Recent advances in the systematics of emission processes

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Outline

I. Introduction

II. Universal law for reduced width

III. Systematics of the two-proton emission

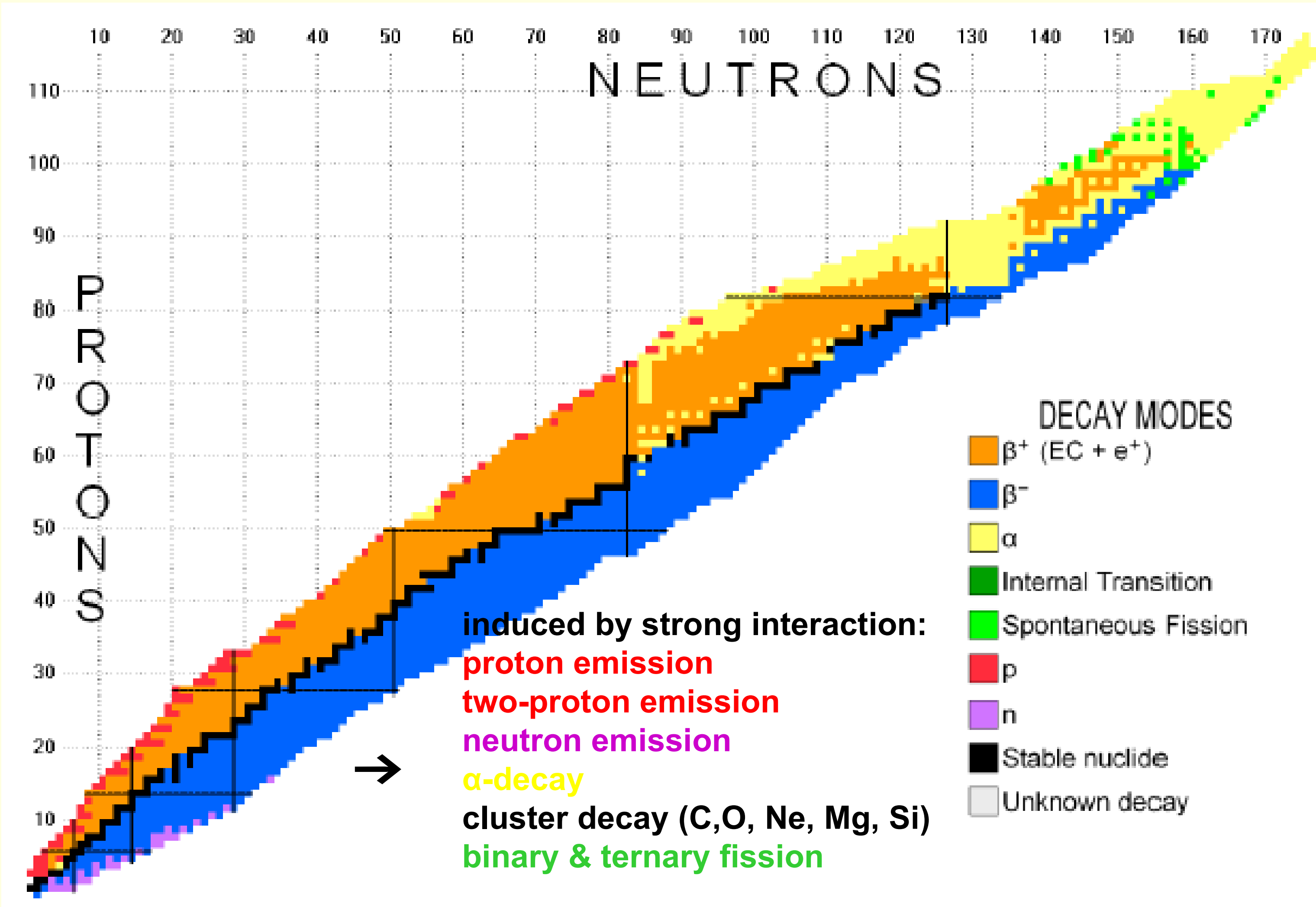
IV. Alpha versus gamma and beta decays

V. Proton-neutron correlations and alpha-clustering

VI. Conclusions

I. Introduction

Most of nuclei are unstable and decay through various modes



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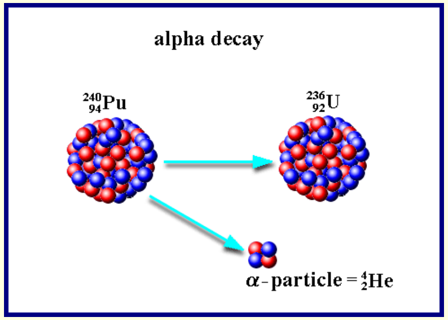
induced by strong interaction:
proton emission
two-proton emission
neutron emission
α-decay
cluster decay (C,O, Ne, Mg, Si)
binary & ternary fission

Geiger-Nuttall law for half lives



$$\log_{10} T = a \frac{Z_D}{\sqrt{E}} + b$$

- **H. Geiger and J.M. Nuttall** "The ranges of the α particles from various radioactive substances and a relation between range and period of transformation," *Philosophical Magazine*, Series 6, vol. 22, no. 130, 613-621 (1911).

- **H. Geiger and J.M. Nuttall** "The ranges of α particles from uranium," *Philosophical Magazine*, Series 6, vol. 23, no. 135, 439-445 (1912).

George Gamow in 1909,
two years before
the discovery of the G-N law

… and in 1930,
two years after
his explanation

Decay width

The number radioactive of nuclei at a certain moment is exponentially decreasing. Therefore the probability to find a decaying nucleus at a certain point is given by

$$\left|\Phi\left(R,t\right)\right|^2 = \left|\Psi\left(R\right)\right|^2 e^{-\lambda t}$$

where the decay constant is proportional to the decay width

$$\lambda = \frac{\Gamma}{\hbar}$$

Thus, a decaying state is a stationary state (Gamow resonance) with complex energy. The real part is the Q-value (energy release) and imaginary part is proportional to the decay width

$$E = Q - \frac{i}{2}\Gamma$$

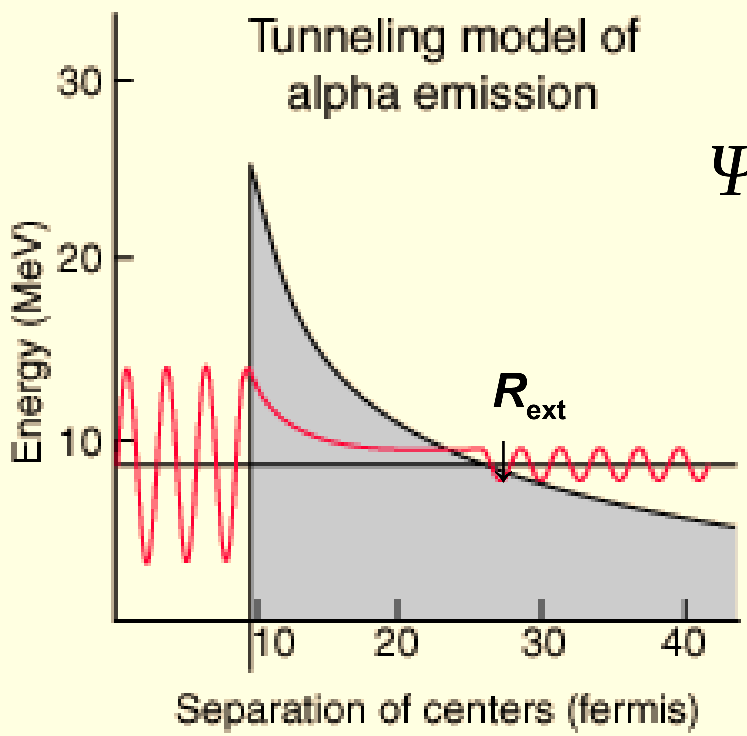
G. Gamow "Zur Quantentheorie des Atomkernes" (On the quantum theory of the atomic nucleus), *Zeitschrift für Physik*, vol. 51, 204-212 (1928).

The first probabilistic interpretation of the wave function

External wave function is an outgoing spherical Coulomb wave

$$\Psi_{ext}(R) = \frac{H_l^{(+)}(kR)}{R}$$

$\Psi_{\text{int}} \rightarrow$



$\leftarrow N\Psi_{ext}$

Internal region **External region**

By using the Schrodinger equation and its conjugate

$$\left(Q - \frac{i}{2}\Gamma\right)\Psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi$$

$$\left(Q + \frac{i}{2}\Gamma\right)\Psi^* = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi^*$$

one obtains the decay width

$$\Gamma = \frac{\dfrac{\hbar}{2mi}\oint\left(\Psi\dfrac{\partial\Psi}{\partial R} - \Psi\dfrac{\partial\Psi}{\partial R}\right)R^2 d\Omega}{\int|\Psi|^2 d^3R} = \hbar v N^2 = \hbar v\left|\frac{\Psi_{\mathrm{int}}(R)}{\Psi_{ext}(R)}\right|^2$$

Decay width can be rewritten

as a product between

$$\Gamma = \hbar v N^2 = 2\gamma^2 P$$

the reduced width

$$\gamma^2 = \frac{\hbar^2}{2mR} \left| \Psi_{\text{int}}(R) \right|^2$$

and
penetrability
on the matching radius *R*

$$P = \frac{\kappa R}{\left| H_0^{(+)}(\chi, kR) \right|^2} = c e^{a\chi}$$

depending exponentially upon
the Coulomb parameter

$$\chi = \frac{2 Z_D Z_C}{\hbar v} = \frac{2 Z_D Z_C}{\hbar \sqrt{2E/m}}$$

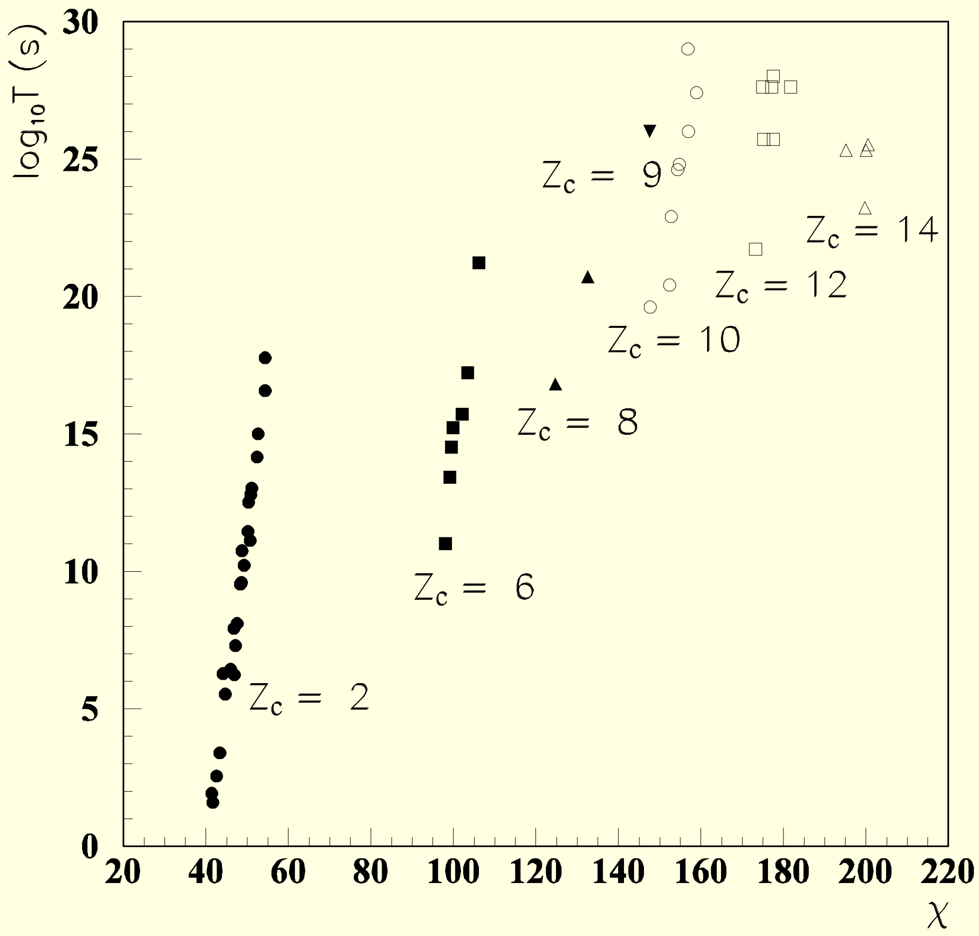
Geiger-Nuttall law relates log(decay width) to the Coulomb parameter

$$\log_{10}\Gamma = \log_{10}P + \log_{10}2\gamma^2$$

$$\log_{10}P = a\chi + b$$

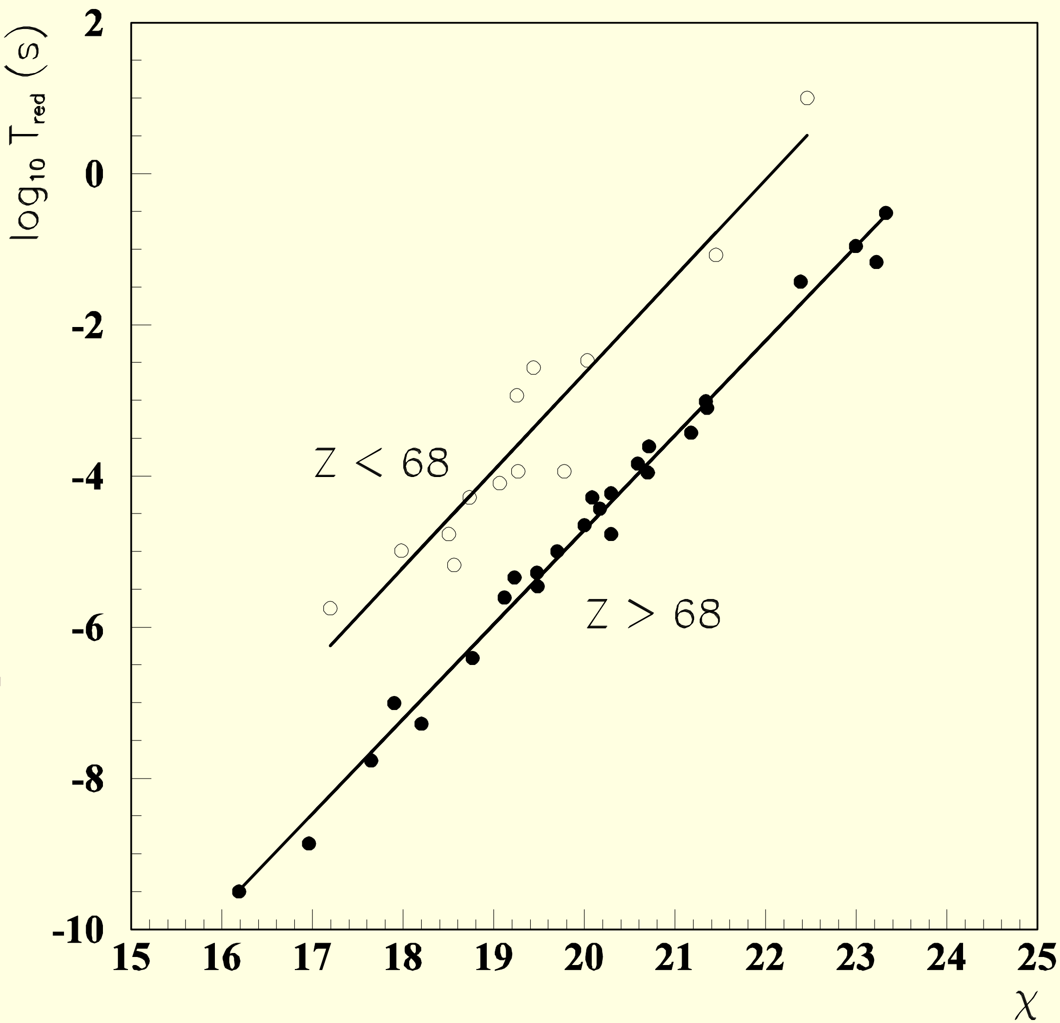
$$\chi = c\frac{Z_D}{\sqrt{E}}$$

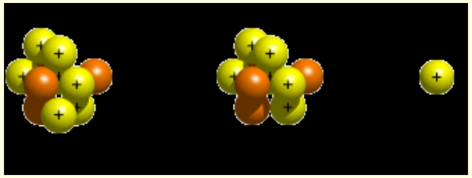
Geiger-Nuttall law for alpha and cluster-decays



Geiger-Nuttall law for proton emission

D.S. Delion, R.J. Liotta, R. Wyss, Systematics of proton emission, Physical Review Letters 96, 072501 (2006)





