

# ALPHA CLUSTERIZATION FOR MEDIUM AND HEAVY NUCLEI FROM SHELL MODEL

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*Frontiers in Nuclear Structure Theory  
**In honor to Prof. Jan Blomqvist***

*May 23, 2022. Stockholm, Sweden*

# ALPHA CLUSTERIZATION FOR MEDIUM AND HEAVY NUCLEI FROM SHELL MODEL

*Motivations*

*Formalisms*

*Applications*

*Outlooks*

*Frontiers in Nuclear Structure Theory*  
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# MOTIVATIONS

- Obtain four-body clustering from the single-particle degree of freedom
- Towards an unified description of alpha decay from SM:
  - \* Estructure(Cluster) + Reaction(Decay)
- Build an alpha-like wave function for the description of alpha decay at the drip line:
  - \* Four-body correlations in the continuum

# FORMALISM

- Single-particle representations from finite-range potentials, with continuum:
  - \* Berggren basis
- Effective interaction with all spin-isospin channels:
  - \* Four-body correlations
- Weak-coupling approach:
  - \* Correlated two like-nucleons bases
  - \* Five-body Hamiltonian diagonalized in the above bases

# FORMALISM

## Single-particle representation

$$h(\mathbf{r})\psi_{am_a}(\mathbf{r}) = \varepsilon_a \psi_{am_a}(\mathbf{r})$$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{\mathbf{r}}) \right]_{j_a m_a}$$

$$h(\mathbf{r}) = -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{r}}^2 + V_{WS}(r) + V_{so}(r) + V_{Coul}(r)$$

# FORMALISM

## Single-particle representation

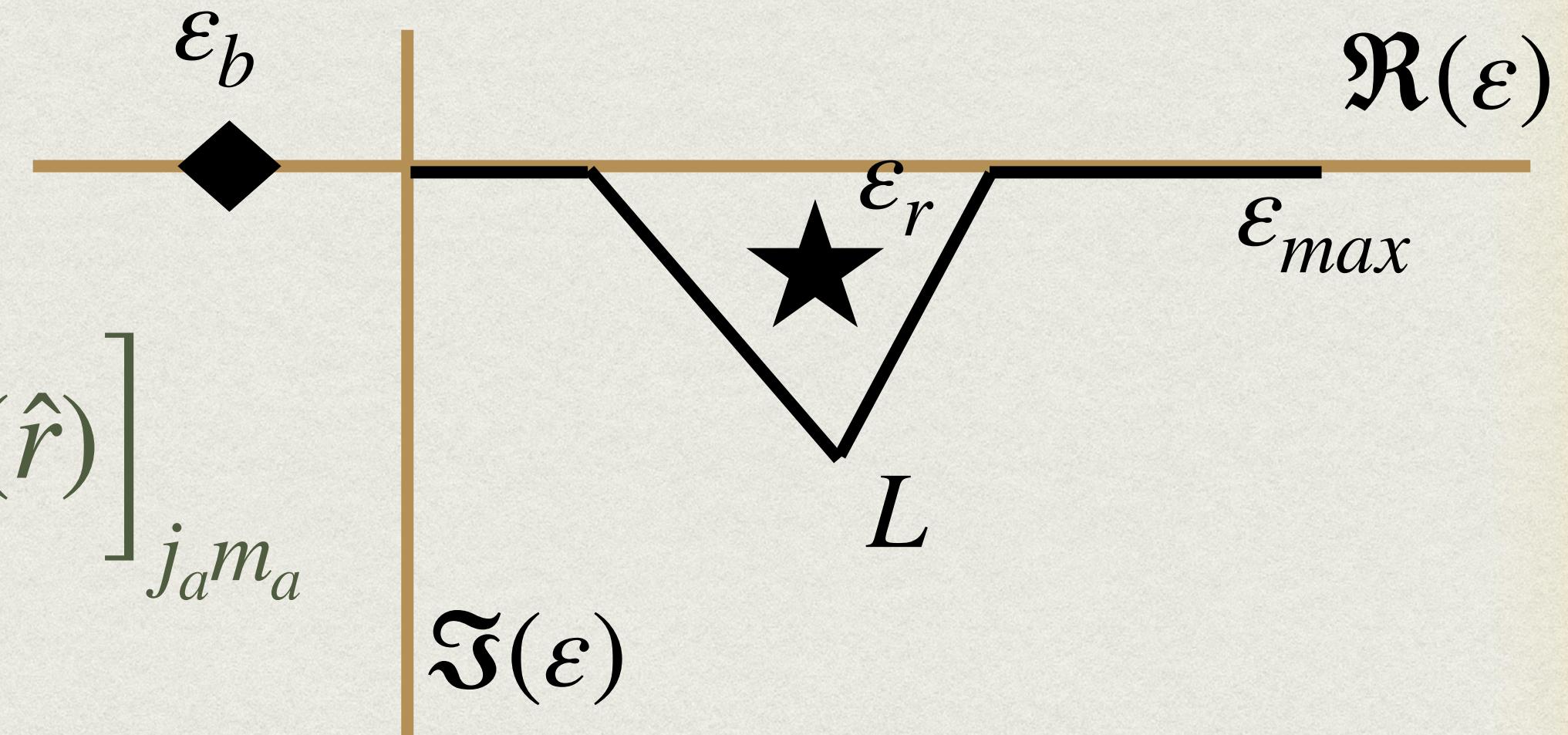
$$h(\mathbf{r})\psi_{am_a}(\mathbf{r}) = \varepsilon_a \psi_{am_a}(\mathbf{r})$$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{\mathbf{r}}) \right]_{j_a m_a}$$

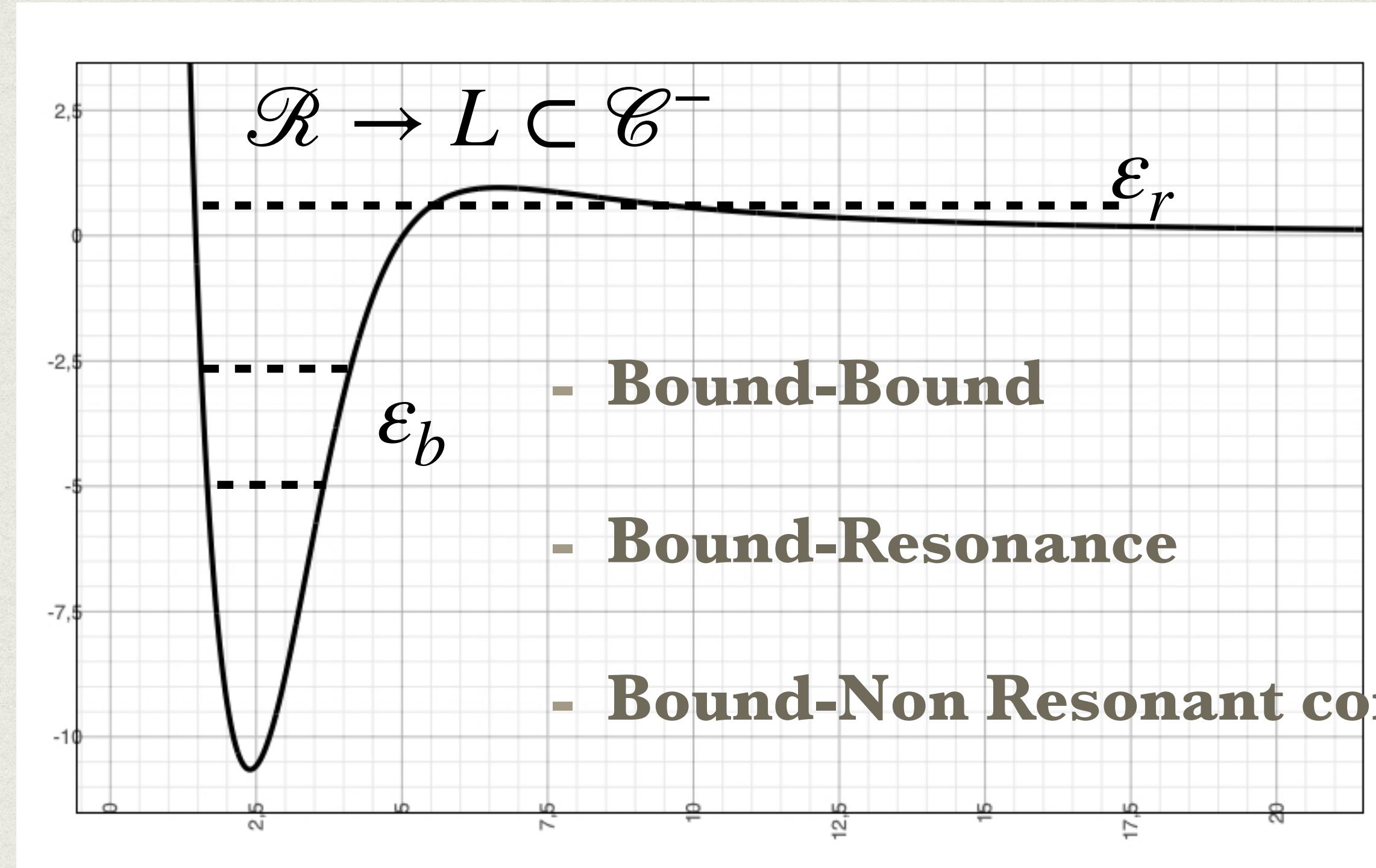
$$h(\mathbf{r}) = -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{r}}^2 + V_{WS}(r) + V_{so}(r) + V_{Coul}(r)$$

$$\delta(r - r') = \sum_{n=n_b, n_r} u_n(r) u_n(r') + \int_{L \subset \mathcal{C}^-} u(\varepsilon, r) u(\varepsilon, r') d\varepsilon$$

Berggren basis

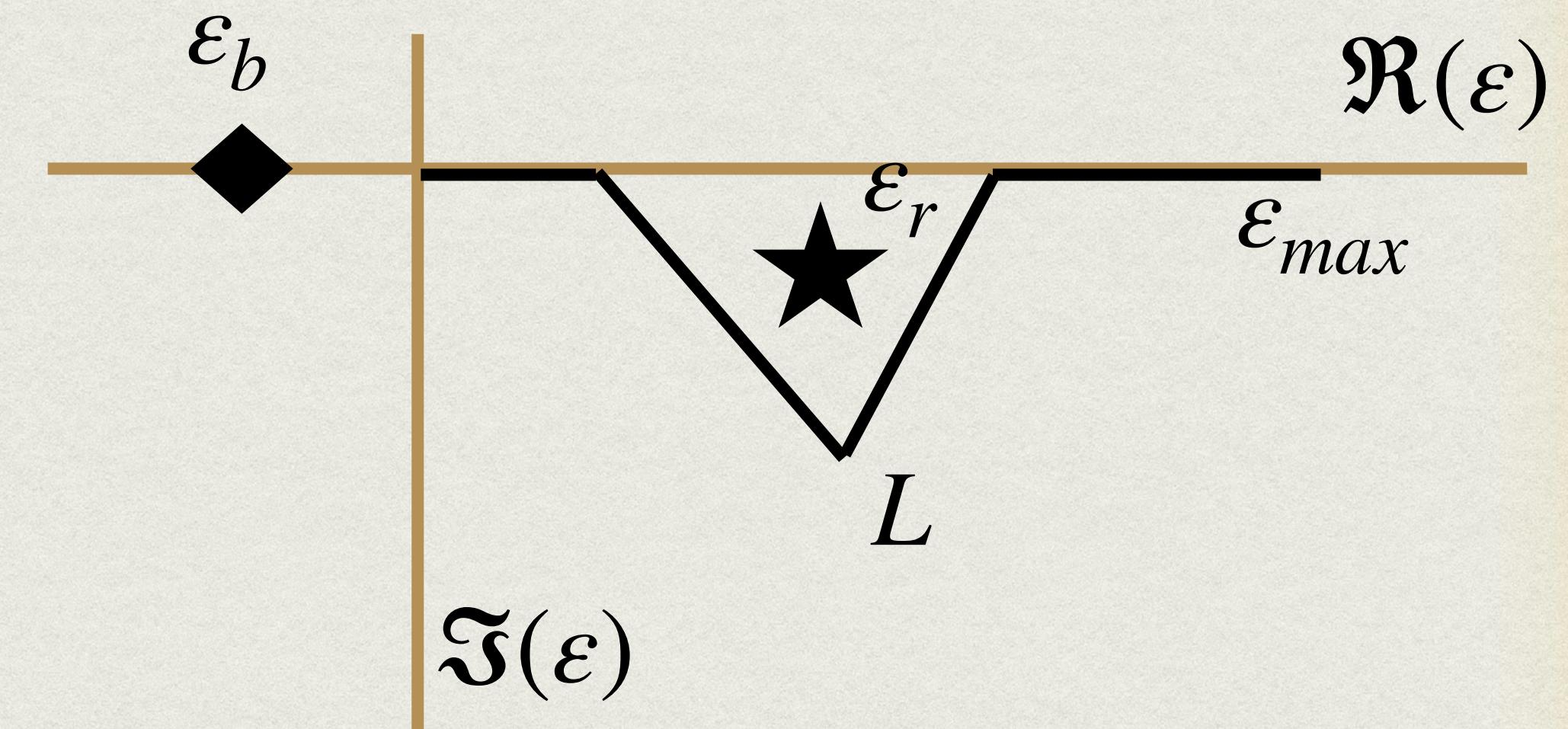


# Correlations



# FORMALISM

## Single-particle representation



- Resonant-Resonant
- Resonant-Non Resonant
- Non Resonant-Non Resonant

# FORMALISM

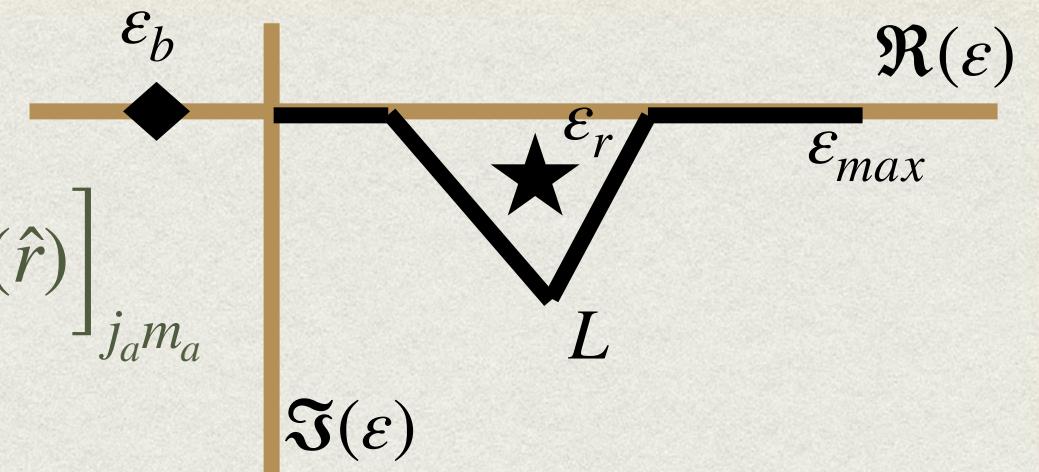
## Two-nucleon interaction

### Central Interaction

$$V(r) = V_{se}(r) P_{se} + V_{to}(r) P_{to} + V_{te}(r) P_{te} + V_{so}(r) P_{so}$$

$T$	$S$	$L$	name	$P$	Definition
1	0	even	singlet-even	$P_{se}$	$\frac{1}{4} (1 - P^\sigma) (1 + P^\tau)$
1	1	odd	triplet-odd	$P_{to}$	$\frac{1}{4} (1 + P^\sigma) (1 + P^\tau)$
0	1	even	triplet-even	$P_{te}$	$\frac{1}{4} (1 + P^\sigma) (1 - P^\tau)$
0	0	odd	singlet-odd	$P_{so}$	$\frac{1}{4} (1 - P^\sigma) (1 - P^\tau)$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{\mathbf{r}}) \right]_{j_a m_a}$$



## Two-nucleon basis

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab}^J \sum_P (-1)^P P \left[ \psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2) \right]_{JM}$$

# FORMALISM

## Two-nucleon ME

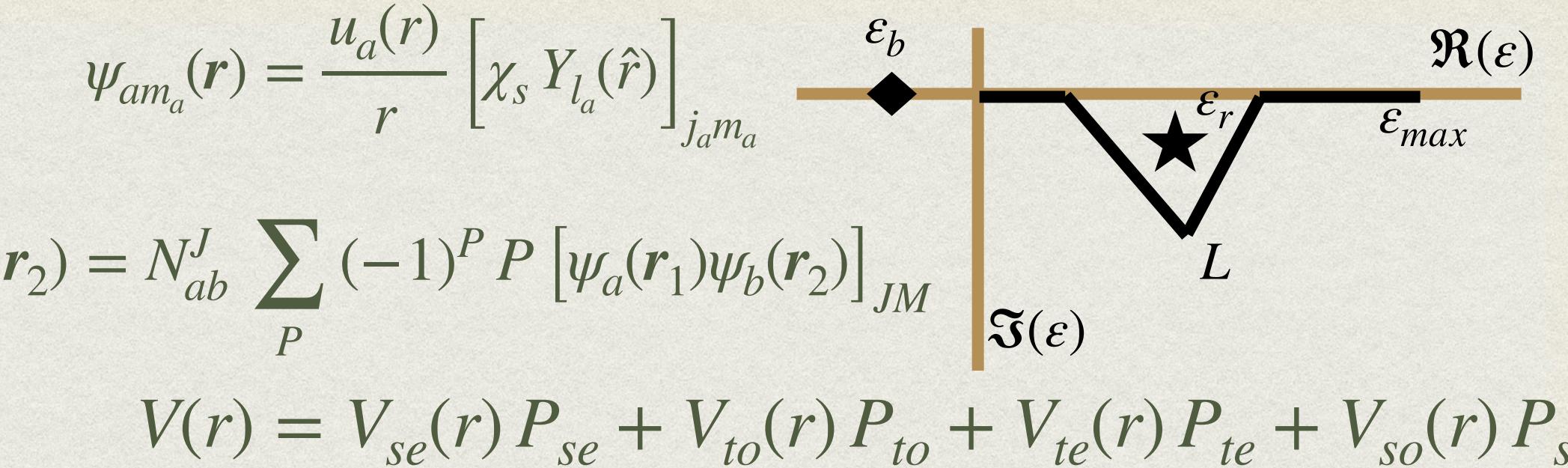
## Two-nucleon ME

$$\langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - (-)^{j_c + j_d - J} P_{cd}] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$

$$P_{cd} | j_c j_d; JM \rangle = | j_d j_c; JM \rangle$$

np ME

$$2N_{ab} N_{cd} [1 - (-)^{j_c + j_d - J} P_{cd}] = 1$$



**Wigner-Singlet projectors**

$$P_{se} = \frac{1}{2} [1 - (-)^{j_c + j_d + J} P_{cd}] P_S$$

$$P_{to} = \frac{1}{2} \left\{ [1 - (-)^{j_c + j_d + J} P_{cd}] - [1 - (-)^{j_c + j_d + J} P_{cd}] P_S \right\}$$

$$P_{te} = \frac{1}{2} \left\{ [1 + (-)^{j_c + j_d + J} P_{cd}] - [1 + (-)^{j_c + j_d + J} P_{cd}] P_S \right\}$$

$$P_{so} = \frac{1}{2} [1 + (-)^{j_c + j_d + J} P_{cd}] P_S$$

# FORMALISM

## Two-nucleon ME

### Effective interaction

$$V_\tau(r) = V_\tau e^{-\frac{r^2}{\beta_\tau^2}}$$

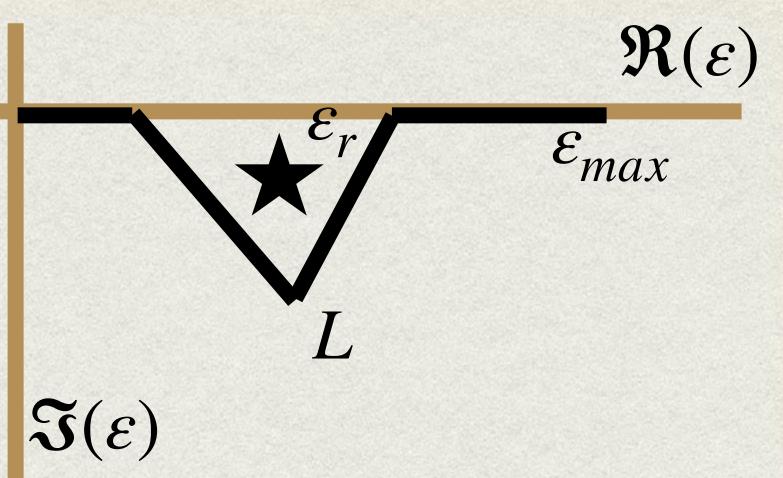
$$r = |\bar{r}_1 - \bar{r}_2|$$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P \left[ \psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2) \right]_{JM}$$

$$V(r) = \sum_\tau V_\tau(r) P_\tau$$

$$\langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} \left[ 1 - (-)^{j_c + j_d - J} P_{cd} \right] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$



