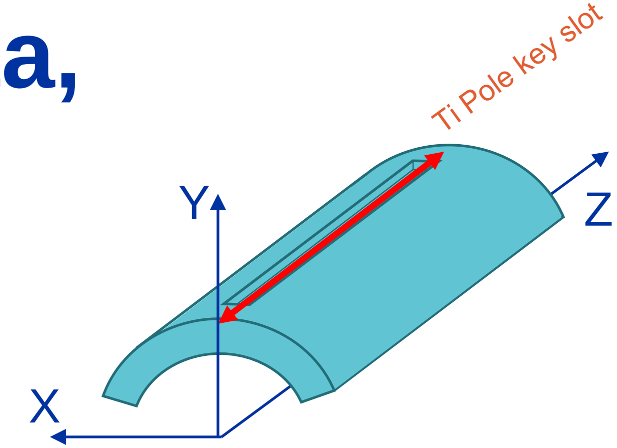
Longitudinal strain of inner pole (SG) fonction of longitudinal position, gravity at 77K



Comparison with photogrammetry data, displacement in Z direction

E isotropic
CTE isotropic



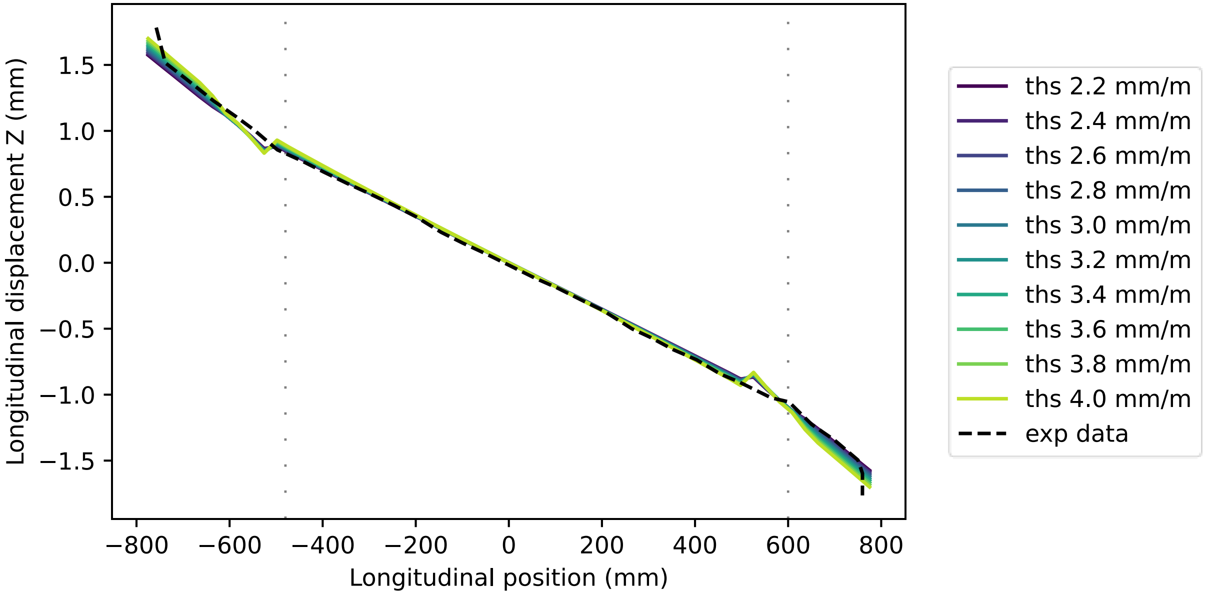


Experimental data and FEA results are consistent.

Sanity check: CTE has low influence

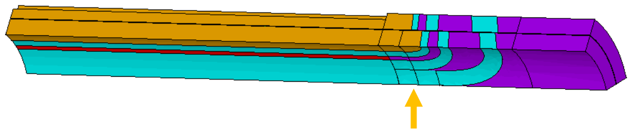
Two behaviours: Ti pole in the middle, heads outside.