Reduced half-life corrected by the centrifugal barrier

$$T_{red} = \frac{T}{C_l^2}$$

satisfies a G-N rule with two regions divided by Z=68

$$\log_{10} T_{red} = a\frac{Z_D}{\sqrt{E}} + b(Z)$$

II. Universal law for reduced widths

(a) schematic approach : D.S. Delion

Universal decay rule for reduced widths

Physical Review **C80** (2009) 024310

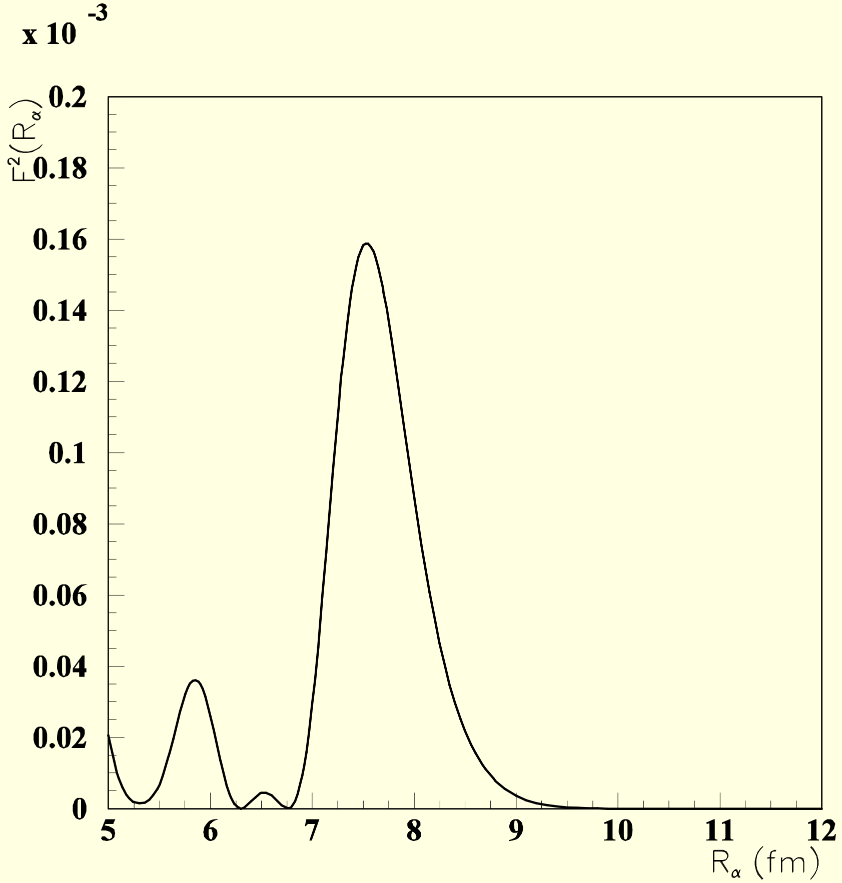
(b) realistic approach: D.S. Delion and A. Dumitrescu

Realistic analytical approach of the alpha-decay and clustering

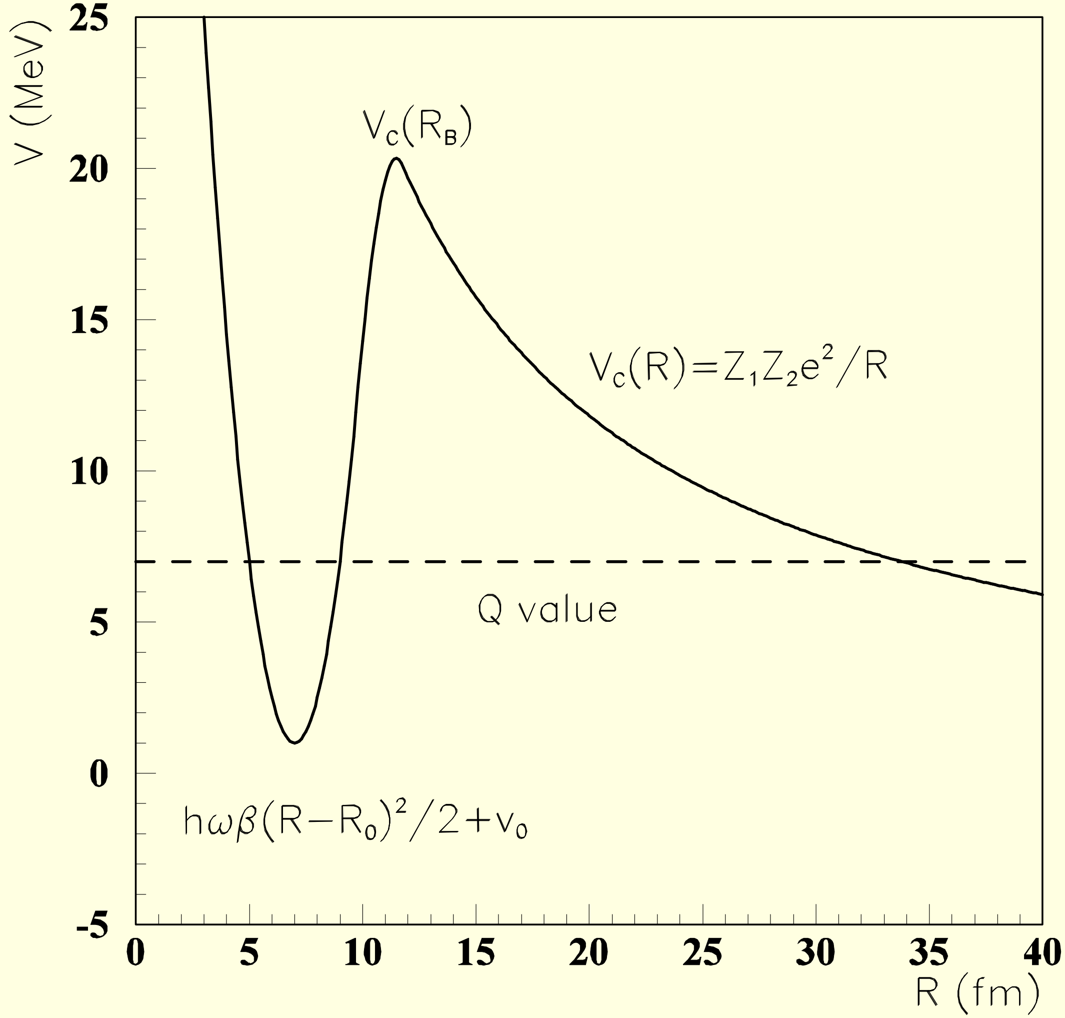
Physical Review **C102** (2020) 014327

Microscopic α-particle formation probability within the mean field + pairing approach is peaked on the nuclear surface

$$\mathscr{F}(\mathbf{R}_\alpha) = \langle \alpha D | P \rangle = \int d\mathbf{x}_\alpha d\mathbf{x}_D \left[\psi_\alpha^{(\beta_\alpha)}(\mathbf{x}_\alpha) \Psi^{(D)}(\mathbf{x}_D) \right]^* \Psi^{(P)}(\mathbf{x}_P)$$



Therefore the cluster-daughter interaction should be a pocket-like potential on the nuclear surface



(a) Schematic approach: ho oscillator matched to a Coulomb potential

Conditions for an α-particle moving in a shifted harmonic oscillator potential

1) The first eigenstate energy is the Q-value

$$Q = E = \frac{1}{2}\hbar\omega$$

2) Its wave function is given by

$$\Psi(R) = A_0 e^{-\beta(R-R_0)^2/2}$$

where the oscillator parameter is

$$\beta = \frac{m\omega}{\hbar}$$

One obtains an analytical universal law for the reduced width in terms of the fragmentation potential V_{frag} and cluster amplitude A_0

$$\log_{10}\gamma^2 = -\frac{\log_{10} e^2}{\hbar\omega} V_{frag} + \log_{10}\frac{\hbar^2 A_0^2}{2emR_B}$$

It does not depend on the pocket radius and remains valid for any pocket potential,

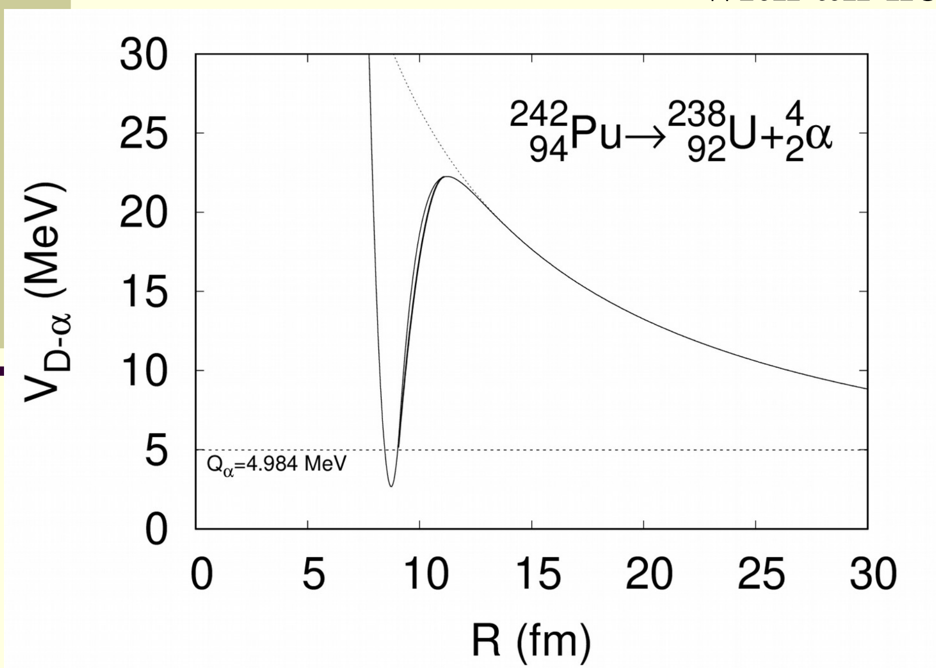
The fragmentation potential is given by the difference between the Coulomb barrier and Q-value:

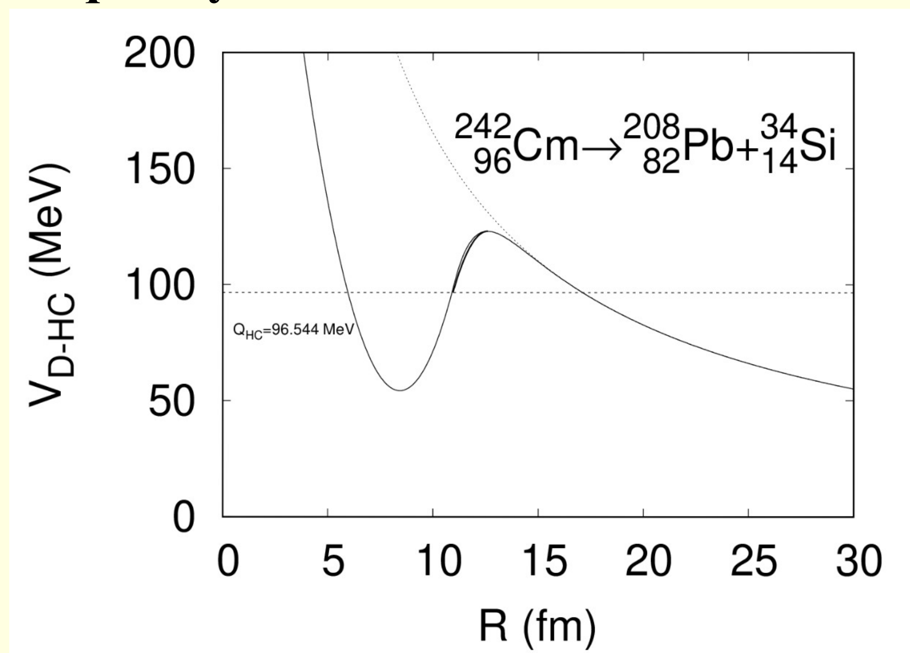
$$V_{frag} = \frac{Z_D Z_C}{R_B} - Q$$

THE SLOPE SHOULD BE NEGATIVE!

(b) Realistic approach: inverted ho oscillator matched to a Coulomb potential

Realistic nuclear cluster-core interaction between minimal and barrier values estimated within the double-folding approach can be approximated by an inverted parabola with an ho frequency hω





Parameter of the inverted ho oscillator

emission	ho frequency (MeV)	error (MeV)
Proton emission A<145	11.389	0.259 (2.27%)
Proton emission A>145	12.580	0.265 (2.10%)
Alpha-decay	9.080	0.246 (2.71%)
Cluster-decay	5.619	0.204 (3.63%)

The internal wave function

can be approximated at the barrier radius R_B

by the Hill-Wheeler ansatz

$$\Psi_{\text{int}}\left(R_B\right) \xrightarrow[WKB]{} A_0 e^{-S_N}, \tag{1}$$

in terms of the nuclear action

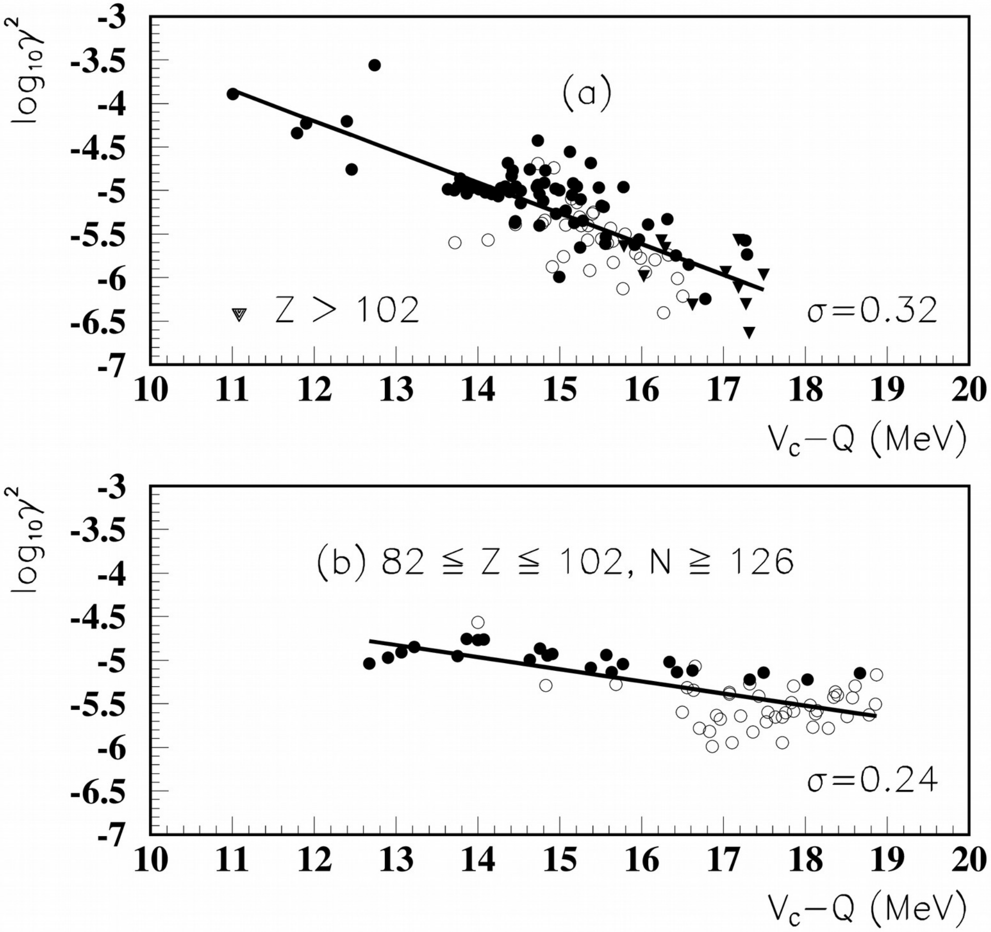
$$S_N = \frac{\pi V_{\text{frag}}}{2\hbar\omega}. \tag{2}$$