### Two-nucleon WF

$$\Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2)$$

$$\sum_{b \leq a} (X_{ab}^{J^\pi})^2 = 1$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M}$$

Berggren 'metric'

$$H_n = h_n(\mathbf{r}_1) + h_n(\mathbf{r}_2) + V(\mathbf{r}_1, \mathbf{r}_2)$$

$$H_p = h_p(\mathbf{r}_3) + h_p(\mathbf{r}_4) + V(\mathbf{r}_3, \mathbf{r}_4) + V_{Coul}$$

# FORMALISM

**Weak-coupling approach**

Glendenning & Harada, NP72(1965)481

## Four-body Hamiltonian

$$\mathcal{H} = H_n + H_p + V_{np}$$

$$V_{np} = V(\mathbf{r}_1, \mathbf{r}_3) + V(\mathbf{r}_1, \mathbf{r}_4) + V(\mathbf{r}_2, \mathbf{r}_3) + V(\mathbf{r}_2, \mathbf{r}_4)$$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_\tau V_\tau(r) P_\tau \quad \langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

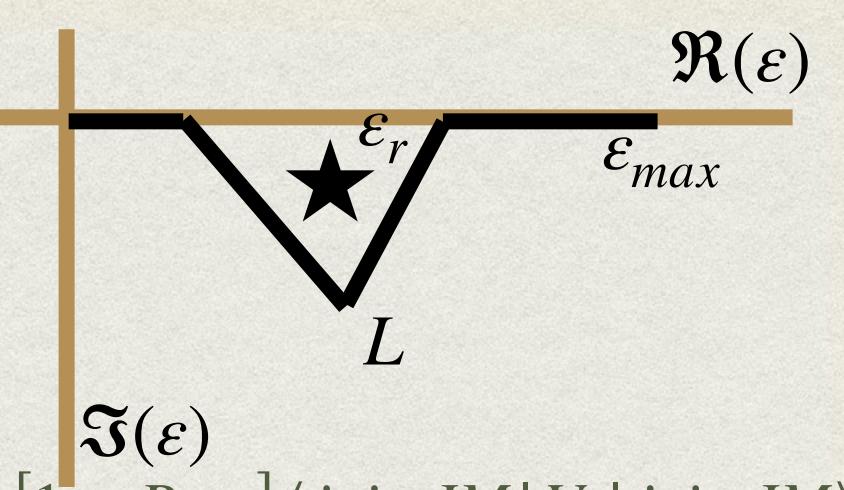
## Four-body basis

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^{\pi_n}} \Psi_{J_p^{\pi_p}}]_{J^\pi M} = | J_n J_p, J^\pi \rangle$$

## Four-body WF

$$| \Psi_{J^\pi M} \rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} | J_n J_p, J^\pi \rangle$$

$$\mathcal{H} | \Psi_{J^\pi M} \rangle = E_{J^\pi} | \Psi_{J^\pi M} \rangle$$



# FORMALISM

**Weak-coupling approach**

## Four-body Hamiltonian

$$\mathcal{H} = H_n + H_p + V_{np}$$

$$V_{np} = V(\mathbf{r}_1, \mathbf{r}_3) + V(\mathbf{r}_1, \mathbf{r}_4) + V(\mathbf{r}_2, \mathbf{r}_3) + V(\mathbf{r}_2, \mathbf{r}_4)$$

$$\sum_{J'_n J'_p} \left[ (E_{J_n} + E_{J_p}) \delta_{J'_n J_n} \delta_{J'_p J_p} + \langle J_n J_p, J^\pi | V_{np} | J'_n J'_p, J^\pi \rangle \right] Z_{J'_n J'_p}^{J^\pi} = E_{J^\pi} Z_{J_n J_p}^{J^\pi}$$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_\tau V_\tau(r) P_\tau \quad \langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

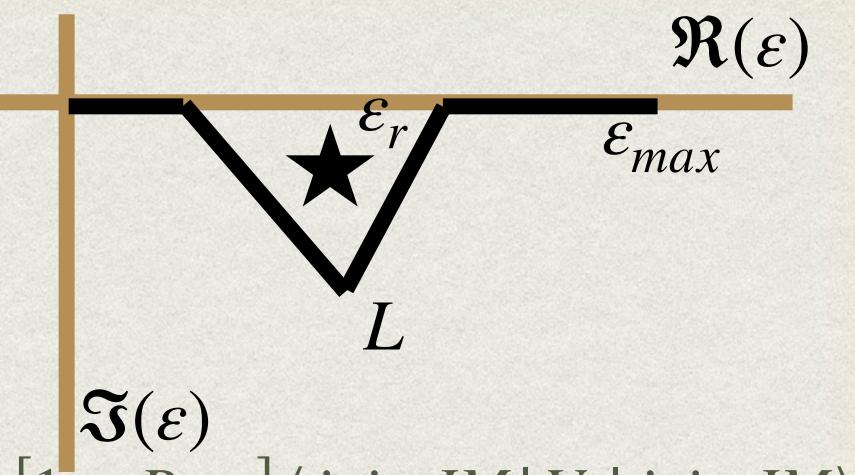
## Four-body basis

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^{\pi_n}} \Psi_{J_p^{\pi_p}}]_{J^\pi M} = | J_n J_p, J^\pi \rangle$$

## Four-body WF

$$| \Psi_{J^\pi M} \rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} | J_n J_p, J^\pi \rangle$$

$$\mathcal{H} | \Psi_{J^\pi M} \rangle = E_{J^\pi} | \Psi_{J^\pi M} \rangle$$



# FORMALISM

## Optimization

## Single-particle

$$h(\mathbf{r}) = -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{r}}^2 + V_{WS}(r) + V_{so}(r) + V_{Coul}(r)$$

$\{r_0, r_c\} \rightarrow \{\text{matter } r_{rms}, \text{ proton } r_{rms}\}$

$\{a, V_{WS}, V_{so}\} \rightarrow \chi^2$  minimization

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi$$

$$H = h + h + V + V_{Coul}$$

## Two-particle

$$V(r) = V_{se}(r) P_{se} + V_{to}(r) P_{to} + V_{te}(r) P_{te} + V_{so}(r) P_{so}$$

$$V_{\tau}(r) = V_{\tau} e^{-\frac{r^2}{\beta_{\tau}^2}}$$

$\{\beta_{\tau}\} \rightarrow \{\text{Ref. } 208\text{Pb, } 40\text{Ca by residues}\}$

$\{P_{\tau}\} \rightarrow \chi^2$  optimization  $nn, pp, np$

$\{P_{\tau}\} \rightarrow \chi^2$  optimization isospin 'conserving'

T	S	L	V	Affects to...	$(-)^{S+T+L} = -1$
1	0	even	$V_{se}$	$J$ even in $nn, pp$ y $np$	
1	1	odd	$V_{to}$	$J$ even and odd in $nn, pp$ y $np$	
0	1	even	$V_{te}$	$J$ even and odd in $np$	
0	0	odd	$V_{so}$	$J$ odd in $np$	

# FORMALISM

## Optimization

### Penalty function

$$\chi^2(p) = \sum_i \left[ O_i^{\text{cal}}(p) - \mathcal{O}_i^{\text{exp}} \right]$$

$$\frac{\partial \chi^2}{\partial p} \rightarrow \frac{\partial O_i}{\partial p_\alpha} = \frac{\partial \langle \phi_i | O | \phi_i \rangle}{\partial p_\alpha}$$

$$= \langle \phi_i | \frac{\partial O}{\partial p_\alpha} | \phi_i \rangle$$

### Method

Levenberg-Marquardt

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_\tau V_\tau(r) P_\tau \quad \langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{\text{exch}}] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi$$

### Single-particle(s.p.)

$$O \rightarrow h(\mathbf{r}) = -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{r}}^2 + V_{WS}(r) + V_{so}(r) + V_{Coul}(r)$$

$$p \rightarrow \{a, V_{WS}, V_{so}\} \rightarrow \chi^2 \text{ optimized}$$

### Two-particle(t.p.)

$$O \rightarrow V(r) = V_{se}(r) P_{se} + V_{to}(r) P_{to} + V_{te}(r) P_{te} + V_{so}(r) P_{so}$$

$$V_\tau(r) = V_\tau e^{-\frac{r^2}{\beta_\tau^2}}$$

$$p \rightarrow \{P_\tau\} \rightarrow \chi^2 \text{ optimized}$$

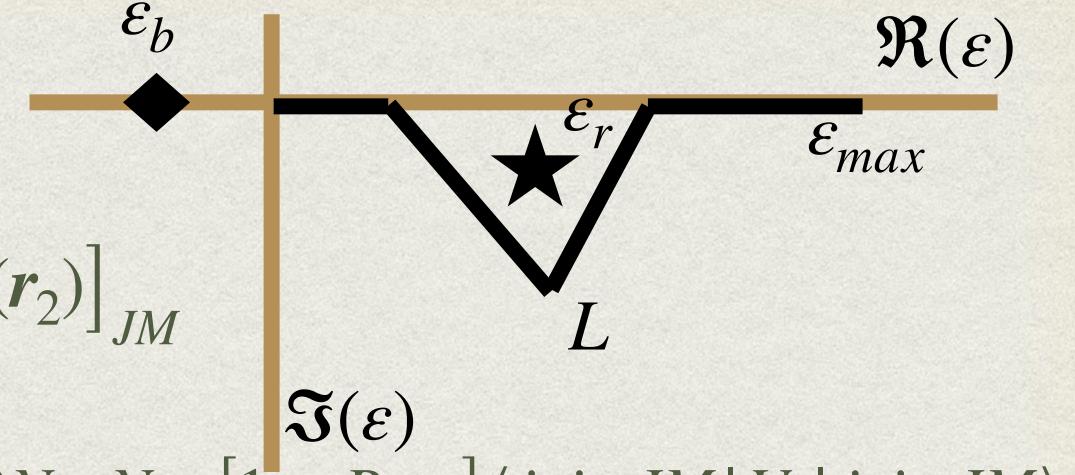
# APLICACIONES

- $^{212}\text{Po}$
- $^{44}\text{Ti}$

# APPLICATIONS

## $^{212}\text{Po}$ Mean-field

→

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$


$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^{\pi_n}} \Psi_{J_p^{\pi_p}}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

N. K. Glendenning and K. Harada, Nuclear Physics 72 (1965) 481

Blomqvist and Wahlborn<sup>7)</sup> have calculated the eigenfunctions in a Woods-Saxon potential for the single-particle levels around  $^{208}\text{Pb}$ . To determine  $\nu$ , we postulated the following condition:

$$\int r^2 \psi_{\text{h.o.}}^2 d\mathbf{r} = \int r^2 \psi_{\text{BW}}^2 d\mathbf{r}, \quad (1)$$

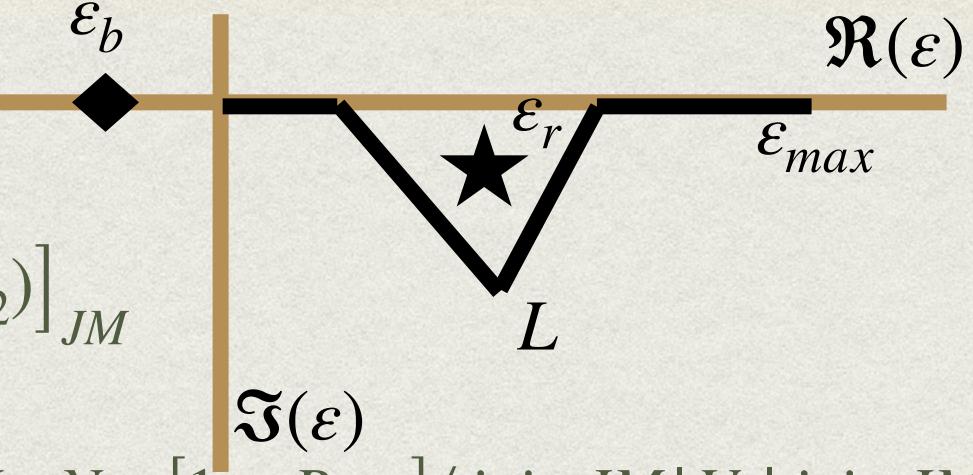
where  $\psi_{\text{h.o.}}$  and  $\psi_{\text{BW}}$  are respectively the harmonic and Blomqvist-Wahlborn wave

<sup>†</sup> In calculating  $\gamma^2$  we use the Blomqvist-Wahlborn radial functions which are needed at  $R_0$ , to avoid the incorrect asymptotic behaviour of the harmonic oscillator functions.

# APPLICATIONS

## $^{212}\text{Po}$ Single-particle energies

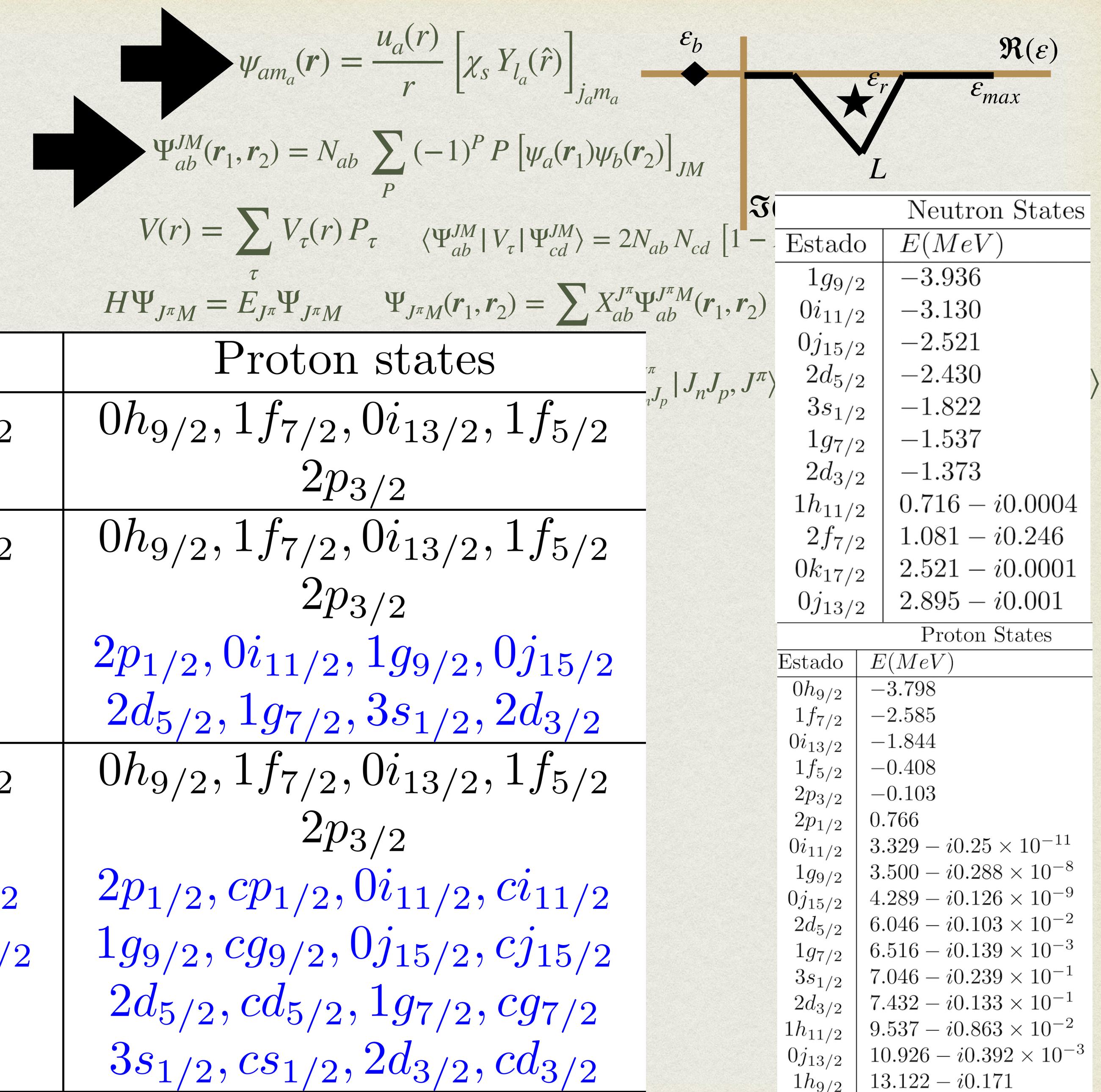
Neutron States			Proton States		
Estado	$E(\text{MeV})$	$\text{Rel. Err.}$	Estado	$E(\text{MeV})$	$\text{Rel. Err.}$
$1g_{9/2}$	-3.936	0.03%	$0h_{9/2}$	-3.798	0.02%
$0i_{11/2}$	-3.130	0.9%	$1f_{7/2}$	-2.585	0.05%
$0j_{15/2}$	-2.521	0.3%	$0i_{13/2}$	-1.844	0.05%
$2d_{5/2}$	-2.430	2.4%	$1f_{5/2}$	-0.408	0.25%
$3s_{1/2}$	-1.822	4.5%	$2p_{3/2}$	-0.103	84%
$1g_{7/2}$	-1.537	6.4%	$2p_{1/2}$	0.766	
$2d_{3/2}$	-1.373	2%	$0i_{11/2}$	$3.329 - i0.25 \times 10^{-11}$	
$1h_{11/2}$	$0.716 - i0.0004$		$1g_{9/2}$	$3.500 - i0.288 \times 10^{-8}$	
$2f_{7/2}$	$1.081 - i0.246$		$0j_{15/2}$	$4.289 - i0.126 \times 10^{-9}$	
$0k_{17/2}$	$2.521 - i0.0001$		$2d_{5/2}$	$6.046 - i0.103 \times 10^{-2}$	
$0j_{13/2}$	$2.895 - i0.001$		$1g_{7/2}$	$6.516 - i0.139 \times 10^{-3}$	
			$3s_{1/2}$	$7.046 - i0.239 \times 10^{-1}$	
			$2d_{3/2}$	$7.432 - i0.133 \times 10^{-1}$	
			$1h_{11/2}$	$9.537 - i0.863 \times 10^{-2}$	
			$0j_{13/2}$	$10.926 - i0.392 \times 10^{-3}$	
			$1h_{9/2}$	$13.122 - i0.171$	