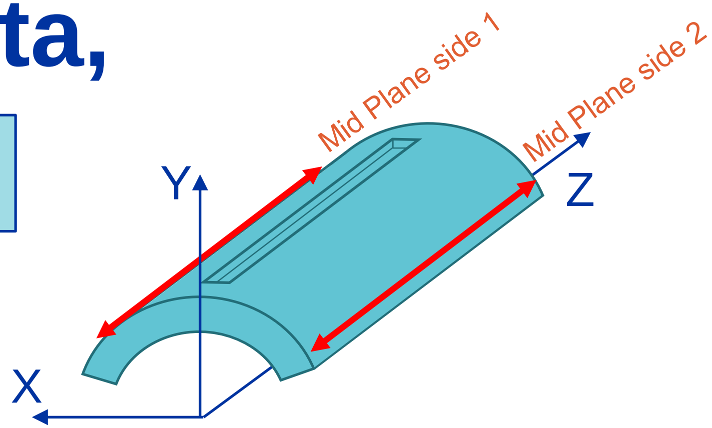
CTE parametric study compared with experimental data (longitudinal displacement at 77K)



Comparison with photogrammetry data, displacement in Z direction

E isotropic
CTE isotropic

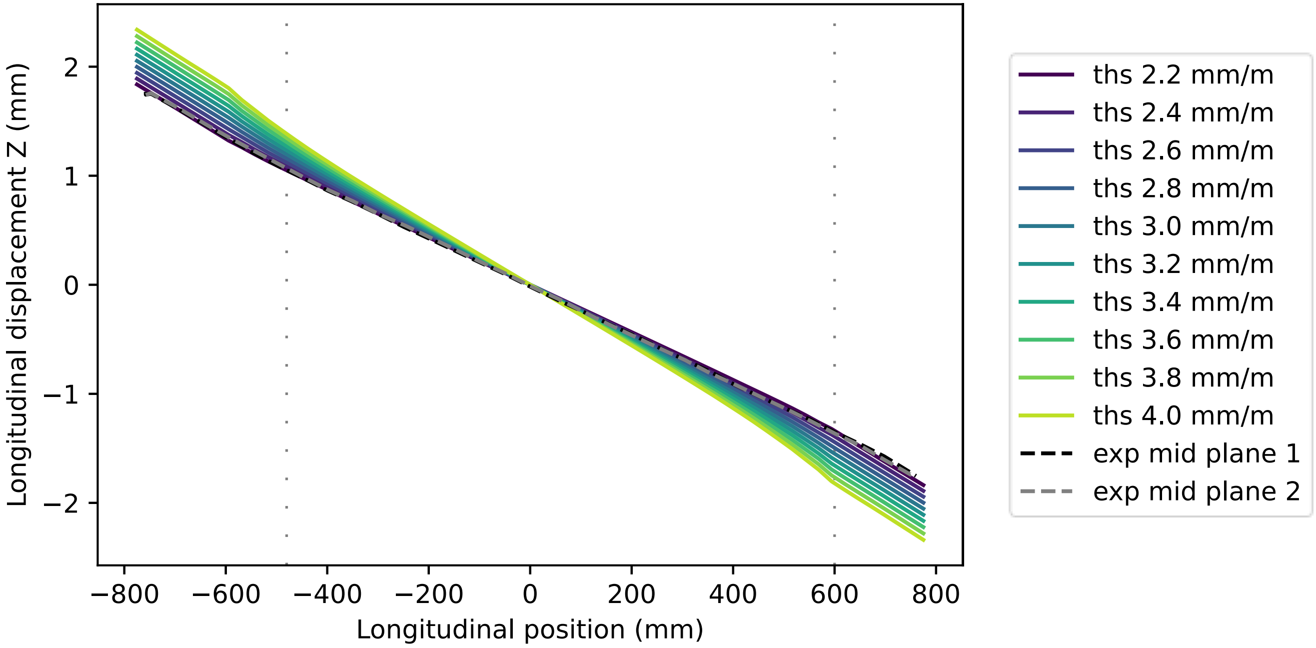




Two behaviour are observed in experimental data:

- **In the endshoes FEA is consistent with experimental data.**
- **In the centre part, composed of the coil bloc, a low CTE seems closer to experimental data.**

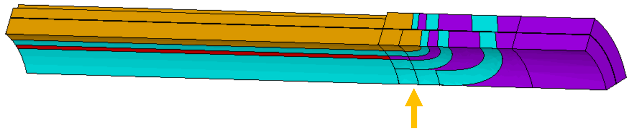
CTE parametric study compared with experimental data (longitudinal displacement at 77K)