The universal law for the reduced width

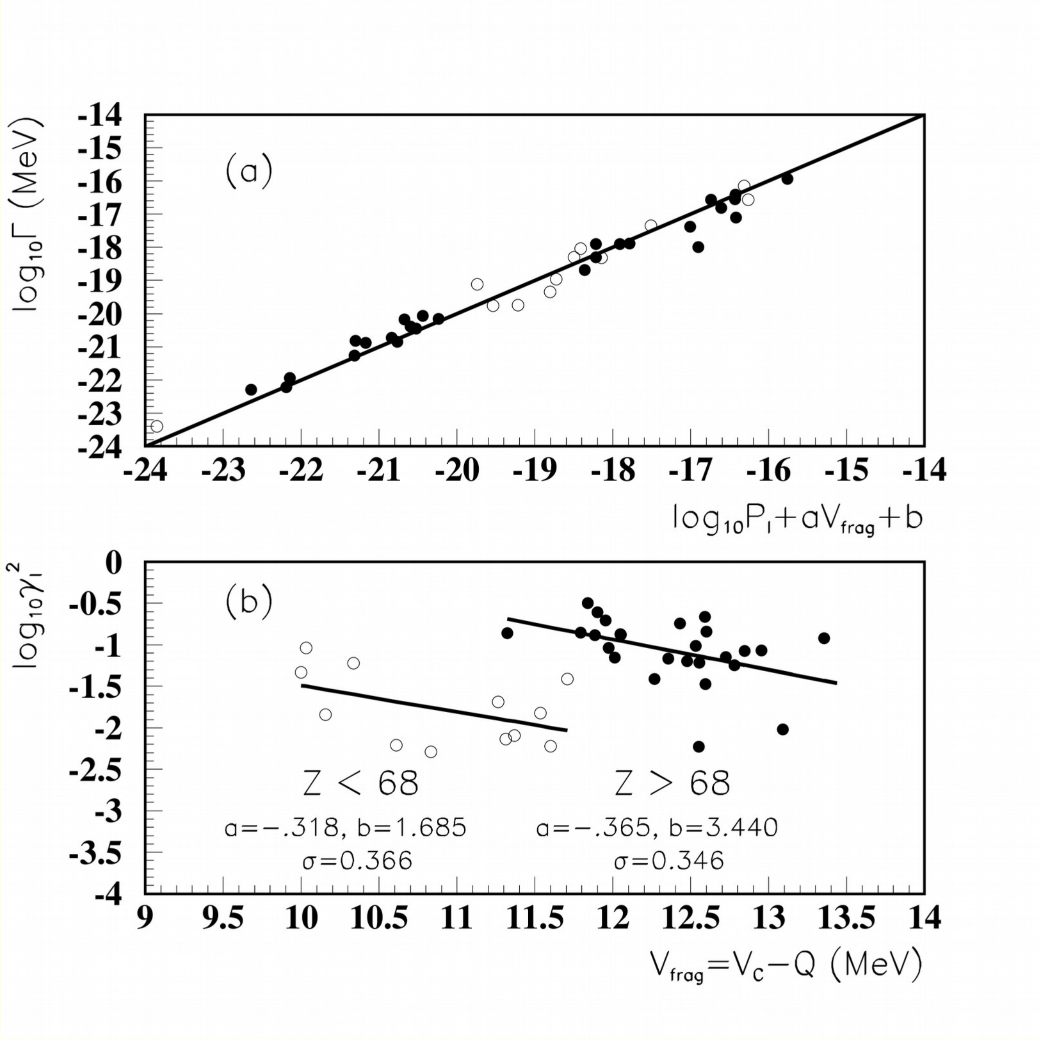
becomes

$$\log_{10}\gamma^2(R_B) = -\frac{\pi \log_{10} e}{\hbar\omega} V_{\text{frag}} + \log_{10}\frac{\hbar^2 A_0^2}{2mR_B}. \tag{3}$$

Experimental universal law for alpha-decay from even-even nuclei has indeed a negative slope and two main regions for spectroscopic factor, divided by ^{208}Pb

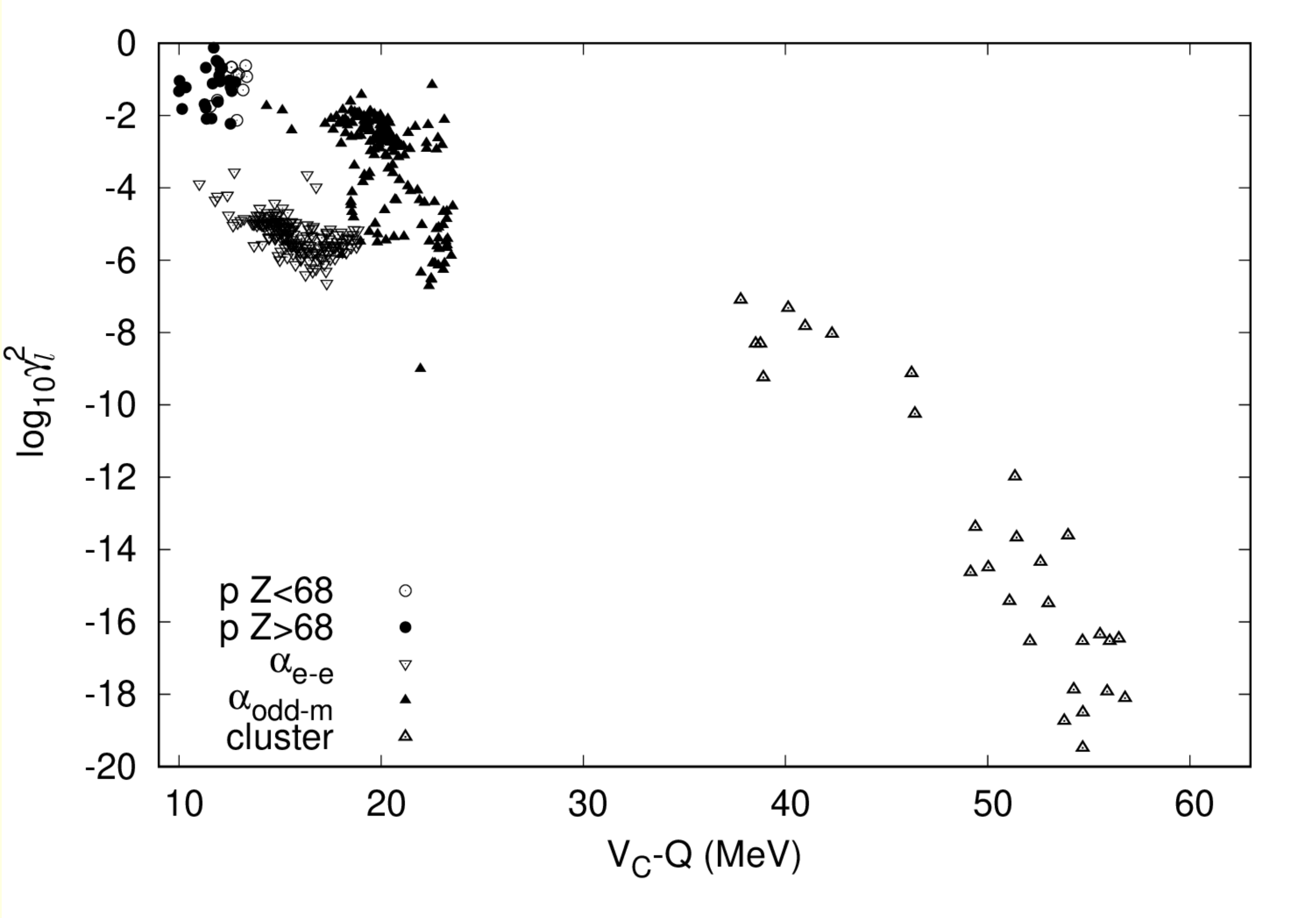


One obtains similar dependencies for proton emission. Universal law (b) explains the two lines in the systematics, corresponding to two regions of the fragmentation potential.

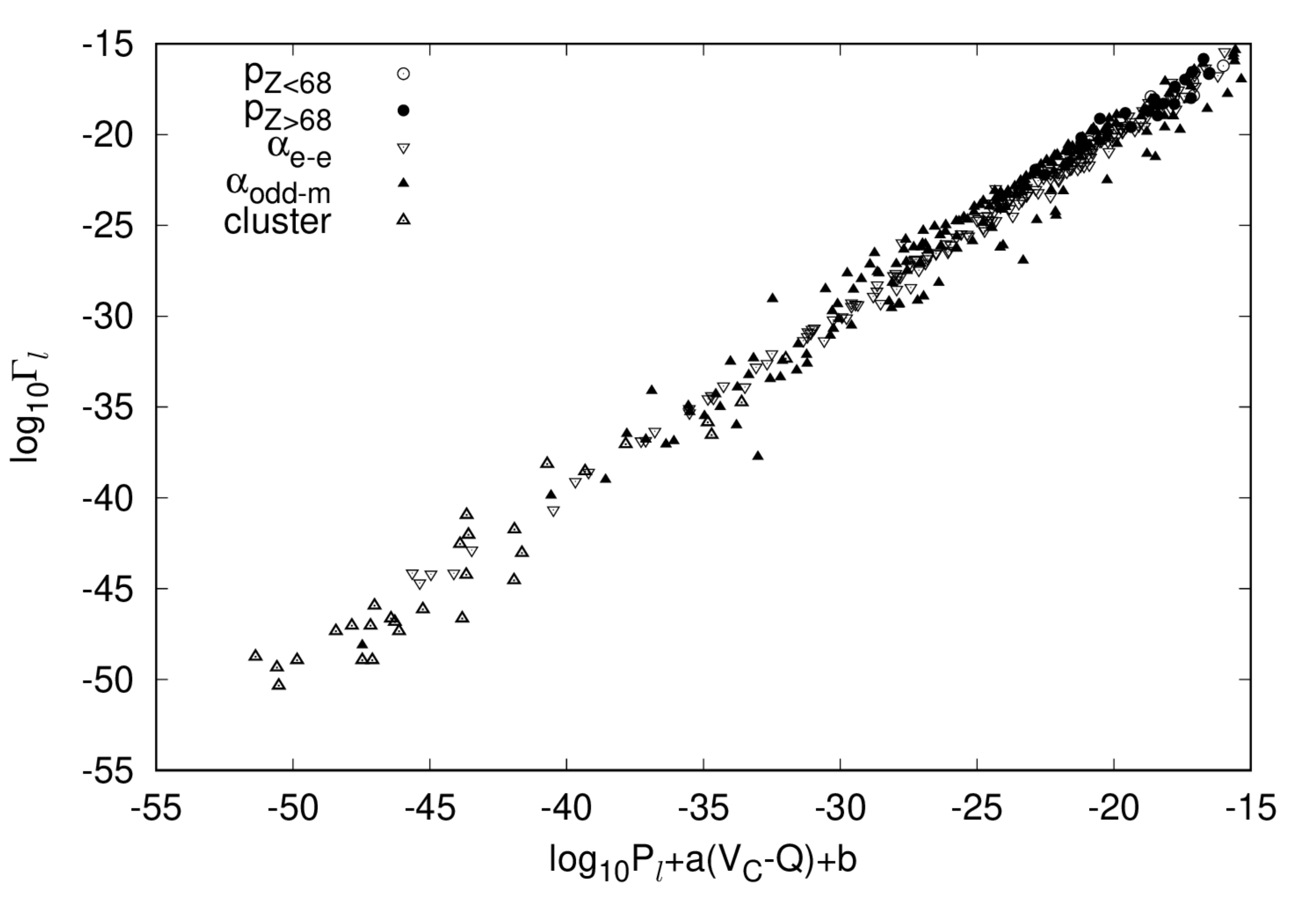


log(width) dependence upon log(penetrability) plus linear dependence of the reduced width

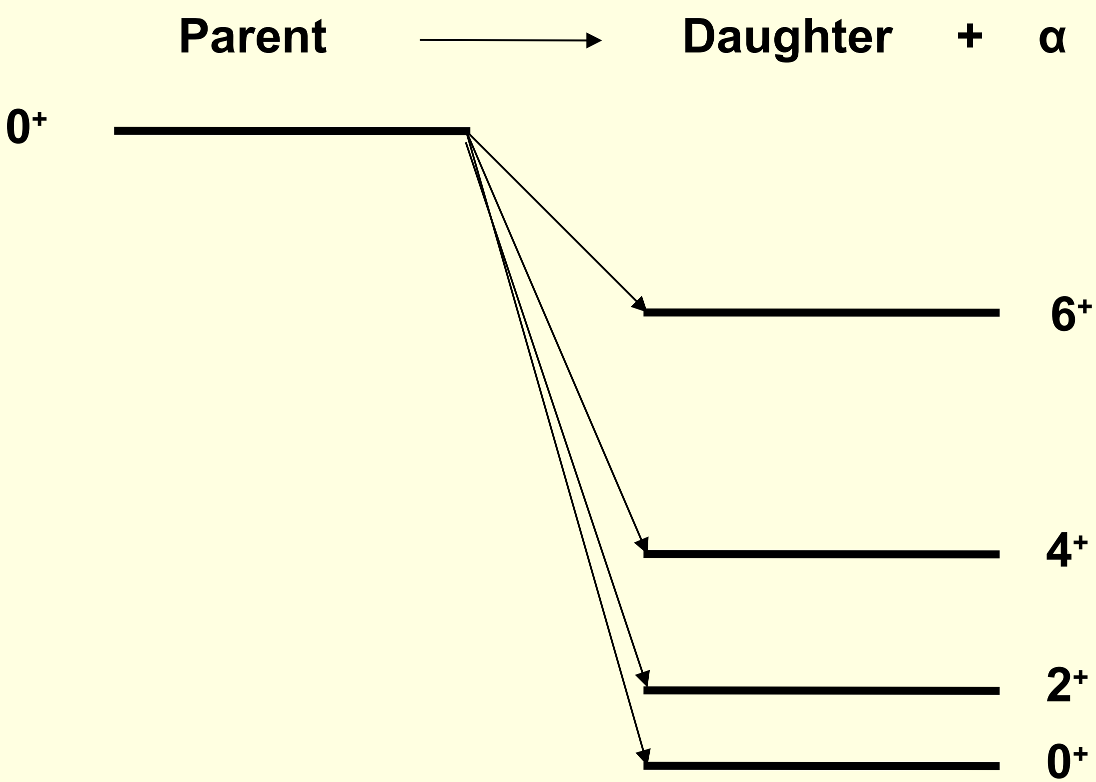
Universal law for reduced widths is valid for all emission processes: proton, even-even, odd-mass alpha & cluster decays



One obtains a general log(width)-log(penetrability) dependence for all emission processes by using the corresponding fit parameters



Universal law for reduced width and α-spectroscopy (fine structure)



Transitions to the ground band in even-even nuclei

$$P \to D(J) + \alpha$$

Observables describing the fine structure

Hindrance factor

$$HF_J = \frac{\gamma_0^2}{\gamma_J^2} = \frac{\Gamma_0}{\Gamma_J}\frac{P_J}{P_0}$$

Intensity

$$I_J = \log_{10}\frac{\Gamma_0}{\Gamma_J} = \log_{10} HF_J + \log_{10}\frac{P_0}{P_J}$$

Ratio of penetrabilities has an almost constant value for considered energies

By using the law for the reduced width one obtains a law for hindrance factors in terms of the excitation energy in the daughter nucleus

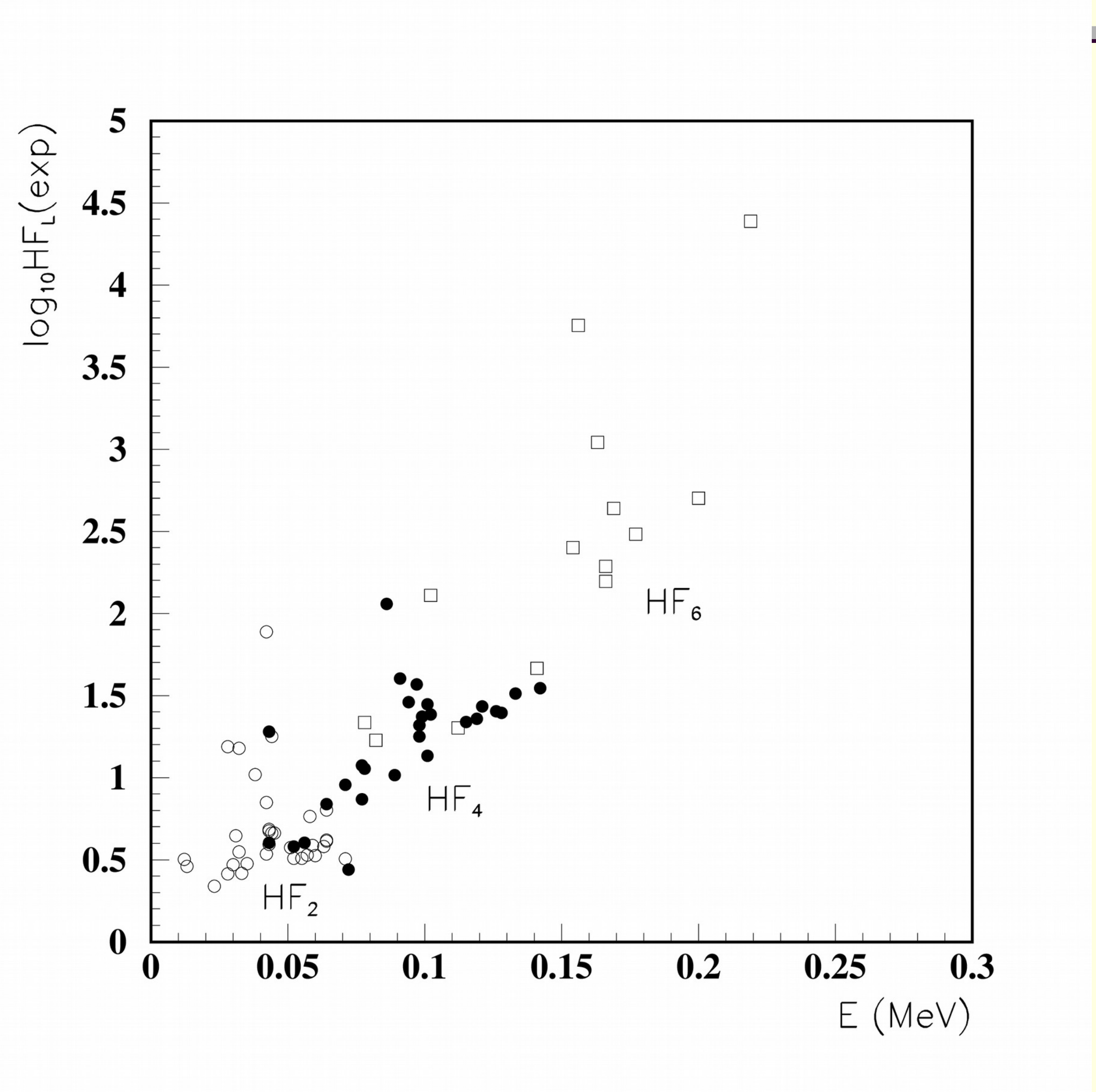
$$\log_{10} HF_J = \frac{\log_{10} e^2}{\hbar\omega} E_J + \log_{10} \frac{A_0^2}{A_J^2}$$