$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$       
  
 $\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$ 
  
 $V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau}$      $\langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$ 
  
 $H\Psi_{...} = E_{...} \Psi_{...}$      $\Psi_{\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J\pi} \Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2)$      $H = h + h + V + V_{Coul}$ 
  
 $|\Psi_{J\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J\pi} |J_n J_p, J^{\pi}\rangle$      $\mathcal{H} |\Psi_{J\pi M}\rangle = E_{J\pi} |\Psi_{J\pi M}\rangle$

	$V_0$	$V_{so}$	$a$	$r_0$
$n$	33.44	17.29	0.84	1.50
$p$	52.83	17.71	0.74	1.36

# APPLICATIONS

## $^{212}\text{Po}$ Single-particle basis



# APPLICATIONS

## $^{212}\text{Po}$ Two-body interactions

Nucleus	Pot	Ene(MeV)	Diff(keV)
$^{210}\text{Pb}$	$V_{se} = -24.538$	$0^+ = -9.135$	12
	$V_{to} = -7.218$	$2^+ = -8.230$	-93
		$4^+ = -8.043$	18
		$6^+ = -7.984$	56
		$8^+ = -7.956$	111
$^{210}\text{Po}$	$V_{se} = 7.043$	$0^+ = -8.749$	-33
	$V_{to} = 45.712$	$2^+ = -7.749$	67
		$4^+ = -7.469$	113
		$6^+ = -7.330$	31
		$8^+ = -7.218$	-8
$^{210}\text{Bi}$	$V_{se} = -114.968$	$1^- = -8.493$	-89
	$V_{te} = -131.368$	$0^- = -8.881$	-424
		$9^- = -8.137$	5
	$V_{so} = 44.12$	$2^- = -8.105$	21
	$V_{to} = -49.895$	$3^- = -8.035$	-22
		$7^- = -7.964$	-4

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$J_p^M = [\Psi_{J_n^{\pi_n}} \Psi_{J_p^{\pi_p}}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

$\{P_{\tau}\} \rightarrow \chi^2$  optimization  $nn, pp, np$

Ranges ( $\beta$ )[fm $^2$ ]

	$nn$	$pp$	$np$
$se$	1.8	1.4	1.0
$to$	1.8	2.0	1.4
$so$	—	—	1.4
$te$	—	—	1.0

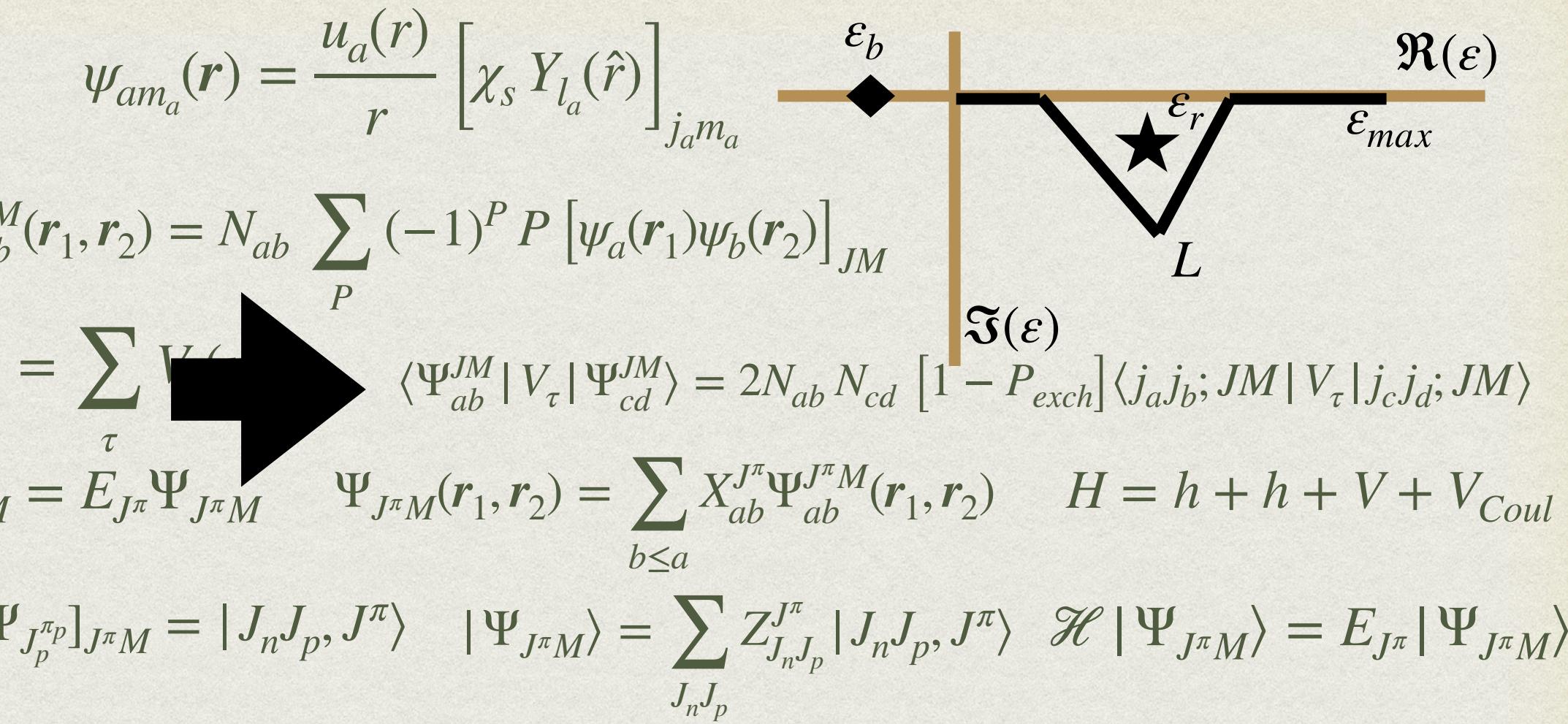
# APPLICATIONS

## $^{212}\text{Po}$ Two-body interactions

### ME comparison $^{208}\text{Pb}$

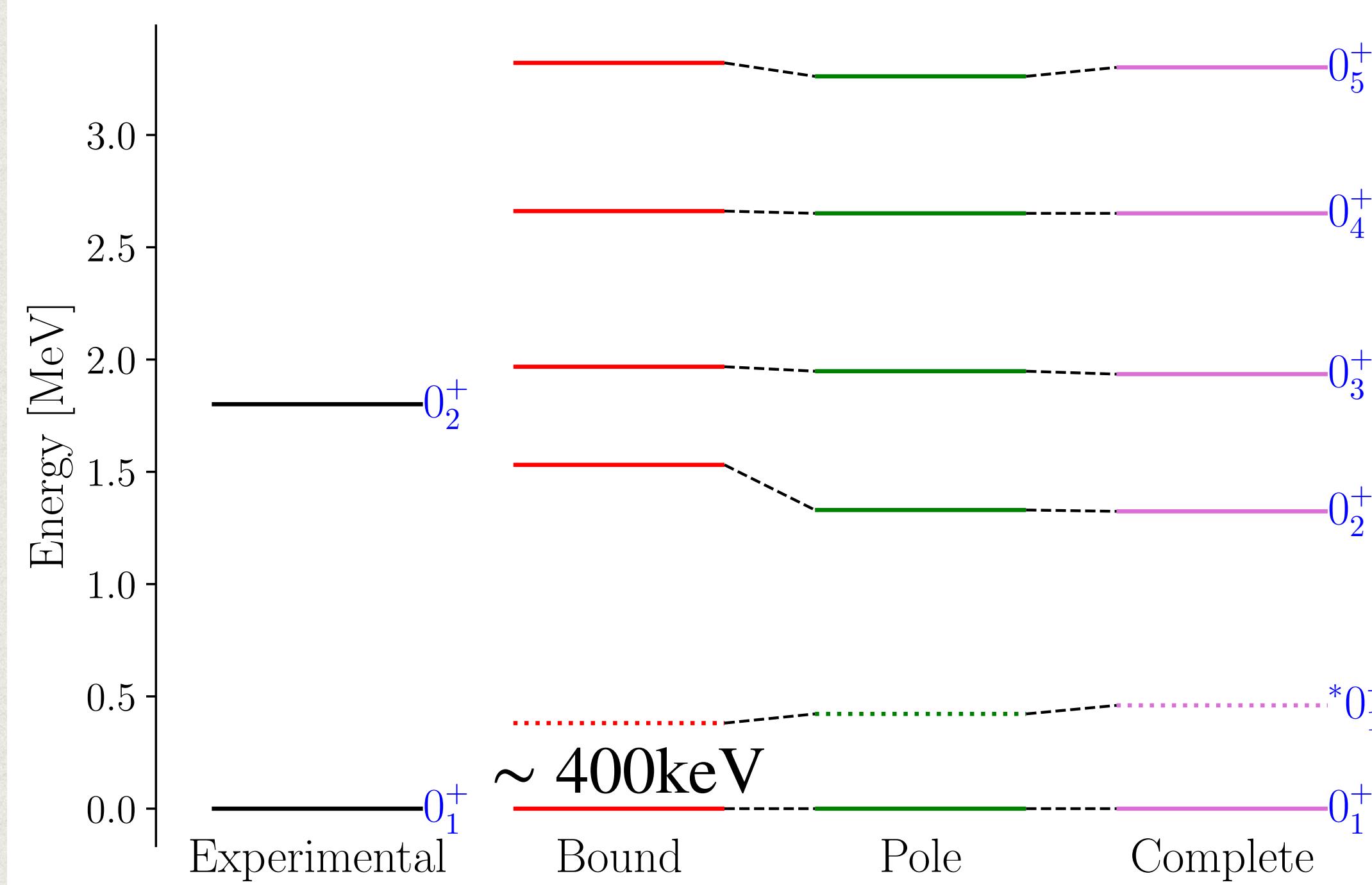
E. K. Warburton, and B. A. Brown, Phys.Rev. C 43, 602 (1991)  
Appraisal of the Kuo-Herling SM interaction and application to A=210-212

Space	Average	Range	$G_{Bare}$	$G_{Bare} + G_{1p1h} + G_{2p2h}$	Rosario
$nn$	$\langle G \rangle$	$J \neq 0^+$	-0.021	-0.022	-0.064
$nn$	$\langle  G  \rangle$	$J \neq 0^+$	0.047	0.078	0.091
$nn$	$\langle G \rangle$	$J = 0^+$	-0.112	-0.217	-0.260
$nn$	$\langle  G  \rangle$	$J = 0^+$	0.272	0.427	0.316
$np$	$\langle G \rangle$	$J \neq 0^+$	-0.041	-0.039	-0.036
$np$	$\langle  G  \rangle$	$J \neq 0^+$	0.082	0.119	0.059
$np$	$\langle G \rangle$	$J = 0^+$	-0.121	-0.109	-0.182
$np$	$\langle  G  \rangle$	$J = 0^+$	0.369	0.447	0.272



# APPLICATIONS

## $^{212}\text{Po}$ $0^+$ states



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^n} \Psi_{J_p^p}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

Representation	Neutron states	Proton states
Bound	$1g_{9/2}, 0i_{11/2}, 0j_{15/2}, 2d_{5/2}$ $3s_{1/2}, 2d_{3/2}, 1g_{7/2}$	$0h_{9/2}, 1f_{7/2}, 0i_{13/2}, 1f_{5/2}$ $2p_{3/2}$
Pole	$1g_{9/2}, 0i_{11/2}, 0j_{15/2}, 2d_{5/2}$ $3s_{1/2}, 2d_{3/2}, 1g_{7/2}$ $1h_{11/2}, 2f_{7/2}, 0k_{17/2}$ $0j_{13/2}$	$0h_{9/2}, 1f_{7/2}, 0i_{13/2}, 1f_{5/2}$ $2p_{3/2}$ $2p_{1/2}, 0i_{11/2}, 1g_{9/2}, 0j_{15/2}$ $2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 2d_{3/2}$
Complete	$1g_{9/2}, 0i_{11/2}, 0j_{15/2}, 2d_{5/2}$ $3s_{1/2}, 2d_{3/2}, 1g_{7/2}$ $1h_{11/2}, ch_{11/2}, 2f_{7/2}, cf_{7/2}$ $0k_{17/2}, ck_{17/2}, 0j_{13/2}, cj_{13/2}$	$0h_{9/2}, 1f_{7/2}, 0i_{13/2}, 1f_{5/2}$ $2p_{3/2}$ $2p_{1/2}, cp_{1/2}, 0i_{11/2}, ci_{11/2}$ $1g_{9/2}, cg_{9/2}, 0j_{15/2}, cj_{15/2}$ $2d_{5/2}, cd_{5/2}, 1g_{7/2}, cg_{7/2}$ $3s_{1/2}, cs_{1/2}, 2d_{3/2}, cd_{3/2}$

$$V_{te} = -131.368$$

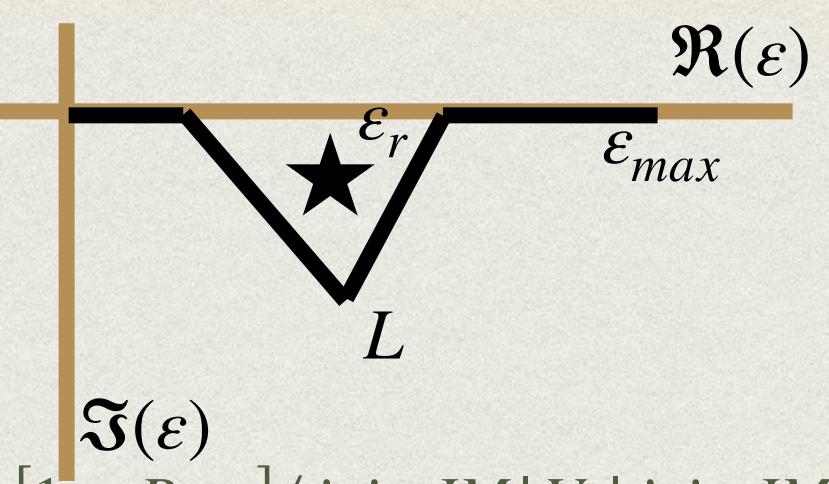
MeV	Bound	Pole	Complete
$V_{te}$	-192.2	-186.1	-194