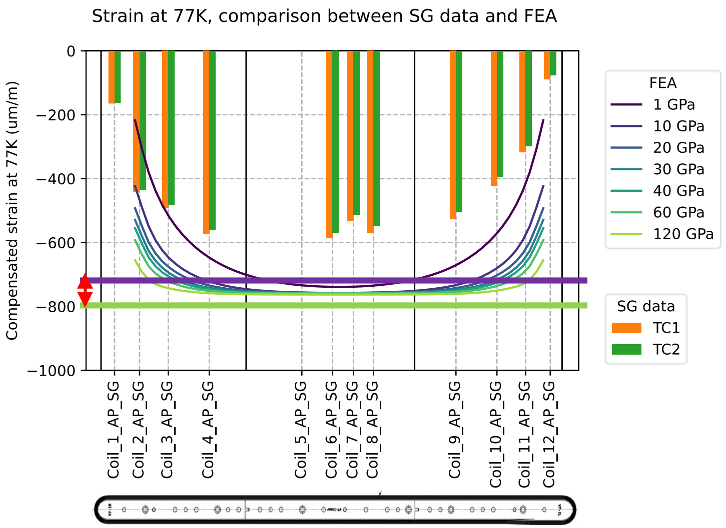
Parametric on elastic modulus, changing the CTE

Pole strain

E isotropic
CTE isotropic

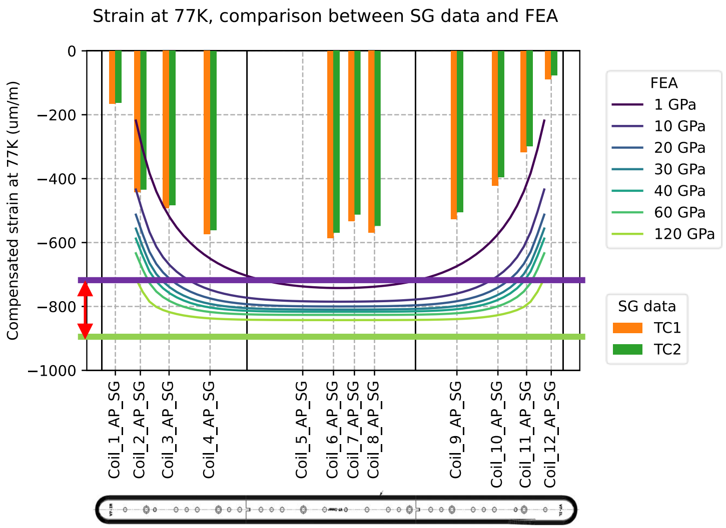


$\varepsilon_{th\ coil} = 2.4\ mm/m$



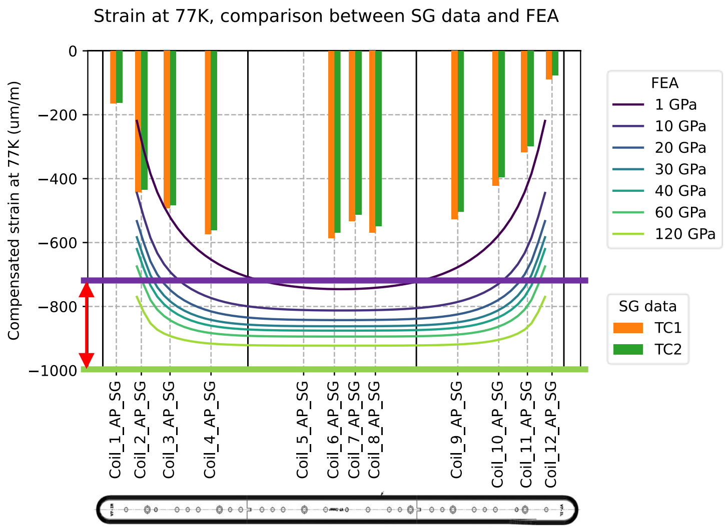
$\varepsilon_{th\ coil} - \varepsilon_{th\ titanium} = 0.8\ mm/m$

$\varepsilon_{th\ coil} = 2.5\ mm/m$



$\varepsilon_{th\ coil} - \varepsilon_{th\ titanium} = 0.9\ mm/m$

$\varepsilon_{th\ coil} = 2.6\ mm/m$



$\varepsilon_{th\ coil} - \varepsilon_{th\ titanium} = 1\ mm/m$

Lower boundary: ~[800 ; 1000] um/m when E_{coil} get closer to $E_{titanium} = 130\ GPa$.

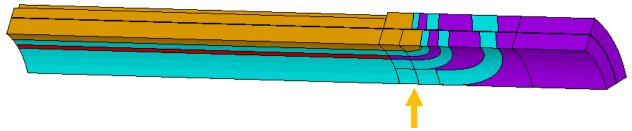
Upper boundary: ~750 um/m at low E_{coil} (spacer + wedge loop contribution).