and intensities

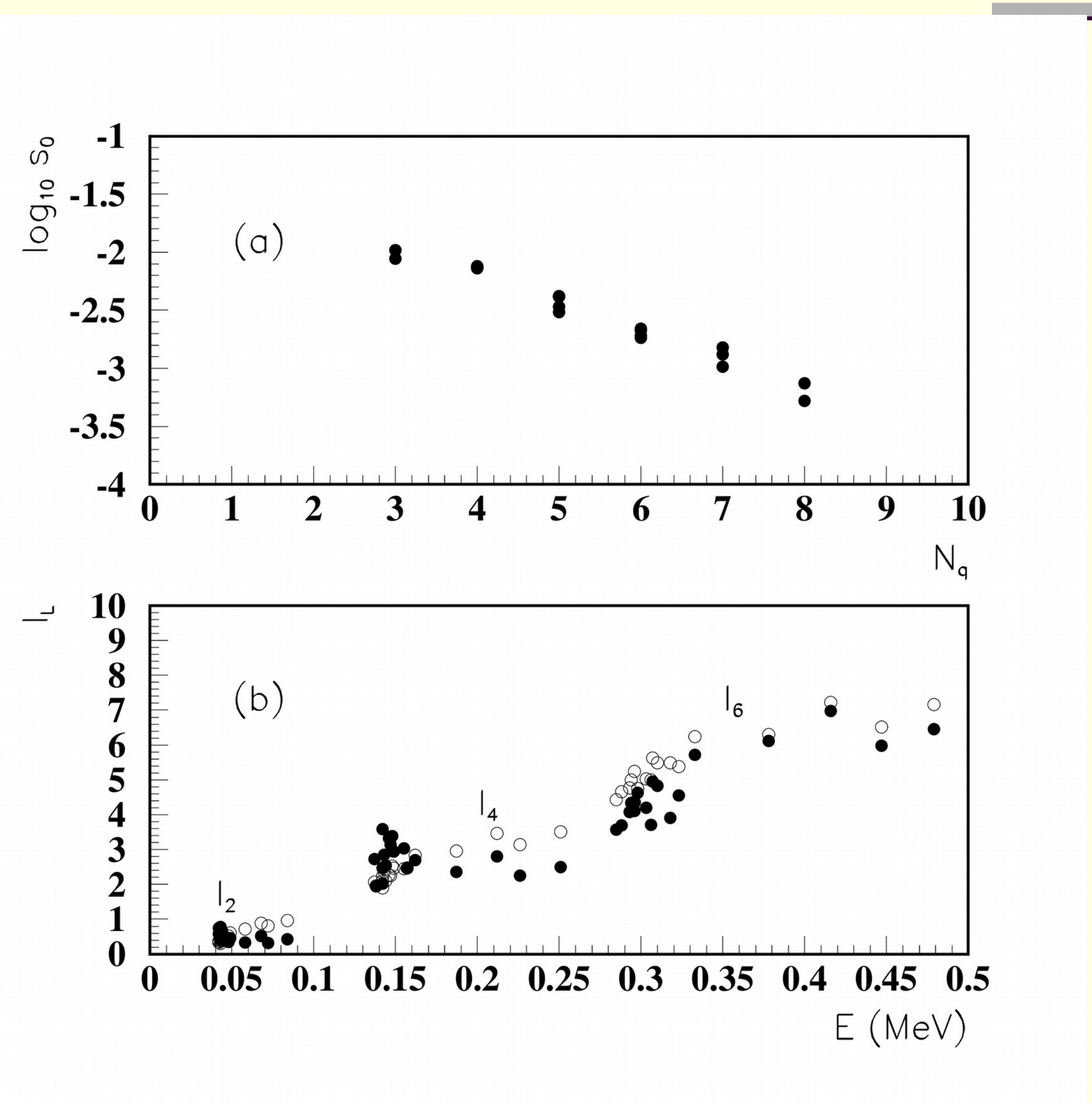
$$I_J = \frac{\log_{10} e^2}{\hbar\omega} E_J + \log_{10} \frac{A_0^2}{A_J^2} + \log_{10} \frac{P_0}{P_J}$$

THE SLOPE SHOULD BE POSITIVE !

Universal law for hindrance factors to excited states in even-even nuclei has a positive slope



Universal law for intensities to excited states (b) in even-even nuclei has a similar behavior Spectroscopic factor decreases a a function of alpha-cluster above magic nuclei (a)

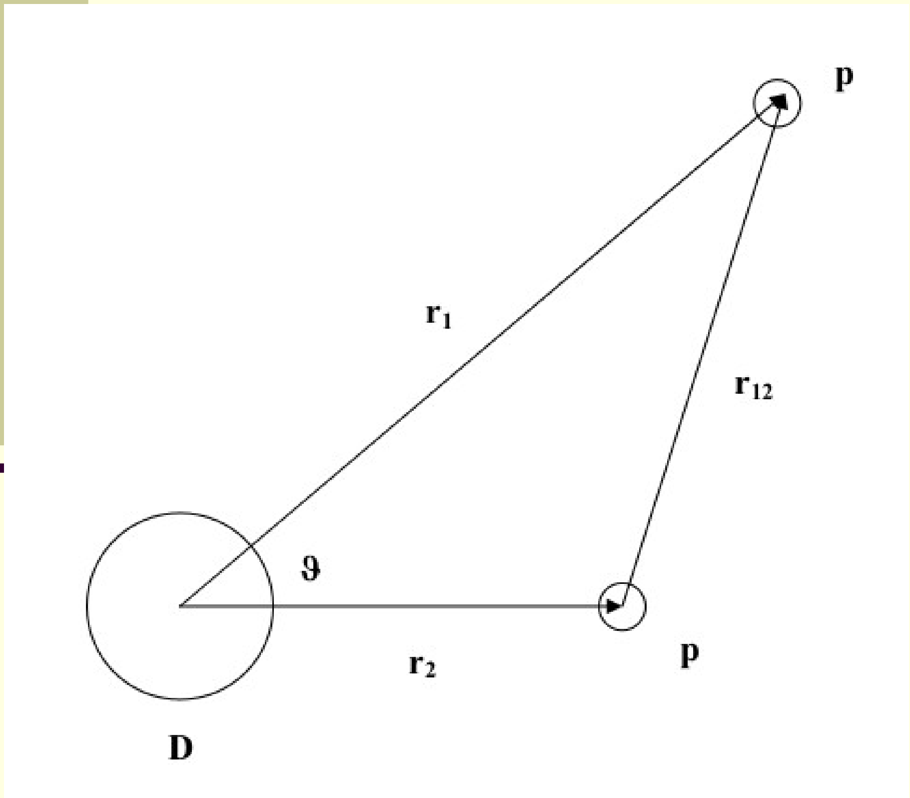


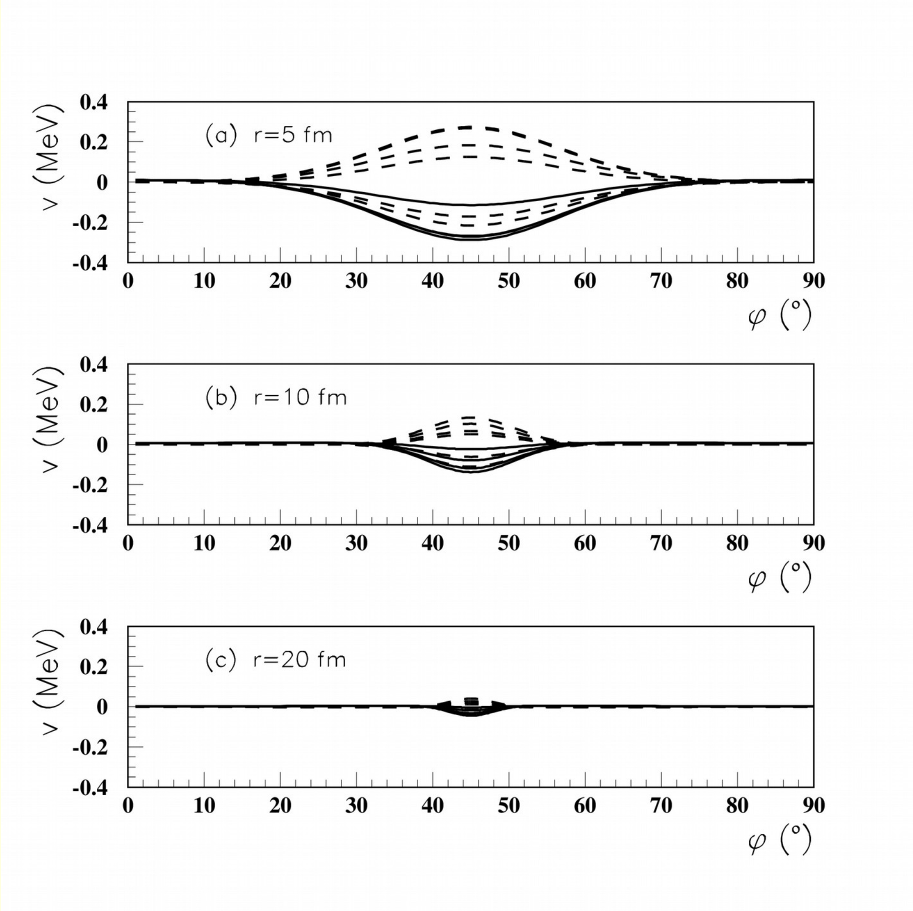
III. Systematics of the two-proton emission

Simultaneous emission of two protons is a three-body process

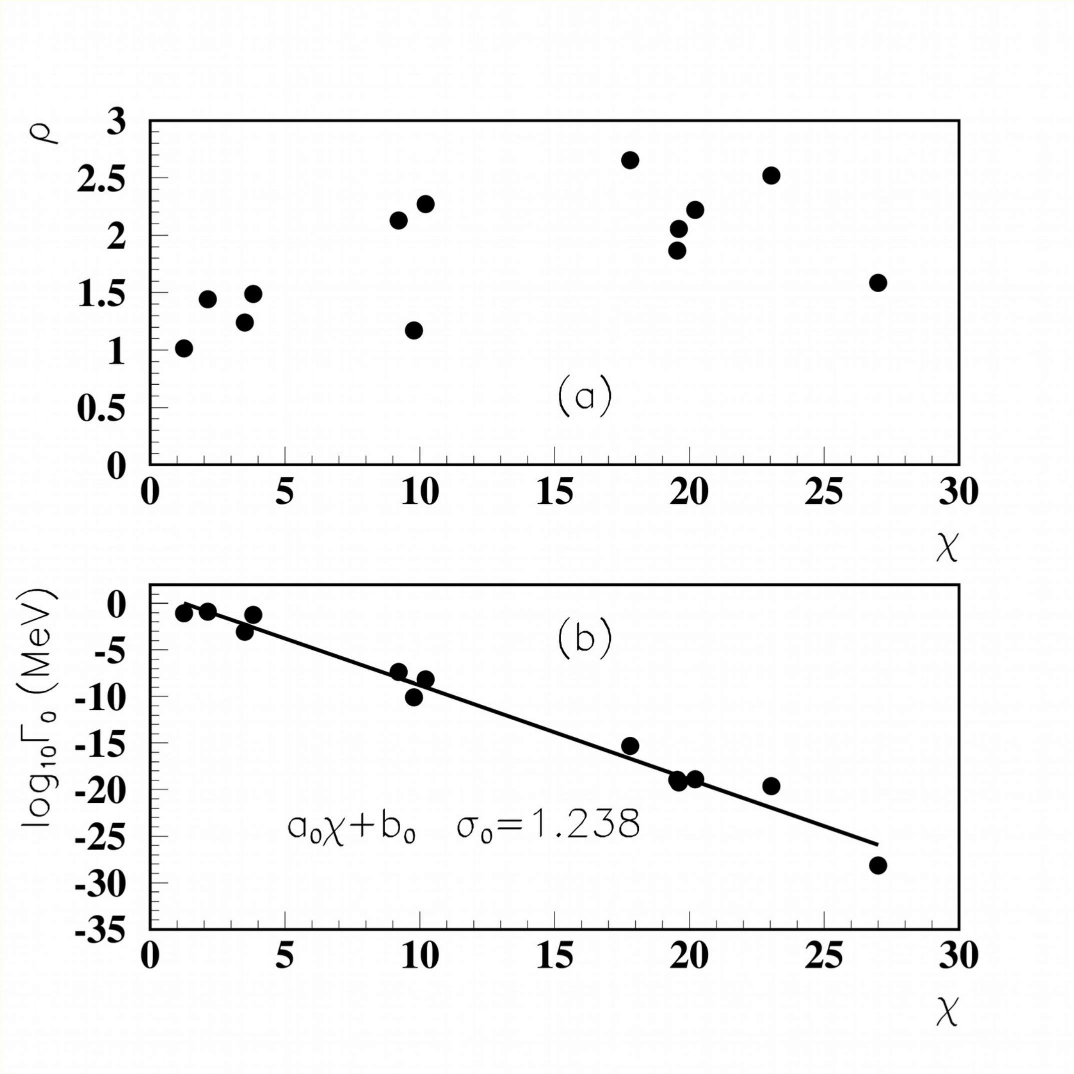
Hypershperical (polar) coordinates: r_1-r cos φ, r_2-r sinφ

Inter-proton gaussian potential components versus φ for various distances r





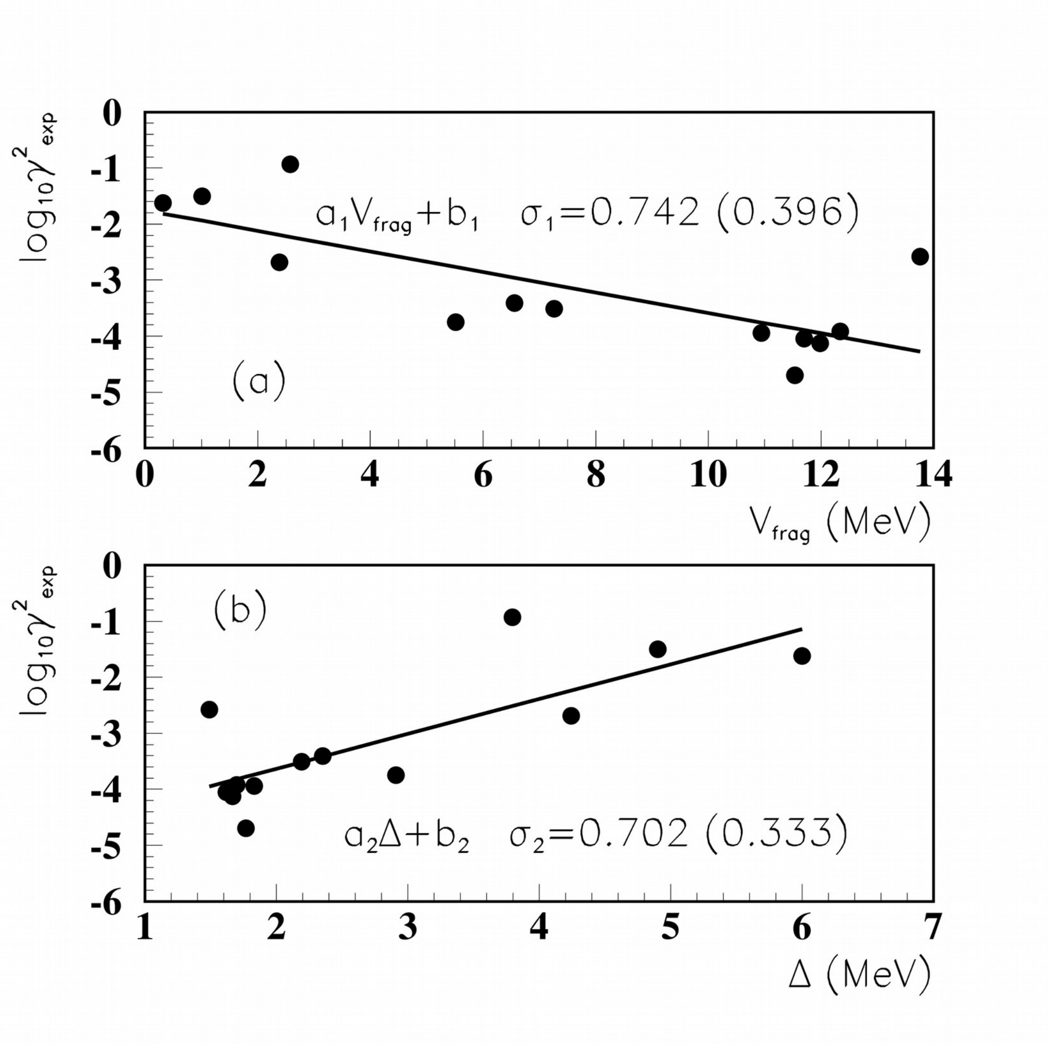
Di-proton approach (bound system of two protons)