$$V_{se} = -114.968$$

$$V_{te} = -131.368$$

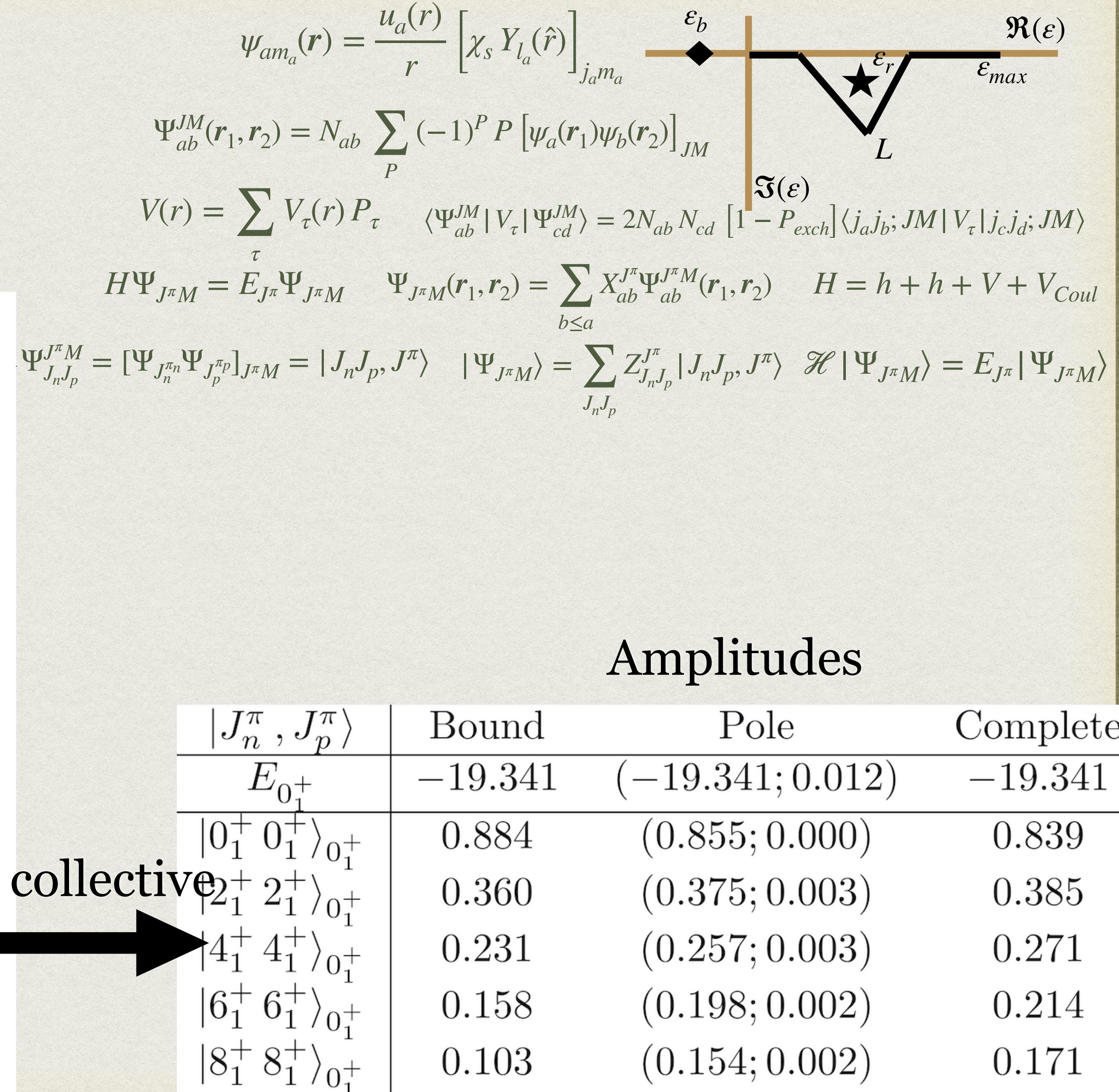
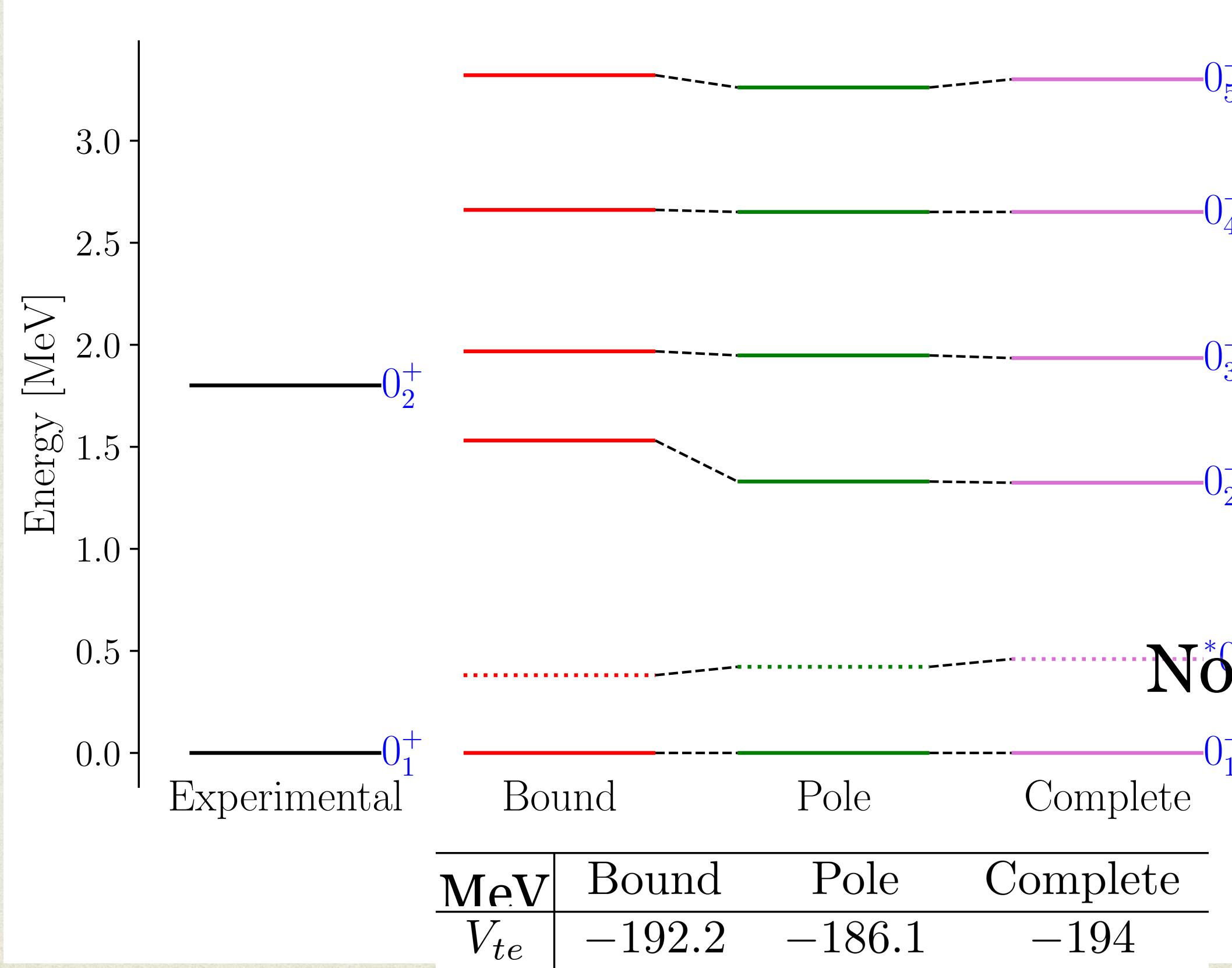
$$V_{so} = 44.12$$

$$V_{to} = -49.895 \text{ MeV}$$



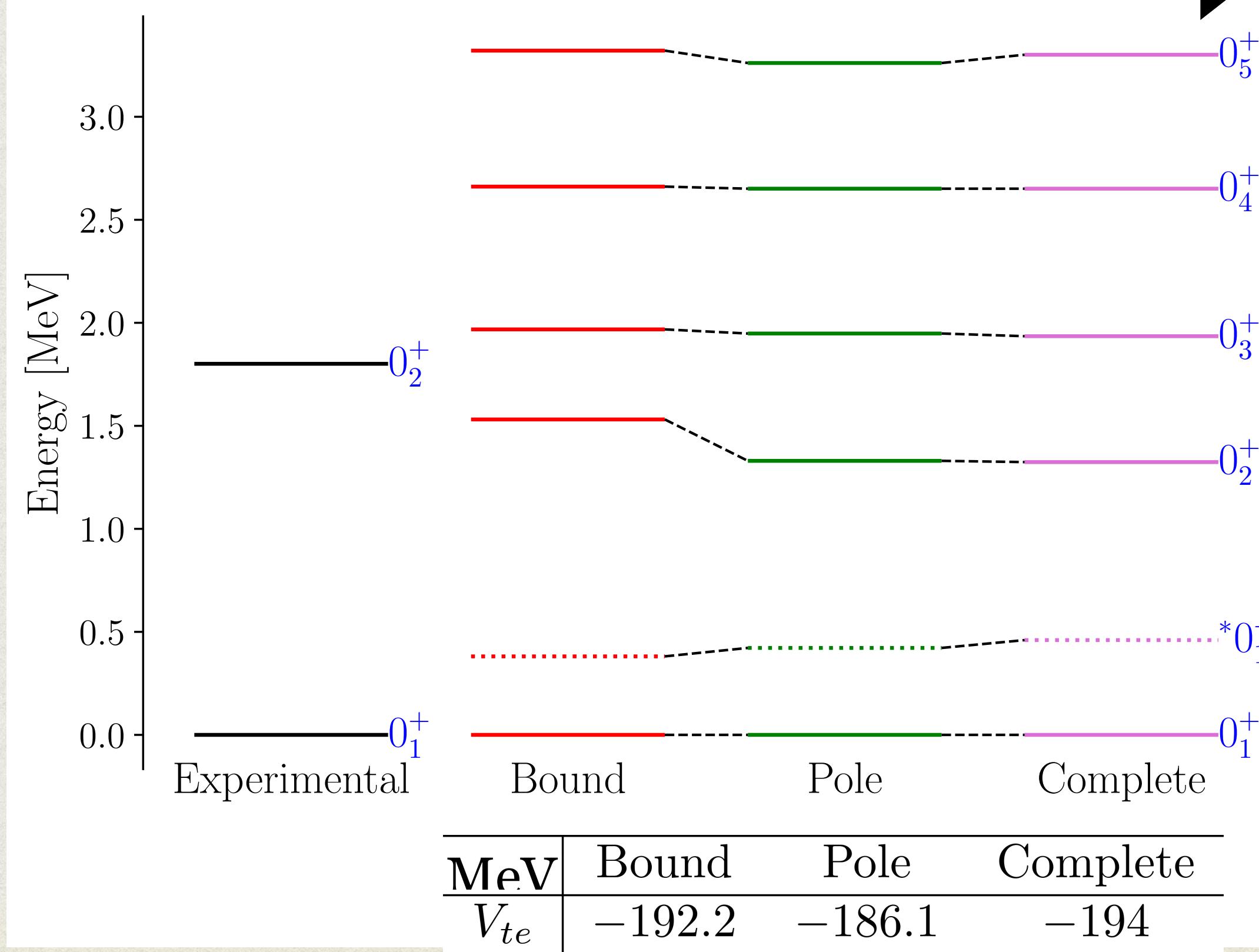
# APPLICATIONS

## $^{212}\text{Po}$ $0^+$ states



# APPLICATIONS

$^{212}\text{Po}$   $0^+$  states



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

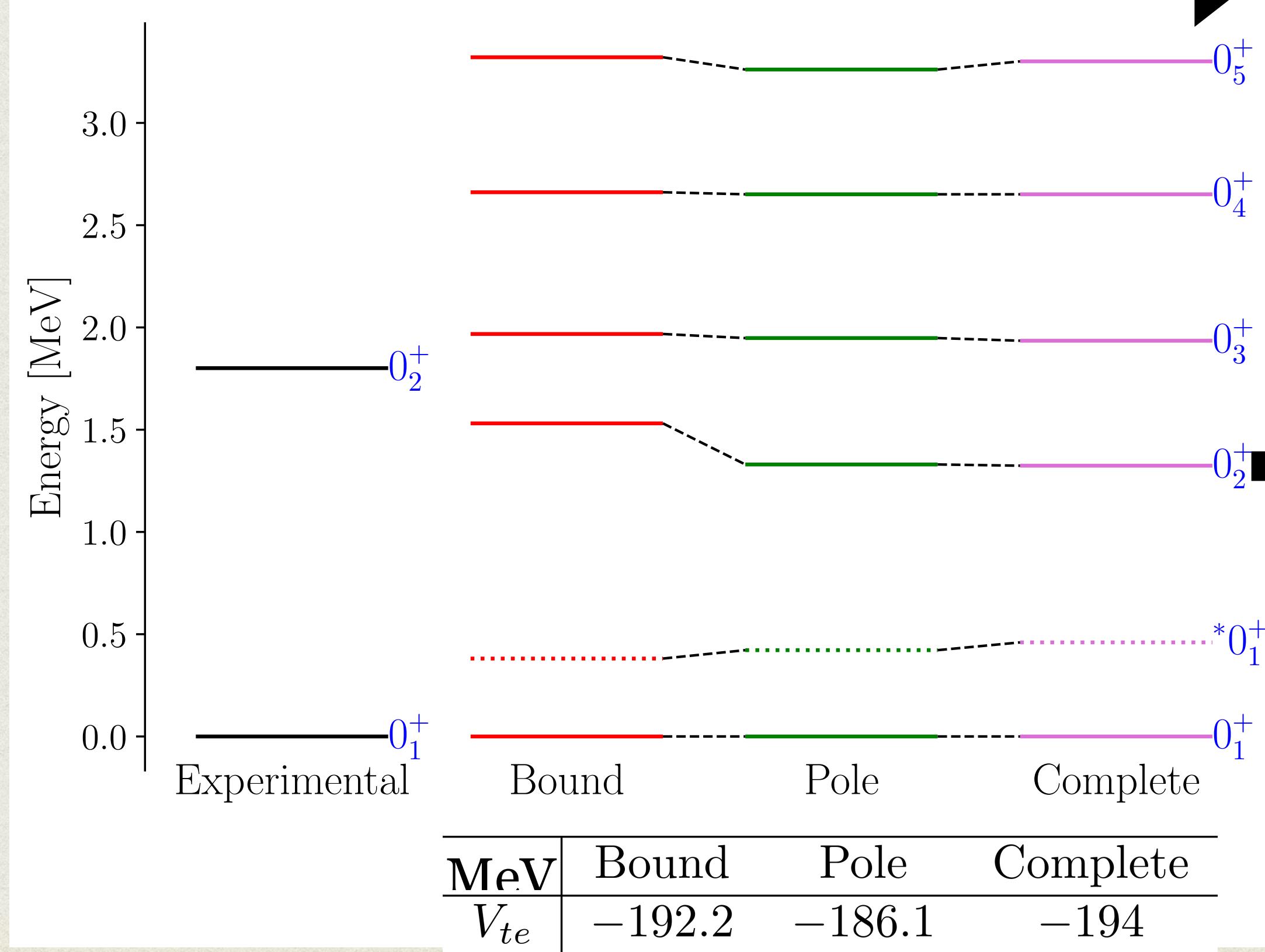
$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^{\pi n}} \Psi_{J_p^{\pi p}}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

Amplitudes

	AB	GH	Complete
$ J_n^\pi, J_p^\pi\rangle$			$-19.341$
$E_{0_1^+}$			
$ 0_1^+ 0_1^+\rangle_{0_1^+}$	0.872	0.898	0.839
$ 2_1^+ 2_1^+\rangle_{0_1^+}$	0.436	0.313	0.385
$ 4_1^+ 4_1^+\rangle_{0_1^+}$	0.300	0.148	0.271
$ 6_1^+ 6_1^+\rangle_{0_1^+}$	0.077	0.095	0.214
$ 8_1^+ 8_1^+\rangle_{0_1^+}$	0.138	0.084	0.171

# APPLICATIONS

$^{212}\text{Po}$   $0^+$  states



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H \Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^{\pi n}} \Psi_{J_p^{\pi p}}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

Collective

$E_{0_2^+}$	-17.810	(-18.011; 0.004)	-18.017
$ 0_1^+ 0_1^+\rangle_{0_2^+}$	-0.416	(-0.474; -0.002)	-0.491
$ 2_1^+ 2_1^+\rangle_{0_2^+}$	0.390	(0.332; -0.005)	0.286
$ 4_1^+ 4_1^+\rangle_{0_2^+}$	0.431	(0.413; -0.002)	0.396
$ 6_1^+ 6_1^+\rangle_{0_2^+}$	0.479	(0.475; -0.001)	0.478
$ 8_1^+ 8_1^+\rangle_{0_2^+}$	0.510	(0.519; 0.000)	0.540

# APPLICATION $^{44}\text{Ti}$

# APPLICATIONS

## $^{44}\text{Ti}$ Nucleons mean-field

### Spectroscopic weighted s.p. energies

N. Schwierz, I. Wiedenhöver, and A. Volya, arXiv: 0709.3525. 2007

state	$^{41}\text{Ca}$		$^{41}\text{Sc}$	
	$\varepsilon_{\text{Exp}}$	$\varepsilon$	$\varepsilon_{\text{Exp}}$	$\text{Re}(\varepsilon)$
$0f_{7/2}$	-8.36	-8.309	-1.09	-1.109
$1p_{3/2}$	-5.84	-6.017	0.69	0.760
$1p_{1/2}$	-4.20	-3.995	2.38	2.291
$0f_{5/2}$	-1.56	-1.626	4.96	4.982

### Mean-field parameters

nucleon	$V_0(\text{MeV})$	$V_{so}(\text{MeV fm})$	$a(\text{fm})$	$r_0(\text{fm})$
neutron	52.052(1.0)	16.915(4.2)	0.811(0.2)	1.274
proton	51.440(1.1)	16.190(4.9)	0.791(0.2)	1.278

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P \left[ \psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2) \right]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{\text{exch}}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{\text{Coul}}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^n} \Psi_{J_p^{\pi p}}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

### Pole bases $\Gamma < 1 \text{ MeV}$

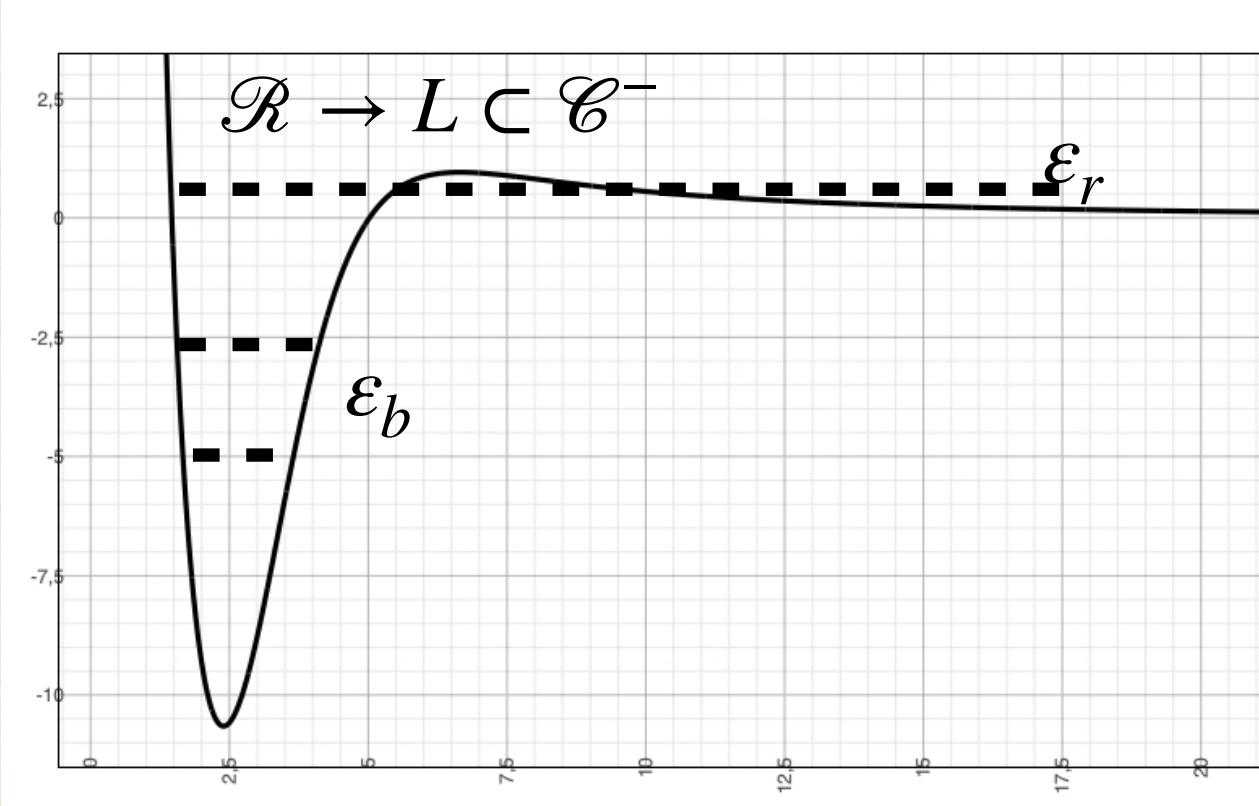
state	$\varepsilon_{\nu}$ (MeV)	$\varepsilon_{\pi}$ (MeV)
$0f_{7/2}$	-8.309	-1.109
$1p_{3/2}$	-6.017	$(0.760, -0.671 \times 10^{-5})$
$1p_{1/2}$	-3.995	$(2.291, -0.494 \times 10^{-1})$
$0f_{5/2}$	-1.626	$(4.982, -0.789 \times 10^{-1})$
$0g_{9/2}$	$(1.658, -0.405 \times 10^{-2})$	$(7.961, -0.222)$
$0g_{7/2}$	$(8.321, -1.541)$	$(14.433, -2.788)$
$2d_{5/2}$	$(0.895, -0.188)$	$(6.066, -1.735)$
$2d_{3/2}$	$(1.954, -1.335)$	$(6.952, -3.848)$
$0h_{11/2}$	$(10.568, -1.129)$	$(16.560, -2.075)$
$0h_{9/2}$	$(17.876, -7.300)$	$(24.171, -9.195)$