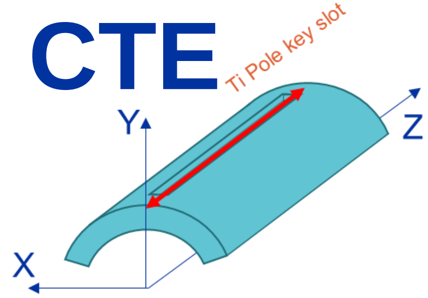
→ Margin of adaptation of FEA with E_{coil} is limited.

Parametric on elastic modulus, changing the CTE

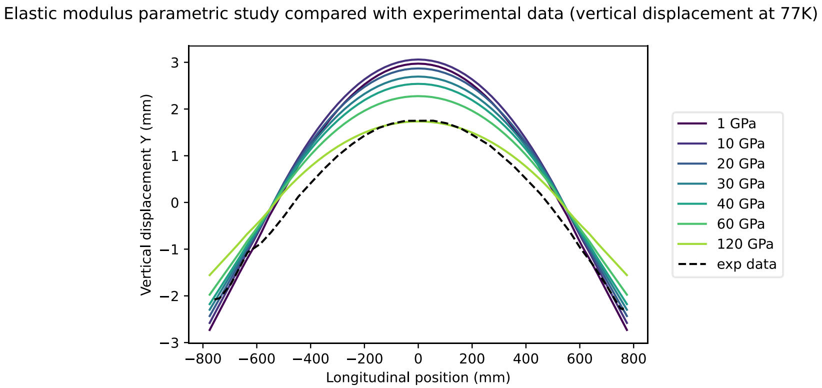
Coil bending

E isotropic
CTE isotropic



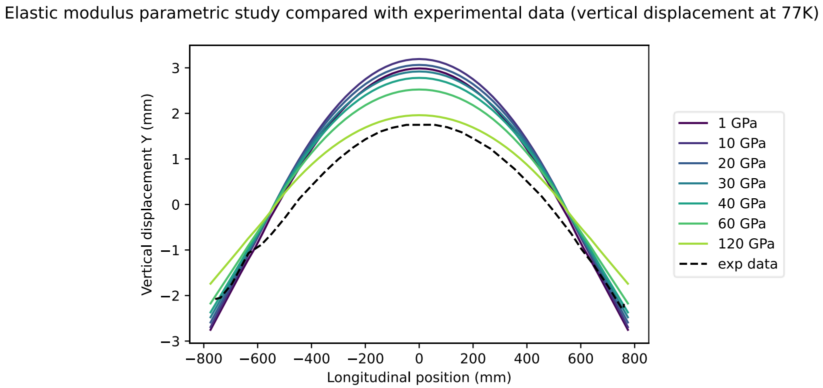


$\varepsilon_{th\,coil} = 2.4\,mm/m$


Elastic modulus parametric study compared with experimental data (vertical displacement at 77K)

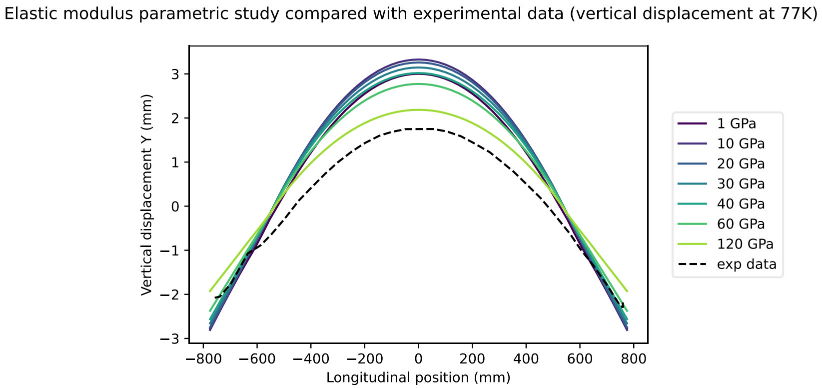
‘Equilibrium’ at 10 GPa

$\varepsilon_{th\,coil} = 2.5\,mm/m$


Elastic modulus parametric study compared with experimental data (vertical displacement at 77K)