Reduced radius
versus
Coulomb parameter
is quasi-linear
except two lower points

Geiger-Nuttall law
for the monopole
di-proton decay width
follows the main trend
but it has a rather poor
predictive power

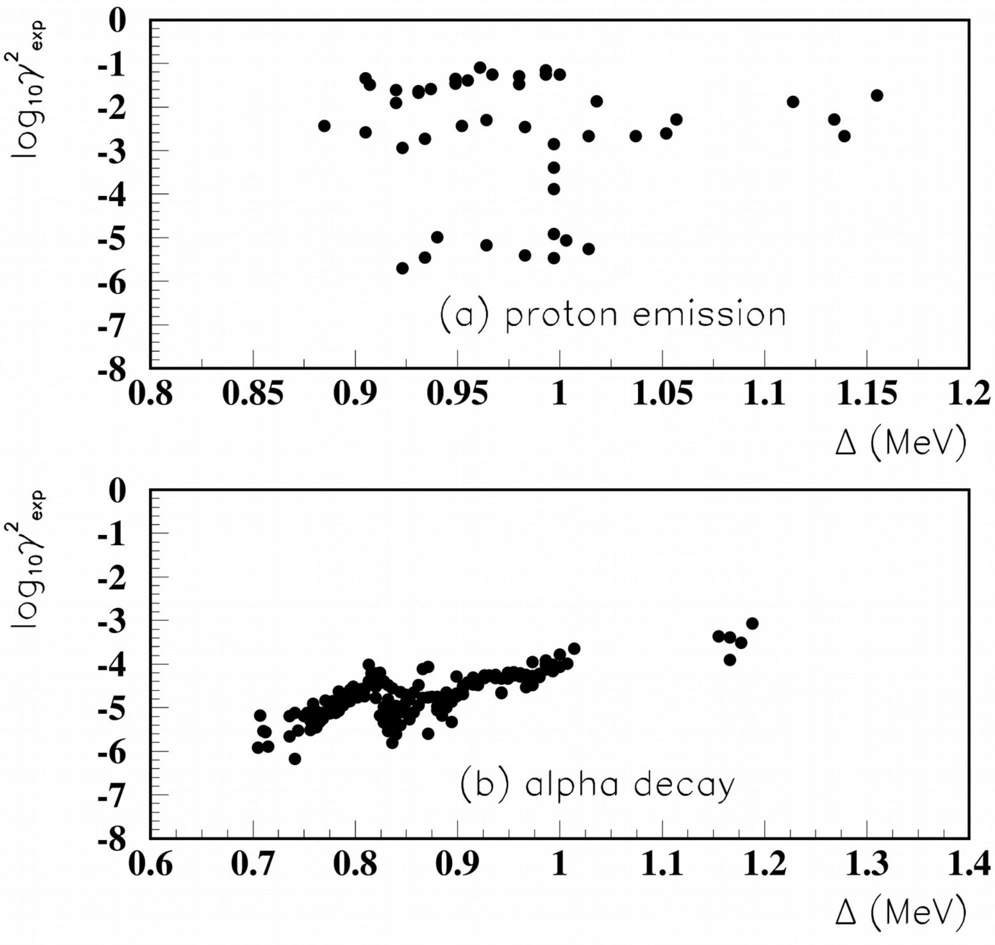
Two-proton reduced width versus (a) fragmentation potential and (b) pairing gap



These laws are similar to alpha-decay, but the experimental reduced width overestimates by two orders of magnitude the pairing value, due to the additional "dissociation" width of the di-proton

The exact solution in terms of hyperspherical harmonics confirms this.

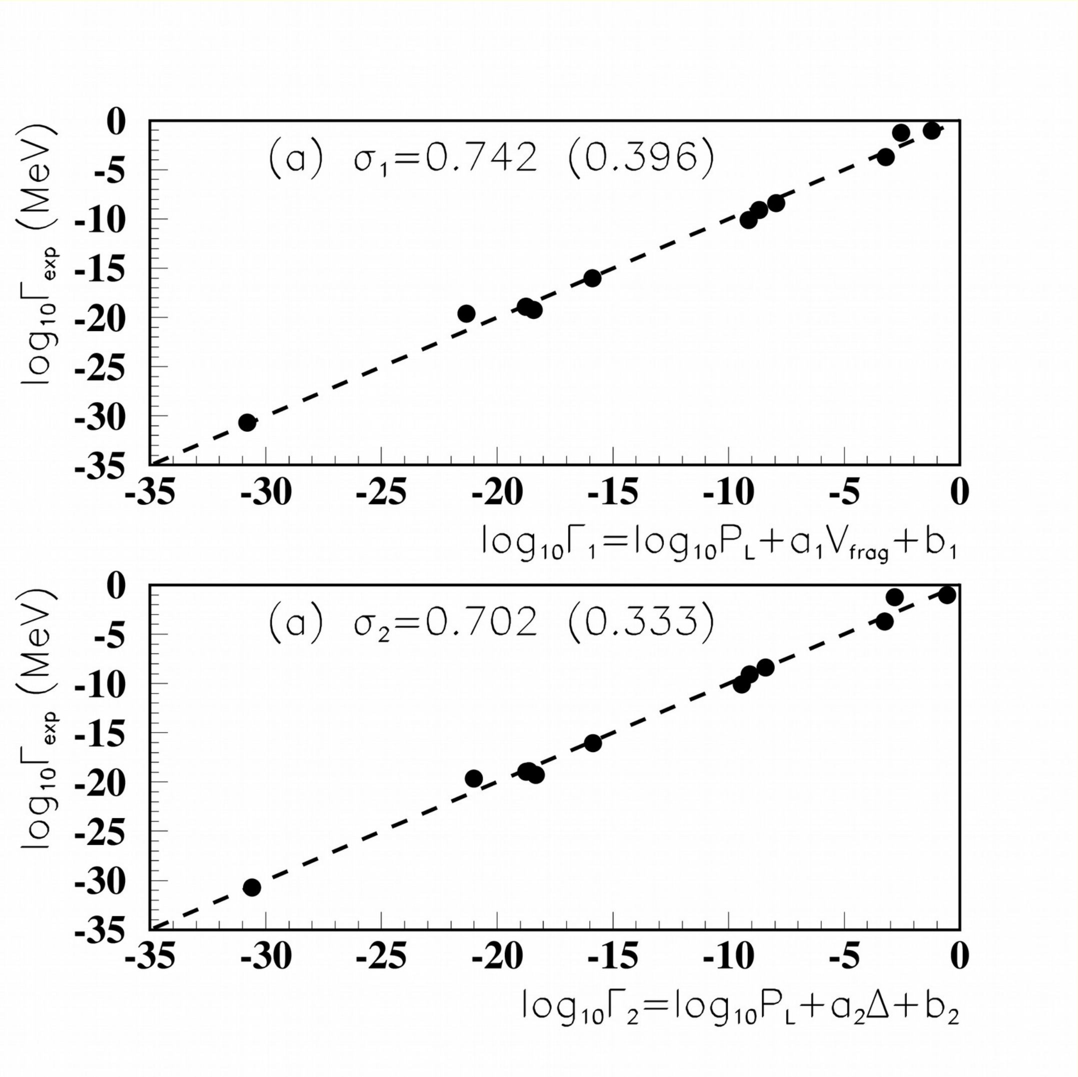
Dependence of the reduced width upon the pairing gap for (a) proton emission and (b) alpha decay



In one-proton emission the reduced width is constant for two regions divided by A=145

In alpha emission the behavior si similar, due to the clustering nature of both processes

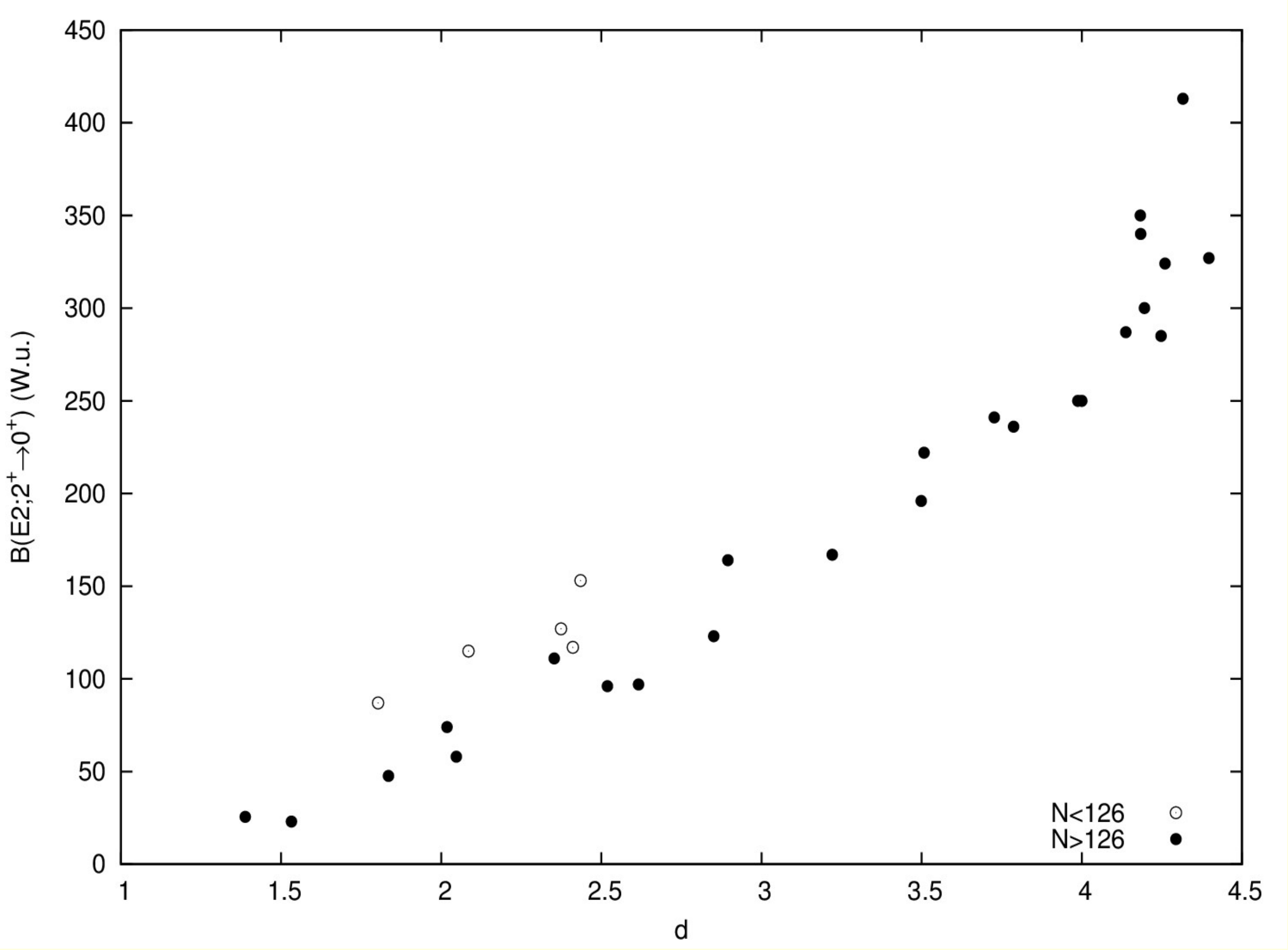
log-log dependence of the decay width by using (a) fragmentation potential and (b) pairing gap



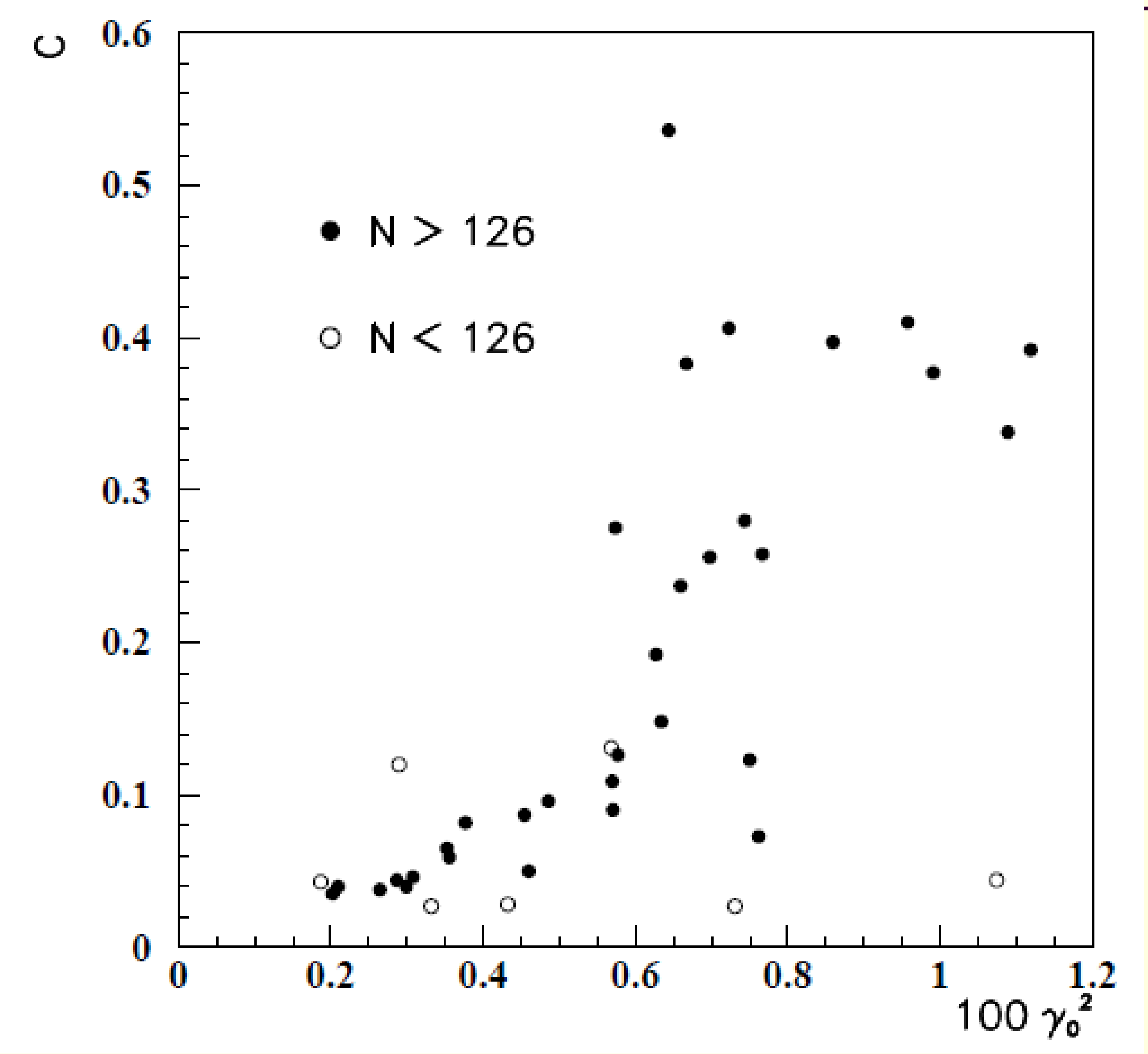
This law has the best predictive power

IV. Alpha versus gamma and beta decays

Electromagnetic B(E:2^+-->0^+) value is proportional with respect to the quadrupole deformation parameter