Bound bases	4	1
Pole bases	6	5

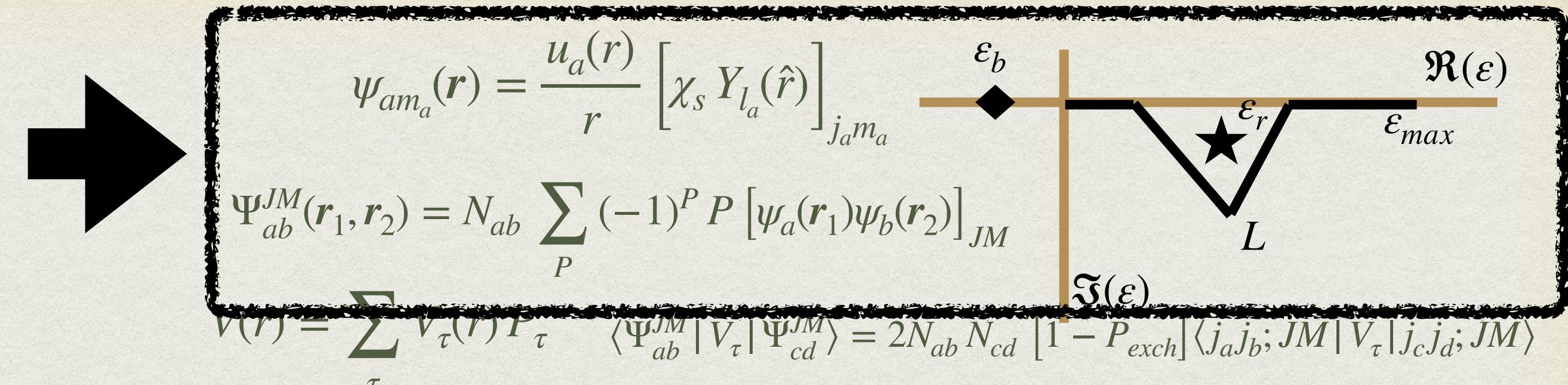
# APPLICATIONS

## $^{44}\text{Ti}$ Model spaces

Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}$ $0f_{5/2}$ $cf_{5/2}, 0g_{9/2}, cg_{9/2}$ $cs_{1/2}$



- Bound-Bound
- Bound-Resonance
- Bound-Non Resonant continuum
- Resonant-Resonant
- Resonant-Non Resonant
- Non Resonant-Non Resonant



122  
discretized  
non-RC

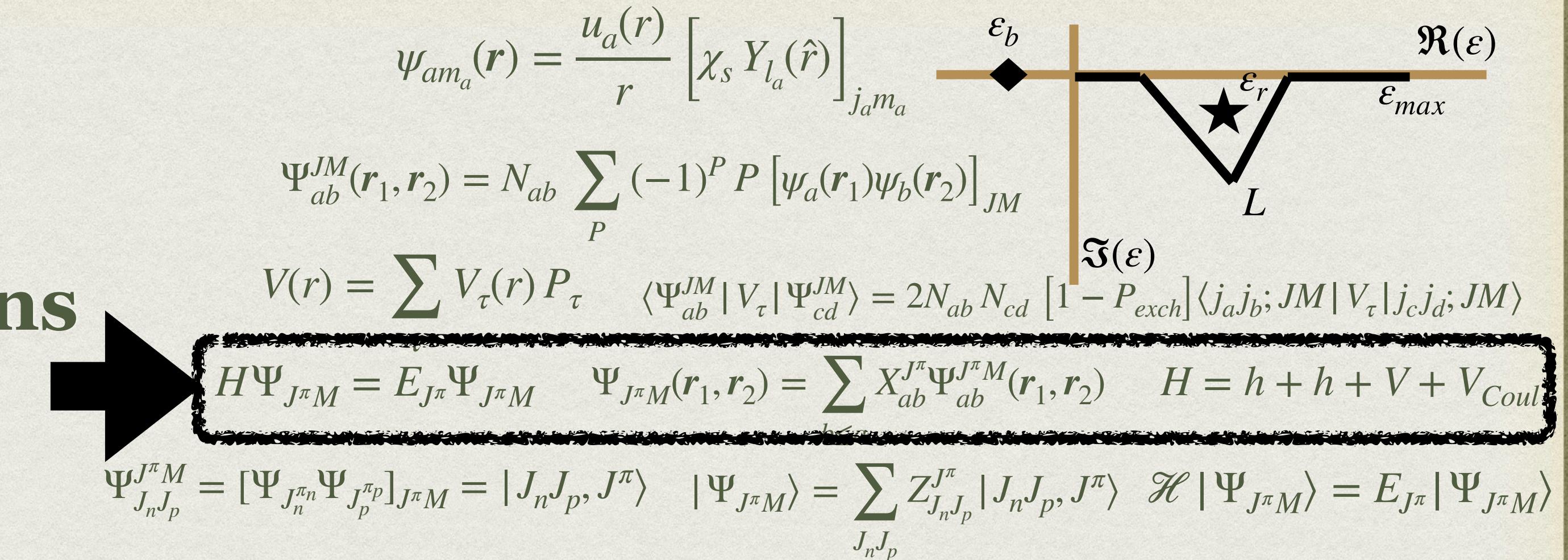
state	$\varepsilon_\nu$ (MeV)	$\varepsilon_\pi$ (MeV)
$0f_{7/2}$	-8.309	-1.109
$1p_{3/2}$	-6.017	(0.760, $-0.671 \times 10^{-5}$ )
$1p_{1/2}$	-3.995	(2.291, $-0.494 \times 10^{-1}$ )
$0f_{5/2}$	-1.626	(4.982, $-0.789 \times 10^{-1}$ )
$0g_{9/2}$	(1.658, $-0.405 \times 10^{-2}$ )	(7.961, -0.222)
$0g_{7/2}$	(8.321, -1.541)	(14.433, -2.788)
$2d_{5/2}$	(0.895, -0.188)	(6.066, -1.735)
$2d_{3/2}$	(1.954, -1.335)	(6.952, -3.848)
$0h_{11/2}$	(10.568, -1.129)	(16.560, -2.075)
$0h_{9/2}$	(17.876, -7.300)	(24.171, -9.195)

# APPLICATIONS

## $^{44}\text{Ti}$ Two-nucleon interactions

Optimization isospin 'conserving'

$J^\pi$	Exp.	Bound	Pole	Complete
$^{42}\text{Ca}$				
$0^+$	-19.843	-19.211	$-19.113 + i0.006$	-19.161
$2^+$	-18.319	-18.213	$-18.276 + i0.003$	-18.306
$^{42}\text{Ti}$				
$0^+$	-4.836	-4.145	$-4.072 + i0.031$	-4.085
$2^+$	-3.282	-3.257	$-3.349 + i0.016$	-3.365
$^{42}\text{Sc}$				
$0^+$	-10.411	-11.825	$-11.789 + i0.022$	-11.813
$1^+$	-9.799	-9.798	$-9.799 + i0.032$	-9.799



Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}, 0f_{5/2}$ $cf_{5/2}, 0g_{9/2}, cg_{9/2}$ $cs_{1/2}$

122 discretized non-RC

## Strengths

$V_\tau$	$\beta$	Bound	Pole	Complete
se	1.8	-31.449	-31.092	-31.092
to	1.8	-34.712	-40.090	-40.090
so	1.4	16.029	12.691	3.150
te	1.4	-123.971	-56.910	-42.273

T=1

T=0

# APPLICATIONS

## $^{44}\text{Ti}$ Two-nucleon bases

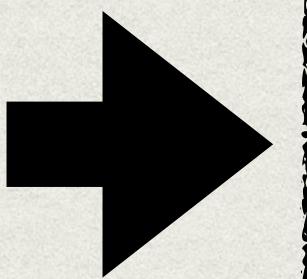
$$E_n^{\max} = 3\text{MeV} \quad (\text{S}_{2n}(^{42}\text{Ca}) = 19.843\text{MeV})$$

$$E_p^{\max} = 2.7\text{MeV} \quad (\text{S}_{2p}(^{42}\text{Ti}) = 4.836\text{MeV})$$

### Two-body bases

$$E_n : 0^+, 2^+, 4^+, 6^+$$

$$E_p : 0^+, 2^+, 4^+, 6^+$$



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum V_\tau(r) P_\tau \quad \langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n}^{\pi_n} \Psi_{J_p}^{\pi_p}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}, 0f_{5/2}$ $1p_{1/2}, cd_{5/2}, cs_{1/2}$ $cf_{5/2}, 0g_{9/2}, cg_{9/2}$ $cs_{1/2}$

122 discretized non-RC

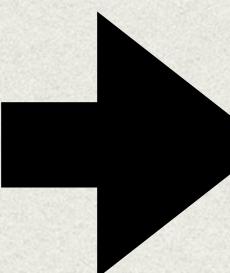
### Four-body basis

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n}^{\pi_n} \Psi_{J_p}^{\pi_p}]_{J^\pi M} = |J_n J_p, J^\pi\rangle$$

# APPLICATIONS

## $^{44}\text{Ti}$ Four-body interaction

$$\mathcal{H} = H_n + H_p + V_{np}$$



$$E_{\text{gs}}^{(0)}(^{44}\text{Ti}) = E_{\text{gs}}(^{42}\text{Ca}) + E_{\text{gs}}(^{42}\text{Ti}) = -24.679 \text{ MeV}$$

$$E_{\text{gs}}(^{44}\text{Ti}) = -33.423 \text{ MeV}$$

$$E_{\text{corr}} = 8.744 \text{ MeV}$$

$$V_{te}, V_{so} \rightarrow \chi V_{te}, \chi V_{so}$$

$$\chi_B = 1.2152$$

$$\chi_P = 2.7772$$

$$\chi_C = 3.5092$$

$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{\text{exch}}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b \leq a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{\text{Coul}}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^n} \Psi_{J_p^{\pi p}}]_{J^\pi M} = |J_n J_p, J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \boxed{\mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle}$$

Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}, 0f_{5/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$ $0g_{9/2}, cg_{9/2}$ $cs_{1/2}$

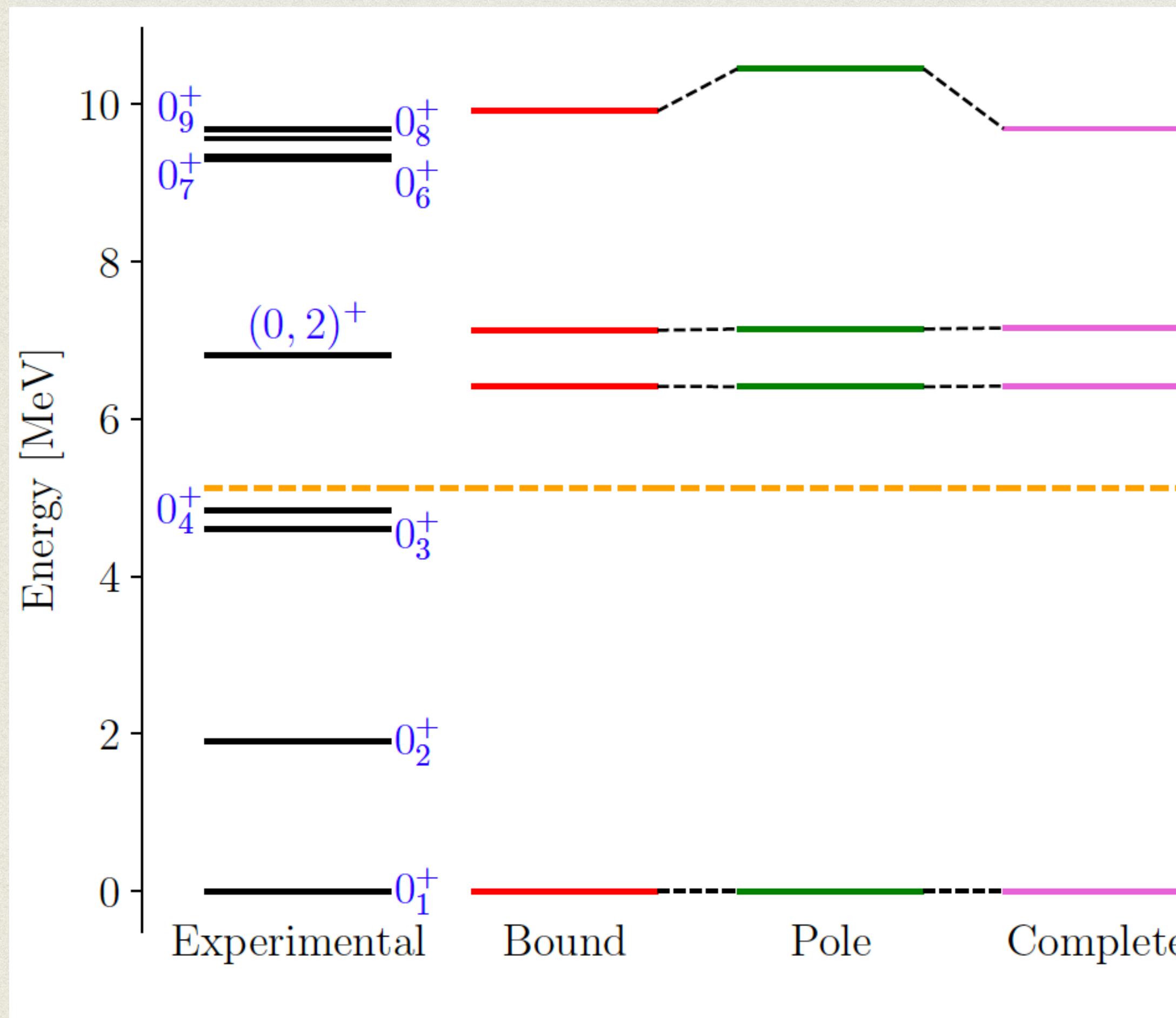
$^{122}$  discretized non-RC

## Four-body strengths

$V_{\tau}$	$\beta$	Bound	Pole	Complete
		se	to	so
$se$	1.8	-31.449	-31.092	-31.092
$to$	1.8	-34.712	-40.090	-40.090
$so$	1.4			
$te$	1.4	-		
		Bound basis		
		$so$	19.479	35.245
		$te$	-150.650	-148.344
				11.054

# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

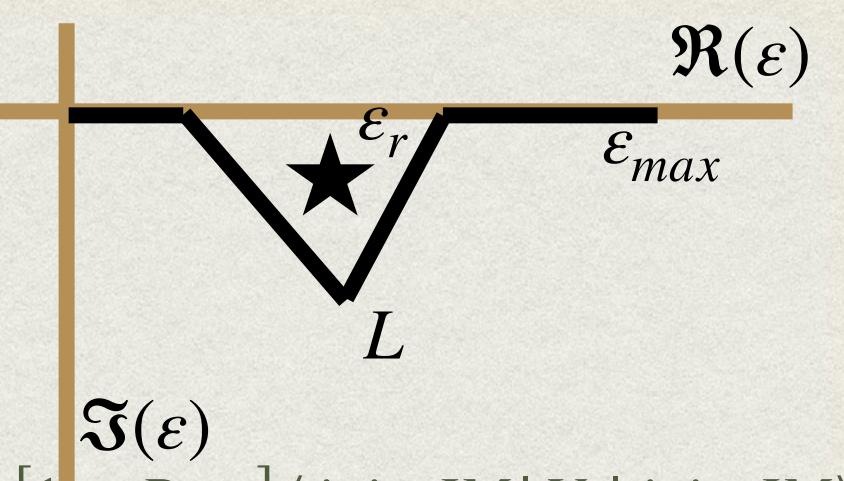
$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n}^{\pi_n} \Psi_{J_p}^{\pi_p}]_{J^\pi M} = |J^\pi\rangle$$

$$|\Psi_{J^\pi M}\rangle = \sum_{J, J'} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

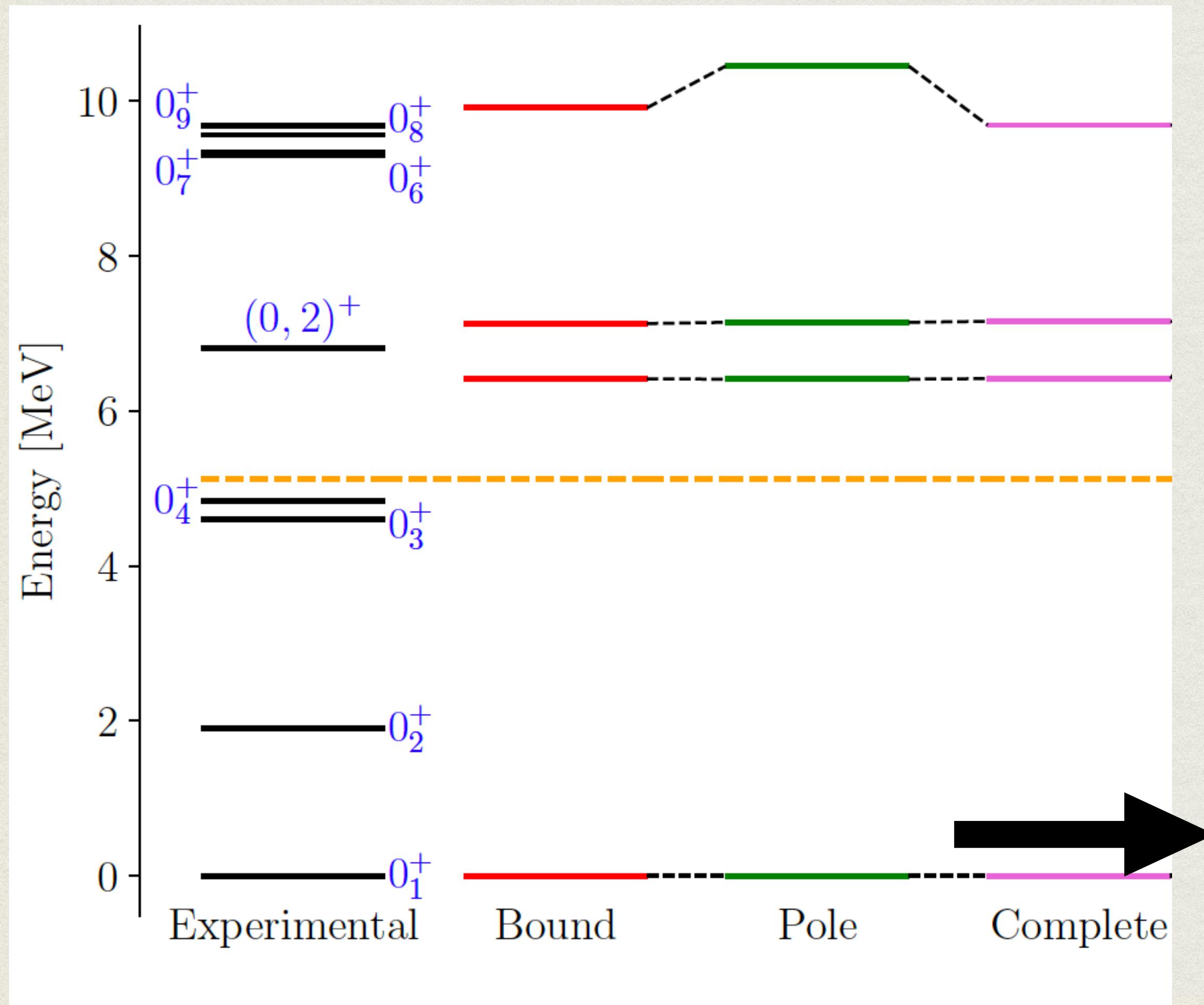
Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}, 0f_{5/2}$ $cf_{5/2}, 0g_{9/2}, cg_{9/2}$ $cs_{1/2}$