‘Equilibrium’ at 10 GPa

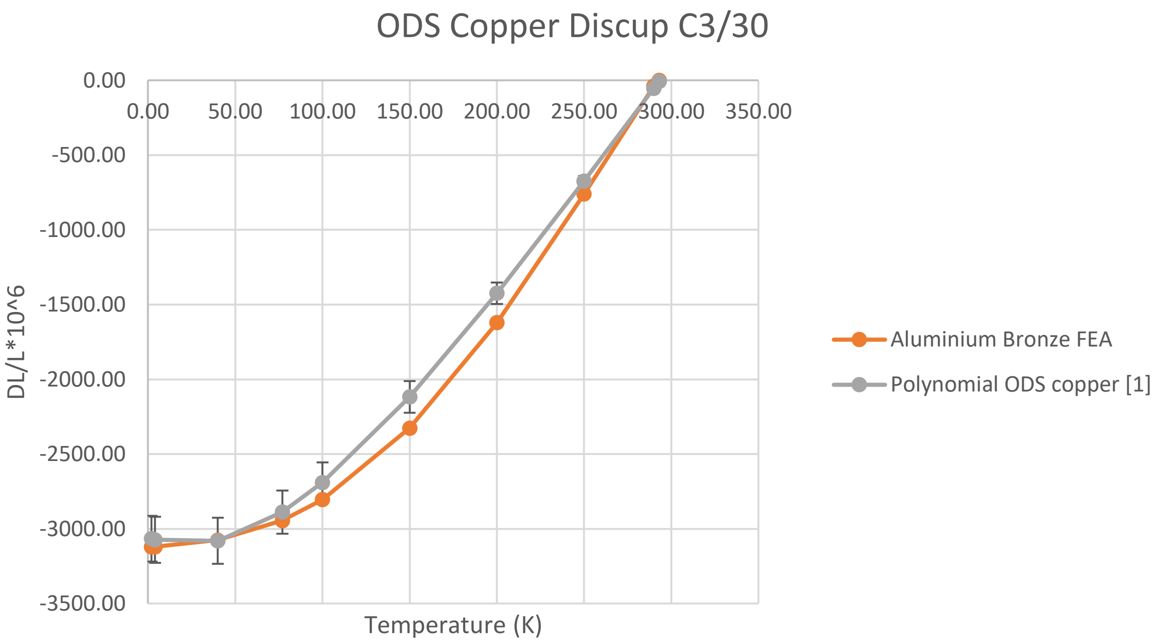
$\varepsilon_{th\,coil} = 2.6\,mm/m$


Elastic modulus parametric study compared with experimental data (vertical displacement at 77K)

‘Equilibrium’ at 10 GPa

Saturation close to the expected differential thermal contraction when E_{coil} get closer to $E_{titanium} = 130\,GPa$.

CTE of the wedges



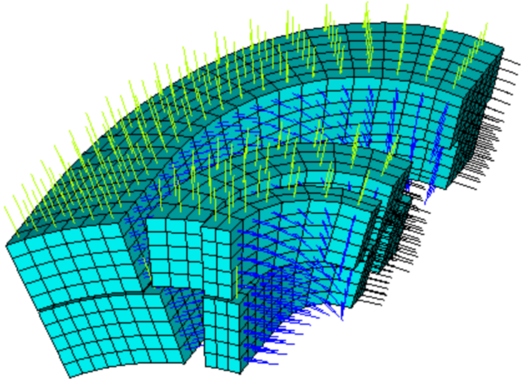
Previously in FE code: Aluminum bronze

Real material: *ODS copper Discup C3/30*, CTE measurement made by CEA Grenoble [1]

→ 5% of error on the measurements, ±145 um/m at 77 K.

[1] J.P. Arnaud, *ODS Copper Discup C3/30 Thermal Expansion,* technical note SBT/CT12-28, INAC CEA Grenoble, July 2012.

Orthotropic mechanical behaviour



Generalized Hooks's law: $[\varepsilon] = \underline{\underline{S}}[\sigma] + \Delta\mathrm{T}.\underline{\underline{\alpha}}$ **with** $[\varepsilon] = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}$; $[\sigma] = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}$

Orthotropic : $\underline{\underline{S}} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & & & \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & & 0 & \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & & & \\ & & & 1/G_{23} & & \\ & 0 & & & 1/G_{13} & \\ & & & & & 1/G_{12} \end{bmatrix}$ **and** $\underline{\underline{\alpha}} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Shear modulus: orders of magnitude

Material	Shear modulus (GPa)
Steel	79.3 [1]
Iron	52.5 [1]
Copper	44.7 [1]
Titanium	41.4 [1]
Aluminum	25.5 [1]
Composite (epoxy+Kevlar)	[4 ; 21] depending on the directions

Nb3Sn coil block	Shear modulus (GPa)
Isotropic: $E = 20GPa$ and $\nu = 0.3$	$G = E/(2+2\nu) = 7.7$

→ parametric study on shear modulus: from 1 GPa to 40 GPa.

[1] Wikipedia, https://en.wikipedia.org/wiki/Shear_modulus