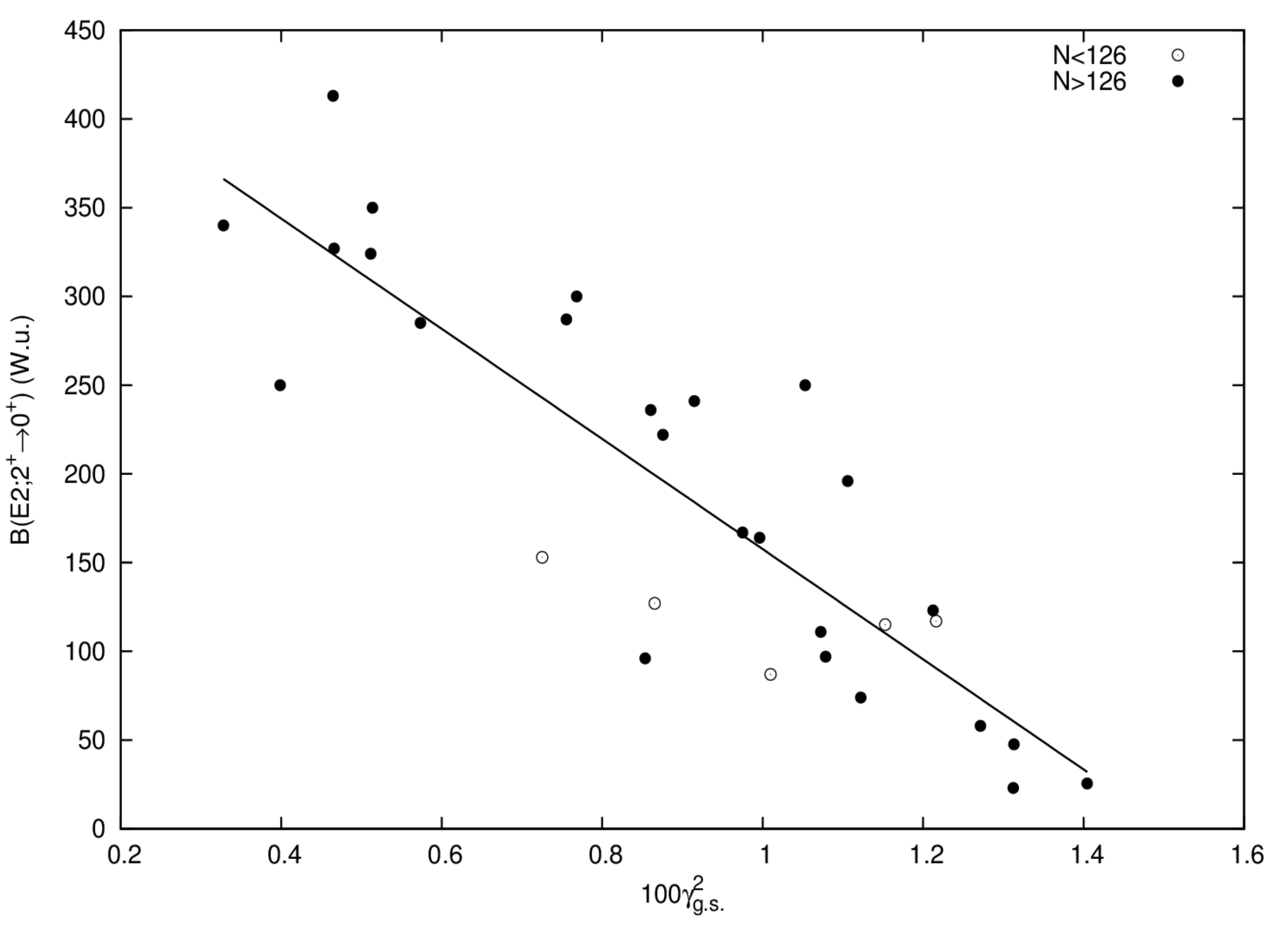


α-daughter QQ coupling strength C reproducing the decay width to the excited 2^+ states is proportional to the reduced width



$$\gamma_0^2 = \frac{\Gamma_0}{2P_0}$$

As a consequence, the collectivity (given by BE2 values) decreases when clustering (given by reduced width) increases toward the magic numbers

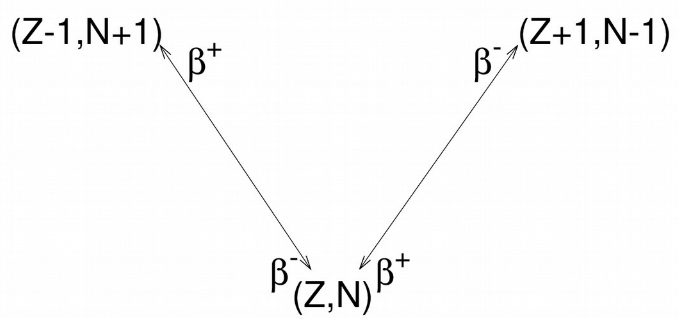


Gamow-Teller beta versus alpha decays

PHYSICAL REVIEW C **100**, 024331 (2019)

Effective axial-vector strength within proton-neutron deformed quasiparticle random-phase approximation

D. S. Delion,[1,2,3] A. Dumitrescu,[1,2] and J. Suhonen[4]



Exp. beta matrix element in terms of the ft-value and axial-vector strength g_A

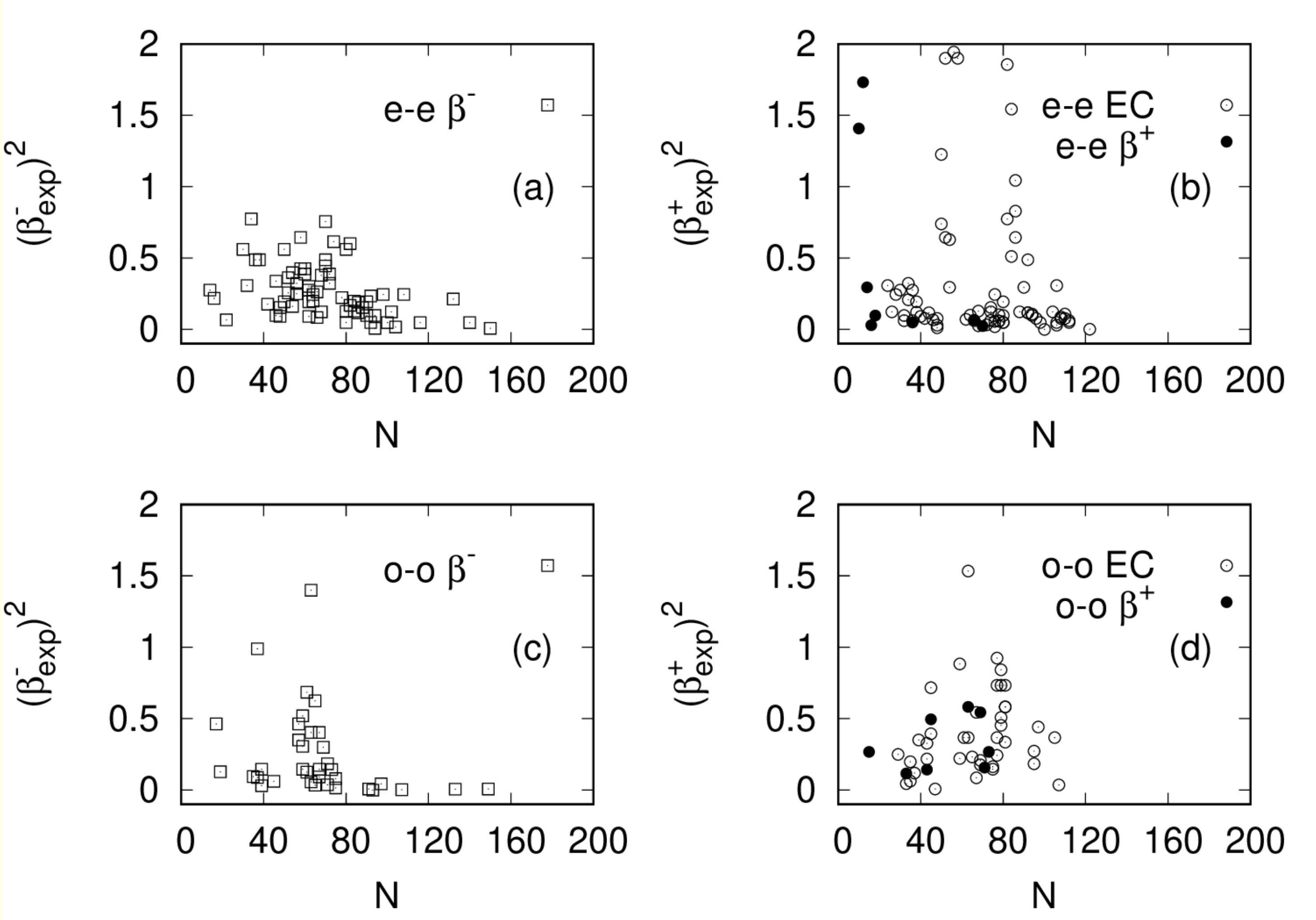
$$g_A \beta^{\pm}_{\rm exp} = \sqrt{\frac{6147\,(2J_i+1)}{10^{\log ft}}},$$

The GT operators are given by

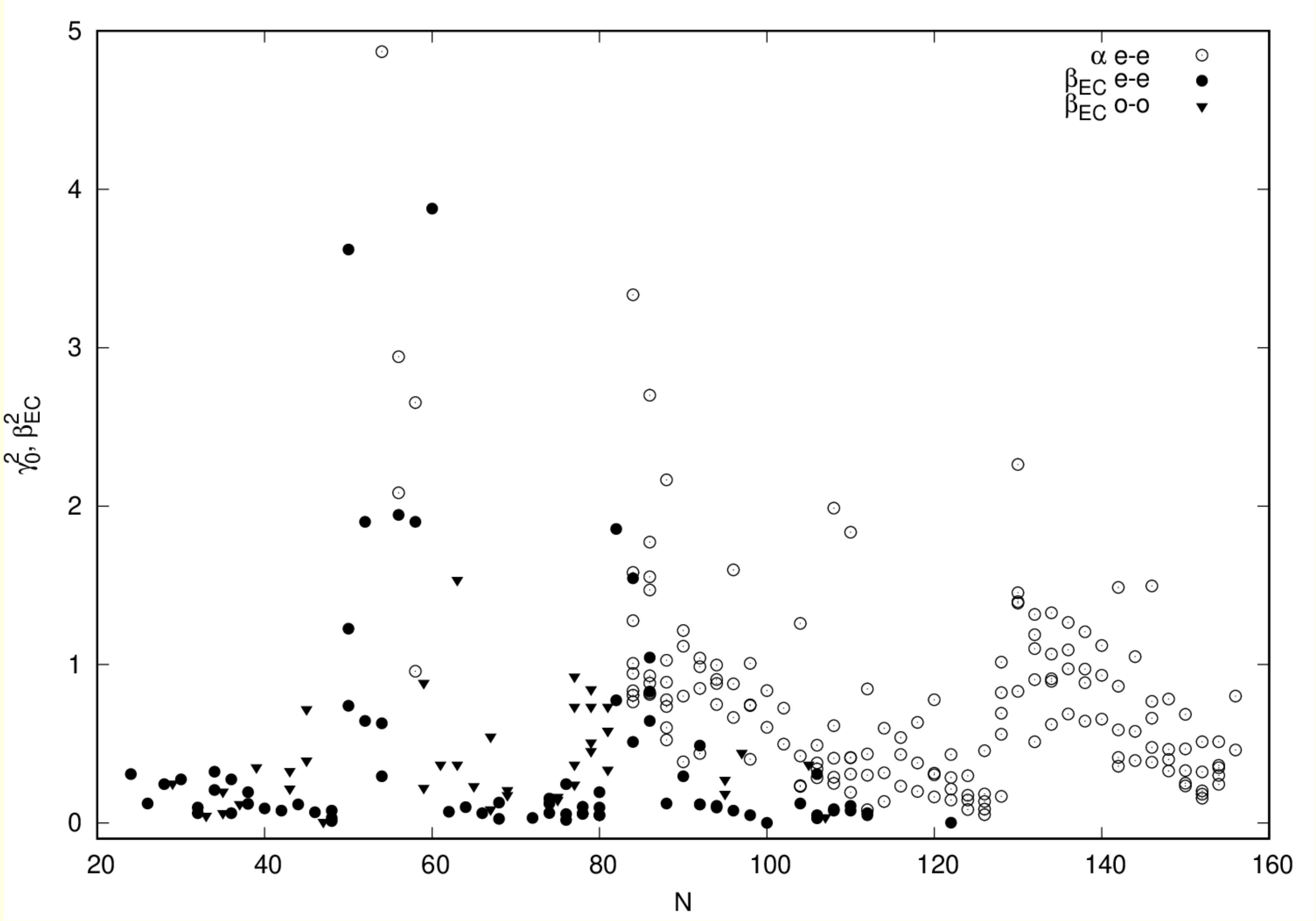
$$D^{-}_{1\mu} = \frac{1}{\sqrt{3}} \sum_{pn} (p\|\sigma\|n) \left[a_p^{\dagger} \otimes \tilde{a}_n\right]_{1\mu}$$

Beta matrix element squared is the analog of the alpha decay reduced width (Coulomb effect is extracted from the decay width)

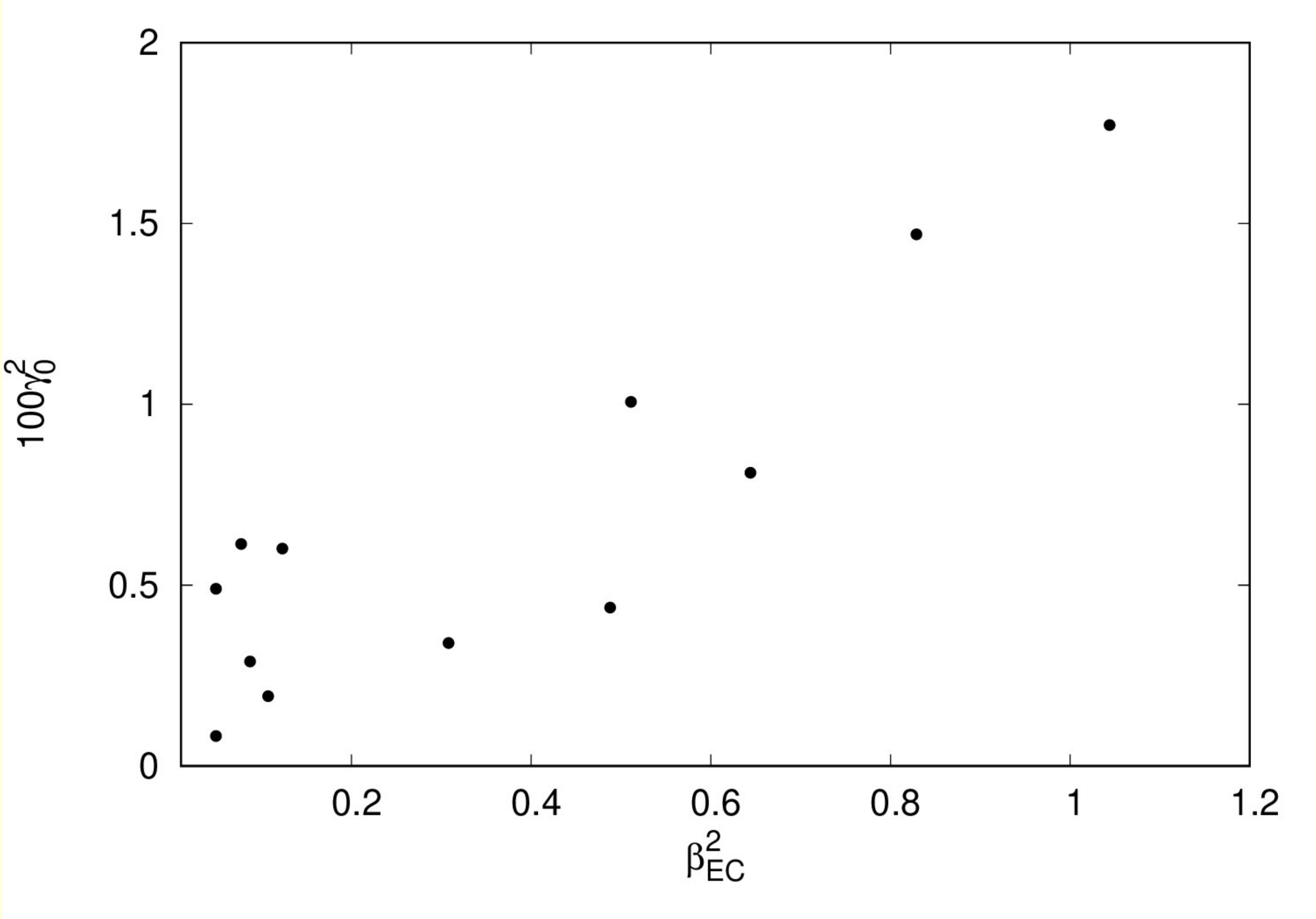
Exp. beta matrix elements squared connecting 0^+ (even-even nuclei) to 1^+ (odd-odd nuclei) are larger above magic numbers N=50, 82