122 discretized non-RC



# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

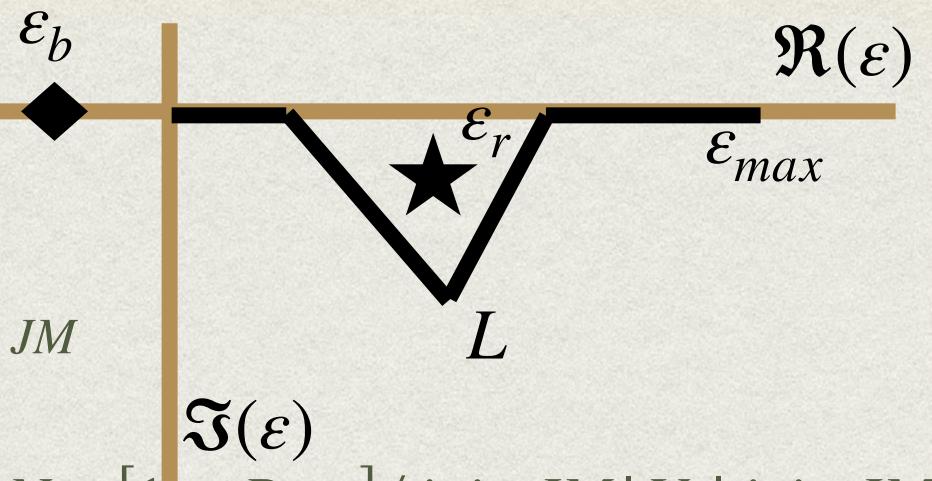
$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^n} \Psi_{J_p^p}]_{J^\pi M} = \rightarrow | \Psi_{J^\pi M} \rangle$$

$$| \Psi_{J^\pi M} \rangle = \sum_{J,J'} Z_{J_n J_p}^{J^\pi} | J_n J_p, J^\pi \rangle \quad \mathcal{H} | \Psi_{J^\pi M} \rangle = E_{J^\pi} | \Psi_{J^\pi M} \rangle$$

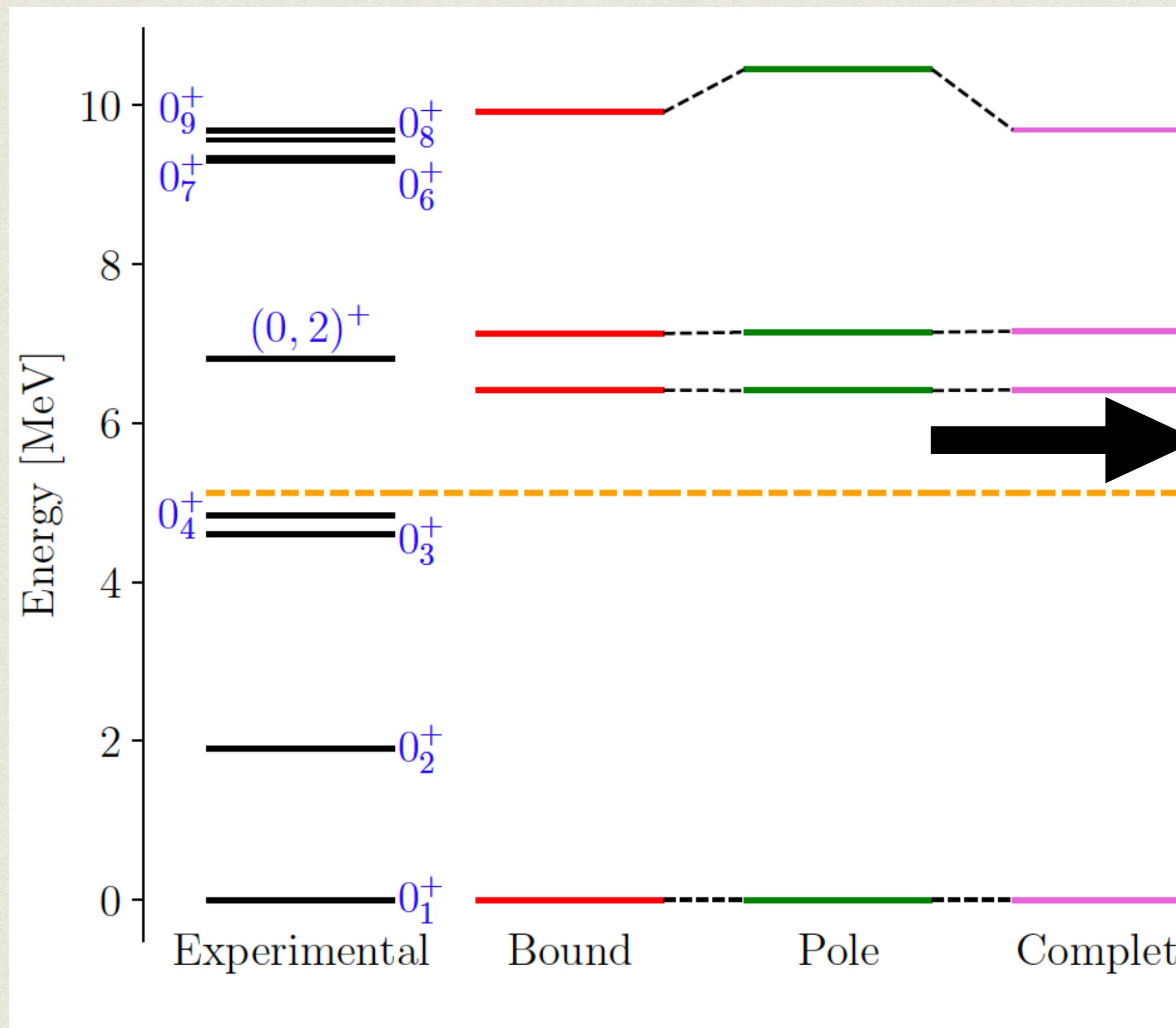
Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}, 0f_{5/2}$

$ J_n^\pi, J_p^\pi\rangle$	Bound basis	Pole basis	Complete basis
$E_{0_1^+}$	(-33.423, 0.000)	(-33.423, 0.059)	(-33.423, 0.000)
$ 0_1^+ 0_1^+\rangle_{0_1^+}$	(0.771, 0.000)	(0.765, 0.000)	(0.759, 0.000)
$ 2_1^+ 2_1^+\rangle_{0_1^+}$	(0.591, 0.000)	(0.594, 0.001)	(0.600, 0.000)
$ 4_1^+ 4_1^+\rangle_{0_1^+}$	(0.194, 0.000)	(0.201, -0.001)	(0.214, 0.000)
$ 6_1^+ 6_1^+\rangle_{0_1^+}$	(0.134, 0.000)	(0.145, 0.000)	(0.132, 0.000)



# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

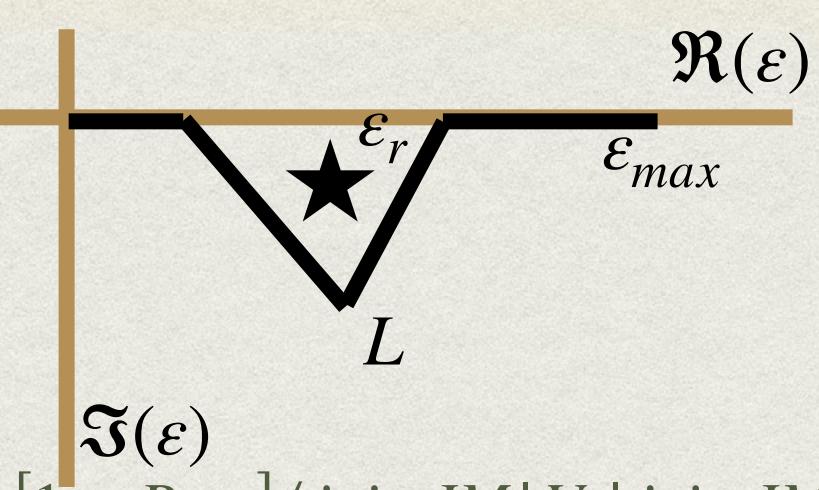
$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n}^{J^\pi n} \Psi_{J_p}^{J^\pi p}]_{J^\pi M} = |\Psi_{J^\pi M}^{J^\pi}\rangle$$

$$|\Psi_{J^\pi M}^{J^\pi}\rangle = \sum_{J_n J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}^{J^\pi}\rangle = E_{J^\pi} |\Psi_{J^\pi M}^{J^\pi}\rangle$$

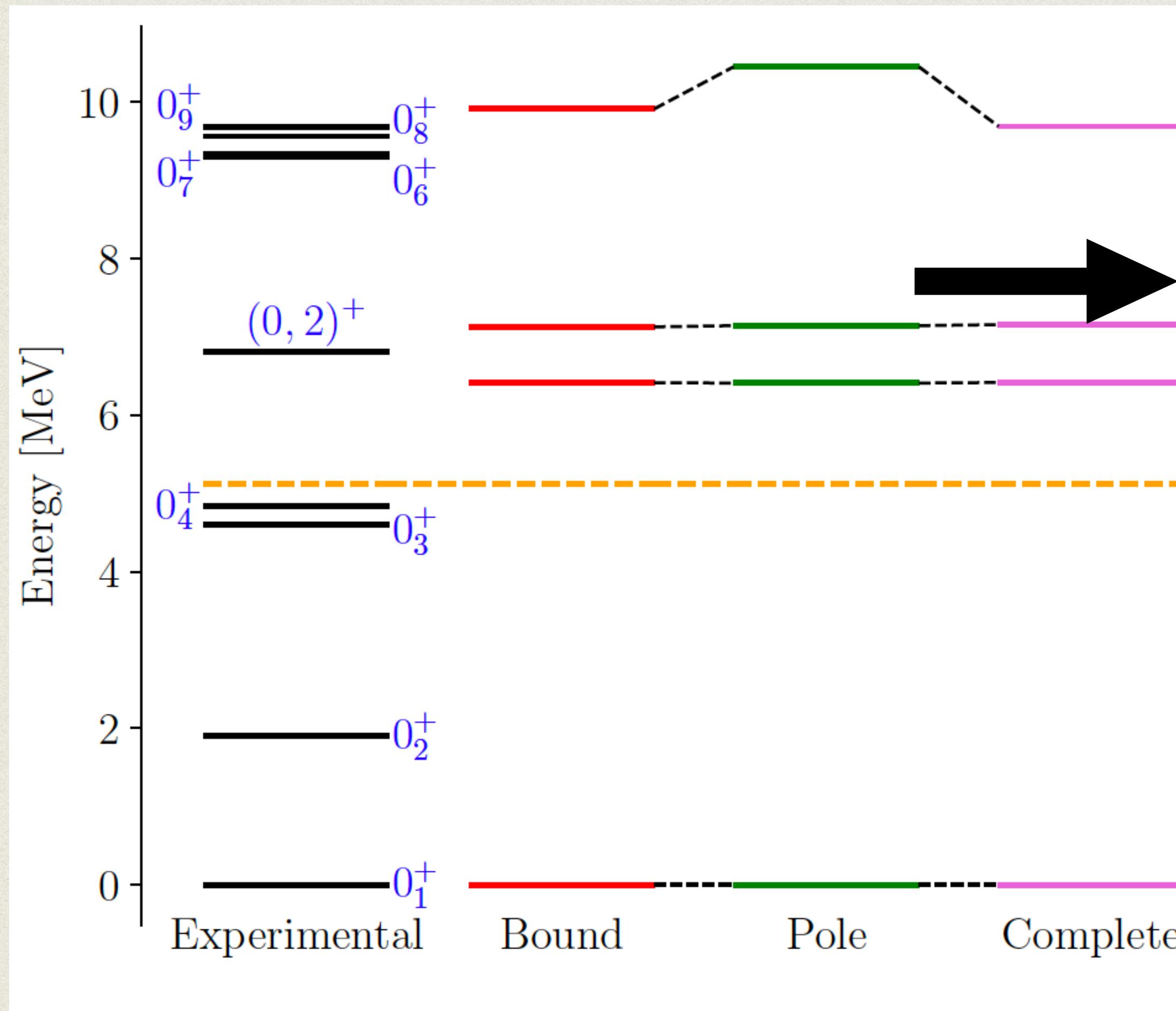
Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$	$0f_{7/2}$

$ J_n^\pi, J_p^\pi\rangle$	Bound basis	Pole basis	Complete basis
$E_{0_2^+}$	(-27.009, 0.000)	(-27.013, 0.022)	(-27.006, 0.000)
$ 0_1^+ 0_1^+\rangle_{0_2^+}$	(0.257, 0.000)	(0.226, 0.000)	(0.208, 0.000)
$ 2_1^+ 2_1^+\rangle_{0_2^+}$	(-0.628, 0.000)	(-0.608, 0.002)	(-0.590, 0.000)
$ 4_1^+ 4_1^+\rangle_{0_2^+}$	(0.617, 0.000)	(0.655, 0.000)	(0.644, 0.000)
$ 6_1^+ 6_1^+\rangle_{0_2^+}$	(0.398, 0.000)	(0.388, 0.000)	(0.440, 0.000)



# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

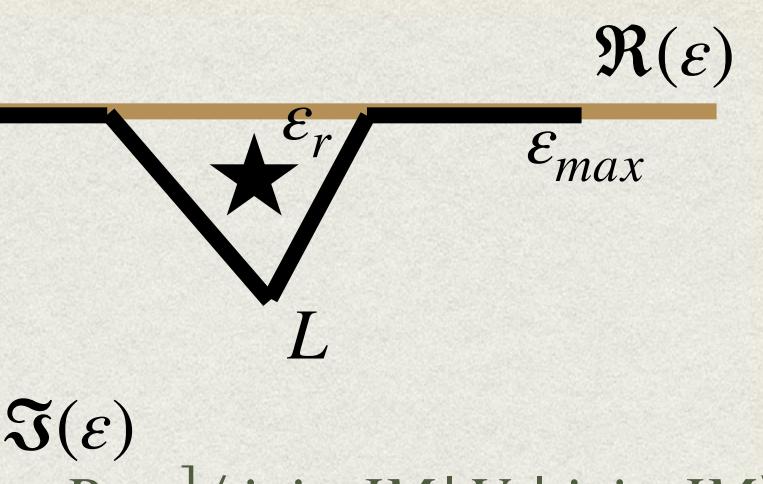
$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^{\pi_n}} \Psi_{J_p^{\pi_p}}]_{J^\pi M} = |J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

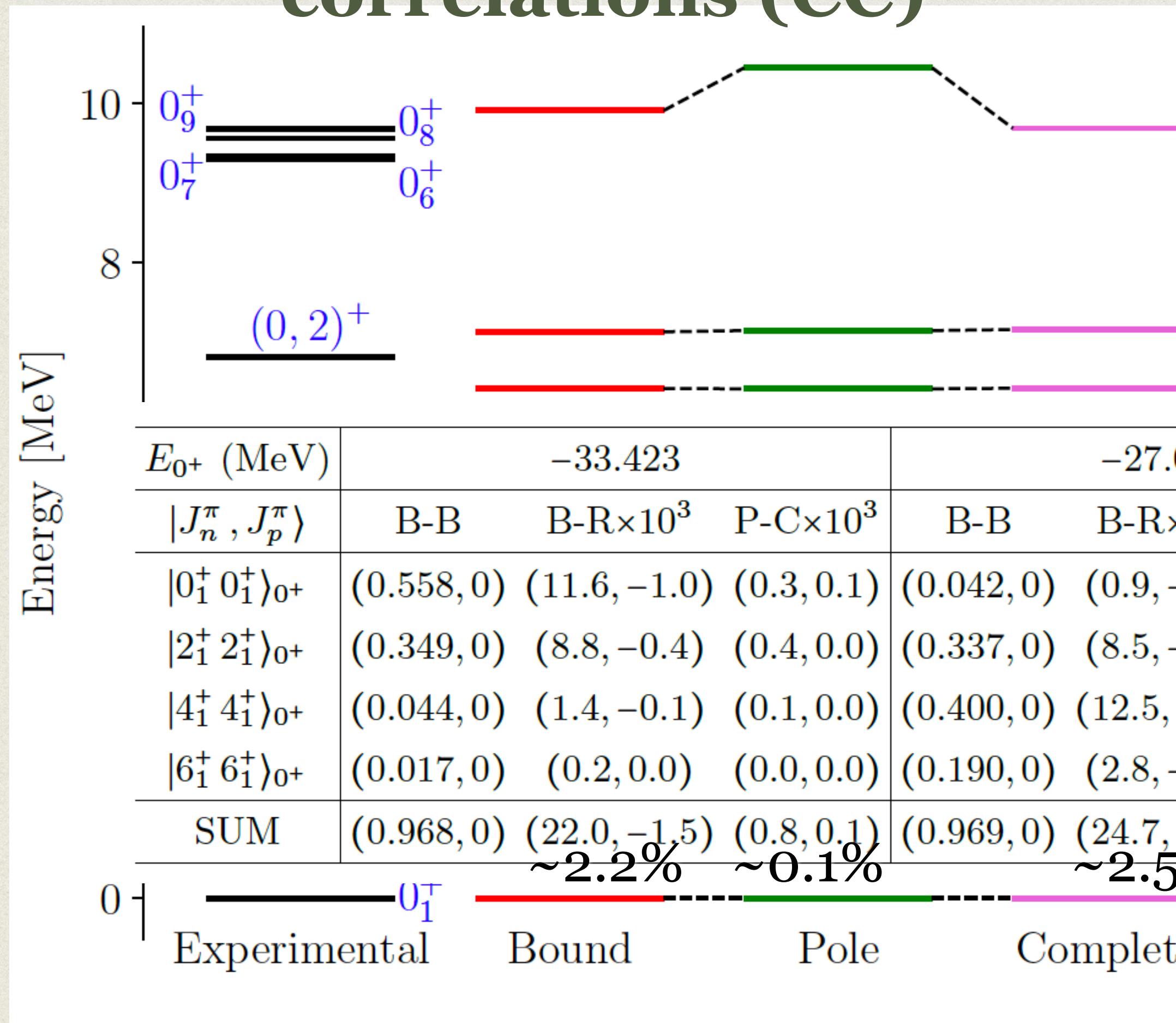
$ J_n^\pi, J_p^\pi\rangle$	Bound basis	Pole basis	Complete basis
$E_{0_3^+}$	(-26.296, 0.000)	(-26.281, 0.033)	(-26.297, 0.000)
$ 0_1^+ 0_1^+\rangle_{0_3^+}$	(0.565, 0.000)	(0.584, 0.000)	(0.598, 0.000)
$ 2_1^+ 2_1^+\rangle_{0_3^+}$	(-0.489, 0.000)	(-0.503, 0.000)	(-0.527, 0.000)
$ 4_1^+ 4_1^+\rangle_{0_3^+}$	(-0.383, 0.000)	(-0.355, 0.002)	(-0.331, 0.000)
$ 6_1^+ 6_1^+\rangle_{0_3^+}$	(-0.543, 0.000)	(-0.529, 0.000)	(-0.505, 0.000)

Collective state



# APPLICATIONS

## $^{44}\text{Ti}$ Four-body Continuum correlations (CC)



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{\mathbf{r}}) \right]_{j_a m_a}$$

The diagram illustrates the energy spectrum  $\mathfrak{R}(\varepsilon)$  and  $\Im(\varepsilon)$  for a system. It shows energy levels  $\varepsilon_b$ ,  $\varepsilon_r$ , and  $\varepsilon_{max}$  along with a star symbol indicating a resonance. The energy levels are represented by horizontal lines of different colors (black, red, green, pink) with vertical dashed lines indicating transitions.

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_\tau V_\tau(r) P_\tau \quad \langle \Psi_{ab}^{JM} | V_\tau | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_\tau | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b<1} X_{ab}^{J^\pi} \Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$\Psi_{J_n J_p}^{J^\pi M} = [\Psi_{J_n^n} \Psi_{J_p^p}]_{J^\pi M} = \rightarrow |J^\pi\rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J,J'} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

Complete bases

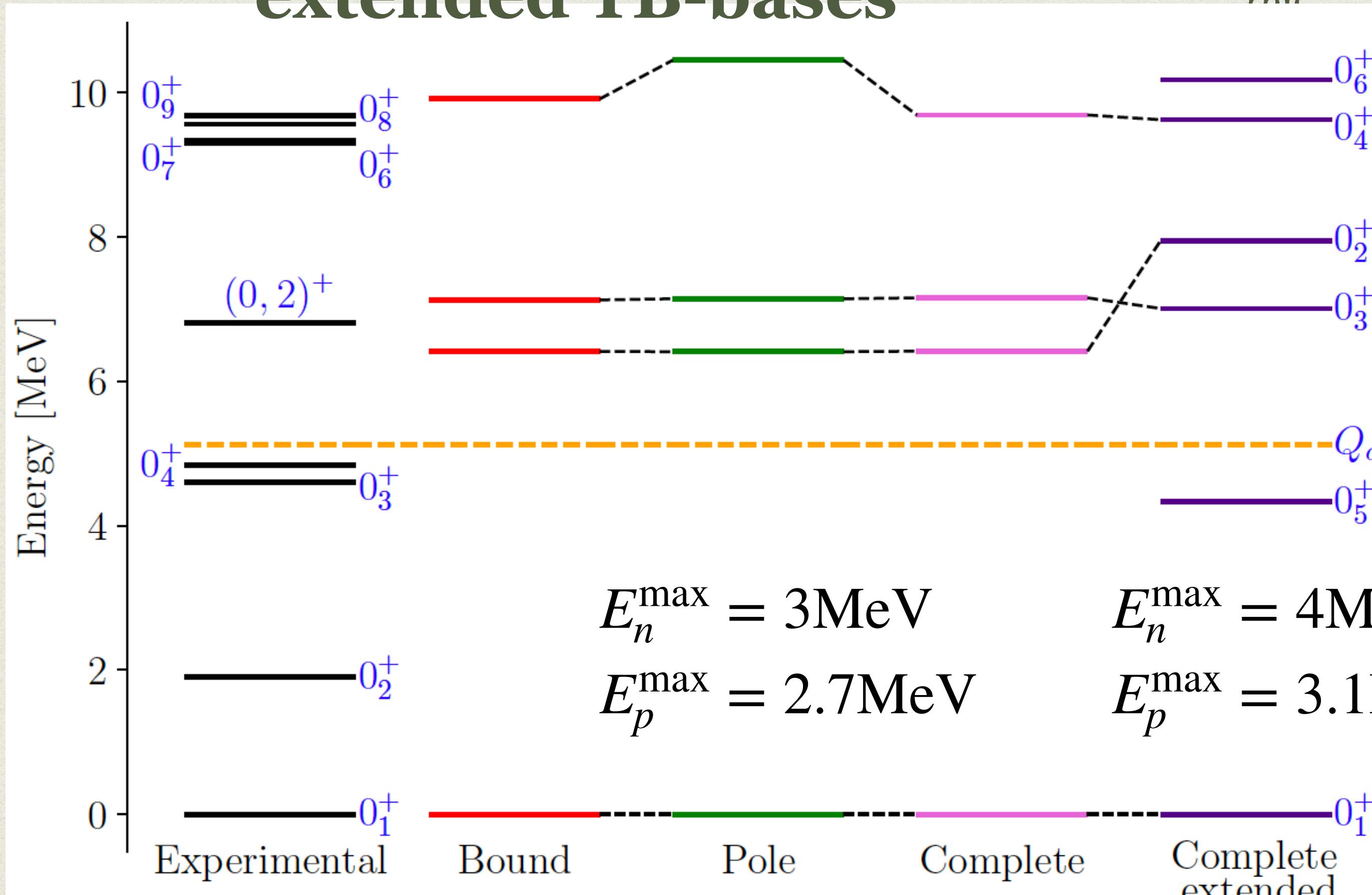
Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$

Bound-Resonance: 2.1-2.5%  
Pole-NonRC: 0.1-0.3 %

CC ~ 3 %

# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum in extended TB-bases



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

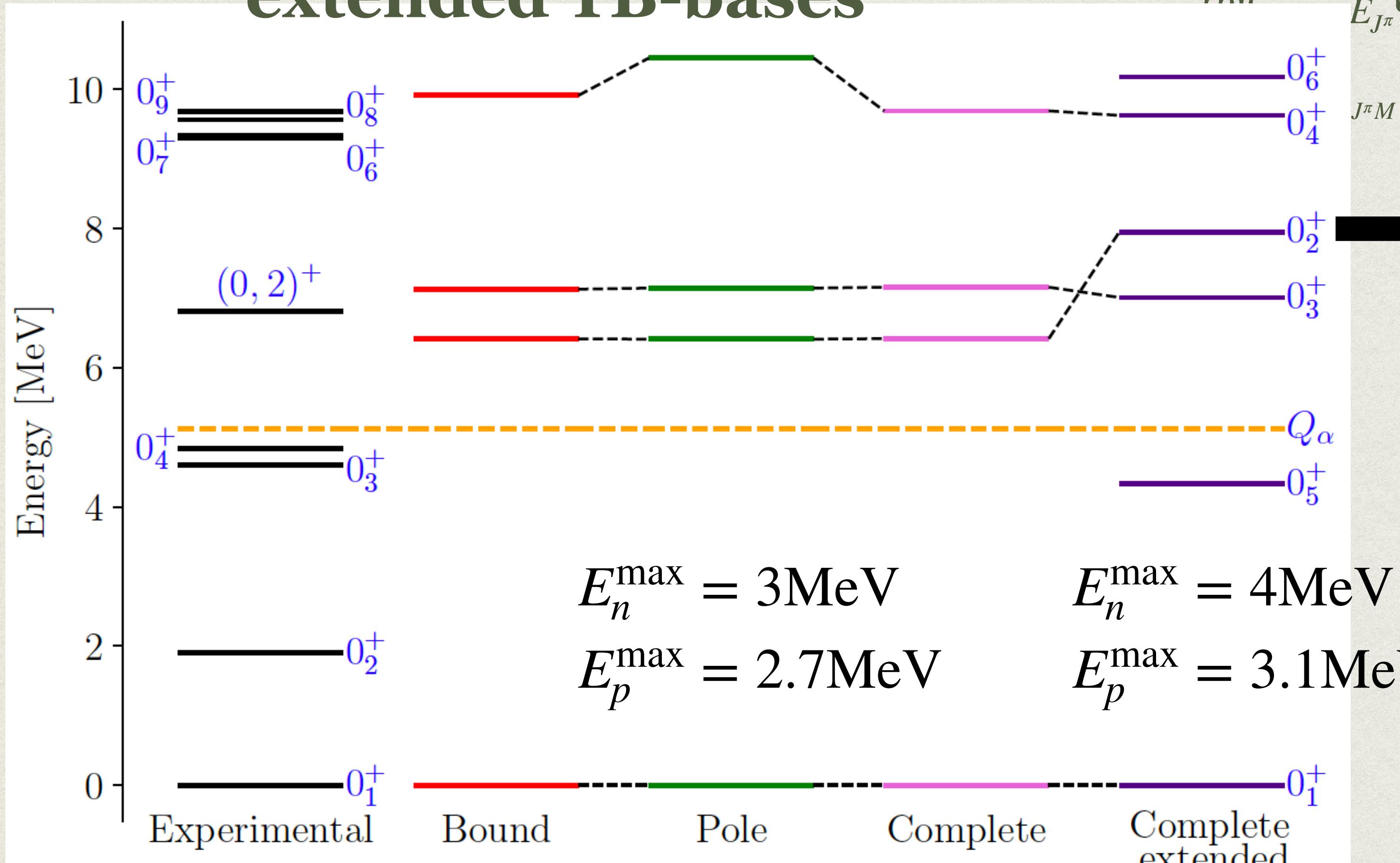
$$| \Psi_{J^\pi M} \rangle = \sum_{J, J_p} Z_{J_n J_p}^{J^\pi} | J_n J_p, J^\pi \rangle \quad \mathcal{H} | \Psi_{J^\pi M} \rangle = E_{J^\pi} | \Psi_{J^\pi M} \rangle$$

Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}, 0f_{5/2}$ $cf_{5/2}, 0g_{9/2}, cg_{9/2}$ $cs_{1/2}$

122 discretized non-RC

# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum in extended TB-bases



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

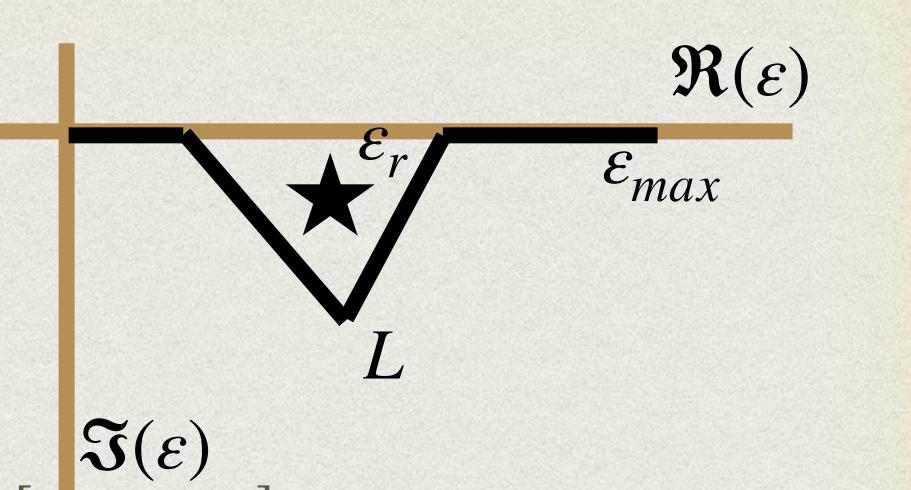
$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{ab} X_{ab}^{J^\pi} \Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$J^\pi M = \sum_{n,p} |J_n^\pi, J_p^\pi\rangle$$

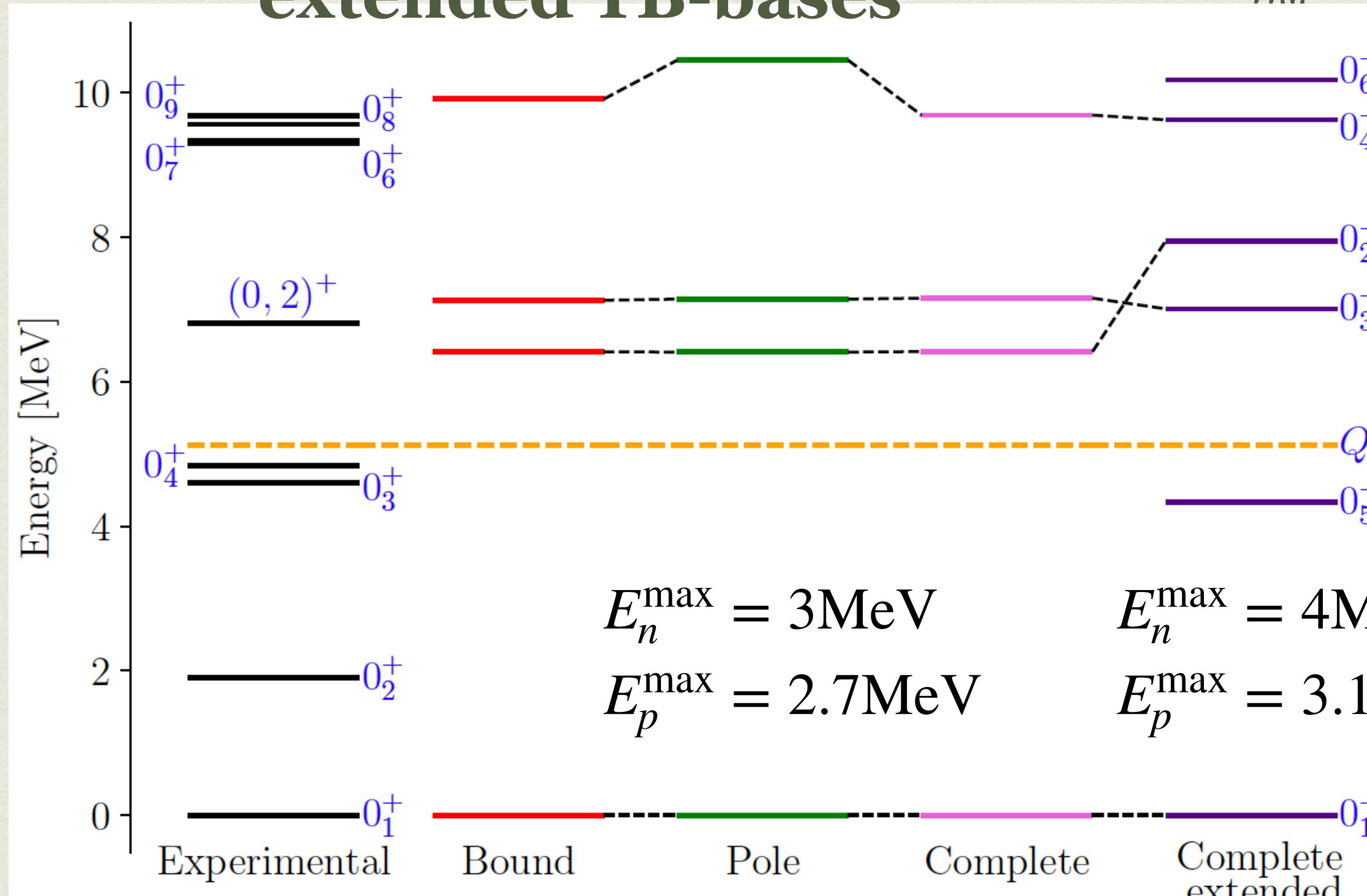
$$|J_n^\pi, J_p^\pi\rangle \quad \text{Complete} \quad \text{Complete extended}$$

$ 0_2^+\rangle_{0_2^+}$	-27.006	-25.396
$ 0_1^+ 0_1^+\rangle_{0_2^+}$	0.208	0.320
$ 2_1^+ 2_1^+\rangle_{0_2^+}$	-0.590	-0.401
$ 2_1^+ 2_2^+\rangle_{0_2^+}$	-	-0.380
$ 2_2^+ 2_1^+\rangle_{0_2^+}$	-	-0.354
$ 2_2^+ 2_2^+\rangle_{0_2^+}$	-	-0.441
$ 4_1^+ 4_1^+\rangle_{0_2^+}$	0.644	0.399
$ 4_2^+ 4_1^+\rangle_{0_2^+}$	-	0.237
$ 6_1^+ 6_1^+\rangle_{0_2^+}$	0.440	0.240