Comparison to alpha reduced widths

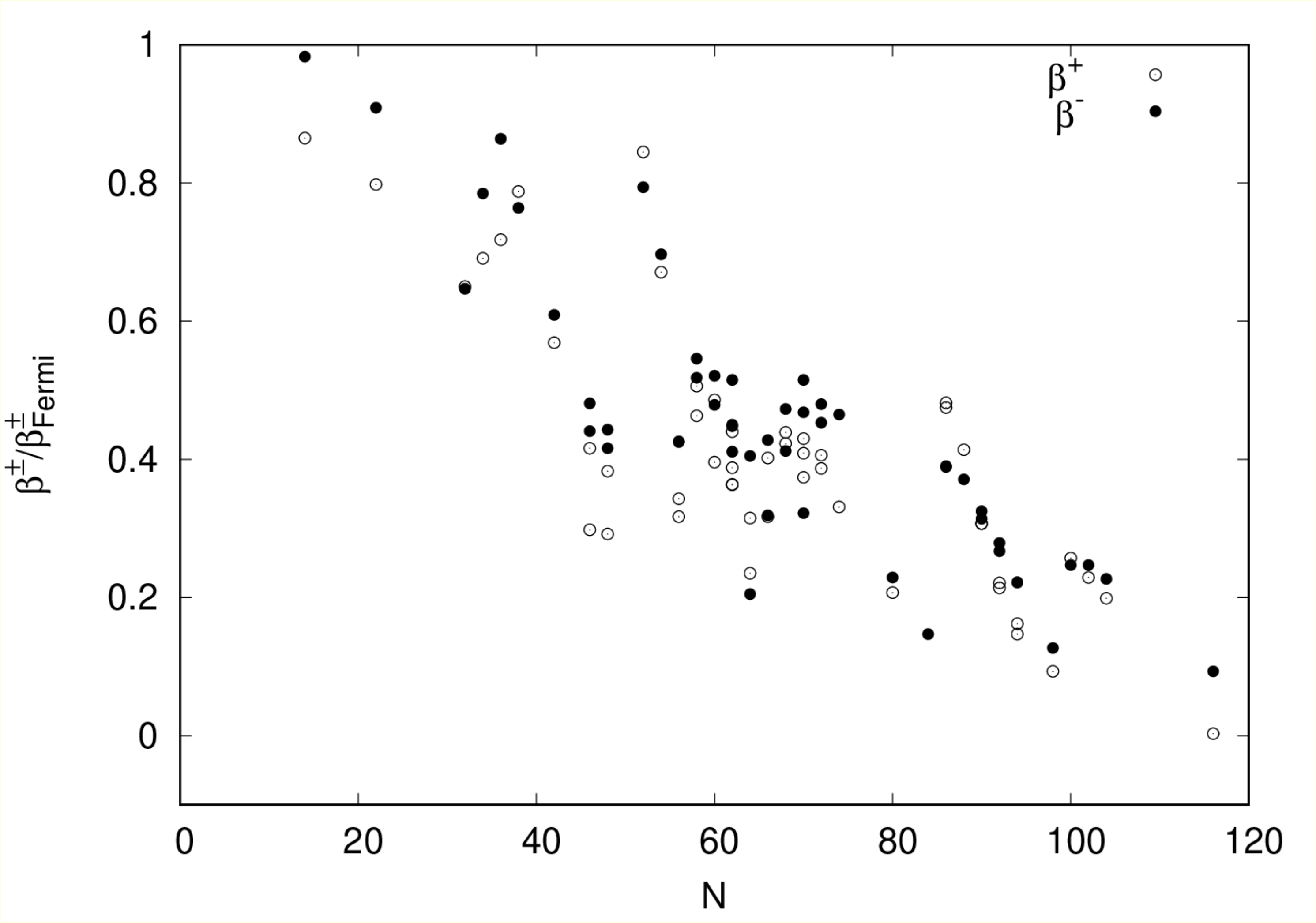


Exp. beta matrix elements squared are proportional to the corresponding alpha reduced widths

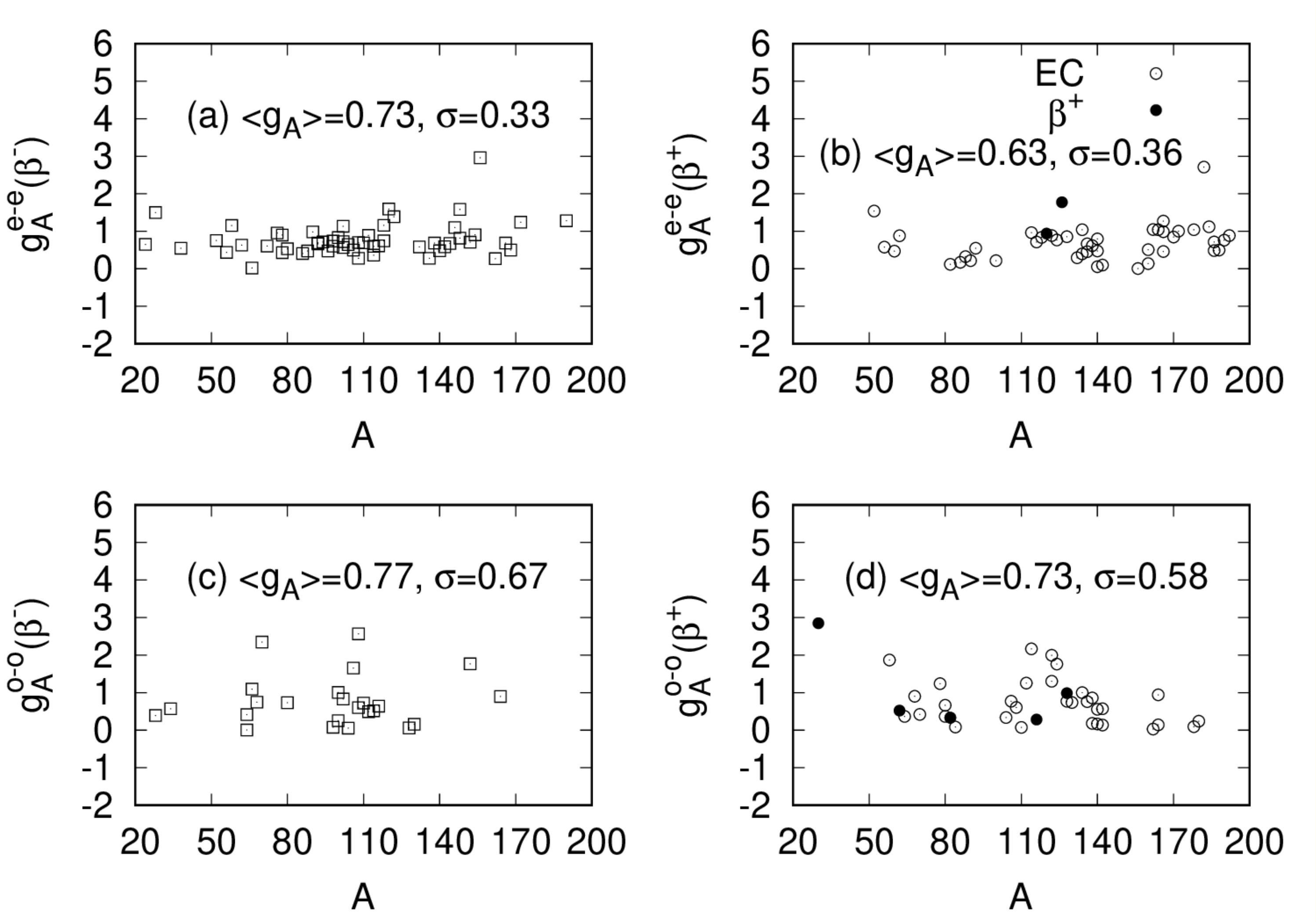


Above magic numbers beta transitions within pn-QRPA are mainly given by the closest to the Fermi level p-n pair

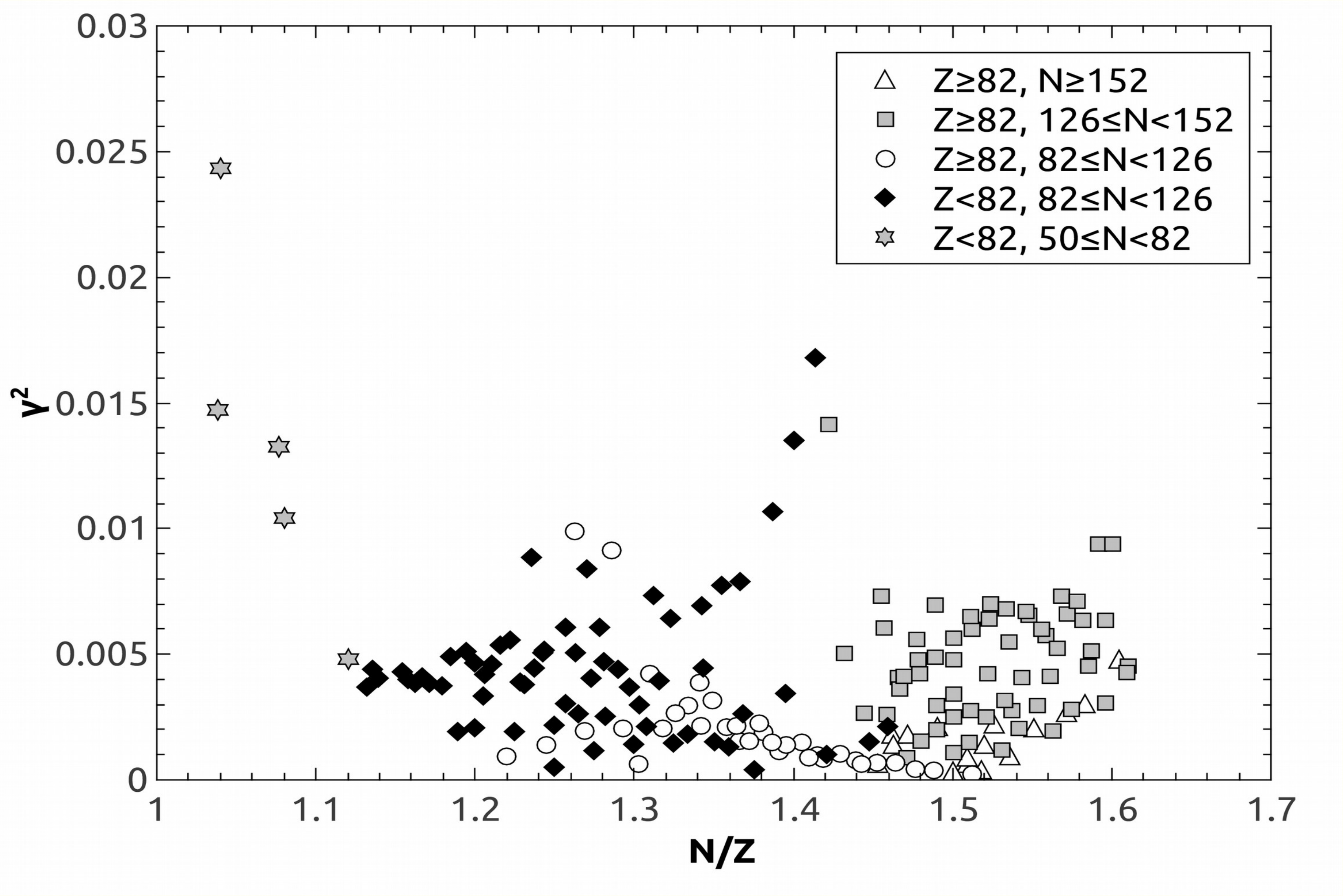
Therefore the few valence nucleons above closed shells mainly contribute to both alpha clustering and beta transitions. Otherwise Pauli antisymmetrisation hinders clustering/beta transitions



As a consequence, the effective axial-vector strength decreases from $g_A \sim 1$ above magic numbers (vacuum value is $g_A=1.25$) to $g_A \sim 0.7$ between shells



V. Proton-neutron correlations
are larger in N~Z nuclei



Formation amplitude is the overlap between parent and daughter * alpha wave functions

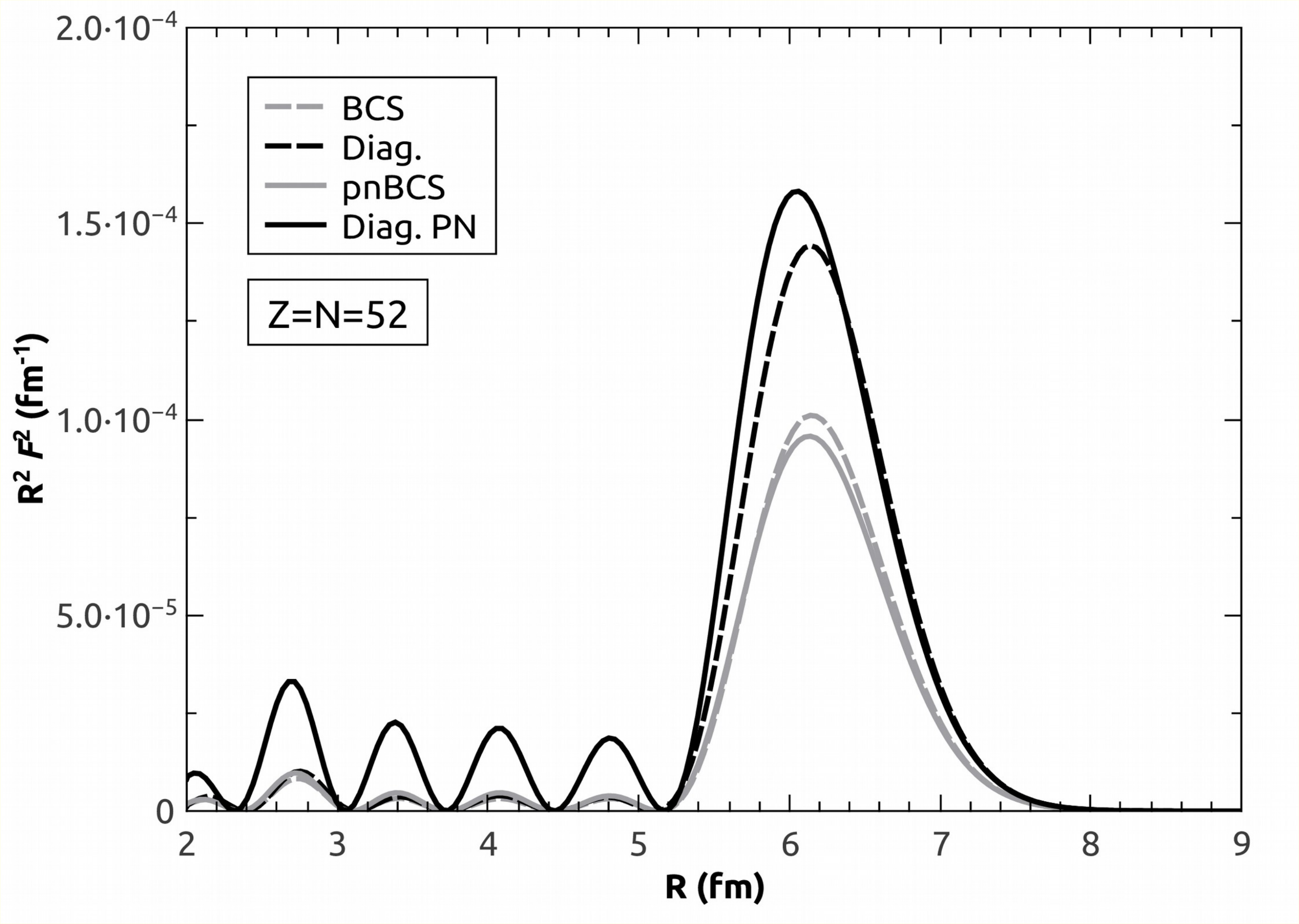
$$\mathscr{F}(\mathbf{R}_\alpha) = \langle \alpha D | P \rangle = \int d\mathbf{x}_\alpha d\mathbf{x}_D \left[\psi_\alpha^{(\beta_\alpha)}(\mathbf{x}_\alpha) \Psi^{(D)}(\mathbf{x}_D) \right]^* \Psi^{(P)}(\mathbf{x}_P)$$

By using the cm and relative coordinates it becomes a superposition of ho orbitals depending on alpha-core radius with four times sp ho parameter 4β

$$\mathscr{F}_\alpha(\mathbf{R}) = \sum_{L_\alpha} \mathscr{F}_{L_\alpha}^{(\alpha)}(\mathbf{R}) = \sum_{L_\alpha} \sum_{N_\alpha} W(N_\alpha L_\alpha) \phi_{N_\alpha L_\alpha M_\alpha}^{(4\beta)}(\mathbf{R}).$$

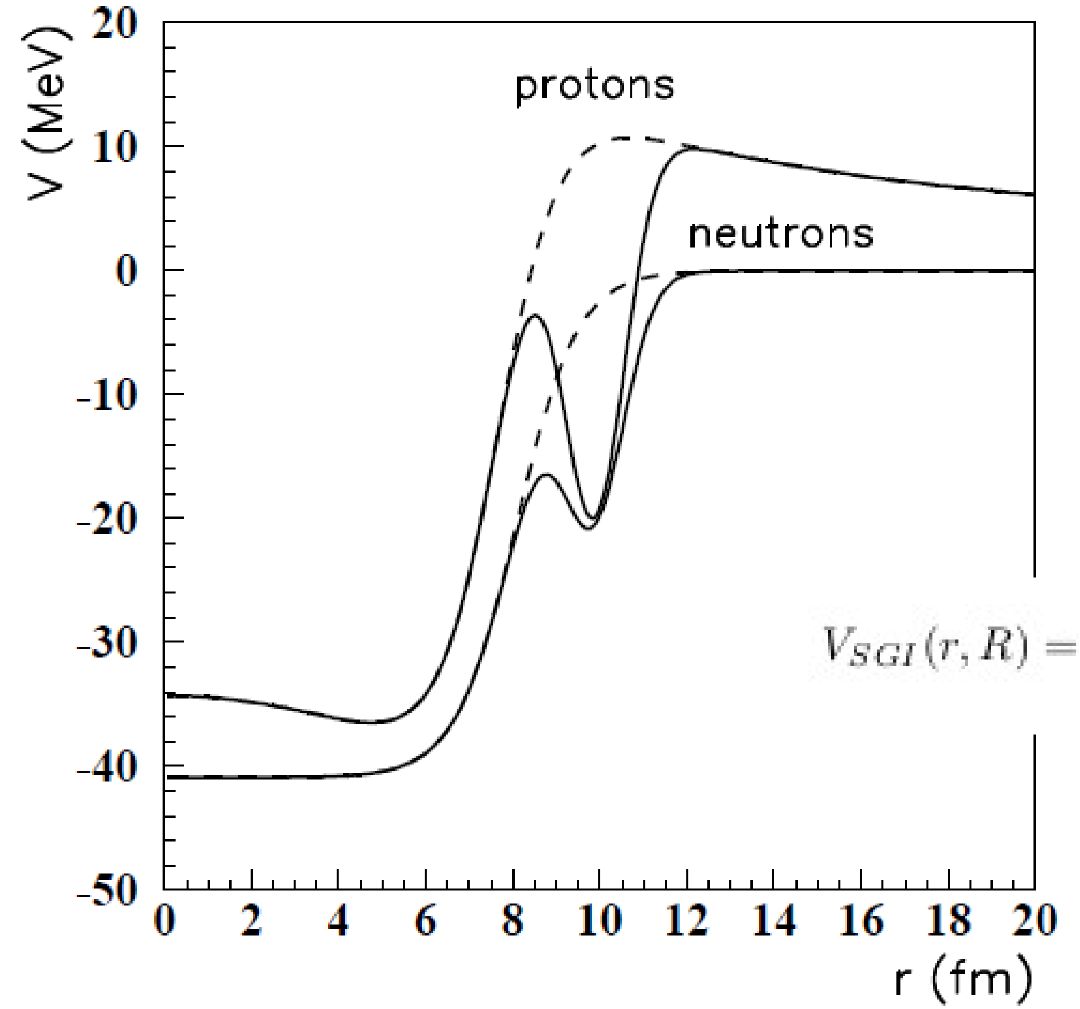
where W-coefficients depend on Nilsson expansion coefficients and BCS amplitudes

Above N=Z=50 formation amplitude is similar for various theoretical approaches. Influence of the proton-neutron pairing on the alpha-formation probability is small, but still the experimental decay width is underestimated.



Woods-Saxon mean field plus a Gaussian surface cluster component enhances the tail of sp orbitals

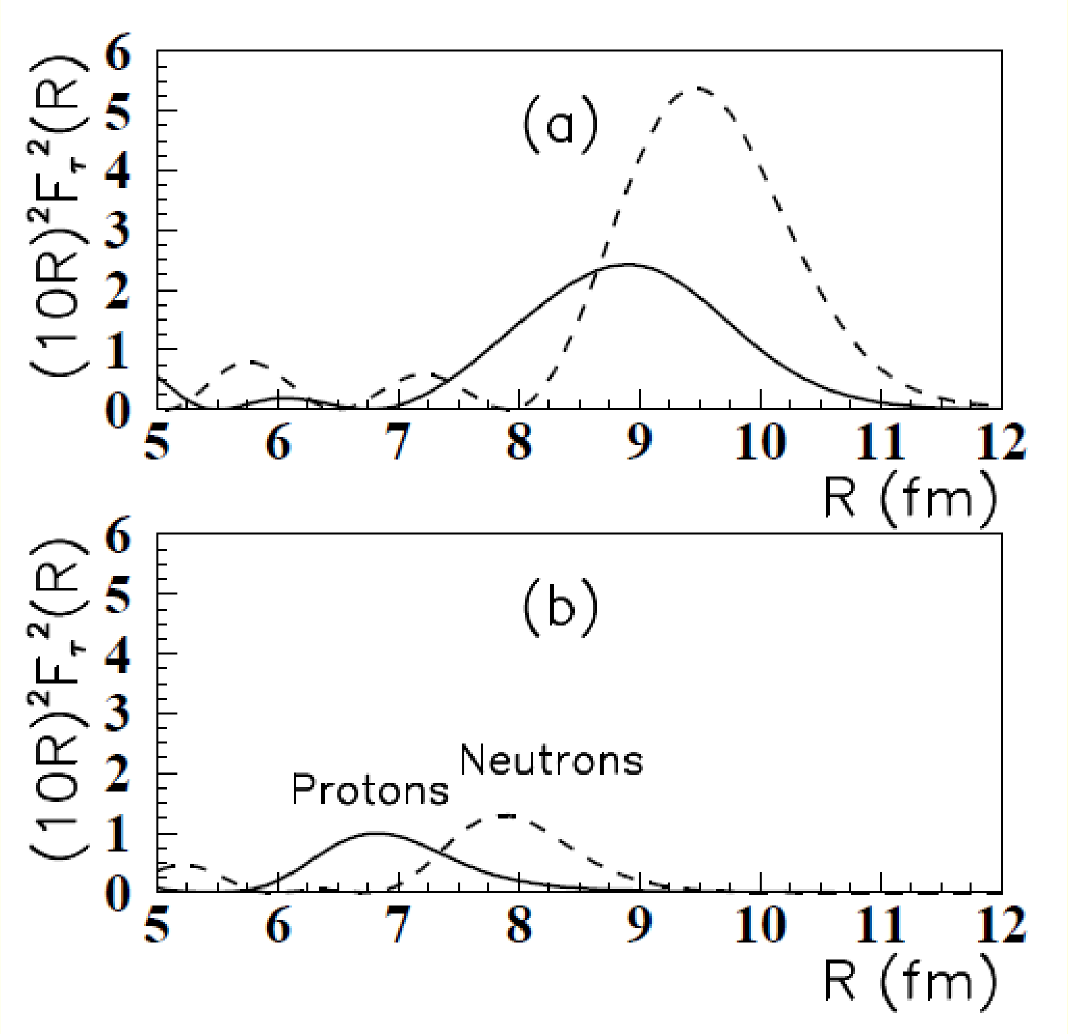
D.S. Delion and R.J. Liotta, **Shell model representation to describe alpha-decay**, Phys. Rev. **C78**, 041302R (2013)



This shape of the potential can be obtained by using the Hartree-Fock procedure with a Surface Gaussian Interaction (SGI) depending on relative (r) and cm (R) coordinates

$$V_{SGI}(r,R) = -v_0 \exp\left(-\frac{r^2}{b_{rel}^2}\right) \exp\left(-\frac{(R-R_0)^2}{b_{cm}^2}\right)$$

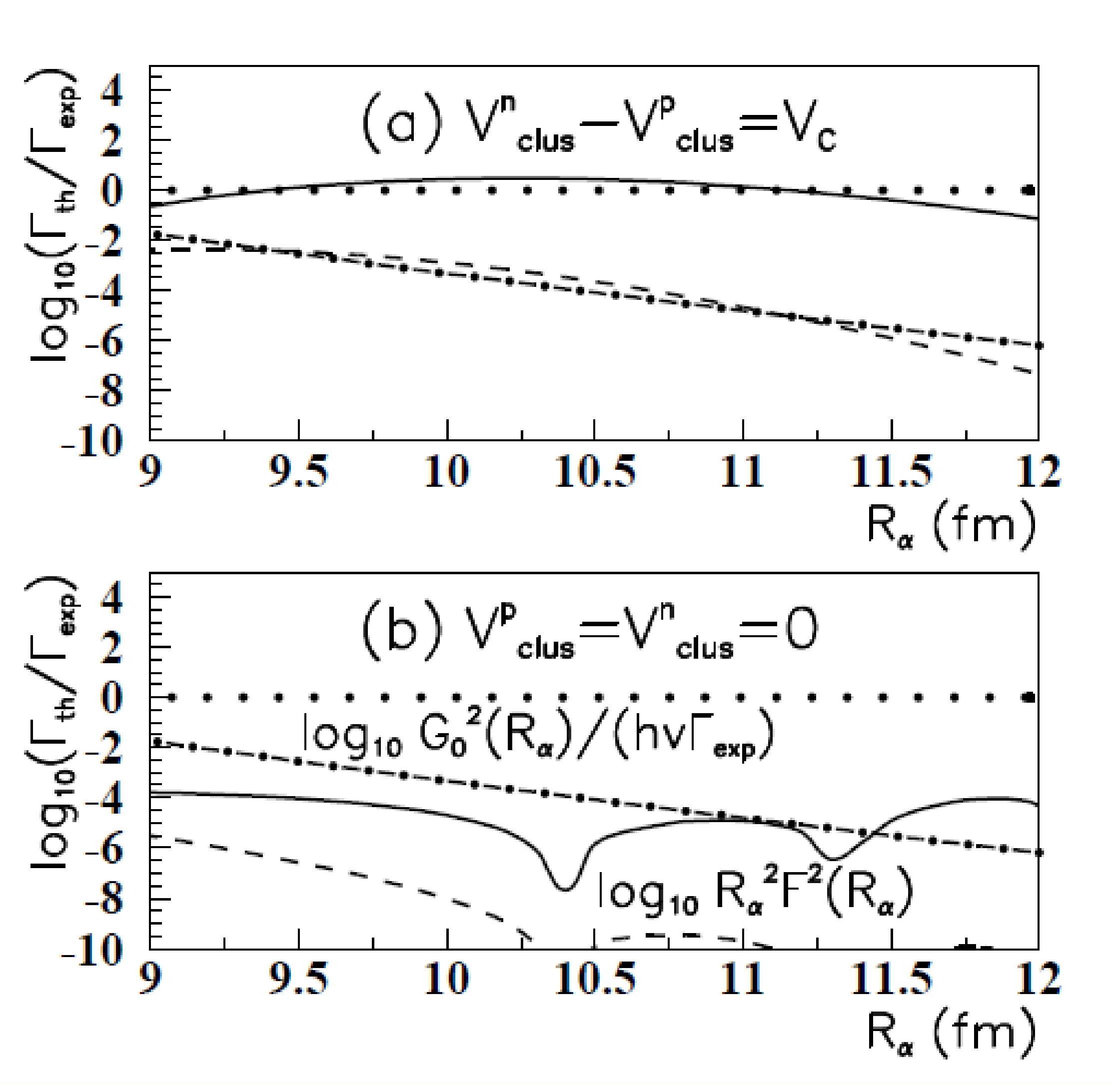
Proton and neutron formation probabilities with cluster component (a) and without cluster component (b)



Cluster component increases the p-n overlap by creating p & n orbitals with the same principal quantum number.

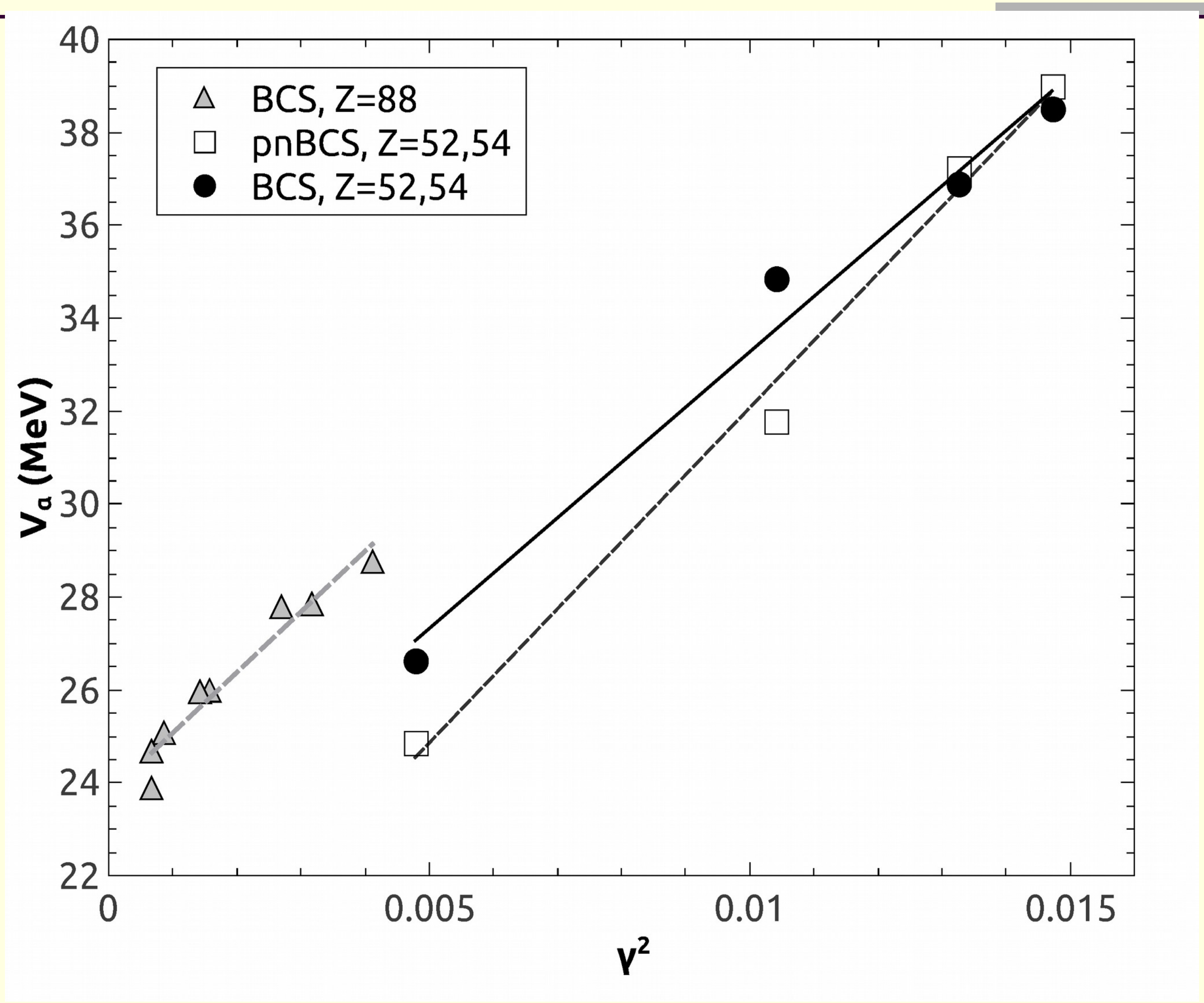
Thus, the effective p-n correlation increases.

Decay width with the cluster component reproduces the exp. value and weakly depends on the cm radius (a). Decay width without the cluster component is by 2 orders of magnitude smaller than the exp. value (b).



Universal behavior of the surface Gaussian potential strength, proportional to the reduced width, for Z>50 and Z>82 regions

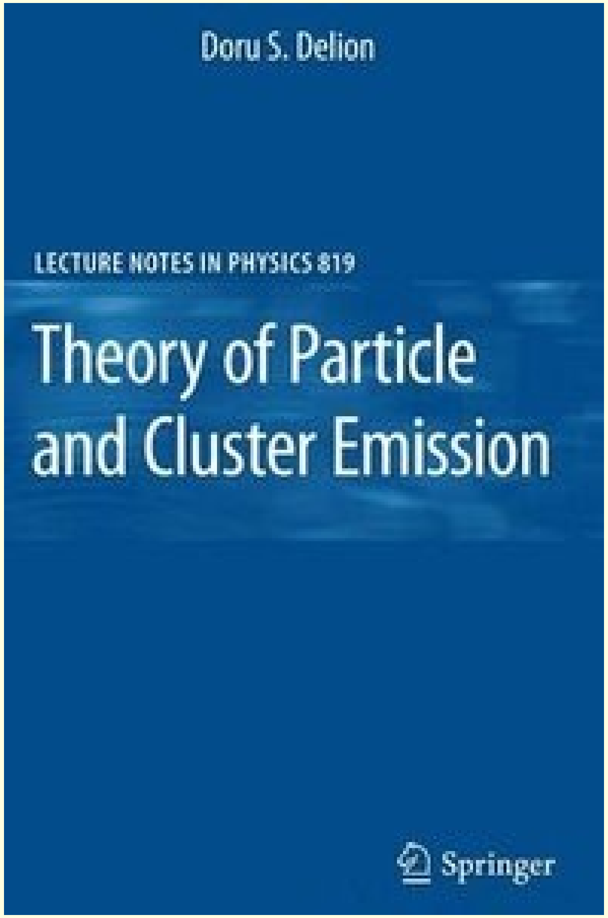
V.V. Baran and D.S. Delion, **Proton-neutron versus alpha-like correlations above ^{100}Sn**, Phys. Rev. **C94**, 034319 (2016)



VI. Conclusions

1) Surface α-daughter interaction leads to
   the universal law for reduced width versus the fragmentation potential
   and for hindrance factors versus the excitation energy
2) Two-proton reduced width satisfies the same decay rules for two-body emission processes
3) Nuclear collectivity linearly decreases when alpha-clustering increases
4) Reduced widths are proportional to beta matrix elements squared:
   clustering and p-n transitions are given by few valence nucleons
   above magic numbers and are hindered by exchange effects between shells
5) Absolute decay widths can be described microscopically by using a mixed
   sp basis, containing additional clustering components
6) Proton-neutron corrrelations have a small influence on the alpha-clustering.
7) Alpha-clustering induced by the surface interaction has an universal behavior
   for both Z>50 and Z>82 regions

THANK YOU !


Doru S. Delion
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