# APPLICATIONS

## $^{44}\text{Ti}$ $0^+$ spectrum in extended TB-bases



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$E_{J^\pi} \Psi_{J^\pi M}$$

$$\Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

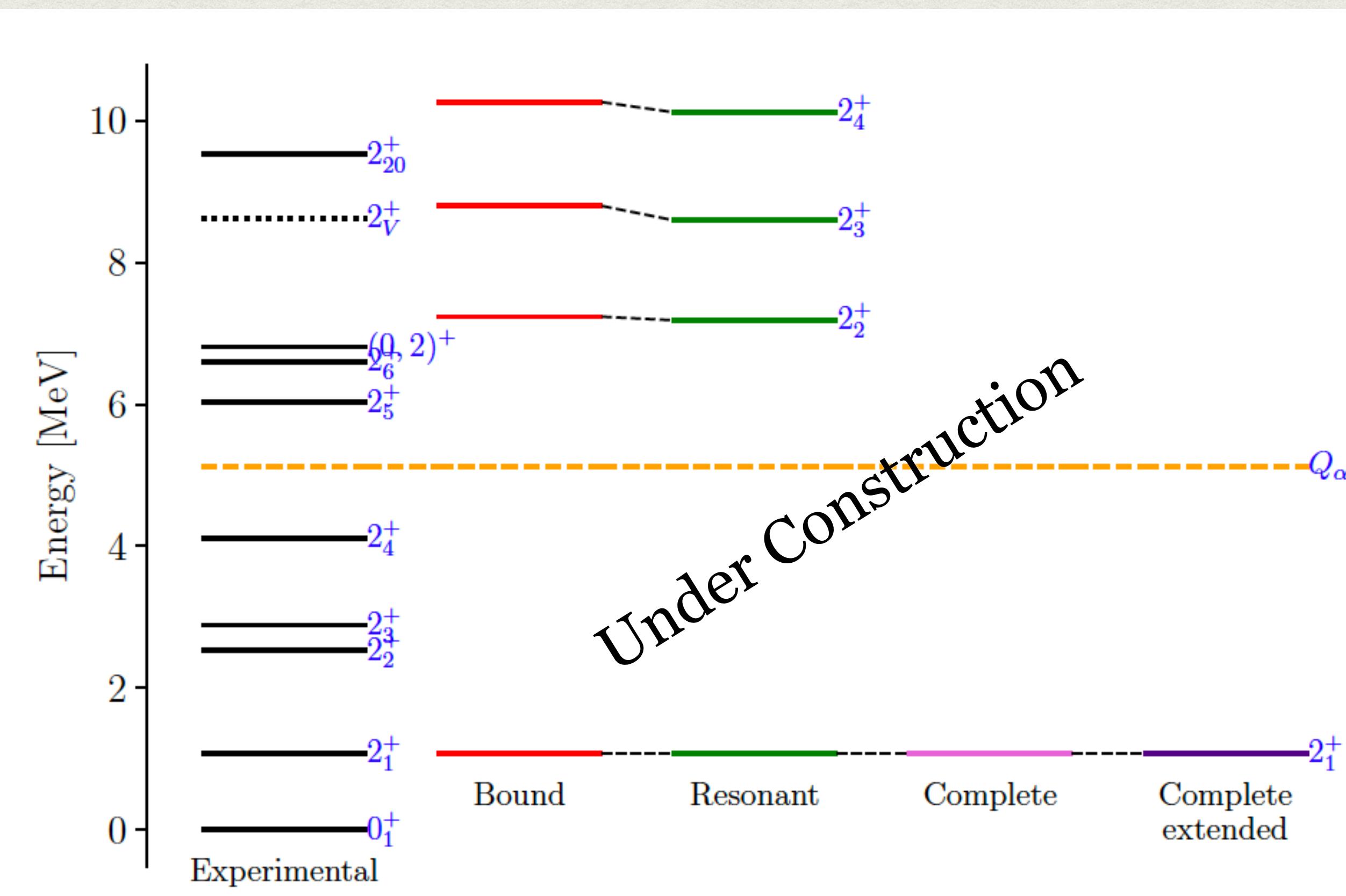
$$| \Psi_{J^\pi M} \rangle = \sum_{J J'} Z_{J_n J_p}^{J^\pi} | J_n J_p, J^\pi \rangle \quad \mathcal{H} | \Psi_{J^\pi M} \rangle = E_{J^\pi} | \Psi_{J^\pi M} \rangle$$

$ J_n^\pi, J_p^\pi\rangle$	Complete	Complete	extended
$E_{0_3^+}$	-26.297	-26.339	
$ 0_1^+ 0_1^+\rangle_{0_3^+}$	0.598	0.540	
$ 2_1^+ 2_1^+\rangle_{0_3^+}$	-0.527	-0.388	
$ 2_1^+ 2_2^+\rangle_{0_3^+}$	-	0.028	
$ 2_2^+ 2_1^+\rangle_{0_3^+}$	-	0.050	
$ 2_2^+ 2_2^+\rangle_{0_3^+}$	-	-0.049	
$ 4_1^+ 4_1^+\rangle_{0_3^+}$	-0.331	-0.455	
$ 6_1^+ 6_1^+\rangle_{0_3^+}$	-0.505	-0.587	

Collective state

# APPLICATIONS

# $^{44}\text{Ti}$ $2^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{\mathbf{r}}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P \left[ \psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2) \right]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2 N_{ab} N_{cd} \left[ 1 - P_{exch} \right] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$P_{J_p^{\pi p}}]_{J^\pi M} = \rightarrow^{J^\pi} \rangle$$

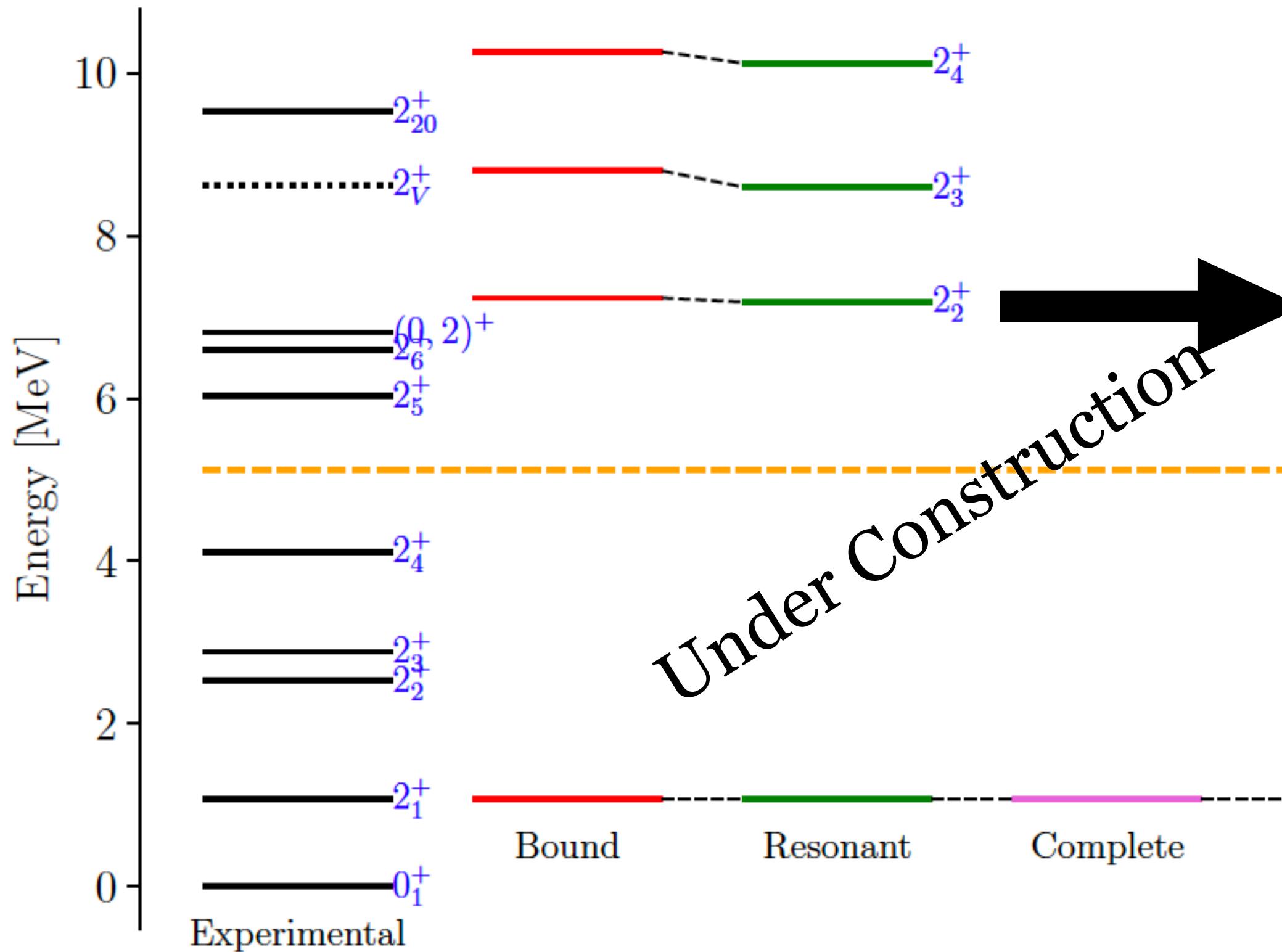
$$|\Psi_{J^\pi M}\rangle = \sum_{JJ} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

Basis	Neutron states	Proton states
bound	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}$	$0f_{7/2}$
pole	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, 1d_{5/2}$	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}$
complete	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$ $0f_{5/2}, 0g_{9/2}, cg_{9/2}$ $1d_{5/2}, cd_{5/2}, cs_{1/2}$ $cp_{1/2}, cp_{3/2}$	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$ $1p_{1/2}, cp_{1/2}$ $0f_{5/2}, cf_{5/2}$ $0g_{9/2}, cg_{9/2}$ $cs_{1/2}$

122 discretized non-RC

# APPLICATIONS

## $^{44}\text{Ti}$ $2^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{r}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

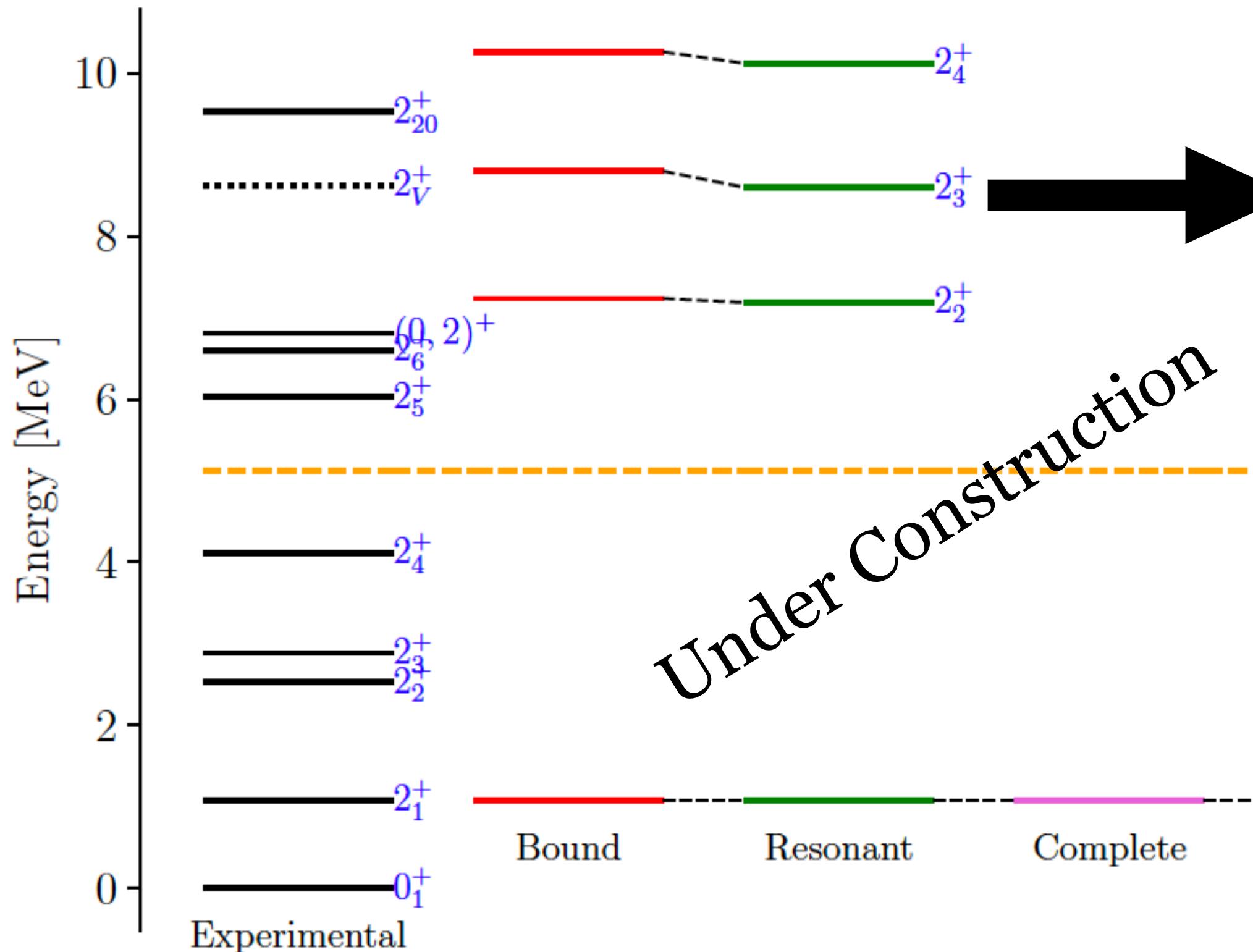
$$P_{J_p^{\pi p}}]_{J^\pi M} = \rightarrow | J^\pi \rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J,J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

	Basis	Neutron states	Proton states
$E_{2_2^+}$	(-26.187, 0.00)    (-26.237, 0.025)		$0f_{7/2}$
$ 0_1^+ 2_1^+\rangle_{2_2^+}$	-0.046	(-0.054, 0.000)	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$
$ 2_1^+ 0_1^+\rangle_{2_2^+}$	0.034	(0.055, 0.001)	$0f_{5/2}, 0g_{9/2}$
$ 2_1^+ 2_1^+\rangle_{2_2^+}$	-0.205	(-0.193, 0.000)	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$
$ 2_1^+ 4_1^+\rangle_{2_2^+}$	0.138	(0.126, 0.001)	$1p_{1/2}, cp_{1/2}, 0f_{5/2}$
$ 4_1^+ 2_1^+\rangle_{2_2^+}$	0.768	(0.782, 0.000)	$cf_{5/2}, 0g_{9/2}, cg_{9/2}$
$ 4_1^+ 4_1^+\rangle_{2_2^+}$	-0.055	(-0.061, 0.000)	$cs_{1/2}$
$ 4_1^+ 6_1^+\rangle_{2_2^+}$	0.148	(0.136, 0.001)	seed non-RC
$ 6_1^+ 4_1^+\rangle_{2_2^+}$	0.375	(0.367, 0.001)	
$ 6_1^+ 6_1^+\rangle_{2_2^+}$	0.425	(0.415, 0.001)	

$Z_{J_n J_p} > 0.3 \Rightarrow Z^2 \gtrsim 0.1$

# APPLICATIONS

## $^{44}\text{Ti}$ $2^+$ spectrum



$$\psi_{am_a}(\mathbf{r}) = \frac{u_a(r)}{r} \left[ \chi_s Y_{l_a}(\hat{\mathbf{r}}) \right]_{j_a m_a}$$

$$\Psi_{ab}^{JM}(\mathbf{r}_1, \mathbf{r}_2) = N_{ab} \sum_P (-1)^P P [\psi_a(\mathbf{r}_1) \psi_b(\mathbf{r}_2)]_{JM}$$

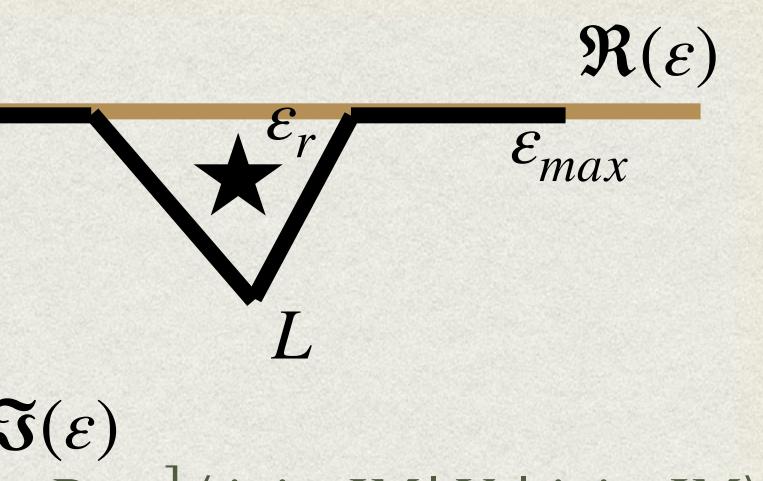
$$V(r) = \sum_{\tau} V_{\tau}(r) P_{\tau} \quad \langle \Psi_{ab}^{JM} | V_{\tau} | \Psi_{cd}^{JM} \rangle = 2N_{ab} N_{cd} [1 - P_{exch}] \langle j_a j_b; JM | V_{\tau} | j_c j_d; JM \rangle$$

$$H\Psi_{J^\pi M} = E_{J^\pi} \Psi_{J^\pi M} \quad \Psi_{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{b < a} X_{ab}^{J^\pi} \Psi_{ab}^{J^\pi M}(\mathbf{r}_1, \mathbf{r}_2) \quad H = h + h + V + V_{Coul}$$

$$P_{J_p^{\pi p}}]_{J^\pi M} = \rightarrow | J^\pi \rangle \quad |\Psi_{J^\pi M}\rangle = \sum_{J,J_p} Z_{J_n J_p}^{J^\pi} |J_n J_p, J^\pi\rangle \quad \mathcal{H} |\Psi_{J^\pi M}\rangle = E_{J^\pi} |\Psi_{J^\pi M}\rangle$$

	$E_{2_3^+}$	$(-24.625, 0.00)$	$(-24.824, 0.018)$	s Proton states
$1/2$	$ 0_1^+ 2_1^+\rangle_{2_3^+}$	-0.007	(-0.012, 0.000)	$0f_{7/2}$
$1/2$	$ 2_1^+ 0_1^+\rangle_{2_3^+}$	0.040	(0.042, 0.000)	$0f_{7/2}, 1p_{3/2}, 1p_{1/2}$
$3/2$	$ 2_1^+ 2_1^+\rangle_{2_3^+}$	-0.307	(-0.315, 0.001)	$0f_{5/2}, 0g_{9/2}$
$1/2$	$ 2_1^+ 4_1^+\rangle_{2_3^+}$	-0.021	(-0.015, 0.000)	$0f_{7/2}, 1p_{3/2}, cp_{3/2}$
$1/2$	$ 4_1^+ 2_1^+\rangle_{2_3^+}$	-0.347	(-0.340, 0.000)	$1p_{1/2}, cp_{1/2}, 0f_{5/2}$
$1/2$	$ 4_1^+ 4_1^+\rangle_{2_3^+}$	0.542	(0.530, 0.001)	$cf_{5/2}, 0g_{9/2}, cg_{9/2}$
$1/2$	$ 4_1^+ 6_1^+\rangle_{2_3^+}$	0.212	(0.206, 0.001)	$cs_{1/2}$
$1/2$	$ 6_1^+ 4_1^+\rangle_{2_3^+}$	0.659	(0.671, 0.000)	ed non-RC
$1/2$	$ 6_1^+ 6_1^+\rangle_{2_3^+}$	-0.103	(-0.092, -0.001)	

$$Z_{J_n J_p} > 0.3 \Rightarrow Z^2 \gtrsim 0.1$$



# OUTLOOKS

# OUTLOOKS

- Formation amplitude calculation for half live determination
- Four-neutron systems. Ej.  $^{14}\text{C}$  (T. Faestermann, et al., Physics Letters B 824 (2022) 136799 )
- Tensor force

**THANK YOU ALL**  
**FOR YOUR ATTENTION**